This PDF is a selection from a published volume from the National Bureau of Economic Research

Volume Title: Analyses in the Economics of Aging

Volume Author/Editor: David A. Wise, editor

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-90286-2

Volume URL: http://www.nber.org/books/wise05-1

Publication Date: August 2005

Title: Individual Subjective Survival Curves

Author: Li Gan, Michael D. Hurd, Daniel L. McFadden

URL: http://www.nber.org/chapters/c10367

12

Individual Subjective Survival Curves

Li Gan, Michael D. Hurd, and Daniel McFadden

12.1 Introduction

Many economic models are based on the forward-looking behavior of economic agents. Although it is often said that "expectations" about future events are important in these models, it is the probability distributions of future events that influence the models. For example, an individual's consumption and saving decisions are believed to depend upon concerns regarding future interest rates, the likelihood of dying, and the risk of substantial future medical expenditures. According to our theories, decision makers have subjective probability distributions about these and other events in their lives and, moreover, use them to make decisions about their saving practices.

A typical objective of empirical models on intertemporal decision making is to estimate responses to changes in variable levels, such as changes in saving due to an anticipated change in the interest rate. A second objective is to find the extent of an individual's risk aversion; namely, what is his or her response to changes in outcome variability? For instance, do changes in the variability of future income lead to changes in saving practices?

Li Gan is an assistant professor of economics at the University of Texas, Austin, and a faculty research fellow of the National Bureau of Economic Research. Michael D. Hurd is a senior economist at RAND, director of the RAND Center for the Study of Aging, and a research associate of the National Bureau of Economic Research. Daniel McFadden is the E. Morris Cox Professor of Economics at the University of California, Berkeley, 2000 Nobel Laureate in Economics, and a research associate of the National Bureau of Economic Research (NBER).

We gratefully acknowledge the research assistance from Malathi Velamuri, Victoria Vernon, and especially Guan Gong. Research support for this paper was provided to NBER by a grant from the National Institute of Aging. The views expressed in this paper are those of the authors and not necessarily those of NBER.

These are worthwhile objectives due to the importance of choices that depend on uncertain future events in our society. For example, poverty in old age depends partly on an individual's consumption choices at a younger age. Consequently, how is consumption influenced by mortality risk and the uncertainty of medical expenditures? Why do some individuals purchase adequate insurance against unfavorable outcomes while others do not? Why do many reach retirement age with inadequate financial provisions for postretirement living expenses? Is it due to misperceptions about the probabilities of reaching old age? Do people maintain excessive housing into old age as a hedge against inflation risk? The answers to these and similar questions depend on our understandings of decision maker reactions to future uncertainty. Moreover, creating policies that alleviate the consequences of such decision-making processes depends on answers to the aforementioned questions.

In a few economic models, we have data on probability distributions that are assumed to approximate those required by decision-making models under uncertainty. Life-cycle models of consumption, in which mortality risk helps determine savings, have been estimated by assuming that individuals have subjective probability distributions on mortality risk that are the same as those found from life tables (Hurd 1989). A precautionary motive for saving thus depends on the risk of future medical outlays. It therefore seems reasonable that the distribution of outlays as estimated from microdata represents a good approximation of the subjective probability distributions used by decision makers (Hubbard, Skinner, and Zeldes 1995). More generally, Manski (1993) has proposed using observed outcome probabilities in panel data as estimations of the subjective probability distributions for individuals on the panel, on the grounds that the sampling exercise can itself be taken as a model of the subjective probability process.

In most applications, however, we do not have adequate data for probability distributions—thus requiring the use of unverifiable assumptions in estimations. For example, in macroeconomic models expectations are assumed to be rational. Yet the rationality assumption cannot be tested outside of the model's immediate context. In life-cycle models on saving, a cohort's average mortality risk may not be well approximated by the mortality risk found on life tables since the cohort may not believe that the mortality experience of older cohorts will be the same as his or her own. Furthermore, individuals within the same cohort will have different subjective evaluations of probability distribution and its influence on their behavior, even if it is systematically incorrect. However, such evaluations are not generally observable. These individual heterogeneities often become problematic in parameter estimates. For example, consider a typical individual utility function,

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

where c_t is the consumption at time t, and γ is the risk aversion parameter. The first-order condition in a common formulation is

$$\frac{1}{c_t} \cdot \frac{dc_t}{dt} = \frac{-h_t + r - \rho}{\gamma} + \beta' X_t,$$

where h_t is the individual subjective hazard rates, while X_t represents certain sociodemographic variables. In this framework, if h_t is not observed but correlated with X_t, we will have a typical endogeneity problem. If h_t is poorly measured, estimations of γ will subsequently be biased.

Previous studies have typically obtained individual mortality risks through two different approaches: either by using life tables or by using well-known variations in mortality rates by economic status. Since mortality risk life tables only vary by age, race, and sex, there are not enough variations from which to calculate mortality risks. If subjective mortality risks of individuals with different economic status vary in the same way as observed mortality rates, model estimations using standard life tables will lead to biased estimates. Moreover, forecasts of economic status distributions will be incorrect such that poorer individuals who believe that their mortality risk is higher will spend money faster than what is predicted by the model. Although mortality risk variations can, in principle, be calculated from some given variables, individuals surely have subjective probability distributions that are only partly related to observable variables.

Two recent surveys have posed questions regarding individual subjective probabilities, including Asset and Health Dynamics among the Oldest Old (AHEAD) and the Health and Retirement Survey (HRS). Hurd and McGarry (1995) reveal that average survival probabilities are very close to those presented in life tables. In a more recent paper, Hurd and McGarry (2002) use panel data from HRS and find that respondents modify their probabilities in response to new information, such as the onset of a new illness. Their findings are consistent with an earlier study of Hamermesh (1985), who surveys a selected sample of economists about their survival probabilities. On average, self-reported survival probabilities are consistent with life tables; at the personal level, however, these probabilities face a serious problem. In all age groups, we find that a large fraction of respondents give what we call focal-point responses: 0.0 and 1.0. These responses cannot represent the respondents' true probabilities, as the distribution of true probabilities should be continuous, and moreover, true probabilities cannot literally equal 0 or 1. Thus, the main focus of this paper is to recover the true subjective survival curve for each respondent. To do so, we develop a Bayesian update model to accomplish this objective.

In our model, for individuals at age a, we let the prior survival probability distribution at a future point in time ($a + t$) be a truncated normal between 0 and 1 (we do not include 0 and 1). The conditional density of the observed survival probability is assumed to be a censored normal between 0 and 1, allowing for the focal points. In addition, we suggest two approaches that model the deviations of each individual's belief from the life table.

We use the posterior density mean as an individual's estimated subjective survival probabilities, and estimate the model using the observed death record. Our model produces optimistic indices to measure the deviation of his or her subjective belief from the life table. Consequently, the survival curves for each individual produced by the optimistic indexes do not encounter problems associated with focal points and have considerable variations. These subjective survival curves are readily applicable to life-cycle models and other economic models that require individual subjective mortality risk.

The remainder of the paper is organized as follows: section 12.2 introduces the self-reported subjective survival probabilities including their consistency with the life table and problems associated with individual responses. Next, section 12.3 introduces a Bayesian method that helps us to recover underlying subjective survival curves. Section 12.3 also introduces two approaches that are used to represent individual deviations from life tables. In section 12.4, we estimate the model and conduct the out-of-sample prediction. Lastly, we present the paper's conclusions in section 12.5.

12.2 Individual Subjective Mortality Risk

In the AHEAD sample, each respondent is asked a series of questions about how likely it is that various presented future events will occur. These future events include an income that is consistent with changes in inflation, major medical expenses, leaving a bequest, receiving financial help from family members, moving to a nursing home, and surviving for another ten to fourteen years.[1] In particular, the survival probability question AHEAD posed to respondents is as follows:

> [Using any] number from 0 to 100 where "0" means that you think there is absolutely no chance and "100" means that you think the event is absolutely sure to happen . . . What do you think are chances that: You will live to at least A? (A is an age that is 11–15 years older than the respondent's current age.)

To examine whether these survival probabilities carry useful informa-

1. Bassett and Lumsdaine (2001) find that all responses contain a common component.

Table 12.1 **Self-reported and life table survival probabilities**

	Target age									
	Male					Female				
	80	85	90	95	100	80	85	90	95	100
	Wave 1									
Means										
AHEAD	0.557	0.510	0.382	0.332	0.302	0.570	0.510	0.386	0.307	0.289
Life table	0.593	0.422	0.252	0.114	0.037	0.716	0.605	0.432	0.232	0.081
Median										
AHEAD	0.500	0.500	0.400	0.250	0.100	0.500	0.500	0.450	0.100	0.100
Life table	0.593	0.422	0.252	0.115	0.037	0.723	0.603	0.433	0.232	0.076
N	90	951	631	436	175	575	1,334	978	664	309
	Wave 2									
Means										
AHEAD	0.524	0.279	0.622	0.278	0.574	0.516	0.283	0.692	0.296	0.559
Life table	0.614	0.457	0.284	0.138	0.078	0.736	0.633	0.464	0.260	0.138
Median										
AHEAD	0.500	0.200	0.600	0.200	0.600	0.500	0.200	0.600	0.200	0.600
Life table	0.629	0.456	0.285	0.140	0.051	0.746	0.632	0.465	0.261	0.100
N	95	1,044	675	451	223	620	1,436	1,090	807	498

Note: N = number of observations.

tion, we compare the subjective survival probabilities with the life tables. Table 12.1 lists the average and median survival probabilities from AHEAD and the 1992 life tables for the target ages used in the AHEAD survival questions, as calculated by the first two waves of AHEAD (e.g., eighty-five years of age for subjects aged seventy to seventy-four, ninety years of age for subjects aged seventy-five to seventy-nine, etc.). In general, younger AHEAD respondents have average subjective probabilities that closely mirror life table averages, while older respondents have averages that are substantially higher.[2] In general, AHEAD medians are closely related to those in the life table.

Table 12.2 lists the percentage of those respondents who gave continuous responses, focal responses, and no responses in the two waves. Table 12.2 also lists the transition probabilities of different response modes between the two waves. In wave 1, only 50 percent of respondents gave continuous responses, with about 25 percent of them providing either 0 or 1 as their answers. The subjective probabilities for the remainder of the population are not available. In wave 2, about 53 percent of respondents who

2. Several reasons are suggested in Hurd, McFadden and Gan (1998) for this finding. One reason is that the AHEAD survey does not include respondents who reside in nursing homes or other institutional care facilities. Thus, AHEAD represents a healthier population than is represented by a life table.

