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CHAPTER II

THE STATISTICAL MEASUREMENT

1. The Time Unit

Statistical measures of seasonal variations are obtained from time series, that is, from quantitative records arranged in chronological sequence. Time series, suitable for this purpose, may record the given aspect of economic activity by days, weeks, months or quarters. Since the choice of the statistical technique to be used depends upon the time unit adopted, the first step in our statistical analysis was to decide upon a standard time unit.

The decision was largely predetermined by the fact that most records available in units of less than a year are monthly series. The number of weekly measures of industry and trade is negligible, while daily time series are almost entirely lacking. The statistical technique is, therefore, discussed only with respect to monthly data. And the analysis in chapters to follow confines itself, with the exception of a few quarterly records brought in to fill gaps, to monthly series.[1]

2. The Statistical Definition of Seasonal Variations

In Chapter I seasonal variations have been defined as changes in rate of activity attributable to the influence of climatic and conventional seasons. Although these seasons and their influences recur from year to year, the time of their appearance is not precise in terms of months. Though in every year in the temperate zones higher temperatures prevail in summer than in winter the relative differences in temperature between months do not remain the same year in, year out. Similarly, and partly as a consequence, the bulk of the wheat crop may be harvested in July this year and in August next.

[1] For the problems raised in measuring seasonal variations in weekly data see *The Review of Economic Statistics*, January 1927, Weekly Fluctuations in Outside Bank Debits, by W. L. Crum, pp. 30-36, and February 1931, Seasonal Variations in Selected Series of Weekly Data, by B. Fox, pp. 26-33.

A like shifting is characteristic of the conventional seasons. In some years Easter occurs in March, in others in April, and the number of Sundays and holidays in any month of one year is not necessarily equal to that in the corresponding month of another year.

Thus, strictly defined, monthly seasonal variations do not possess statistical characteristics precise enough to make the establishment of a typical seasonal change easy. The strict definition, if followed, would lead to the investigation of the records of variable precipitation, temperature and other climatic elements, the dates of planting and maturity of crops. These data would be combined with indexes measuring variations in the conventional seasons and then correlated with the series that describe economic activity. The correlation of measures of both natural and conventional seasons with those of business activity might establish the amount of variation in economic activity associated with variations in all seasonal factors; and from these segregated variations typical seasonal changes could be derived.

The difficulties of such a procedure are only too obvious. First, continuous data on variations in seasonal factors are seldom available for the entire area involved in the correlation. Second, the attempt to correlate economic series with a series describing variations in seasonal influences is beset with the usual grave difficulties of correlation analysis in time sequences, especially since in the projected analysis the dependent variable, an economic time series, is affected by many more factors than are enumerated in the independent variable, the seasonal changes. Both of the series involved are affected by factors other than seasonal and these are often the preponderant influence. Longer cycles, secular movements and random changes must be segregated. The complex correlation analysis would be extremely laborious and the validity of its segregation of changes due specifically to seasonal factors would be very dubious.[2]

[2] The difficulty of correlation analysis may be reduced considerably by eliminating from the series, through a moving average, the secular movements and a large part of the cyclical swing. Chapter X presents an attempt to use the mechanism of correlation for testing the persistence from year to year of the average seasonal variation. But there is no simple way of removing the difficulty arising from the absence of data on variations in seasonal factors for the entire area involved in the correlation.

The usual solution is easier, although much more crude and approximate in result than the analysis just suggested. The most widely used statistical technique for measuring seasonal variations is based upon the assumption that they are recurrent by months within each twelve-month period. This assumption is largely valid. The calendar is constructed in such a way that a year encompasses the full succession of climatic seasons; there is also within the year a rough recurrence of climatic changes by months. Conventional seasons, including holidays, are, for the most part, definitely dependent upon the calendar. On the other hand, although the assumption of recurrence by months is an over-simplification, it makes possible a precise statistical definition upon which to base the technique.[3] Moreover, the concept may be modified somewhat when the problem of changing seasonal swings arises, for recurrence may shift gradually within the period, altering the relative position of the months; or amplitude may change gradually; or the relative position of a pair of months may alternate from year to year. With these qualifications, then, seasonal variations are changes that recur monthly within every year, for a given number of years. Their recurrence and the limited duration of their swing serve to distinguish them clearly from other significant changes in a series. Their oscillating movement differentiates them from the secular trend and their confinement within the limits of the calendar year from cyclical fluctuations. The condition of their recurrence distinguishes them from random movements which do not appear in the same calendar month in successive years.

