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Volume Author/Editor: Harold Barger

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Chapter Author: Harold Barger

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Chapter III

THE LEVEL OF THE NATIONAL PRODUCT

ESTIMATES such as those presented in this volume, which not only purport to be global in character, but also extend over a considerable period of time, may be viewed from two different standpoints. For they provide information, first, about the *absolute level* of the national product in money terms (as we have chosen to define it); and, second, about its *variability through time.* In this chapter attention will be confined to the annual estimates of outlay and income, viewed as measures of the absolute level of the product; outlay and income, regarded as measures of its variability, will be considered in Chapter VI, where it will be possible to present quarterly as well as annual series.

§1. *Comparison of Outlay and Income*

The outlay totals in Table 3 and the income totals in Table 5 have been rendered comparable, so far as it is possible to do so, and are now brought together in the first two columns of Table 6. It is likely both that flaws can be found in the interpretation of the conceptual framework described in the preceding chapter, and that the estimates themselves can be further improved. I shall be content, however, if I can persuade the reader that the two estimates really are intended to measure the same quantity, and that there is no room, in terms of the framework mentioned, for any *conceptual* difference be-

TABLE 6

COMPARISON OF OUTLAY AND INCOME

Year	*National Product as Measured by*		*Outlay Minus Income* (*d*)	*Outlay as Percent of Income*[a]
	Outlay (from Table 3)	Income (from Table 5)		
	millions of current dollars			
1921	59,869	56,978	2,891	105.1
1922	60,221	58,714	1,507	102.6
1923	71,236	68,974	2,262	103.3
1924	70,181	69,289	892	101.3
1925	77,045	73,324	3,721	105.1
1926	79,921	78,244	1,677	102.1
1927	78,968	76,589	2,379	103.1
1928	79,422	78,496	926	101.2
1929	85,026	83,554	1,472	101.8
1930	73,836	73,900	−64	99.9
1931	59,638	59,047	591	101.0
1932	45,200	43,205	1,995	104.6
1933	43,402	41,630	1,772	104.3
1934	51,739	49,264	2,475	105.0
1935	58,855	54,991	3,864	107.0
1936	68,962	64,930	4,032	106.2
1937	72,005	66,917	5,088	107.6
1938	62,772	62,811	−39	99.9
		Mean:	2,080	103.4

[a] Theoretically the percentages in this column should equal 100 in every case. The fact that they do not is an indication that the outlay or income estimates, or both, are subject to error.

tween them. This is why the discrepancies do *not* provide us with a measure of bad debts, unclaimed dividends, kickbacks, checks which get lost in the mail, or any other such residuals. These, and all similar items, are supposed to have been accounted for by appropriate treatment in the derivation of the estimates themselves. The differences between the outlay and income totals, revealed in Table 6, are due solely to defects in the data, and must be regarded—insofar as they are a measure of anything—as a measure of these defects.

The two sets of estimates are substantially independent

of each other in respect of the source material from which each is drawn. It is true that in making estimates for certain services to consumers it was necessary to compromise this independence.[1] Again, the outlay and income totals sometimes draw upon material which, although not the same in the two cases, nevertheless comes ultimately from the same source. A case in point is the Census of Manufactures which furnishes data on wages and salaries and on outlay for commodities as well. But these qualifications are not of fundamental importance. The fact that in their derivation outlay and income are substantially independent enables us to view the discrepancies between them as a measure, however imperfect, of their precision when they are regarded as estimates of the product.

It will be seen that the outlay data have a rather consistent tendency to run ahead of the figures for income, the excess ranging from negligible amounts to around $5 billion, or some 7½ percent of the latter.[2] This result in itself requires some explanation. Of course with estimates of this character a tendency to exaggerate in one year is likely to be associated with a similar tendency in other years; the same is true of a tendency toward understatement. We have only to remind ourselves that certain types of data are available only for one or for a very few years during the period. For example, data on the distribution of sales of manufacturing plants, and on the number of persons engaged in many occupations, are available only for 1929; an error in one direction in that year might easily lead to an error in the same direc-

[1] See Appendix A.

