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Volume Title: Econometric Models of Cyclical Behavior, Vols. 1 and 2

Volume Author/Editor: Bert G. Hickman, ed.

Volume Publisher: UMI

Volume ISBN: 0-870-14232-1

Volume URL: http://www.nber.org/books/hick72-1

Publication Date: 1972

Chapter Title: Effects Of Aggregation Over Time On Dynamic Characteristics Of An Econometric Model

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Chapter URL: http://www.nber.org/chapters/c2787

Chapter pages in book: (p. 673 - 738)

EFFECTS OF AGGREGATION OVER TIME ON DYNAMIC CHARACTERISTICS OF AN ECONOMETRIC MODEL

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The paper examines the biases in estimates of the average lag and long-run marginal propensity which result when a model of the general Koyck-Nerlove variety is estimated with data which is aggregated over time. The problem is analyzed in simple terms, and then the approach is generalized by using spectral analysis. These theoretical calculations are then compared with the experience of aggregating Liu's (1969) monthly model of the U.S. economy to quarterly and annual forms, and estimating these with two-stage least squares and a nonlinear regression program designed to compensate for serial correlation in the disturbance. The single-equation results are compared with the theoretical analysis to substantiate the validity of the analytical approach. The systems of equations are then compared by examining dynamic interim multipliers and eigenvalues of the different sets of regression coefficients.

MODERN economic theory, in its effort to forecast the future and to describe the operation of complex economic systems, has become much concerned with the temporal interrelationships between economic variables. The empirical analysis of this problem has been pursued most successfully by using distributed-lag models as tools for econometric research. These models are often rich in dynamic structure, since difference-equation analysis on the estimated coefficients can yield insight into short- and long-run cyclical and noncyclical characteristics

NOTE: The authors are grateful to F. M. Fisher and E. Kuh for helpful comments on an early draft. The theoretical analysis in Section 2 is a summary of the results obtained by Engle in his Ph.D. thesis, *Biases from Time Aggregation of Distributed Lag Models*, Cornell University, 1969, under the supervision of Liu. The empirical research in Section 3 has been formulated jointly by the authors.

of complicated economic systems. In particular, knowledge about speeds of adjustment and natural frequencies of oscillation can be very helpful to policy-makers contemplating measures designed to change certain economic variables.

The statistical problems associated with estimating distributed-lag models have brought forth a plethora of new estimating techniques, which are designed to compensate for the inadequacy of the ordinary least squares approach. Unfortunately, none of these techniques is capable of dealing with the errors inherent in the manner in which a distributed-lag model is specified. In particular, there has been very little discussion of the consequences of assuming a particular unit-period for measurement which may not be the same as the unit period of the decision process. For instance, if a firm makes its basic production decisions at a weekly meeting, then the true model of its behavior should be weekly; any more highly aggregated model will have a specification bias. In many cases, it is, of course, not possible to specify a model with the proper unit-period, because the data either are not available or are so unreliable that the effort would be worthless. Nevertheless, it is extremely important that one be aware of the biases to be expected from any particular choice of basic period. The purpose of this paper is to investigate the magnitude and direction of biases which result from estimating econometric models with different unit-time periods.

1 STATEMENT OF THE PROBLEM

ECONOMIC variables are generally classified as either stock or flow variables, and the interpretation of time aggregation depends on the type of variable. Defining the lag operator L such that

$$Lx_t = x_{t-1}, \qquad L^2x_t = x_{t-2} \tag{1-1}$$

we can express both forms of aggregation by an aggregation operator $R(L)$. Letting capital letters refer to aggregated data, and small letters refer to disaggregated data, we can write

$$R(L)x_t = X_t \tag{1-2}$$

where $R(L) = (1 + L + L^2 + \ldots + L^{n-1})/n$ if x is a flow variable and the period of aggregation is n. Alternately, if x is a stock variable defined at the beginning of the period, then $R(L)$ would have the simple form: $R(L) = 1$. The bulk of this paper will be concerned with the aggregation of flow variables, since this is the more difficult case and is more relevant for the model considered, because some variables which might ordinarily be treated as stocks (such as prices and the money supply) are averaged over the measured period and treated as flows. The only variables which are still aggregated as stocks are the measures of capital stocks.

In order to distinguish between aggregates for which data might be available and those for which it would not, it is convenient to define a "proper" aggregate. When monthly data is aggregated into quarters, the three-period aggregate including January, February, and March is the proper quarter, while those from February to April, or from March to May, are improper quarters. Because of the possible ambiguity between proper and improper aggregates, the subscripts on all variables will always be given in the most disaggregated time period. For example, X_t and X_{t-3} would be successive quarterly observations when the framework of the analysis is monthly.

Time aggregation is very easy in models which have no lagged variables, because it follows that if the model is true in one period, and again in the next, then it should be true for the two together. For instance,

$$y_t = \sum_i a_i x_{it} + u_t \tag{1-3}$$

will aggregate directly by applying $R(L)$ to both sides of the equation giving

$$Y_t = \sum_i a_i X_{it} + U_t \tag{1-4}$$

The model appears to be exactly the same, although there is a hidden difference. The process of the disturbance U is not necessarily the same as that of u. If u is serially independent, then U will also be; but if, for example, u is a first-order auto-regressive stochastic process, then U is more complicated, and cannot be described in this simple fashion. This phenomenon has consequences for estimation of the model, in that although procedures can be found to estimate the coefficients in the presence of first-order serial correlation, it is difficult to find an estimating technique which will work efficiently in the presence of the complicated process of U. In addition, Theil (1965, p. 40) shows that if only uncorrelation of x and u is assumed, then the aggregated model might have correlation between X and U.

Static models can always be aggregated exactly into models which use only proper time periods. If the exogenous variables are independent of the disturbances, then unbiased but perhaps inefficient estimators are easily constructed. However, it does not follow that a static model can always be disaggregated; such an assertion requires the assumption that the model, as specified, holds exactly for each of the disaggregated time-periods. An example will make this asymmetry clear. Even if consumption this year is assumed to be a function of income this year, there is no guarantee that consumption today is only a function of income today. However, it is true that if car accidents today are a function of the number of car-miles driven today, then the number of accidents this year will be a function of the number of car-miles driven this year. In the latter example, the asymptotic values of the estimated coefficients should be the same in the annual and the daily models, but the standard errors in the annual model for any finite sample would, presumably, be much larger, because the cross effects of one day's car-miles and another's accidents would contribute to the variance of the estimate. The above examples illustrate why the approach used here is, first, to assume a particular form for the disaggregated model and, then, to examine the estimated values of the coefficients in the aggregated model.

When there are lagged values of independent or dependent variables in the model, it then becomes a dynamic model, or a distributed-lag model. The process of time aggregation is much more involved in this class of models, because the form of direct aggregation which was used in the static models described above, yields, in general, an aggregated model which contains improper aggregates. For example, the very important Koyck-Nerlove Model can be written in monthly form as

(1-5) $$y_t = \alpha y_{t-1} + \beta x_t + u_t$$

If the aggregation operator $R(L)$ is applied to both sides of the equation to obtain a quarterly aggregate, the following will result

(1-6) $$Y_t = \alpha Y_{t-1} + \beta X_t + U_t$$

Notice that although Y_t and X_t are proper quarters, Y_{t-1} is not, and therefore (1-6) is not a useful aggregate form of (1-5). The quarterly form of (1-5) is simply

(1-7) $$Y_t = AY_{t-3} + BX_t + V_t$$

where Y_{t-3} is the observation on the previous quarter. Now V has become more complicated than U, since it includes parts of the lagged dependent variable as a result of the mis-specification of the model. This additional component is very systematic, since it incorporates some of the process of the economic variable and is, therefore, most unlikely to be serially independent. Thus, the disturbance in the aggregated form of a distributed-lag model is complicated both by the aggregation of the true disaggregated disturbance and by the inclusion of additional terms due to the almost inevitable mis-specification of the model.

Naturally, we will want to compare the coefficients of (1-5) with those of the aggregated form (1-7). There is, however, no reason to expect, or even desire, that the coefficients be the same, since the dynamic properties of the quarterly model will necessarily be different from those of the monthly model. In order to describe this and other distributed lag-models—and the differences between models—more concisely, it is necessary to choose unit-period invariant parameters to characterize the models. Several have been used in the literature; but before choosing one to measure the effects of aggregation on a lag model, it is revealing to analyze lag distributions in two different ways.

In terms of the lag operator L, all rational distributed-lag models can be written as

(1-8) $$A(L)y = B(L)x + u$$

where $A(L)$ and $B(L)$ are polynomials in L, and $A(L)$ must have an inverse in order for the model to be stable. The last condition merely means that the difference equation which results when the error is set to zero, is not explosive. See for example Griliches (1967, p. 22). Thus, the model can be rewritten as follows

(1-9)
$$y = A(L)^{-1}\, B(L)x + A(L)^{-1}u$$
$$y = W(L)x + A(L)^{-1}u$$

This equation now describes y in terms of a perhaps infinite series of lagged x's and will be called the moving average form of a distributed-lag model. For example

(1-5)
$$y_t = \alpha y_{t-1} + \beta x_t + u_t$$

can be rewritten as

(1-10)
$$y_t = \beta \sum_{i=0}^{\infty} \alpha^i x_{t-i} + \sum_{i=0}^{\infty} \alpha^i u_{t-i}$$

Setting the error equal to zero, we see immediately the contribution of an increase in x to the value of y for any number of periods in the future. Plotting the weights in (1-9) as a function of time, we have a complete description of how y changes with a unit change in x, which is analogous to a discrete probability density function—although not yet normalized (Chart 1).

The total area under the curve gives the total change in y resulting from a sustained change in x. We could also have written this as a distribution function by integrating the previous curve.

An alternative way of describing this information is in terms of a phase diagram. If the exogenous variable is assumed to vary cyclically, then this is a more instructive approach. In particular, if x is allowed to vary with a certain frequency, the phase diagram will indicate how

CHART 1

Weight Diagram of Distributed-Lag Model

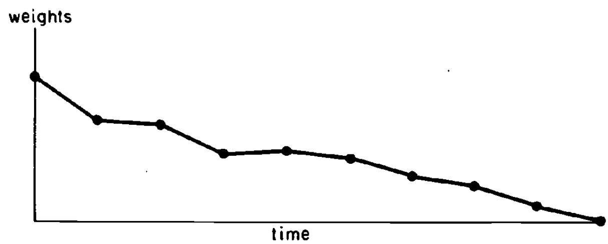
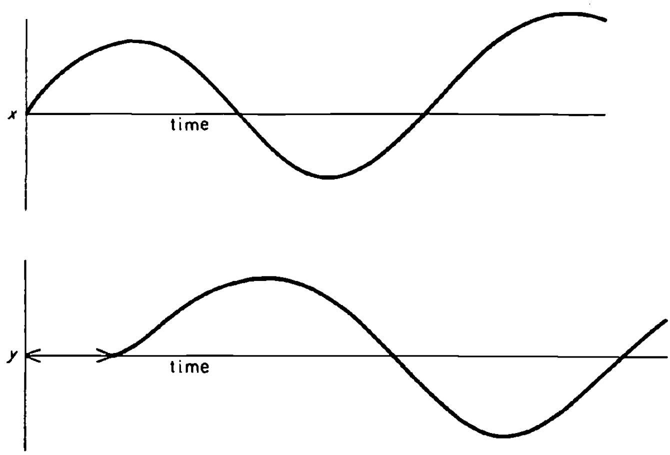


CHART 2

Phase Shift

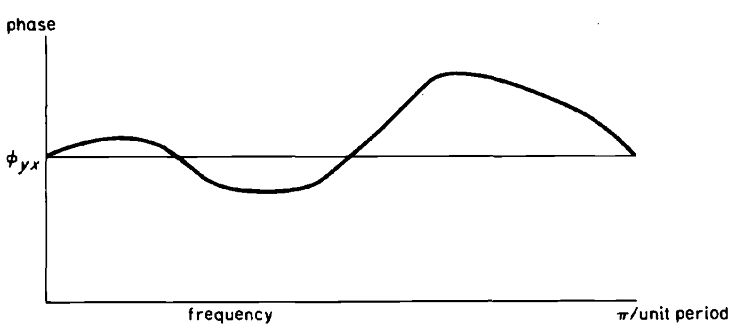


much later y will begin to vary in this frequency. The phase diagram ignores the relative sizes of the two fluctuations (the gain); but as time lags are the primary concern of distributed-lag analysis, the phase diagram is a useful summary of the information in a lag model. Again setting the error term in (1-9) to zero, we can identify the phase shift between x and y as the arrow in Chart 2.

The plot of the phase shift as a function of frequency is called the phase diagram and might look like Chart 3. See, for example, Fishman (1968, p. 260).

CHART 3

Phase Diagram of Distributed-Lag Model



The phase diagram is an important tool in spectral analysis, and it is crucial to notice that it is defined for frequencies only to π/unit period, the Nyquist frequency, which is the shortest period that can be distinguished in the spectrum. It has a period twice the unit-time period, while all higher frequencies are confounded in the sense that they are indistinguishable from some frequency below the Nyquist frequency, perhaps with opposite phase. See Fishman (1968, p. 50). With quarterly data it is impossible to distinguish any components with periods of less than six months from those of more than six months.

When we try to compare distributed-lag models with different time periods in terms of the phase diagram, confounding becomes very important. The quarterly phase diagram only extends one-third as far as the monthly, since it terminates at a six-month period rather than at the two-month oscillation of the monthly diagram. It is, therefore, impossible to compare a monthly and quarterly model on this basis, because there is no natural correspondence. Similarly, it is impossible to compare the models of different aggregates on the basis of the weight diagram, because the two models do not have weights on the same points; and with fewer weights, each will have to be larger, so again there is no natural correspondence between the two diagrams. Consequently, it is crucial to be able to specify some characteristics of a lag distribution in terms of parameters which do not depend on the unit period.

The motivation for the most common choice of a parameter to use in comparing different aggregates of a distributed-lag model stems from the analysis of a simple auto-regressive model with no exogenous variable, such as the following

$$y_t = \alpha y_{t-1} + u_t \tag{1-11}$$

This model implies

$$Y_t = \alpha Y_{t-1} + U_t \tag{1-12}$$

which, in turn, can be repeatedly substituted to obtain

$$Y_t = \alpha^n Y_{t-n} + U_t + \alpha U_{t-1} + \alpha^2 U_{t-2} + \ldots + \alpha^{n-1} U_{t-n+1} \tag{1-13}$$

From this model, Ironmonger (1959, p. 626), Nerlove (1959, p. 632), and Telser (1967, p. 486) immediately compare the coefficient of the lagged variable in the aggregated model

$$Y_t = AY_{t-n} + V_t \tag{1-14}$$

with α^n. This comparison, it is argued, gives the best measure of the dynamic properties of the model in its different forms. Unfortunately, the approach gives no insight into the method of comparison when other lag schemes are introduced or exogenous variables are incorporated into equation (1-11). Furthermore, it is misleading to say that because (1-13) and (1-14) agree except for the error term, they are the same model. The structure of the error term is very important for any estimation procedure and cannot be ignored in the specification of the model. Nevertheless, at several places in this paper, the comparison between $\hat{A}$ and α^n will be used to measure the effects of aggregation on this simple model.