[3] Some of the current seasonal studies attempt to eliminate the variable influence of conventional seasons by dividing production or volume of output by the number of working days in each month, this number fluctuating from month to month and from year to year. While such an operation improves the seasonal measure obtained, it is beset with difficulties. To establish the number of working days in an industry is often impossible, the number being at best an estimate. In many economic processes it is difficult to assume that volume of activity is directly proportional to the number of working days (for example, bank clearings or retail sales). Besides, the amount of variation eliminated by such an analysis is usually small when compared with the magnitude of the swings in the seasonally affected industries.

3. The Measurement of Average Seasonal Variations

a. *The General Problem*

Since seasonal changes are assumed to be recurrent, the problem of statistical measurement is that of discovering the seasonal variation that characterizes a given time series over a given period of years. Considering for the present the measurement of only the *average* seasonal change, we find that each of the procedures customarily followed may be subdivided into two steps: (1) treatment of the data in order to eliminate completely or partly the non-seasonal elements; (2) striking an average from the residuals of step (1) to obtain the typical seasonal variation.

As has already been pointed out, the non-seasonal elements in a time series consist of secular movements, cyclical fluctuations and random changes. Over a long period the elimination of the cyclical and random elements seems at first to present no difficulties. When successive Januaries, successive Februaries, etc., are each added over a long period, the random changes and the cyclical movements will be cancelled, since random perturbations tend to be distributed symmetrically about the other components of a time series and since pure cycles, by definition, add up to zero within the limits of each complete cycle. Even if the latter condition does not hold strictly for any cycle in the series, it will be more valid when several cycles are taken, that is, when a long period is covered. The only possibility of distortion will then be afforded by any incomplete cycles that the period may contain and by the presence of secular movements. The elimination of the latter from the resulting seasonal averages presents little difficulty.

Hence, it would appear that in a series covering a long period any preliminary treatment of the data might be dispensed with and the stable seasonal measure computed directly from the original series, by averaging successive Januaries, successive Februaries, etc., and then correcting these averages for the secular trend. But the basic difficulty of such a procedure lies in the assumption that seasonal variation remains constant over a long period. Such an assumption may be valid for a short interval, such as six to twelve years, but it is obviously dangerous to assume that seasonal variations persist, without modification, long enough to secure the proper cancellation of cycles and of random changes.

In addition to this theoretical consideration, there is the practical circumstance that many of the monthly series on production and trade have become available only recently. Some go back to 1919, many to 1920 and 1921, and quite a few begin between 1923 and 1925. For such series it would be dangerous to rely upon the length of the period for the complete elimination of the cyclical elements. Thus, both theoretical and practical considerations require a preliminary treatment of data to remove as much of the non-seasonal element as possible. After this is done the establishment of the typical seasonal variations by the process of averaging will obviously yield more reliable results.

b. *Standard Method of the Study*

The method adopted in the present study was that of relative deviations from a properly centered twelve-month moving average. For each series a two-item total of a twelve-month moving total was taken and the final sum divided by twenty-four. This amounts to taking a thirteen-month moving average with the two end months at half weight. The reason for thus modifying the simple, twelve-month moving average is two-fold: (1) the average may be properly centered at the seventh month of the original twelve; (2) the resulting line of moving averages is slightly more elastic than the line of a simple, twelve-month moving average; this tends to reduce the erratic elements in the relative deviations.

These relatives were obtained by dividing the original data by the ordinates of the moving average and multiplying the ratio by 100. Only in the rare instances where the original series contained items of small absolute size were absolute instead of relative deviations taken. (In these series relatives would have given distorted results.) Relative rather than absolute deviations were taken because the underlying moving average represents both the secular movement and at least a large part of the cycle. The secular trend affects the size of the absolute deviations from it and the cyclical movement has, if in less degree, a somewhat similar influence.[4]

The relatives of the original data to the moving average were then plotted separately for successive Januaries, succes-

[4] This choice between absolute and relative deviations is not as simple as it may at first appear. For some of its interesting implications for the constancy of seasonal amplitude see Chapter XI.

sive Februaries, etc. The resulting small charts, one for each month, served as a graphic check upon the final index.