[2] It is interesting to notice that in a similar comparison for the United Kingdom for the year 1932 Colin Clark reports an estimate for outlay which exceeds the corresponding estimate for income by about 3 percent (*National Income and Outlay*, Macmillan, 1937, Ch. VII). Clark is inclined to attribute this discrepancy to tax evasion; but it should be observed that a much larger fraction of his income total is derived from income tax statistics than is the case with the totals presented here, our reliance on this type of data being practically confined to the estimates for residual income.

tion in most or all other years. Again, estimates for the level (as distinct from the movement) of some consumer services depend entirely upon the Census of 1935, while in other cases data on consumer allocation cannot be tied to any particular year, but must at best be labeled "at present" or "ten years ago." Even where annual material is available, as in the case of Bureau of Internal Revenue figures for corporate income,[3] a more or less continuous bias can by no means be excluded.

Consequently it is not altogether surprising that the discrepancy shown in Table 6 is almost uniformly in the same direction.[4] We cannot say unequivocally that in each year, or even in a majority of years, the outlay estimates are too high and the income estimates too low, for both may be underestimates, or both overestimates. Indeed in two years of the period the outlay falls short of the income estimate by a small amount. Only if we have strong reasons for believing that the two sets of estimates, compiled by different methods, are equally reliable, can we say with confidence that the truth lies probably somewhere between them, and that their mean in any year is the best available estimate of the product in that year. It is therefore important to inquire, in the case of each of the calculations, in which direction errors are most likely to occur. Naturally it is impossible to reconcile the two calculations by any detailed comparison among the items of which each is composed, for the two

[3] Corporate income is almost certainly understated for all years in the period owing to the fact that the totals of the *Statistics of Income* are derived from unaudited returns. See §3 below; also Appendix B, §19.

[4] Just what the result would be, if the compiler of every component of each of the two sets of estimates were asked to reconsider his work in detail from the ground up, and carry out such revisions (upward in the one case, downward in the other) as he felt able to do in every doubtful case, it is impossible to say. In fact the whole of the work has been carried out on exactly the opposite plan. Not only were all of the National Bureau estimates of income and capital formation carried out (by Kuznets), but also the estimates for consumers' services and subsidiary adjustments (by myself), before even a preliminary comparison was made between the resulting totals.

breakdowns are evidently entirely different in character. We must endeavor nevertheless to form some opinion, however tentative, of the relative worth of the two series. To do this it will be necessary to consider separately, and in some detail, the more important factors affecting the precision of the totals for outlay and for income.

§2. *Precision of the Outlay Estimates*

In the derivation of outlay probably the largest area of doubt surrounds the segregation of finished from unfinished goods. Consumers are assumed to receive no pig iron, producers to purchase no packaged foods.[5] Between these extremes lies a wide range of commodities and services whose output must be allocated, sometimes more or less arbitrarily, between consumption and business use. The extent to which the outlay totals depend upon estimates resulting from the allocation of "mixed" commodities and services may be seen from Table 7, which relates to the year 1929. In that year about one quarter of consumers' outlay was devoted to such commodities and services: an error (for example) of 20 percent in estimating this part of the total would lead to an error of some 5 percent in our estimate for consumption, and to a slightly smaller error in our figure for outlay as a whole. Clearly, only an error of this order would account for the whole of the discrepancy between outlay and income in Table 6.

The uncertainty concerning the allocation of the gross output of services between final consumers on the one

[5] It is difficult to think of any single commodity or service whose entire output is absorbed by final consumers, but in many cases the fraction of the supply going to business users is so small that it can be neglected. Small amounts of some commodities which are treated as entirely unfinished may also reach consumers. Cf. Kuznets, *Commodity Flow and Capital Formation*, Vol. I (National Bureau of Economic Research, 1938), p. 14.

TABLE 7

FRACTION OF OUTLAY ESTIMATED BY ALLOCATION BETWEEN CONSUMER AND BUSINESS USE, 1929

	Total (Tables 1 and 3)	*Amounts Resulting from Allocation*	*Percentage Subject to Allocation*
	billions of current dollars		
Consumers' services[a]	27.2	6.1	22.3
Commodities:[b]			
Consumers' perishable	28.6	10.0	35.2
Consumers' semidurable	12.4	2.3	18.9
Consumers' durable	9.9	1.1	11.1
Total consumers' outlay	78.0	19.6	25.1
Producers' durable commodities[b]	6.5	.1	1.0

[a] For consumers' services, the items which result from allocation between consumers and business users are: laundering; moving, repairs, and storage; telephones; the entire "transportation and recreation" group, except recreation and amusement; postage; and legal expenses. Data from Appendix A, Table 22.