The basic measure of a distributed-lag structure throughout the work will be taken to be the average, or mean, lag. This parameter is employed repeatedly by Griliches (1967, p. 19) and has additional characteristics which are relevant for this study. The average lag is closely related to the point in time at which half the adjustment from initial to final value of the dependent variable has occurred and if the weight-diagram is symmetric and unimodal, it will be exactly that point. This measure is chosen in preference to either 5 per cent or 95 per cent lag parameters, which emphasize, respectively, speed of impact and speed of long-run adjustment, because it better represents the over-all structure of the distributed-lag model. The mean lag is easily calculated as the mean of the weights of (1-9), illustrated in Chart 1, giving

$$\text{Average Lag} = \sum_i iw_i \Big/ \sum_i w_i \tag{1-15}$$

A more convenient computational definition is

$$\text{Average Lag} = dW(L)/dL|_{L=1}/W(1) \tag{1-16}$$

which is easily shown to be equivalent.

The average lag has another advantage for our applications. It can be represented also on the phase diagram in a fundamental way. From differentiation of the phase with respect to frequency, it can be shown that the average lag is the slope of the phase diagram at the origin. In order to see this, define the phase of a lag distribution $W(L)$, which may be an infinite polynomial, as[1]

$$\text{phase} = \phi = \arctan \frac{Im[W(e^{i\theta})]}{Re[W(e^{i\theta})]} \tag{1-17}$$

Noticing that the imaginary part contains only sine functions, and the real part includes only cosines, we have directly that $Im[W(e^{i\theta})]|_{\theta=0} = 0$ and $Re[W(e^{i\theta})]|_{\theta=0} = \sum_j w_j$ where w_j is the coefficient of L^j. Differentiating the phase with respect to θ and setting $\theta = 0$, we get

$$d\phi/d\theta\bigg|_{\theta=0} = \frac{d[Im(W(e^{i\theta}))]/d\theta}{Re[W(e^{i\theta})]}\bigg|_{\theta=0} \tag{1-18}$$

which becomes (1-15) when we recall that $d \sin (n\theta)/d\theta|_{\theta=0} = n \cos (n\theta)|_{\theta=0} = n$

If Granger (1966, p. 150) is correct in his assertion that most economic time series have their power concentrated at the low frequency end of the power spectrum, then this is the most important part of the phase diagram, and the slope of the phase at the origin should be a very good parameter to describe that region. Again, this is a justification for comparing distributed-lag models in terms of their average lag. It is a particularly relevant argument for this paper; because, repeatedly, it will be necessary to assume that the power of a variable is concentrated at the origin.

The strategy to be followed in this analysis is now clear. Each model under consideration will be examined at different levels of aggregation, and the resulting distributions will be compared in terms of the average lag estimates computed from the coefficients. For example, in the simple Koyck-Nerlove model of equation (1-5), the average lag is

$$\text{Average Lag} = \alpha/(1-\alpha) \tag{1-19}$$

For the aggregated model, the same relation must hold true; but in terms of the disaggregated time period it becomes

$$\text{Average Lag} = nA/(1-A) \tag{1-20}$$

[1] $W(e^{i\theta})$ is the frequency response function corresponding to a lag distribution $W(L)$. To see this and the definition of phase, consult, for example, Nerlove (1964, p. 256 and 277).

The bias in the average lag of the aggregated model will be

$$\text{(1-21)} \qquad \text{Bias in Average Lag} = nA/(1-A) - \alpha/(1-\alpha)$$

It can easily be seen that $A = \alpha^n$ is neither necessary nor sufficient for the bias to be zero. Consequently, the focus of the analysis will be the bias in the average lag which results from aggregation over time.

This study aspires to discover the effects of time aggregation on distributed-lag models as a function of several simple parameters of the underlying model. In this way, hopefully, it will be possible for an econometrician to evaluate the probable biases to be expected from using aggregated data when an underlying model is assumed to be the true one.

Two hypotheses about the effects of time-aggregation seem intuitively appealing. The first says: if the lag is short with respect to the basic time period over which the data are collected, it is unlikely that the model will accurately reveal the lag structure. For example, if the true lag is three months, we would expect an annual model to miss it, and the coefficient of the lagged terms would be statistically insignificant—or perhaps, even negative—due to the uncertainties of measurement. As this model is aggregated from monthly to annual data, the estimates of the average lag would go from three months to no lag, implying a negative bias. So, in any model, if the aggregation is carried far enough, the bias should become negative.

The second hypothesis is exactly contrary to both the first one and to the prevalent belief that the process of the disturbance is more highly autocorrelated in a model of short unit period of observation than in models of longer unit periods. As pointed out in Liu (1969, p. 2), as the model is aggregated, increasing portions of the variation of the observed variables are passed into the disturbances. Consequently, the disturbances would become more highly serially correlated and the estimated coefficients of the lagged variables would be more seriously biased upwards.[2] This would generate increasing biases in the average lags as the model is aggregated.

In the remainder of this paper, both empirical and analytical evi-

[2] If the serial correlation coefficient is positive, then the estimate of the coefficient of the lagged endogenous variable will be biased consistently upwards. See Griliches (1961, p. 67).

dence is uncovered to support each of the hypotheses; and the goal of the study will be to determine which effect dominates in any particular situation.

2 THEORETICAL ANALYSIS

THE purpose of this section is to establish estimates of the biases to be expected from time aggregation of distributed-lag models, based on specific stochastic specifications for the models. These results will then be compared with the empirical findings of the following section.

There have been several attempts to explore the effects of time aggregation on distributed-lag models from a theoretical point of view; all have used the Koyck-Nerlove Model almost exclusively. The earliest occurred in the debate between Ironmonger (1959) and Nerlove (1959) over the biases resulting from the use of annual data in consumer-demand regressions with lagged variables. The basic methodology used in the analysis was to hypothesize some particular time path of the exogenous variable and then compare the estimated value of A with α^n. Mundlak (1961) tried to improve the argument by using more systematic analysis, but the main conclusion of his study is that the aggregated model will almost always be mis-specified. He asserted that it will, in general, give an overestimate of α^n, but this does not seem to follow from the argument.

The most sophisticated analysis of the problem of time-aggregation appears in Telser (1967). He computes the size of the regression coefficients in the aggregated model in terms of the disaggregated parameters for the simple auto-regressive case which is to be discussed here. However, he does not extend this computation in a useful way—to include exogenous variables or more complicated assumptions about the disturbances. His main concern is with developing a technique whereby the underlying model can be consistently estimated by the aggregate data. Unfortunately, this procedure requires exact knowledge of the form of the underlying model, but even given this, Telser admits that it is not a viable econometric technique.

The task of deriving estimates of the effects of time-aggregation in useful cases remains to be undertaken. In this paper, a simple case will be computed in detail, while the more complicated and useful cases will be drawn from Engle (1969). The reason for merely citing these cases is that the mathematics is substantially more difficult, yet the approach is, in principle, quite similar, and the mathematics would add little to the understanding of the mechanism of time-aggregation. The results themselves are very useful, however, and will be employed in the next section in order to compare the behavior of the empirical models with the theoretical conclusions.

A. SIMPLE ANALYSIS OF FIRST-ORDER AUTO-REGRESSIVE SCHEME

In this analysis, as well as in that of Engle (1969), it is necessary to assume that all the variables are covariance stationary, an assumption neither unreasonable nor unusual in economics. Most asymptotic results are meaningless unless the variables are, at least, stationary in this sense; otherwise, sample moments would not converge to the true covariances. Although economic time series often have a strong trend in mean, it is not unreasonable to assume that the covariances are constant, since, of course, they depend only on deviations from the mean, which might very well not grow over time. In addition, it is not fully possible to distinguish between pure trend and very long cycles, owing to the finite lengths of all series. The analysis will always be conducted under the assumption that the regression constant is zero; this implies, however, no loss of generality, since the sample means can always be subtracted first from the variables.

The model to be considered now is the same as (1-11), which can be formulated in monthly terms as

$$y_t = \alpha y_{t-1} + u_t \tag{2-1}$$

where u is assumed to be uncorrelated with preceding y's and serially independent. In this model, ordinary least squares will be consistent and asymptotically efficient, since the regressors are uncorrelated with the disturbance. However, if the model is estimated in the aggregate form (see (1-14)),

$$Y_t = AY_{t-n} + V_t \tag{2-2}$$

we can no longer make that assertion. The asymptotic bias in $\hat{A}$ can be derived by computing the value of OLS estimates of the aggregated model. For convenience, the subscripts will all be in months, as described in Section 1, page 675, and the index t will be suppressed. The estimator of A in the quarterly aggregate is given by

$$\hat{A} = (Y'_{-3}Y_{-3})^{-1}(Y'_{-3}Y) \tag{2-3}$$

$$\hat{A} = (Y'Y)^{-1}(Y'_{-3}Y) \tag{2-4}$$

where we have used the assumption of covariance stationarity to obtain (2-4). From the true relation (2-1), we can compute the autocovariances of y asymptotically as

$$\text{plim } y'y_{-s}/T = \alpha^s\sigma_y^2 \tag{2-5}$$

This information and the definition of the aggregated variables are sufficient to compute the asymptotic value of $\hat{A}$.

It is perhaps helpful to consider the stock variable case first so that $Y_{-s} = y_{-s}$.

$$\text{plim } \hat{A} = (\text{plim } y'y)^{-1} \text{ plim } y'_{-3}\, y = \alpha^3 \tag{2-6}$$

In this simple model, the appealing result indicates that in the stock variable case with no exogenous variable, aggregation over time should give a consistent estimate of α^3 or, in general, of α^n. As stated before, this does not imply a consistent estimate of the average lag.

The flow variable case is only slightly more difficult in this example.

$$\text{plim } \hat{A} = \frac{\text{plim } (y_{-3} + y_{-4} + y_{-5})'(y + y_{-1} + y_{-2})}{\text{plim } (y + y_{-1} + y_{-2})'(y + y_{-1} + y_{-2})}$$

$$= \frac{\text{plim } (y'y_{-1} + 2y'y_{-2} + 3y'y_{-3} + 2y'y_{-4} + y'y_{-5})}{\text{plim } (3y'y + 4y'y_{-1} + 2y'y_{-2})}$$

$$\text{plim } \hat{A} = \frac{\alpha + 2\alpha^2 + 3\alpha^3 + 2\alpha^4 + \alpha^5}{3 + 4\alpha + 2\alpha^2} \tag{2-7}$$

The numerator and denominator are in a general form which will subsequently be defined, respectively, as the polynomials $\alpha N(\alpha)$ and $Q(\alpha)$. Relation (2-7) is somewhat more complicated than (2-6), but some useful conclusions can be reached by noting that α is always assumed to be greater than zero and must be less than 1. Factoring out an

α in the numerator, it is easy to group terms so that each term in the denominator is equal to, or greater than, the corresponding term in the numerator. Thus, plim $\hat{A}$ is less than α, just as one would expect. By multiplying and dividing by α^3 instead, the terms can again be grouped; this time the denominator is everywhere less than, or equal to, the numerator. One possible grouping is the following

$$\text{plim } A = \frac{(3\alpha^3) + (2\alpha^2 + 2\alpha^4) + (\alpha + \alpha^5)}{3\alpha^3 + 4\alpha^4 + 2\alpha^5} \alpha^3$$

which implies that plim $\hat{A}$ is greater than α^3 or, in general, the same argument can be established to show that plim $\hat{A}$ is greater than α^n. Finally, it can be shown by a considerable degree of algebraic manipulation that for this example

$$\frac{3 \text{ plim } \hat{A}}{1 - \text{plim } \hat{A}} \geqslant \frac{\alpha}{1 - \alpha}$$

but this result is not true for all α when n is greater than 3.

B. GENERALIZATION TO MORE COMPLICATED MODELS

The previous section illustrates the general procedure to be followed in analyzing the biases due to time-aggregation; an underlying model will be assumed to be true, while an aggregated model is estimated. The underlying model will be given, in general, by

$$\alpha(L)y = \beta(L)x + \gamma(L)u \tag{2-8}$$

where $\alpha(L)$, $\beta(L)$, and $\gamma(L)$ are rational polynomials in L. It will be natural to let $\alpha(0) = \gamma(0) = 1$, and to require that they have inverses; these assumptions imply no loss of generality. In addition, it will always be assumed that u is independent of all x's.

The model which is actually estimated is an aggregated Koyck-Nerlove Model such as

$$Y = AY_{-n} + BX + V \tag{2-9}$$

where n is the period of aggregation. In order to obtain the estimates of A and B under any set of stochastic assumptions embodied in (2-8), the true model is aggregated by operating on both sides with the aggregation operator $R(L)$ to get

$$\alpha(L)Y = \beta(L)X + \gamma(L)U \tag{2-10}$$

and then this true model is projected on the space of the mis-specified model (2-9). This gives the least squares estimators of the aggregate model. Only these estimates of the aggregate model are derived analytically for four reasons: least squares is the most common technique; it is often the first stage of a more complicated technique; it is the closest approximation to the true model in the sense that it is the projection under the assumption that nothing is known about the process of the disturbance; and finally, it is easily obtained analytically in closed form. This approach is just that followed by Theil (1965, p. 53).

In order to compute the magnitude of the estimated coefficients under the particular stochastic assumptions of (2-8), it is necessary to compute the covariances between various aggregated variables. It was found that this could be done more easily if the variables were described in spectral representation, since, then, the asymptotic value of any covariance is merely the integral of the cross-spectral density function. A second advantage of using spectral representation is that two quite different stochastic specifications can be analyzed in the same notation—deterministic and random. The assumption which was found most useful in the analysis is that the exogenous variable is deterministic with frequency ω; that is, it oscillates with a period of $2\pi/\omega$. In this case, the integral takes its value from only one point and is easy to evaluate. It may be noted that Engle finds that the assumption of a first-order auto-regressive process for the exogenous variable does not appreciably change the character of the results.

Because of the importance of this assumption, it is helpful to elaborate on exactly what is being assumed. In general, the power spectrum decomposes the total variance of a time series into the variances which appear in each particular frequency-band. Covariances are integrals of cross-spectral density functions over the frequency domain, where each point is weighted by its variance. Thus, the assumption that the power spectrum of x is concentrated at ω means that the variance at this point is sufficiently great for this component to dominate the total integral. If there are two components of similar power, it then turns out that the combined solution will be a linear combination of the two estimates. The further assumption that the power is concentrated at $\omega = 0$ implies that the variable has its variance primarily in long periodicities, such as trend. This assumption seems valid for many economic time-series (see Granger (1966, p. 150)), although it effectively eliminates from the calculations the interesting short-run fluctuations which might, indeed, give a significant perturbation to the results presented here.

