Corruption and Collusion in Procurement Tenders*

Ariane Lambert[†]and Konstantin Sonin[‡]

April 12, 2003

Abstract

There is a mounting body of evidence that collusive agreements between bidders in large multiple-object procurement tenders are often supported by a corrupt administrator. In a first-price multiple-object auction, if the auctioneer has some legal discretion to allow bidders to readjust their offers prior to the official opening, he also has incentives to extract bribes from agents in exchange for abusing this discretion. In particular, corrupt agent's incentives to receive bribes are closely linked with that of creating a 'bidding ring' as the agent's discretionary power gains value when firms collude. Thus, corruption generates focal equilibria where bidders fully refrain from competing with each other. Additional flexibility of the auction format such as the possibility to submit package bids, which is often considered to be efficiency-enhancing in theoretical literature, increases the risk of collusion in the presence of corruption. Such problems are more likely to arise in tenders, where participating firms are not too close competitors.

Keywords: auctions, corruption, collusion.

JEL: D 44, H 57, K 42.

1 Introduction

Recent advances in economic theory allow to obtain deep insights into the design and structure of complicated selling schemes in different environments.[1] Still, major problems of real-life auction mechanisms, such as procurement tenders, received only limited attention in the theoretical literature. Klemperer (2002) argues strongly that collusion between bidders should be a main concern for auction designers. In this paper, we investigate how collusive agreement can be sustained in the presence of a corrupt auctioneer (a government employee). The main motivation for the paper was mounting body of evidence that collusion and corruption often go hand-in-hand in public procurement.

In France, practitioners and investigators in courts of accounts, in competition authorities, and in the judiciary have long been aware of the close links between collusion and corruption in public tenders. The testimony of J. C. Mery provides suggestive evidence of those links (*Le Monde*, September 22 and 23, 2000).[2] A recent judgment in the court case concerned with corruption in 'Les Yvelinnes' (Cour d'Appel de Versaille, January 2002) illustrates how corrupt politicians and agents actually were the initiators to both collusion and corruption. According to a judge currently investigating a major collusion/corruption case in Paris, there exists in France, almost not a single case of large stake collusion in procurement tenders, without corruption.[3] Besides empirical evidence, theoretical arguments also motivate the investigation of the links between collusion and corruption. First, a cartel usually has to solve a series of problems including information revelation, agreeing on how to share the spoils, enforcement and entry deterrence (see McAfee and McMillan, 1992). A

---

[1]For a most up-to-date collection of both classic and recent articles on auctions, see Klemperer (2000); for general auction theory, see Krishna (2002) and Milgrom (2003).

[2]J. C. Mery, a City Hall official, admitted that for ten years (1985-994) he organized and arbitrated market sharing in the allocation of most construction and maintenance contracts for the Paris City Hall. In exchange, firms were paying bribes used to finance political parties. The contracts in question were on average very profitable: they generated up to 30 percent profit in an industry that averages 5 percent. Mr. Mery also claimed that he had always managed to allocate the contracts to the lowest price bidder. Both these features suggest that the firms were not competing with each other, but were instead implementing a market sharing agreement.

[3]The case concerns the procurement of a 4.3 billion euros construction market (see *Le Monde*, January 26, 2000).

corrupt agent can contribute to solving some of those problems e.g. by providing means of retaliation or creating barriers to entry. Second, corrupt agents seek to extract rents. They may support collusion to create rents they can appropriate.

This paper shows that, in one-shot first-price multiple-object auction, corruption may induce collusive market sharing. A main assumption is that the auctioneer (the agent) has discretion to let firms simultaneously readjust their bids. If the agent is honest, a collusive market-sharing agreement is not sustainable, since each bidder benefits from defection. However, when the agent is corrupt, collusion can be sustainable. The intuition is that a defection from collusive bidding offers an occasion for the agent to extract rents from his discretionary power. He exploits that occasion by letting the firms compete in bribes for influence on his decision. Thus, defection becomes less profitable for a bidder. Now the defector must overbid victims in bribe of his defection as they are willing to pay the agent so all firms readjust to a low payoff equilibrium.

Tender procedures in procurement often include various provisions that allow the auctioneer to intervene, e.g. providing all bidders with updated information to correct an undue informational advantage or to clear an ambiguity in tender documents. In connection with those interventions, submitted offers can be taken back (before the official opening) for readjustments (the submission deadline is extended). In the World Bank guidelines, 'Procurements under IBRD loans and IDA credits', one can read "*Additional information, clarification, correction of errors or modification in bidding documents* shall be sent to each recipient of the original bidding documents in sufficient time before the deadline. *If necessary the deadline shall be extended.*" article 2.18.[4] We show that the discretion connected with these seemingly innocuous features of the procedural design can be exploited to defeat competition.

A second result of our paper is that flexibility of bidding rules may be detrimental to the seller: package bidding facilitates collusion. With bids on individual tasks only, the enforcement power of corruption is drastically reduced. The intuition is that package bidding gives opportunities for firms to select collusive bidding strategies that imply a credible

---

[4]To secure fair treatment, when the deadline is extended those who have already submitted are allowed to submit a new offer

commitment to retaliate. This result mitigates somehow the recent emphasis on the value of package bidding.[5]

The analysis predicts that collusive market sharing is more likely to occur in tenders, where firms are not too close competitors. This is an immediate consequence of relying on the corrupt agent's self-interest to deter defection. The cost imposed on defection reflects the cheated firm's willingness to bribe which is bounded from above by the non-cooperative payoff. When these payoffs are too small (because of fierce competition), defection cannot be deterred.

The situation is modelled as a sealed bid multiple-object first-price auction with package bidding. There is an inefficient public firm (the government) and two private firms. Following Bernheim and Whinston (1986), symmetric information among bidders (but not between bidders and the auctioneer) is assumed. The agent who administers the auction can decide, on the basis of a private signal, on an extension of the submission deadline so firms can readjust their offers. In the absence of corruption, any Nash equilibrium is characterized by price competition between the private firms. We then introduce corruption by assuming that after submission of offers, the agent discloses the offers to the firms and invites them to compete in bribes for the 'right to decide' on the deadline. We show that the effect of corruption is to impose a cost on defection from collusive bidding. The defector must overbid (in bribes) the displaced bidder in order to avoid an extension of the deadline which triggers a low payoff Nash equilibrium. When the bribe needed to overbid the displaced bidder is sufficiently high, defection is deterred.

In the package auction, defection from collusive bidding profile implies that the displaced bidder earns zero payoff: his stake in the 'right to decide' is equal to his non-cooperative payoff. In contrast, in the single item bids auction, a defection from the collusive market sharing generally only reduces the other bidder's payoff (he is not fully displaced). That bidder may not be willing to pay much to revert to a low payoff Nash equilibrium. As a consequence, corruption may not suffice to deter defection and collusion fails. Our results are consistent with early conjectures that package bidding may facilitate collusion (e.g., CRA 1998). To the best of our knowledge, the argument has only been made on examples of

---

[5]See for instance Cybernomics (2000) and Ausubel and Milgrom (2002).

second price auction however.[6] In contrast, in the context of first price sealed bid package auction, existing results (Bernheim and Whinston 1986, Milgrom 2003) indicate difficulties for bidders to enforce collusive agreement.[7]

There exists a significant body of theoretical literature on collusion in auctions initiated by Graham and Marshall (1987) and McAfee and McMillan (1992). However, only recently have economists started to investigate corruption in auctions. In contrast to collusion, there exists currently no general approach to this problem. The emerging literature suggests that abuses of discretionary power can take two distinct forms depending on the nature of discretion. The first type of corruption is often referred to as favoritism. It relates to deals whereby the agent biases competition in favor of some individual firm(s) (see Laffont and Tirole, 1993, Burguet and Che, 2000). The second type of corruption targets competition per se. Discretion is abused in 'support' of collusion (Compte *et al*, 2000a).