Table 12.2 Focal responses

	Wave 2					
	Continuous	0	1	Dead	n.a.	Total
Wave 1						
Continuous	2,728	223	327	227	592	4,097
	(66.6)	(5.4)	(8.0)	(5.5)	(15.5)	(49.9)
0	508	329	44	178	265	1,324
	(38.4)	(24.9)	(3.3)	(13.4)	(20.0)	(16.1)
1	306	18	244	35	119	722
	(42.4)	(2.5)	(33.8)	(4.9)	(16.4)	(8.8)
n.a.	403	106	78	372	1,116	2,075
	(19.4)	(5.1)	(3.8)	(17.9)	(53.8)	(25.3)
Total	3,945	676	693	812	2,092	8,218
	(48.0)	(8.2)	(8.4)	(9.9)	(25.5)	

Note: Numbers in parentheses are percentages. n.a. = not available.

were alive gave continuous responses, whereas approximately 19 percent of the population responded either 0 or 1. A continuous respondent in wave 1 is much more likely to give continuous response again in wave 2. If a respondent gave focal response of 1 in wave 1, he or she is much more likely to give a focal response of 1 than 0 in wave 2. A person who is a nonrespondent in wave 1 is more likely to be a nonrespondent again in wave 2. Finally, a person who gave a focal response of 0 in wave 1 is much more likely to die than other persons, indicating that responses themselves do carry information about the actual survival probabilities, as suggested in Hurd and McGarry (1995, 2002). Thus the prevalence of focal-point responses indicates that subjective probability measurements in AHEAD cannot represent the respondents' true probabilities. Without correcting for focal responses of 0 or 1, it is impossible to derive a survival curve that varies over time. In the next section, we develop a Bayesian update model to correct focal responses.

12.3 Modeling Individual Subjective Survival Curves

Before we present the model, it is necessary to define the notations that we use throughout this paper.

- a: age
- t: time at risk
- $L_0(t)$: life table survival probability from birth
- $S_{0a}(t) = L_0(a + t)/L_0(a)$: life table survival probability from age a
- $\Lambda_0(t)$: life table integrated mortality hazard rate
- $\lambda_0(t)$: life table mortality hazard rate

- T: an age at which $L_0(T) = 0$, say $T = 108$
- i: individual
- $S_{ia}(t)$: personal survival probability from age a to target age $a + t$ for subject i. Since survival probabilities differ for different people at the same age a, we let $S_{ia}(t)$ be a random variable with a density $\pi(s_{ia}[t])$, or $\pi(s_{iat})$
- $\Lambda_{ia}(t)$: personal integrated mortality hazard rate at age a
- $\lambda_{ia}(t)$: personal mortality hazard rate at age a
- τ: time at risk in interview survival question
- p_{iat}: response to interview survival question. We assume that p_{iat} is measured with an error. The density of p_{iat}, conditional on personal survival probability from age a to age $a + t$, is given by $f(p_{iat} \mid S_{ia\tau} = s_{ia\tau})$

By definition, an individual i's survival curve is

$$s_{ia}(t) = \exp[-\Lambda_{ia}(a + t) + \Lambda_{ia}(a)] = \exp\left[-\int_0^t \lambda_{ia}(a + r)dr\right]. \tag{1}$$

It is first necessary to specify the plausible families of $\lambda_{ia}(a + t)$ that satisfy this equation. We propose to use the population hazard function $\lambda_{0a}(a + t)$ as a base, while minimally modifying it to calculate individual $\lambda_{ia}(a + t)$. Two alternative ways to specify the $\lambda_{ia}(a + t)$ function include

$$\lambda_{ia}(a + t) = \gamma_i \lambda_{0a}(a + t). \tag{2}$$

The parameter γ_i is an individual "optimism" parameter. In comparison with the life table, if $\gamma_i > 1$, then the person is "pessimistic"; if $\gamma_i < 1$, then the person is "optimistic." Since this model in equation (2) scales the population hazard, we will refer to it as a "hazard-scaling" model from now on.

The second model specification is given as

$$\lambda_{ia}(a + t) = \lambda_{0a}\left(a + \frac{t}{\gamma_{ia}}\right)\frac{1}{\gamma_{ia}}. \tag{3}$$

This model represents an accelerated failure time frame where the individual thinks of himself or herself as aging forward from his or her current age more or less rapidly than the average person. If a large γ_i corresponds to slow future aging, that is, $\gamma_i > 1$, then the person is "optimistic"; if $\gamma_{ia} < 1$, then the person is "pessimistic." Similarly, we refer to the model in equation (3) as the "age-scaling" model, as it scales ages to represent individual optimism.

If $p_{ia\tau}$ has no response error or focal bias, the models in equations (2) and (3) are accurately identified with no free parameters. We can then take these models as actual survival information and subsequently decide which model works best. Since a response error or focal bias in $p_{ia\tau}$ is present, the personal survival curve is not forced through $p_{ia\tau}$ at age $a + \tau$. To solve this problem, we use a model of Bayesian update.

The prior belief for the personal survival curve density S_{iat} in the Bayesian model is $\pi(s_{iat})$. The mean for prior density is $\exp(-\psi\Delta\Lambda_{0at})$, where ψ represents a parameter for measuring the population's average subjective optimistic degree. When $\psi = 1$, the mean of prior distribution S_{iat} corresponds with the life table value. Given $S_{iat} = s_{iat}$, the self-reported survival probability p_{iat} has a conditional density of $f(p_{iat} \mid s_{iat})$. The difference between the self-reported survival probability p_{iat} and the subjective survival probability S_{iat} is the measurement error.

The primary objective of this paper is to use the observed $p_{ia\tau}$ to update the prior density $\pi(s_{ia\tau})$ and to obtain the posterior density $\pi(s_{ia\tau} \mid p_{ia\tau})$. After we observe $p_{ia\tau}$, the posterior density of s_{iat} is given by

$$\pi(s_{ia\tau} \mid p_{ia\tau}) = \frac{f(p_{ia\tau} \mid s_{ia\tau})\pi(s_{ia\tau})}{f(p_{ia\tau} \mid s_{ia\tau})\pi(s_{ia\tau})ds_{ia\tau}}.$$

If the loss function is given by $L(S_{it}, \hat{S}_{it}) = E(S_{it} - \hat{S}_{it})^2$, the best estimator for $S_{i\tau}$ is $\hat{S}_{i\tau} = E(S_{i\tau} \mid p_{ia\tau})$. We apply $\hat{S}_{i\tau}$ to the observed death record to obtain the model's parameter values. The log-likelihood function is given by

$$\ln L = \sum_{\text{alive}} \ln \hat{S}_{it} + \sum_{\text{dead}} \ln(1 - \hat{S}_{it}). \tag{4}$$

Maximizing the likelihood function in equation (4) requires specifying the distribution functions. For the population of agents who share the same age a, their surviving probabilities to age $a + t$ are different. The random variable S_{iat} is used to represent such differences. Let the prior distribution for the random variable S_{iat}, $\pi(s_{ia\tau})$ be the truncated normal distribution with the truncation range being $0 < s_{ia} < 1$. We also let the mean of S_{iat} be $\exp(-\psi\Delta\Lambda_{0at})$, variance σ_2^2. The prior distribution is given by

$$\pi(s_{ia}; \psi) = \frac{\dfrac{1}{\sigma_2}\phi\left(\dfrac{\mathrm{s_{ia}} - \nu_{\mathrm{ia}}}{\sigma_2}\right)}{\Phi\left(\dfrac{1 - \nu_{\mathrm{ia}}}{\sigma_2}\right) - \Phi\left(-\dfrac{\nu_{\mathrm{ia}}}{\sigma_2}\right)}, \tag{5}$$

where ν_{ia} and σ_2 satisfy the equation

$$\exp(-\psi\Delta\Lambda_{0at}) = \nu_{iat} - \sigma_2\eta(0,1,\nu_{iat}, \sigma_2).$$

The right-hand side of equation (6) represents the mean of the truncated normal in equation (5). The functional form of $\eta(0,1,\nu_{iat}, \sigma_2)$ in equation (6) is provided in equation (A2) in the appendix. We let the conditional density of the responses to interview survival questions follow a censored normal distribution (including 0 and 1).

$$f(p_{ia\tau} \mid s_{ia\tau}) = \phi\left(\frac{p_{ia\tau} - \mu_{ia\tau}}{\sigma_1}\right) \text{ when } 0 < p_{ia\tau} < 1;$$

$$\Pr(p_{ia\tau} = 0 \mid s_{ia\tau}) = 1 - \Phi\left(\frac{\mu_{ia\tau}}{\sigma_1}\right) \text{ and } \Pr(p_{ia\tau} = 1 \mid s_{ia\tau}) = 1 - \Phi\left(\frac{1 - \mu_{ia\tau}}{\sigma_1}\right).$$

Furthermore, we assume that the expectation of the conditional distribution is s_{ia}. Thus, μ_{ia} and σ_1 satisfy the following equation:

$$s_{ia} = \left[\Phi\left(\frac{1 - \mu_{ia}}{\sigma_1}\right) + \Phi\left(\frac{\mu_{ia}}{\sigma_1}\right) - 1\right][\mu_{ia} - \sigma\eta(e, f, \mu_{ia}, \sigma_1)]$$
$$+ \left[1 - \Phi\left(\frac{1 - \mu_{ia}}{\sigma_1}\right)\right]$$

The formula for the mean of the censored normal is given in equation (A3) in the appendix. The censored normal captures the idea that many observations may be at 0 or 1. Given p_{iat}, the posterior distribution is given by

$$\text{(7)} \qquad \pi(s_{ia} \mid p_{ia\tau}) = \frac{\phi\left[\dfrac{p_{ia\tau} - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\phi\left[\dfrac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]}{\int \phi\left[\dfrac{p_{ia\tau} - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\phi\left[\dfrac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]ds_{ia}}.$$

The distribution in equation (7) is no longer a normal or a censored normal. The best estimator for s_{ia} under a mean square loss function is its mean:

$$\text{(8)} \qquad \hat{S}_{ia} = \int_0^1 s_{ia}\pi(s_{ia} \mid p_{ia\tau})ds_{ia}$$
$$= \frac{\int_0^1 s_{ia}\phi\left[\dfrac{p_{ia\tau} - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\phi\left[\dfrac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]ds_{ia}}{\int \phi\left[\dfrac{p_{ia\tau} - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\phi\left[\dfrac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]ds_{ia}}.$$

When $p_{iat} = 0$, we have

$$\Pr(s_{ia} < s \mid p_{ia\tau} = 0) = \frac{\int_0^s \Pr(p_{ia\tau} = 0 \mid s_{ia})\pi(s_{ia}; \psi)ds_{ia}}{\int_0^1 \Pr(p_{ia\tau} = 0 \mid s_{ia})\pi(s_{ia}; \psi)ds_{ia}}$$

$$= \frac{\int_0^S \left\{1 - \Phi\left[\dfrac{\mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\dfrac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]ds_{ia}}{\int_0^1 \left\{1 - \Phi\left[\dfrac{\mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\dfrac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]ds_{ia}}.$$