Engle computed the probability limits of the coefficients of the aggregated model under six different stochastic specifications, which include alternative assumptions about the original process of the disturbance. In particular, he considered the cases where the disturbance is first-order auto-regressive before, as well as after, both sides of (2-8) are multiplied by $\alpha(L)^{-1}$. The most fruitful of all these models for empirical applications was the model with a deterministic exogenous variable and first-order serial correlation in the disturbance. More complicated disturbances must be approximated by truncating the process at first order. In particular, this model assumes that $\alpha(L) = 1 - \alpha L$, $\beta(L) = \beta$, and $\gamma(L) = 1/(1 - \gamma L)$. The result of this complicated calculation is presented below as the probability limit of the parameter A

$$\text{(2-11)}\quad \operatorname{plim} \hat{A} = \frac{\dfrac{MG(\omega)}{(1-\alpha^2)T(\omega)}\left[\dfrac{N(\alpha)\alpha^2(1-\gamma^2) - N(\gamma)\gamma^2(1-\alpha^2)}{(1-\alpha\gamma)(\alpha-\gamma)}\right] + \cos n\omega \dfrac{(1-\alpha\cos\omega)[\cos n\omega - \alpha\cos(n-1)\omega]}{G(\omega)}}{\dfrac{MG(\omega)}{(1-\alpha^2)T(\omega)}\left[\dfrac{Q(\alpha)\alpha(1-\gamma^2) - Q(\gamma)\gamma(1-\alpha^2)}{(1-\alpha\gamma)(\alpha-\gamma)}\right] + 1 - \dfrac{[\cos n\omega - \alpha\cos(n-1)\omega]^2}{G(\omega)}}$$

where

$N(\alpha) = [1 + 2\alpha + 3\alpha^2 + \cdots + n\alpha^{n-1} + (n-1)\alpha^n + \cdots + \alpha^{2(n-1)}]/n^2$

$Q(\alpha) = [n + 2(n-1)\alpha + 2(n-2)\alpha^2 + \cdots + 2\alpha^{n-1}]/n^2$

$G(\omega) = 1 + \alpha^2 - 2\,\alpha\cos\omega$

$$T(\omega) = [n + 2(n - 1) \cos \omega + 2(n - 2) \cos (2\omega) + \cdots + 2 \cos (n - 1)\omega]/n^2$$
$$M = \sigma_u^2/\beta^2\sigma_x^2$$

This relationship is so complicated that no general tendencies can be observed until simplifying approximations are made. The most important one is that the power spectrum be concentrated at such low frequencies that we can approximate the entire spectrum by a spike at $\omega = 0$. That a small spread from this point is unimportant can be seen by differentiating (2-11) with respect to ω at $\omega = 0$. This derivative is zero, which means that to the first approximation, deviations from $\omega = 0$ do not change the estimate of A. We appeal to Granger (1966, p. 150) for empirical support for this assumption, noting that the scale of the power spectrum is generally a log scale; and thus, the low-frequency components can be very strongly dominant, the typical spectral shape of an economic variable having a peak at the low-frequency end.

Making the approximation that $\omega = 0$ or, equivalently, that M is very large, we obtain an important simplification

$$\text{(2-12)} \qquad \text{plim } \hat{A} = \frac{N(\alpha)\alpha^2(1 - \gamma^2) - N(\gamma)\gamma^2(1 - \alpha^2)}{Q(\alpha)\alpha(1 - \gamma^2) - Q(\gamma)\gamma(1 - \alpha^2)}$$

which, however, is not so simple that it is easy to tell the effect of time-aggregation. If, in addition, $\gamma = 0$, thus eliminating serial correlation of the disturbance, then the result is merely

$$\text{(2-13)} \qquad \text{plim } \hat{A} = \frac{N(\alpha)\alpha}{Q(\alpha)}$$

which is the general case of equation (2-7), and has the same property of giving an estimate greater than α^n but smaller than α.

When γ is not zero, (2-12) is hard to analyze, but it can be seen that if γ is close to one, it will dominate the expression, leading to an estimate $\hat{A}$, which is independent of α, and which, consequently, will lead to an overestimate of the average lag.

To discern the behavior of plim $\hat{A}$ in terms of α, γ, n and ω, it was necessary to resort to computations. For fixed values of α, γ, and ω (and $M = .5$), the magnitude of the estimated average lag was computed, as n varied from one to twelve. Thus, a profile of the effects of aggregation on the estimated average-lag was generated for a whole series of combinations of parameters. The graphs in Charts 4 and 5 illustrate the profiles which are observed under various assumptions. Chart 4 is calculated with $\alpha = .55$, i.e., a true average lag of 1.22 mo.; while Chart 5 assumes $\alpha = .90$, a true average lag of 9.0 mo. The top, middle, and lower profiles in each case have γ equal to 0.5, 0.0, and −0.5, respectively, while the three curves drawn in each section relax the assumption that $\omega = 0$ by plotting the profile for $\omega = 0.0, 0.05$, and 0.1. The last two values of ω correspond to oscillations of 125 and 63 periods.

Several important facts can be seen by studying these two figures. First, the effect of serial correlation is strongest in relatively disaggregated models and leads to an over-all increase or decrease in the estimated average-lag, depending on whether the serial correlation is negative or positive, respectively.[3] Second, in the more highly aggregated models, the size of the serial correlation coefficient seems less important than the deviations of ω from zero in explaining the size of the estimate of the average lag. In particular, the larger is ω, the larger will be the aggregated average-lag. If ω gets even larger, other calculations show that the average lag goes to infinity, then becoming negative and large; that is, it goes through a singular point.[4]

The third important observation is that the consequences on the estimated average-lag of having $\omega \neq 0$, seem to be much stronger when α is close to one. In a monthly model, one would expect the true α to

[3] One important source of negative serial-correlation in the disturbance is described by Liu (1969, p. 11). It results from a first-order truncation of the disturbance in the case where the auto-regressive parameter is greater than the serial correlation coefficient. Specifically if

$$y = \frac{\beta x}{1 - \alpha L} + \frac{\epsilon}{1 - \gamma L}$$

and $\alpha > \gamma$, then the following model will have a negative first-order serial correlation coefficient

$$\begin{aligned} y - \alpha y_{-1} &= \beta x + \frac{1 - \alpha L}{1 - \gamma L} \epsilon \\ &= \beta x + [1 + (\gamma - \alpha)L + \cdots]\epsilon \end{aligned}$$

[4] This could, perhaps, explain Bryan's very large results (1967, p. 855). For weekly data, $\omega = .1$ corresponds to a period of 1.2 years, and so if annual fluctuations for his bank data are actually stronger than the trend, it could be that the effective ω in his problem was somewhat greater than 0.1.

CHART 4

Inconsistency of Average Lag
($A = .55$)

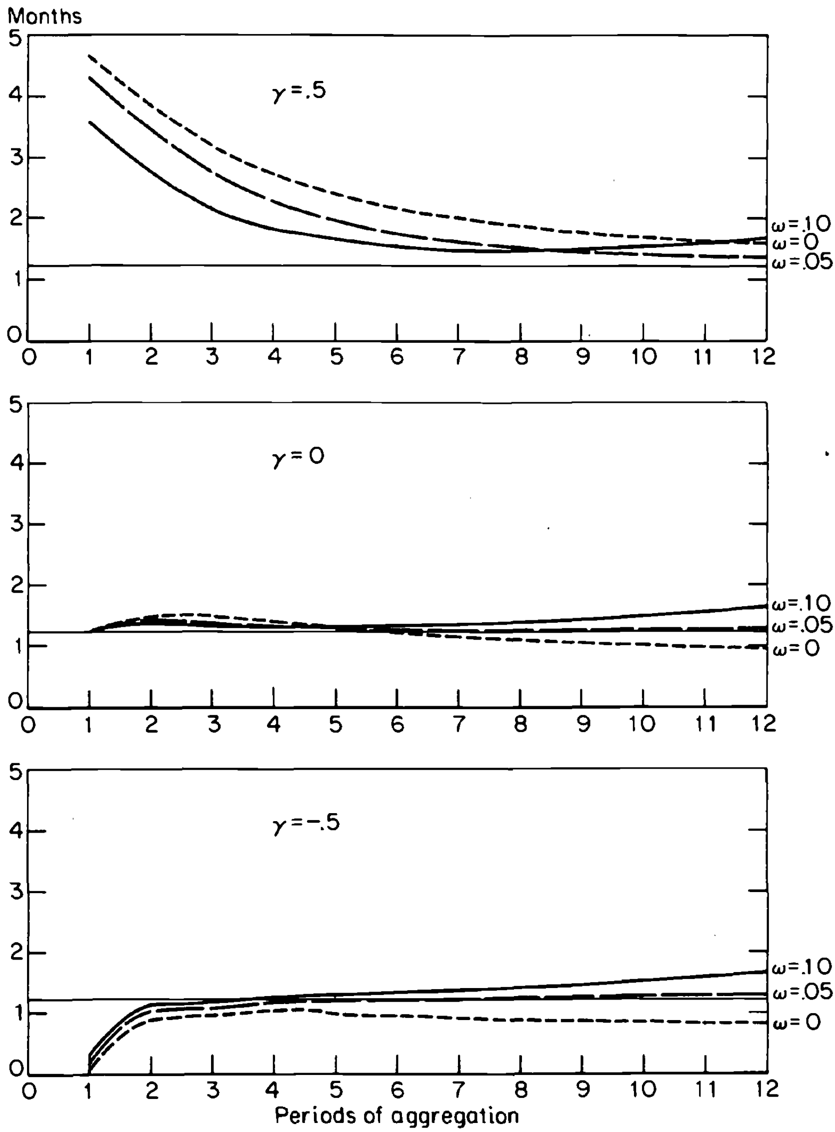
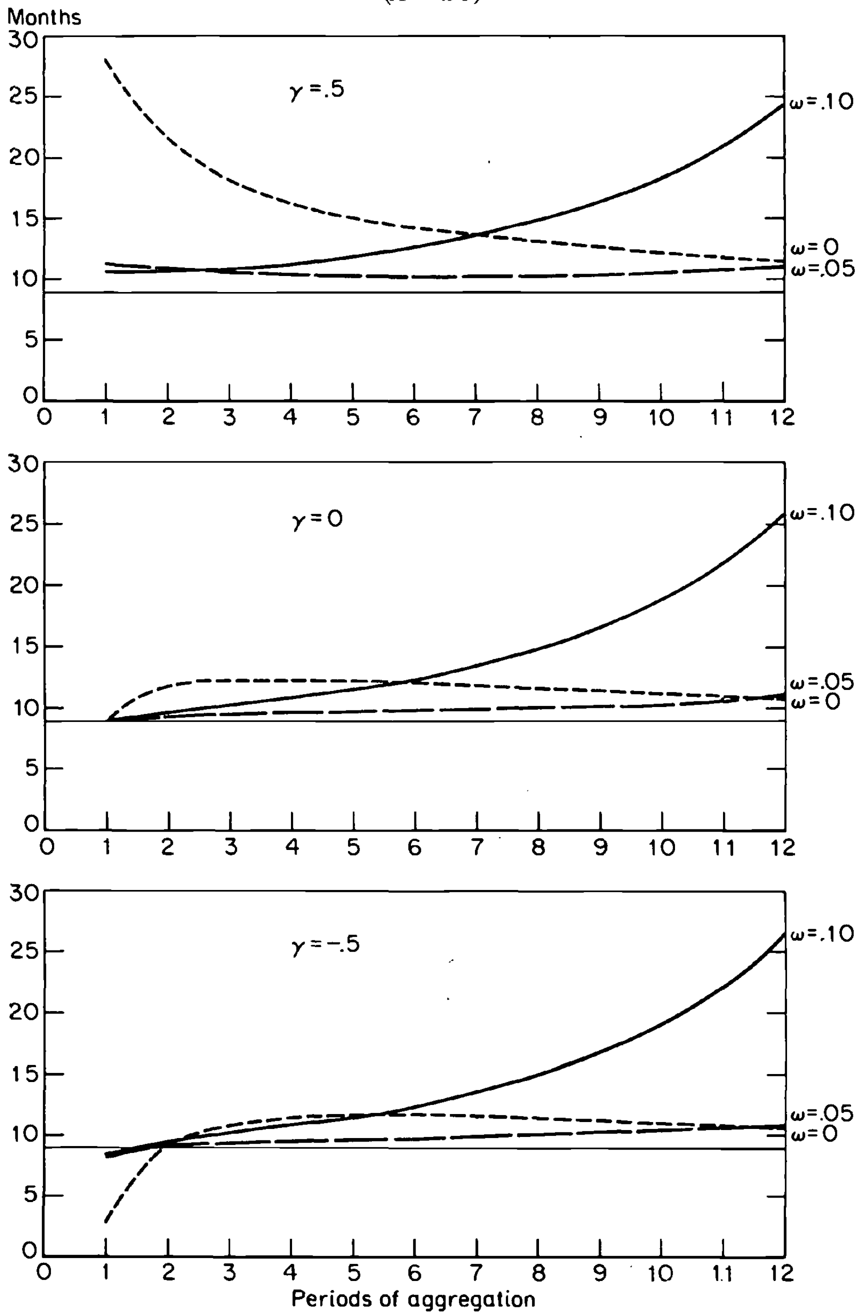


CHART 5

Inconsistency of Average Lag
(A = .90)



be very large and, thus, deviations from $\omega = 0$ to have a strong effect. It must be remembered, however, that all of these calculations were made under the assumption that $M = .5$, and therefore any comparison between the strength of the disturbance terms and the systematic terms in the expression (3-3) can be altered if M is different from .5.

In summary, there are three basic characteristics in the behavior of this model as it is aggregated over time. First, the average lag will increase or decrease if the serial correlation is, respectively, negative or positive. Second, when $\omega \neq 0$, the estimate of the lag will be greater, especially in highly aggregated models. And third, if α is close to one, the size of ω will be more important in determining the average lag in aggregated models.

In this model it is also possible to obtain a probability limit of the estimator $\hat{B}$. The expression is quite complicated, but under the assumption that $\omega = 0$, becomes very simply

$$\text{plim } \hat{B} = \frac{\beta}{1-\alpha} [1 - \text{plim } \hat{A}] \tag{2-14}$$

Thus the long-run marginal propensity which is given by

$$LRMP = \frac{\hat{B}}{1 - \hat{A}} \tag{2-15}$$

has probability limit

$$\text{plim } LRMP = \frac{\beta}{1-\alpha}$$

which is the same as that estimated in the disaggregated model. Thus, if the approximation $\omega = 0$ is valid, *time-aggregation does not bias the long-run marginal propensities.*

Finally, one additional remark must be made in order to apply these results directly to the macroeconomic model in the next section. A very useful form of the Koyck-Nerlove Model is the "inverted-V" model, which differs from that used above only in that the exogenous variable is replaced by its own moving average of several periods. This means that the peak effect of any change in the exogenous variable is not felt in the first period, but rather, somewhat later, depending on the length of the moving average used. Such a model is more appealing from an econometric point of view, especially for models using very short time-periods.