The present analysis addresses the second type of corruption in the context of multiple-object auctions. Compte *et al* (2000a) show that in a first price single object auction, collusion may obtain in equilibrium when the agent is corrupt. Corruption is modelled as follows: the agent provides *one* firm with an *illegal* opportunity to secretly resubmit a bid in exchange for a bribe. A key feature in that model is that while firms compete in price and in bribe (for the opportunity to resubmit), competition in bribe is imperfect. In our model a similar result is obtained: corruption induces collusion. Our result however does not rely on any imperfection in bribe competition. Also, our focus is on the role of *legal* provisions in tender procedures. Two legal aspects might be singled out: (i) provisions that give discretion for the agent to give *all* bidders a chance to readjust their offer and (ii) rules pertaining to the formulation of bids. The issue of bid formulation is of major interest when dealing with multiple-item auction. The multiple-item context also appears to be a most relevant context to address issues related to links between corruption and collusion. Indeed, a majority of the corruption cases in e.g. France do pertain to situations where the market is made out of a number of contracts. In the 'Les Yvelinnes' case mentioned above, 88 con-

---

[6] In that context, collusion can be sustained in weakly dominated strategies in a single auction context..

[7] In particular, this follows from the fact that in the first price package auction, the best reply set always includes the *profit target* strategy - independantly of the others (pure) strategies (Theorem 8.6, Milgrom 2003). The *profit target* strategy equilibria also correspond to the coalition proof Nash equilibria.

struction/maintenance contracts were simultaneously allocated, and 9 firms were involved, together with civil servants and politicians. On the other hand, we are not aware of real life (procurement) tenders that provide the flexibility of the full-fledged combinatorial auction we investigate. Most often real-life mechanisms combine ex-ante bundling with some extent of packaging.[8] By contrasting a full-fledged combinatorial auction with a single item bid auction, our analysis can provide insights of relevance for these real life mechanisms.

The paper proceeds as follows. In the next section a simple numerical example is proposed to illustrate our basic story. In section 3, we present out theoretical model. Section 4 first establishes some results in the benchmark case with no corruption, proceeds to introduce corruption and the main results are derived. Those findings are contrasted with the results that obtain when the auction mechanism only allows for single item bids. Section 5 suggests some policy recommendations and discusses key assumptions. Section 6 concludes.

2 A Simple Example

In this section, we present a simple example to illustrate our basic story. A tender procedure allocates a project composed of four tasks (A, B, C, D). There are two private firms indexed 1 and 2, and a public firm (hereinafter, the government) indexed 0. The cost structure for all firms is given in Table 1. Each entry $c_i(S)$ reports firm i's cost of fulfilling the tasks consisting of $|S|$ units: firm i's cost for a package S depends on the quantity of tasks, $|S|$, but not on the identity of individual tasks in the package.

	$|S|=1$	$|S|=2$	$|S|=3$	$|S|=4$
$c_1(S)$	10	14	22	∞
$c_2(S)$	10	14	26	∞
$c_0(S)$	20	40	60	80

Table 1.

This example features some important elements of our general setup. Both private firms, 1 and 2, have U-shaped marginal cost curves, while the government's marginal cost is constant. The private firms are individually 'small' relative to the project. On the other hand,

---

[8] For an analysis of corruption risks in situations with ex-ante bundling see Compte et al. 2000b).

the market is small relative to the firms' total demand for tasks (priced at the government's unit cost). In other words, efficiency requires that both firms be allocated some tasks and we may expect competition between the private firms to obtain public contracts.

The firms are competing in a first-price sealed-bid auction with package bidding. Bids must be put on collections of identified tasks, not just quantities. Each firm i has a possibility to specify a bid $p_i(S)$ on any package S. For example, if firm i submits the following collection of bids $B_i = \{(AB, p_i(AB) = 12), (ABC, p_i(ABC) = 23), (BCD, p_i(BCD) = 25)\}$, it means that firm i is willing to fulfill tasks A and B for the payment of 12, or tasks A, B and C for 23, or tasks B, C and D for 25 (to simplify matters, only integer bids are allowed).

After all offers are submitted, the auctioneer opens the envelopes and selects a collection of packages that minimizes the total expenditure for the project. The winning firms are paid their bid and awarded the corresponding package from the winning collection. We depict three Nash equilibria of this game[9]: (a), (b), and (c).

	(a)	(b)	(c)
$p_1(D)$			28
$p_1(AB)$	25		32
$p_1(ABC)$	31	36	40

Table 2.

	(a)	(b)	(c)
$p_2(D)$		20	24
$p_2(CD)$	26		28
$p_2(BCD)$	31		40

Table 3.

Equilibrium (a) is efficient. It yields a total cost, $TC(\Omega) = 28$. Bidders 1 and 2 get two tasks each. The total expenditure for the government is $TE(\Omega) = 51$. The respective payoffs are 11 for bidder 1, and 12 for bidder 2. Equilibrium (b) is inefficient, $TC(\Omega) = 32$ ($TE(\Omega) = 56$). The payoffs in (b) are 14 for bidder 1 and 10 for bidder 2, totaling 24. This is more than the bidders' aggregate payoff in equilibrium (a). Equilibrium (c) yields a payoff of 14 to bidder 1 and of 18 for bidder 2. It is a particular case of what Bernheim and Whinston (1986) called a truthful strategy equilibrium.[10] These equilibria are of particular interest to our analysis.

We are now in a position to demonstrate how corruption allows the bidders to sustain prices higher than those in the Nash equilibria described above. Let us assume, for the sake

---

[9] For a complete specification of the auction game see section 3.

[10] A complete specification of a firm's truthful strategy requires that bids be defined for all packages. In our case we would e.g. have $p_1(A) = p_1(B) = p_1(C) = 28$.

of simplicity, that firms 1 and 2 share the market equally (in principle, any division-of-rents rule might be incorporated into our study). Market–sharing offers[11] corresponding to the partitioning $\pi^c = \{AB, CD\}$ include a single (serious) package bid for each one of the large bidders: $B_1^c = \{(AB, \ p_1(AB))\}$, $B_2^c = \{(CD, p_2(CD))\}$, with $p_1(AB) = p_2(CD) = 40$. Of course, this bidding profile is not a Nash equilibrium. Indeed, firm 1 has incentives to defect and bid $p_1(ABC) = 59$ to earn 37 instead of 26 in the proposed market-sharing scheme.

Suppose now that the agent has discretion to give firms an opportunity to readjust their offer. When the auctioneer is known be corrupt, market sharing becomes sustainable. To see that assume that equilibrium (c) is selected as the threat equilibrium i.e. the equilibrium that is being played if firms are given a chance to readjust. The following condition which compares the potential gains of defection (left-hand side) with the displaced bidder's willingness to bribe the agent to trigger readjustments (right-hand side), assures that bidding $p_1(ABC) = 59$ is not a profitable defection:

$$v_1\left(\{(ABC, 59), (CD, 40)\}\right) - v_1\left(\{(AB, 40), (CD, 40)\}\right) \leq v_2\left(\{(AB, 32), (CD, 28)\}\right).$$

Computing the figures in the inequality above, we obtain $37 - 26 \leq 14$. This means that to avoid readjustments the deviator must overbid the displaced bidder with a bribe larger that 14 which is more than his gain from defection. It is easy to show that in our example no defection is profitable. So we find that the corrupt agent incentives to extract rents from his discretionary power makes the market-sharing allocation sustainable. The net gain to the coalition amounts to 20. Bidder 2's net gain from collusion is 12 while bidder 1 gains 8. In this example the auctioneer receives no bribe in the equilibrium. With uncertainty about the agent's type (corrupt or honest), both collusion and bribes characterize the equilibrium.