The index itself was computed by the method of positional arithmetic means. For each month the successive relative deviations from the moving average were arrayed and the arithmetic mean of the two or three middle items, four or five middle items, six or seven middle items and so on, was taken. Such a mean removes the influence of extreme values and is not subject to the vagaries of a median chosen from a small number of items.

For each number of middle items the twelve means (one for each of the twelve months) were centered about 100 to allow for a better comparison of the means derived for one and the same month from successive numbers of middle items. This comparison for each month usually revealed some one figure about which the means for successive combinations of middle items tended to cluster. The final seasonal index was then chosen as the series of twelve means for that combination of middle items for which the largest number of means showed values identical or close to the central tendency for each month.

The seasonal index thus obtained was already centered at 100. The value for each month was plotted as a horizontal straight line on the small chart that gives for this particular month in successive years the relatives of the original data to the moving average. This graphical comparison of the seasonal index with the successive relatives tested the validity of the index as a description of the central tendency. Inspection sometimes caused a decision to alter the index by one or two points but the change was never large.

As a final step, the original data were divided by the seasonal index except when absolute deviations from the moving average were taken. In these few instances the seasonal index was subtracted from the original data.

To recapitulate the procedure:

1. For the original data a two-item moving total of a twelve-month moving total is computed. The original data, and this total, divided through by 24, are plotted for comparison on one chart. The original data are divided by the moving average and the ratio multiplied by 100.

2. The relatives for each month for successive years are plotted, yielding a set of twelve charts for the twelve months.

3. These relatives for each month are then arrayed and an arithmetic mean of a successively increasing number of middle items is taken.

4. The means for the twelve months for each combination of middle items are centered at 100.

5. The means for some one combination of central items show in the greatest number of months a value close to the central tendency for each month. This set of means is chosen as the seasonal index.

6. A line is drawn on each chart (under 2) representing the arithmetic mean obtained from the selected number of central items (chosen under 5). Sometimes, as a result of the graphical comparison, these means may be changed slightly.

7. This set of twelve means yields the final, average seasonal index. The original data are divided through by this index and plotted on another chart, analogous to the chart giving the uncorrected data with the moving average. A graphical comparison of these two charts affords an approximate test of the success of the correction by the stable index and reveals roughly the significance of regularly recurring seasonal variations in the series under analysis.

The method of measuring the average seasonal variations described above is substantially similar to that first suggested by Dr. F. R. Macaulay and adopted by the Research Division of the Federal Reserve Board.[5] The minor differences are as follows: (a) instead of the unweighted average, a two-item total of the twelve-month moving average was used in the present study; (b) no allowance was made for the number of working days in the month; (c) the means for each successive number of middle items were centered to afford better comparisons.

For reasons discussed in Chapter X few moving seasonal indexes were computed. Instead, whenever the behavior of the

[5] Aryness Joy and Woodlief Thomas, The Use of Moving Averages in the Measurement of Seasonal Variations, *Journal of the American Statistical Association,* September 1928, pp. 241-52.

series necessitated it, average seasonal indexes were established for rather short periods. Hence, the brevity of the period used in the analysis has often been determined not by lack of data for a longer interval but by the necessity of taking care of changes in the seasonal variations.