Thus, the posterior distribution S_{ia} given $p_{ia\tau} = 0$ is

$$\pi(s_{ia} \mid p_{ia\tau} = 0) = \frac{\left\{1 - \Phi\left[\frac{\mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]}{\int_0^1 \left\{1 - \Phi\left[\frac{\mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right] ds_{ia}}.$$

Then, the best predictor for S_{ia} when $p_{iat} = 0$ is

$$\hat{S}_{ia} = \frac{\int_0^1 s_{ia}\left\{1 - \Phi\left[\frac{\mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right] ds_{ia}}{\int_0^1 \left\{1 - \Phi\left[\frac{\mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right] ds_{ia}}. \tag{9}$$

Similarly, when $p_{iat} = 1$,

$$\pi(s_{ia} \mid p_{ia\tau} = 1) = \frac{\left\{1 - \Phi\left[\frac{1 - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right]}{\int_0^1 \left\{1 - \Phi\left[\frac{1 - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right] ds_{ia}},$$

with the best predictor being given by

$$\hat{S}_{ia} = \frac{\int_0^1 s_{ia}\left\{1 - \Phi\left[\frac{1 - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right] ds_{ia}}{\int_0^1 \left\{1 - \Phi\left[\frac{1 - \mu_{ia}(s_{ia}, \sigma_1)}{\sigma_1}\right]\right\}\phi\left[\frac{s_{ia} - \nu_{ia}(\psi, \sigma_2)}{\sigma_2}\right] ds_{ia}}. \tag{10}$$

In equations (8), (9), and (10), we obtain the predicted $\hat{S}_{ia}$ given the observed subjective survival probability of $p_{ia\tau}$. In the next section, we discuss how to estimate our model. We also present the estimation results and out-of-sample predictions.

12.4 Estimation and Out-of-Sample Prediction

Since respondents are interviewed every two years, we update information regarding whether they are still alive, accordingly. The likelihood function in equation (4) should be changed to: $\ln L = \Sigma_{\text{alive}} \ln \hat{S}_{ia2} + \Sigma_{\text{dead}} \ln(1 - \hat{S}_{ia2})$. However, the self-reported survival probability is not the survival probability during a two-year period. Rather, it typically represents a survival probability ten to fifteen years in the future.

To derive a survival probability in two years, it is necessary to get individual optimistic indexes γ_i. First, we consider the hazard-scaling model in equation (2). Plug equation (2) into equation (1), and we get

$$(11) \qquad s_{ia}(t) = \exp[-\gamma_i \Lambda_{0a}(t)].$$

In equation (11), by letting $t = \tau$, we can solve for the individual optimistic index γ_i:

$$(12) \qquad \hat{\gamma}_i = -\frac{\ln(\hat{S}_{ia\tau})}{\Delta\Lambda_{0a\tau}}.$$

If we plug equation (12) into equation (11), but let $t = 2$, we can get the survival probability in a two-year period:

$$(13) \qquad \hat{S}_{ia2} = \hat{S}_{ia\tau}^{\Delta\Lambda_{0a2}/\Delta\Lambda_{0a\tau}},$$

Therefore, the log-likelihood function in equation (4) should be rewritten to accommodate the observed data. The new log-likelihood function is given by

$$(14) \qquad \ln L = \sum_{\text{alive}} \ln \hat{S}_{ia\tau}^{\Delta\Lambda_{0a2}/\Delta\Lambda_{0a\tau}} + \sum_{\text{dead}} \ln(1 - \hat{S}_{ia\tau}^{\Delta\Lambda_{0a2}/\Delta\Lambda_{0a\tau}}).$$

Second, we consider the age-scaling model in equation (3). Although we cannot arrive at the explicit expression of γ_i, we can numerically solve the following equation to obtain the value of γ_i for each individual:

$$(15) \qquad \hat{S}_{ia\tau} = \exp[-\Lambda_0\left(a + \frac{\tau}{\hat{\gamma}_i}\right) + \Lambda_0(a)].$$

Similarly to the hazard-scaling model, the numerically obtained $\hat{\gamma}_i$ is then used to calculate the survival probability in a two-year period:

$$(16) \qquad \hat{S}_{ia2} = \exp\left[\Lambda_0\left(a + \frac{\tau}{\hat{\gamma}_i}\right) - \Lambda_0\left(a + \frac{2}{\hat{\gamma}_i}\right)\right]\hat{S}_{ia\tau}.$$

Plugging equation (16) into equation (14), we have a likelihood function. Maximizing the likelihood function yields the parameter estimates of the model.

In sum, we let the prior survival probability distribution from age a to age $a + t$ be a truncated normal (between 0.0 and 1.0). The conditional density of observed survival probabilities is assumed to be a censored normal, allowing for the focal points 0.0 and 1.0. The posterior density of the survival probabilities has, therefore, a distribution that does not allow for the focal points 0.0 and 1.0. In order to obtain the model's parameter values, we apply the posterior distribution mean to actual death records between wave 1 and wave 2 in order to estimate a person's survival probability.

Both the hazard-scaling model in equation (2) and the age-scaling model in equation (3) are estimated. In each model, we first let $\psi = 1$, constrain-

Table 12.3 Estimation results

	Hazard-scaling		Age-scaling	
	$\psi = 1$	ψ is a parameter	$\psi = 1$	ψ is a parameter
Standard deviation of conditional density: σ_1 (censored normal)	.3255	.1837	.5434	.2793
	(.1197)	(.0154)	(.0012)	(.0312)
Standard deviation of prior density: σ_2 (truncated normal)	.2045	.1165	.3159	0.1083
	(.0045)	(.0176)	(.0000011)	(.0304)
Average optimistic parameter: ψ		.7226		0.6590
		(.0507)		(.0011)
Maximum likelihood value[a]	–1,495	–1,483	–1,500	–1,491
Log likelihood for out-of-sample prediction[b]	–1,692.9	–1,532.4	–1,644.1	–1,533.4

Note: Standard errors are in parentheses.

[a]The likelihood value calculated from life table survival rates is –1,533.4.

[b]The likelihood value for out-of-sample prediction using life table survival rates is –2,559.0.

ing the mean of the prior density to be the same as that of the life table. Whenever we do this, we refer to the model as the constrained model. In addition, we let ψ be an estimated parameter. In this case, we let the data determine if the prior density mean is the same as the life tables. We refer to such a model as an unconstrained model. Table 12.3 lists the results of four different specifications: constrained hazard-scaling model, unconstrained hazard-scaling model, constrained age-scaling model, and, finally, unconstrained age-scaling model. All four specifications yield reasonable estimates that are highly significant. Moreover, likelihood ratio tests favor unconstrained models over constrained models.

Since we use a survey that currently has three waves of data available, we can apply the estimated parameters to actual survival experiences in wave 3 observations and compare the log-likelihood of each model for model selection. We select the sample that comprises individuals who are still alive in wave 2, then calculate the log-likelihood values separated by those who are alive in wave 3 and those who are dead between waves 2 and 3. The log-likelihood from the out-of-sample prediction is given by

$$\ln L = \sum_{\text{alive in wave 3}} \ln \hat{S}_{ia4} + \sum_{\text{dead b/w waves 2\&3}} \ln(1 - \hat{S}_{ia4}).$$

The log-likelihood values from the out-of-sample predictions are reported in table 12.3. The two unconstrained models perform much better than the corresponding two constrained models. Between the two unconstrained models, the hazard-scaling model yields slightly better likelihood values than the age-scaling model. Finally, we calculate the maximum likelihood value if the life-table survival rates are used. The value is –1,533.4, much smaller than any of the four specifications. Further, using the life table survival rates to predict the observed survival experience yields the maximum likelihood of –2,599.0. Again, this value is much smaller than the likelihood values from out-of-sample predictions from any of the four specifications in this paper. Clearly, the subjective survival probabilities can predict the observed survival record much better than the life tables.

For each specification, we calculate the optimistic indexes γ_i for each individual. The formula to calculate γ_i in the hazard-scaling model is given by equation (12), while the implicit formula to calculate γ_i in the age-scaling model is provided in equation (15). Table 12.4 presents the summary statistics of the indexes and the correlation coefficients from the four different models' indexes.

From table 12.4, we find that the correlation coefficients among different indexes are very high. The lowest correlation coefficient between the unconstrained hazard-scaling model and the age-scaling model is –.8975. The negative signs for the correlation coefficients between the two models are expected; that is, in the hazard-scaling model, the larger the index, the less optimistic a person is. The inverse result can be found for the age-scaling model: namely, the larger the index, the more optimistic a person is. The highest correlation coefficient between the unconstrained and the constrained hazard-scaling models is .9887, which is very close to 1.

Although the correlations among the four different specifications are very high, the means of estimated γ_i from the four different models differ significantly. These means are also reported in table 12.4. The estimated $\hat{\gamma}_i$ for unconstrained specifications portrays a more optimistic picture than those for constrained specifications. In the hazard-scaling model, the average $\hat{\gamma}_i$ in the constrained specification is 1.020, indicating that an individual's subjective survival probability on average is very close to the life table.

Table 12.4 **Correlation coefficients among four optimistic indexes**

	Hazard-scaling		Age-scaling	
	Constrained	Unconstrained	Constrained	Unconstrained
Hazard-scaling (constrained)	1	.9887	–.9000	–.9019
		(.00024)	(.0014)	(.0018)
Hazard-scaling (unconstrained)		1	–.8975	–.9284
			(.0017)	(.0016)
Age-scaling (constrained)			1	.9479
				(.0015)
Age-scaling (unconstrained)				1
Means	1.040	.822	1.051	1.271
	(.375)	(.296)	(.227)	(.186)

Notes: "Constrained" indicates $\psi = 1$; "unconstrained" indicates ψ is a parameter to be estimated. Standard errors are in parentheses, calculated from bootstrapping 1,000 times of the sample.

In the unconstrained version of the hazard model, the average $\hat{\gamma}_i$ is .822, indicating that people are generally optimistic about their survival probabilities. Similar patterns occur in the two specifications of the age-scaling model.

In the constrained specification, the means of the prior densities (truncated normal) are constrained according to life-table survival probabilities. The Bayesian update model only changes its σ_2, that is, the standard deviation of the original normal density that generates the truncated normal density (see equations [5] and [6]). Although updating σ_2 may have some effects on the means of the prior densities, the effects are relatively small. Therefore, it is not surprising to see that the constrained versions of both models are very similar to life tables. In the unconstrained specification, in addition to obtaining the value of σ_2, the updated Bayesian model also changes the mean of the prior density through ψ.

Although different specifications yield different levels of optimistic indexes, an important feature of all these indexes is that a significant heterogeneity exists among all individuals. The individual heterogeneity in γ_i can be summarized by a simple regression that uses the optimism indexes to regress certain demographic variables. In this regression, four different optimism indexes represent dependent variables, while independent variables include a constant, the person's age, a male dummy, an African American dummy, a Hispanic dummy, a marriage status dummy, a high school graduate dummy, a some-college dummy, a college (including postcollege) dummy, mother's age at the time of death, father's age at the time of death, and lastly, the wealth level at wave 1 in \$1,000. The last column in table 12.5 lists the summary statistics of the variables we used in the regression. The first four columns in table 12.5 report the estimation results.