3   A Model

There is a large project denoted Ω to be procured. The project is divided into k different tasks indexed with superscript $j : \omega^j$. We denote $S \subseteq \Omega$ a subset of tasks or a package.

---

[11]Market sharing offers, defined precisely below are offers that maximize the coalition payoff for a given partitioning.

There exists $2^k - 1$ possible combinations (packages) of tasks. The packages are indexed with a superscript h. The government can implement the project at a cost of a unit (1) per task.[12] There are two private firms 1 and 2. They have private costs for implementing tasks, $c_i : N \to R$, $c_i\left(S^h\right) = c_i\left(\left|S^h\right|\right)$ where $\left|S^h\right|$ denotes the number of tasks in package S^h.[13] They are more efficient than the government over some range: $\left.\Delta c_i\left(x\right)\right|_{x < n_i} \leq 1$, for $x < n_i$, and $\left.\Delta c_i\left(x\right)\right|_{x > n_i} > 1$, $x > n_i$, $i = 1, 2$.

Following Bernheim and Whinston (1986) we assume symmetric information among firms: the firms' cost for all packages are known to both firms. While the government and its agent (see below) only know the cost distribution of the private firms.

The auction procedure views each task as *unique*. A package is defined as a set of identified tasks (as opposed to a quantity of tasks). We denoted B_i an offer made by firm i. It is a collection of bids. A bid is a pair $\left(S^h, p_i^h\right)$ where S^h is a set of tasks (a package) and p_i^h is the minimum price firm i requires, so $B_i = \left\{\left(S^h, p_i^h\right)\right\}_{h \leq 2^k - 1}$. So, the bids are mutually exclusive (such bids are called XOR bids, see Nisan, 2000).

We consider a first-price sealed-bid auction with package bidding. An agent administers the procedure. His role is to publicly open the envelopes and select the cost minimizing collection of packages. In case of tie with the government, the agent favors the private firms. In case of a tie between the private firms, the agent randomizes with equal probability.

Let S_i^* denote i's package in the winning collection of packages, firm i's payoff only depends on S_i^* (no externalities)

$$v_i = p_i\left(S_i^*\right) - c_i\left(S_i^*\right). \tag{1}$$

Discretionary power

The agent that administers the procedure has some discretion over its implementation.[14] He can offer *simultaneously to both firms* an opportunity to readjust their offers - prior to

---

[12] This assumption is closely related to the free disposal assumption in standard auction.

[13] The *multiple unit* character of the cost structure is not determinant to any of the (qualitative) results in the paper. It only simplifies the presentation of the arguments. The *multiple object* character of the auction, as opposed to multiple unit, is captured by the auction rule.

[14] The public buyer is ultimately the tax payer. The agent is a player who has been delegated the power to administer the allocation procedure.

the official opening. This corresponds to extension of the deadline in real-life public tenders described above. The usual motivation is as follows. The agent may privately notice an ambiguity in a tender document or learn that some firm has an undue information advantage. The procedure requires that the agent clears the ambiguity (information advantage) and offers all firms an opportunity to readjust their offer. Most often, rules allowing for readjustments are motivated appealing to an objective of fairness in competition. Not seldom, the declared objective is to combat favoritism (see the discussion in Section 6).[15]

The firms and the agent (but not the government) share information about the relevance for competition of the alleged default (information advantage). In the analysis, we neglect all instances where the deadline is extended for good reasons. Instead, the decision to extend the deadline for readjustments always refers to an abuse of discretion.

The agent may be of two types: either honest, or corrupt. When he is honest, he never takes bribes. In contrast, the corrupt agent can be bribed to abuse discretion. His payoff is equal to the sum of the bribes he receives. When the agent is indifferent between abusing discretion or not, he chooses not to. This captures the idea that there is a cost to abusing discretion, normalized here to zero.

4 Analysis

The no-corruption case

As a benchmark, we first investigate the case when the agent is known to be honest. The allocation process is given by the timing of the auction game as follows:

$\tau = 0$: The project $\Omega = \{\omega^j\}_{j=1}^k$ is announced, both bidders learn the costs for all the packages of tasks.

$\tau = 1$: The firms submit their offers, a collection of prices and associated packages, in sealed envelopes.

---

[15] Another rationale for such rules can be found in the seminal paper by Milgrom and Weber (1982). They show that the auctioneer can reduce his expenditure by improving bidders' information on a common value component. (See, however, Mikusheva and Sonin, 2002).

10

$\tau = 2$: The agent opens the envelopes and selects the cost minimizing collection of packages and the winning firms are paid their bid, which ends the game.

In the single object first-price auction with symmetric information, the question of allocation efficiency entails no subtleties whatsoever: in equilibrium, the contract goes to the firm which has the lowest costs. The equilibrium price then corresponds to the second lowest cost. In contrast, a multiple object auction with package bidding may have multiple equilibria some of which are inefficient (e.g., as in the example in Section 2)[16]. Bernheim and Whinston (1986) established a few key results applying to symmetric information first price 'menu auctions'.[17] In particular, they show that any first price menu auction has a truthful equilibrium which also is coalition-proof and yields an efficient allocation.[18] When there are only two bidders, the coalition-proof Nash equilibrium is unique and the equilibrium net payoffs are identical to the extended Vickrey auction's payoff .

We start with the following straightforward results :

Proposition 1 *If $n_1 + n_2 \leq k$, then there exists a Nash equilibrium of the first-price auction that yields a total expenditure equal to k. Otherwise, any Nash equilibrium yields a total expenditure for the government $TE(\Omega) < k$.*

Proposition 1 simply states that when the market is large relative to private firms' demand for tasks (given the public alternative), there exist equilibria where they do not compete with each other. In particular, the Vickrey payoffs imply a compensation of a unit per task. When the market is too small, i.e. $n_1 + n_2 > k$, there exists no partition of the market such that the private firms do not compete with each other. Any Nash equilibrium entails a total compensation for the winning packages of tasks with an average price per task less that 1. This simple result is the starting point for our questioning as to whether and how corruption can help bidders to avoid costly competition in this context.

---

[16]Bernheim and Winston (1986) discuss the introduction of uncertainty to conclude that it does not eliminate the inferior equilibria.

[17]In a menu auction, the bidders put bids on the whole allocation (a decision) while in the package auction they only bid on their own packages. This distinction is not relevant in our context.

[18]As mentioned in the introduction this result suggests difficulties sustaining collusive equilibria.

In what follows we assume that $n_1 + n_2 > k$, and that the non-cooperative outcome of the package auction is the coalition-proof Nash equilibrium. It is efficient and yields the Vickrey payoffs. Given this non-cooperative outcome, the paper focuses on the issue of existence of equilibria in a game extended with corruption such that they yield a larger (than Vickrey) payoff to the winners. Specifically, we focus on equilibria where bidder 1 and 2 collude to share the market at the 'reserve price' corresponding to the public cost of production i.e. a unit per task. We assume that side transfers between firms are precluded.

Corruption

We now consider a situation where the agent may be either honest or corrupt. When he is of the corrupt type, he uses his discretionary power to extract bribes. Our first objective is to exhibit a complementarity between the corrupt agent's self-interest and the bidders' interest to avoid competition. To this end, we extend the benchmark model with a corruption stage. At the corruption stage, the agent uses discretion to extract rents. Specifically, he secretly reveals the winning collection of bids to bidders. Thereafter, he selects a procedure and 'sells' for bribes his 'right to decide' over the deadline.

Formally, our game has four stages:

(i) *Revelation stage*

Each firm secretly offers a fixed 'initial fee' f to the agent. The agent simultaneously accepts or refuses the fees.

(ii) *First submission of offers:* each firm submits its offer B_i a collection of bids including a price and an associated package.

(iii) *Corruption game:*

a. The agent discloses the winning collection of bids to the firms.

b. He announces a procedure and the bidders make their bribe offers.

c. The agent selects a winner. If the winner chooses not to extend the deadline, the agent proceeds to (vi).

d. If the winner decides to extend the deadline, the firms are invited to readjust their offers.