In terms of our analysis, estimation with an inverted-V model merely means that (2-9) would be replaced by

$$Y = AY_{-n} + B\frac{1}{r}\sum_{i=0}^{r-1} X_{-in} + V \tag{2-16}$$

If, however, X is expressed in spectral representation with $\omega = 0$, this composite variable will be indistinguishable from the original X, and the estimate of A and B will be unchanged. The estimate of the average lag will, though, be changed, because in (2-16) it is given by

$$\text{Average Lag} = n\left(\frac{A}{1-A} + \frac{r-1}{2}\right) \tag{2-17}$$

so the estimate will be greater by $n(r-1)/2$ than that computed before.

Thus, the description of the effects of time-aggregation on Koyck-Nerlove Models is valid for the inverted-V models as well, if the approximation $\omega = 0$ is valid, and if the additional contribution is taken into account.

3 EMPIRICAL ANALYSIS

IN ORDER to test the validity of the results of the preceding section and to explore the consequences of time-aggregation empirically, an estimation of a multiple-equation model of the U.S. economy, with corresponding monthly, quarterly, and annual data, was decided on.

A. FORMULATION OF THE PROBLEM

Since a monthly econometric model is needed as a basis for comparison, a recent model of the U.S. economy from 1948 to 1964, by Liu (1969), was chosen to provide the framework. The model was taken exactly as published, and the same data were aggregated to give quarterly and annual observations for the same set of variables. This approach insured honesty, since the equation structure was fixed and could not be changed to correct for wrong signs or other unpleasant behavior. Also, the most disaggregated model is reasonably sensible—as required by the analytic section—since, in each case, the assumption of a valid underlying model was crucial. The use of Liu's model also suggests answers to the natural question which must have occurred to readers of this paper, which makes the first advance into the field of model-building with monthly data: What do we learn, or accomplish, by the additional disaggregation?

In order to simplify the computations, the model is truncated so that it examines only the real-goods sector of the economy. This is done by considering the monetary variables as given, and removing from the model the corresponding explanatory equations for these variables. Eleven equations remain to be estimated, and with the seven identities, they give a reasonable explanation of income determination. Specific interactions between exogenous variables can be added to the model at all levels of aggregation to improve the realism of the multiplier analysis. In fact, this principle will be used to incorporate tax effects into the model.

The method of aggregating the model presents some interesting problems. Most of the monthly equations are distributed-lag equations with an inverted-V lag-distribution; when they are aggregated, it is necessary to decide how the equation is to be expressed. The decision was made to keep the length of time of the moving average the same, rather than maintaining the same number of periods. However, in the multiple-equation model, there is an additional problem which has important consequences. The monthly model is a recursive model, and consequently, all the variables are lagged by at least one month when used as regressors. This formulation insures the immunity of ordinary least squares estimators to simultaneous-equation bias and allows concentration on the problem of estimating distributed-lag models in the presence of serial correlation. However, when the model is aggregated, it is unreasonable to lag all variables one period and estimate the models recursively; thus, the quarterly and annual forms automatically become simultaneous. This choice seems reasonable, since intuition suggests that annual models—and, probably, quarterly models—should have simultaneous determination of the endogenous variables, because in this amount of time, there should be repercussions between the equations.

Several estimation procedures were used to fit the models described above, but none was completely satisfactory. The monthly model was not reestimated, as the following method, employed in Liu (1969), seemed consistent with the approach of this study. The serial correlation coefficient of the residuals was computed from the estimated parameters of ordinary least squares regressions and the asymptotic bias was then corrected. If the coefficient was significant, the variables were differenced by this parameter and the regression rerun. The second time, the serial correlation coefficient, again estimated as above, never appeared significant; and so, a third iteration was unnecessary.

For the quarterly and annual models, the basic estimation technique was two-stage least squares, since the model was simultaneous. However, the differences between TSLS and OLS estimates were often so small that it appeared plausible to assume that the biases due to serial correlation were more serious than the simultaneous-equation bias.

To examine this possibility, a nonlinear estimation-program, written by Martin (1968), was used to estimate each of the equations in the presence of serial correlation of the disturbance. The program estimates the serial correlation coefficient at the same time as the regression coefficients by including additional lagged values for the regressors, while enforcing the constraints between the coefficients. The program is iterative and maintains the constraints within .001. It can be used to estimate the relationship under either first- or second-order serial correlation of the disturbance, but there are many common second-order processes for which the program will not work. For example, when the process is oscillatory, the roots will be complex, and the program will not generate any solution which satisfies the constraints. This program has been used to estimate all the equations in the presence of serial correlation of first-order (with parameter γ_1) and second-order (with roots γ_1, γ_2), or, equivalently, with a process $[(1 - (\gamma_1 + \gamma_2)L + \gamma_1\gamma_2L^2)\epsilon]$ whenever the latter converges. The results of this estimation will be called consistent estimates, because in a properly specified model, in the presence of first- or, sometimes, second-order serial correlation, the program should yield consistent results.

In both theory and practice, it is possible to combine these programs by first estimating the reduced form separately, using the nonlinear program, and then taking the estimated $\hat{y}$'s and using them as regressors in a nonlinear estimation of the structural equation. For this study it was not deemed necessary to go through all these calculations in order to get an idea of the effects of time-aggregation in econometric models.

B. REGRESSION RESULTS: INDIVIDUAL EQUATIONS

The regression results are presented in the Appendix as Tables 5 to 15, corresponding to each of the eleven estimated relations. A brief interpretation of these tables will immediately precede them.

In examining the mass of regression results, it is difficult to sort out general principles and tendencies. In order to find our way through this complexity, it is helpful to categorize the equations. Seven of the equations (*CN*, *CS*, *BC*, *Q*, *DIV*, *CP*, *CP-DC*) are of the form discussed previously, while the other four are not. The depreciation equation has no lagged dependent-variable and, thus, is not appropriate for our analysis. The housing, inventory, and consumer durable equations are of the stock adjustment variety of distributed-lag models, which behave very differently from the models that we have been considering. In response to an income change, spending will increase, but eventually, the stock of the variable will also rise, driving the level of spending back down again. Such a relation has this characteristic: not only does the incremental effect peak (as in the inverted-V model), but the absolute level of spending peaks and then falls to a lower level. The model often gives rise to long-delayed effects which gradually damp the system. The analysis of Section 2 has not handled this type of lag model.

Looking specifically at these three stock adjustment relations, it is clear that since the housing equation has no income variable, a change in government spending will not affect the level of housing; and so, the propensities will be zero. In the inventory equation, the rate of depreciation of inventories is assumed to be zero, so the only way a steady state can be reached (which is the argument leading to long-run propensities) is for the stock of inventories to rise sufficiently to damp out the change in inventories, thus implying a long-run propensity of zero. The equation is, nonetheless, important in the interim periods and contributes to the cyclic nature of the model. The third stock adjustment equation is consumer durables, very important in determining long-run levels of income. Consequently, for this equation, the long-run propensities have been provided in the table, but no discussion of them can be relevant to the basic models of this paper.

This leaves us with only the seven equations mentioned above. These must be carefully studied for implications about the effects of aggregation. A condensation of the average-lag estimates of Tables 5 to 15 is presented in Table 1, which shows for each equation the estimate of the average lag obtained from OLS, TSLS, and the consistent regression – if one exists and is significant. The test for the significance of a nonlinear regression is that the estimated serial correlation coefficient exceed its standard error. The estimate of the original monthly serial correlation coefficient approximately corrected for asymptotic bias, following Nerlove-Wallis (1966, p. 236) and Liu (1969, p. 7), is given for comparison with the results of Section 2. Since many of these equations are of the inverted-V form, an estimate of the average lag, minus the contribution due to the moving average of the exogenous variable, is presented in parentheses. If the approximation $\omega = 0$ is valid, the analytic results of the previous section would apply directly to the OLS (ordinary least squares) or TSLS (two-stage least squares)[5] figures in parentheses; although, of course, they are not the relevant lags for actual behavior of the variables.

Comparing the data in Table 1 with the profiles in Charts 4 and 5, we see some important corroboration of the analytic results. There are four equations with positive serial correlations, and all of them show a decline in the average lag, just as shown in the top section of the figures. Unfortunately, sometimes they also go below the estimated true value of the average lag, which, theoretically, should be unlikely. Of the three equations with negative (or nearly zero) serial correlations, two rise from monthly to quarterly, and then fall with the annual

[5] Two-stage least squares can be seen analytically as just ordinary least squares with some of the correlation between regressors and the disturbance eliminated; therefore, the theory applies as well, or better, to this estimation procedure.

model; just as is most commonly observed in the lower two frames of Charts 4 and 5 when $\omega = 0$. The third, *CS*, however, does not follow the patterns; it first falls and then rises slightly. It appears that other factors, such as mis-specification of the underlying model, are important in this equation. Thus, six out of seven equations exhibit a profile of the average lag which is qualitatively the same as that derived analytically from knowledge of the serial correlation; consequently, an idea

TABLE 1

Average Lag

Equation	Serial Correlation	Monthly	Quarterly	Annual
CN				
OLS	−.18	3.2 (1.2)	3.2	−.8
TSLS			3.6	−.9
Consistent			1.4	−4.2
CS				
OLS	−.25	36.6 (34.6)	22.4	26.8
TSLS			25.8	27.5
Consistent			26.6	22.6
BC				
OLS	.32	45.5 (39.0)	39.5 (35.0)	25.5
TSLS			37.2 (32.7)	25.2
Consistent		29.3 (22.8)	21.0 (16.5)	
Q				
OLS	−.09	9.1 (2.6)	9.3 (4.8)	3.7
TSLS			8.9 (4.4)	3.7
Consistent			6.4 (1.9)	
DIV				
OLS	.66	18.3 (16.3)	9.1	4.0
TSLS			8.6	1.6
Consistent		11.7 (9.7)	6.6	3.7
CPDC				
OLS	.69	22.2	11.5	1.0
TSLS			11.9	1.0
Consistent		11.0	5.1	

NOTE: All estimates are in months. The figures in parentheses are average lags without the extra contribution of the inverted-V form. (See the end of Section 2.) These numbers are only for comparison with the analytic theory.

of the biases due to time-aggregation can be obtained from the analytic formulation.

One might hope that the use of the nonlinear regression would give an estimate of the lag distribution uncolored by the serial correlation which can result from time-aggregation; and therefore, that it would give an estimate of the average lag which is closer to the true value. Assuming that the consistent estimate of the monthly model yields the true average lag, the nonlinear estimation program is unsatisfactory for reproducing this value in the quarterly or annual cases.

The other parameter of great interest in economic theory is the long-run propensity. In Table 2, the long-run propensities for the seven equations are presented (along with the propensities for *CD*) for each of the estimation procedures used. The impression given upon inspection of this table is that the propensities in several cases differ very little, as anticipated by our theory; while in others, the patterns are quite irregular. In no equation, however, is the largest estimate more than twice the smallest (except one which is barely more); and so, if the true standard errors of the long-run propensities are roughly half of the size of the coefficients, the 95 per cent confidence intervals will be almost completely overlapping. These standard errors will be greater than those of the individual regression coefficients, because the bias resulting from serial correlation in the disturbance leads to an underestimate of the standard errors; and because each is divided by a number less than one, which is itself a random variable; and thus the variance of the ratio will be increased. These conditions are so undemanding that it does not seem that the regression results are sufficient to invalidate the theoretical prediction that the long-run propensity would be unchanged by aggregation over time. Examination of the consistent estimates indicates that in several cases they are better than least squares, but, in general, they are not.

C. REGRESSION RESULTS: DYNAMIC CHARACTERISTICS

The most informative type of simulation which can be used on a multiple-equation econometric model is multiplier analysis; but this is only applicable when the model is strictly linear. The multipliers of each of the exogenous variables can be computed separately, and the effects

TABLE 2

Long-Run Propensities

Equation	Serial Correlation	Monthly	Quarterly	Annual
CN				
OLS	−.18	.093	.155	.099
TSLS			.097	.095
Consistent			.201	.195
CS				
OLS	−.25	.430	.450	.480
TSLS			.442	.480
Consistent			.445	.500
BC				
OLS	.32	.480	.486	.281
TSLS		.350	.465	.283
Consistent			.312	
Q				
OLS	−.09	.381	.712	.674
TSLS			.735	.684
Consistent			.634	
DIV				
OLS	.66	.242	.223	.210
TSLS		.213	.221	.209
Consistent			.216	.208
CP				
OLS	.56	.269	.211	.167
TSLS		.194	.204	.167
Consistent				
CPDC				
OLS	.69	.178	.136	.080
TSLS		.088	.139	.080
Consistent			.094	
CD				
OLS	.03	.170	.166	.167
TSLS			.165	.167
Consistent			.155	.139

of linear combinations of changes in the policy variables is just the same linear combination of the multipliers. As the truncated version of the Liu model is linear, the use of multipliers seems ideal. In addition, there is no possibility that the particular assumed path of the exogenous variables would bias the results in one way or another.

In addition to the impact and long-run multipliers which are commonly computed by economics freshmen, the interim dynamic multipliers have also been computed, using approximately a technique from Theil and Boot (1962, p. 139). If the model is formulated as (3-1), where y is the column vector of endogenous variables, and x is the column vector of exogenous variables

$$Ay = B_1 y_{-1} + B_2 y_{-2} + \cdots + B_r y_{-r} + C_1 x + C_2 x_{-1} + \cdots + C_s x_{-s} \tag{3-1}$$

and where A's, B's and C's are matrices of estimated coefficients, then Theil and Boot say that the relation can be rewritten as

$$y^* = E y^*_{-1} + F x^* \tag{3-2}$$

where

$$E = \begin{bmatrix} A^{-1}B_1 & A^{-1}B_2 & \cdots & A^{-1}B_r \\ I & O & \cdots & O \\ O & I & O & \cdot \\ \cdot & O & I & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ O & \cdot & I & O \end{bmatrix} \tag{3-3}$$

$$F = \begin{bmatrix} A^{-1}C_1 & A^{-1}C_2 & \cdots & A^{-1}C_s \\ O & \cdot & \cdot & O \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ O & \cdot & \cdot & O \end{bmatrix}$$

$$y^* = \begin{bmatrix} y \\ y_{-1} \\ \cdot \\ \cdot \\ \cdot \\ y_{-r+1} \end{bmatrix} \qquad x^* = \begin{bmatrix} x \\ x_{-1} \\ \cdot \\ \cdot \\ \cdot \\ x_{-s} \end{bmatrix}$$

This relationship will be used to compute values of the cumulative dynamic interim multiplier (as opposed to the instantaneous multiplier used by Theil and Boot), which is defined for period k as the level of y which results from a sustained unit increase in x which occurred k periods ago. This multiplier is especially desirable for comparison of models with different levels of aggregation, because with all variables in annual rates, the multipliers should be the same. That is, the multiplier in year k of the annual model should be the average of the four quarterly, or the twelve monthly, multipliers of the same period. When $k = 1$, we get the impact multiplier; and when k goes to infinity, we get the long-run multiplier; so these fit nicely into the analysis.