From the estimates reported in table 12.5, the coefficients for African American dummies are negative for the hazard-scaling model specifications and positive for the age-scaling model specifications. All coefficients indicate that African Americans are more optimistic than white respondents. No difference exists between Hispanic and White respondents in terms of their optimism indexes. Neither does a person's marriage status make any difference in his or her optimism indexes. Another pattern that can be found in all four specifications is that male respondents are more optimistic than female respondents. In addition, older respondents are generally more optimistic than younger respondents in three specifications. The only exception is the unconstrained age-scaling model, where the age coefficient is insignificant. Parents' ages of death also affect people's subjective optimistic indexes: people whose parents died at older ages are more optimistic than people whose parents died at younger ages. Finally, as expected, richer people are more optimistic than poorer people.

Tables 12.6 through 12.9 provide the predicted survival probabilities of four different specifications, the stated survival probabilities, and the life

Table 12.5 **Summary regressions of four optimistic indices**

	Hazard-scaling		Age-scaling		Summary statistics
	Constrained	Unconstrained	Constrained	Unconstrained	
Constant	2.193	1.738	.638	.809	
	(.071)	(.056)	(.044)	(.035)	
Age	–.0116	–.0093	.0037	.0045	75.46
	(.00081)	(.00064)	(.0005)	(.0004)	(6.09)
Male	–.173	–.130	.100	.088	.371
	(.010)	(.0079)	(.0062)	(.0051)	(.483)
Black	–.085	–.068	.066	.052	.107
	(.015)	(.012)	(.0094)	(.0076)	(.309)
Hispanic	.039	.024	.0078	.0038	.045
	(.022)	(.018)	(.014)	(.011)	(.207)
Married	.0153	.014	–.012	–.00074	.579
	(.010)	(.0082)	(.0064)	(.0052)	(.494)
High school	–.031	–.021	.0037	.0066	.323
graduate	(.011)	(.0089)	(.0070)	(.0057)	(.468)
Some college	–.059	–.043	.0172	.0168	.166
	(.014)	(.011)	(.0085)	(.0069)	(.372)
College	–.070	–.051	.0157	.0186	.136
graduate	(.015)	(.012)	(.0093)	(.0075)	(.343)
Mom's age	–.0012	–.00096	6.13E-04	5.08E-04	72.08
of death	(.00031)	(.00024)	(1.90E-04)	(1.53E-04)	(14.83)
Dad's age	–.00124	–.00098	5.80E-04	4.48E-04	74.04
of death	(.00029)	(.00024)	(1.76E-04)	(1.42E-04)	(15.97)
Wealth/1,000 in	–4.34E-05	–3.64E-05	1.03E-05	1.51E-05	192.3
wave 1	(1.24E-05)	(9.82E-06)	(7.69E-06)	(6.21E-06)	(383.0)
No. of observations	6,092	6,092	6,092	6,092	6,092
R^2	.109	.104	.069	.092	

Note: Standard errors are in parentheses.

table survival probabilities. The predicted survival probabilities in the unconstrained specifications are higher than those based on constrained specifications. This derives from the fact that the unconstrained specifications produce indexes that indicate more optimism than those based on constrained specifications.

In figure 12.1, we produce two fitted probability histograms for males and females between the ages of seventy and seventy-four at the time the survey is conducted for the constrained hazard-scaling model. The histograms for all other age groups and all other models are the same save for their location. From this figure, all focal responses of 0 and 1 have moved away from 0 and 1. For example, for males who are between seventy and seventy-four years old at the time of the survey, the predicted probabilities of surviving to age eighty-five are .22 and .61 if the responses are 0 and 1, respectively. Figure 12.2 has various survival curves for both males and fe-

Table 12.6 **Fitted survival probabilities (constrained hazard-scaling model)**

Age group	Target age	Life table	Nonfocal respondents: Predicted	Nonfocal respondents: Stated	Focal respondents: $p_{ia\tau} = 0$ (predicted)	Focal respondents: $p_{ia\tau} = 1$ (predicted)	All respondents: Predicted	All respondents: Stated
Female								
70–74	85	0.5880	0.5565	0.5001	0.3571	0.7592	0.5604	0.5095
			(0.0696)		(0.0218)	(0.0213)	(0.1215)	
75–79	90	0.4250	0.4426	0.4616	0.2486	0.6584	0.4107	0.3885
			(0.0745)		(0.0155)	(0.0348)	(0.1319)	
80–84	95	0.2240	0.2904	0.4139	0.1398	0.4806	0.2485	0.3029
			(0.0666)		(0.0176)	(0.0213)	(0.1113)	
Male								
70–74	85	0.3970	0.4293	0.4845	0.2270	0.6342	0.4383	0.5103
			(0.0680)		(0.0225)	(0.0250)	(0.1199)	
75–79	90	0.2500	0.3086	0.4127	0.1466	0.5091	0.2936	0.3820
			(0.0651)		(0.0133)	(0.0446)	(0.1079)	
80–84	95	0.1130	0.1848	0.3960	0.0771	0.3208	0.1645	0.3324
			(0.0543)		(0.0119)	(0.0561)	(0.0845)	

Note: Standard errors are in parentheses.

Table 12.7 **Fitted survival probabilities (unconstrained hazard-scaling model)**

Age group	Target age	Life table	Nonfocal respondents: Predicted	Nonfocal respondents: Stated	Focal respondents: $p_{ia\tau} = 0$ (predicted)	Focal respondents: $p_{ia\tau} = 1$ (predicted)	All respondents: Predicted	All respondents: Stated
Female								
70–74	85	0.5880	0.6266	0.5001	0.4659	0.8062	0.6322	0.5095
			(0.0708)		(0.0193)	(0.0207)	(0.1084)	
75–79	90	0.4250	0.5171	0.4616	0.3602	0.7069	0.4927	0.3885
			(0.0769)		(0.0169)	(0.0323)	(0.1162)	
80–84	95	0.2240	0.3634	0.4139	0.2117	0.5551	0.3213	0.3029
			(0.0790)		(0.0242)	(0.0158)	(0.1167)	
Male								
70–74	85	0.3970	0.5042	0.4845	0.3339	0.6838	0.5129	0.5103
			(0.0710)		(0.0304)	(0.0227)	(0.1092)	
75–79	90	0.2500	0.3814	0.4127	0.2224	0.5777	0.3667	0.3820
			(0.0750)		(0.0199)	(0.0377)	(0.1109)	
80–84	95	0.1130	0.2673	0.3960	0.1191	0.4503	0.2390	0.3324
			(0.0771)		(0.0171)	(0.0380)	(0.1146)	

Note: Standard errors are in parentheses.

Table 12.8 **Fitted survival probabilities (constrained age-scaling model)**

Age group	Target age	Life table	Nonfocal respondents		Focal respondents		All respondents	
			Predicted	Stated	$p_{ia\tau} = 0$ (predicted)	$p_{ia\tau} = 1$ (predicted)	Predicted	Stated
Female								
70–74	85	0.5880	0.5561	0.5001	0.5589	0.7554	0.5589	0.5095
			(0.0565)		(0.1167)	(0.0195)	(0.1167)	
75–79	90	0.4250	0.4452	0.4616	0.2516	0.6629	0.4136	0.3885
			(0.0611)		(0.0141)	(0.0342)	(0.1276)	
80–84	95	0.2240	0.2917	0.4139	0.1505	0.4750	0.2529	0.3029
			(0.0524)		(0.0165)	(0.0249)	(0.1024)	
Male								
70–74	85	0.3970	0.4306	0.4845	0.2320	0.6399	0.4407	0.5103
			(0.0552)		(0.0209)	(0.0250)	(0.1154)	
75–79	90	0.2500	0.3112	0.4127	0.1572	0.5070	0.2974	0.3820
			(0.0524)		(0.0132)	(0.0476)	(0.0998)	
80–84	95	0.1130	0.1788	0.3960	0.0850	0.2879	0.1602	0.3324
			(0.0431)		(0.0123)	(0.0646)	(0.0717)	

Note: Standard errors are in parentheses.

Table 12.9 **Fitted survival probabilities (unconstrained age-scaling model)**

Age group	Target age	Life table	Nonfocal respondents		Focal respondents		All respondents	
			Predicted	Stated	$p_{ia\tau} = 0$ (predicted)	$p_{ia\tau} = 1$ (predicted)	Predicted	Stated
Female								
70–74	85	0.5880	0.6689	0.5001	0.5850	0.7728	0.6733	0.5095
			(0.0384)		(0.0202)	(0.0241)	(0.0606)	
75–79	90	0.4250	0.5531	0.4616	0.4720	0.6592	0.5413	0.3885
			(0.0446)		(0.0184)	(0.0369)	(0.0649)	
80–84	95	0.2240	0.3825	0.4139	0.3007	0.4864	0.3598	0.3029
			(0.0457)		(0.0286)	(0.0179)	(0.0662)	
Male								
70–74	85	0.3970	0.5342	0.4845	0.4427	0.6329	0.5392	0.5103
			(0.0394)		(0.0351)	(0.0258)	(0.0613)	
75–79	90	0.2500	0.4043	0.4127	0.2812	0.5122	0.3956	0.3820
			(0.0458)		(0.0684)	(0.0429)	(0.0657)	
80–84	95	0.1130	0.2715	0.3960	0.1738	0.3719	0.2509	0.3324
			(0.0488)		(0.0239)	(0.0417)	(0.0728)	

Note: Standard errors are in parentheses.