In view of such brevity of the periods, it might be suggested that a graduation line more flexible than the thirteen-month average should have been used to eliminate the non-seasonal elements. But this criticism is of limited significance. The moving average of the type used eliminates the major portion of the secular and cyclical movements and the averaging of selected relatives may be trusted to remove the remainder. In any case, the brevity of the series makes impracticable the computation of any more refined graduation lines, for those based on averages involve the loss of a period at both ends, which is the larger the more complex the type of averaging. In the present study the moving average used involves the loss of six months at each end of the series. In some of the very short series it was found advisable to compute for these months relatives to the average for the last twelve-month period or to a moving average extrapolated free hand. Any more complex graduation of the moving average type would involve a still greater loss of the period. And such methods as the Whittaker-Henderson, while giving the graduation for the full period of the series, are too labor-consuming even to be thought of for use in a study of the present scope.[6]

c. *Other Methods Used*

The procedure of measuring average seasonal swings, just described, was applied to most of the series presented in the study. However, a somewhat different procedure was followed for two groups:

(1) A number of series had already been analyzed by the National Bureau in preparation for the second volume of Professor Mitchell's *Business Cycles*. This analysis involves the establishment and elimination of seasonal variations. The method used was that of a positional arithmetic mean of ratios to ordinates, the ordinates being an arithmetic mean established for each complete cyclical swing in general business conditions. It was not considered important to recompute the

[6] For the discussion of these graduation methods, see Frederick R. Macaulay, *The Smoothing of Time Series* (National Bureau of Economic Research 1931).

series since the differences between the results of this procedure and the one used in the study would be, on the whole, rather small.[7]

(2) In several series, especially where the seasonal swing was very pronounced, a short-cut method was used, that of positional arithmetic means of relatives to annual averages. These means, when corrected for secular trend, yielded the average seasonal index. This method was applied usually where data were available for one and the same branch of activity in several regions. The large number of series then to be analyzed made it important to employ a time-saving procedure.

4. Testing the Significance of Seasonal Indexes

In the procedure described above of establishing average seasonal measures it is almost always possible to obtain results deviating from 100. The probability of a perfect cancellation of cyclical and random items is so remote that even a series that by supposition is not subject to seasonal influences is likely to yield a set of indexes differing from 100, indicating thereby the presence of some seasonal variation. To be sure, such variations will be small, but it is important to have some criterion that will indicate in fairly familiar terms the significance of the seasonal indexes obtained.

For this purpose, several tests, of varying rigidity and involving different assumptions concerning the nature of the statistical universe from which seasonal indexes are obtained, may be used. The choice depends largely upon the judgment of the investigator as to which assumptions he accepts as valid or rejects as unjustified by the character of the data. Consequently, it is advisable to present briefly the various tests and then to indicate which have been employed in the present study.

a. *Standard Error of the Arithmetic Mean*

This test may be applied on the basis of the following assumptions:

(1) That the value that forms the seasonal index for each month may be taken as the true arithmetic mean of the scatter for the corresponding month.

[7] For a description of the ratio-to-ordinate method see Helen D. Falkner, The Measurement of Seasonal Variations, *Journal of the American Statistical Association,* June 1929, pp. 167-79. For the dates demarcating cycles in general business conditions in the country see *Recent Economic Changes* (National Bureau of Economic Research 1929) II, 892.

(2) That the scatter from which these means have been derived contains, in addition to the constant seasonal, only random changes. The standard error of the mean for any sample is given by the formula:

$$\sigma_{\text{A. M.}} = \frac{\sigma \text{ of the sample}}{\sqrt{n}}$$

(n being the number of cases in the sample)

Since seasonal analysis deals with small samples, two modifications should be introduced:

(1) In computing the standard deviation of the sample the sum of the squares of the deviations from the mean is to be divided by the number of items less one.

(2) In evaluating the probability significance of the standard error allowance should be made for the smallness of the sample. This is done most conveniently by utilizing the table of t given by R. A. Fisher for this purpose.[8]

In our procedure for computing the seasonal index this test may easily be applied. The seasonal index for a given month, for example, January, is assumed to be the true arithmetic mean of the scatter, the items in this scatter being the relatives of the original data to the moving average for successive Januaries. There will, therefore, be as many items in the scatter as there are years in the period for which the seasonal index is computed.

The standard error of the index for January is the standard deviation of the relatives for successive Januaries from the index divided by the square root of the number of items (years in the period). In order to apply this standard error to test the significance of any difference, t must be computed and its value looked up in Fisher's Table IV.