If we label the first m columns (where m is the number of exogenous variables) of F as F_1 and the next m as F_2 so that F can be written as

$$F = (F_1, F_2, \ldots, F_s) \tag{3-4}$$

we can then express the cumulative dynamic interim multipliers ($CDIM$) in terms of the instantaneous dynamic interim multipliers ($IDIM$), where the latter are defined as the level of y corresponding to a process whereby x is zero at all times—except for one period k intervals ago, when $x = 1$. It follows directly from the linearity of the model that

$$CDIM(k) = \sum_{j=1}^{k} IDIM(j) = CDIM(k-1) + IDIM(k) \tag{3-5}$$

In turn, the $IDIM$ can be expressed in terms of E and F

$$\begin{aligned} IDIM(1) &= F_1 \\ IDIM(t) &= E \times IDIM(t-1) + F_t \qquad \text{if } t \leq s \\ & E \times IDIM(t-1) \qquad\quad\ \ \text{if } t > s \end{aligned} \tag{3-6}$$

In all cases, the only interesting multipliers are found in the first n rows (where n is the number of equations), because other elements reflect multipliers on other than current levels of y, and therefore are not actually kth period multipliers.

A model which behaves like equation (3-2) can be solved directly, and the solution will have a standard form for each of the variables. In particular (see, for example, Baumol (1959, p. 347)), it will be a linear combination of the eigenvalues of E, all raised to the power t; therefore, after several periods, all but the largest terms will have dropped from the solution. This observation makes it easy to see cyclic effects which are built into the system by the regression coefficients, since any complex pair of eigenvalues is automatically associated with a characteristic frequency of oscillation; and the larger is the absolute value of the eigenvalue, the more persistent is the cyclic component.

Cumulative dynamic interim multipliers were computed for each of six different sets of regression coefficients: Monthly, Quarterly TSLS (Two-Stage Least Squares), Annual TSLS, Quarterly Consistent, Annual Consistent, and Annual No Negative Lag. Eigenvalues were computed for only the last five models, because the monthly equations generated too large a matrix for our eigenvalue program (250 $\times$ 250). The quarterly and annual consistent models were composed of nonlinear regression estimates of each equation if: (*a*) it converged, (*b*) the serial correlation coefficients exceeded their standard errors, and (*c*) the equation was not pure economic nonsense. In Tables 5 to 15, the equation chosen in each case is indicated with an asterisk. For two equations the annual TSLS estimates implied a negative average lag. These equations were reestimated without the lagged dependent variable, and these coefficients were combined with the other TSLS estimates, giving the annual no negative lag model.

Three policy variables are available in the model to stimulate *GNP:* government spending, increases in personal holdings of liquid assets (which will be somewhat inaccurately referred to as the money supply), and decreases in the interest rate. In the analysis, these are independent policy tools, although in practice the latter two are not obviously independent. Changes in interest rates may very well influence the total personal holding of liquid assets, as well as the split between demand and savings deposits. Consequently, any policy which uses some of one tool may well counteract it by some of the other. This effect does not invalidate the analysis; it merely makes it more complicated to see the over-all effects of changes in monetary policy.

The regression results are shown in Tables 5 to 15 in the Appendix, while the important long-run multipliers and eigenvalues are displayed in Tables 3 and 4 respectively. In Charts 6 through 11, the cumulative dynamic interim multipliers are plotted for twenty years for each of the three policy variables and for each of the six models.

Turning first to the government spending multipliers in Chart 6, we note that the quarterly and annual TSLS estimates are very close but are overestimates of the monthly multipliers—although in the long run, the monthly and quarterly estimates are identical. The paths are roughly similar with the eventual peak produced by the delayed stock adjustment equations occurring in the third month of the fifteenth year in the monthly model; the last quarter of the ninth year in the quarterly; and the eleventh year in the annual model. For the first few years, the annual model has strong and unreasonable fluctuations, which indicate that it is not useful for short-term forecasts. This oscillation is represented by the 4.2 year cycle eigenvalue, which has absolute value .855. The frequency is excellent corroboration of the inherent tendency of the economy to feel business cycles of four to five years duration in response to shocks from exogenous variables. The damping of this component is insured by the fact that the eigenvalue is less than one;

TABLE 3

Long-Run GNP Multipliers

	Policy Variable		
Type of Estimation	Government Spending	Money Supply	Interest Rate
Monthly	2.40	.450	11.33
Quarterly TSLS	2.41	.324	6.40
Annual TSLS	2.64	.222	2.39
Quarterly Consistent	3.06	.312	6.01
Annual Consistent	2.53	.137	2.46
Annual No Negative Lag	2.75	.228	2.40

TABLE 4

Eigenvalues

Model	Absolute Value	Period in Years if Complex
Quarterly TSLS	.985	–
	.9589	162.0
	.953	4.18
	.906	–
	.857	–
	.848	46.7
	.799	2.9
	.741	48.7
Quarterly Consistent	.985	–
	.958	113.8
	.878	3.56
	.874	3.0
	.873	122.5
	.765	–
	.673	–
	.582	30.2
Annual TSLS	.946	–
	.855	4.2
	.840	314.8
	.678	–
	.567	15.3
	.487	3.7
Annual Consistent	.946	–
	.918	4.6
	.916	2.0
	.855	–
	.797	–
Annual No Negative Lag	.945	–
	.855	4.2
	.844	181.3
	.678	–
	.573	15.3
	.498	3.8

however, it is not as rapidly damped as the quarterly model. To see this, note merely that $(.953)^4 = .823$, which is smaller than .855, and therefore, over a year, the quarterly oscillation will be more strongly damped.

A further observation can be made about the multipliers in Chart 6: in the monthly model, seasonal effects are discernible. Even though the variables are presumably seasonally adjusted, the monthly model seems to be capable of describing residual seasonal effects, whereas the quarterly and annual ones are not. This might be an important advantage of monthly models.

In Chart 7, three more sets of dynamic multipliers are plotted on the same scale to indicate the effects of different estimation procedures on the dynamic behavior of the model. The quarterly consistent model gives larger multipliers and is more violent in the initial periods than the quarterly TSLS. The eigenvalues show more cyclic effects, in that there are two short period components, but the magnitudes are smaller, so such elements should be more transient than in the TSLS model.

