Volume Title: Annals of Economic and Social Measurement, Volume 4, number 1

Volume Author/Editor: Sanford V. Berg, editor

Volume Publisher: NBER

Volume URL: http://www.nber.org/books/aesm75-1

Publication Date: 1975

Chapter Title: The Structure of Consumer Preferences

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Chapter URL: http://www.nber.org/chapters/c10218

Chapter pages in book: (p. 49 - 101)

Annals of Economic and Social Measurement, 4/1, 1975

THE STRUCTURE OF CONSUMER PREFERENCES

BY

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AND

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The purpose of this paper is to present an econometric methodology for selecting among alternative specifications of the structure of consumer preferences in statistical demand analysis. We first derive parametric restrictions for direct and indirect transcendental logarithmic utility functions corresponding to restrictions on the form of consumer preferences and on changes in preferences over time. We consider restrictions corresponding to groupwise separability in goods and in time, groupwise homotheticity, groupwise linear logarithmic utility, and groupwise equality of rates of commodity augmentation. Second, we formulate statistical tests of these restrictions based on the likelihood ratio principle. Finally, we present empirical tests of each set of restrictions for U.S. time series data on personal consumption expenditures for the period 1947–1971.

1. INTRODUCTION

The purpose of this paper is to present an econometric methodology for characterizing the structure of consumer preferences and changes in preferences over time.[1] For this purpose we introduce new representations of consumer preferences. Our approach is to represent the underlying utility function by functions that are quadratic in the logarithms of the quantities consumed and time. Similarly, we represent the underlying indirect utility function by functions that are quadratic in the logarithms of ratios of prices to total expenditure and time. These representations of consumer preferences do not require the assumptions of additivity, homotheticity, and stationarity of preferences implicit in the traditional approach to statistical demand analysis.

We refer to our representation of the utility function as the direct transcendental logarithmic utility function with time-varying preferences, or more simply, the *direct translog utility function*. The utility function is a transcendental function of the logarithms of the quantities consumed and of time.[2] Similarly, we refer to our representation of the indirect utility function as the indirect transcendental logarithmic utility function with time-varying preferences or, more simply, the *indirect translog utility function*. Direct and indirect translog utility functions

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The research of Dale W. Jorgenson was supported by The National Science Foundation under Grant GI-43097. The research of Lawrence J. Lau was supported by the John Simon Guggenheim Memorial Foundation and by the National Science Foundation through Grant GS-40104 to the Institute for Mathematical Studies in the Social Sciences, Stanford University. We are grateful to Christophe Chamley and Paul Swaim for expert research assistance and to Laurits Christensen for helpful advice. Responsibility for any remaining deficiencies rests entirely with the authors.

[1] Direct and indirect utility functions with time-varying preferences are discussed by Lau [1969a].

[2] A function $U = F(X)$ is an algebraic function if U can be defined implicitly by an equation $G(U, X) = 0$, where G is a polynomial in U and X. All functions which are not algebraic are transcendental. See Courant [1936], p. 119.

without time-varying preferences were introduced by Christensen, Jorgenson, and Lau and used by them to test the theory of demand and to characterize substitution patterns among commodity groups.[3] Lau and Mitchell and Christensen and Manser have employed homothetic indirect translog utility functions to characterize substitution patterns.[4]

As an illustration of the traditional approach to demand analysis, we can consider the double logarithmic demand functions employed in the pioneering studies of consumer demand by Schultz, Stone, and Wold.[5] If the theory of demand is valid and demand functions are double logarithmic with time trends, the utility function is neutral linear logarithmic. A neutral linear logarithmic utility function is additive, homothetic, and stationary. Elasticities of substitution among all pairs of commodities are constant and equal to unity. All expenditure proportions are constant for all values of prices, total expenditure, and time. Similarly, the Rotterdam system of demand functions with time trends employed by Barten and Theil is consistent with utility maximization only if the utility function is neutral linear logarithmic.[6] We conclude that the double logarithmic and Rotterdam demand systems implicitly maintain the hypotheses of additivity, homotheticity, and stationarity.

Houthakker and Stone have developed alternative approaches to demand analysis that retain the assumption of additivity while dropping the assumption of homotheticity.[7] Stone has employed a linear expenditure system, based on a utility function that is linear in the logarithm of quantities consumed less a constant for each commodity, representing initial commitments of expenditure. Nonzero commitments permit expenditure proportions to vary with total expenditure. Houthakker has employed a direct addilog system, based on a utility function that is additive in functions that are homogeneous in the quantity consumed for each commodity. The degree of homogeneity may vary from commodity to commodity, again permitting expenditure proportions to vary with total expenditure. Parallelling the direct addilog demand system, Houthakker has also employed an indirect addilog system, based on an indirect utility function that is additive in the ratios of prices to total expenditure.

Basmann, Johansen, and Sato have combined the approaches of Houthakker and Stone, defining each of the homogeneous functions in the direct addilog utility function on the quantity consumed less a constant for each commodity.[8] The resulting utility function is additive but not homothetic. We conclude that

[3] See Christensen, Jorgenson and Lau [1975]. Earlier Christensen, Jorgenson and Lau [1971, 1973] introduced transcendental logarithmic functions into the study of production.

[4] See Lau and Mitchell [1971] and Christensen and Manser [1974a, 1974b].

[5] See Schultz [1938], Stone [1954a], and Wold [1953]. For a proof that an integrable system of double logarithmic demand functions with time trends implies neutral linear logarithmic utility, see Jorgenson and Lau [1974].

[6] See Barten [1964, 1967, 1969], McFadden [1964], and Theil [1965, 1967, 1971]. For a proof that an integrable Rotterdam system with time intercepts implies explicit neutral linear logarithmic utility, see Jorgenson and Lau [1974].

[7] See Houthakker [1960] and Stone [1954b]. The linear expenditure system was originally proposed by Klein and Rubin [1947–1948].

[8] See Basmann [1969], Johansen [1969] and Sato [1972]. For an empirical application, see Brown and Heien [1972]. A recent survey of econometric studies of consumer demand is given by Brown and Deaton [1972].

the linear expenditure system, the direct and indirect addilog systems, and the combined systems introduced by Basmann, Johansen, and Sato maintain the hypotheses of direct or indirect additivity. By employing direct and indirect translog utility functions with time-varying preferences we can test additivity, homotheticity, and stationarity restrictions rather than maintaining these re-restrictions on preferences as part of our econometric model.

In the following section we introduce direct and indirect translog utility functions with time-varying preferences and the corresponding systems of indirect and direct demand functions. We consider restrictions on the demand functions implied by utility maximization. We impose these restrictions as part of our maintained hypothesis. In Section 3 we consider demand systems associated with restrictions on the structure of consumer preferences and changes in preferences over time. We begin with groupwise separability and groupwise homotheticity of preferences. For each set of restrictions on preferences, we derive parametric restrictions on the corresponding system of demand functions. These parametric restrictions provide the basis for statistical test of alternative hypotheses about the structure of consumer preferences.

We consider two alternative sets of restrictions on the variation of consumer preferences over time. The first set corresponds to separability of goods and time: a commodity group is separable from time if the ratios of any pair of demand functions for all commodities within the group are independent of time. An alternative set of restrictions on changes in preferences is associated with commodity augmentation; commodity augmentation by itself is not a testable hypothesis since any change in preferences over time can be regarded as commodity augmenting or commodity diminishing. We impose restrictions on the variation of preferences with time by imposing restrictions on rates of augmentation of commodities within a given group: in particular, we formulate tests of equality of rates of commodity augmentation within a group. Groupwise separability from time and groupwise equality of rates of commodity augmentation are not mutually exclusive; however, they coincide only under additional restrictions such as neutral linear logarithmic utility.

We present empirical results of tests of alternative sets of restrictions on consumer preferences and changes in preferences over time in Section 4. Our tests are based on time series data for U.S. personal consumption expenditures of three commodity groups--durables, non-durables, and energy—for the period 1947–1971. Our concept of personal consumption expenditures differs from the corresponding concept in the U.S. national income and product accounts in the treatment of consumers' durables.[9] We treat expenditure on consumers' durables as part of gross private domestic investment rather than personal consumption expenditures. We add an imputed flow of services from consumers' durables to personal consumption expenditures, so that our concept of durables services is perfectly analogous to the national accounting concept of housing services.

[9] A detailed reconciliation of our concept of personal consumption expenditures and the national accounting concept is given by Christensen and Jorgenson [1973], pp. 331–348.

2. Transcendental Logarithmic Utility Functions with Time-Varying Preferences

2.1. *The direct translog utility function*

A *direct utility function* U with time-varying preferences can be written in the form:

$$-\ln U = F(X_1, X_2, X_3, t), \tag{2.1}$$

where $X_i (i = 1, 2, 3)$ is the quantity consumed of the ith commodity and t is time. At each time the consumer maximizes utility, subject to the budget constraint,

$$\sum p_i X_i = M, \tag{2.2}$$

where $p_i (i = 1, 2, 3)$ is the price of the i-th commodity and M is the value of total expenditure.

Maximizing utility, subject to the budget constraint, we obtain the identity:

$$\frac{\partial \ln U}{\partial \ln X_j} = \frac{p_j X_j}{M} \sum \frac{\partial \ln U}{\partial \ln X_i}, \qquad (j = 1, 2, 3). \tag{2.3}$$

This identity gives the ratios of prices to total expenditure as functions of the quantities consumed:

$$\frac{p_j}{M} = \frac{\dfrac{\partial \ln U}{\partial \ln X_j}}{X_j \sum \dfrac{\partial \ln U}{\partial \ln X_i}}, \qquad (j = 1, 2, 3). \tag{2.4}$$

We refer to these functions as *indirect demand functions*.

Utility is nondecreasing in the quantities consumed, so that the negative of the logarithm of utility is nonincreasing in the logarithms of the quantities consumed. A necessary and sufficient condition for monotonicity of the negative of the logarithm of the utility function at a particular point is that the budget shares are non-negative at that point. The utility function is quasiconcave, so that the negative of the logarithm of the utility function is quasiconvex. Monotonicity and quasiconvexity of the negative of the logarithm of the utility function are the basic assumptions of the theory of demand.

We approximate the negative of the logarithm of the utility function by a function quadratic in the logarithms of the quantities consumed and t:

$$\begin{aligned} -\ln U = {} & \alpha_0 + \sum \alpha_i \ln X_i + \alpha_t \cdot t + \tfrac{1}{2} \sum\sum \beta_{ij} \ln X_i \ln X_j \\ & + \sum \beta_{it} \ln X_i \cdot t + \tfrac{1}{2} \beta_{tt} \cdot t^2. \end{aligned} \tag{2.5}$$

Using this form of the utility function we obtain:

$$\alpha_j + \sum \beta_{ji} \ln X_i + \beta_{jt} \cdot t = \frac{p_j X_j}{M} \sum (\alpha_k + \sum \beta_{ki} \ln X_i + \beta_{kt} \cdot t), \qquad (j = 1, 2, 3). \tag{2.6}$$

To simplify this notation we write:

$$\alpha_M = \sum \alpha_k, \qquad \beta_{Mi} = \sum \beta_{ki}, \qquad \beta_{Mt} = \sum \beta_{kt}, \qquad (i = 1, 2, 3), \tag{2.7}$$

so that:

$$\frac{p_j X_j}{M} = \frac{\alpha_j + \sum \beta_{ji} \ln X_i + \beta_{jt} \cdot t}{\alpha_M + \sum \beta_{Mi} \ln X_i + \beta_{Mt} \cdot t}, \qquad (j = 1, 2, 3). \tag{2.8}$$

We note that the parameters α_t and β_{tt} have no effect on the utility-maximizing quantities consumed. These two parameters cannot be identified from data on prices and quantities.

The budget constraint implies that:

$$\sum \frac{p_i X_i}{M} = 1, \tag{2.9}$$

so that, given the parameters of any two equations for the budget shares, $p_j X_j/M$ $(j = 1, 2, 3)$, the parameters of the third equation can be determined from the definitions of α_M, β_{Mj} $(j = 1, 2, 3)$, and β_{Mt}.

Since the equations for the budget shares are homogeneous of degree zero in the parameters, normalization of the parameters is required for estimation. A convenient normalization for the direct translog utility function is:

$$\alpha_M = \sum \alpha_i = -1 \tag{2.10}$$

We estimate only two of the equations for the budget shares, subject to normalization of the parameter α_M appearing in each equation at minus unity. Unrestricted, there are eighteen unknown parameters to be estimated from the two equations. If the equations are generated by utility maximization, the parameters $\beta_{Mj}(j = 1, 2, 3)$ and β_{Mt} appearing in each equation must be the same. This results in a set of restrictions relating the four parameters appearing in each of the two equations, a total of four restrictions. We refer to these as *equality restrictions.*

The negative of the logarithm of the direct translog utility function is twice differentiable in the logarithms of the quantities consumed, so that the Hessian of this function is symmetric. This gives rise to a set of restrictions relating the parameters of the cross-partial derivatives:

$$\beta_{ij} = \beta_{ji}, \qquad (i \neq j : i, j = 1, 2, 3). \tag{2.11}$$

There is one restriction of this type among the parameters of the two equations we estimate directly and two such restrictions among the parameters of the equation we estimate indirectly from the budget constraint. We refer to these as *symmetry restrictions.* The total number of symmetry restrictions is three.

If equations for the budget shares are generated by maximization of a direct translog utility function, the parameters satisfy equality and symmetry restrictions. There are seven such restrictions. Given the seven equality and symmetry restrictions, eleven unknown parameters remain to be estimated. Our approach to the analysis of consumer demand takes as assumptions the restrictions on expenditure allocation implied by utility maximization and the existence of the three commodity groups—durables, non-durables, and energy—as well-defined economic aggregates. Given these assumptions, we estimate the unknown parameters of our complete demand system simultaneously.

Given the hypothesis of consistency between our system of indirect demand functions and the maximization of utility and the grouping of commodities into three aggregates, we could proceed to impose further constraints on the allocation of personal consumption expenditures, such as constant price and income elasticities of demand or constant elasticities of substitution among commodity groups.[10] However, such an approach would frustrate our primary research objective of characterizing the pattern of consumer demand empirically. This approach would convert hypotheses about budget allocation and patterns of substitution into assumptions rather than hypotheses to be tested. Instead we propose to test all further restrictions on the structure of the direct utility function.

2.2. *The indirect translog utility function.*

An *indirect utility function* V with time-varying preferences can be written in the form:

$$\ln V = G\left(\frac{p_1}{M}, \frac{p_2}{M}, \frac{p_3}{M}, t\right), \tag{2.12}$$

where V is the maximum level of utility corresponding to the prices $p_i (i = 1, 2, 3)$ and the level of total expenditure M.

We can determine the budget share from the j-th commodity from the identity:[11]

$$\frac{\partial \ln V}{\partial \ln p_j/M} = \frac{p_j X_j}{M} \sum \frac{\partial \ln V}{\partial \ln p_i/M}, \qquad (j = 1, 2, 3). \tag{2.13}$$

This identity gives the quantities consumed as functions of the ratios of prices to total expenditures:

$$X_j = \frac{\dfrac{\partial \ln V}{\partial \ln p_j/M}}{\dfrac{p_j}{M} \sum \dfrac{\partial \ln V}{\partial \ln p_i/M}}, \qquad (j = 1, 2, 3). \tag{2.14}$$

We refer to these functions as *direct demand functions.*

Utility is nonincreasing in the prices, so that the logarithm of utility is nonincreasing in the logarithms of the prices. A necessary and sufficient condition for monotonicity of the logarithm of the indirect utility function at a particular point is that the budget shares are non-negative at that point. The indirect utility function is quasiconvex, so that the logarithm of this function is quasiconvex.

The system consisting of the negative of the logarithm of the direct utility function and the indirect demand functions is dual to the system consisting of the logarithm of the indirect utility function and the direct demand functions. One system can be obtained from the other by simply interchanging the quantities consumed X_i $(i = 1, 2, 3)$ with the ratios of prices to total expenditure $p_i/M (i = 1, 2, 3)$. All the properties of one system carry over to the other system with the role of these two sets of variables interchanged.

[10] Systems of direct and indirect demand functions with these properties are discussed by Christensen, Jorgenson and Lau [1975].

[11] This is the logarithmic form of Roy's Identity. See Roy [1943].

We approximate the logarithm of the indirect utility function by a function quadratic in the logarithms of the ratios of prices to the value of total expenditure and t:

$$\text{(2.15)} \qquad \ln V = \alpha_0 + \sum \alpha_i \ln \frac{p_i}{M} + \alpha_t \cdot t + \frac{1}{2}\sum\sum \beta_{ij} \ln \frac{p_i}{M} \ln \frac{p_j}{M} + \sum \beta_{it} \ln \frac{p_i}{M} \cdot t + \frac{1}{2}\beta_{tt} \cdot t^2.$$

Using this form of the indirect utility function we obtain:

$$\text{(2.16)} \qquad \alpha_j + \sum \beta_{ji} \ln \frac{p_i}{M} + \beta_{jt} \cdot t = \frac{p_j X_j}{M}\left(\sum \alpha_k + \sum \beta_{ki} \ln \frac{p_i}{M} + \beta_{kt} \cdot t\right), \qquad (j = 1, 2, 3).$$

As before, we simplify notation by writing:

$$\text{(2.17)} \qquad \alpha_M = \sum \alpha_k, \qquad \beta_{Mi} = \sum \beta_{ki}, \qquad \beta_{Mt} = \sum \beta_{kt} \qquad (i = 1, 2, 3),$$

so that:

$$\text{(2.18)} \qquad \frac{p_j X_j}{M} = \frac{\alpha_j + \sum \beta_{ji} \ln p_i/M + \beta_{jt} \cdot t}{\alpha_M + \sum \beta_{Mi} \ln p_i/M + \beta_{Mt} \cdot t}, \qquad (j = 1, 2, 3).$$

The parameters α_t and β_{tt} cannot be identified.

The budget constraint implies that, given the parameters of any two equations for the budget shares, the parameters of the third equation can be determined from the definitions of α_M, $\beta_{Mj}(j = 1, 2, 3)$, and β_{Mt}. As before, we normalize the parameters of the indirect translog utility function so that:

$$\text{(2.19)} \qquad \alpha_M = \sum \alpha_i = -1.$$

As in the case of the direct translog utility function with time-varying preferences, we estimate only two of the equations for the budget shares, subject to normalization of the parameter α_M appearing in each equation at minus unity. We also maintain the assumptions of utility maximization and the existence of the three aggregates. The equality and symmetry restrictions resulting from these assumptions are strictly analogous to those for the direct translog utility function with time-varying preferences.