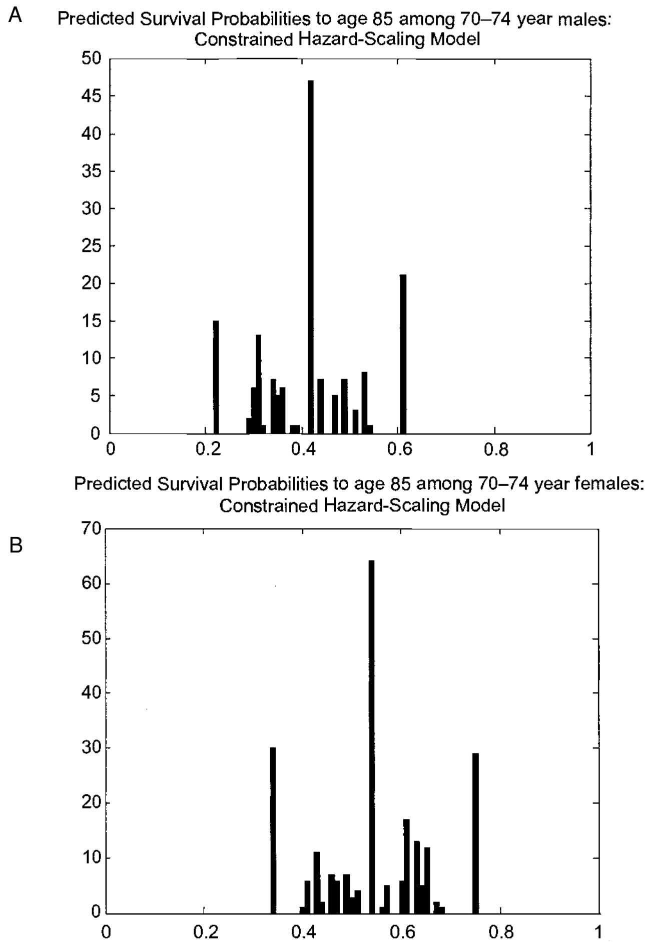
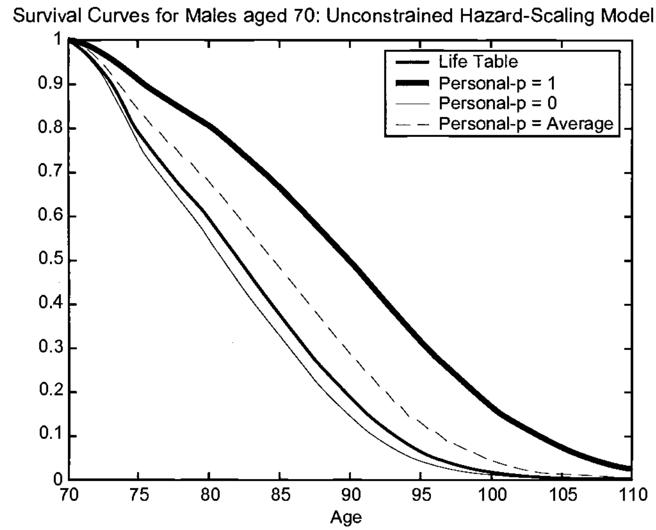


Fig. 12.1 Histograms of predicted survival probabilities: *A,* predicted survival probabilities to age 85 among 70–74 year males (constrained hazard-scaling model); *B,* predicted survival probabilities to age 85 among 70–74 year females (constrained hazard-scaling model)

males at age seventy for both constrained and unconstrained specifications in the hazard-scaling model. Graphs based on other models at other age categories look similar. In figure 12.2, the lines "personal-p = 1" and "personal-p = 0" represent the survival curves if the response is 1 and 0, respectively. The line "personal-p = Average" represents the survival curve if the response represents the average of all responses. Not surprisingly, a person whose response is 1 typically has the highest survival curve, thus demonstrating the highest survival probabilities, while a person whose response is 0 has the lowest survival curve.

A



B

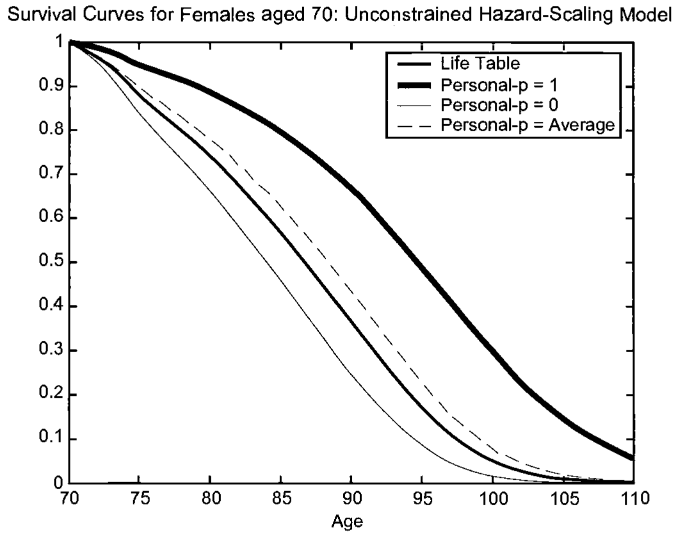
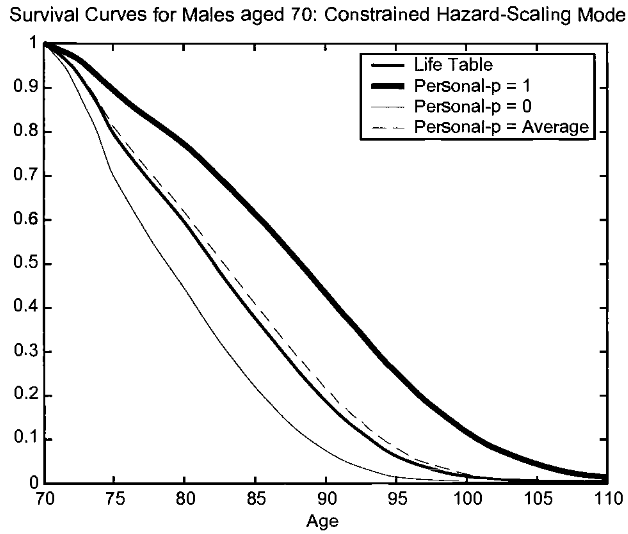


Fig. 12.2 Survival curves: *A,* survival curves for males aged 70 (unconstrained hazard-scaling model); *B,* survival curves for females aged 70 (unconstrained hazard-scaling model); *C,* survival curves for males aged 70 (constrained hazard-scaling model); *D,* survival curves for females aged 70 (constrained hazard-scaling model)

The densities of prior and posterior distributions are illustrated in figure 12.3. The first panel in figure 12.3 shows the prior and posterior densities if the response is 1, with the posterior density lying to the right of the prior density. Similarly, in the second panel in figure 12.3, the posterior density lies to the left of the prior density if the response is 0. This is

C



D

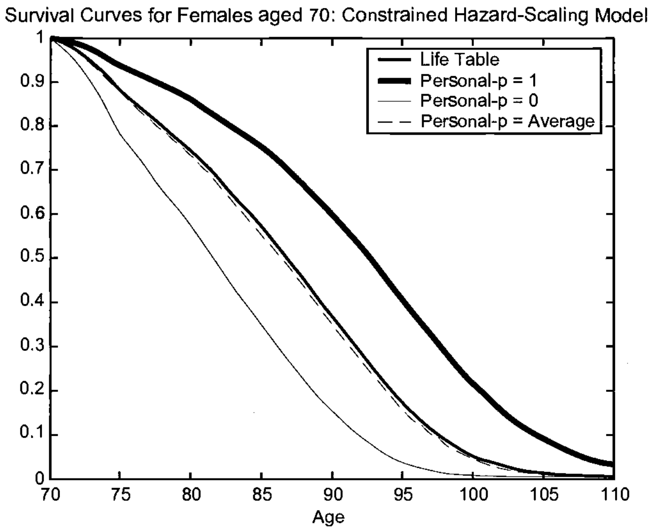


Fig. 12.2 (cont.) Survival curves: *A*, survival curves for males aged 70 (unconstrained hazard-scaling model); *B*, survival curves for females aged 70 (unconstrained hazard-scaling model); *C*, survival curves for males aged 70 (constrained hazard-scaling model); *D*, survival curves for females aged 70 (constrained hazard-scaling model)

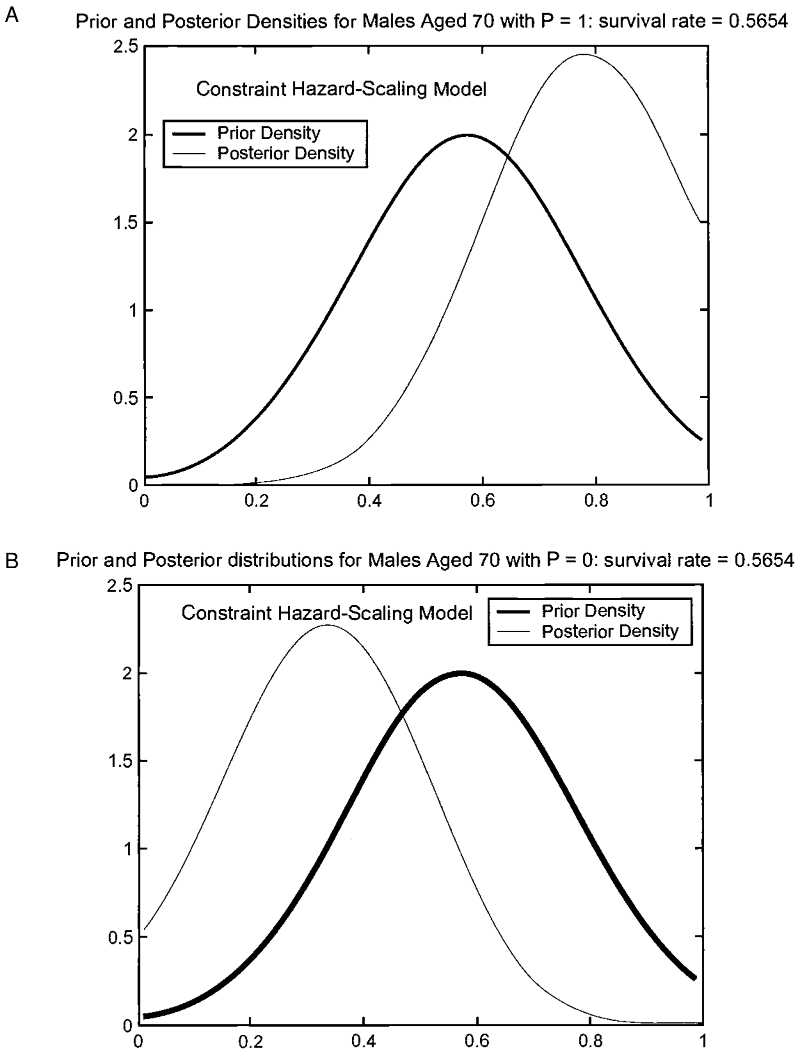


Fig. 12.3 Densities of prior, conditional, and posterior distributions: *A*, prior and posterior densities for males aged 70 with $P = 1$: survival rate = 0.5654; *B*, prior and posterior distributions for males aged 70 with $P = 0$: survival rate = 0.5654; *C*, prior and posterior densities for males aged 70 with $P = 0.5$: survival rate = 0.5654

what one would expect from the Bayesian update model. The third panel in figure 12.3 illustrates a case where the response is 0.5. In this case, it is unclear a priori that the response would pull the prior to the left or the right.