[b] For commodities, the percentages are taken from Kuznets' *Commodity Flow and Capital Formation*, Vol. I, Table I-a, pp. 20–21. The absolute figures for amounts resulting from allocation have been obtained by application of these percentages to the data in Tables 1 and 3. This procedure is not exact, since the percentages apply only to manufactured commodities, and refer to manufacturers' prices, but the error involved cannot be large.

hand and business users on the other is perhaps particularly acute. For instance, it is impossible to say exactly how much revenue from passenger transportation is consumers' outlay and how much a business expense. A similar uncertainty surrounds many other elements of the estimates for consumer services shown in Table 1 and derived in Appendix A. It may well be that some of the allocations chosen tend to exaggerate consumers' use of the facilities concerned.[6] As will be seen from Table 7,

[6] The allocations mentioned are given in detail in the notes to Table 22 (Appendix A). They follow closely those used by Lough. That Lough obtains better agreement between the outlay and income figures he presents in *High-Level Consumption* (McGraw-Hill, 1935), Table 4, does not prove that his (and therefore our) allocations are appropriate, and that the solution of the problem must consequently be sought elsewhere. For, as is noted in the discussion in Appendix F, the closer agreement obtained by Lough in the comparison he undertook is largely illusory.

about $6 billion, or slightly more than one fifth, of the total outlay for services shown in Table 1 for 1929 is subject to this uncertainty to a greater or less degree. If the allocations we have used contain an element of exaggeration as large as one third of their true level the entire discrepancy (in 1929, for example, $1.5 billion) would be explained. That would be an extreme assumption. It is quite possible, however, that the estimates for consumers' services run as much as, say, half a billion dollars too high on this account.

The same difficulty is encountered in the case of the commodity estimates. Many goods (for instance coal, stationery) are used partly by final consumers and partly by business enterprises, for further fabrication or otherwise. As regards their economic characteristics, such commodities are partly finished and partly unfinished. It will be seen from Table 7 that this difficulty is most serious in the estimates for the perishable group. William H. Shaw, who was responsible for much of the detailed work on the National Bureau's commodity estimates, and is at present engaged in compiling fresh estimates of the same kind for the Department of Commerce, tells me he now thinks that the allocations upon which the National Bureau estimates were based may have led to the inclusion of some unfinished goods in the totals, to some extent perhaps through incomplete elimination of the duplication caused by interplant transfers.[7] Again Kuznets has himself suggested that the distributive margins used in the commodity estimates may have led to an overstatement in the totals of as much as $2 billion (1929), although probably considerably less.[8] It may be noted in passing that Lough's estimate for consumers' outlay on commodities, for the years for which it is avail-

[7] Cf. *Commodity Flow and Capital Formation*, Vol. I, pp. 17–19.
[8] *Ibid.*, Part III, especially p. 176.

able,[9] runs from $1 to $2 billion below the National Bureau commodity estimates used in this volume. Apart from minor differences of definition, different allowances for transportation costs and distributive mark-ups also may have given rise to variations in the results of the two studies.

There are evidently some grounds for supposing that the National Bureau estimates for consumable commodities are too high. It is possible, however, to think of reasons why they may be too low. Thus all food is valued in ordinary (food store) retail prices, no allowance being made for the higher value of food sold in hotels and restaurants. A rough estimate of my own indicates that the understatement in the commodity totals on this account may well have been about half a billion dollars in 1929.[10] Again, the absence from the estimates of any explicit allowance for manufacturers' selling costs, and the omission of a few commodities for which data were unobtainable (e.g. cut flowers), point to a conclusion opposite to that suggested in the preceding paragraph. Having made no commodity estimates myself, and being acquainted with the problem only at second-hand, I can offer no opinion as to whether or not there may be, on balance, some overstatement in the commodity totals in *Commodity Flow and Capital Formation*. Should there be

[9] Through 1931. See W. H. Lough, *op. cit.*, p. 28.