2.3. *Stochastic specification*

The first step in implementing an econometric model of demand based on the direct translog utility function with time-varying preferences is to add a stochastic specification to the theoretical model based on equations for the budget shares $p_j X_j/M (j = 1, 2, 3)$. Given the disturbances in any two equations, the disturbance in the remaining equation can be determined from the budget constraint. Only two equations are required for a complete econometric model of demand. We assume that the noncontemporaneous disturbances, whether from the same or different equations, have zero covariance. No additional restrictions are placed on the disturbances, other than the requirement that disturbances from the three equations must add up to zero. We also assume that the right hand side variables of the equations for the budget shares are uncorrelated with the stochastic disturbances. This latter assumption facilitates the use of the method of maximum likelihood in estimation of the parameters.

In implementing an econometric model of demand based on the indirect utility function with time-varying preferences the first step, as before, is to add a stochastic specification to the theoretical model based on equations for the budget shares $p_jX_j/M (j = 1, 2, 3)$. Only two equations are required for a complete model. The assumptions that we make here are strictly analogous to those for the direct translog utility function with time-varying preferences. We note, however, that the implications of the stochastic specification are different for the direct and indirect models and hence the results for the two models are not directly comparable.

To summarize: We have derived models for the allocation of personal consumption expenditures from direct and indirect translog utility functions with time-varying preferences. We take the hypothesis of utility maximization to be an assumption rather than a hypothesis to be tested. Utility maximization implies that the parameters of equations for the budget shares in each model satisfy seven equality and symmetry restrictions that enable us to reduce the number of unknown parameters from eighteen to eleven. These parameters are further constrained by certain inequalities that embody monotonicity and quasiconvexity restrictions on the negative of the logarithm of the direct utility function and the logarithm of the indirect utility function. We estimate the parameters of our models of consumption subject to the equality and symmetry restrictions; at a later stage we incorporate the monotonicity and quasiconvexity restrictions.[12]

3. Preference Structure

3.1. *Approximation*

The primary objective of our research is to ascertain and characterize the structure of consumer preferences empirically, without maintaining restrictive assumptions on the specific form of the utility function other than monotonicity and quasiconvexity. We wish, first, to determine the effects of changes in total expenditures and changes in preferences over time on the allocation of the consumer budget among commodity groups and, second, to determine the effects of changes in relative prices on the allocation of the consumer budget, that is, to characterize the patterns of substitution among commodities.

In the remainder of this section, we develop tests of a series of possible restrictions on the underlying structure of consumer preferences. First, we consider groupwise separability of preferences in commodities and in time. Second, we consider overall homotheticity and groupwise homotheticity of preferences.

[12] Monotonicity and quasiconvexity restrictions are discussed by Lau [1974]. See also Jorgenson and Lau [1974].

Third, we consider groupwise linear logarithmic utility as a possible restriction on preferences. Finally, we consider groupwise equal rates of commodity augmentation as a possible restriction on changes in the structure of preferences over time.

The transcendental logarithmic utility function with time-varying preferences can be interpreted as a local second-order Taylor's series approximation of an arbitrary utility function with time-varying preferences that is differentiable at least up to the third order. In practical applications the latter condition is hardly any restriction as any utility function can be approximated arbitrarily closely by an infinitely differentiable function. Using this local approximation property, the translog utility function can be used to test specific hypotheses on the structure of the underlying utility function.

The parameters of the translog utility function can be identified with the coefficients in a Taylor's series expansion to the underlying utility function. They take the values of the first and second partial logarithmic derivatives of the negative of the logarithm of the underlying utility function at the point of expansion. Specific hypotheses on the structure of preferences imply restrictions on the Hessian of the negative of the logarithm of the utility function and can be tested by imposing these restrictions on the parameters of the translog utility function.

Restrictions on the structure of preferences do not necessarily imply the corresponding restrictions on the translog utility function itself. Properties of the underlying utility function and its translog approximation agree up to and including second-order derivatives at the point of approximation. We distinguish between situations where the translog utility function provides an approximation to an underlying utility function with a certain property and situations where the translog utility function also possesses that property. In the latter case, we say that the translog utility function possesses the property intrinsically.

3.2. *Groupwise separability*

The first set of restrictions on consumer preferences that we propose to test are *groupwise separability* restrictions. A direct utility function U with time-varying preferences that is separable in X_1 and X_2 from X_3 can be written in the form:

$$-\ln U = F(-\ln U^1(X_1, X_2, t), X_3, t), \tag{3.1}$$

where the function $-\ln U^1$ depends only on X_1, X_2 and time and is nonincreasing and quasiconvex in X_1 and X_2. A necessary and sufficient condition for groupwise separability of the direct utility function in X_1 and X_2 from X_3 is that the ratio of the indirect demand functions for X_1 and X_2 is independent of the quantity of X_3. A direct utility function that is groupwise separable in X_1 and X_2 from time can be written in the form:

$$-\ln U = F(-\ln U^1(X_1, X_2, X_3), X_3, t), \tag{3.2}$$

which is analogous to equation (3.1) with the roles of X_3 and t interchanged. A necessary and sufficient condition for groupwise separability of X_1 and X_2 from time is that the ratio of the indirect demand functions of X_1 and X_2 is independent of time. Groupwise separability in time is also referred to as *groupwise neutrality*.

Partially differentiating equation (3.1) first with respect to $\ln X_3$ and then with respect to $\ln X_1$ and $\ln X_2$ separately, we obtain:

$$\text{(3.3)} \qquad \frac{\partial^2 - \ln U}{\partial \ln X_1 \partial \ln X_3} = \frac{\partial^2 F}{\partial - \ln U^1 \partial \ln X_3} \cdot \frac{\partial - \ln U^1}{\partial \ln X_1},$$

$$\frac{\partial^2 - \ln U}{\partial \ln X_2 \partial \ln X_3} = \frac{\partial^2 F}{\partial - \ln U^1 \partial \ln X_3} \cdot \frac{\partial - \ln U^1}{\partial \ln X_1}.$$

By observing that:

$$\text{(3.4)} \qquad \frac{\partial - \ln U}{\partial \ln X_1} = \frac{\partial F}{\partial - \ln U^1} \cdot \frac{\partial - \ln U^1}{\partial \ln X_1},$$

$$\frac{\partial - \ln U}{\partial \ln X_2} = \frac{\partial F}{\partial - \ln U^1} \cdot \frac{\partial - \ln U^1}{\partial \ln X_2},$$

we can rewrite:

$$\frac{\partial^2 - \ln U}{\partial \ln X_1 \partial \ln X_3} = \frac{\partial^2 F/(\partial - \ln U^1 \partial \ln X_3)}{\partial F/(\partial - \ln U^1)} \cdot \frac{\partial - \ln U}{\partial \ln X_1},$$

so that:

$$\text{(3.5)} \qquad \frac{\partial^2 - \ln U}{\partial \ln X_2 \partial \ln X_3} = \frac{\partial^2 F/(\partial - \ln U^1 \partial \ln X_3)}{\partial F/(\partial - \ln U^1)} \frac{\partial - \ln U}{\partial \ln X_2}.$$

Given groupwise separability, equations (3.5) must hold everywhere; in particular, they must hold at the point of approximation, in this case, $\ln X_i = 1$ $(i = 1, 2, 3)$, $t = 0$, where we can identify the first and second partial derivatives with the parameters of the direct translog utility function with time-varying preferences:

$$\frac{\partial^2 - \ln U}{\partial \ln X_1 \partial \ln X_3} = \beta_{13}, \qquad \frac{\partial^2 - \ln U}{\partial \ln X_2 \partial \ln X_3} = \beta_{23},$$

$$\frac{\partial - \ln U}{\partial \ln X_1} = \alpha_1, \qquad \frac{\partial - \ln U}{\partial \ln X_2} = \alpha_2.$$

Thus, given groupwise separability of X_1 and X_2 from X_3, the parameters of the direct translog utility function must satisfy the restrictions:

$$\text{(3.6)} \qquad \beta_{13} = \rho_3 \alpha_1, \qquad \beta_{23} = \rho_3 \alpha_2,$$

where ρ_3 is a constant given by:

$$\rho_3 = \frac{\partial^2 F/(\partial - \ln U^1 \partial \ln X_3)}{\partial F/(\partial - \ln U^1)},$$

at the point of approximation.

Similarly, in a manner strictly analogous to the derivation of equation (3.6), it can be shown that given groupwise separability of X_1 and X_2 from time, the parameters of the direct translog utility function must satisfy the restrictions:

$$\text{(3.7)} \qquad \beta_{1t} = \rho_t \alpha_1, \qquad \beta_{2t} = \rho_t \alpha_2.$$

We note that there are no analogous restrictions on the direct translog parameters for groupwise separability of the type X_1 and time from X_2 because the parameter α_t cannot be identified.

We distinguish among three commodity groups. Each pair of commodities, such as X_1 and X_2, can be separable from the remaining commodity, X_3 in this instance, and time. Corresponding to the three possible pairs of commodities, there are six possible sets of groupwise separability restrictions analogous to equation (3.6) or equation (3.7). Each set of two restrictions involves the introduction of one new parameter—ρ_3 and ρ_t in the examples given above. Under each set of such restrictions, maintaining the symmetry and equality restrictions, ten unknown parameters remain to be estimated.

The translog approximation to a groupwise separable utility function is not necessarily groupwise separable. For a direct translog utility function to be groupwise separable in X_1 and X_2 from X_3, the ratio of the indirect demand functions generated by the direct translog utility function must be independent of X_3. We refer to a direct translog utility function as *intrinsically groupwise separable* if it is groupwise separable. Two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for intrinsic groupwise separability of the direct translog utility function. The first set consists of the restrictions given in equation (3.6) and the additional restriction:

$$\rho_3 = 0. \tag{3.8}$$

This restriction implies that the cross partial derivatives of the direct translog utility function with respect to X_1 and X_3 and X_2 and X_3, respectively, are identically zero at the point of approximation. Thus the indirect demands of X_1 and X_2 do not depend on X_3. We refer to this set of restrictions as *explicit groupwise separability* restrictions.

A second set of restrictions that implies intrinsic groupwise separability of the direct translog utility function is that ρ_3 is different from zero, but that the ratio of the budget shares of X_1 and X_2 is constant for all prices, total expenditure and time. This means that the parameters of the direct translog utility function must satisfy the restrictions:

$$\alpha_1\beta_{12} = \alpha_2\beta_{11}, \qquad \alpha_1\beta_{22} = \alpha_2\beta_{12}, \qquad \alpha_1\beta_{23} = \alpha_2\beta_{13}, \qquad \alpha_1\beta_{2t} = \alpha_2\beta_{1t}; \tag{3.9}$$

that is, the second order translog parameters corresponding to the first and second commodities must be in the same proportion as the first order translog parameters. If the ratio of the optimal budget shares of X_1 and X_2 is constant, the direct utility function takes the form:

$$-\ln U = F(\delta_1 \ln X_1 + \delta_2 \ln X_2, X_3, t),$$

where δ_1 and δ_2 are constants. This utility function is both *groupwise linear logarithmic* in X_1 and X_2 and *groupwise separable* in X_1 and X_2 from time. We say that such a utility function is *groupwise neutral linear logarithmic*. This condition is much more restrictive than groupwise separability or explicit groupwise separability; we will discuss it in more detail in Section 3.4 below.

Similarly, two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for intrinsic groupwise separability of X_1 and X_2 from time. The first set consists of the restrictions given in equation (3.7) above and the additional restriction:

$$\rho_t = 0. \tag{3.10}$$

that is, the direct translog utility function is *explicitly groupwise separable* in X_1 and X_2 from time. A second set of restrictions that also implies *intrinsic groupwise separability* of X_1 and X_2 from time are the restrictions of groupwise neutral linear logarithmic utility.

We can show that restrictions analogous to equations (3.8) and (3.10) must hold for any one of the six possible types of explicit groupwise separability, given groupwise separability. Under each set of explicit groupwise separability restrictions, nine unknown parameters remain to be estimated.

A direct utility function with time-varying preferences is *additive* in X_1, X_2 and X_3 if it can be written in the form:

$$-\ln U = F(-(\ln U^1(X_1, t) + \ln U^2(X_2, t) + \ln U^3(X_3, t)), t). \tag{3.11}$$

A necessary and sufficient condition for additivity in commodities is that the direct utility function is groupwise separable in any pair of commodities from the remaining commodity. In particular, since there are only three commodities, groupwise separability of any two pairs of commodities from the third is sufficient for additivity. A direct translog utility function with time-varying preferences is *explicitly additive* if it can be written in the form:

$$-\ln U = -\ln U^1(X_1, t) - \ln U^2(X_2, t) - \ln U^3(X_3, t), \tag{3.12}$$

where each function $-\ln U^i (i = 1, 2, 3)$ is nonincreasing and convex. The translog approximation to an explicitly additive utility function is necessarily explicitly additive. A necessary and sufficient condition for explicit additivity in commodities is that the direct translog utility function is explicitly groupwise separable in any pair of commodities from the remaining commodity. Since there are only three commodities, explicit groupwise separability for any two pairs of commodities from a third commodity is sufficient for explicit additivity.

A direct utility function with time-varying preferences is *neutral* if it can be written in the form:

$$-\ln U = F(-\ln U^1(X_1, X_2, X_3), t),$$

where $-\ln U^1$ is independent of time. A necessary and sufficient condition for neutrality is that the direct utility function is groupwise separable in any pair of commodities from time. In particular, since there are only three commodities, groupwise separability of any two pairs of commodities from time is sufficient for neutrality. A direct utility function with time-varying preferences is *explicitly neutral* if it can be written in the form:

$$-\ln U = -\ln U^1(X_1, X_2, X_3) + F(t). \tag{3.13}$$

The translog approximation to an explicitly neutral utility function is necessarily explicitly neutral. A necessary and sufficient condition for explicit neutrality is that the direct translog utility function is explicitly groupwise separable in any pair of commodities from time. In particular, since there are only three commodities, explicit groupwise separability of any two pairs of commodities from time is sufficient for explicit neutrality.

3.3. *Groupwise homotheticity and homogeneity*

The second set of functional restrictions on consumer preferences that we propose to test are *homotheticity* restrictions. First, we consider overall homotheticity of preferences. A direct utility function with time-varying preferences that is *homothetic* can be written in the form:

$$-\ln U = F(\ln H(X_1, X_2, X_3, t), t), \tag{3.14}$$

where H is homogeneous of degree one in the quantities X_1, X_2, and X_3. Under homotheticity, the optimal budget shares for all three commodities depend only on prices and time and are independent of total expenditure. An equivalent characterization of homotheticity is that the ratios of indirect demand functions are all homogeneous of degree zero in X_1, X_2 and X_3.

Partially differentiating equation (3.14) with respect to $\ln X_j (j = 1, 2, 3)$, we obtain:

$$\frac{\partial - \ln U}{\partial \ln X_j} = \frac{\partial F}{\partial \ln H} \frac{\partial \ln H}{\partial \ln X_j}, \qquad (j = 1, 2, 3). \tag{3.15}$$

Second, differentiating again with respect to $\ln X_k (k = 1, 2, 3)$, we obtain:

$$\frac{\partial^2 - \ln U}{\partial \ln X_k \partial \ln X_j} = \frac{\partial^2 F}{\partial \ln H} \frac{\partial \ln H}{\partial \ln X_k} \cdot \frac{\partial \ln H}{\partial \ln X_j} + \frac{\partial F}{\partial \ln H} \frac{\partial^2 \ln H}{\partial \ln X_k \partial \ln X_j}, \qquad (j, k = 1, 2, 3). \tag{3.16}$$

Finally, summing over k and using homogeneity of degree one of the function H, we can write:

$$\sum \frac{\partial^2 - \ln U}{\partial \ln X_k \partial \ln X_j} = \frac{\partial^2 F}{\partial \ln H} \frac{\partial \ln H}{\partial \ln X_j}, \qquad (j = 1, 2, 3). \tag{3.17}$$

Given homotheticity, equations (3.17) must hold everywhere; in particular, they must hold at the point of approximation, where we can identify the first and second partial derivatives with the parameters of the direct translog utility function with time-varying preferences:

$$\sum \frac{\partial^2 - \ln U}{\partial \ln X_k \partial \ln X_j} = \sum \beta_{kj} = \beta_{Mj}, \qquad (j = 1, 2, 3),$$

and:

$$\frac{\partial - \ln U}{\partial \ln X_j} = \alpha_j, \qquad (j = 1, 2, 3).$$

Given homotheticity, the parameters of the direct translog utility function must satisfy the restrictions:

$$\beta_{M1} = \sigma\alpha_1, \qquad \beta_{M2} = \sigma\alpha_2, \qquad \beta_{M3} = \sigma\alpha_3, \tag{3.18}$$

where σ is a constant given by:

$$\sigma = \frac{\partial^2 F/(\partial \ln H)}{\partial F/(\partial \ln H)}$$

We introduce one new parameter, σ, so that these restrictions reduce the number of parameters by two, leaving nine unknown parameters to be estimated.

The translog approximation to a homothetic direct utility function is not necessarily homothetic, even though it must satisfy the restrictions given in equation (3.18) above. For a direct translog utility function to be homothetic, the ratios of the indirect demand functions generated by the direct translog utility function must be homogeneous of degree zero in the quantities consumed. We refer to a direct translog utility function as *intrinsically homothetic* if it is itself homothetic. Two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for intrinsic homotheticity of the direct translog utility function. The first set consists of the restrictions given in equation (3.18) above and the additional restriction:

$$\sigma = 0. \tag{3.19}$$

We refer to this set of restrictions as *explicit homotheticity* restrictions. Under the explicit homotheticity restrictions, only eight unknown parameters remain to be estimated.

A second set of restrictions that implies intrinsic homotheticity of the direct translog utility function is that σ is different from zero, but that the ratios of all pairs of optimal budget shares are constant for all prices, total expenditure and time. This means that the parameters of the direct translog utility function must satisfy:

$$\begin{aligned}
&\alpha_1\beta_{12} = \alpha_2\beta_{11}, \quad && \alpha_1\beta_{13} = \alpha_3\beta_{11}, \quad && \alpha_2\beta_{13} = \alpha_3\beta_{12}, \\
&\alpha_1\beta_{22} = \alpha_2\beta_{12}, && \alpha_1\beta_{23} = \alpha_3\beta_{12}, && \alpha_2\beta_{23} = \alpha_3\beta_{22}, \\
&\alpha_1\beta_{23} = \alpha_2\beta_{13}, && \alpha_1\beta_{33} = \alpha_3\beta_{13}, && \alpha_2\beta_{33} = \alpha_3\beta_{23}, \\
&\alpha_1\beta_{2t} = \alpha_2\beta_{1t}, && \alpha_1\beta_{3t} = \alpha_3\beta_{1t}, && \alpha_2\beta_{3t} = \alpha_3\beta_{2t},
\end{aligned} \tag{3.20}$$

not all of which are independent. In other words, the second order parameters of each commodity must be in the same proportion as the first order parameters. If the ratios of all pairs of optimal budget shares are constant, the direct utility function takes the form:

$$-\ln U = F(\delta_1 \ln X_1 + \delta_2 \ln X_2 + \delta_3 \ln X_3, t) \tag{3.21}$$

where δ_1, δ_2, and δ_3 are constants. We refer to such a utility function as *neutral linear logarithmic*. This condition is much more restrictive than homotheticity or explicit homotheticity and we will discuss it in more detail in Section 3.4.