Example:

For the series of the number of shares sold on the New York Stock Exchange the seasonal index for December for the period 1919-30 is 111. The standard error of this mean is 5.96. If it is desired to ascertain whether or not the difference of the December index from 100 is significant, t is equal to 11 divided by $5.96 = 1.85$. The probability value for a t of 1.85 and

[8] See his *Statistical Methods for Research Workers* (Edinburgh 1925) pp. 106-10 and Table IV of t appended at the end of the book.

for n of 11 (there are 12 items in the scatter, but 11 degrees of freedom) is slightly less than 0.1, signifying that in conditions of random sampling the probability is slightly less than one out of ten that the mean of another sample will depart by as much as 11 from the given mean of 111. These are rather large odds and the difference of the index 111 from 100 may be considered significant.

If the same standard errors are computed for the seasonal index for each month, it is possible, utilizing Fisher's Table IV of t, to estimate:

(1) the significance of the difference of the seasonal index for each month from 100 or from any other value;

(2) the significance of the difference between the seasonal index for one month and that for any other month.

b. *Correlation of the Average Seasonal Index with Relatives to the Moving Average*

In the procedure just described a test is provided for the significance of the seasonal index for each single month or of the difference between seasonal indexes for any pair of months. In order to test the significance of the seasonal index as a whole, the following procedure is suggested:

(1) The sum of the squared deviations of the relatives to the moving average from the seasonal index for the twelve months yields the total squared error about the seasonal index. Dividing this sum by the number of years yields the mean squared error about the seasonal index

$$\left(\frac{\Sigma d^2}{y}\right).$$

(2) Were there no seasonal variations in the data, the mean for every month would have tended to be 100. Let us compute the mean squared error from a seasonal index in which the standing for each of the twelve months is 100. Upon the assumption that the actually computed seasonal index is a set of true arithmetic means the mean squared error about a set of means of 100 can be obtained by adding to the mean squared error under (1) the square of the *correction* (for we are actually computing a squared standard deviation about a set of assumed means). This squared correction is the sum of the

squares of the deviations of the computed seasonal index from 100. The total obtained then is

$$\frac{\Sigma d^2}{y} + c^2,$$

$$c^2 = (\text{January seasonal} - 100)^2 + (\text{February seasonal} - 100)^2 + \ldots . (\text{December seasonal} - 100)^2.$$

(3) The ratio of the squared correction to the total under (2) yields a fairly precise idea of the significance of the total seasonal index. This ratio, varying from zero to one, is a coefficient of determination in a correlation between the seasonal index and the relatives of the original data to the moving average, *upon the assumption that the coefficient of regression involved is one.* It is obvious that since the computed seasonal index is so chosen as to represent precisely the *average* amplitude of seasonal variations in the series for the period covered, the coefficient of regression of the dependent variable (relatives of original data) on the independent variable (average seasonal index) is always close to one (see Chapter XI).

(4) A coefficient of determination measures the proportion of the variance in the dependent variable accounted for by the variations in the independent variable. It is equal to the square of the coefficient of correlation. The square root of the ratio under (3) yields the coefficient of correlation between the seasonal index and the relative of the original data to the moving average for the total period covered by the analysis.[9]

(5) If this correlation is significant, the seasonal index is obviously significant. In most instances the number of pairs

[9] Another way of conceiving this coefficient is as a correlation ratio between the deviation of the data from the moving average (y) and time (x), the latter being taken in its seasonal order (that is, Januaries, Februaries, etc. being grouped together) rather than in a continuous chronological order. The correlation ratio is a ratio of the standard deviation of the means of columns from the central mean to the standard deviation of the ungrouped scatter around the central mean. If we arrange time (the independent variable) in its seasonal order, then the means of the columns are the seasonal indexes for each month. And the correlation ratio is the ratio of the standard deviation of the seasonal index from 100 to the standard deviation from 100 of the original data adjusted for the moving average.

For the significance of the seasonal index the correlation ratio and the coefficient of correlation are about equivalent since the standard error of the correlation ratio is approximately the same as that for the coefficient of linear correlation.

involved is sufficiently large so that recourse need not be taken to R. A. Fisher's tables for small samples.[10]

Example:

For the same series of number of shares sold on the New York Stock Exchange the quantities involved are as follows:

1. The mean squared error around the seasonal index = 4,992.
2. Square of Correction, that is, the sum of the squares of the deviations of the seasonal index from 100 = 1,496.
3. Total Variance about a Set of Means of 100 (1) + (2) = 6,488.
4. The Coefficient of Determination $\frac{(2)}{(3)} = 0.2306$.
5. The Coefficient of Correlation = $\sqrt{(4)} = 0.48$.