(iv) *Selection:*

The agent selects from among the last submitted offers the cost minimizing collection of bids. If the price for the package in the winning collection is lower than the public cost, the firm is awarded the package and paid its bid.

Definition 1 *The offers $\{B_i^c\}_{i=1,2}$ form a market sharing bidding profile (in a first-price package auction) if and only if $B_i^c = \{(S_i^c, p_i^c), (S_i', p_i')\}$ where $S_i^c \cap S_j^c = \emptyset$ and $p_i^c = |S_i^c|$ for $i \neq j$, $i, j = 1, 2$ and $p_i' > |S'|$ for $S' \neq S_i^c$.*

Given any partition of the market $\pi^c = \{S_i^c\}_{i=1,2}$, the corresponding market-sharing strategies maximize the coalition's payoff: the private firms do not compete with each other. A key feature of a market-sharing offer is that it includes a single 'serious' bid, the one on the collusive package. The other bids are 'non-serious' bids: they just exceed the government reserve price.

Let $\mathbf{B}^0 = \{B_i^0\}_{i=1,2}$ denote the truthful bidding profile relative to π^* (an efficient allocation) so $v_i(\mathbf{B}^0)$, $i = 1, 2$ are the corresponding Vickrey payoffs. Let f denote the 'initial fee' such that $f < p v_i(\mathbf{B}^c)$ where $p \leq 1$ denotes the firms' (common) priors about the agent being of the honest type. We have the following result:

Proposition 2 *Any market-sharing strategy profile $\{B_i^c\}_{i=1,2}$ such that $v_i(\mathbf{B}^c) \geq v_i(\mathbf{B}^0) + f$, $i = 1, 2$ can be sustained in a Bayesian perfect equilibrium of the game with corruption provided that*

$$(COR): \ v_i\left(\widehat{B}_i, B_j^c\right) - v_i(\mathbf{B}^c) \leq v_j(\mathbf{B}^0), i \neq j, \ i, j = 1, 2$$

for any offer $\widehat{B}_{i(j)}$.

Proposition 2 tells us that, under condition (COR), market sharing is sustainable when the agent is corrupt. The agent's own incentives to exploit defection to extract rents secures his contribution to the ring which is to deter defection. In the appendix, we show that the agent's 'allocation problem' i.e. how to maximize the revenue from the selling of his 'right to decide', can be solved by a simple auction mechanism.[19] The agent lets the firms

---

[19]In our context, a first price auction is an optimal mechanism. This is because the agent's choice of mechanism is constrained by a requirement that implementation be incentive compatible. In particular, when offering different bribes, firms expect that the auctioneer selects the firm that offered the largest bribe.

compete in bribe for obtaining the right to decide over the deadline.[20] This auction in bribes implies punishments for deviation from the collusive agreement as follows. When bidders play market-sharing strategies, defection from one bidder implies that the other earns zero as his single serious bid is being displaced. Therefore, he has incentives to bribe the agent to extend the deadline, so he can readjust his offer, and subsequently earn the non-cooperative payoff. The defector also offers a bribe to counter the displaced bidder's proposal, that is *to avoid* that the agent extends the deadline. Under condition (COR), the cost of overbidding the displaced bidder is so large that no profitable defection exists. In this sense, condition (COR) establishes a connection between the market structure and collusion.

Proposition 3 *(i) When one firm dominates the market, collusion to share the market cannot be sustained by the mechanism described in Proposition 2.(ii) Market sharing is more likely to be sustainable, if competition between private firms is less severe.*

Proposition 3 captures the main predictions of our model. Market sharing is more likely in tenders where no firm clearly dominates the market and when the large firms are not too close competitors. These predictions may at first appear counter intuitive. Collusion may fail when the collusive rents are high but succeed when they are lower. The reason for this is as follows. In the equilibrium described in Proposition 2, the punishment for defection is not the (low) Nash equilibrium payoffs *but the bribe the defector has to pay to avoid it.* The highest bribe the displaced bidder is willing to pay is equal to his non-cooperative payoff. This payoff (and the maximum bribe) is larger, the less intensive the competition is. The proof of Proposition 3 makes use of the fact that the non-cooperative outcome is a coalition-proof truthful equilibrium. As in many other cases when dealing with package auction, we cannot prove the generality of this result. Bernheim and Winston (1986) show that although the truthful equilibria are not the only stable equilibria, the truthful outcomes are the only stable outcomes.

The mechanism of Proposition 2 ascribes corruption (abuse of discretion in exchange for a bribe) a crucial role, out of the equilibrium path however. In equilibrium, no defection occurs and the agent's rents are equal to $2f$. The payment of the fee is due to the firms'

---

[20]The bidders cannot collude in the corruption mechanism since there is no way for them to deter defection.

incomplete information about the agent's type. The agent's acceptance of the fee is a signal that he his corrupt, so firms learn that he is able to punish deviations. In the absence of uncertainty about the type, the agent earns no rents in equilibrium.[21] To understand better the issue of equilibrium rent -sharing, consider a slight modification of the model: in addition to his discretion to extend the deadline, the agent has an option to ex-post costlessly 'alert a control agency'. Intuitively one expects that the agent be able to appropriate some of the ring's rents under the threat of denouncing the successful ring. Assume reasonably that an agent who accepts a bribe but denounces the ring gets fired (the firms denounce him). It can be shown that there exists no equilibrium in pure strategies where both firms pay a bribe. But then if there exists an equilibrium (in mixed strategies) with some collusion, it must be that in some cases no firm pays.[22] This suggests that the threat of denouncing is not sufficient to secure the agent strictly positive rents in equilibrium.

The result in Proposition 2 relies on several key assumptions: (i) The agent has some discretion to give all firms a chance to readjust their bid; (ii) the agent knows the content of the offers; (iii) the agent's objective is to extract rents.

We discuss them in turn. (i) As mentioned in the Introduction there is ample evidence of discretionary rules in procurement laws and guidelines that in effect give the agent the right to let firms readjust their offer (before the official opening). (ii) It might seem questionable to assume that the agent knows the content of the offers so he can disclose it. Indeed, a main rule of public tenders is that no one should have access to that information before the official opening. However, there is empirical evidence the agent has been able to learn the offers before the official opening. One example is in the court case concerned with the construction of the High Speed line North in France (Cartier Bresson, 1998).[23] (iii) The assumption on self-interest is supported by widespread empirical evidence of corruption in procurement around the world (e.g., Transparency International Global Report, 2002). In

---

[21] Note that with complete information about the type the corrupt agent deters deviation all the same.

[22] This follows from the conjunction of the following features: i. the agent cannot credibly commit to refuse a bribe when only one firm pays; ii. it is sufficient that one firm pays to secure his silence; iii. both firms prefers the other one to pay.

[23] An SNCF (French Railroad) agent was convicted for having opened and disclosed the content of offers to members of a cartel. He also gave them two (?!) opportunities to readjust their offer.

the proposition, the bribing game maximizes the agent's rents. We below show that similar results obtain with a bribing mechanism that is not optimal.

The proof of Proposition 2 does make use of the fact that there are only two private firms. In an earlier version of this paper, it was shown that a similar result can be obtained with n firms. In the $n-$firms case, the first-bribe auction is associated with a free-riding problem: if there are several displaced bidders, who pays for readjustments? The agent uses a mechanism that first selects two firms out of the $n-$firms, and thereafter let only those two compete in bribes.

Alternative bribing game

Consider the following alternative formulation of the corruption game: After the offers have been made, the agent discloses them to the displaced bidder (if any) and makes a take-it-or-leave-it offer: he offers to extend the deadline in exchange for the payment of a bribe equal to a fee z. If the firm rejects the offer, the agent proceeds to the official opening. As in proposition 2, we assume $f < pv_i(\mathbf{B}^c)$ and $v_i(\mathbf{B}^c) \geq v_i(\mathbf{B}^0) + f$. The timing is the same as in Section 4 except for the corruption game which is given by the formulation above.