A direct utility function with time-varying preferences is *homogeneous* if it can be written in the form:

$$-\ln U = \ln H(X_1, X_2, X_3, t), \tag{3.22}$$

where H is a homogeneous function of degree one in X_1, X_2 and X_3. Homogeneity is, of course, a specialization of homotheticity. Under homogeneity the parameters of the direct translog utility function must satisfy the explicit homotheticity restrictions given in equation (3.19) above and the additional restriction:

$$\beta_{Mt} = 0. \tag{3.23}$$

We refer to this set of restrictions as *homogeneity* restrictions. Under these restrictions only seven unknown parameters remain to be estimated. We note that the translog approximation to a homogeneous direct utility function is necessarily homogeneous.

An alternative form of homotheticity of preferences is *groupwise homotheticity*. A direct utility function with time-varying preferences that is groupwise homothetic in X_1 and X_2 can be written in the form:

$$-\ln U = F(\ln H(X_1, X_2, X_3, t), X_3, t) \tag{3.24}$$

where H is homogeneous of degree one in the quantities X_1 and X_2. Under groupwise homotheticity in X_1 and X_2 the ratio of the indirect demand functions of X_1 and X_2 is homogeneous of degree zero in X_1 and X_2. In other words, the ratio of the indirect demands remains invariant under proportional changes in the quantities consumed of X_1 and X_2. Under groupwise homotheticity the parameters of the direct translog utility function must satisfy the restrictions:

$$\beta_{11} + \beta_{12} = \sigma_{12}\alpha_1, \qquad \beta_{12} + \beta_{22} = \sigma_{12}\alpha_2. \tag{3.25}$$

This set of two restrictions involves the introduction of one new parameter, σ_{12}, so that only ten unknown parameters remain to be estimated. Corresponding to the three possible pairs of commodities, there are three possible sets of groupwise homotheticity restrictions. Restrictions analogous to those given in equations (3.25) above must hold for any one of the three possible sets of groupwise homotheticity restrictions.

The translog approximation to a groupwise homothetic direct utility function is not necessarily groupwise homothetic. For a direct translog utility function to be groupwise homothetic, the ratio of the indirect demand functions of X_1 and X_2 generated by the direct translog utility function must be homogeneous of degree zero in X_1 and X_2. We shall refer to a direct translog utility function as *intrinsically groupwise homothetic* if it is itself groupwise homothetic. Two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for intrinsic groupwise homotheticity of the direct translog utility function. The first set consists of the restrictions given in equations (3.25) above and the additional restriction:

$$\sigma_{12} = 0. \tag{3.26}$$

We refer to this set of restrictions as *explicit groupwise homotheticity* restrictions.

Under the explicit groupwise homotheticity restrictions, only nine unknown parameters remain to be estimated.

A second set of restrictions that implies intrinsic groupwise homotheticity of the direct translog utility function is that σ_{12} is different from zero, but that the ratio of the optimal budget shares of X_1 and X_2 is constant for all prices, total expenditure and time. This is precisely the case of groupwise neutral linear logarithmic utility discussed in Section 3.2 above with the restrictions given in equation (3.9). Corresponding to the three possible pairs of commodities, there are three possible sets of explicit groupwise homotheticity restrictions. Resirictions analogous to those given in equation (3.26) above must hold for any one of the three possible sets of explicit groupwise homotheticity restrictions.

A direct utility function with time-varying preferences is *inclusively groupwise homothetic* in X_1 and X_2 if it can be written in the form:

$$-\ln U = F(\ln H(X_1, X_2, X_3, t), t), \tag{3.27}$$

where H is homogeneous of degree one in the quantities X_1 and X_2. Given groupwise homotheticity, this condition implies in addition that the ratios of all the indirect demand functions are homogeneous of degree zero in the quantities X_1 and X_2. Under *inclusive groupwise homotheticity* in X_1 and X_2 the parameters of the direct translog utility function must satisfy the groupwise homotheticity restrictions given in equation (3.25) above and the additional restriction:

$$\beta_{13} + \beta_{23} = \sigma_{12}\alpha_3. \tag{3.28}$$

Under the inclusive groupwise homotheticity restrictions, only nine unknown parameters remain to be estimated. Again, there are three possible sets of inclusive groupwise homotheticity restrictions corresponding to the three possible sets of groupwise homotheticity restrictions. Restrictions analogous to those given in equation (3.28) must hold for any one of the three possible sets of groupwise homotheticity restrictions.

The translog approximation to a inclusively groupwise homothetic direct utility function is not necessarily inclusively groupwise homothetic. For a direct translog utility function to be inclusively groupwise homothetic, the ratios of all pairs of indirect demand functions generated by the direct translog utility function must be homogeneous of degree zero in X_1 and X_2. As before, two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for inclusive groupwise homotheticity of the direct translog utility function. The first set consists of the restrictions given in equations (3.28) above and the additional restriction:

$$\sigma_{12} = 0. \tag{3.29}$$

We refer to this set of restrictions as *explicit inclusive groupwise homotheticity restrictions*. Under this set of restrictions, only eight unknown parameters remain to be estimated.

A second set of restrictions that implies intrinsic inclusive groupwise homotheticity of the direct translog utility function is that σ_{12} is different from zero but that the direct utility function is groupwise neutral linear logarithmic. Corresponding to the three possible pairs of commodities, there are three possible sets of explicit inclusive groupwise homotheticity restrictions. Restrictions analogous to those given in equation (3.9) above must hold for any one of the three possible sets of explicit inclusive groupwise homotheticity restrictions.

Finally, direct utility function with time-varying preferences is *groupwise homogeneous* if it can be written in the form:

$$(3.30) \qquad -\ln U = \ln H(X_1, X_2, X_3, t),$$

where H is homogeneous of degree one in the quantities X_1 and X_2. Groupwise homogeneity is, of course, a specialization of inclusive groupwise homotheticity which is in turn a specialization of groupwise homotheticity. Under groupwise homogeneity the parameters of the direct translog utility function must satisfy the explicit inclusive groupwise homotheticity restrictions given in equation (3.29) above and the additional restriction:

$$(3.31) \qquad \beta_{1t} + \beta_{2t} = 0.$$

We refer to this set of restrictions as *groupwise homogeneity* restrictions. Under these restrictions only seven unknown parameters remain to be estimated. We note that the translog approximation to a groupwise homogeneous direct utility function is not necessarily groupwise homogeneous. Corresponding to the three possible pairs of commodities, there are three possible sets of groupwise homogeneity restrictions. Restrictions analogous to those given in equation (3.31) must hold for any one of the three possible sets of groupwise homogeneity restrictions.

We conclude this section by noting that groupwise homotheticity in all possible groups is neither necessary nor sufficient for homotheticity of the direct utility function. Even explicit groupwise homotheticity in all possible groups is not sufficient for homotheticity of the direct utility function. On the other hand, inclusive groupwise homotheticity in all possible groups is sufficient, but not necessary, for homotheticity. Inclusive groupwise homotheticity in all possible groups implies linear logarithmic utility. Finally, explicit inclusive groupwise homotheticity in all possible groups implies explicit linear logarithmic utility and groupwise homogeneity in all possible groups implies neutral linear logarithmic utility.

3.4. *Groupwise linear logarithmic utility*

A direct utility function with time-varying preferences that is *groupwise homothetically separable* in X_1 and X_2 from X_3 can be written in the form:

$$(3.32) \qquad -\ln U = F(\ln H(X_1, X_2, t), X_3, t),$$

where H is a homogeneous function of degree one and depends only on X_1, X_2 and time. A necessary and sufficient conditions for a direct utility function to be groupwise homothetically separable in X_1 and X_2 from X_3 is that the function is both groupwise separable and groupwise homothetic in X_1 and X_2.

Groupwise homothetic separability implies that the ratio of the indirect demand functions is independent of X_3 and is homogeneous of degree zero in X_1 and X_2. The translog approximation to a groupwise homothetically separable direct utility function is not necessarily groupwise homothetically separable.

For a direct translog utility function to be itself groupwise homothetically separable, the ratio of the indirect demand functions of X_1 and X_2 generated from a direct translog utility function must be independent of X_3 and homogeneous of degree zero in X_1 and X_2. We refer to a direct translog utility function as *intrinsically groupwise homothetically separable* if it is groupwise homothetically separable.

As before, two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for intrinsic groupwise homothetic separability of the direct translog utility function. The first consists of the combination of the explicit groupwise separability. given in equation (3.8) above, and explicit groupwise homotheticity, given in equation (3.26) above. We refer to the conjunction of these two sets of restrictions as the *explicit groupwise homothetic separability* restrictions. A second set of restrictions that implies intrinsic groupwise homothetic separability of the direct translog utility function is that of groupwise neutral linear logarithmic utility, given in equation (3.9) above.

A direct utility function U with time-varying preferences is *groupwise linear logarithmic* if it can be written in the form:

$$-\ln U = F(\delta_1(t)\ln X_1 + \delta_2(t)\ln X_2, X_3, t), \tag{3.33}$$

where $\delta_1(t)$ and $\delta_2(t)$ are functions only of time. A necessary and sufficient condition for groupwise linear logarithmic utility in X_1 and X_2 is that the ratio of the optimal budget shares of X_1 and X_2 is independent of all prices and total expenditure and depends only on time. Given groupwise homothetic separability in X_1 and X_2 from X_3, groupwise linear logarithmic utility in X_1 and X_2 requires the additional restriction:

$$\alpha_1\beta_{12} = \alpha_2\beta_{11}. \tag{3.34}$$

Under these restrictions only eight unknown parameters remain to be estimated. There are three possible sets of groupwise linear logarithmic utility restrictions and restrictions analogous to those given in equation (3.34) must hold for any one of them.

The translog approximation of a groupwise linear logarithmic direct utility function is not necessarily groupwise linear logarithmic. For a direct translog utility function to be itself groupwise linear logarithmic, the ratio of the optimal budget shares of X_1 and X_2 generated from a direct translog utility function must depend only on time. We shall refer to a direct translog utility function as *intrinsically groupwise linear logarithmic* if it is itself groupwise linear logarithmic. As before, two alternative sets of restrictions on the parameters of the direct translog utility function are jointly necessary and sufficient for intrinsic groupwise linear logarithmic utility. The first consists of the explicit groupwise homothetic separability restrictions and the additional restriction:

$$\beta_{12} = 0. \tag{3.35}$$

Under these restrictions only six unknown parameters remain to be estimated. We refer to these restrictions as *explicit groupwise linear logarithmic utility* restrictions. A second set of restrictions that implies intrinsic groupwise linear logarithmic utility is that of groupwise neutral linear logarithmic utility, given in equation (3.9) above. Corresponding to the three possible pairs of commodities, there are three possible sets of *explicit groupwise linear logarithmic utility* restrictions. Restrictions analogous to those given in equation (3.35) must hold for any one of them.

A direct utility function with time-varying preferences is *linear logarithmic* in X_1, X_2, and X_3 if it can be written in the form:

$$(3.36) \qquad -\ln U = F(\delta_1(t)\ln X_1 + \delta_2(t)\ln X_2 + \delta_3(t)\ln X_3, t),$$

where $\delta_1(t)$, $\delta_2(t)$ and $\delta_3(t)$ are functions only of time. A necessary and sufficient condition for linear logarithmic utility is that the direct utility function is groupwise linear logarithmic in every pair of the three commodities. In particular, since there are only three commodities, groupwise linear logarithmic utility for any two pairs of commodities is sufficient for linear logarithmic utility.

A direct utility function U with time-varying preferences is *explicitly linear logarithmic* if it can be written in the form:

$$(3.37) \qquad -\ln U = \delta_1(t)\ln X_1 + \delta_2(t)\ln X_2 + \delta_3(t)\ln X_3 + F(t).$$

The translog approximation to an explicitly linear logarithmic utility function is necessarily explicitly linear logarithmic. A necessary and sufficient condition for *explicit linear logarithmic utility* is that the direct translog utility function is explicitly groupwise linear logarithmic in every pair of the three commodities. In particular, since there are only three commodities, explicit groupwise linear logarithmic utility for any two pairs of commodities is sufficient. Given linear logarithmic utility, explicit groupwise linear logarithmic utility in any one of the three possible pairs implies that the direct utility function is explicitly linear logarithmic. For an explicitly linear logarithmic utility function the budget shares of all commodities are independent of prices and total expenditure, depending only on time.

Finally, a direct utility function U with time-varying preferences is *neutral linear logarithmic* if it can be written in the form:

$$(3.38) \qquad -\ln U = F(\delta_1 \ln X_1 + \delta_2 \ln X_2 + \delta_3 \ln X_3, t),$$

where δ_1, δ_2 and δ_3 are constants. Two alternative sets of conditions are jointly necessary and sufficient for neutral linear logarithmic utility. First, the direct translog utility function is both neutral and linear logarithmic and it is either explicitly neutral, explicitly linear logarithmic, or both. Alternatively, the direct translog utility function satisfies the restrictions given in equation (3.20), that is, the neutral linear logarithmic utility restrictions. In either case, the empirical implications are identical—the budget shares of all commodities are constant.

3.5. *Groupwise equal rates of commodity augmentation*

As an alternative point of departure for the analysis of time-varying preferences, we suppose that the quantities consumed of X_1, X_2 and X_3 are augmented by factors $A_1(t)$, $A_2(t)$ and $A_3(t)$ respectively, where the augmentation factors are functions only of time. A direct utility function with commodity-augmenting time-varying preferences can be written in the form:

$$(3.39) \qquad -\ln U = F(A_1(t)X_1, A_2(t)X_2, A_3(t)X_3).$$

Without loss of generality, the augmentation factors can be normalized so that they all take the value unity for $t = 0$. Without further restrictions on the function F commodity augmentation is not a testable hypothesis, since it has no empirical implications that can be refuted. Even if one restricts each augmentation factor to be drawn from the family of one-parameter algebraic functions, commodity augmentation is still not a testable hypothesis since the parameters α_t and β_{it} are not identified.

A direct utility function with time-varying preferences that is characterized by *groupwise equal rates of commodity augmentation* can be written in the form:

$$-\ln U = F(A(t)X_1, A(t)X_2, A_3(t)X_3). \tag{3.40}$$

The cross partial derivatives of $-\ln U$ with respect to time and $\ln X_1$, $\ln X_2$ or $\ln X_3$ are given by:

$$\begin{aligned}
\frac{\partial^2 -\ln U}{\partial \ln X_1 \, \partial t} &= \frac{\partial^2 F}{\partial \ln X_1^2} \cdot \frac{\dot{A}}{A} + \frac{\partial^2 F}{\partial \ln X_1 \, \partial \ln X_2} \cdot \frac{\dot{A}}{A} \\
&\quad + \frac{\partial^2 F}{\partial \ln X_1 \, \partial \ln X_3} \cdot \frac{\dot{A}_3}{A_3}, \\
\frac{\partial^2 -\ln U}{\partial \ln X_2 \, \partial t} &= \frac{\partial^2 F}{\partial \ln X_1 \, \partial \ln X_2} \cdot \frac{\dot{A}}{A} + \frac{\partial^2 F}{\partial \ln X_2^2} \cdot \frac{\dot{A}}{A} \\
&\quad + \frac{\partial^2 F}{\partial \ln X_2 \, \partial \ln X_3} \cdot \frac{\dot{A}_3}{A_3}, \\
\frac{\partial^2 -\ln U}{\partial \ln X_3 \, \partial t} &= \frac{\partial^2 F}{\partial \ln X_1 \, \partial \ln X_3} \cdot \frac{\dot{A}}{A} + \frac{\partial^2 F}{\partial \ln X_2 \, \partial \ln X_3} \cdot \frac{\dot{A}_3}{A} \\
&\quad + \frac{\partial^2 F}{\partial \ln X_3^2} \cdot \frac{\dot{A}_3}{A_3}.
\end{aligned} \tag{3.41}$$

By observing that:

$$\frac{\partial^2 F}{\partial \ln X_i \, \partial \ln X_j} = \frac{\partial^2 -\ln U}{\partial \ln X_i \, \partial \ln X_j}, \qquad (i, j = 1, 2, 3), \tag{3.42}$$

and the fact that equation (3.41) must hold everywhere, in particular, at the point of approximation where $t = 0$, we can identify the first and second partial derivatives of $-\ln U$ with the parameters of the direct translog utility function with time-varying preferences. Groupwise equal rates of commodity augmentation in X_1 and X_2 implies the following sets of restrictions:

$$\begin{gathered}
\beta_{1t} = \beta_{11}\lambda + \beta_{12}\lambda + \beta_{13}\lambda_3, \qquad \beta_{2t} = \beta_{12}\lambda + \beta_{22}\lambda + \beta_{23}\lambda_3, \\
\beta_{3t} = \beta_{13}\lambda + \beta_{23}\lambda + \beta_{33}\lambda_3,
\end{gathered} \tag{3.43}$$

where:

$$\lambda = \frac{\dot{A}}{A}, \qquad \lambda_3 = \frac{\dot{A}_3}{A_3},$$

are the rates of commodity augmentation at the point of approximation. We note that this set of three restrictions involves the introduction of two new parameters. λ and λ_3 in the example given above. Hence under groupwise equal rates of commodity augmentation only ten unknown parameters remain to be estimated. Restrictions analogous to those given in equation (3.43) must hold for the two remaining possible sets of groupwise equal rates of commodity augmentation restrictions.

A necessary and sufficient condition for groupwise equal rates of commodity augmentation of the direct utility function in X_1 and X_2 is that there exist two scalars η and η_3 for every t such that:

$$\text{(3.44)} \qquad \frac{[\partial U/\partial X_1](X_1, X_2, X_3, t)}{[\partial U/\partial X_2](X_1, X_2, X_3, t)} = \frac{[\partial U/\partial X_1](\eta X_1, \eta X_2, \eta_3 X_3, 0)}{[\partial U/\partial X_2](\eta X_1, \eta X_2, \eta_3 X_3, 0)}.$$

In other words, at every t there exist a proportional scaling of X_1 and X_2, and a scaling for X_3, so that the ratio of the indirect demands at time zero is the same as the ratio of the indirect demands at time t. We can verify directly that a translog approximation to a direct utility function with time-varying preferences characterized by groupwise equal rates of commodity augmentation is always characterized by groupwise equal rates of commodity augmentation.