[10] The figure used by Kuznets for the ratio of expenses to sales for retail food stores is 19.5 percent (*Fifteenth Census, 1930, Distribution*, Vol. I, p. 51). This leads to a retail margin for the food group of 20.7 percent (*Commodity Flow and Capital Formation*, Vol. I, p. 209) and a retail markup of 26.1 percent (*ibid.*, p. 204). The ratio of expenses to sales for restaurants and eating places (not used by Kuznets) is given as 40.0 percent (*Distribution*, Vol. I, p. 52): this suggests a mark-up of at least 67 percent for food sold in restaurants. Net sales of restaurants and eating places for 1929 are reported at $2,125 million (*ibid.*, p. 48); the cost of this food (before mark-up) must have been about one and a quarter billion dollars, and the understatement in our totals consequently about half a billion dollars (67 percent − 20.7 percent = 46 percent; 46 percent of $1,250 million = $575 million).

such an overstatement, it would help to explain the discrepancies revealed in Table 6.

So much for consumption. Other elements of outlay may likewise be subject to error and to possible overstatement. For producers' durable goods the fraction affected by difficulties of allocation is unimportant (see Table 7). The construction estimates, too, are free from this difficulty. All new construction, whether for residential or for business purposes, comprises part of outlay, so that incomplete segregation is a matter of secondary interest. On the other hand, the line of demarcation between expenditures for new construction and equipment and expenditures for repair work is of great importance and unknown reliability.

This consideration warrants an inquiry into the appropriateness of our measures of depreciation. In particular, the comparability of the figures for the output of producers' durable goods and for construction, and the related figures for depreciation, may be called in question. These various sets of data, it will be remembered, come from widely differing sources. Producers' goods are covered by the Census of Manufactures, and their reliability is increased by the fact that almost no distributive mark-ups are necessary in this field. The construction estimates (worked out by Chawner and Dennis of the Department of Commerce) are based mainly on F. W. Dodge contracts data. The depreciation estimates (taken from Fabricant's study of *Capital Consumption and Adjustment*) come principally from the *Statistics of Income* and from published corporate accounts.[11] The difference between the first two items and the third is the most important constituent of

[11] These estimates make no allowance for write-downs due to obsolescence, or for depreciation reported to the Bureau of Internal Revenue under "cost of goods sold." See above p 49.

private net investment shown in Table 3. If producers' goods or new construction are charged to current expenses, we might say either that gross investment has been exaggerated, or that depreciation has been underestimated: in either case the outlay totals will be too high. The servicing of capital goods, included by Kuznets with his other estimates,[12] has of course been deliberately excluded from our estimates of outlay. But the real test of the exclusion—the manner in which the cost of the operation is charged—may not have been adequately applied. Again, I can offer no opinion on this question.

While there seems to be no special reason to suppose that the estimates for producers' durable commodities and for construction are too high, it is possible that they may err in the opposite direction. Thus no allowance is made in *Commodity Flow and Capital Formation* for installation charges on machinery, or for the value of machinery built and used by the same concern. Such defects would lead toward too low a value for gross, and presumably also for net, investment.

Neither the inventory change, nor the foreign balance, although each is a constituent of net investment (Table 3), has sufficient influence upon the general level of outlay to show in which direction the totals are most likely to be in error.

The general impression left by the foregoing discussion is that the totals, at any rate for gross investment, are perhaps more likely to be too low than too high. Figures compiled by R. W. Goldsmith for the net volume of individual and business savings for 1933–38 run substantially below our own data for net investment.[13] Even if the net investment figures are near the truth, compensating

[12] See *Commodity Flow and Capital Formation*, Table V-10.

[13] See Table 47, Appendix F, below.

errors may of course remain in the estimates for gross investment and for depreciation.

Unfortunately the commodity totals we use, both for consumers' and for producers' goods, include a certain amount of finished commodities purchased by governmental units. Consequently the data in Table 3 do not correspond perfectly to our notion of outlay by *private* consumers and investment by *private* enterprises. There is no way of telling just how important this defect is, but it must lead to some duplication in the outlay totals.[14] The construction estimates, on the other hand, cover private construction only, although the exclusion of public construction may not have been complete during the earlier years of the period.