There are 108 pairs in this correlation (9 years with 12 months in each) and hence 106 degrees of freedom. Since a p of 0.01 for $n = 100$ is given for a coefficient as low as 0.254, the correlation in the example is undoubtedly significant.

a-b. *Validity of the Tests*

The standard error test, applied to seasonal indexes for single months, and the correlation coefficient, available as a test of seasonal indexes as a whole, provide in combination a rigid assay of the validity of the seasonal indexes computed. The answers yielded by these procedures are definite and precise, once the investigator accepts certain probability values as indicating significance. Brief reflection will show, however, that the assumptions of these tests are far from generally valid and that consequently the utility of the precise results that they yield is drastically limited.

Both tests assume that the seasonal index is the true arithmetic mean of the scatter for each month. This assumption does not hold whenever the extreme items excluded for the given month do not average out to its seasonal index, or whenever the selected mean is changed as a result of the graphical comparison. Thus, one of the valuable features in the procedure of establishing the seasonal index, namely, the rejection

[10] See Table Va, Values of the Correlation Coefficient for Different Levels of Significance, in his *Statistical Methods for Research Workers*.

of extreme items through the selection of a positional arithmetic mean and the graphical comparison, is completely neglected by these tests. Since it is only the true mean that yields the smallest standard deviation, the result in most instances is that the standard error is larger and the coefficient of determination smaller than they should have been.

Both tests assume that the scatter is composed of random changes. This implies that the moving average used eliminates all of the secular and cyclical movements and that the time series contains, in addition to the average seasonal index, only random elements. But the moving average never eliminates the cyclical swing entirely and a further analysis of the series indicates that in addition to average seasonal variations there are variations related to the average seasonal swing that are far from random in character (see Chapter XI).

Inspection of the scatter in a few series indicates that for single months, and even for months combined into quarters, the frequency distribution of deviations from the seasonal index of the relatives to the moving averages is highly irregular in character. Even when the scatter is combined for all the months for the period as a whole the total frequency distribution still shows considerable departures from normality in the case of some series, for which, owing to the sharp turns in the cycles and the presence of large random disturbances, the moving average is not a satisfactory graduation line (for example, number of shares sold on the New York Stock Exchange). Consequently, the standard errors are invalid even in their purely descriptive function.

But another even more fundamental question is in respect of the imputation of probability significance to the standard errors and the correlation coefficient. Even if the time series under analysis were adjusted for all of the secular and cyclical changes; even if, with the help of refinements upon the average seasonal index, the seasonal variations were described as precisely as possible, can the residual changes be treated as random in the meaning given to this term by the statistical theory of random sampling? Are these changes, taken out of the context of a continuous historical process, similar in nature to variations under conditions of random sampling, strung out in time?

This question opens up a field in which, heretofore, there has been little thinking and study. It seems that current rapid progress of the statistical technique of time series analysis is likely to result in the segregation of certain continuous movements in what at present appear to be casual changes. Moreover, the residual changes found in many time series are certainly not random in the sense of being produced by a large number of small, independent disturbing factors (consider, for example, strikes, and changes in foreign trade as a result of modification in tariffs). Great caution must, therefore, be exercised in accepting any tests of probability significance for measurements derived from time series.

The precision of the tests by standard errors and the coefficient of correlation is, for reasons just stated, of limited value. Consequently in the present study standard errors were computed for a few series as a matter of experiment. The coefficient of correlation, by the procedure just described, was also applied to only a few series. But the same coefficients were obtained as a result of an attempt to measure short-time changes in the amplitude and pattern of seasonal variations. Readers interested in the value of these coefficients will find them in Tables XXXVI and XXXVIII. Brief inspection will reveal that in the large number of cases where such coefficients were computed the values are so high as to admit no question regarding their significance, even if the assumptions necessary to assure their validity are accepted.

c. *Size of Seasonal Variation and Graphical Inspection*

These procedures are rough in character and their conclusions are merely an approximation to the more precise results yielded by the two tests described above.