Proposition 4 *Under the alternative bribing game for z ; $z \leq v_i(\mathbf{B}^0)$ for $i = 1, 2$, the FPAP cum corruption game has perfect Bayesian equilibrium where the private firms play market-sharing strategies.*

The proof is immediate. When $z > v_i(\mathbf{B}^0)$ for $i = 1, 2$, the displaced bidder rejects the offer. He prefers to earn zero $v_i\left(B_i^c, \widehat{B}_j\right) = 0$, $i \neq j$. By Proposition 1, defection is profitable and collusion fails. With $z \leq v_i(\mathbf{B}^0)$ for $i = 1, 2$, the displaced bidder accepts to pay the bribe which yields him a payoff of $v_i(\mathbf{B}^0) - z \geq 0$. The deadline is extended which makes defection non profitable.

5 Package Bidding

This section aims at illustrating the role of package bidding in facilitating corruption in procurement tenders. For this purpose we compare the vulnerability to market sharing of the package auction with that of a first-price multiple-item auction that allows for single-item bids only.

In view of the limited objective of this section, we shall perform this comparison in a simpler environment than that of the benchmark model. Indeed, we know from Milgrom (2002) that in an environment characterized by price complementarities there may not exist single-item equilibrium prices. The 'single-item-bid' auction may not have any Nash equilibrium at all.[24] We thus instead consider an environment where the tasks are price substitutes. Costs are additive over the whole set Ω. More precisely the cost structure is given by:

$$c_i \left(S^h \right) = \sum_{\omega^j \in S^h} c_i \left(\omega^j \right),$$

$$\left| \left\{ \omega^j; c_i \left(\omega^j \right) \le 1 \right\} \right| = n_i, \ n_1 + n_2 > k.$$

As in the benchmark model we assume symmetric information among firms.

A single-item bid auction is defined by the following bidding rule. The firms submit an offer in a sealed envelope. An offer is a collection of *non-exclusive* single-task bids. The agent selects the price minimizing collection of bids subject to the constraint that the private alternative is cheaper than the public alternative. The winners are paid their bid. To simplify the presentation of the result, we assume that the government (indexed 0) puts in a bid of a unit on each task.

Proposition 5 *A first-price auction, where bidders are allowed to bid on single items only, has an efficient Nash equilibrium characterized by* $i^* \left(\omega^j \right) = \arg \min_{i=0,1,2} c_i \left(\omega^j \right), \ p^* \left(\omega^j \right) = \min \left\{ 1, \ \min_{i \ne i^*, i=1,2} c_i \left(\omega^j \right) \right\}.$

---

[24]Consider the following example, firm 1's technology exhibits increasing returns to scale over AB while firm 2's technology exhibits decreasing returns to scale:

	A	B	AB
$c_1 (.)$	10	15	20
$c_2 (.)$	7	12	22

Player 1 cannot make offers on A and B that sum up to less than 20. But there exists no combination of single task bids such that player 1 is not outcompeted by player 2 on either A or B. But then 1 wins the other task at too low a price and it cannot be optimal for him to do so. If Player 2 makes a combination of two bids that sum up to at least 22, player 1 can outcompete with a combination that sum up to 21.

In such an auction, the allocation of each task can be viewed as an independent first-price auction. With symmetric information, the lowest cost bidder wins and is paid the second lowest cost unless that cost is above 1. The outcome is the same as in the second price (Vickrey) auction. Let $\mathbf{B}^0$ denote the non-cooperative equilibrium offer profile. We now ask whether or not bidders can sustain a market-sharing agreement such that they only compete with the government, but not with each other. Market sharing-offers in a single-item auction are defined as follows.

Definition 2 *The offers* $\{B_i^c\}_{i \in M}$ *form a market-sharing bidding profile in a first-price single-item auction if* $B_i^c = \left\{ \left(\omega^j, p_i^j \right)_{j=1,..k} \right\}$ *;* $p_i^j = 1$ *for* $\omega^j \in S_i^c$ *and* $p^j > 1$ *for* $\omega^j \notin S_i^c$ *with* $S_i^c \cap S_k^c = \emptyset$*, for* $i = 1, 2$ *.*

For any partition of the market $\pi^c = \{S_i^c\}_{i=1,2}$, the corresponding market-sharing offers maximize the firms' payoff. Note that each bidder only puts a serious bid on the tasks belonging to his collusive 'package'. Despite the similarities with the market-sharing strategies in the first-price auction, it is much more difficult to sustain collusion in the single-item format as compared with the format allowing for package-bidding. To see that, we extend the single-item auction with a corruption game essentially identical to the one introduced in the preceding section.

Proposition 6 *With corruption, a market-sharing strategy profile* $\{B_i^c\}_{i=1,2}$ *,* $v_i(\mathbf{B}^c) \geq v_i(\mathbf{B}^0) + f$*,* $i = 1, 2$ *can be sustained in a Bayesian perfect equilibrium of the first-price auction with bids restricted to single items provided that*

$$(COR') : \ v_j \left(\widehat{B}_j, B_{-j}^c \right) - v_j(\mathbf{B}^c) \leq v_i(\mathbf{B}^0) - v_i \left(\widehat{B}_j, B_{-j}^c \right)$$

$i \neq j$*,* $i, j = 1, 2$ *for any offer* $\widehat{B}_j(\widehat{B}_i)$*.*

Comparing with the format that allowed for package-bidding, we immediately see that that the new condition (COR') is far more restrictive than the (COR) condition of Proposition 2. The right-hand side of the inequality includes a negative term $-v_i \left(\widehat{B}_j, B_{-j}^c \right)$. This term is responsible for a main distinction between the two auction formats with respect to their vulnerability to collusion. In the package auction the market-sharing strategies are

designed so as to give maximum incentives for the displaced bidder to bribe the agent: in the absence of readjustments he earns zero, $v_i \left(\widehat{B}_j, B_{-j}^c \right) = 0$. Such strategies are not available in the auction, because the tasks are not bundled into packages. When the defector overlaps on the collusive set of tasks of the other bidder, the later is only partially displaced $v_i \left(\widehat{B}_j, B_{-j}^c \right) \geq 0$. The displaced bidder earns a smaller set of tasks paid at the high collusive price however. Therefore, he may not be willing to pay a bribe to extend the deadline and readjust to the Nash equilibrium. But then defection cannot be deterred and collusion fails.

We illustrate this insight in an example. The costs are depicted in Table 4, and the non-cooperative allocation in Table 5:

	ω^1	ω^2	ω^3	ω^4
$c_1(\omega)$	8	15	10	15
$c_2(\omega)$	14	5	14	10
$c_0(\omega)$	20	20	20	20

$i^*(\omega^1) = 1$	$p^*(\omega^1) = 14$
$i^*(\omega^2) = 2$	$p^*(\omega^2) = 15$
$i^*(\omega^3) = 1$	$p^*(\omega^3) = 14$
$i^*(\omega^4) = 2$	$p^*(\omega^4) = 15$

Table 4   Table 5

The non-cooperative payoffs are respectively: $v_1(\mathbf{B}^0) = 8 + 4 = 12$, $v_2(\mathbf{B}^0) = 10 + 5 = 15$. The collusive payoffs associated with the efficient partition are: $v_1(\mathbf{B}^c) = 12 + 10 = 22$, $v_2(\mathbf{B}^c) = 15 + 10 = 25$.

First, we see that in the auction where bids are allowed on single-items only, player 2 has an incentive to displace player 1 on ω_3 by bidding 19. If doing so, he earns 30. Since $v_1 \left(B_2^{def}, B_1^c \right) = 12 = v_1(\mathbf{B}^0)$, player 1 has no incentives to retaliate and the collusive agreement cannot be sustained.