CHART 6

GNP Multipliers of Government Spending

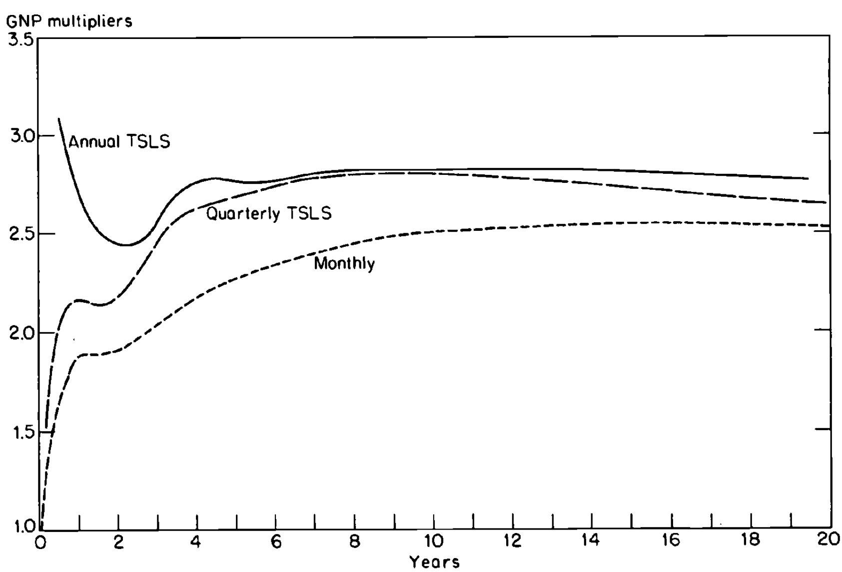


CHART 7

GNP Multipliers of Government Spending, Additional Estimators

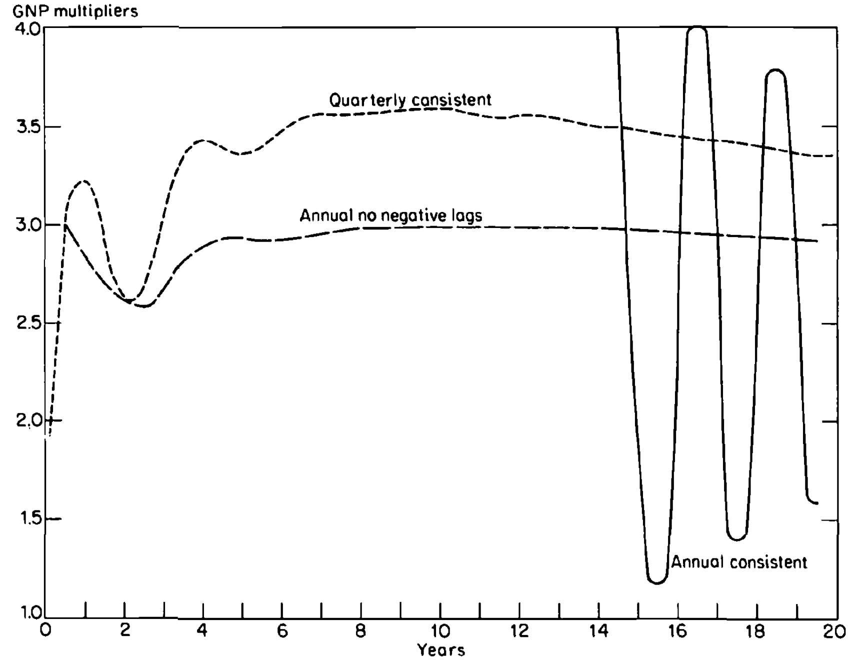


CHART 8

GNP Multipliers of Money Supply

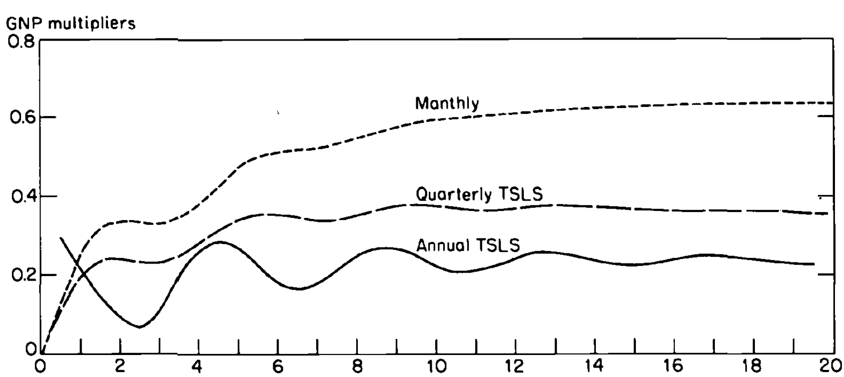


CHART 9

GNP Multipliers of Money Supply, Additional Estimators

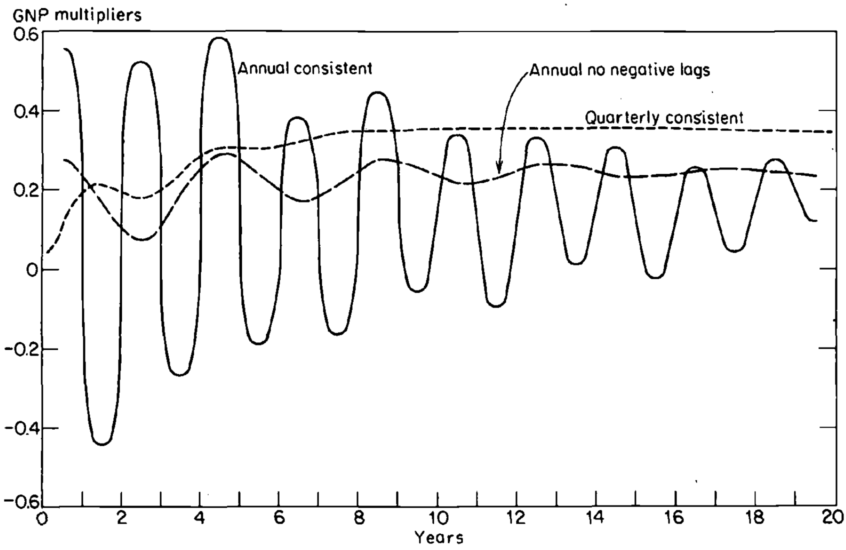


CHART 10

GNP Multipliers of Interest Rate Reduction

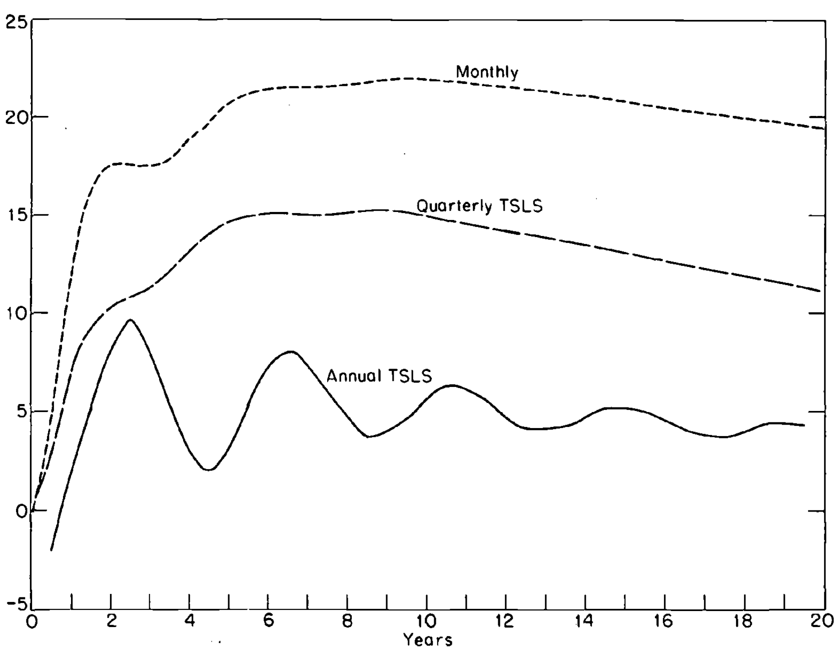
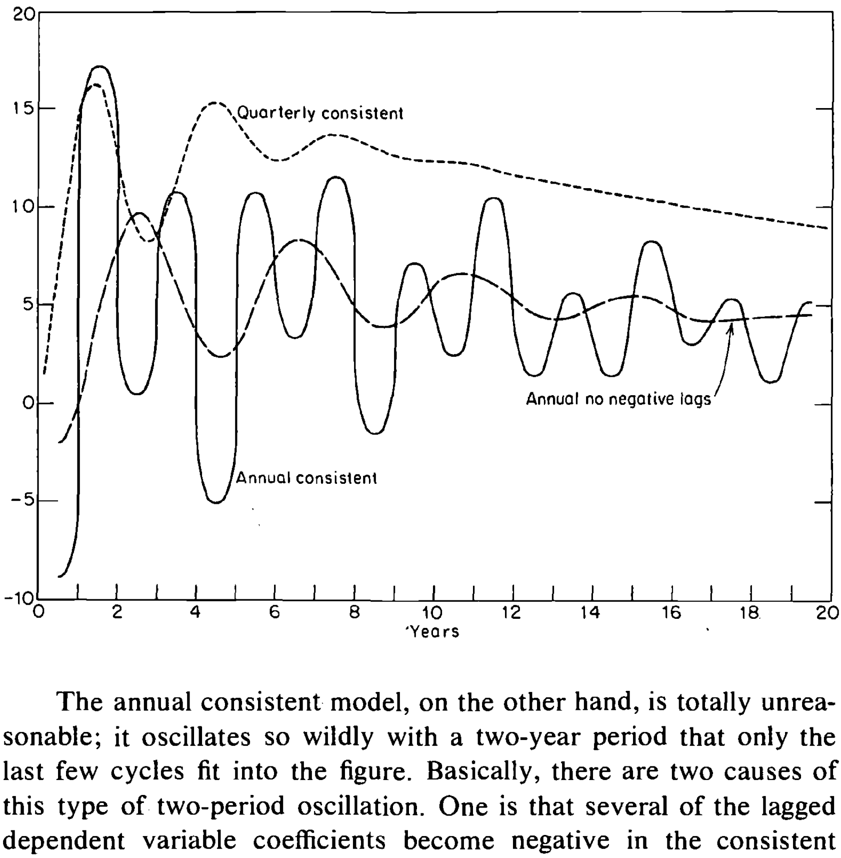


CHART 11

GNP Multipliers of Interest Rate Reduction, Additional Estimators



The annual consistent model, on the other hand, is totally unreasonable; it oscillates so wildly with a two-year period that only the last few cycles fit into the figure. Basically, there are two causes of this type of two-period oscillation. One is that several of the lagged dependent variable coefficients become negative in the consistent model; the other is that the stock adjustment mechanism is very unstable when the stock coefficient is large and negative. For a negligible depreciation rate, the model is unstable if the sum of the stock coefficient and the lagged dependent variable coefficient is not between minus one and plus one. Even if this sum is within these bounds but negative, the equation will alternate in direction, emphasizing the two-year cycle. Clearly, in the annual consistent model, this type of effect dominates the result.

The third curve is obtained by the common econometric practice of eliminating a lagged variable when its coefficient is negative, thereby making the equation static. Two equations fall into this category, consumer nondurables and corporate profits. The resulting model gives somewhat larger multipliers, especially in the long run, and slightly less violent oscillations in the first few years. The important eigenvalues are virtually unchanged.

In similar fashion, Charts 8 through 11 examine the effects of changes in the money supply and interest rates on *GNP* in each of the same six models. It is clear that monetary policy in each of its forms has much greater and more rapid effects in the monthly model than in all others. The best approximation to this underlying model is given by the quarterly TSLS, which has roughly the same shape but is somewhat smaller. The annual models exhibit far more cyclic behavior than either the quarterly or monthly ones. In particular, the annual consistent model is again dominated by a two-period oscillation, which makes the model economically untenable. There seems to be very little difference between the other two annual models, TSLS and No Negative Lag, either in terms of long-run multipliers or cyclic behavior. In general, the multipliers of the aggregated models are not good approximations to the monthly model, regardless of the estimation technique. The results for monetary policy seem to be worse than those for government spending, but perhaps this is to be expected from the way the underlying model has been truncated.

The over-all impression derived from examining the multipliers and eigenvalues is that the best estimates are still the TSLS, although the quarterly consistent estimator is almost as good. The more aggregated models are likely to have more cyclic behavior, particularly in the business-cycle frequencies, and sometimes with two-year periods; this often leads to meaningless models or, at least, strange short-run effects. The behavior of the aggregated models is somewhat better in the long run, as predicted by our theory; but, in general, the less highly aggregated models are better approximations to the underlying model.

4 CONCLUSIONS

THE GOAL of this paper has been to explore the consequences of time-aggregation of econometric models. The biases in the average lag and in the long-run marginal propensity were calculated analytically and estimated empirically to obtain relations between the various parameters of aggregated forms of any particular model. In both cases, the analysis was restricted to models of the Koyck-Nerlove genre because of their simplicity, prevalence, and econometric usefulness. The single-equation empirical results were then combined to examine the dynamic behavior of multiple-equation systems.

It was found that the best way to evaluate the biases in average lag and long-run marginal propensity analytically was to assume a particular stochastic specification for the underlying model, including the process of the exogenous variable, and then aggregate the variable, performing the mathematical estimation. After numerous approximations which appear to be empirically justified, it was found that the bias in the estimated average lag would be likely to increase or decrease with aggregation according to whether the serial correlation coefficient was negative or positive, respectively. If the exogenous variable had strong cyclical components, then it might be expected that the estimated average lag would eventually increase with aggregation in either case. The inverted-V form of the Koyck-Nerlove Model fits nicely into this formulation and can be directly compared with these results. A particularly important additional result is that the long-run marginal propensities should be unaffected by time-aggregation.

The empirical test of these analytic results consisted of reestimating the quarterly and annual versions of the seven equations from the monthly model of Liu (1969), which conformed to the analytic models considered. The results were quite gratifying in that six of the seven behaved much as predicted by theory. This corroboration substantiates the empirical validity of the assumptions which were used to obtain the analytic results and suggests that this type of analysis can be fruitfully used to anticipate the effects of time-aggregation.

Finally, the eleven quarterly and annual estimated relations from Liu's model were combined with the identities to construct dynamic interim multiplier simulations of the economic system under three policy parameters. In addition, eigenvalues for the system were computed in an effort to focus on some of the important cyclical components. The analysis of the regression results indicates five basic conclusions which seem to be true for this model:

(1) The more a model is aggregated, the worse it is as an approximation to the underlying monthly model.

(2) The aggregated models are especially inept in the short run, exhibiting strong fluctuations of business-cycle frequency; and often, of even shorter components. These models are therefore less stable, in the sense that they respond to a shock in a policy variable by protracted and sometimes violent oscillations.

(3) The long-run multipliers do not differ much among the models—even though the interim dynamic behavior may have quite bizarre variations.

(4) TSLS estimates of the equations are generally better approximations to the monthly model than are the nonlinear regression estimates.

(5) In this particular system of equations, the aggregated model overestimated the effects of government spending on *GNP* and underestimated the effects of monetary policy.

The validity of these observations is necessarily limited to this model, because there seems to be no easy way to extend the analytic single-equation results to entire systems, especially when stock adjustment equations are so important. Nevertheless, the behavior of this system gives insight into the phenomena which can be expected from aggregation over time.

APPENDIX: REGRESSION RESULTS

IN THE following eleven tables are the regression results for all of the models in the different levels of aggregation. The definition of the variables and the identities are also included. A brief discussion on the interpretation of these tables should be helpful. When the name of a regressor is appended by *L*, this implies a lagged value of the variable; while the use of *Q* or *A* implies a quarterly observation (in the monthly and quarterly models, but not in the annual ones) and an annual observation on the variable. In the monthly model, all explanatory variables are lagged one period, while in the other models this is not the case. In the monthly case, the estimate of serial correlation, γ_1, is

TABLE 5

Equation 1

Consumer Nondurables

Type of Estimation	*CNL*	*YQ*	*M*	*T*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly	.550	.042	.022	.282	34.8	.992	3.2
($\gamma = -.18$)	(.06)	(.024)	(.008)	(.02)			.093
Quarterly TSLS	.545	.044	.022	.283	34.6	.994	3.6
	(.106)	(.038)	(.012)	(.095)		(2.12)	.097
Quarterly OLS	.516	.075	.017	.249	31.2	.994	3.2
	(.105)	(.035)	(.012)	(.093)		(2.07)	.115
Quarterly First Order	.560	.072	.015	.222	29.9	.995	3.8
($\gamma = -.083$ (.187))	(.145)	(.034)	(.011)	(.116)		(1.97)	.164
*Quarterly Second Order[b]	.320	.137	.009	.310	20.9	.995	1.4
($\gamma_1 = .63$ (.186))	(.237)	(.053)	(.018)	(.165)		2.04	.201
($\gamma_2 = -.51$ (.139))							
*Annual TSLS	−.082	.102	.035	.703	86.5	.994	−.9
	(.275)	(.086)	(.029)	(.329)		1.95	.095
Annual OLS	−.075	.107	.034	.686	84.6	.994	−.8
	(.272)	(.077)	(.027)	(.310)		1.93	.099
Annual First Order	−.547	.301	−.054	.933	44.8	.998	−4.2
($\gamma = .52$ (.118))	(.143)	(.055)	(.028)	(.193)		2.17	.195
Annual Second Order	−.531	.305	−.057	.924	46.7	.998	−4.1
($\gamma_1 = -.12$ (.387))	(.167)	(.061)	(.030)	(.208)		2.06	.199
($\gamma_2 = .55$ (.173))							
Annual TSLS		.111	.031	.616	76.3	.995	0.0
		(.077)	(.024)	(.151)		2.11	.111
Annual OLS		.114	.030	.610	75.6	.995	0.0
		(.070)	(.023)	(.138)		2.08	.114

[a]Average Lag/Long-Run Propensity.

[b] Asterisk indicates equation chosen. (See p. 705.)

TABLE 6

Equation 2

Consumer Durables

Type of Estimation	*CDL*	*YQ*	*KCD*	Const	R^2/dw	LR Prop[a]
Monthly	.759	.067	−.029	−6.03	.914	
($\gamma = -.029$)	(.047)	(.020)	(.020)			.170
Quarterly TSLS	.717	.090	−.050	−7.8	.918	
	(.085)	(.032)	(.032)		2.05	.165
Quarterly OLS	.705	.101	−.060	−9.2	.919	
	(.085)	(.031)	(.031)		2.02	.166
Quarterly First Order	.726	.100	−.063	−9.8	.919	
($\gamma = -.069$ (.167))	(.102)	(.031)	(.030)		1.92	.166
*Quarterly Second Order[b]	.459	.185	−.124	−6.5	.923	
($\gamma_1 = -.55$ (.135))	(.212)	(.078)	(.074)		1.91	.155
($\gamma_2 = .72$ (.180))						
Annual TSLS	.319	.240	−.145	−23.1	.857	
	(.254)	(.072)	(.090)		1.90	.167
Annual OLS	.319	.239	−.143	−22.9	.857	
	(.254)	(.071)	(.089)		1.90	.167
*Annual First Order	−.066	.416	−.365	−5.02	.890	
($\gamma = .79$ (.328))	(.266)	(.110)	(.358)		1.67	.139
Annual Second Order[c]						

[a] Long-Run Propensity.

[b] Asterisk indicates equation chosen. (See p. 705.)

[c] Does not converge.

TABLE 7

Equation 3

Consumer Services

Type of Estimation	*CSL*	*YQ*	*M*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly	.972	.012	.004	−1.19	.999	36.6
($\gamma = -.248$)	(.017)	(.007)	(.002)			.430
Quarterly TSLS	.896	.046	.009	−4.17	.999	25.8
	(.041)	(.018)	(.006)		2.43	.442
Quarterly OLS	.882	.053	.009	−4.7	.999	22.4
	(.039)	(.017)	(.006)		2.39	.450
*Quarterly First Order[b]	.899	.045	.009	−4.9	.999	26.6
($\gamma = -.217$ (.125))	(.032)	(.014)	(.005)		2.08	.445
Quarterly Second Order[c]						
Annual TSLS	.696	.146	.016	−13.3	.999	27.5
	(.067)	(.030)	(.011)		1.60	.480
Annual OLS	.690	.149	.016	−13.5	.999	26.8
	(.065)	(.028)	(.011)		1.56	.480
*Annual First Order	.653	.174	.008	−11.3	.999	22.6
($\gamma = .293$ (.274))	(.082)	(.047)	(.019)		1.94	.500
Annual Second Order[c]						

[a]Average Lag/Long-Run Propensity.
[b] Asterisk indicates equation chosen. (See p. 705.)
[c]Does not converge.

TABLE 8

Equation 4

Business Construction

Type of Estimation	*BCL*	*CP-CTA*	*RL*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly Stage 1	.975	.012	−.101	.28	.990	45.5
($\gamma = .324$)	(.014)	(.004)	(.042)			.480
Monthly Stage 2	.958	.015	−.097	.27	.980	29.3
($\gamma = -.035$)	(.020)	(.006)	(.059)			.350
Quarterly TSLS	.916	.039	−.295	.77	.963	37.2
	(.045)	(.013)	(.143)		1.41	.465
Quarterly OLS	.921	.037	−.281	.76	.963	39.5
	(.045)	(.013)	(.143)		1.41	.468
*Quarterly First Order[b]	.846	.048	−.231	.82	.967	21.0
($\gamma = .365$ (.161))	(.086)	(.021)	(.209)		2.02	.312
Quarterly Second Order	.818	.051	−.225	.91	.966	18.0
($\gamma_1 = .463$ (.325))	(.117)	(.024)	(.224)		2.01	.280
($\gamma_2 = -.087$ (.272))						
*Annual TSLS	.678	.091	−.396	2.78	.828	25.2
	(.209)	(.048)	(.710)		2.03	.283
Annual OLS	.680	.090	−.383	2.78	.828	25.5
	(.209)	(.047)	(.708)		2.03	.281
Annual First Order	.605	.093	−.393	4.05	.850	18.4
($\gamma = .028$ (.486))	(.360)	(.051)	(.709)		2.25	.235
Annual Second Order[c]						

[a] Average Lag/Long-Run Propensity.
[b] Asterisk indicates equation chosen. (See p. 705.)
[c] Does not converge.