A direct utility function U with time-varying preferences that is characterized by *groupwise zero rates of commodity augmentation* can be written in the form:

$$\text{(3.45)} \qquad -\ln U = F(X_1, X_2, A_3(t)X_3).$$

The corresponding restrictions on the parameters of the indirect translog utility function with time-varying preferences can be obtained from equation (3.43) above by setting λ equal to zero. Under groupwise zero rates of commodity augmentation the parameters must satisfy the restrictions:

$$\text{(3.46)} \qquad \beta_{1t} = \beta_{13}\lambda_3, \qquad \beta_{2t} = \beta_{23}\lambda_3, \qquad \beta_{3t} = \beta_{33}\lambda_3.$$

Under these restrictions, only nine unknown parameters remain to be estimated. Restrictions analogous to those given in equation (3.46) must hold for the two remaining possible sets of groupwise zero rates of commodity augmentation. As before, we can show that the translog approximation to a direct utility function with time-varying preferences characterized by groupwise zero rates of commodity augmentation is always characterized by groupwise zero rates of commodity augmentation.

A direct utility function with time-varying preferences is characterized by equal rates of commodity augmentation in X_1, X_2 and X_3 if it can be written in the form:

$$\text{(3.47)} \qquad -\ln U = G(A(t)X_1, A(t)X_2, A(t)X_3).$$

A necessary and sufficient condition for equal rates of commodity augmentation of the direct utility function is that the direct utility function is characterized by groupwise equal rates of commodity augmentation in every pair of the three commodities. In particular, since there are only three commodities, groupwise equal rates of commodity augmentation for any two pairs of commodities is sufficient for equal rates of commodity augmentation.

Finally, a direct utility function with time-varying preferences is characterized by *zero rates of commodity augmentation* if and only if it is characterized by groupwise zero rates of commodity augmentation in every pair of the three commodities. In particular, since there are only three commodities, groupwise zero rates of commodity augmentation for any two pairs of commodities is sufficient. In fact, given equal rates of commodity augmentation, zero rates of commodity augmentation for any pair of commodities implies zero rates of augmentation. In this case the direct utility function is also explicitly neutral.

3.6. *Duality*

The implications of separability, homotheticity, linear logarithmic utility and equal rates of commodity augmentation for the indirect utility function with time-varying preferences are strictly analogous to the corresponding properties for the direct utility function with time-varying preferences. They impose restrictions on the direct demand functions as opposed to the indirect demand functions. Similarly, the parametric restrictions implied by these properties of the indirect translog utility functions are strictly analogous to the parametric restrictions implied by the corresponding properties of the direct translog function. The roles of quantities consumed and ratios of prices to total expenditure are, of course, interchanged.

However, a given property of the direct utility function need not imply the same property of the indirect utility function. For example, a groupwise homothetic direct utility function does not correspond to a groupwise homothetic indirect utility function. The direct utility function is inclusively groupwise homothetic if and only if the indirect utility function is inclusively groupwise homothetic. Since homotheticity implies groupwise inclusive homotheticity for the group consisting of all commodities, direct homotheticity is equivalent to indirect homotheticity. An alternative sufficient condition for groupwise homotheticity of both the direct and indirect utility functions is groupwise separability (either direct or indirect) in the same group of commodities.

Similarly, a groupwise commodity separable direct utility function does not correspond to a groupwise commodity separable indirect utility function. Direct and indirect utility functions are groupwise commodity separable in the same group of commodities if and only if the utility function (either direct or indirect) is also groupwise homothetic in the same group of commodities. In addition, the direct utility function is groupwise homothetically commodity separable if and only if the indirect utility function is groupwise homothetically commodity separable.

In general, a groupwise time-separable direct utility function does not correspond to a groupwise time separable indirect utility function. Two alternative sufficient conditions for groupwise time separability of both the direct and the indirect utility functions in the same group of commodities are, first, inclusive groupwise homotheticity of the utility function (either direct or indirect) in the same group of commodities and, second, groupwise homothetic commodity separability of the utility function (either direct or indirect) in the same group of commodities.

An additive direct utility function does not correspond to an additive indirect utility function. Direct and indirect utility functions are simultaneously additive only if the utility function (either direct or indirect) is homothetic or if the utility function (either direct or indirect) is linear logarithmic in all but one of the commodities.[13] In addition, the direct utility function is additive and homothetic if and only if the indirect utility function is additive and homothetic. On the other hand, a neutral direct utility function always corresponds to a neutral indirect utility function. A groupwise linear logarithmic direct utility function always corresponds to a groupwise linear logarithmic indirect utility function. Since a groupwise linear logarithmic utility function is groupwise homothetically commodity separable, a groupwise neutral linear logarithmic direct utility function always corresponds to a groupwise neutral linear logarithmic indirect utility function.

Moreover, a direct utility function with time-varying preferences characterized by groupwise equal rates of commodity augmentation always corresponds to an indirect utility function with time-varying preferences characterized by groupwise equal rates of commodity augmentation. Likewise, a direct utility function with time-varying preferences characterized by groupwise zero rates of commodity augmentation always corresponds to an indirect utility function with time-varying preferences characterized by groupwise zero rates of commodity augmentation.[14]

Finally, a utility function is *self-dual* if both the direct and the indirect utility functions (corresponding to the same preferences) have the same functional form.[15] The only translog utility function which is self-dual is the neutral linear logarithmic utility function. Neutral linear logarithmic utility functions are the only intrinsically additive, homothetic, and stationary direct or indirect translog utility functions. Direct and indirect translog utility functions can represent the same preferences if and only if they are neutral linear logarithmic. Unless this stringent condition is met, the direct and indirect translog approximations to a given pair of direct and indirect utility functions correspond to different preferences, so that the properties of these approximations are not fully comparable.

4. Empirical Results

4.1. *Summary of tests*

Tests of the restrictions on preferences we have considered can be carried out in many sequences. We propose to test restrictions on the structure of preferences, given equality and symmetry restrictions, but not monotonicity and quasiconvexity restrictions. Monotonicity and quasiconvexity restrictions take the form of inequalities rather than equalities, so that these restrictions do not affect the asymptotic distributions of our statistics for tests of restrictions on the structure of preferences.[16] These distributions are the same with or without

[15] See Samuelson [1965] and Houthakker [1965]. We may also mention the "self-dual addilog system" introduced by Houthakker [1965]. This system is not generated by additive utility functions except for special cases.

[13] This is the special case introduced by Hicks [1969]. See also Samuelson [1969].

[14] For some of these results on the duality of direct and indirect utility functions, see Houthakker [1960], Samuelson [1960] and Lau [1969b].

[16] See Malinvaud [1970], pp. 366–368.

imposing the restrictions associated with monotonicity and quasiconvexity. After the set of acceptable restrictions on the structure of preferences is determined, we can impose the constraints implied by monotonicity and quasiconvexity of the direct or indirect utility function.

Our proposed test procedure is presented in diagrammatic form in a series of five figures. We propose to test the restrictions derived from groupwise separability, homotheticity, groupwise homotheticity, and commodity augmenting change in preferences, in parallel. Given groupwise homothetic separability for any group, we proceed to test the additional restrictions implied by groupwise linear logarithmic utility, conditional on the restrictions implied by groupwise homothetic separability. Given the outcome of these tests we can determine the set of acceptable restrictions on the structure of preferences.

Beginning with separability, we recall that, first, groupwise separability for two of the three possible groups of two commodities from the third commodity implies groupwise separability for the third group and additivity of the utility function. Likewise, explicit groupwise separability for two of the three possible groups implies explicit groupwise separability for the third and explicit additivity of the utility function. Second, groupwise separability for two of the three possible groups of two commodities from time implies groupwise separability of the third group from time and neutrality of the utility function. Likewise, explicit groupwise separability for two of the three possible groups from time implies explicit groupwise separability of the third group from time and explicit neutrality of the utility function.

We first test groupwise separability restrictions for each possible group. If we accept groupwise separability for any group, we proceed to test explicit groupwise separability for that group. If we accept the hypothesis of groupwise separability from the third commodity for any two of the three possible groups, we accept the hypothesis of additivity. If we accept the hypothesis of explicit groupwise separability from the third commodity for any two of the three groups, we accept the hypothesis of explicit additivity. If we accept the hypothesis of groupwise separability from time for any two of the three possible groups, we accept the hypothesis of neutrality. If we accept the hypothesis of explicit groupwise separability from time for any two of the three groups, we accept the hypothesis of explicit neutrality.

Our test procedure for separability is presented diagrammatically in Figure 1. There are three sets of tests of this type; the diagram gives only one set of such tests. For each group we test groupwise separability from the third commodity and from time. Conditional on the corresponding groupwise separability restrictions, we proceed to test the hypothesis of explicit groupwise separability from the third commodity and from time. Combining results from the tests for each of the three commodity groups, we can test the hypotheses of additivity, explicit additivity, neutrality, and explicit neutrality.

Continuing with homotheticity, we first test groupwise homotheticity restrictions for each possible group. In parallel we test homotheticity restrictions for the group consisting of all three commodities. If we accept homotheticity for all three commodities, we proceed to test explicit homotheticity. If we accept explicit homotheticity for all three commodities, we proceed to test homogeneity.

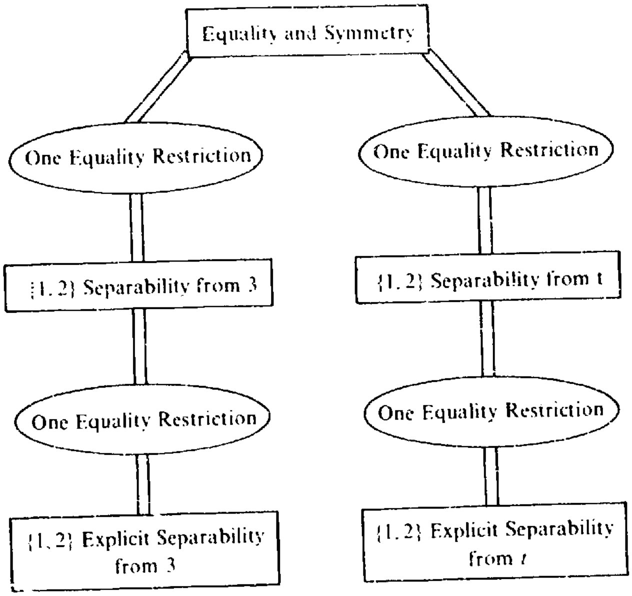

Figure 1 Tests of Separability. (There are three sets of tests of this type: this diagram gives only one set of such tests corresponding to the group {1, 2}.)

Our test procedure for homotheticity, homotheticity, explicit homotheticity, and homogeneity is presented diagrammatically in Figure 2.

If we accept groupwise homotheticity for any group, we proceed to test explicit groupwise homotheticity and inclusive groupwise homotheticity for that group in parallel. If we accept both explicit groupwise homoetheticity and inclusive groupwise homotheticity for any group, we accept the hypothesis of explicit groupwise inclusive homotheticity. Conditional on explicit groupwise homotheticity for any group, we proceed to test groupwise homogeneity for that group. Our test procedure for explicit and inclusive groupwise homotheticity is presented diagrammatically in Figure 3. There are three sets of tests of this type; the diagram gives only one set of such tests.

We observe that a utility function with time-varying preferences is characterized by linear logarithmic utility if it is groupwise linear logarithmic in all three possible groups consisting of two commodities each. Inclusive groupwise homotheticity for all three groups implies that the utility function is linear logarithmic; if we accept inclusive groupwise homotheticity for all three groups, we accept the hypothesis of linear logarithmic utility. If we accept explicit inclusive groupwise homotheticity for all three groups, we accept the hypothesis of explicit linear logarithmic utility. Finally, if we accept groupwise homogeneity for all three groups, we accept the hypothesis of neutral linear logarithmic utility.

We can combine the results of our parallel tests of separability and homotheticity in order to draw conclusions about homothetic separability. If we accept the hypothesis of groupwise separability for a group consisting of two commodities from the third, and for the same group we accept the hypotheses of groupwise

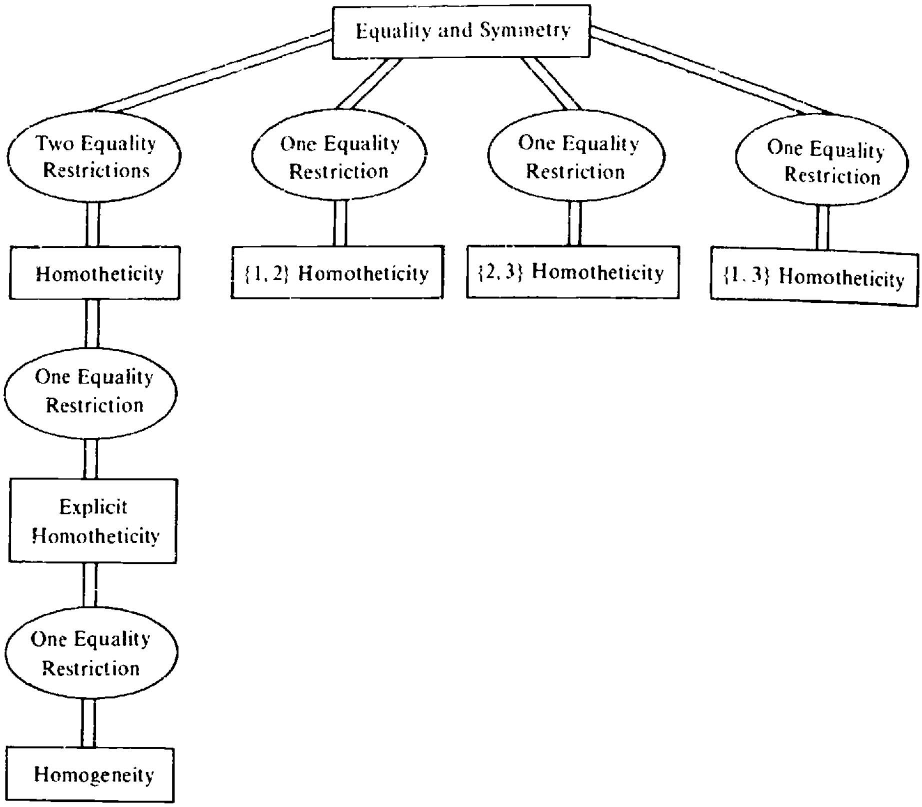

Figure 2 Tests of Homotheticity.

homotheticity, explicit groupwise homotheticity, inclusive groupwise homotheticity, or groupwise homogeneity, we accept the hypotheses of groupwise homothetic separability, groupwise explicitly homothetic separability, groupwise inclusive homothetic separability, or groupwise homogeneous separability, respectively, for that group. Similarly, if we accept the hypothesis of explicit groupwise separability for a given group, and for the same group we accept the hypothesis of groupwise homotheticity, explicit groupwise homotheticity, inclusive groupwise homotheticity and groupwise homogeneity, we accept the hypotheses of groupwise homothetic explicit separability, explicit groupwise homothetic separability, groupwise inclusive homothetic explicit separability and explicit groupwise homogeneous separability, respectively, for that group. Finally, if we accept the hypotheses of additivity and homotheticity, we accept the hypothesis of homothetic additivity. If we accept the hypotheses of explicit additivity and either explicit homotheticity or homogeneity, we accept the hypotheses of explicit linear logarithmic utility and neutral linear logarithmic utility, respectively.

Proceeding under the hypothesis of additivity, if we accept inclusive groupwise homotheticity of any one of the three possible groups of two commodities each, we accept the hypothesis of groupwise linear logarithmic utility for that group. If we accept inclusive groupwise homotheticity of any two of the three possible

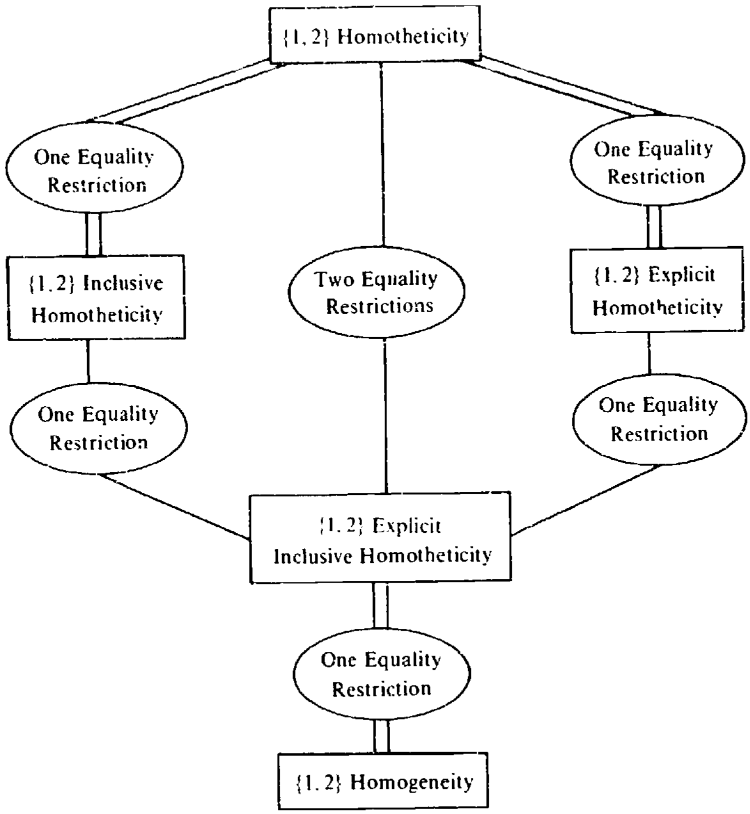

Figure 3 Tests of Groupwise Homotheticity. (There are three sets of tests of this type: this diagram gives only one set of such tests corresponding to the group {1, 2}.)

groups of two commodities each, we accept linear logarithmic utility of the utility function. If we accept explicit inclusive groupwise homotheticity of any one of the three possible groups of two commodities each, we accept the hypothesis of explicit groupwise linear logarithmic utility for that group. If we accept explicit inclusive groupwise homotheticity of any two of the three possible groups of two commodities each, we accept the hypothesis of explicit linear logarithmic utility of the utility function.

Alternatively, proceeding under the hypothesis of explicit additivity, if we accept inclusive groupwise homotheticity of any one of the three possible groups of two commodities each, we also accept the hypothesis of explicit groupwise linear logarithmic utility for that group. If we accept inclusive groupwise homotheticity of any two of the three possible groups of two commodities each, we accept the hypothesis of explicit linear logarithmic utility.