Finally, we produce histograms of the estimated optimistic parameters γ_i

C

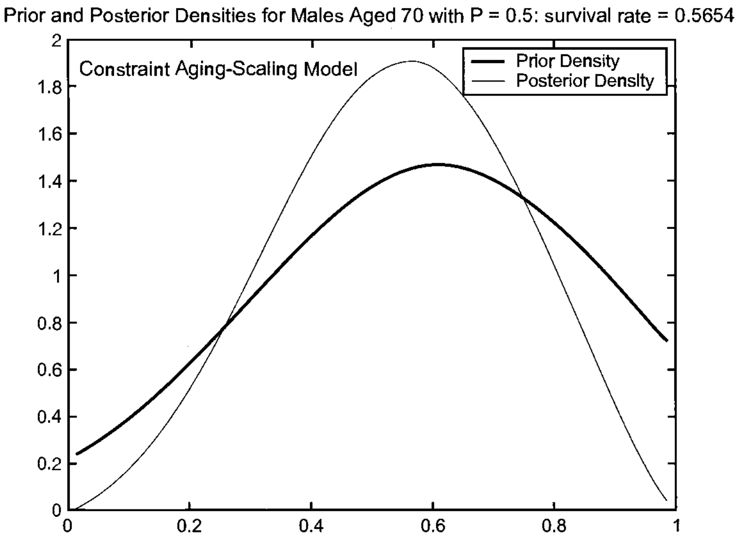


Fig. 12.3 (cont.) Densities of prior, conditional, and posterior distributions: *A*, prior and posterior densities for males aged 70 with $P = 1$: survival rate = 0.5654; *B*, prior and posterior distributions for males aged 70 with $P = 0$: survival rate = 0.5654; *C*, prior and posterior densities for males aged 70 with $P = 0.5$: survival rate = 0.5654

for all four models in figure 12.4. The average and standard deviations of γ_i are also given in the histograms. There are significant variations between these indexes. The significant variations in optimistic indexes produce significant variations in individual survival curves.

12.5 Conclusions

Many economic models are based on forward-looking behavior on the part of economic agents. Surveys such as HRS and AHEAD ask individuals for their expectations on the probability of given future events' occurring in their lifetime. On average, the subjective probability of a future event is consistent with the observed probability that the event does occur. For example, average individual survival probabilities are consistent with those from the life tables.

However, at the micro level, the subjective probability responses in HRS and AHEAD suffer serious problems of focal responses of 0.0 and 1.0. Consequently, applications of subjective probabilities will be extremely limited if "true" subjective survival probabilities are not recovered.

In this paper, we suggest a Bayesian update model to account for problems caused by focal responses of 0.0 and 1.0. As a result, individual sur-

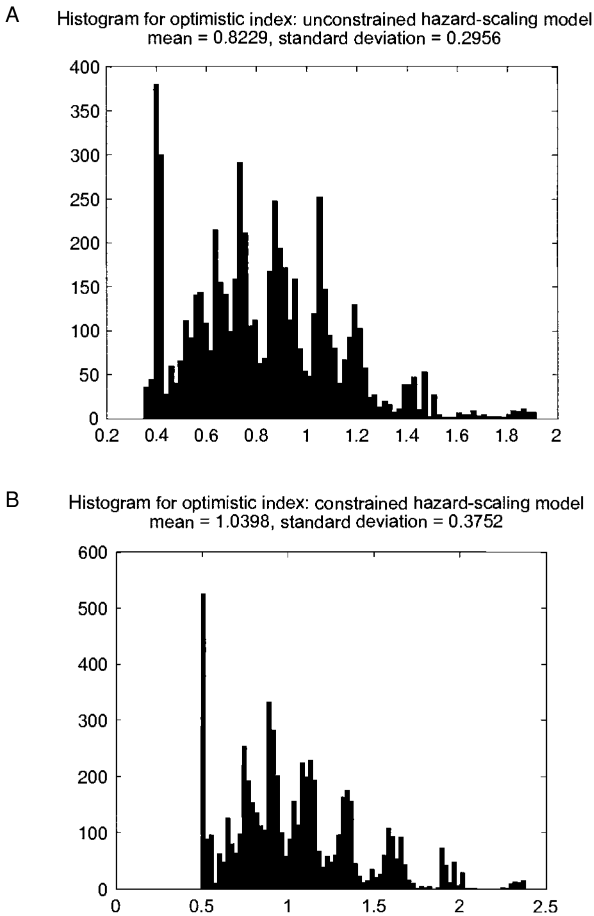


Fig. 12.4 Histogram for optimistic indices: *A,* histogram for optimistic index: unconstrained hazard-scaling model (mean = 0.8229, standard deviation = 0.2956); *B,* histogram for optimistic index: constrained hazard-scaling model (mean = 1.0398, standard deviation = 0.3752); *C,* histogram for optimistic index: unconstrained age-scaling model (mean = 1.2708, standard error = 0.1855); *D,* histogram for optimistic index: constrained age-scaling model (mean = 1.0617, standard deviation = .2049)

vival curves derived from the model do not suffer the problems of focal responses. We also propose two approaches to model the individual heterogeneities of their subjective survival curves. One approach modifies the life table hazard rates, while another approach models the subjective aging process, which is different from the life table aging process. The model is estimated from the observed survival information of our sample. From the

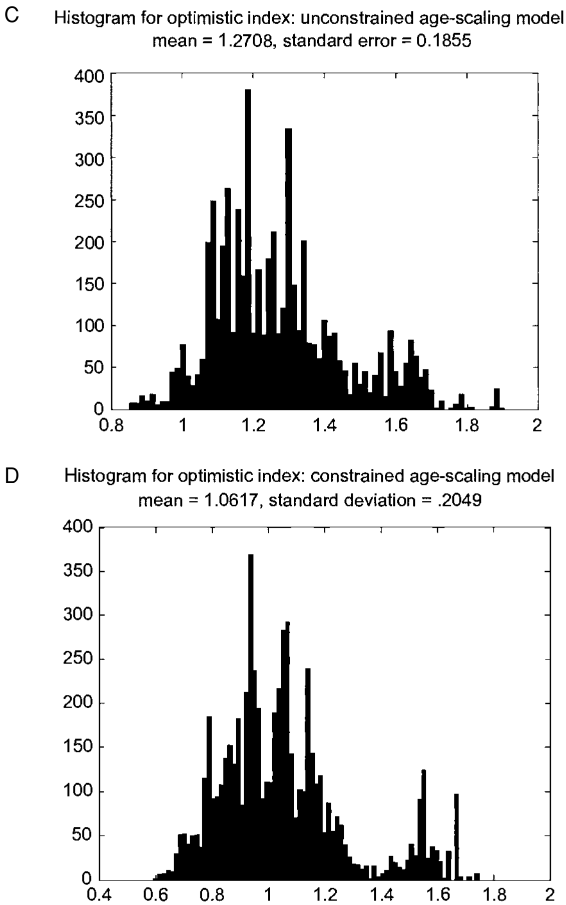


Fig. 12.4 (cont.) Histogram for optimistic indices: *A*, histogram for optimistic index: unconstrained hazard-scaling model (mean = 0.8229, standard deviation = 0.2956); *B*, histogram for optimistic index: constrained hazard-scaling model (mean = 1.0398, standard deviation = 0.3752); *C*, histogram for optimistic index: unconstrained age-scaling model (mean = 1.2708, standard error = 0.1855); *D*, histogram for optimistic index: constrained age-scaling model (mean = 1.0617, standard deviation = .2049)

estimated model, we construct several optimistic indexes for each individual and conduct a test that is based on out-of-sample prediction. These optimistic indexes are used to create individual subjective survival curves that have considerable variations and are readily applicable to economic models that require individual subjective survival curves. In a companion paper, we apply these individual subjective survival curves to a life-cycle model of savings and bequests.

Appendix

Mean of the Truncated Normal Distribution

If $x \sim N[\mu, \sigma^2]$, the density of the truncated normal distribution is then

$$g(x \mid e < x < f) = \frac{\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{f-\mu}{\sigma}\right) - \Phi\left(\frac{e-\mu}{\sigma}\right)}.$$

The mean is

$$\text{(A1)} \quad E[x \mid e < x < f] = \int_e^f x g(x \mid e < x < f)dx = \frac{\int_e^f x \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)dx}{\Phi\left(\frac{f-\mu}{\sigma}\right) - \Phi\left(\frac{e-\mu}{\sigma}\right)}$$

$$= \mu - \sigma \frac{\phi\left(\frac{f-\mu}{\sigma}\right) - \phi\left(\frac{e-\mu}{\sigma}\right)}{\Phi\left(\frac{f-\mu}{\sigma}\right) - \Phi\left(\frac{e-\mu}{\sigma}\right)}$$

$$= \mu - \sigma\eta(e, f, \mu, \sigma),$$

where

$$\text{(A2)} \quad \eta(e, f, \mu, \sigma) = \frac{\phi\left(\frac{f-\mu}{\sigma}\right) - \phi\left(\frac{e-\mu}{\sigma}\right)}{\Phi\left(\frac{f-\mu}{\sigma}\right) - \Phi\left(\frac{e-\mu}{\sigma}\right)}.$$

Mean of the Censored Normal Distribution

If $x^* \sim N[\mu, \sigma^2]$ and $x = e$ if $x^* \leq e$; $x = x^*$ if $e \leq x^* \leq f$; and $x = f$ if $x^* \geq f$, where e and f are constant, then

$$\text{(A3)} \quad E[x] = \Pr(x = e)E\lfloor x \mid x = e \rfloor + \Pr(e < x < f)E\lfloor x \mid e < x < f \rfloor + \Pr[x \geq f]E\lfloor x \mid x = f \rfloor$$

$$= \Pr(x^* \leq e)e + \Pr(e < x^* < f)E[x^* \mid e < x^* < f] + \Pr[x^* \geq f]f$$

$$= \Phi\left(\frac{e-\mu}{\sigma}\right)e + \left[\Phi\left(\frac{f-\mu}{\sigma}\right) - \Phi\left(\frac{e-\mu}{\sigma}\right)\right][\mu - \sigma\eta(e, f, \mu, \sigma)] + \left[1 - \Phi\left(\frac{f-\mu}{\sigma}\right)\right]f$$

where $\eta(e, f, \mu, \sigma)$ is defined in equation (A2).

References

Bassett, William, and Robin Lumsdaine. 2001. Probability limits—are subjective assessments adequately accurate? *Journal of Human Resources* 36 (2): 327–63.

Hamermesh, Daniel. 1985. Expectations, life expectancy, and economic behavior. *Quarterly Journal of Economics* 10 (2): 389–408.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen Zeldes. 1995. Precautionary saving and social insurance. *Journal of Political Economy* 103 (2): 360–99.

Hurd, Michael D. 1989. Mortality risks and bequests. *Econometrica* 57 (4): 779–813.

Hurd, Michael D., and Kathleen McGarry. 1995. Evaluation of subjective probability distributions in the Health and Retirement Study. *Journal of Human Resources* 30 (5): S268–S292.

———. 2002. The predictive validity of subjective probabilities of survival. *The Economic Journal* 112 (482): 966–85.