The absolute size of the seasonal variation, as reflected in the average index, is in itself a rough guide to its significance, especially when compared to the average absolute size of the deviations of the original data from the moving average. If the seasonal index for a given month is 75 (in terms of the average for the year as 100), while the relative deviations from the moving average for the same month range between -35 and -15, the seasonal index is obviously significant. In such instances the precise measurement of the standard error is not necessary to confirm the significance of the seasonal measurement. Its added precision is of little value, in view of the above mentioned doubts as to its validity.

A similar rough test is afforded by a graphical comparison of the original data with the seasonal index. If this comparison indicates the presence of a recurrent variation in the original data, a variation similar to that shown by the seasonal index, the latter is obviously significant. In such instances the refinement introduced by the coefficient of correlation described above is unnecessary and its precision of relatively small value.

These two rough tests may be applied not only to each average seasonal index but also to comparisons between indexes for single months or for a full year. Of course, in such instances the approximate character of the procedures renders them valid only for rather obvious and substantial differences.

The tests just presented are used widely by investigators who deal with the measurement of seasonal variations. Rough as they are, they are yet sufficient in many series and have been used extensively in the present study.

d. *Similarity of Seasonal Variations in Successive Periods and Among Related Series*

If, for a given series, average seasonal indexes are computed for more than one period a comparison of the resulting measures may afford a test of significance. Close similarity for two periods is substantial evidence of the significance of the indexes, unless for some reason a difference was expected, for the probability of obtaining closely similar variations of twelve items each, in a series that contains random variations only, is very small. The test becomes still more reliable when more than two periods are available for comparison and close similarity is observed not merely for two but for a larger number of periods.

Similarly, if seasonal indexes are computed for related branches of activity, as for example, production, shipments and stocks of one and the same commodity, or consumption of several commodities all used in the same industry, comparison affords a good basis for estimating the significance of the measures. Substantial similarity, or the presence of an *expected* difference, lends weight to the recognition of the seasonal indexes involved as significant, for recurrence under variable conditions is proof of reliability.

Both comparisons may be applied to test the significance of differences between seasonal indexes but the tests are valid within a much more circumscribed area.

In the present study the technique employed led to the computation of an average seasonal index for each of several short periods. Consequently, the first of the two comparisons could be and was applied to a very large proportion of the series analyzed. Likewise, the detailed consideration of seasonality at the successive stages of the flow of a commodity made feasible numerous comparisons of seasonal measures in related branches of activity.

e. *Conformity to Non-Statistical Data*

This test is most important from the point of view of the present study and was applied, as far as was possible, to each seasonal index obtained. The purpose of statistical analysis is to measure more precisely the influence of seasonal factors on a given branch of economic activity. The nature of these factors and the general character of their influence is known or should be ascertained in all seasonally affected economic series. General conformity of the statistical index to this otherwise known effect of seasonal factors is an important test in a study dealing not only with description but also with explanation.

Failure of a seasonal index to pass this test is an indication either of the unreliability of the index or of the insufficiency of the non-quantitative knowledge of the factors and influences involved. Then an attempt may be made to 'explain' the seasonal index better, that is, to add further non-statistical data to the stock of knowledge and thus to restore the conformity between statistical measurement and non-quantitative data. Or upon investigation of the reliability of the statistical index the conclusion may be reached that it is not sufficiently significant.

The tests presented above to estimate the significance of average seasonal indexes vary from precise ones, such as the standard error and the correlation coefficient, to the more approximate ones listed under c, d and e. The precision of the exact tests is conditioned by the assumptions that they imply concerning the nature of the data from which the indexes are obtained. The validity of the more approximate tests derives from wider and more easily acceptable assumptions in regard to the data upon which they may be based.

The choice among these tests depends upon the investigator's judgment as to the validity of the assumptions implicit in each. The present study employs almost exclusively tests c, d and e, while measures for test b are available for several series analyzed for the persistence of the average seasonal swing from year to year. Standard errors were computed merely as an experiment for a few series.