TABLE 9

Equation 5

Business Equipment

Type of Estimation	*QL*	*CP-CTA*	*RL*	*T*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly	.711	.110	−1.58	.026	7.81	.940	9.06
($\gamma = -.086$)	(.066)	(.064)	(.413)	(.012)			.381
Quarterly TSLS	.593	.299	−1.36	−.005	3.34	.929	8.87
	(.095)	(.078)	(.564)	(.036)		1.42	.735
Quarterly OLS	.616	.273	−1.38	.004	3.67	.929	9.32
	(.094)	(.076)	(.563)	(.036)		1.45	.712
*Quarterly First Order[b]	.382	.391	−1.93	.015	3.73	.943	6.35
($\gamma = .420$ (.26))	(.248)	(.154)	(1.08)	(.059)		1.73	.634
Quarterly Second Order[c]							
*Annual TSLS	.237	.522	−.914	.062	2.98	.869	3.72
	(.231)	(.156)	(1.68)	(.120)		1.92	.684
Annual OLS	.237	.514	−.953	−.057	3.32	.869	3.72
	(.231)	(.149)	(1.67)	(.117)		1.93	.674
Annual First Order	.625	.799	2.11	−.381	−21.1	.957	20.0
($\gamma = .0079$ (.173))	(.171)	(.131)	(1.38)	(.119)		2.22	2.13
Annual Second Order[c]							

[a] Average Lag/Long-Run Propensity.

[b] Asterisk indicates equation chosen. (See p. 705.)

[c] Does not converge.

TABLE 10

Equation 6

Housing

Type of Estimation	*HL*	*KHL*	*MA*	*RL*	*T*	Const	R^2/dw
Monthly Stage 1	.971	−.199	.005	−.031	.735	58.3	.982
($\gamma = .382$)	(.014)	(.020)	(.002)	(.114)	(.075)		
Monthly Stage 2	.945	−.196	.006	−.125	.732	39.2	.963
($\gamma = .069$)	(.020)	(.028)	(.002)	(.165)	(.105)		
Quarterly OLS	.912	−.563	.012	−.167	2.09	157.5	.935
	(.048)	(.065)	(.005)	(.377)	(.239)		.96
*Quarterly First Order[a]	.775	−.728	.012	−1.23	2.75	59.9	.957
($\gamma = .713$ (.156))	(.085)	(.161)	(.014)	(1.17)	(.624)		1.98
Quarterly Second Order	.760	−.687	.016	−.887	2.58	64.4	.953
($\gamma_1 = .646$ (.302))	(.101)	(.199)	(.013)	(1.15)	(.766)		2.02
($\gamma_2 = .074$ (.279))							
Annual OLS	.744	−1.59	.044	1.98	5.77	445.9	.725
	(.302)	(.421)	(.024)	(2.04)	(1.47)		2.17
*Annual First Order	.859	−1.48	.056	2.91	5.29	562.6	.794
($\gamma = -.377$ (.374))	(.333)	(.376)	(.021)	(1.88)	(1.31)		1.35
Annual Second Order[b]							

[a] Asterisk indicates equation chosen. (See p. 705.)

[b] Does not converge.

TABLE 11

Equation 7

Change in Inventories

Type of Estimation	*DVL*	*GNPQ*	*RSP*	*VL*	Const	R^2/dw
Monthly	.389	.197	−.369	−.907	−60.5	.495
($\gamma = -.230$)	(.059)	(.032)	(.124)	(.156)		
Quarterly TSLS	.486	.199	−.085	−.946	−61.9	.720
	(.070)	(.027)	(.148)	(.134)		2.26
Quarterly OLS	.485	.201	−.081	−.955	−62.5	.720
	(.070)	(.027)	(.147)	(.132)		2.25
Quarterly First Order	.556	.182	−.117	−.860	−70.0	.747
($\gamma = .235$ (.144))	(.068)	(.023)	(.128)	(.112)		1.95
*Quarterly Second Order[a]	.521	.193	−.168	−.904	−64.5	.758
($\gamma_1 = .255$ (.268))	(.086)	(.029)	(.152)	(.139)		1.90
($\gamma_2 = -.432$ (.199))						
Annual TSLS	.140	.202	−.194	−.946	−63.4	.860
	(.105)	(.026)	(.208)	(.132)		1.43
Annual OLS	.140	.203	−.193	−.948	−63.5	.860
	(.105)	(.025)	(.207)	(.129)		1.43
Annual First Order[b]	.269	.228	−.014	−1.245	−10.7	.899
($\gamma = .836$ (.170))	(.096)	(.022)	(.150)	(.153)		2.29
*Annual Second Order	.346	.208	−.136	−1.31	−30.4	.899
($\gamma_1 = .944$ (.063))	(.118)	(.027)	(.269)	(.159)		2.31
($\gamma_2 = -.310$ (.310))						

[a] Asterisk indicates equation chosen. (See p. 705.)
[b] Does not converge.

TABLE 12

Equation 8

Dividends

Type of Estimation	*DIVL*	*CP-CTQ*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly Stage 1	.942	.014	.098	.995	18.3
($\gamma = .658$)	(.018)	(.004)			.242
Monthly Stage 2	.906	.020	.085	.981	11.7
($\gamma = .177$)	(.032)	(.007)			.213
Quarterly TSLS	.747	.056	.482	.981	8.6
	(.054)	(.011)		1.58	.221
Quarterly OLS	.752	.056	.480	.981	9.1
	(.051)	(.010)		1.59	.223
*Quarterly First Order[b]	.689	.067	.50	.982	6.6
($\gamma = .267$ (.148))	(.074)	(.014)		2.03	.216
Quarterly Second Order	.676	.069	.52	.981	6.3
($\gamma_1 = .394$ (.293))	(.087)	(.016)		2.01	.212
($\gamma_2 = -.136$ (.269))					
Annual TSLS	.119	.184	2.02	.948	1.6
	(.173)	(.033)		2.49	.209
Annual OLS	.249	.158	.176	.951	4.0
	(.160)	(.030)		2.63	.210
*Annual First Order	.235	.157	2.67	.962	3.7
($\gamma = -.353$ (.322))	(.188)	(.034)		1.12	.208
Annual Second Order[c]					

[a] Average Lag/Long-Run Propensity.

[b] Asterisk indicates equation chosen. (See p. 705.)

[c] Does not converge.

TABLE 13

Equation 9

Corporate Profits

Type of Estimation	*CPL*	*GNP*	*PW*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly Stage 1	.948	.014	.170	−33.3	.993	19.2
($\gamma = .560$)	(.021)	(.004)	(.032)			.269
Monthly Stage 2	.897	.020	.107	−9.26	.975	9.7
($\gamma = -.004$)	(.034)	(.006)	(.041)			.194
Quarterly TSLS	.720	.058	5.18	−100.8	.966	7.7
	(.076)	(.013)	(1.18)		1.23	.204
*Quarterly OLS[b]	.716	.060	5.26	−102.5	.966	7.6
	(.076)	(.013)	(1.18)		1.23	.211
Quarterly First Order[c]						
Quarterly Second Order[c]						
*Annual TSLS	−.200	.200	7.22	−139.2	.934	−2.0
	(.233)	(.033)	(3.84)		1.90	.167
Annual OLS	−.208	.202	7.24	−139.7	.933	−2.1
	(.233)	(.033)	(3.84)		1.88	.167
Annual First Order	−.221	.202	7.05	−128.1	.932	−2.2
($\gamma = .055$ (.635))	(.491)	(.073)	(4.01)		1.91	.165
Annual Second Order[c]						
Annual TSLS		.175	7.89	−153.1	.935	0.0
		(.015)	(3.72)		2.20	.175
Annual OLS		.175	7.97	−154.4	.935	0.0
		(.015)	(3.73)		2.20	.175

[a] Average Lag/Long-Run Propensity.
[b] Asterisk indicates equation chosen. (See p. 705.)
[c] Does not converge.

TABLE 14

Equation 10

Corporate Profits After Depreciation

Type of Estimation	*CP-CDL*	*GNP*	*PW*	Const	R^2/dw	Av Lag/ LR Prop[a]
Monthly Stage 1	.955	.008	.161	−30.6	.975	22.2
(γ = .689)	(.017)	(.002)	(.030)			.178
Monthly Stage 2	.909	.008	.041	−1.97	.883	11.0
(γ = .231)	(.034)	(.003)	(.037)			.088
Quarterly TSLS	.798	.028	5.04	−93.7	.863	11.9
	(.067)	(.006)	(1.19)		1.29	.139
Quarterly OLS	.794	.028	5.11	−95.0	.863	11.5
	(.067)	(.006)	(1.19)		1.29	.136
*Quarterly First Order[b]	.629	.035	3.37	−28.3	.889	5.1
(γ = .517 (.322))	(.279)	(.019)	(1.54)		1.75	.094
Quarterly Second Order[c]						
*Annual TSLS	.079	.074	7.40	−122.7	.543	1.0
	(.289)	(.021)	(4.82)		2.08	.080
Annual OLS	.076	.074	7.46	−123.9	.543	1.0
	(.289)	(.021)	(4.82)		2.07	.080
Annual First Order	.284	.062	9.29	−207.5	.587	4.7
(γ = −.286 (.521))	(.480)	(.027)	(4.17)		2.14	.087
Annual Second Order	−.320	.384	1.59	−6.67	.758	−2.9
(γ_1 = .975 (.061))	(.169)	(.088)	(4.81)		1.76	.291
(γ_2 = .204 (.356))						

[a] Average Lag/Long-Run Propensity.
[b] Asterisk indicates equation chosen. (See p. 705.)
[c] Does not converge.

TABLE 15

Equation 11
Depreciation

Type of Estimation	*KL*	Const	R^2/dw
Monthly Stage 1	.071	−20.4	.977
(γ = .795)	(.009)		
Monthly Stage 2	.060	−2.20	.822
(γ = −.060)	(.002)		
Quarterly OLS	.075	−24.1	.991
	(.0009)		.21
*Quarterly First Order[a]	.072	−2.53	.998
(γ = .884 (.056))	(.003)		.94
Quarterly Second Order[b]			
*Annual OLS	.075	−23.3	.992
	(.002)		1.02

[a] Asterisk indicates equation chosen. (See p. 705.)
[b] Does not converge.

obtained *after* the regression, not simultaneously with the variable coefficients, so the first stage is presumably inconsistent as long as the serial correlation coefficient is γ_1, not zero.

In the second stage, the serial correlation coefficient is again estimated after the regression is run and, consequently, if it is significantly different from zero, this stage, too, would be inconsistent. In the last column of most of the tables, the calculated values of the average lag in months and the long-run propensities for the equation by itself are to be found. When the equation has several regressors, long-run propensities can be computed for each of them; and in fact, when the exogenous variables are observed in different periods, the average lag may be different with respect to different variables. In order to avoid this ambiguity, in each equation the variable which is tabulated second—immediately after the lagged dependent variable—is used as the basis for the long-run propensity and average lag. This variable is chosen because it reflects the effects of *GNP* on the particular spending category and is, therefore, the relevant propensity for computing *GNP* multipliers from an exogenous change in government spending.

The reason for focusing on these multipliers is that by truncating the model so that it deals only with real goods, we have artificially made government spending the most important policy tool, and it is the lag and multiplier with respect to this process of income determination which the model should be best equipped to analyze. It is also possible to obtain changes in *GNP* from changes in monetary or interest-rate variables, but the model should not be as good at predicting such effects.

DEFINITIONS OF VARIABLES

(Unless specified otherwise, all variables are in 1958 dollars in annual rates.)

CN	Personal consumption expenditure on nondurable goods
Y	Personal disposable income
M	Personal liquid assets in billions of 1958 dollars
T	Time trend (initial quarter = 1)
CD	Personal consumption expenditures on durable goods
KCD	Stock of capital in consumer durables, in billions of 1958 dollars
CS	Personal consumption expenditures on services
BC	Gross private domestic investment in business construction
CP	Gross corporate profits and inventory valuation adjustments
CT	Corporate profit tax liability
RL	Long-term rate of interest represented by domestic corporate bond yields (Moody's) in per cent per year, annual averages
Q	Gross private domestic investment in producers' durable equipment
H	Gross private domestic investment in residential housing construction
KH	Stock of capital in residential housing structures, in billions of 1958 dollars
DV	Change in nonfarm business inventory

GNP	Gross national product
RS	Short-term rate of interest, represented by the rate on prime commercial paper (4–6 months) in per cent per year
P	Rate of change of the GNP deflator in per cent per year
V	Stock of nonfarm inventory in billions of 1958 dollars
PW	$PW = (P/W)/(GNP)$
DIV	Dividends
P	GNP deflator, index numbers, 1958 = 100
W	Salary and wage payments, in billions of current dollars
DC	Corporate depreciation allowance
D	Capital consumption allowance
KBC	Stock of capital in business structure, 1958 dollars
KQ	Stock of capital in producers' durable equipment, in 1958 dollars
DVF	Change in farm inventory
G	Government purchases
E	Net exports of goods and services
RES	Net income tax plus residual from accounting identity
DQ	Ratio of capital consumption allowance on producers' durable equipment to the stock of durable equipment at beginning of year (*assume* = .1224)
DBC	Ratio of capital consumption allowance on business structures to the stock of business structures at beginning of year (*assume* = .0488)
DH	Ratio of capital consumption allowance on residential housing structures to the stock of housing structures at beginning of year (*assume* = .01721)
DCD	Rate of depreciation of consumer durables (*assume* = .1904)

Identities to Complete Model

$$GNP = CN + DC + DS + BC + Q + H + DV + DVF + G + E$$
$$Y = GNP - D - (CP - DC) + DIV - RES$$
$$V = DV/(12/n) + V_{-n}$$
$$KBC = BC/(12/n) = [1 - DBC/(12/n)]\ KBC_{-n}$$
$$KQ = Q/(12/n) + [1 - DH/(12/n)]\ KCD_{-n}$$
$$KH = H/(12/n) + [1 - DH/(12/n)]\ KH_{-n}$$
$$KCD = CD/(12/n) + [1 - DCD/(12/n)]\ KCD_{-n}$$

NOTE: n = period of aggregation.

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DISCUSSION

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This paper by Engle and Liu represents the first serious attempt to investigate the impact on the dynamic characteristics of econometric models of a change in their time reference. While the first econometric models were based on annual observations, over the past seven years or so, quarterly models have become more popular. Thus, it is indeed odd that this problem was not addressed at an earlier time.

A look at the Engle-Liu paper will supply part of the answer. The problem is indeed difficult. The contribution made by the paper rests mainly on an experiment with the Liu Model, whose dynamic characteristics are examined under successive schemes of time-aggregation.

The analytical aspects of the paper are mainly concerned with the simple model

$$y_t = \lambda y_{t-1} + u_t \tag{1}$$

where $(u_t{:}t = 1,2, \ldots)$ is a sequence of independent, identically distributed random variables (which was also investigated by others: [2], [3], [4]) and an extension to the case

$$y_t = \frac{\alpha I}{I - \lambda L} x_t + \frac{I}{I - \rho L} \epsilon_t \tag{2}$$

where now $(\epsilon_t{:}t = 1,2, \ldots)$ is a sequence of independent, identically distributed random variables. The bulk of this, however, appears in Engle's dissertation to which, regrettably, I had no access.