Finally, the estimates for public outlay in Table 3 probably involve some overstatement. Expenditure by government for the purchase of existing assets has no counterpart in the income stream, and it is therefore proper to exclude such expenditure in computing public outlay, as we have defined that term. The purchase by government of its own obligations for redemption has of course been excluded in the derivation of the figures shown in Table 3. But the government also purchases land, buildings, and perhaps second-hand equipment. Nevertheless there is no way of measuring such expenditures, and no deduction has been made for them in the computation of public outlay.

[14] The importance of the matter may be roughly gauged from data to be found in *Government Purchasing*, Temporary National Economic Committee Monograph 19 (Washington, 1940), by C. C. Linnenberg Jr. and D. M. Barbour. According to a tabulation reproduced by these authors (p. 3), the Federal government spent some $913 million on commodities of all kinds from December 1937 through November 1938, of which perhaps as much as one third would have been classified by Kuznets as finished. A minimum estimate of $1,064 million is given (p. 20) for aggregate State and local commodity purchases during fiscal years ending in 1938, but much of this amount must of course have been devoted to construction materials and other unfinished commodities.

§3. *Precision of the Income Estimates*

Like the outlay estimates, the income estimates in Table 6 may be either too high or too low; but the comparison undertaken in that table suggests that the latter is more probable than the former. Except for minor adjustments made in Table 5, the income totals used here are taken directly from Kuznets' *National Income and its Composition*. Since their precision and coverage are discussed at considerable length by Kuznets in that work,[15] only a few of the more important points will be selected for consideration here.

The data for residual income are derived mainly from the *Statistics of Income*, i.e. from reports for tax purposes.[16] Tax evasion by corporations would, of course, lead to an understatement of residual income; about this we can necessarily know almost nothing. Apart from the question of successful evasion, we may note that the data come from unaudited reports. Figures published in the *Annual Reports of the Commissioner of Internal Revenue* suggest that substantial amounts of additional taxes are assessed as a result of audit and litigation, but no breakdown between corporations and individuals has so far been published.[17] Moreover an allocation of additional assessments among the tax years to which they apply has not hitherto been available. A study now in progress at the Bureau of Internal Revenue, under the direction of Thomas C. Atkeson, is designed to fill this gap. The results of some tentative calculations I have made on the basis of unpublished figures supplied to me

[15] National Bureau of Economic Research, 1941; see especially Chs. 9, 11 and 12.

[16] However, residual income in the Public Utilities, Steam Railroads and Communication groups is derived from other sources, and so is not subject to the understatement discussed below.

[17] Underreporting of individual incomes by the *Statistics of Income* does not concern us, for in the National Bureau income estimates salaries and wages are derived from other sources.

by Mr. Atkeson are included in Appendix B (Table 30). These suggest that corporate incomes, business savings, and the national income totals involve an understatement ranging from as much as $1,000 million in 1929 to perhaps $400 million in 1932. About the character of this understatement we know little, except that sizable amounts of depreciation are disallowed during the auditing process.[18] An overstatement of depreciation by estimates which rely upon the *Statistics of Income* does not in itself contribute any explanation of the shortfall of income disclosed in Table 6, for such an overstatement would merely involve an equal understatement of outlay. Insofar as the auditing process reduces depreciation allowances, both sets of estimates—for outlay and for income alike—will tend to understate the national product. Our estimates for net private investment (Table 3) also will be too small. It seems likely, however, that considerably less than half the additional income mentioned can result from the disallowance of depreciation, and that the major part of it must be due to other causes, reflecting upon the accuracy of the income but not of the outlay totals. As explained in Appendix B, §19, the data in Table 30 have not actually been used to make corrections to any of our figures because of the difficulties which stand in the way of such an adjustment. We may conclude that our use of the unaudited data in the *Statistics of Income* leads to some understatement of outlay, and a considerably larger understatement of income.