Consider now the following package bids: (p_1^c, S_1^c) where $p_1(S_1^c) = 40$ and $S_1^c = \{\omega_1, \omega_3\}$ and (p_1^c, S_1^c) where $p_1(S_1^c) = 40$ and $S_2^c = \{\omega_2, \omega_4\}$. By construction the most profitable defection is to displace the other bidder on both tasks. If player 2 displaces 1 e.g. with a bid of 39 on $\{\omega_1, \omega_3\}$, condition (COR) writes $36 - 25 < 12$, the condition for player 1 is $31 - 22 < 15$. In sum, market-sharing (absence of competition between bidders) is sustainable in the first-price auction with package-bidding, but not in the auction, where bidders are required to bid on single items only.

The main lesson from this section is that while package bidding is potentially efficiency

improving as suggested by experimental work (e.g., Cybernomics 2000), the flexibility of package bidding may have a substantial flip side. Flexibility rules designed to improve competition can be exploited to defeat it. With package bidding, this happens because with a richer set of bidding strategies, the bidders can select an offer that implies a credible commitment to retaliate in case of defection. This result is consistent with remarks concerning the risks connected with flexibility with regards to other aspects of competitive procedures. One example concerns the right to withdraw bids discussed in connection with the elaboration of the FCC package auction design. Plott and Salmon (2000) argue that the right to withdraw bids open up gaming opportunities detrimental to competition.

6   Policy Recommendations

Let us summarize the main insights of our analysis: First, corruption and collusion in procurement tenders exhibit strategic complementarity. In particular, seemingly innocuous features of the procedures that in effect give opportunities for bid readjustments can be exploited to defeat competition. Second, package bidding, a heavily promoted idea aimed to enhance allocative efficiency, facilitates collusion. Third, in the presence of corruption, collusion is more likely when firms are not too close competitors.

A first insight of the analysis in terms of policy implications is that the agent's discretion to intervene in the process with e.g. new information and to give firms a chance to readjust their offer accordingly is potentially very disruptive. Thus, one should aim at reducing the agent's discretion in that respect. One possibility is to make the agent liable for ambiguities and other defaults in the bidding documents. Another measure is to submit the agent's motivation to serious examination before a readjustment of the bids is allowed. Note however that this restrictiveness runs against arguments that view favoritism as the main issue. These arguments emphasize that the agent must be held responsible for avoiding a situation in which a firm has an undue information advantage. How unduly advantageous a piece of information is, typically difficult to assess which is why, e.g. the World Bank is not restrictive on motivations.

A second insight of the analysis is that package bidding facilitates collusion to share the

market. An immediate recommendation is to limit the use of package bidding to situations, where significant complementarities are expected. Where the patterns of complementarity are similar among firms, ex-ante bundling of objects maybe preferable. When those patterns are not identical, ex-ante bundling generates a risk of favoritism however: the agent bundles tasks to favor one firm at the expense of others. Wherever this is feasible it seems clearly desirable that the procedure requires that any package bid also includes serious sub-package bids. This may allow the retention of some of the benefits of package bidding while reducing the risks for collusion.[25] The requirement cannot always be imposed however. Indeed there may be significant cost associated with the evaluation of the subpackages.

Our third insight invite us to be particularly vigilant to the risk of collusion when competition between firms is not too fierce. This corresponds to situations when the market is characterized by a small numbers of medium size (relative to the whole market) firms of comparable productive efficiency. This insight is expected to be relevant in contexts where interaction between firms is one-shot or the market is finite (but allocation may involve a finite series of multiple object auctions). In those cases side-transfers are likely not to be enforceable and some form of outside enforcement (e.g. corruption) may be necessary. As we argue below the argument can partly be extended to contexts characterized be repeated interaction.

The analysis is performed in the context of a single auction. Yet, it is often argued that collusion in public tenders is enforced by repeated interaction. There are both theoretical arguments and empirical evidence suggesting that corruption also plays an important role for collusion in procurement. A first argument is that there is often significant uncertainty and variation in the profitability of future contracts. This creates tensions in the cartel. This kind of problem is similar to the one encountered by a price cartel when the market is subject to demand shocks (see Rotember and Saloner 1986, Green and Porter 1984). In procurement, relying on corruption may turn out to be an optimal solution when uncertainty about he profitability of future contracts is large. As mentioned in the Introduction a main reason for why we should expect links between collusion and corruption is that a corrupt agent's

---

[25] Such procedures are sometimes used, most often with other objectives in mind. One example is in the auction for selling portfolio of the Portland General Electric Company (see Milgrom (2000)).

objective is to extract rents. He therefore has his own incentives in collusion: by helping avoid competition, he contributes to creating rents that he can appropriate. As illustrated by the case of 'Les Yvelinnes' mentioned in the Introduction. The details of the judgment clearly show that the corrupt politician and the civil servants initiated and fully arbitrated the cartel. They selected the firms that were to participate. They divided the market among the firms. And they punished deviators. This was made possible by communicating selectively an information that could be used to formulate a winning bid. In case a bidder deviated (tried to obtain more), the information could however be made worthless by the politician. The scheme used in Les Yvelinnes is different from the one investigated in the paper. The agent(s) had much more discretion. The main point however is that the agent's incentives to extract rents from his discretion did lead to collusion among firms. The collusive scheme relied primarily on corruption despite repeated interaction between the firms. This appears also to have been the case in the earlier mentioned corruption/collusion case in the procurement of the Paris City Hall's construction projects. Nine of France's largest construction firms (including Bouygues, Dumez, SGE, SAEP etc..) are involved.

These examples illustrates a strong community of interests between firms and the corrupt agent in defeating competition. Clearly, there is also competition between the ring and the agent for the rents. Our next step in this research program includes the analysis of corruption in a repeated collusion context.

7   Conclusion

This paper has developed a theoretical argument showing that collusion and corruption in auction are linked. Corruption and collusion in procurement tenders are strategic complements: Firms can use corruption to enforce collusive bidding. The agent can use collusion to extract rents from his discretionary power, e.g., if auction rules give him an opportunity for bid readjustments.

That firms agree to share the market instead of competing is a common form of collusion in the procurement of divisible contracts. Empirical evidence suggests that corruption often accompanies collusive market sharing. This paper proposes an explanation appealing to an

agency problem in the administration of the auction procedure. Specifically, we assumed that agent's incentives are not aligned with those of the public. The auctioneer may be corrupt and seek to extract rents from discretion in the management of the auction procedure. We focused attention on features of discretion that imply that the agent can offer, simultaneously to all, a chance to readjust offers before the official opening. Our first result shows that the agent's incentive to extract rents from his discretion can induce collusive bidding. Collusion becomes sustainable because the agent has private incentives to exploit defection to earn rents, which makes defection non-profitable. A second result is that package bidding facilitates collusion. Package bidding allows for collusive strategies which include a credible commitment to retaliate. We compare with an auction format with single item bids only. We show that generally the deterrence power of corruption is severely reduced. The analysis predicts that collusion is more likely on markets where firms are not too close competitors.

Our main policy implications are focused the design of tender procedures. The results suggest that one should be careful when giving the agent discretion to intervene in the procurement process even with seemingly innocuous powers. The results also suggests that the potential benefits of the use of package bidding must be traded-off against increased risk for collusion. Finally, an implication of our results is that control agencies should pay special attention to markets where firms are not too close competitors.

Ausubel, L. and Cramton, P. (1998) 'Demand Reduction and Inefficiency in Multi-unit Auctions', University of Maryland.