If we accept the hypothesis of groupwise homothetic separability for all three possible groups of two commodities each and, in addition, we accept the hypothesis of inclusive groupwise homotheticity of any one of the three possible groups of two commodities each, we accept the hypothesis of linear logarithmic utility. If either of these two hypotheses are strengthened to hold explicitly, we accept the hypothesis of explicit linear logarithmic utility.

If we accept the hypothesis of groupwise homothetic separability for any group of two commodities from the third, we proceed to test the hypothesis of groupwise linear logarithmic utility for that group, conditional on groupwise homothetic separability. If we accept the hypothesis of groupwise linear logarithmic utility for group consisting of two commodities, and for that group we accept any two of the three hypotheses of explicit groupwise separability, explicit groupwise homotheticity, and inclusive groupwise homotheticity, we accept the hypothesis of explicit linear logarithmic utility for that group. If, in addition, we accept the hypothesis of groupwise homogeneity for that group, we accept the hypothesis of explicit neutral linear logarithmic utility for that group. If we accept the hypothesis of groupwise linear logarithmic utility for any two of the three possible commodity groups, we accept the hypothesis of linear logarithmic utility. Our test procedure for groupwise linear logarithmic utility, given groupwise homothetic separability restrictions, is presented digrammatically in Figure 4.

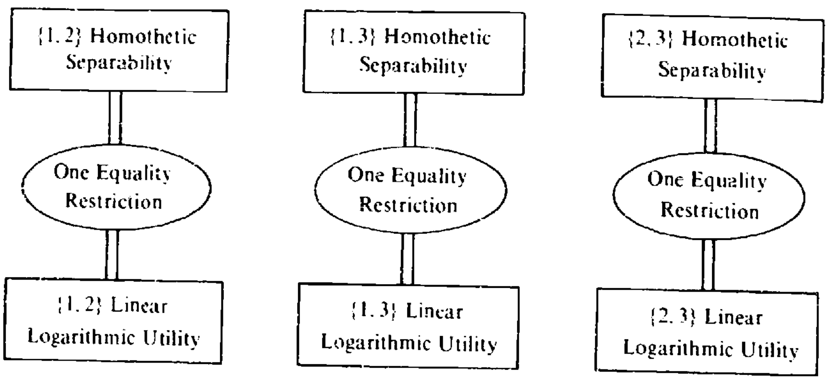

Figure 4 Tests of Linear Logarithmic Utility.

Finally, we consider tests of restrictions associated with commodity augmenting changes of preferences over time. First we test the hypothesis of groupwise equal rates of commodity augmentation for all three possible groups of two commodities each. If we accept the hypothesis of equal rates of commodity augmentation for any two of the three groups, we accept the hypothesis of equal rates of augmentation for all three commodities, and hence for all three groups. There is then no need to test zero rates because equal zero rates for all commodities is implied by explicit neutrality, which has been tested under separability. If we accept the hypothesis of equal rates of commodity augmentation for only a single group of two commodities, we proceed to test the hypothesis that the rate of augmentation for that group is equal to zero. Our test procedure for equal rates of commodity augmentation is presented diagrammatically in Figure 5.

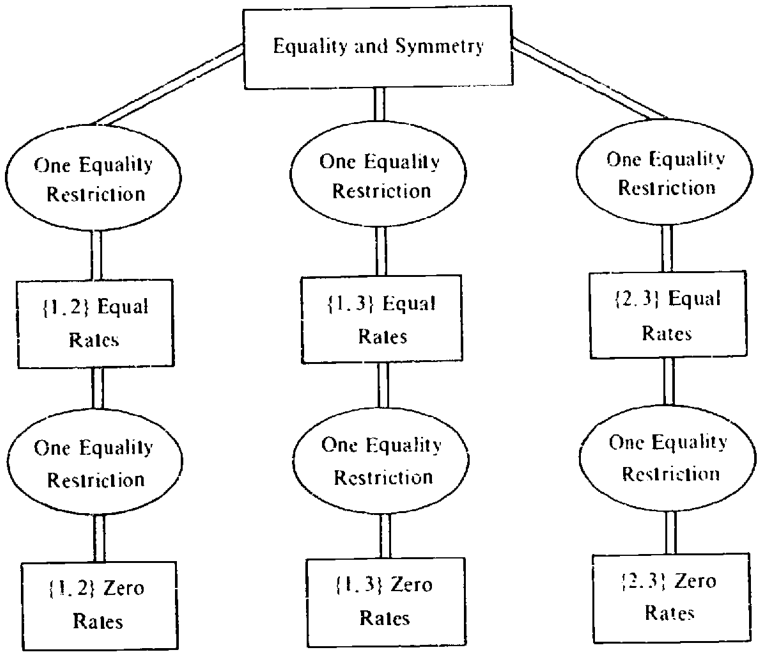

Figure 5 Tests of Commodity-Augmenting Change in Preferences.

4.2. *Estimation*

Our empirical results are based on time series data for prices and quantities of durables, non-durables, and energy and time. We have fitted the equations for budget shares generated by direct and indirect translog utility functions with time-varying preferences, using the stochastic specification outlined above. Under this specification only two equations are required for a complete econometric model of demand. We have fitted equations for durables and for energy.[17] For both direct and indirect specifications we impose the hypothesis that the model of demand is consistent with utility maximization, so that the parameters of this model satisfy equality and symmetry restrictions. Given these restrictions, and the normalization of α_M at minus unity, eleven unknown parameters remain to be estimated in our econometric model. Estimates of these parameters for the direct translog utility function with time-varying preferences are given in the first column of Table 1. Estimates of these parameters for the indirect translog utility function with time-varying preferences are presented in the first column of Table 2.

[17] We employ the maximum likelihood estimator discussed, for example, by Malinvaud [1970], pp. 338–341. For the direct series of tests we assume that the disturbances are independent of the quantities consumed. For the indirect series of tests we assume that the disturbances are independent of the ratios of prices to the value of total expenditure.

TABLE !

PARAMETER ESTIMATES, DIRECT TRANSLOG UTILITY FUNCTION

Parameter	1. Equality and Symmetry		2. Commodity Augmentation		3. Additivity		4. Explicii Additivity		5. Neutrality		6. Explicit Neutrality	
α_1	−0.238	(0.00183)	−0.238	(0.00198)	−0.248	(0.00269)	−0.238	(0.00244)	−0.237	(0.00174)	−0.225	(0.00549)
β_{11}	−0.962	(0.0414)	−0.961	(0.310)	0.0608	(0.0494)	0.101	(0.768)	−1.03	(0.260)	−0.514	(0.0505)
β_{12}	−3.03	(0.102)	−3.03	(0.833)	−0.108	(0.101)			−3.08	(0.728)	−3.35	(0.298)
β_{13}	−0.203	(0.00890)	−0.202	(0.0684)	−0.00805	(0.00752)			−0.210	(0.0587)	−0.0992	(0.0219)
β_{1t}	−0.0705	(0.00253)	0.0705	(0.0705)	−0.0145	(0.00233)	−0.0145	(0.00000)	0.0760	(0.0270)		
α_2	−0.710	(0.00193)	−0.710	(0.00203)	−0.700	(0.00266)	−0.711	(0.00170)	−0.711	(0.00183)	−0.727	(0.00620)
β_{21}	−3.03	(0.102)	−3.03	(0.833)	−0.108	(0.101)			−3.08	(0.728)	−3.15	(0.298)
β_{22}	−8.35	(0.237)	−8.34	(0.236)	−1.51	(0.226)	0.332	(0.105)	−8.37	(2.23)	−5.63	(0.485)
β_{23}	−0.509	(0.0202)	−0.508	(0.164)	−0.0227	(0.0213)			−0.556	(0.156)	−0.477	(0.0556)
β_{2t}	0.217	(0.00000)	0.217	(0.0885)	0.00281	(0.00000)	−0.0481	(0.00523)	0.228	(0.0804)		
α_3	−0.0520	(0.000441)	−0.0520	(0.00047)	−0.0521	(0.00039)	−0.527	(0.00441)	−0.0516	(0.00057)	−0.0484	(0.00111)
β_{31}	−0.203	(0.00890)	−0.202	(0.0684)	−0.00805	(0.00752)			−0.210	(0.0587)	−0.0992	(0.0219)
β_{32}	−0.509	(0.0202)	−0.508	(0.164)	−0.0227	(0.0213)			−0.556	(0.156)	−0.477	(0.0556)
β_{33}	−0.0360	(0.0131)	−0.0360	(0.0153)	−0.0152	(0.0105)	−0.103	(0.0242)	−0.0750	(0.0171)	−0.0811	(0.0212)
β_{3t}	0.0132	(0.000575)	0.0186	(0.0143)	−0.00104	(0.00061)	−0.00558	(0.00273)	0.0165	(0.00590)		
λ_1			−0.0403	(0.0520)								
λ_2			−0.0206	(0.0377)								
λ_3			0.152	(0.290)								
ρ					−0.623	(0.530)						
ρ_t									−0.320	(0.103)		

TABLE 1 (continued)

Parameter	7. Homotheticity		8. Explicit Homotheticity		9. Homogeneity		10. Linear Logarithmic Utility		11. Explicit Linear Logarithmic Utility		12. Neutral Linear Logarithmic Utility	
α_1	−0.240	(0.00127)	−0.242	(0.00258)	−0.244	(0.00318)	−0.242	(0.00201)	−0.244	(0.00193)	−0.236	(0.00619)
β_{11}	−0.593	(0.226)	−0.0250	(0.0461)	−0.105	(0.0284)	−0.00057	(0.00076)				
β_{12}	−2.16	(0.623)	0.0352	(0.048)	0.131	(0.0275)	−0.0943	(0.0242)				
β_{13}	−0.141	(0.0594)	−0.0102	(0.0164)	−0.0251	(0.0118)	−0.00673	(0.00175)	−0.0166	(0.00102)		
β_{1t}	0.0380	(0.0214)	−0.108	(0.00530)	−0.00148	(0.00053)	−0.0150	(0.00120)	−0.707	(0.00193)	−0.714	(0.00673)
α_2	−0.710	(0.00121)	−0.706	(0.00273)	−0.703	(0.00328)	−0.707	(0.00194)				
β_{21}	−2.16	(0.623)	0.0352	(0.048)	0.131	(0.0275)	−0.0943	(0.0242)				
β_{22}	−5.99	(1.91)	−0.0711	(0.0610)	−0.178	(0.0284)	−0.275	(0.716)				
β_{23}	−0.417	(0.417)	0.0359	(0.00848)	0.0476	(0.00438)	−0.0197	(0.00519)	−0.0365	(0.00366)		
β_{2t}	0.135	(0.0630)	−0.0208	(0.0126)	0.00068	(0.00053)	−0.0259	(0.00507)	−0.0495	(0.00057)	−0.0488	(0.00077)
α_3	−0.0506	(0.00058)	−0.519	(0.00063)	−0.0525	(0.00046)	−0.0505	(0.00049)				
β_{31}	−0.141	(0.0594)	−0.0102	(0.0164)	−0.0251	(0.0118)	−0.00673	(0.00175)				
β_{32}	−0.417	(0.147)	0.0359	(0.00848)	0.0476	(0.00438)	−0.0197	(0.00519)				
β_{33}	−0.0531	(0.0148)	−0.0257	(0.0146)	−0.0225	(0.0130)	−0.00140	(0.00038)				
β_{3t}	0.00982	(0.00504)	−0.00085	(0.00104)	0.00081	(0.00022)	−0.00057	(0.00076)	−0.0560	(0.00514)		
σ	12.1	(3.76)					−0.551	(0.134)				
ρ												
ρ_t												

TABLE 1 (continued)

Parameter	13. {1, 2} Separability from 3		14. {1, 3} Separability from 2		15. {2, 3} Separability from 1		16. {1, 2} Separability from 3		17. {1, 3} Separability from 2		18. {2, 3} Separability from 1	
α_1	−0.239	(0.00153)	−0.248	(0.00298)	−0.238	(0.00220)	−0.242	(0.00242)	−0.248	(0.00278)	−0.242	(0.00255)
β_{11}	−0.652	(0.0345)	0.354	(0.0525)	−1.09	(0.337)	−0.00425	(0.0462)	0.0653	(0.0831)	0.0480	(0.0394)
β_{12}	−2.35	(0.0639)	0.920	(0.143)	−3.24	(1.05)	−0.366	(0.0864)				
β_{13}	−0.130	(0.00498)	0.0506	(0.00934)	−0.239	(0.0772)			−0.00957	(0.0167)		
β_{1I}	0.0417	(0.00171)	−0.0392	(0.00320)	0.0834	(0.0298)	−0.0139	(0.00266)	−0.0140	(0.00758)	−0.0168	(0.00229)
α_2	−0.709	(0.00159)	−0.699	(0.00302)	−0.710	(0.00230)	−0.706	(0.00254)	−0.699	(0.00277)	−0.705	(0.00266)
β_{21}	−2.35	(0.0639)	0.920	(0.143)	−3.24	(1.05)	−0.366	(0.0864)				
β_{22}	−6.51	(0.0900)	1.38	(0.358)	−9.00	(2.61)	−1.19	(0.196)	−1.27	(0.183)	−0.228	(0.290)
β_{23}	−0.384	(0.0147)	0.195	(0.0316)	−0.553	(0.184)					0.0737	(0.0146)
β_{2I}	0.146	(0.00000)	−0.0754	(0.00000)	0.248	(0.0835)	−0.0143	(0.00618)	0.00178	(0.0156)	−0.0301	(0.00769)
α_3	−0.0515	(0.00039)	−0.0526	(0.00046)	−0.0523	(0.00040)	−0.0519	(0.00039)	−0.0523	(0.00051)	−0.0524	(0.00037)
β_{31}	−0.130	(0.00498)	0.0506	(0.00934)	−0.239	(0.0772)			−0.00957	(0.0167)		
β_{32}	−0.384	(0.0147)	0.195	(0.0316)	−0.553	(0.184)					0.0737	(0.0146)
β_{33}	−10.5	(0.442)	0.0105	(0.0144)	11.2	(0.626)	−0.0111	(0.00772)	−0.00910	(0.0140)	0.00967	(0.00863)
β_{3I}	0.00813	(0.00051)	−0.00688	(0.00064)	0.0155	(0.00669)	−0.00306	(0.00056)	−0.00117	(0.00149)	−0.00429	(0.00073)
ρ_3	−10.5	(0.397)										
ρ_2			5.30	(0.741)								
ρ_1					−19.2	(5.62)						

TABLE 1 (continued)

Parameter	19. {1, 2} Separability from t		20. {1, 3} Separability from t		21. {2, 3} Separability from t		22. {1, 2} Explicit Separability from t		23. {1, 3} Explicit Separability from t		24. {2, 3} Explicit Separability from t	
α_1	−0.239	(0.00255)	−0.240	(0.00170)	−0.240	(0.00201)	−0.239	(0.00252)	−0.242	(0.00231)	−0.244	(0.00339)
β_{11}	−0.172	(0.0228)	−0.603	(0.184)	0.166	(0.0555)	−0.172	(0.0226)	−0.140	(0.0259)	−0.0898	(0.0676)
β_{12}	−0.450	(0.144)	−2.20	(0.523)	−0.150	(0.0627)	−0.449	(0.142)	−0.676	(0.168)	0.143	(0.0905)
β_{13}	−0.0381	(0.00685)	−0.128	(0.0435)	0.0117	(0.0112)	−0.0381	(0.00671)	−0.0381	(0.00703)	−0.0159	(0.0135)
β_{1t}	0.00000	(0.00012)	0.0388	(0.0192)	−0.0229	(0.00377)					−0.00334	(0.00414)
α_2	−0.708	(0.00276)	−0.708	(0.00171)	−0.708	(0.00212)	−0.708	(0.00273)	−0.706	(0.00243)	−0.703	(0.00350)
β_{21}	−0.450	(0.144)	−2.20	(0.523)	−0.150	(0.0627)	−0.449	(0.142)	−0.676	(0.168)	0.143	(0.0905)
β_{22}	−0.947	(0.215)	−6.24	(1.58)	0.0841	(0.0642)	−0.945	(0.212)	−2.20	(0.426)	−0.206	(0.0926)
β_{23}	0.00663	(0.0225)	−0.374	(0.111)	1.17	(0.227)	−0.00670	(0.0224)	−0.0723	(0.0288)	0.0203	(0.193)
β_{2t}	0.00000	(0.00035)	0.140	(0.0507)	−0.0953	(0.0128)			0.0255	(0.00626)		
α_3	−0.0523	(0.00048)	−0.0516	(0.00043)	−0.0520	(0.00046)	−0.0523	(0.00046)	−0.0522	(0.00045)	−0.0527	(0.00051)
β_{31}	−0.0381	(0.00685)	−0.128	(0.0435)	0.0117	(0.0112)	−0.0381	(0.00671)	−0.0381	(0.00703)	−0.0159	(0.0135)
β_{32}	0.00663	(0.0225)	−0.374	(0.111)	1.17	(0.0227)	−0.00670	(0.0224)	−0.0723	(0.0288)	0.0203	(0.193)
β_{33}	0.00659	(0.0135)	−0.0436	(0.0114)	−1.06	(0.231)	0.00652	(0.0132)	−0.0121	(0.0106)	0.0221	(0.190)
β_{3t}	−0.00173	(0.00047)	0.00834	(0.00415)	−0.00700	(0.00095)	−0.00173	(0.00044)				
ρ_t	0.00000	(0.00045)	−0.162	(0.0717)	0.135	(0.0162)						

TABLE I (continued)