Hurd, Michael D., Daniel McFadden, and Li Gan. 1998. Subjective survival curves and life cycle behavior. In *Inquiries of economics of aging,* ed. David Wise, 259–305. Chicago: University of Chicago Press.

Manski, Charles. 1993. Dynamic choice in social settings: Learnings from the experiences of others. *Journal of Econometrics* 58 (1–2): 121–36.

Comment

Robert J. Willis

Cognitive capacity, personality, physical abilities, motivation, and other factors influence the willingness of individuals to participate in surveys, their willingness to answer any given question, and the quality of the information they provide in their answers to that question. Survey designers attempt to minimize survey and item nonresponse and try to ask questions in a way that will elicit "true" answers. While much progress has been made in increasing the quality of data produced by surveys such as the Health and Retirement Study (HRS) used in the Gan, Hurd, and McFadden (GHM) paper, there remains considerable heterogeneity in the quality of responses across respondents. Because of this, I believe that economists (and other survey users) need to develop theories of survey response that provide a link between observed responses and the underlying true values. It is also important to recognize that many of the factors that influence the quality of an individual's survey responses may also influence his or her behavior in the real world. This suggests that it may be useful to model behavior in the real world and on surveys jointly. The paper by GHM under discussion can be understood as an important contribution to this general agenda.

The HRS has pioneered asking questions about subjective probability beliefs on a wide variety of topics, including survival probabilities. These

Robert J. Willis is a professor of economics at the University of Michigan.

questions depart from the conventional approach to expectations in economics. Specifically, as Dominitz and Manski (1999) argue, "Economists have typically assumed that expectations formation is homogeneous; all persons condition their beliefs on the same variables and process their information in the same way" (Dominitz and Manski 1999, p. 16). Within the conventional approach, probability beliefs are treated as unobservable, and assumptions about beliefs, such as rational expectations, along with assumptions about unobservable preference parameters, such as risk aversion or time preference, are embedded in optimizing models from which (hopefully) testable relations between observable variables may be derived. An important motivation for asking directly about subjective probabilities in a survey is to allow relaxation of the assumption of homogeneous expectations by converting probabilities from an unobservable to an observable quantity that may vary across respondents and thus capture individual heterogeneity in expectations.

Early analysis of the survival probabilities questions in HRS by Hurd and McGarry (1995) showed that, on average, there was (to me, surprising) agreement between these subjective reports and life table estimates of survival, including covariation with health status and health behaviors such as smoking. However, the probability reports are quite "noisy," with a large number of "focal" answers at 0 percent, 100 percent, and 50 percent. In terms of the original purpose of asking these probability questions, this is quite troublesome. On the one hand, there is clear evidence that the survey questions are successful in capturing important information about people's probability beliefs. On the other hand, it is also clear that answers at the individual level do not necessarily measure a given person's probability beliefs.

GHM attempt to clarify the link between survey responses and an individual's subjective survival probability by utilizing three pieces of information: (a) the individual's survey response to a question about the probability of survival to a given age, (b) the life table estimate of the survival probability of a person whose demographic characteristics (age, race, sex) match those of the respondent, and (c) observations of the subsequent mortality experience of members of the AHEAD cohort in the HRS. These three pieces of information are combined in an elegant Bayesian model in order to recalibrate the range of individual answers to conform to a range consistent with the life table, and to estimate an individual-specific "optimism" parameter indicating the degree to which a given person believes that he or she is more or less likely to survive than observationally identical individuals.[1] If this approach is successful, the recalibrated probabilities

1. The survival probability questions ask about survival to a given age, which varies with the current age of the respondent. GHM rescale these probabilities to measure one minus the annual mortality hazard.

can be used as direct measures of individual subjective survival probabilities in optimizing models that allow for individual-specific hetereogeneity in beliefs.

In my comments, I first use a geometric approach to explain how the GHM model works. I then argue that GHM fail to take into account the uncertainty (or ambiguity or imprecision) that an individual may have about his or her mortality risks and that this may lead to a bias in the estimated probabilities implied by their model. Using a theory of survey response to probability questions developed in Lillard and Willis (2001), I sketch out why the GHM model should be generalized to include the degree of uncertainty about probability beliefs and discuss briefly why heterogeneity in the precision of probability beliefs as well as heterogeneity in optimism may be important for behavior.

One basic problem in relating survey responses on subjective survival probabilities to an objective measure of survival probabilities is illustrated in figure 12C.1. The lower panel of figure 12C.1 shows a histogram of subjective survival probabilities based on responses in wave 1 of AHEAD. The histogram shows that responses cover the entire range from 0 to 100 percent. The histogram shows heaping at focal points of 0 and 100, which GHM regard as improper responses, because it is not possible for probabilities to be truly 0 or 1. There is also a focal response at 50, which produces a larger spike than those at 0 or 100. The focal response at 50 is ignored by GHM, a point to which I return later.

The upper panel of figure 12C.1 depicts a stylized "calibration curve"

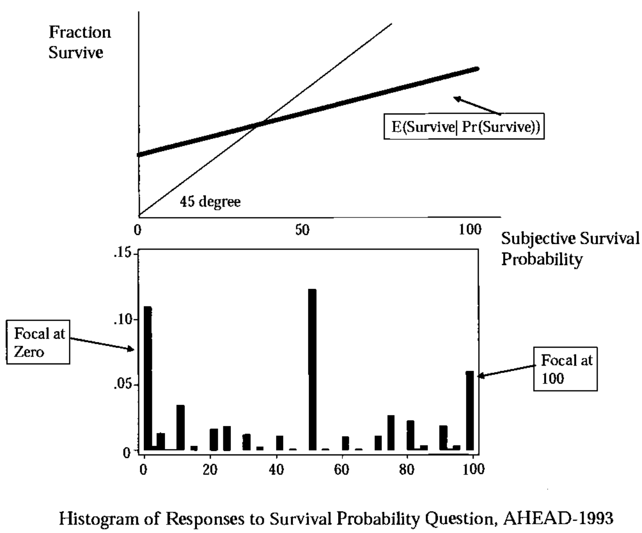


Fig. 12C.1 Calibration of survival probabilities: Histogram of responses to survival probability question, AHEAD 1993

(Camerer 1995), which relates subjective probability reports, measured on the horizontal axis, to relative frequencies such as life table estimates, measured on the vertical axis. If the elicitation of subjective survival probabilities on the survey is completely successful and if respondents have rational expectations, the calibration curve would correspond to a 45-degree line. For example, consider a set of sixty-five-year-old respondents who give the answer 80 percent when asked about their chance of survival to age seventy-five. These respondents would be judged as "well calibrated" if 80 percent of them actually survived after ten years.

In practice, calibration curves usually tend to be flatter than the 45-degree line, like the thick line labeled E(survive | Pr(survive)) in figure 12C.1. In the behavioral economics and psychological literature surveyed by Camerer (1997), this pattern is usually explained by suggesting a cognitive bias. Thus, Camerer (p. 591) states, "In general subjects are overconfident. They are insufficiently regressive in judging the likelihood of events. Events they say are certain happen only 80 percent of the time." Conversely, "Events judged to be impossible happen 20 percent of the time." To an economist reared on Milton Friedman's explanation for a flat consumption function, an obvious alternative explanation for a flat calibration curve is that the survey measure of subjective probability contains measurement error, implying that mean response errors tend to be negative for persons who report relatively low probabilities and positive for people who report high probabilities.[2] Still another possible explanation is that people vary in their degree of optimism or pessimism about their survival chances. On average, the pessimists will be found among the people giving low survival probabilities and the optimists among those with high probabilities. To the extent that optimism or pessimism is unwarranted, this would also produce a flat calibration curve.[3]

These three alternative reasons for a flat calibration curve represent substantively different hypotheses about the nature of subjective probability beliefs. The first suggests that people tend to have biased beliefs, with the bias becoming worse and worse as the objective probability of the event tends toward small or large values. The second is consistent with individuals having rational expectations that are not elicited with complete accuracy on a survey. It is also consistent, as I argue later, with the possibility that people are uncertain about what the true probability really is. The third reason emphasizes the possibility of considerable individual heterogeneity in beliefs that may or may not be warranted by their private information. Obviously, these possibilities are not mutually exclusive. However, it is of interest to know the extent to which each is operative, since they may

2. This possibility has also occurred to psychologists. See Soll (1996).

3. Physicians, including Alan Garber at this meeting, tend to think that optimism is a good trait because they find that among patients who present equivalent clinical data the optimists are more likely to recover than the pessimists.

have different implications for the way in which people behave in the real world.

GHM emphasize the role of focal responses at 0 and 100 in creating a miscalibration of individual responses. They suggest modeling the relationship between the individual's subjective survival probability and his survey response as a truncated normal distribution, with truncation points at 0 and 1. Two versions of this model are considered. The "constrained version" assumes that the mean survival probability across respondents is the same as the life table value, while the "unconstrained" version allows for the average of the survey responses to differ from the life table estimates.

A geometric interpretation of how their model works is presented in figure 12C.2, which replicates figure 12C.1 except for labels and scale. The vertical axis in figure 12C.2 measures survival probabilities by the actual mortality experience of a sample of AHEAD respondents (treated in the diagram as observationally identical) between wave 1 and wave 2, while the horizontal axis represents a recalibrated (or rescaled) version of the subjective probabilities such that the expected fraction of survivors in wave 2, conditional on the rescaled subjective probabilities, falls on the 45-degree line. The mean of the rescaled probabilities may either be constrained to be equal to the life table value or be unconstrained and left to be estimated from the response data. After rescaling, the histogram of survey responses is compressed, as shown in the bottom panel of figure 12C.2, so that persons who answer 0 have a substantial positive probability of survival and

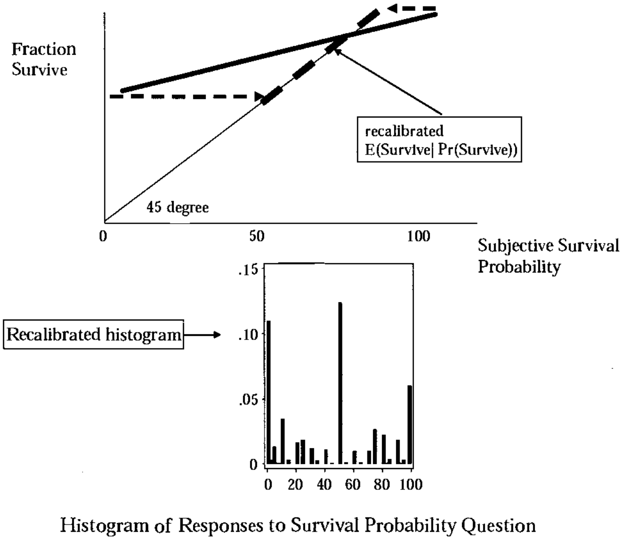


Fig. 12C.2 Recalibration of survival probabilities

those who answer 100 have a significantly positive probability of death. (This histogram is conceptually equivalent to those shown in figure 12.1 of the GHM paper.)