Ausubel, L. and Milgrom, P. (2002) "Ascending Auctions with Package Bidding", *Frontiers of Theoretical Economics*: Vol. 1: No. 1, Article 1. http://www.bepress.com/bejte/frontiers/vol1/iss1/

Bernheim, D. and Whinston, M.(1986) 'Menu Auction, Resource Allocation and Economic Influence' *Quarterly Journal of Economics* Vol CI 1-31, 1-31.

Brusco, S. and Lopomo, G. (1999) 'Collusion via Signaling in Open Ascending Auction with Multiple Objects and Complementarities' mimeo NYU

Burget, P. and Che, J. (2000) 'Competitive procurement with Corruption', working paper SSRI 2023.

Crampton, P. and Schwartz, J. (1999) 'Collusive Bidding in the FCC Spectrum Auctions' University of Maryland *mimeo.*

Compte O., Lambert-Mogiliansky, A. and Verdier, T. (2000a) 'Corruption and Competition in Public Market Auctions' CEPR *Discussion paper series N.* 2434.

Compte O. and Lambert-Mogiliansky, A. (2000b) 'Efficacité et Transparence dans les Procédures de Spécification sur les Marchés Publics'. RCB report, Ministère de l'Economy et des Finances, Paris.

Charles Rivers Associates (1998), 'Package bidding for Spectrum licences' CRA Report 1B, n.1351-00.

Cour D'Appel de Versaille 'Requisitoire définitif, instruction n: 5/95/83' January 2002.

Cybernomics (2000), 'An experimental Comparison of the simultaneous Multiple Round Auction and the CRA Combinatorial Auction' Report to the Federal Communication Commission.

Graham, D. and Marshall, R. (1987) 'Collusive Bidder Behavior at Single Object Second price and English Auctions' *Journal of Political economy* vol. 95/6, 1217- 1239.

Green, E and Porter, R. (1984), 'Noncooperative Collusion under Imperfect Price Information' *Econometrica* 52-1, 87-100.

Hendricks, K. and Porter, R. (1989) 'Collusion in Auctions' *Annales d'Economie et de Statistiques* 15/16.

Klemperer, P. (2002) 'Using and Abusing Economic Theory' Alfred Marshal Lecture EEA,

mimeo.

Laffont J. and Tirole, J.(1993) Incentives in Regulation and Procurement, MIT Press.

Ledyard, J. Porter, R., and Rangel, A. (1997) 'Experiments Testing Multiobject Allocation Mechanisms' *Journal of Economics and Management Strategy* 6/3, 639–675.

Mc Afee, P. and McMillan, J. (1992) 'Bidding Rings' *American Economic Review* 82, 579-599.

Milgrom, P. and Weber, R (1982) 'A theory of Auctions and Competitive Bidding' *Econometrica* 50 1089-1122.

Milgrom, P. (2003) Putting Auction To Work, Cambridge, Cambridge University Press.

Nisan, N. (2000) 'Bidding and Allocation in Combinatorial Auctions' Mimeo, Institute of Computer Sciences Hebrew University U, Jerusalem.

Plott, C. and Salmon, T. 'Comments Sought on Modifying the Simultaneous Multiple Round Auction Design to Allow for Combinatorial (package) Bidding' FCC report N AUC 00-31-G.

Rothember, J.. and Saloner, G. (1986), 'A Supergame Theoretical Model of Price War during Booms', *American Economic Review* 76:3 pp 390-407.

Vickrey, W. (1961) 'Counter-speculation, Auctions and Competitive Sealed tenders' *Journal of Finances*, 8-37.

Wilson, R. (1979) 'Auction with Shares' *Quarterly Journal of Economics* 93, 675-89.

. **APPENDIX**

Proof of Proposition 1

Our auction game allocates k tasks. The public alternative: home production at a cost 1 per task is common knowledge. The home production alternative acts as a price cap and secures that all tasks be allocated. The auction game is played among the two large firms. We use results from Berhneim Whinston (1986) and derive for the unique coalition-proof Nash equilibrium payoffs.

Let $\Omega \backslash S^h$ denote the complement in Ω of S^h. Recall the agent breaks tie in favor of the private firms.

i. When $n_1 + n_2 \leq k$, any efficient partition $\pi^* = \{S_1^*, S_2^*, S_0^*\}$ where S_0^* denotes the package that is allocated to the government, satisfies: $|S_1^*| = n_1$, $|S_2^*| = n_2$, $|S_0^*| = k - n_1 - n_2$.

The corresponding Vickrey prices are:

$$
\begin{aligned}
p_i\left(S_i^*\right) &= \left[p_j\left(n_j\right) + k - n_j\right] - \left[p_j\left(n_j\right) + k - n_i - n_j\right] \\
p_i\left(S_i^*\right) &= n_i, \ i = 1, 2.
\end{aligned}
$$

Bidder i's equilibrium offer includes a bid of n_i on package S_i^* and by definition of the truthful strategy it includes bids on all the other packages which either yields a payoff of $v_i\left(S_i^*\right) = n_i - c_i\left(S_i^*\right)$ or are null. The total expenditure for the project is $TE\left(\Omega\right) = p_1\left(S_1^*\right) + p_1\left(S_1^*\right) + k - n_1 - n_2 = k$.

ii. When $n_1 + n_2 > k$, there is an efficient partition $\{S_1^*, S_2^*, S_0^*\}$ with $|S_0^*| = 0$. The corresponding Vickrey prices are:

$$
\begin{aligned}
p_i\left(S_i^*\right) &= \left[p_j\left(n_j\right) + \left|\Omega \backslash S^h\right|\right] - \left[c_i\left(S_i^*\right) + c_j\left(S_j^*\right)\right] \\
p_i\left(S_i^*\right) &\leq n_i \ i = 1, 2 \text{ and } v_i\left(S_i^*\right) < n_i - c_i\left(S_i^*\right) \text{ for at least some } i.
\end{aligned}
$$

In this equilibrium the total expenditure $\sum p_i\left(S_i^*\right) < k$. We next show that there exists no equilibrium with $TE\left(\Omega\right) = k$. Assume by contradiction that $\sum p_i\left(S_i^*\right) = k$. Since the government never pays for a package more than it costs for a public firm to produce it, we must have $p_i\left(S_i^*\right) = |S_i^*|$, $i = 1, 2$. Next since $n_1 + n_2 < k$, either $|S_1^*| < n_1$ or $|S_2^*| < n_2$ or both. Suppose $|S_1^*| < n_1$, bidder 1 could submit an offer including a bid on $S_1' = \{S_1^*, \omega^j\}$, $\omega^j \in S_2^*$ at price

$|S_1'| - \varepsilon$. The cost minimizing collection would then be $\{S_1', S_0\}$, $S_0 = \Omega \backslash S_1'$. Bidder 1 would thus win S_1' and obtain a payoff $v_1(S_1') = |S_1'| - \varepsilon - c_1(|S_1'|)$. Since $\Delta c_i(x)|_{x<n_i} \le 1$, for $x < n_i$, and $\Delta c_i(x)|_{x>n_i} > 1$, $x > n_i$, $i = 1,2$, we have that $v_1(S_1') > |S_1^*| - c_1(S_1^*) = v_1(S_1^*)$. Hence, the offers that yield a minimum cost of $\sum p_i(S_i^*) = k$, are not equilibrium offers. When $\sum_{i=1,2} n_i > k$, there exists no Nash equilibrium such that $TE(\Omega) = k$ but there exists an equilibrium which yields $TE(\Omega) < k$. $\blacksquare$

Proof of Proposition 2

We show that the following strategies form a Bayesian Nash equilibrium of the FPAP cum corruption game. We assume common knowledge of a standing partition π^c.

The agent

i. If the agent is honest he rejects the bribe offers and administers the procedure as an automata. If the agent is corrupt, he accepts the fees;

ii. After he received the offers he discloses them to the firms and proposes his rtd in a first bribe auction;

iii. He awards the rtd to the highest bribe bidder. If the later decides to extend the deadline, the agent does so whereafter he proceeds to vi. If not extension is demanded,

vi. he proceeds to the official opening of the last submitted offers, computes the cost minimizing collection of package and allocates the contracts accordingly.