The question asked is the following: if we sum the model over n periods to obtain

$$Y_t = \frac{\alpha I}{I - \lambda L} X_t + \frac{I}{I - \rho L} \epsilon_t^*, \; Y_t = \sum_{i=0}^{n-1} Y_{t-i}, \; X_t = \sum_{i=0}^{n-1} x_{t-i} \tag{3}$$

$$\epsilon_t^* = \sum_{i=0}^{n-1} \epsilon_{t-i}$$

and if we apply OLS to the reduced model, *using only the feasible data inputs*, what are the properties of the resulting estimators and how are their probability limits related to the parameters indicated in (3)?[1]

Thus, in the problem above, if we have only "n-spaced" (if $n = 12$ and t refers to months, then annual data), the estimation procedure can be carried out only with respect to the model

(4) $$Y_{n\tau} = \frac{\alpha I}{I - \lambda L} X_{n\tau} + \frac{I}{I - \rho L} \epsilon^*_{n\tau} \qquad \tau = 1, 2, \ldots, k$$

The difficulty arises because the quantity $\frac{I}{I - \lambda L} X_{n\tau}$ cannot be computed from the available data, since

(5) $$\frac{I}{I - \lambda L} X_{n\tau} = \sum_{i=0}^{\infty} \lambda^i X_{n\tau-i} = \lambda^{n\tau}\alpha_o^* + \sum_{i=0}^{n\tau-1} \lambda^i X_{n\tau-i}\alpha_o^* = \sum_{j=0}^{\infty} \lambda^j X_{-j}$$

and we can *only compute* $\sum_{i=0}^{\tau-1} \lambda^{ni} X_{n(\tau-i)}$.

Thus, if we further reduce the model

(6) $$Y_{n\tau} = \lambda Y_{n\tau-1} + \alpha X_{n\tau} + \frac{I - \lambda L}{I - \rho L} \epsilon^*_{n\tau}$$

we see that we can operate only with the model

(7) $$Y_{n\tau} = \lambda^* Y_{n(\tau-1)} + \alpha^* X_{n\tau} + \text{error}$$

The analysis in the paper applies OLS to (7) and then seeks to determine the probability limits of $\hat{\lambda}^*$, $\hat{\alpha}^*$ and $\frac{\hat{\alpha}^*}{1 - \hat{\lambda}^*}$, the latter being the "long-run multiplier" of the explanatory variable relative to the conditional expectation of the dependent variable.

The nature of inconsistencies involved is twofold.

(*i*) There is specification inconsistency due to the fact that $Y_{n(\tau-1)}$ is used instead of $Y_{n\tau-1}$.

(*ii*) There is inconsistency because $Y_{n(\tau-1)}$ is correlated with the error term of the equation. This is occasionally called, somewhat unsatisfactorily, "least squares bias."

[1] Throughout the paper, the authors have the unfortunate habit of referring to probability limits as "asymptotic expectations," and to inconsistencies as "asymptotic biases."

In examining the effects of these inconsistencies on the parameter estimates, the authors concentrate on the "average lag," which, in the model above, may be shown to be

$$\text{(8)} \qquad \text{Average Lag} = \frac{\lambda}{1-\lambda}$$

Allegedly, this was chosen because it indicates "the point in time at which half the adjustment from initial to final value of the dependent variable has taken place." I believe this to be in error. If the explanatory variable changes by one unit at time 1, and remains constant thereafter, then using the representation

$$\text{(9)} \qquad \frac{I}{I-\lambda L} x_t = \lambda^t \alpha_o^* + \sum_{i=0}^{t-1} \lambda^i x_{t-i}$$

we see that the impact at time t is given by

$$\text{(10)} \qquad \sum_{i=0}^{t-1} \lambda^i = \frac{1-\lambda^t}{1-\lambda}$$

Setting this equal to $\frac{1}{2}\frac{1}{1-\lambda}$ we find

$$\text{(11)} \qquad \frac{\lambda^t}{1-\lambda} = \frac{1}{2}\frac{1}{1-\lambda} \quad \text{or} \quad t = -\frac{\ln 2}{\ln \lambda}$$

which is certainly not the average lag.

Mean lag is computed by analogy to the first moment of the probability distribution and will fail to have meaning when some lag coefficients are negative.

In connection with this problem, I wonder if there is no way in which the basic parameters of the model can be estimated consistently. It would appear to me that if we know that $\rho = 0$, the parameters of (4) can be estimated consistently by spectral techniques. In this procedure we would have to use a certain *alias* of the spectrum of x and the cross-spectrum of x and y, but I would conjecture that α and λ can be estimated consistently, although we would have to solve a nonlinear system. If $\rho \neq 0$, a difficulty would arise, since initial consistent estimators of α, λ would be unavailable; and thus, no estimators of the spectrum of the transformed error in (6) could be obtained.

The dependence of the results regarding inconsistencies on the specific form of the explanatory variable is quite interesting, and I wish the authors had provided a better discussion of them. Do I understand correctly in thinking that they have taken the explanatory variable as

$$x_t = A \cos (wt + \beta) \tag{12}$$

where A, β are appropriate constants? Is this not a very odd choice?

Turning now to the empirical part of the paper, I find a number of problems that I hope the authors will comment upon in the course of the ensuing discussion.

(*a*) In the monthly model of Liu, the typical equation is of the form

$$y_t = \lambda y_{t-1} + \sum_{i=1}^{n-1} \beta_i x_{ti} + u_t \tag{13}$$

where the x_{ti} may be exogenous or *other* lagged endogenous variables. The method of estimation indicated in the foregoing paper is generally inconsistent if the error is subject to serial correlation. "*Correcting*" *for serial correlation ex post will not eliminate the problem, since the "estimate" of the autocorrelation parameter is also inconsistent.* Some recent work that I have done [1] indicates that a two-step procedure based on a first-stage use of instrumental variables will be inefficient relative to simultaneous estimators of the parameters and the autocorrelation coefficient. Presumably, a spectral estimation scheme is most appropriate here; and with monthly observations, this is certainly feasible. In fact, I hope that with the forthcoming plentitude of monthly data, spectral analytic estimation techniques will make single-equation estimation of such models quite attractive.

(*b*) In the quarterly and annual models, the authors use a simultaneous-equations estimation scheme. *If serial correlation in the monthly model is negative,* it is conceivable that aggregating, as one does, to a quarterly or annual basis will render (first-order) serial correlation zero or near zero (or positive). Thus, it is difficult to see what it is that we are comparing when the time paths of various "multipliers" are considered. Ideally, we would want to see the impact of the aggregation inconsistencies only, not that of the inconsistencies incurred through estimation techniques which are inappropriate—but whose inappropriateness varies from instance to instance. I hope that the authors will discuss this aspect of the problem.

All in all, this is a significant paper. Although hard, palpable results are few, still it opens up an extremely important and interesting area of research. Moreover, as econometric models are brought more closely into the decision-making apparatus of policy-makers, the implications of this type of research will have enormous practical implications as well.

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Engle and Liu pose a very hard problem and reach very few general conclusions. I am afraid this really reflects the underlying structure of the problem: the consequences of aggregation depend crucially on the internal interrelationships of the items to be aggregated, and hence they will be different for different models and types of time series.

In the models examined by Engle and Liu, three different effects occur and bias the results in different directions, with a net effect which is very difficult to evaluate

(1) If the model is given by

$$y = ax_{-1} + bx_{-2} + u$$

and we aggregate to

$$y + y_{-1} = ax_{-1} + (a + b)x_{-2} + bx_{-3} + u + u_{-1}$$

but estimate

$$y + y_{-1} = B(x_{-2} + x_{-3}) + v$$

we commit the errors of leaving out a relevant x_{-1} variable and of forcing the x_{-2} and x_{-3} variables to have the same coefficients. In general, this will lead to an *attenuation* in the estimated effect of a unit change in x on y. That is, $EB < a + b$. In this particular case

$$EB = \frac{a(r_1 + r_2) + (a + b)(1 + r_1) + b(1 + r_1)}{2(1 + r_1)} = a + b - \frac{a(1 - r_2)}{2(1 + r_1)}$$

where r_i is the ith serial correlation coefficient of the (stationary) x process. The magnitude of the bias does, in fact, depend on the relative size of the second serial correlation coefficient. If x is itself first-order Markov, the bias is $-a(l - r)/2$ and vanishes as $r \rightarrow 1$. In the latter case, where $r \rightarrow 1$, the spectrum would have most of its power concentrated at $w = 0$, which is the case examined by Engle and Liu.

To put it differently, the true aggregated equation can be rewritten as

$$y_t + y_{t-1} = (a + b)(x_{-2} + x_{-3}) + a(x_{-1} - x_{-3}) + u + u_{-1}$$

If x is, in fact, a deterministic trend, as assumed in parts of Section II, then $x_{-1} - x_{-3}$ is a constant, and its omission causes no bias to our estimates.

That is, by assuming that x's are trendlike, we dispose of much of the aggregation problem, since almost no information about the x's is lost either by aggregating them, or even by getting them out of the correct phase. Aggregation of the x's is really only a problem when the x's are *not* smooth, and when there are real fluctuations (or cycles) *within* the aggregation span. If the x's are trendlike, there is little point in knowing $W(L)$ (in the model $y = W(L)x$) and even less point for the economic actors to behave adaptively, to form expectations about the future of x according to $W(L)$. When the x's are trends, distributed lags make little sense as models of economic behavior.

(2) In dynamic models of the form $y = ay_{-1} + e$, even if e is a

well-behaved independent random process, aggregation over time will induce serial correlation in the aggregated disturbance where there was none to start with. This will result in an upward bias in the estimated coefficient of the lagged dependent variable, and hence, also in an upward bias in the estimated average lag. For the simple model

$$y = ay_{-1} + e \qquad Eee_{-i} = 0 \text{ for } i \neq 0$$

aggregating it to

$$y + y_{-1} = A(y_{-1} + y_{-2}) + e + e_{-1} = (1 + a)y_{-1} + e$$

leads to an estimated least squares coefficient

$$\text{plim } \hat{A} = \frac{1 + a}{2} = a + \frac{1 - a}{2} > a$$

and an estimated average lag (θ)

$$\text{plim } \hat{\theta} = \frac{2 \text{ plim } A}{1 - \text{plim } A} = \frac{2(1 + a)}{1 - a} > \frac{a}{1 - a}$$

If, in addition, the e's were also positively serially correlated, this would only strengthen whatever biases there would have been had simple least squares been used to estimate the disaggregated equation.

(3) What is considered here, however, is not fitting $y + y_{-1}$ on $y_{-1} + y_{-2}$, but on $y_{-2} + y_{-3}$. This, in general (for uncorrelated or first-order Markov e's with positive ρ's) will *reduce* the serial correlation bias in the estimated coefficient of the lagged y's which had been induced by the aggregation procedure; i.e., if in the model above

$$y + y_{-1} = A'(y_{-2} + y_{-3}) + v$$

is estimated, then

$$\text{plim } A' = \frac{a(1 + a)}{2} = a - \frac{a(1 - a)}{2} < a$$

$$\text{plim } \hat{\theta} = \frac{a(1 + a)}{1 - \frac{a(1 + a)}{2}} < \frac{2(1 + a)}{1 - a}$$

That is, in this simple model, lagging the y's more than is implied by the correct model, leads to a downward bias in the estimated coefficient, and a smaller bias in the estimated mean lag (with not very high a's).

To summarize, aggregating and then choosing an analogous, but not the correct, aggregate variable will usually tend to attenuate the estimated coefficient of that variable. In a dynamic context (i.e., using lagged y's as "independent" variables), time-aggregation per se will introduce or increase serial correlation bias and hence lead to an overestimate of the average lag. Using a lagged "improper" analogous y_{-h} variable, will however, undo some or all of this damage. The net effect of all this cannot be derived without a more intensive investigation of the time structure of the x's.

Difficulties arise, I think, when we try to interpret the empirical results as if the monthly equations were the correct ones. This is not to disparage Liu's pioneering effort at building a monthly model, but only to remind you that it was presented as a "test of feasibility" rather than as a "well shaken down" final version. In some cases, it is not clear that the monthly results give very good estimates of the dynamic characteristics of the model, or could conceivably do so. For example, in the consumer durables equation, it makes little sense to have a constant monthly rate of replacement on the basis of an interpolated consumer durables stock series. First, the assumption of a constant rate of replacement is only reasonable in some "averaged" sense, and a month may be too short for that. Moreover, a relevant series does not really exist monthly. It is not surprising, therefore, that the coefficient of the stock variable becomes larger (in absolute value) as the level of aggregation increases. Similarly, in the consumer services equation, at the monthly level, the estimated mean lag is not "significantly" different from infinity. Also, the series used to interpolate monthly consumer services expenditures are wages and salaries in the service industries. But while this may be the best we can do, it prevents us from picking up the most important dynamic elements at the monthly level (if there are such): the discrepancies in the timing of changes in demand and supply.

To echo a point initially conceded by Liu, the original monthly model is "more a model constructed on monthly data, than a monthly model as such, in the sense that the model would probably predict the long-run levels better than the month-to-month changes."[1] It is not

[1] Liu (1969), p. 12. Taking care of serial correlation does not really dispose of this criticism. The problem is in the economic content of the behavior relations rather than in the properties of the disturbances.

surprising, therefore, that in many cases the quarterly or annual versions "look better" than the original monthly estimates. Thus, it is probably not a very good basis for the type of analysis attempted by Engle and Liu.

The basic difficulty here is that most of our economic theory, including even Koyck-Nerlove lags, is directed toward the elucidation of relatively longer-run phenomena. Our theory does not pretend to explain the daily pattern of certain purchases, nor does the Koyck-Cagan-Nerlove theory of distributed lags make much sense for very short time periods.[2] In a sense, our theories fit best some intermediate levels of aggregation, and applying them to monthly data may involve us in significant mis-specification.

Theil's framework for analyzing the aggregation problem was extremely valuable in clarifying the issues in this field, but to make further progress we shall probably have to break out of it. Beyond some point, it is no longer useful to assume that "truth" exists at some level, and that an *analogous* system may be fitted at another level, followed by an inquiry into the connection between the fitted values of the analogous system and the underlying "truth." A seminal idea contained in Houthakker's (1955) paper, and still largely unexploited, suggests that there are different "truths" at different levels of aggregation, and that they are connected by *both* the aggregation rules and the properties of the distribution of the microvariables. I think that when we come to know more, we shall find that good monthly and annual models do not really look alike, and that there is rhyme and reason for this difference.

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[2] I have not seen, for example, a successful integration of seasonality with partial-adjustment models. Does one partially adjust from the actual or the seasonality-adjusted levels, and if the latter, why?