In addition to allowing the population as a whole to be optimistic or pessimistic relative to life table probabilities, GHM also allow for an individual-specific optimism parameter. This is depicted in figure 12C.3 for persons who give survey responses that are above or below the average of the answers given by observationally identical members of the AHEAD sample.

In their empirical work, GHM find evidence that, on average, the AHEAD respondents believe that their survival probabilities are higher than life table estimates and, in addition, that there is considerable individual-specific variation in their degree of optimism or pessimism, based on answers to survival probability questions in wave 1 of AHEAD and observed mortality between wave 1 and wave 2. Using the estimated parameters to make out-of-sample predictions using data on the mortality experience between wave 2 and wave 3 of individuals who survived to wave 2, they find that the unconstrained model is a statistically significant improvement on a model constrained to conform to the life table and that the individual-specific parameters are significant predictors.

These findings suggest to me that it might be better to choose some other term than "optimism" or "pessimism" to describe these parameters. Apparently, individual beliefs contain considerable predictive information that is not contained in the life table. However, as illustrated in table 12.5

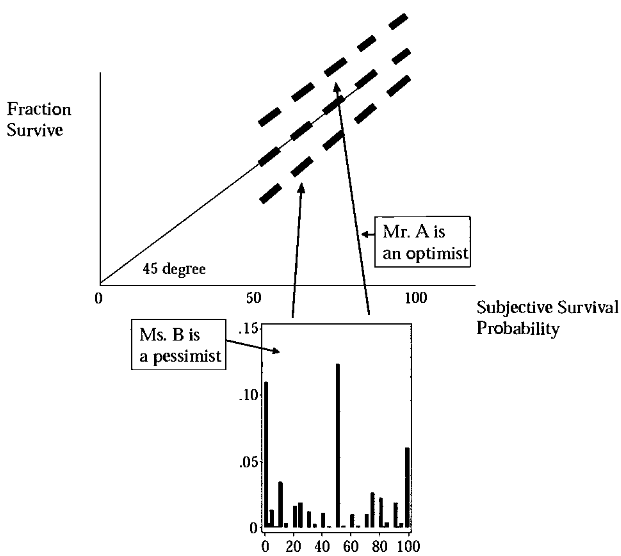


Fig. 12C.3 Optimism and pessimism

of GHM, a significant amount of the heterogeneity in individual beliefs is associated with economic and demographic characteristics of the individual in the same direction that these characteristics are correlated with actual health and mortality. It would be useful to have some additional discussion and, if possible, analysis of the degree to which heterogeneity in subjective beliefs about survival is rational versus the extent to which it is biased in an optimistic or pessimistic direction. It would also be interesting to ask whether the data available in the HRS could be used to address the physicians' hypothesis, noted earlier in note 3, that optimism has a beneficial causal effect on survival.

Although their earlier paper on subjective survival curves (Gan, Hurd, and McFadden 1998) gave equal attention to focal answers at 0, 50, and 100, the current paper ignores focal answers at 50, even though these are more numerous than those at the endpoints of the distribution. In the concluding part of my comments, I briefly describe a theory of survey response to probability questions developed in Lillard and Willis (2001) that attempts to relate the pattern of focal and "exact" responses to an individual's subjective probability beliefs and discuss some possible implications of this theory for the GHM model.

In each wave of the HRS survey, respondents are asked a series of subjective probability questions about the likelihood of a variety of events. Topics range from questions about general events (e.g., likelihood that economy will experience major depression, of an increase in stock prices, that tomorrow will be a sunny day), events with private knowledge (e.g., survival to a given age), and events subject to personal control (e.g., leaving an inheritance, working past age sixty-two). On average, respondents take about fifteen seconds to listen to each probability question and give a response by choosing a number on a scale ranging from 0 to 100.

What is the relationship between the chosen survey response and the respondent's subjective probability belief about the event in question? Lillard and Willis (2001) hypothesize that an individual's subjective belief about the likelihood of a given event is represented by a subjective density function, $g(p)$. If the individual has precise beliefs, this density has a mass point at a particular value of p. For example, the person may believe that the probability that a coin will come up heads is exactly one-half or the probability that a die will come up 5 is exactly one-sixth. For many events, including the chance of survival, it is reasonable to assume that a respondent is not completely sure about the value of the probability. This uncertainty or imprecision is represented by the spread of $g(p)$.

In giving a single number in response to a probability question in HRS, what aspect of $g(p)$ does the respondent report? One possibility is the mean: $\overline{p} = \int_0^1 g(p)dp$. In addition to being cognitively difficult, assuming that an answer to a subjective probability question measures $\overline{p}$ is inconsistent with the evidence that a large fraction of answers are focal, since $\overline{p}$

would tend to have a nonfocal value no matter how uncertain the individual might be. An alternative suggested by Lillard and Willis (2001) is the "modal choice hypothesis." According to this hypothesis, the respondent answers by giving that probability, denoted by p^{mode}, which is most likely among all possible values. This, of course, is the mode of $g(p)$. If the person has relatively precise beliefs, $g(p)$ is unimodal, the value of p^{mode} tends to be close to $\bar{p}$, and we expect the respondent to give an "exact" (i.e., nonfocal) answer. As the person's degree of uncertainty increases, we show that that $g(p)$ tends to become J-shaped with a mode at 0 if $\bar{p} < 1/2$ or at 1 if $\bar{p} > 1/2$. In these cases, the modal choice hypothesis implies that the individual will give a focal answer at 0 or 100. As uncertainty increases still further, $g(p)$ tends to become bimodal at 0 and 1. In this case, the individual is truly ambivalent about the most likely value of the probability and gives a focal answer of 50, reflecting his or her "epistemic uncertainty" (Bruine de Bruin et al. 2000).

One implication of the modal choice hypothesis that is of relevance to the GHM paper concerns the proper treatment of focal answers versus nonfocal or exact answers. As is illustrated in figure 12C.2, the recalibration of the subjective probabilities in the GHM model compresses the range of the subjective probabilities, moving the focal answers at 0 and 1 away from these extremes. Their model also compresses the range of the nonfocal answers so as to maintain the same ordering in the recalibrated probability as in the raw survey answer. Thus, a person who gives a 99 percent chance of survival to the survey question will be assigned a smaller probability of survival by GHM than the person who gives a focal answer of 100 percent. Conversely, a person who gives a very low nonfocal answer will have that answer raised in order to assign a sufficiently positive probability to those who give a focal answer of 0 percent. This is problematic if people who give focal answers are less certain about their beliefs than those who give nonfocal answers.

I examine this issue empirically in a simple regression model relating survival between the first two waves of AHEAD to answers to the subjective survival probability question in wave 1 of AHEAD.[4] The (unweighted) fraction of survivors between waves was 0.917. The implied average subjective survival probability including both focal and nonfocal answers is 0.692, considerably below the actual survival rate.[5] The average excluding those who gave focal answers is 0.894, which is much closer to the actual

4. I thank Jody Schimmel for carrying out this analysis.

5. The raw answer to the survival probability question is first rescaled into an annual rate, using the time distance between the individual's age and the target age in the question under the simple but somewhat inaccurate assumption of a constant mortality hazard. (GHM are more sophisticated in their adjustment for time distance.) This rate is then used to form the subjective probability of survival from the date of interview at wave 1 to the date of interview at wave 2. This procedure assigns a probability of 0 or 1 to those who give focal answers at 0 or 100. This rescaled variable is used on the right-hand side of the regressions in table 12C.1.

Table 12C.1 Linear regression of survival to AHEAD wave 2

	Model 1	Model 2
Two-year self-reported survival probability	0.105	0.262
	(0.009)	(0.041)
Focal at 0		0.136
		(0.036)
Focal at 50		–0.008
		(0.009)
Focal at 100		–0.038
		(0.013)
Constant	0.844	0.713
	(0.007)	(0.035)
No. of observations	6,140	6,140
R^2	0.0206	0.0232

Note: Standard errors are in parentheses.

rate. Table 12C.1 reports two regression models. Model 1 simply regresses actual survival on the (rescaled) subjective survival probability. The slope coefficient is only 0.105, indicating that the empirical counterpart of the calibration curve in figure 12C.1 is extremely flat. Model 2 adds dummy indicators for focal answers at 0, 50, and 100. This causes the slope coefficient to increase dramatically from 0.105 to 0.264, which, however, is still far flatter than the 45-degree line in figure 12C.1. The dummy variables for focal answers at 0 and 100 are both highly significant, while the dummy for focal at 50 is insignificant. Model 2 implies that a person who gives a focal answer of 0 has an actual survival probability that is 13 percentage points higher than a person who gives very low nonfocal answer. Similarly, a person who gives a focal answer of 100 has a survival chance that is 3.8 percentage points lower than a person who gives a nonfocal answer near 100.

These results suggest that hetereogeneity across respondents in the precision of their subjective beliefs is important and should be taken into account in recalibrating raw answers to subjective survival probability questions into values that can be treated as a direct measure of individual-specific subjective mortality risk in behavioral models. In addition, Lillard and Willis (2001) argue that imprecision of beliefs is of direct significance for behavior. For example, they show that imprecision of subjective beliefs about rates of return to stocks causes individuals to behave more risk aversely. I would encourage GHM to extend their elegant model of survival probabilities to address this issue.

References

Bruine de Bruin, Wandi, Baruch Fischhoff, Susan G. Millstein, and Bonnie L. Halpern-Felscher. 2000. Verbal and numerical expressions of probability: “It’s a

fifty-fifty chance." *Organizational Behavior and Human Decision Processes* 81 (1): 115–31.
Camerer, Colin. 1995. Individual decision making. In *Handbook of experimental economics,* ed. John H. Kagel and Alvin E. Roth, 587–703. Princeton: Princeton University Press.
Dominitz, Jeff, and Charles F. Manski. 1999. The several cultures of research on subjective expectations. In *Wealth, work and health: Innovations in measurement in the social sciences,* ed. James P. Smith and Robert J. Willis, 15–35. Ann Arbor: University of Michigan Press.
Hurd, M., D. McFadden, and L. Gan. 1998. Subjective survival curves and life cycle behavior. In *Inquiries in the economics of aging,* ed. David A. Wise, 259–305. Chicago: University of Chicago Press.
Hurd, M. D., and K. McGarry. 1995. Evaluation of the subjective probabilities of survival in the health and retirement study. *Journal of Human Resources* 30: S268–92.
Lillard, Lee A., and Robert J. Willis. 2001. Cognition and wealth: The importance of probabilistic thinking. University of Michigan, Michigan Retirement Research Center. Working Paper no. WP2001-007.
Soll, Jack B. 1996. Determinants of overconfidence and miscalibration: The roles of random error and ecological structure. *Organizational Behavior and Human Decision Processes* 65 (2): 117–37.