Parameter	25. {1, 2} Homotheticity		26. {1, 3} Homotheticity		27. {2, 3} Homotheticity		28. {1, 2} Homotheticity		29. {1, 3} Explicit Homotheticity		30. {2, 3} Explicit Homotheticity	
α_1	−0.240	(0.00145)	−0.239	(0.00174)	−0.241	(0.00176)	−0.241	(0.00218)	−0.242	(0.00251)	−0.243	(0.00192)
β_{11}	−0.761	(0.0250)	−0.754	(0.224)	0.217	(0.0439)	0.0159	(0.0366)	0.00933	(0.0101)	0.158	(0.0661)
β_{12}	−2.5436	(0.105)	−2.55	(0.574)	0.214	(0.0448)	−0.0159	(0.0331)	−0.149	(0.124)	0.136	(0.0706)
β_{13}	−0.164	(0.00821)	−0.158	(0.0463)	−0.192	(0.0500)	−0.00304	(0.0129)	−0.00933	(0.00911)	−0.115	(0.0700)
β_{1t}	0.0536	(0.00125)	0.0521	(0.0185)	−0.0182	(0.00414)	−0.0153	(0.00391)	0.0133	(0.00166)	−0.0175	(0.00680)
α_2	−0.708	(0.00151)	−0.709	(0.00180)	−0.656	(0.0120)	−0.707	(0.00231)	−0.706	(0.00268)	−0.672	(0.0181)
β_{21}	−2.54	(0.105)	−2.55	(0.574)	0.214	(0.0448)	−0.0159	(0.0331)	−0.149	(0.124)	0.136	(0.0706)
β_{22}	−7.21	(0.256)	−7.08	(1.70)	0.620	(0.121)	0.0159	(0.0366)	−0.628	(0.406)	−0.0444	(0.00980)
β_{23}	−0.422	(0.0218)	−0.426	(0.120)	0.124	(0.0129)	0.0817	(0.00963)	0.0458	(0.0213)	0.0444	(0.00888)
β_{2t}	0.177	(0.00000)	0.171	(0.0543)	−0.0520	(0.00838)	−0.0343	(0.00866)	−0.0192	(0.00968)	−0.0828	(0.0130)
α_3	−0.0520	(0.00044)	−0.0518	(0.00042)	−0.104	(0.0125)	−0.0523	(0.00043)	−0.0524	(0.00045)	−0.0845	(0.0192)
β_{31}	−0.164	(0.00821)	−0.158	(0.0463)	−0.192	(0.0500)	−0.00304	(0.0129)	−0.00933	(0.00911)	−0.115	(0.0700)
β_{32}	−0.422	(0.0218)	−0.426	(0.120)	0.124	(0.0129)	0.0817	(0.00963)	0.0458	(0.0213)	0.0444	(0.00888)
β_{33}	−0.0301	(0.0130)	−0.0395	(0.0127)	−0.00607	(0.00940)	0.0895	(0.0171)	0.00933	(0.0101)	−0.0444	(0.00980)
β_{3t}	0.0101	(0.00051)	0.0102	(0.00444)	−0.00541	(0.00085)	−0.00438	(0.00089)	0.00364	(0.00041)	−0.00195	(0.00143)
σ_{12}	13.8	(0.464)										
σ_{13}			3.82	(1.03)								
σ_{23}					−1.13	(0.167)						

TABLE 1 (continued)

Parameter	31. {1, 2} Inclusive Homotheticity		32. {1, 3} Inclusive Homotheticity		33. {2, 3} Inclusive Homotheticity		34. {1, 2} Homogeneity		35. {1, 3} Homogeneity		36. {2, 3} Homogeneity	
α_1	−0.240	(0.00122)	−0.239	(0.00212)	−0.241	(0.00138)	−0.243	(0.00315)	−0.241	(0.00358)	−0.241	(0.00309)
β_{11}	−0.608	(0.259)	−0.821	(0.256)	−0.422	(0.0222)	−0.106	(0.0290)	0.0405	(0.0113)	−0.163	(0.0517)
β_{12}	−2.22	(0.652)	−2.53	(0.703)	−1.74	(0.0677)	0.106	(0.0267)	−0.0300	(0.0105)	0.0103	(0.00900)
β_{13}	−0.159	(0.0572)	−0.176	(0.0509)	−0.0946	(0.00818)	−0.0609	(0.00853)	−0.0405	(0.0105)	−0.0103	(0.00970)
β_{1t}	0.0404	(0.0223)	0.0596	(0.0210)	0.0226	(0.00113)	0.00064	(0.00067)	0.00064	(0.00026)	0.00077	(0.00187)
α_2	−0.710	(0.00122)	−0.709	(0.00219)	−0.708	(0.00117)	−0.704	(0.00338)	−0.705	(0.00384)	−0.707	(0.00312)
β_{21}	−2.22	(0.652)	−2.53	(0.703)	−1.74	(0.0677)	0.106	(0.0267)	−0.0300	(0.0105)	0.0103	(0.00900)
β_{22}	−6.16	(2.18)	−7.14	(2.02)	−5.06	(0.197)	−0.106	(0.0290)	−1.53	(0.294)	−0.0457	(0.00711)
β_{23}	−0.436	(0.148)	−0.430	(0.159)	−0.330	(0.0164)	0.0609	(0.00921)	0.0300	(0.0114)	0.0457	(0.00660)
β_{2t}	0.140	(0.0648)	0.187	(0.0625)	0.0990	(0.00000)	−0.00064	(0.00071)	0.0424	(0.00608)	−0.00122	(0.00021)
α_3	−0.0504	(0.00066)	−0.0522	(0.00042)	0.0508	(0.00050)	−0.0529	(0.00053)	−0.0531	(0.00055)	−0.0521	(0.00044)
β_{31}	−0.159	(0.0572)	−0.176	(0.0509)	−0.0946	(0.00818)	−0.0609	(0.00854)	−0.0405	(0.0105)	−0.0103	(0.00970)
β_{32}	−0.436	(0.148)	−0.430	(0.159)	−0.330	(0.0164)	0.0609	(0.00921)	0.0300	(0.0114)	0.0457	(0.00660)
β_{33}	−0.0299	(0.0155)	−0.0414	(0.0165)	−0.0573	(0.00918)	0.0310	(0.0169)	0.0405	(0.0113)	0.0457	(0.00711)
β_{3t}	0.00957	(0.00493)	0.0120	(0.00498)	0.00688	(0.00033)	−0.165	(0.00115)	0.190	(0.00640)	0.234	(0.00193)
σ_{12}	11.8	(3.70)										
σ_{13}			4.17	(1.19)								
σ_{23}					7.62	(0.273)						

TABLE I (continued)

Parameter	37. {1, 2} Homothetic Separability		38. {1, 3} Homothetic Separability		39. {2, 3} Homothetic Separability		40. {1, 2} Linear Logarithmic Utility		41. {1, 3} Linear Logarithmic Utility		42. {2, 3} Linear Logarithmic Utility	
α_I	−0.239	(0.00122)	−0.252	(0.00294)	−0.241	(0.00226)	−0.242	(0.00165)	−0.252	(0.00284)	−0.235	(0.00172)
β_{11}	−0.650	(0.165)	0.229	(0.374)	−0.248	(0.0538)	−0.669	(0.264)	0.124	(0.403)	−0.780	(0.222)
β_{12}	−2.34	(0.444)	0.928	(0.998)	−0.867	(0.165)	−1.95	(0.771)	0.725	(1.07)	−2.47	(0.693)
β_{13}	−0.130	(0.0337)	0.0316	(0.0739)	−0.0644	(0.0125)	−0.122	(0.0548)	0.0258	(0.0840)	−0.175	(0.0489)
β_{1I}	0.0417	(0.0133)	−0.0301	(0.0315)	0.00951	(0.00490)	0.0426	(0.0216)	0.0231	(0.0337)	0.0525	(0.0197)
α_2	−0.709	(0.00123)	−0.695	(0.00300)	−0.706	(0.00238)	−0.706	(0.00154)	−0.695	(0.00689)	−0.714	(0.00185)
β_{21}	−2.34	(0.444)	0.928	(0.998)	−0.867	(0.165)	−1.95	(0.771)	0.725	(1.07)	−2.47	(0.693)
β_{22}	−6.51	(1.45)	1.22	(2.76)	−2.59	(0.642)	−5.69	(2.25)	0.687	(2.95)	−6.09	(1.94)
β_{23}	−0.383	(0.0996)	0.195	(0.209)	−0.0904	(0.0280)	−0.327	(0.160)	0.151	(0.223)	−0.430	(0.137)
β_{2I}	0.146	(0.0396)	−0.0598	(0.0921)	0.0437	(0.0169)	0.140	(0.0637)	−0.0424	(0.0974)	0.152	(0.0565)
α_3	−0.0516	(0.00036)	−0.0529	(0.00048)	−0.0525	(0.00039)	−0.0517	(0.00042)	−0.0527	(0.00041)	−0.0505	(0.00039)
β_{31}	−0.130	(0.0337)	0.0316	(0.0739)	−0.0644	(0.0125)	−0.112	(0.0548)	0.0258	(0.0840)	−0.175	(0.0489)
β_{32}	−0.383	(0.0996)	0.195	(0.209)	−0.0904	(0.0280)	−0.327	(0.160)	0.151	(0.223)	−0.430	(0.137)
β_{33}	−0.0411	(0.00883)	0.0232	(0.0189)	−0.108	(0.0212)	−0.0606	(0.0142)	0.00539	(0.0175)	−0.0304	(0.00970)
β_{3I}	0.00814	(0.00307)	−0.00601	(0.00664)	0.00070	(0.00088)	0.00906	(0.00503)	0.261	(0.143)	0.0103	(0.00431)
ρ_3	−10.5	(2.47)					−8.95	(4.03)				
ρ_2			5.30	(5.16)					4.14	(5.59)		
ρ_1					−5.09	(0.873)					−14.7	(3.80)
σ_{12}	12.5	(2.47)					10.8	(3.92)				
σ_{13}			−1.04	(1.63)					−5.93	(1.77)		
σ_{23}					3.79	(0.868)					9.12	(2.67)

TABLE 1 (continued)

Parameter	43. {1, 2} Explicit Linear Logarithmic Utility	44. {1, 3} Explicit Linear Logarithmic Utility	45. {2, 3} Explicit Linear Logarithmic Utility	46. {1, 2} Neutral Linear Logarithmic Utility	47. {1, 3} Neutral Linear Logarithmic Utility	48. {2, 3} Neutral Linear Logarithmic Utility
α_1	−0.242 (0.00199)	−0.254 (0.00273)	−0.242 (0.00166)	−0.234 (0.00670)	−0.248 (0.00357)	−0.238 (0.00129)
β_{11}			0.152 (0.0221)	−0.559 (0.399)	−0.861 (0.342)	−0.0698 (0.0399)
β_{12}				−1.70 (1.21)	−2.55 (0.965)	−0.591 (0.143)
β_{13}				−0.0934 (0.0838)	−0.178 (0.0703)	−0.0402 (0.00985)
β_{1t}	−0.0154 (0.00120)	−0.0145 (0.00133)	−0.0268 (0.00153)	0.0332 (0.0349)	0.0611 (0.0303)	−0.0122 (0.00404)
α_2	−0.707 (0.00194)	−0.694 (0.00274)	−0.708 (0.00170)	−0.714 (0.00644)	−0.700 (0.00340)	−0.713 (0.00213)
β_{21}				−1.70 (1.21)	−2.55 (0.965)	−0.591 (0.143)
β_{22}		−1.42 (0.167)		−5.18 (3.70)	−8.41 (2.59)	−0.966 (0.500)
β_{23}				−0.284 (0.255)	−0.526 (0.199)	−0.0657 (0.0341)
β_{2t}	−0.0318 (0.00418)	−0.00423 (0.00682)	−0.0514 (0.00304)	−0.101 (0.106)	0.204 (0.0864)	−0.0290 (0.0141)
α_3	−0.0506 (0.00050)	−0.0519 (0.00036)	−0.0502 (0.00044)	−0.0520 (0.00045)	−0.0513 (0.00053)	−0.0485 (0.00166)
β_{31}				−0.0934 (0.0838)	−0.178 (0.0703)	−0.0402 (0.00985)
β_{32}				−0.284 (0.255)	−0.526 (0.199)	−0.0657 (0.0341)
β_{33}	−0.0395 (0.00962)			1.84 (1.17)	−0.0366 (0.0145)	−0.00447 (0.00233)
β_{3t}	−0.00076 (0.00070)	−0.00248 (0.00030)	−0.00397 (0.00018)	0.00546 (0.00670)	0.0126 (0.00625)	−0.00197 (0.00096)
ρ_3				0.398 (0.332)		
ρ_2					10.3 (3.61)	
ρ_1						0.828 (0.187)
σ_{12}				9.64 (6.38)		
σ_{13}					−4.18 (1.54)	
σ_{23}						1.45 (0.695)
ρ_t				−0.142 (0.138)	−0.246 (0.113)	0.0408 (0.0184)

TABLE 1 (concluded)

Parameter	49. {1, 2} Equal Rates	50. {1, 3} Equal Rates	51. {2, 3} Equal Rates	52. {1, 2} Zero Rates	53. {1, 3} Zero Rates	54. {2, 3} Zero Rates
α_1	−0.242 (0.00249)	−0.242 (0.00260)	−0.242 (0.00277)	−0.241 (0.00248)	−0.243 (0.00257)	−0.242 (0.00272)
β_{11}	−0.0836 (0.0486)	−0.104 (0.189)	−0.00638 (0.0291)	−0.0756 (0.0529)	−0.374 (0.158)	−0.0469 (0.457)
β_{12}	−0.427 (0.0854)	−0.309 (0.384)	−0.00409 (0.0830)	−0.407 (0.101)	−0.896 (0.465)	−0.0866 (0.0617)
β_{13}	−0.0207 (0.00497)	−0.0322 (0.0403)	−0.00919 (0.0107)	−0.0187 (0.00574)	−0.0919 (0.0345)	−0.0155 (0.0188)
β_{1t}	−0.00604 (0.00382)	−0.00360 (0.0185)	−0.0124 (0.00311)	−0.00694 (0.00376)	0.0204 (0.0147)	−0.00923 (0.00667)
α_2	−0.706 (0.00261)	−0.705 (0.00275)	−0.705 (0.00293)	−0.706 (0.00259)	−0.704 (0.00269)	−0.705 (0.289)
β_{21}	−0.427 (0.0854)	−0.309 (0.384)	−0.00409 (0.0830)	−0.407 (0.101)	−0.896 (0.465)	−0.0866 (0.0617)
β_{22}	−1.33 (0.224)	−1.05 (1.09)	−0.201 (0.199)	−1.25 (0.147)	−2.77 (1.26)	−0.395 (0.371)
β_{23}	−0.00847 (0.00683)	0.0168 (0.0733)	0.0741 (0.00344)	−0.00400 (0.00957)	−0.0989 (0.0846)	0.0597 (0.0222)
β_{2t}	0.00157 (0.00795)	0.00136 (0.0463)	−0.0242 (0.0102)	−0.00148 (0.00442)	0.0630 (0.0410)	−0.0171 (0.0196)
α_3	−0.0523 (0.00041)	−0.0526 (0.00048)	−0.0525 (0.00045)	−0.0522 (0.00040)	−0.0529 (0.00041)	−0.0525 (0.00045)
β_{31}	−0.0207 (0.00497)	−0.0322 (0.0403)	−0.00919 (0.0107)	−0.0187 (0.00574)	−0.0919 (0.0345)	−0.0155 (0.0188)
β_{32}	−0.00847 (0.00683)	0.0168 (0.0733)	0.0741 (0.00344)	−0.00400 (0.00957)	−0.0989 (0.0846)	0.0597 (0.0222)
β_{33}	−0.00537 (0.00282)	0.0114 (0.0111)	0.0106 (0.0123)	−0.00544 (0.00313)	0.0140 (0.00950)	0.00879 (0.0116)
β_{3t}	−0.00184 (0.00071)	−0.00205 (0.00335)	−0.00362 (0.00053)	−0.00202 (0.00065)	0.00225 (0.00246)	−0.00305 (0.00152)
λ	−0.00261 (0.00385)	0.0794 (0.206)	0.137 (0.825)			
λ_1			1.66 (7.85)			1.97 (7.75)
λ_2	1.97 (7.75)	−0.0233 (0.0134)			−0.0227 (0.00343)	
λ_3	0.356 (0.249)			0.371 (0.276)		

TABLE 2

PARAMETER ESTIMATES. INDIRECT TRANSLOG UTILITY FUNCTION

Parameter	1. Equality and Symmetry		2. Commodity Augmentation		3. Additivity		4. Explicit Additivity		5. Neutrality		6. Explicit Neutrality	
α_1	−0.239	(0.00083)	−0.239	(0.00083)	−0.239	(0.00080)	−0.236	(0.00610)	−0.238	(0.00063)	−0.237	(0.00067)
β_{11}	0.109	(0.133)	0.109	(0.133)	0.164	(0.123)	−0.0274	(0.00204)	0.228	(0.0837)	0.123	(0.00793)
β_{12}	0.640	(0.378)	0.638	(0.378)	0.765	(0.389)			0.929	(0.253)	0.612	(0.0150)
β_{13}	0.0143	(0.0264)	0.0142	(0.0246)	0.0537	(0.0271)			0.0612	(0.0185)	0.0385	(0.00550)
β_{1t}	−0.00077	(0.0106)	−0.00081	(0.0120)	0.00398	(0.00995)	−0.0157	(0.00000)	0.00882	(0.00759)		
α_2	−0.710	(0.00091)	−0.710	(0.00091)	−0.711	(0.00099)	−0.711	(0.00451)	−0.714	(0.00095)	−0.714	(0.00103)
β_{21}	0.640	(0.378)	0.638	(0.378)	0.765	(0.389)			0.929	(0.253)	0.612	(0.0150)
β_{22}	1.40	(0.997)	1.40	(0.997)	1.73	(0.928)	0.0344	(0.0390)	1.89	(0.738)	0.960	(0.0616)
β_{23}	0.0555	(0.0720)	0.0553	(0.0720)	0.160	(0.0811)			0.217	(0.0560)	0.151	(0.0183)
β_{2t}	0.00599	(0.0287)	0.00591	(0.0325)	0.0183	(0.0273)	−0.0381	(0.00185)	0.0265	(0.0239)		
α_3	−0.0513	(0.00046)	−0.0513	(0.00046)	−0.0499	(0.00055)	−0.525	(0.0116)	−0.0489	(0.00059)	−0.0488	(0.00058)
β_{31}	0.0143	(0.0264)	0.0142	(0.0246)	0.0537	(0.0271)			0.0612	(0.0185)	0.0385	(0.00550)
β_{32}	0.0555	(0.0720)	0.0553	(0.0720)	0.160	(0.0811)			0.217	(0.0560)	0.151	(0.0183)
β_{33}	−0.0240	(0.00834)	−0.0240	(0.00833)	−0.0203	(0.00788)	−0.0140	(0.00624)	−0.0204	(0.0119)	−0.0236	(0.0109)
β_{3t}	−0.00259	(0.00231)	−0.00013	(0.00086)	0.00042	(0.00222)	−0.00278	(0.00053)	0.00182	(0.00164)		
$\hat{\lambda}_1$			0.0130	(0.0136)								
$\hat{\lambda}_2$			−0.00576	(0.0120)								
$\hat{\lambda}_3$			0.102	(0.0432)								
ρ					4.50	(2.07)						
ρ_t									−0.0371	(0.0303)		

TABLE 2 (continued)