This conclusion of course rests upon the assumption that the amounts of depreciation actually allowed by the Bureau of Internal Revenue, as distinct from the amounts claimed by taxpaying corporations, do in fact

[18] See *Annual Reports of the Commissioner of Internal Revenue.* The amounts of depreciation disallowed during the fiscal years 1935–36 through 1939–40 were $222, $304, $224, $175 and $103 million respectively. There is no means of distributing these amounts among the tax years to which they apply.

correspond to the amounts needed for the maintenance of capital. Whether or not there is actually such a correspondence, year by year over the principal segments of the economic system, can probably be determined only through a lengthy technological investigation. If the amounts allowed by the Bureau are too small, both our estimates of the national product would tend to be excessive on this account, and vice versa.

So much for the reliability of the estimates of corporate income. In the field of salaries and wages, much of the data comes from Census sources, either directly or via the numbers engaged in different occupations. The only question I would raise here is whether salaries, particularly in the higher brackets, are reported adequately. Admittedly, there does not exist between the Bureau of the Census and the Bureau of Internal Revenue a tie-up of the sort that might lead to deliberate understatement of salaries paid; it is equally true, however, that there is no auditing, as distinct from editing, of Census data. But here again I am not competent to express a definite opinion.

§4. *Precision of Estimates of the National Product*[19]

Unfortunately a review, such as that just undertaken, of the more obvious sources of weakness in the two sets of estimates—for outlay and for income respectively—yields no quantitative information concerning the precision of either set; nor does it indicate which of the calculations is to be preferred. An impression emerges that holes can be picked more easily in the outlay than in the income estimates. This of course is no more than an

[19] In writing this section I have derived material assistance from discussions with Rollin F. Bennett and W. Allen Wallis. Neither is responsible, of course, for the interpretation of the results I have chosen to make.

impression, and one that is quite possibly merely a reflection of the fact that the writer is more familiar with the details of construction of the former than of the latter. In any case we have at present no means of putting such an opinion to the test.

In order to judge the relative reliability of the two series, regarded as estimates of the national product, we need quantitative measures of the errors of estimation associated with the various components of each. Greatly daring, Kuznets has expressed his opinions concerning the accuracy of the income estimates in numerical form. Thus he places the margin of error of the National Bureau income totals at "not much above 10 percent, and perhaps somewhat less."[20] Being merely the expression of an informed opinion, the judgment cited naturally cannot be held to set definite fiducial limits. Yet it does at least convey the notion of an order of precision, with which other such statements may be compared. On the other hand, Kuznets has offered no similar estimate of the accuracy of the commodity totals. Unfortunately the present writer feels no more able to do so in respect of consumers' services, the data for which are derived in Appendix A. Consequently no quantitative comparison is possible between the precision of the outlay and the precision of the income estimates presented in this study. Nevertheless it may be claimed for the latter that unlike the former they are the responsibility of a single individual, who is willing at least to make a numerical guess at their accuracy.

While it appears impossible to establish that one or the other set of estimates is to be preferred, the data in Table 6 can be made to yield some information about the precision of outlay and income taken together. We may notice, for example, that the discrepancies between outlay

[20] *National Income and its Composition*, p. 528.

and income, reported in Table 6, appear to be of the same order of magnitude as the percentage error quoted by Kuznets for the income estimates. This conformity suggests that the measure of agreement obtained is perhaps about as good as one can hope for in compiling estimates of the national product by different methods. It seems worth while, however, to examine the fiducial limits to which the discrepancies lead, once appropriate assumptions are made concerning the character of the errors involved.

The two estimates of the national product furnished by outlay and income respectively may, for any given year, be considered a sample of two observations whose mean constitutes an estimate of the value of the product in that year. The determination of this mean in any year absorbs one degree of freedom; we may use the remaining degree to estimate its precision. Thus if d is the difference between outlay and income, as shown in Table 6, the variance of any annual sample is

$$s^2 = 2\left(\frac{d}{2}\right)^2 = \frac{d^2}{2}.$$

We may obtain an estimate of the variance of the mean of the sample by dividing the variance of the sample by the number of observations it contains, in this case two. For the variance of the mean, therefore, we have

$$s^2_{mean} = \frac{d^2}{4};$$

and for its standard error

$$s_{mean} = \tfrac{1}{2}\sqrt{d^2}$$

These formulae may be explained briefly. We may suppose an infinite population of possible estimates of the national product, say for the year 1926, obtained by different methods or by different investigators. If the

errors are distributed in a random manner, the mean of this population will coincide with the true value of the product. If an infinite number of samples of two estimates each is drawn from such a population, and the mean of each sample computed, a new (infinite) population of sample means will be formed. The mean of this new population also will coincide with the true value of the product. Consequently the standard error of the population of sample means provides us with a measure of the accuracy of any one such mean, regarded as an estimate of the product in the year to which it applies. We may estimate this standard error from the sample in the fashion just indicated. We can then employ the remaining degree of freedom to establish fiducial limits in the usual manner.