Parameter	7. Homotheticity		8. Explicit Homotheticity		9. Homogeneity		10. Linear Logarithmic Utility		11. Explicit Linear Logarithmic Utility		12. Neutral Linear Logarithmic Utility	
α_1	−0.240	(0.00060)	−0.239	(0.00059)	−0.225	(0.00262)	−0.240	(0.00188)	−0.244	(0.00193)	−0.236	(0.00619)
β_{11}	0.00771	(0.0772)	−0.0819	(0.00464)	−0.155	(0.0356)	−0.0316	(0.00346)				
β_{12}	0.358	(0.234)	0.0880	(0.00688)	0.157	(0.0441)	−0.0933	(0.0104)				
β_{13}	0.0137	(0.0191)	−0.00611	(0.00453)	−0.00241	(0.00969)	−0.00666	(0.00076)				
β_{1t}	−0.00780	(0.00665)	−0.0154	(0.00036)	−0.00271	(0.00032)	−0.0157	(0.00108)				
α_2	−0.717	(0.00069)	−0.711	(0.00073)	−0.726	(0.00326)	−0.709	(0.00208)	−0.0166	(0.00102)		
β_{21}	0.358	(0.234)	0.0880	(0.00688)	0.157	(0.0441)	−0.0933	(0.0104)	−0.707	(0.00193)	−0.714	(0.00673)
β_{22}	0.668	(0.690)	−0.133	(0.0154)	−0.191	(0.0550)	−0.276	(0.0317)				
β_{23}	0.0994	(0.0542)	0.0453	(0.0113)	0.0335	(0.128)	−0.0197	(0.00229)				
β_{2t}	−0.0106	(0.0201)	−0.0333	(0.00132)	0.00349	(0.00040)	−0.0361	(0.00352)				
α_3	−0.0496	(0.00047)	−0.0496	(0.00046)	−0.0488	(0.00072)	−0.0506	(0.00040)	−0.0363	(0.00366)		
β_{31}	0.0137	(0.0191)	−0.00611	(0.00453)	−0.00241	(0.00969)	−0.00666	(0.00076)	−0.0495	(0.0057)	−0.0488	(0.00077)
β_{32}	0.0994	(0.0542)	0.0453	(0.0113)	0.0335	(0.128)	−0.0197	(0.00229)				
β_{33}	−0.00184	(0.00153)	−0.0391	(0.00861)	−0.0311	(0.00538)	−0.00141	(0.00017)				
β_{3t}	−0.0347	(0.00948)	−0.00342	(0.00014)	−0.00078	(0.00012)	−0.00441	(0.00028)	−0.0560	(0.00514)		
σ	−1.58	(1.37)										
ρ							−0.0548	(0.0582)				
ρ_t												

TABLE 2 (continued)

Parameter	13. {1, 2} Separability from 3		14. {1, 3} Separability from 2		15. {2, 3} Separability from 1		16. {1, 2} Explicit Separability from 3		17. {1, 3} Explicit Separability from 2		18. {2, 3} Explicit Separability from 1	
α_1	−0.239	(0.00078)	−0.239	(0.00082)	−0.239	(0.00083)	−0.239	(0.00073)	−0.240	(0.00075)	−0.240	(0.00078)
β_{11}	0.159	(0.116)	−0.0414	(0.126)	0.176	(0.138)	0.0394	(0.028)	−0.122	(0.00994)	−0.103	(0.0102)
β_{12}	0.772	(0.334)	0.226	(0.402)	0.802	(0.436)	0.428	(0.0624)				
β_{13}	0.0241	(0.0257)	−0.00605	(0.0265)	0.0564	(0.0307)			−0.0225	(0.00467)		
β_{1t}	0.00290	(0.00941)	−0.0118	(0.0102)	0.00496	(0.0110)	−0.00682	(0.00163)	−0.0182	(0.00037)	−0.0175	(0.00038)
α_2	−0.710	(0.00087)	−0.710	(0.00099)	−0.711	(0.00100)	−0.710	(0.00078)	−0.710	(0.00086)	−0.710	(0.00104)
β_{21}	0.772	(0.334)	0.226	(0.402)	0.802	(9.436)	0.428	(0.0624)				
β_{22}	1.74	(0.890)	0.323	(0.949)	1.84	(1.037)	0.826	(0.182)	−0.257	(0.100)	−0.291	(0.112)
β_{23}	0.0719	(0.0765)	0.0474	(0.0842)	0.162	(0.0747)					0.00867	(0.0244)
β_{2t}	0.0149	(0.0260)	−0.0210	(0.0279)	0.0210	(0.0297)	−0.0118	(0.00497)	−0.0380	(0.00306)	−0.0402	(0.00350)
α_3	−0.0514	(0.00041)	−0.0502	(0.00044)	−0.0500	(0.00070)	−0.0516	(0.00038)	−0.0503	(0.00039)	−0.0500	(0.00071)
β_{31}	0.0241	(0.0257)	−0.00605	(0.0265)	0.0564	(0.0307)			−0.0225	(0.00467)		
β_{32}	0.0719	(0.0765)	0.0474	(0.0842)	0.162	(0.0747)					0.00867	(0.0244)
β_{33}	−0.0196	(0.00533)	−0.0412	(0.00746)	0.422	(0.246)	−0.0227	(0.00400)	−0.0435	(0.00597)	−0.0303	(0.00975)
β_{3t}	−0.00200	(0.00210)	−0.00347	(0.00234)	0.00055	(0.00239)	−0.00397	(0.00015)	−0.00478	(0.00027)	−0.00384	(0.00046)
ρ_3	1.97	(1.89)										
ρ_2			1.33	(2.11)								
ρ_1					4.72	(2.30)						

TABLE 2 (continued)

Parameter	19. {1.2} Separability from t	20. {1.3} Separability from t	21. {2.3} Separability from t	22. {1.2} Explicit Separability from t	23. {1.3} Explicit Separability from t	24. {2.3} Explicit Separability from t
α_1	−0.238 (0.00062)	−0.240 (0.00081)	−0.237 (0.00085)	−0.238 (0.00066)	−0.240 (0.00071)	−0.229 (0.00179)
β_{11}	0.307 (0.0884)	−0.00467 (0.135)	−0.0252 (0.0178)	0.143 (0.00709)	0.105 (0.0104)	0.0969 (0.0357)
β_{12}	1.17 (0.267)	0.330 (0.383)	0.200 (0.0333)	0.679 (0.0139)	0.639 (0.0136)	0.508 (0.0539)
β_{13}	0.0511 (0.0192)	0.0130 (0.0290)	0.0135 (0.00727)	0.0161 (0.00416)	0.0363 (0.00483)	0.0383 (0.00911)
β_{1t}	0.0141 (0.00842)	−0.00875 (0.0121)	−0.0108 (0.00117)			0.00268 (0.00100)
α_2	−0.711 (0.00066)	−0.710 (0.00096)	−0.714 (0.00120)	−0.711 (0.00069)	−0.711 (0.00084)	−0.723 (0.00233)
β_{21}	1.17 (0.267)	0.330 (0.383)	0.200 (0.0333)	0.679 (0.0139)	0.639 (0.0136)	0.508 (0.0539)
β_{22}	2.70 (0.775)	0.666 (1.02)	−0.276 (0.0453)	1.26 (0.0554)	1.48 (0.103)	−0.616 (0.0662)
β_{23}	0.141 (0.0582)	0.0923 (0.0782)	0.180 (0.0723)	0.0367 (0.0151)	0.154 (0.0135)	0.665 (0.0816)
β_{2t}	0.0420 (0.0252)	−0.0117 (0.0295)	−0.0282 (0.00406)		0.0118 (0.00272)	
α_3	−0.0514 (0.00045)	−0.0495 (0.00052)	−0.0487 (0.00065)	−0.0515 (0.00042)	−0.0495 (0.00051)	−0.0485 (0.00068)
β_{31}	0.0511 (0.0192)	0.0130 (0.0290)	0.0135 (0.00727)	0.0161 (0.00416)	0.0362 (0.00483)	0.0383 (0.00911)
β_{32}	0.141 (0.0582)	0.0923 (0.0782)	0.180 (0.0723)	0.0367 (0.0151)	0.154 (0.0135)	0.665 (0.0816)
β_{33}	−0.0136 (0.00674)	−0.0289 (0.0114)	−0.133 (0.0728)	−0.0190 (0.00532)	−0.0234 (0.00935)	−0.0585 (0.0817)
β_{3t}	−0.00003 (0.00180)	−0.00180 (0.00250)	−0.00192 (0.00028)	−0.00296 (0.00035)		
ρ_t	−0.0592 (0.0317)	0.0364 (0.0451)	0.0395 (0.00510)			

TABLE 2 (continued)

Parameter	25. {1, 2} Homotheticity	26. {1, 3} Homotheticity	27. {2, 3} Homotheticity	28. {1, 2} Explicit Homotheticity	29. {1, 3} Explicit Homotheticity	30. {2, 3} Explicit Homotheticity
α_1	−0.239 (0.00059)	−0.239 (0.00084)	−0.237 (0.00083)	−0.239 (0.00060)	−0.239 (0.00075)	−0.242 (0.00075)
β_{11}	0.00618 (0.0882)	0.0633 (0.155)	−0.123 (0.0137)	−0.0841 (0.00541)	−0.00541 (0.00438)	−0.125 (0.0106)
β_{12}	0.359 (0.239)	0.500 (0.393)	0.0508 (0.0171)	0.0841 (0.00490)	0.307 (0.0336)	−0.0186 (0.0186)
β_{13}	−0.00531 (0.0169)	0.0204 (0.0290)	−0.702 (0.0178)	−0.0241 (0.00417)	0.00541 (0.00397)	0.00375 (0.0171)
β_{1t}	−0.00860 (0.00676)	−0.00425 (0.0111)	−0.0176 (0.00052)	−0.0164 (0.00039)	−0.00953 (0.00091)	−0.0182 (0.00041)
α_2	−0.709 (0.00067)	−0.710 (0.00105)	−0.701 (0.00407)	−0.710 (0.00066)	−0.710 (0.00104)	−0.713 (0.00433)
β_{21}	0.359 (0.239)	0.500 (0.393)	0.0508 (0.0171)	0.0841 (0.00490)	0.307 (0.0336)	−0.0186 (0.0186)
β_{22}	0.723 (0.786)	1.14 (1.04)	−0.578 (0.0899)	−0.0841 (0.00541)	0.628 (0.176)	−0.0393 (0.00808)
β_{23}	0.00607 (0.0494)	0.0260 (0.0762)	−0.00499 (0.0108)	−0.0475 (0.0169)	−0.0157 (0.0233)	0.0393 (0.00732)
β_{2t}	−0.0133 (0.0204)	−0.0510 (0.00068)	−0.0621 (0.00465)	−0.0513 (0.00043)	−0.0511 (0.00067)	−0.0452 (0.00498)
α_3	−0.0513 (0.00044)	−0.0510 (0.00068)	−0.0621 (0.00465)	−0.0513 (0.00043)	−0.0511 (0.00067)	−0.0452 (0.00498)
β_{31}	−0.00531 (0.0169)	0.0204 (0.0290)	−0.0702 (0.0178)	−0.0241 (0.00417)	0.00541 (0.00397)	0.00375 (0.0171)
β_{32}	0.00607 (0.0494)	0.0260 (0.0762)	−0.00499 (0.0108)	−0.0475 (0.0169)	−0.0157 (0.0233)	0.0393 (0.00732)
β_{33}	0.0282 (0.00675)	−0.00256 (0.00919)	−0.0466 (0.00794)	−0.103 (0.0195)	−0.00541 (0.00438)	−0.0393 (0.00808)
β_{3t}	−0.00402 (0.00155)	−0.00235 (0.00247)	−0.00471 (0.00032)	−0.0556 (0.00042)	−0.00394 (0.00054)	−0.00372 (0.00019)
σ_{12}	−1.53 (1.33)					
σ_{13}		−0.350 (0.700)				
σ_{23}			0.831 (0.126)			

TABLE 2 (continued)

Parameter	31. {1, 2} Inclusive Homotheticity		32. {1, 3} Inclusive Homotheticity		33. {2, 3} Inclusive Homotheticity		34. {1, 2} Homogeneity		35. {1, 3} Homogeneity		36. {2, 3} Homogeneity	
α_1	−0.239	(0.00059)	−0.237	(0.00119)	−0.241	(0.00069)	−0.225	(0.00254)	−0.233	(0.00492)	−0.231	(0.00169)
β_{11}	−0.0303	(0.0853)	0.385	(0.130)	−0.184	(0.0881)	−0.161	(0.0416)	−0.0237	(0.00209)	−0.233	(0.0383)
β_{12}	0.257	(0.234)	1.27	(0.359)	−0.160	(0.264)	0.161	(0.0386)	0.0269	(0.00570)	0.0226	(0.0100)
β_{13}	−0.00209	(0.0174)	0.0924	(0.0245)	−0.0265	(0.0216)	−0.00747	(0.00829)	0.0237	(0.00194)	−0.0226	(0.0108)
β_{1t}	−0.0109	(0.00667)	0.0185	(0.0102)	−0.0226	(0.00751)	−0.00299	(0.00033)	0.00079	(0.00012)	−0.00923	(0.00082)
α_2	−0.710	(0.00065)	−0.712	(0.00179)	−0.709	(0.00069)	−0.725	(0.00327)	−0.715	(0.00591)	−0.721	(0.00209)
β_{21}	0.257	(0.234)	1.27	(0.359)	−0.160	(0.264)	0.161	(0.0386)	0.0269	(0.00570)	0.0226	(0.0108)
β_{22}	0.414	(0.761)	2.94	(1.02)	−0.553	(0.860)	−0.161	(0.0416)	0.807	(0.610)	−0.0433	(0.00824)
β_{23}	0.0496	(0.0538)	0.170	(0.0807)	0.00481	(0.0552)	0.00747	(0.00894)	−0.0268	(0.00615)	0.0433	(0.00764)
β_{2t}	−0.0188	(0.0201)	0.527	(0.0296)	−0.0461	(0.0227)	0.00299	(0.00035)	0.0318	(0.0150)	0.00164	(0.00024)
α_3	−0.0503	(0.00038)	−0.0510	(0.00088)	−0.0497	(0.00042)	−0.0495	(0.00060)	−0.0516	(0.00108)	−0.0489	(0.00054)
β_{31}	−0.00209	(0.0174)	0.0924	(0.0245)	−0.0265	(0.0216)	−0.00747	(0.00829)	0.0237	(0.00194)	−0.0226	(0.0108)
β_{32}	0.0496	(0.0538)	0.170	(0.0807)	0.00481	(0.0552)	0.00747	(0.00894)	0.0268	(0.00615)	0.0433	(0.00764)
β_{33}	−0.0387	(0.00719)	0.0104	(0.00861)	−0.0432	(0.00900)	0.0322	(0.00325)	−0.0237	(0.00209)	0.0433	(0.00824)
β_{3t}	−0.00729	(0.00154)	0.00224	(0.00232)	−0.00466	(0.00174)	−0.00452	(0.00030)	−0.0585	(0.0162)	−0.00221	(0.00070)
σ_{12}	−0.945	(1.30)										
σ_{13}			−2.02	(0.599)								
σ_{23}					0.773	(1.18)						

TABLE 2 (continued)

Parameter	37. {1, 2} Homothetic Separability	38. {1, 3} Homothetic Separability	39. {2, 3} Homothetic Separability	40. {1, 2} Linear Logarithmic Utility	41. {1, 3} Linear Logarithmic Utility	42. {2, 3} Linear Logarithmic Utility
α_1	−0.239 (0.00058)	−0.241 (0.00082)	−0.239 (0.00082)	−0.241 (0.00207)	−0.241 (0.00087)	−0.238 (0.00080)
β_{11}	−0.0305 (0.0861)	0.0947 (0.146)	0.175 (0.136)	0.242 (0.331)	0.138 (0.144)	0.216 (0.136)
β_{12}	0.422 (0.238)	0.620 (0.417)	0.794 (0.426)	0.711 (0.971)	0.705 (0.409)	0.887 (0.422)
β_{13}	0.00072 (0.0184)	0.0391 (0.0282)	0.0559 (0.0300)	0.989 (0.0367)	0.0279 (0.0291)	0.0622 (0.0297)
β_{1t}	−0.00686 (0.00671)	−0.00097 (0.0107)	0.00479 (0.0109)	0.00613 (0.0258)	0.00114 (0.0106)	0.00752 (0.0109)
α_2	−0.709 (0.00063)	−0.710 (0.00088)	−0.711 (0.00086)	−0.708 (0.00227)	−0.710 (0.00088)	−0.712 (0.00095)
β_{21}	0.422 (0.238)	0.620 (0.417)	0.794 (0.426)	0.711 (0.971)	0.705 (0.409)	0.887 (0.422)
β_{22}	0.919 (0.769)	1.53 (1.01)	1.81 (1.13)	2.09 (2.85)	1.72 (0.997)	1.99 (1.12)
β_{23}	0.00214 (0.0544)	0.128 (0.863)	0.156 (0.0718)	0.108 (0.197)	0.143 (0.0829)	0.139 (0.0787)
β_{2t}	−0.0889 (0.0203)	0.00992 (0.0293)	0.0201 (0.0294)	0.0336 (0.0778)	0.0160 (0.0289)	0.0244 (0.0296)
α_3	−0.0516 (0.0039)	−0.0498 (0.00053)	−0.0501 (0.00048)	−0.0513 (0.00041)	−0.0489 (0.00071)	−0.0499 (0.00057)
β_{31}	0.00072 (0.0184)	0.0391 (0.0282)	0.0559 (0.0300)	0.989 (0.0367)	0.0279 (0.0291)	0.0622 (0.0297)
β_{32}	0.00214 (0.0544)	0.128 (0.0863)	0.156 (0.0718)	0.108 (0.197)	0.143 (0.0829)	0.139 (0.0787)
β_{33}	−0.0222 (0.00443)	−0.0114 (0.00663)	−0.0176 (0.0112)	−0.0313 (0.00921)	0.00566 (0.00591)	0.00977 (0.00553)
β_{3t}	−0.00384 (0.00153)	−0.00021 (0.00221)	0.00045 (0.00229)	−0.00081 (0.00561)	0.00058 (0.00216)	0.00115 (0.00225)
ρ_3	−0.0584 (1.35)			2.97 (4.98)		
ρ_2		3.63 (2.22)			4.12 (2.20)	
ρ_1			4.67 (2.28)			5.23 (2.29)
σ_{12}	−1.89 (1.32)			−3.96 (4.95)		
σ_{13}		−0.556 (0.666)			−0.687 (0.659)	
σ_{23}			−2.76 (1.54)			−2.99 (1.54)

TABLE 2 (continued)