The standard error of the sample mean can be computed for each year taken separately. Or we may summarize the results by averaging the eighteen values of d^2, taking the square root, and dividing by two. This procedure is based upon the assumption that the errors in different years are not really independent. Using the data in Table 6, we obtain as an estimate of the standard error of our measure of the national product in any year

$$s_{product} = \$1{,}251 \text{ million.}$$

The same result can be reached by a slightly different route which yields somewhat more information about the characteristics of the errors with which we are dealing. We may attribute the variation of the 36 observations shown in Table 6 about their mean ($65,532 million) to three sources: (1) variation associated with the point in time to which the observation refers, and reflecting changes in the level of the product from one year to another; (2) variation associated with the choice of method followed in obtaining the observation, i.e. measurement of income on the one hand, or of outlay on the

other, as the case may be; and (3) residual errors of estimate which cannot be attributed separately to the choice of a particular year or to the selection of a particular method of measurement. It so happens that in the present context we are not interested in variation associated with the movement of the product from one year to another. If we assume that variation due to choice of method and to residual errors is independent of the passage of time, we can eliminate variation due to the movement of the product by measuring the deviations

TABLE 8

ANALYSIS OF VARIANCE OF DATA IN TABLE 6[a]

Source of Variation	*Degrees of Freedom*	*Sum of Squares*	*Variance*
Between methods	1	$36\left(\bar{\frac{d}{2}}\right)^2 = 38,939,688$	$v_m = 38,939,688$
Within methods: Residual errors	17	$\Sigma\left(\frac{d}{2} - \bar{\frac{d}{2}}\right)^2 = 17,340,116$	$v_r = 1,020,007$
Total	18	$\Sigma\left(\frac{d}{2}\right)^2 = 56,279,804$	$v = 3,126,656$

[a] After eliminating variation between years. The sign Σ denotes summation over 36 values, 2 for each year.

in each year from their annual means. In this way variation due to errors of measurement may be broken down as shown in Table 8.[21] The importance of difference in method as a source of discrepancy is shown by the high value obtained for the variance between methods. The significance of the difference between this variance (v_m) and the variance associated with residual errors (v_r) can be tested by dividing the first by the second and taking

[21] For a more detailed discussion and illustration of the particular application of the analysis of variance used here, see, e.g., L. H. C. Tippett, *The Methods of Statistics* (Williams and Norgate, London, 1937), especially sections 6.6 and 10.3.

the square root. This yields a value of $t = 6.2$ based on 17 degrees of freedom, which is of course highly significant in a statistical sense.[22] This result merely confirms what we may already guess from an inspection of Table 6, i.e. that there is a strong tendency for estimates of the product derived by the outlay method to come out higher than estimates obtained from income statistics. The effect of this tendency may be measured also. We know that the average difference reported by outlay and by income, i.e. the difference between the method means, is

$$\bar{d} = \$2{,}080 \text{ million.}$$

We may obtain the standard error of $\bar{d}/2$ by dividing the residual variance (v_r) shown above by 36 and taking the square root. By doubling the result we have an estimate for the standard error of the mean difference ($\bar{d}$) shown in Table 6,

$$s_{\bar{d}} = \$336 \text{ million.}$$

The above material may also be made to yield estimates of the population values of the variances associated with choice of method (σ_m^2), and residual errors (σ_r^2) respectively.[23]

$$\sigma_m^2 = \frac{v_m - v_r}{18} = 2{,}156{,}649$$

$$\sigma_r^2 = v_r = 1{,}020{,}007.$$

We may therefore say that differences resulting from the choice of method (measurement of income versus measurement of outlay) are about twice as important, as a source

[22] However, to the extent that we have used interpolations, e.g. for intercensal years, in estimating outlay and income, errors within each method are correlated from one year to another. It might be appropriate therefore to use fewer than 17 degrees of freedom in testing the significance of the difference between v_m and v_r.