Parameter	43. {1, 2} Explicit Linear Logarithmic Utility		44. {1, 3} Explicit Linear Logarithmic Utility		45. {2, 3} Explicit Linear Logarithmic Utility		46. {1, 2} Neutral Linear Logarithmic Utility		47. {1, 3} Neutral Linear Logarithmic Utility		48. {2, 3} Neutral Linear Logarithmic Utility	
α	−0.240	(0.00201)	−0.240	(0.00188)	−0.242	(0.00070)	−0.236	(0.00646)	−0.241	(0.00105)	−0.235	(0.00133)
β_{11}					−0.119	(0.00820)	0.528	(0.516)	0.202	(0.143)	−0.0116	(0.138)
β_{12}							1.60	(1.56)	0.884	(0.408)	0.230	(0.422)
β_{13}							0.107	(0.108)	0.414	(0.0293)	0.0156	(0.0287)
$\beta_{1\iota}$	−0.0129	(0.00127)	−0.0168	(0.00088)	−0.0183	(0.00039)	0.0434	(0.0477)	0.00627	(0.0115)	−0.00879	(0.0119)
α_2	−0.709	(0.00225)	−0.712	(0.00211)	−0.708	(0.00070)	−0.714	(0.00724)	−0.710	(0.00085)	−0.716	(0.00226)
β_{21}							1.60	(1.56)	0.884	(0.408)	0.230	(0.422)
β_{22}			−0.685				4.84	(4.73)	2.22	(1.01)	0.112	(1.13)
β_{23}							0.323	(0.326)	0.181	(0.0837)	0.00759	(0.0767)
$\beta_{2\iota}$	−0.0233	(0.00418)	−0.0531	(0.00690)	−0.0357	(0.00137)	0.131	(0.145)	0.0296	(0.0316)	−0.0248	(0.0326)
α_3	−0.0508	(0.00039)	−0.0486	(0.00075)	−0.0496	(0.00059)	−0.0508	(0.00089)	−0.0493	(0.00092)	−0.0487	(0.00114)
β_{31}							0.107	(0.108)	0.414	(0.0293)	0.0156	(0.0287)
β_{32}							0.323	(0.326)	2.22	(1.01)	0.00759	(0.0767)
β_{33}	−0.0426	(0.00447)					−1.73	(1.51)	0.00848	(0.00601)	−0.00052	(0.00522)
$\beta_{3\iota}$	−0.00386	(0.00028)	−0.00313	(0.00023)	−0.00302	(0.00012)	0.00675	(0.00889)	0.00128	(0.00236)	−0.00169	(0.00222)
ρ_3							−0.453	(0.423)				
ρ_2									−3.67	(1.58)		
ρ_1											−0.321	(0.546)
σ_{12}							−9.02	(8.17)				
σ_{13}									−1.01	(0.665)		
σ_{23}											−0.166	(1.56)
ρ_ι							−0.184	(0.188)	−0.0260	(0.0444)	0.0347	(0.0423)

TABLE 2 (concluded)

Parameter	49. {1, 2} Equal Rates	50. {1, 3} Equal Rates	51. {2, 3} Equal Rates	52. {1, 2} Zero Rates	53. {1, 3} Zero Rates	54. {2, 3} Zero Rates
α_1	−0.239 (0.00085)	−0.240 (0.00090)	−0.239 (0.00060)	−0.238 (0.00066)	−0.240 (0.00079)	−0.238 (0.00068)
β_{11}	−0.0353 (0.0784)	0.104 (0.154)	−0.0817 (0.0473)	0.164 (0.0164)	−0.0758 (0.0371)	0.143 (0.0125)
β_{12}	0.216 (0.207)	0.659 (0.437)	0.0822 (0.00705)	0.748 (0.0351)	0.148 (0.0890)	0.714 (0.0424)
β_{13}	−0.0152 (0.0147)	0.0229 (0.0307)	−0.00605 (0.00423)	0.0200 (0.00543)	−0.00971 (0.00875)	0.0425 (0.00694)
β_{1t}	−0.0127 (0.00658)	0.00047 (0.0138)	−0.0154 (0.00041)	0.00207 (0.00087)	−0.0140 (0.00279)	0.00256 (0.00133)
α_2	−0.710 (0.00093)	−0.710 (0.00104)	−0.711 (0.00075)	−0.711 (0.00065)	−0.709 (0.00095)	−0.713 (0.00089)
β_{21}	0.216 (0.207)	0.659 (0.437)	0.0882 (0.00705)	0.748 (0.0351)	0.148 (0.0890)	0.714 (0.0424)
β_{22}	0.227 (0.464)	1.62 (1.16)	−0.132 (0.0158)	1.45 (0.0884)	0.249 (0.146)	1.41 (0.201)
β_{23}	−0.0227 (0.0417)	0.129 (0.0824)	0.0453 (0.0106)	0.0668 (0.0115)	0.0387 (0.0241)	0.186 (0.0211)
β_{2t}	−0.0278 (0.0153)	0.0162 (0.0373)	−0.0333 (0.00151)	0.00690 (0.00123)	−0.0235 (0.00252)	0.0128 (0.00659)
α_3	−0.0513 (0.00044)	−0.0503 (0.00045)	−0.0496 (0.00048)	−0.0512 (0.00043)	−0.0502 (0.00045)	−0.0488 (0.00055)
β_{31}	−0.0152 (0.0147)	0.0229 (0.0307)	−0.00605 (0.00423)	0.0200 (0.00543)	−0.00971 (0.00875)	0.0425 (0.00694)
β_{32}	−0.0227 (0.0417)	0.129 (0.0824)	0.0453 (0.0106)	0.0668 (0.0115)	0.0387 (0.0241)	0.186 (0.0221)
β_{33}	−0.0302 (0.00614)	−0.0348 (0.00933)	−0.0391 (0.00902)	−0.0226 (0.00610)	−0.0405 (0.00663)	−0.0254 (0.0117)
β_{3t}	−0.00488 (0.00126)	−0.00107 (0.00264)	−0.00331 (0.00078)	−0.00234 (0.00043)	−0.00365 (0.00067)	0.00076 (0.00041)
λ	−0.0512 (0.0789)	0.0315 (0.00609)	−21.7 (0.0448)			
λ_1			−21.6 (0.00000)[a]			0.0179 (0.00738)
λ_2		−0.00534 (0.0130)			−0.0943 (0.0736)	
λ_3	0.225 (0.213)			0.103 (0.0267)		

[a] Singular likelihood function.

Given the validity of the theory of demand, we impose restrictions on the structure of consumer preferences for durables, non-durables, and energy. For each set of restrictions we impose the equality and symmetry restrictions. We then impose the additional restrictions associated with each hypothesis about the form of the utility function. The second column of Table 1 gives estimates under equality and symmetry restrictions, reparametrized to provide estimates of the commodity augmentation factors without imposing further restrictions. The third column gives estimates under additivity restrictions; the fourth column gives estimates under explicit additivity, the fifth under neutrality, and the sixth under explicit neutrality. The seventh column gives estimates under homotheticity, the eighth under explicit homotheticity, the ninth under homogeneity; the tenth column gives estimates under linear logarithmic utility, the eleventh under explicit linear logarithmic utility, and the twelfth under neutral linear logarithmic utility. Corresponding estimates for the indirect translog utility function are presented in the second through twelfth columns of Table 2. Estimates for neutral linear logarithmic utility are identical for direct and indirect translog utility functions.

Our next set of restrictions is associated with groupwise separability of the direct translog utility function with time-varying preferences. The thirteenth column of Table 1 gives restricted estimates for groupwise separability of the group $\{1, 2\}$. This group consists of durables and non-durables. The fourteenth column of Table 1 gives estimates for the group $\{1, 3\}$, consisting of durables and energy. The fifteenth column of Table 2 gives estimates for the group $\{2, 3\}$, non-durables and energy. We present restricted estimates for explicit groupwise separability in these same groups in the sixteenth through eighteenth columns of Table 1. Corresponding estimates for the indirect translog utility function are given in the thirteenth through eighteenth columns of Table 2.

The nineteenth column of Table 1 gives restricted estimates for groupwise separability for the group $\{1, 2\}$ from time. The twentieth column of Table 1 gives estimates for the group $\{1, 3\}$, and the twenty-first column for the group $\{2, 3\}$. We present restricted estimates for explicit groupwise separability from time in these same groups in the twenty-second through twenty-fourth columns of Table 1. Corresponding estimates for the indirect translog utility function are given in the nineteenth through twenty-fourth columns of Table 2.

Our third set of restrictions on functional form is associated with hypotheses of groupwise homotheticity of the direct translog utility function with time-varying preferences. The twenty-fifth column of Table 1 gives restricted estimates for groupwise homotheticity for the group $\{1, 2\}$. Restricted estimates for the groups $\{1, 3\}$ and $\{2, 3\}$ are given in the twenty-sixth and twenty-seventh columns. Columns twenty-eight through twenty-nine give the corresponding restricted estimates for explicit groupwise homotheticity. Columns thirty through thirty-two give the corresponding restricted estimates for inclusive groupwise homotheticity restrictions. Columns thirty-three through thirty-six give the corresponding restricted estimates for groupwise homogeneity. The corresponding restricted estimates for the indirect translog utility function are given in columns twenty-five through thirty-six of Table 2.

Our fourth set of restrictions on functional form is associated with groupwise homothetic separability of the direct translog utility function with time-varying preferences. For each of these hypotheses we impose equality and symmetry restrictions and the corresponding groupwise separability and groupwise homotheticity restrictions. The thirty-seventh column of Table 1 gives restricted estimates for groupwise homothetic separability for the group $\{1, 2\}$. Corresponding estimates for groups $\{1, 3\}$ and $\{2, 3\}$ are given in columns thirty-eight and thirty-nine of Table 1. Restricted estimates for groupwise linear logarithmic utility are given in columns forty through forty-two, for explicit groupwise linear logarithmic utility in columns forty-three through forty-five, and for groupwise neutral linear logarithmic utility in columns forty-six through forty-eight. The corresponding restricted estimates for the indirect translog utility function are given in columns thirty-seven through forty-eight of Table 2.

The fifth and final set of restrictions on functional form is associated with restrictions on the form of commodity augmenting change in preferences for the direct translog utility function with time-varying preferences. We present restricted estimates corresponding to the hypotheses of groupwise equal rates of commodity augmentation in columns forty-nine through fifty-one of Table 1 and restricted estimates corresponding to the hypotheses of zero rates of commodity augmentation in columns fifty-two through fifty-four. Corresponding estimates for the indirect translog utility function is given in columns forty-nine through fifty-four of Table 2.

4.3. *Test statistics*

To test the validity of equality restrictions implied by the theory of demand and restrictions on the form of the utility function, we employ test statistics based on the likelihood ratio Λ, where:

$$\Lambda = \frac{\max\limits_{\omega} \mathscr{L}}{\max\limits_{\Omega} \mathscr{L}}.$$

The likelihood ratio is the ratio of the maximum value of the likelihood function for the econometric model of demand Ω without restriction to the maximum value of the likelihood function for the model ω subject to restriction.

We have estimated econometric models of demand from data on U.S. personal consumption expenditures for 1947–1971. There are twenty-five observations for each behavioral equation, so that the number of degrees of freedom available for statistical tests of the theory of demand is fifty for either direct or indirect specification. For normally distributed disturbances the likelihood ratio is equal to the ratio of the determinant of the restricted estimator of the variance-covariance matrix of the disturbances to the determinant of the unrestricted estimator, each raised to the power $-(n/2)$.

Our test statistic for each set of restrictions is based on minus twice the logarithm of the likelihood ratio, or:

$$-2 \ln \Lambda = n(\ln |\hat{\Sigma}_{\omega}| - \ln |\hat{\Sigma}_{\Omega}|),$$

where $\hat{\Sigma}_{\omega}$ is the restricted estimator of the variance-covariance matrix and $\hat{\Sigma}_{\Omega}$ is the unrestricted estimator. Under the null hypothesis the likelihood ratio test statistic is distributed, asymptotically, as chi-squared with a number of degrees of freedom equal to the number of restrictions to be tested.

To control the overall level of significance for each series of tests, direct and indirect, we set the level of significance for each series at 0.05. We then allocate the overall level of significance among the various stages in each series of tests. We test groupwise separability, homotheticity, groupwise homotheticity, groupwise linear logarithmic utility, and groupwise equal rates of commodity augmentation proceeding conditionally on the validity of the equality and symmetry restrictions implied by the theory of demand. These tests are not "nested" so that the sum of the levels of significance for each of the five sets of hypotheses is an upper bound for the level of significance of tests of the sets of hypotheses considered simultaneously. We assign a level of significance of 0.01 to each of the five sets of restrictions.

There are twelve restrictions associated with groupwise separability and explicit groupwise separability: we assign a level of significance of 0.0008 to each. There are three restrictions associated with homotheticity; we assign 0.0033 to each. There are twelve restrictions associated with groupwise homotheticity; we assign 0.0008 to each. There are three restrictions associated with groupwise linear logarithmic utility; we assign 0.0033 to each of these restrictions. Finally, there are six restrictions associated with groupwise equal rates of commodity augmentation; we assign a level of significance of 0.0017 to each.

For our econometric models of demand based on the direct and indirect translog utility functions with time-varying preferences we have assigned levels of significance to each of our tests of hypotheses about the structure of preferences so as to control the overall level of significance for all tests at 0.05. The probability of a false rejection for one test among the collection of all tests we consider is less than or equal to 0.05. With the aid of critical values for our test statistics given in Table 3, the reader can evaluate the results of our tests for alternative significance levels or for alternative allocations of the overall level of significance among stages of our test procedure. Test statistics for each of the hypotheses we have considered about the structure of preferences are given in Table 4.

TABLE 3

CRITICAL VALUES OF χ^2/DEGREES OF FREEDOM

Degrees of freedom	Level of significance 0.10	0.05	0.01	0.005	0.001	0.0005
1	2.71	3.84	6.64	7.88	10.83	12.12
2	2.30	3.00	4.61	5.30	6.91	7.60

The results of our tests of restrictions on preferences based on the direct translog utility function, as presented in Table 4, are, first, that the group $\{1, 2\}$, durables and non-durables, is separable from commodity 2, energy, and that the group $\{2, 3\}$, non-durables and energy, is separable from commodity 1, durables. These two sets of restrictions imply additivity. Second, the group $\{1, 3\}$, durables

TABLE 4

TEST STATISTICS

Hypothesis	Degrees of Freedom	Critical Values	Test Statistics Direct	Test Statistics Indirect
		Given equality and symmetry		
Groupwise separability				
{1, 2} from 3	1	11.35	4.40	0.55
{1, 3} from 2	1	11.35	27.52	15.14
{2, 3} from 1	1	11.35	1.86	30.35
{1, 2} from t	1	11.35	15.44	3.83
{1, 3} from t	1	11.35	7.08	27.96
{2, 3} from t	1	11.35	4.11	37.73
Homotheticity	2	5.98	28.24	25.37
Groupwise homotheticity				
{1, 2}	1	11.35	1.87	1.08
{1, 3}	1	11.35	1.90	24.68
{2, 3}	1	11.35	3.21	17.65
Groupwise equal rates				
{1, 2}	1	10.32	11.89	2.13
{1, 3}	1	10.32	12.45	16.50
{2, 3}	1	10.32	14.18	30.38
		Given groupwise separability		
Groupwise explicit separability				
{1, 2} from 3	1	11.35	12.61	1.39
{1, 3} from 2	1	11.35	0.88	0.38
{2, 3} from 1	1	11.35	11.61	5.27
{1, 2} from t	1	11.35	0.00	3.99
{1, 3} from t	1	11.35	4.97	0.67
{2, 3} from t	1	11.35	28.16	15.17
		Given homotheticity		
Explicit homotheticity	2	5.98	10.09	1.20
		Given groupwise homotheticity		
Groupwise inclusive homotheticity				
{1, 2}	1	11.35	29.25	13.04
{1, 3}	1	11.35	3.56	21.99
{2, 3}	1	11.35	20.77	13.11
Groupwise explicit homoetheticity				
{1, 2}	1	11.35	12.70	1.63
{1, 3}	1	11.35	10.77	0.16
{2, 3}	1	11.35	26.20	13.99
		Given groupwise equal rates		
Groupwise zero rates				
{1, 2}	1	10.32	0.23	5.12
{1, 3}	1	10.32	2.21	2.60
{2, 3}	1	10.32	0.08	4.90
		Given explicit homotheticity		
Homogeneity	1	9.13	3.21	45.50
		Given groupwise explicit inclusive homotheticity		
Groupwise homogeneity				
{1, 2}	1	11.35	3.69	38.89
{1, 3}	1	11.35	32.01	13.12
{2, 3}	1	11.35	13.82	52.24
		Given groupwise homothetic separability		
Groupwise linear logarithmic utility				
{1, 2}	1	9.13	15.72	27.06
{1, 3}	1	9.13	1.57	20.50
{2, 3}	1	9.13	16.02	10.35

and energy, and the group $\{2,3\}$, non-durables and energy, are separable from time. These two sets of restrictions imply neutrality. Third, all three possible groups of two commodities each are groupwise homothetic; hence, each of these groups is homothetically separable. Fourth, the group $\{1,3\}$, durables and energy, is explicitly inclusive groupwise homothetic, which implies explicit linear logarithmic utility. Finally, the group $\{1,3\}$ is explicitly separable from time, which implies neutral linear logarithmic utility or constant budget shares. This specification is determined by only two unknown parameters.

Turning to the results of our tests of restrictions on preferences based on the indirect translog utility function, as presented in Table 4, we find that the group $\{1,3\}$, consisting of durables and non-durables, is explicitly groupwise separable from commodity 2, energy, and from time. This group is also explicitly groupwise homothetic and has equal rates of commodity augmentation equal to zero. The form of the system of equations corresponding to the indirect utility function is as follows:

$$\frac{p_1X_1}{M} = \frac{\alpha_1 + \beta_{11}(\ln[p_1/M] - \ln[p_2/M])}{-1 + \beta_{33}\ln(p_3/M) + \beta_{3t}\cdot t},$$

$$\frac{p_2X_2}{M} = \frac{\alpha_2 - \beta_{11}(\ln[p_1/M] - \ln[p_2/M])}{-1 + \beta_{33}\ln(p_3/M) + \beta_{3t}\cdot t},$$

$$\frac{p_3X_3}{M} = \frac{\alpha_3 + \beta_{33}\ln(p_3/M) + \beta_{3t}\cdot t}{-1 + \beta_{33}\ln(p_3/M) + \beta_{3t}\cdot t}.$$

This specification is determined by five unknown parameters. We recall that the direct and indirect utility function represent the same preferences only if they are self-dual. The dual of the neutral linear logarithmic direct utility function is the neutral linear logarithmic indirect utility function. We conclude that the test results for the two models do not coincide. This is not surprising, since the stochastic specifications used in the two sets of tests are different.

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