[23] In estimating σ_m^2 we have to reduce v_m by v_r, because our estimates of the method means are subject to error. Cf. Tippett, *op. cit.* section 6.2.

of variation, as are random errors of measurement not apparently associated with one method rather than with the other. This implies that our annual means afford, in a sense, a better estimate of the movement of the product from one year to another than they do of its absolute level. Since we are less interested in the absolute size of the product in a particular year than in its movement over shorter or longer periods of time, a test of the precision of our estimates of level is perhaps more stringent than most purposes would require.

Nevertheless it is worth while also to pool the variances between and within methods, and to compute fiducial limits for our estimate of the product in any given year. To obtain an estimate of the variance of the annual means, we have to divide the variance within years (v) by two, the number of observations in each year. By taking the square root of the result we have for the standard error of any annual estimate of the product, as before,

$$s_{product} = \$1{,}251 \text{ million},$$

which equals 1.91 percent of $65,532 million, the grand mean of the 36 observations in Table 6. For a sample of two observations, possessing one degree of freedom, the expectation that our estimate in any year lies outside these limits, i.e. 1.91 percent on either side of the true value, is about one half. If we wish to reduce this probability to one tenth, we must expand these limits of error to about ± 12 percent; and to reach the .05 level of significance, we have to raise the limits to ± 24 percent.[24]

This result, which of course is based on the assumption that outlay and income are equally good (or bad) estimates of the product, perhaps suggests somewhat wider

[24] These results are obtained by the use of the distribution of t for one degree of freedom. Cf. R. A. Fisher, *Statistical Methods for Research Workers* (7th ed., Oliver and Boyd, Edinburgh, 1938), Ch. V.

limits of error than those given by Kuznets and quoted above for the income totals. Such a result would be in agreement with the impression that income is a somewhat more accurate measure than outlay. For if the precision of income is greater than the average for both measures taken together, the precision of outlay must obviously be less. However, we are now well within the realm of speculation. To interpret Kuznets' estimate of the error of the income totals as setting definite fiducial limits is to place upon it a strain which it was not intended to bear. Again, our own estimate for the standard error of the product proceeds on the assumption that outlay and income have been determined quite independently—an assumption not fully justified. Obviously, therefore, it cannot be claimed for these results that they do more than provide estimates of precision which are themselves highly tentative. I have chosen to present them because they seem to me suggestive, not because they are in any sense conclusive.

Thus far the discussion of the precision of the estimates has been confined entirely to the assumption that the national product which we wish to measure is that so elaborately defined and circumscribed in Chapter II. Lest it be thought that the limits of error quoted above can be applied without reservation to the measurement of the product in any more comprehensive sense, we should perhaps do well to recall the main limitations imposed in Chapter II. It will be remembered that the definition of the product we have chosen credits the government with no services to the final consumer, and substitutes for public investment a balancing item comprising merely the cash deficit of all governmental agencies. But this does not exhaust the list of possible revisions which might be made—mainly in an upward direction—if a different conceptual framework were

chosen. Kuznets presents a catalog of omitted items whose value in 1929 may have run as high as $33 billion, including $23 billion for housewives' services, more than $3 billion each for miscellaneous services (pensions, boarding and odd jobs) and imputed services from durable goods other than houses, and $4 billion for bad debts and hidden payments of various kinds.[25] The list could be expanded still further through the inclusion of the services supplied by other family members, and of nondurable goods.

But while we should not overlook the rather special character of the definition we have chosen for the national product, its extension is of secondary interest, if only because of the immediate inflation which the margin of error undergoes as soon as broader concepts are considered.

As regards measurement of the dollar volume of the national product, as defined here, estimates from the income side appear to offer a more solid basis than those from the side of outlay. As long as no reconciliation is possible, and especially in view of the manifest difficulties in allocating output between consumers and business users, one can do no more than conclude that the measures of outlay are more likely to contain an overstatement than the income measures an understatement.

[25] *National Income and its Composition*, Ch. 9.