The firms

i. Offer the fee f, if the agent rejects it play the non cooperative Nash equilibrium of the FPAP. If the agent accepts,

ii. submit the MS offer corresponding to the standing partition;

iii. If the disclosed offers reveals no defection wait for the official opening. If some firm defected,

vi. submit a bribe bid that is a best response to your opponent in a first bribe auction;

v. If the deadline is extended, submit the non-cooperative Nash offer.

We now check that these strategies are part of an equilibrium.

Selection stage:

The agent has no discretion so it is trivially optimal to compute the cost minimizing

collection of bids out of the last submitted offers and select the winners who are paid their bid.

Corruption game

d. If an extension of the deadline is announced, bidder $i = 1, 2$ readjusts his offer by construction B_i^0 is a best response to B_{-i}^0.

c. The value of the rtd for the bidders is defined for Nash Vickrey payoffs. Recall that when the agent is indifferent he chooses not to extend the deadline. Two cases may present themselves. We may be in a subgame where the disclosed bids revealed no defection. In that case bidder $1(2)$ earns $v_1(\mathbf{B}^c)(v_2(\mathbf{B}^c))$ if the deadline is not extended while he earns $v_1(\mathbf{B}^0)(v_2(\mathbf{B}^0))$ if the deadline is extended. Since $v_1(\mathbf{B}^0) > v_1(\mathbf{B}^c)$ both firms prefer no extension. Since no extension is the outcome the agent chooses by default, the rtd has no value for the firms and it is indeed optimal to wait for the official opening.

Assume instead that we are in a subgame where bidder i defected. When bidder i defects he submits a offer B^{def} including a bid $\left(S^{def}, p^{def}\right)$; $v_i\left(B^{def}\right) > v_i\left(B^c\right)$. Since $n_1 + n_2 > k$ this implies that $S^{def} \cap S_j^c \neq \emptyset$: the defector displaces the bidder $j's$ collusive bid , $i \neq j$, $i, j = 1, 2$. If the procedure selects the defector as the winner he can choose to extend in which case he earns $v_i(\mathbf{B}^0)$ or not to extend in which case he earns $v_i\left(B^{def}, B_j^c\right) > v_i(\mathbf{B}^c) > v_i(\mathbf{B}^0)$. So the defector would choose not to extend. Assume now instead that player j is selected by the procedure. Player j's single serious bid has been displaced so if the deadline is not extended he earns zero $v_j\left(B^{def}, B_j^c\right) = 0$. If the deadline is extended he readjusts and earns the Vickrey payoff $v_j(\mathbf{B}^0) > 0$. So player 2 chooses to extend the deadline. Still supposing that i is the defector (j the displaced bidder), let V denote the value of the rtd: $V^{def} = v_i\left(B^{def}, B_j^c\right) - v_i(\mathbf{B}^0)$ and $V^{dis} = v_j(\mathbf{B}^0)$.

b. The agent has disclosed the offers and learned whether there has been defection or not. He proposes a mechanism. Recall that no extension is the default decision (abusing discretion is costly). Hence, when the firms comply with their collusive agreement there exists no mechanism that can extract rents from the rtd. Proposing a first price auction in bribe is optimal, it yields zero proceeds.

In the subgame where one bidder defected the bidders have opposite interests with respect to the decision, they can be put in competition. The agent's choice of mechanism is

constrained by the absence of commitment technology: we require that implementation be incentive compatible. Since the rtd has no value to the agent, imposing a reserve price strict larger than zero is precluded. Similarly, imposing a bias on one of the bidder is not incentive compatible. Recall also that the bidder know each other's value. Given the constraints, the most the agent can extract is the second highest value and a first price auction trivially achieves that.

The outcome of the mechanism is then for

i. $V^{def} > V^{dis}$, $b_{dis} = V^{dis}$, $b^* = b_{def} = V^{dis} + \varepsilon$, the defector wins, pays the second highest value and decides not to extend.

ii. $V^{def} < V^{dis}$, $b_{dis} = V^{def} + \varepsilon$, $b^* = V^{def}$, the displaced bidder wins and decides to extend.

First submission stage

We may be in one of two subgames. If the agent has turned down the proposed participation bribe, bidder i knows j submits the non-cooperative equilibrium offer B_j^0. By construction it is a best response for i do so as well. If the agent has accepted the bribe, bidder i infers that the agent is corrupt. By the reasoning above he knows that if he defects, he will be invited to bid in bribe and either he will have to pay $b = V^{dis} + \varepsilon$ and by condition $(COR) : v_i\left(\widehat{B}_i, B_j^c\right) - v_i\left(\mathbf{B}^c\right) \leq V^{dis}$, there exists no profitable defection. Or if $V^{def} < V^{dis}$, the displaced bidder wins and extends the deadline. Since $v_i\left(\mathbf{B}^c\right) > v_i\left(\mathbf{B}^0\right) > 0$ $i = 1, 2$ even then defection is not profitable.

Revelation stage

If agent is honest he turns down any bribe by definition. If the agent is corrupt, he accepts the proposed bribes. If he did not he would lose the fee and miss a chance to earn bribes later. The firm hold prior beliefs p that the agent is honest. For $f < pv_{i(j)}\left(\mathbf{B}^c\right)$ and when (COR) holds firm $i(j)$ prefers to pay f and submit the collusive offer, instead of taking the risk or submitting the non-cooperative offer.

Hence, under condition (COR), the market-sharing bidding strategy profile $\mathbf{B}^c$ is part of a Bayesian equilibrium of the auction game extended with corruption.∎

Proof of Proposition 3

i. When the efficient allocation gives all the tasks to one firm, the other large bidder's Vickrey payoff is zero so he would not submit any positive bribe. But the dominant firm has strict incentive to displace the other from any collusive partition (see proposition 1) so the lhs of (COR) is strictly positive. Hence condition (COR) is not satisfied.

ii. Let the degree of competition be captured by the inverse of the Vickrey payoffs. In the proof of proposition 2 we show that the agent chooses a first price auction in bribe for the right to decide. The value of the right to decide to the displaced bidder is given by his Vickrey payoff. The larger those payoffs i.e. the softer competition, the larger the bribe submitted by a displaced bidder and hence by (COR) the less profitable defection.■

Proof of Proposition 4

Firm i's 'payoff-if-win S^h is given

$$v_i\left(S^h\right) = \sum_{\omega^j \in S^h}\left[p_i\left(\omega^j\right) - c_i\left(\omega^j\right)\right], \ i = 1, 2.$$

This function is fully separable in the bids for the individual tasks :$p_i\left(\omega^j\right)$. Hence, the firm views bidding for each task as a separate auction. Each of these auctions are single item first price complete information auction except for the dummy bidder, the government that always bid 1. The outcome is therefore $i^*\left(\omega^j\right) = \arg\min_{i=0,1,2} c_i\left(\omega^j\right)$. The winner is paid the second lowest cost unless the winner is the government in which case it is 'paid'. The equilibrium price is given $p^*\left(\omega^j\right) = \min\left\{1, \ \min_{i \neq i^*, i=1,2} c_i\left(\omega^j\right)\right\}$. Firm i's equilibrium offer is the collection of the unique equilibrium bids on each task.■

Proof of Proposition 5

The argument is identical to proposition 2 expect for the determination of the 'displaced' bidder's value for the rtd. In the subgame where firm i defected and submitted $S^{def} \neq S_i^c$, by $n_1 + n_2 > k$, we have $S^{def} \cap S_j^c \neq \emptyset$. The defection offer overlaps on the bidder j's collusive bid $S^{dis} = S_j^c \backslash S^{def} \cap S_j^c$ and $v_j\left(B^{def}, B_j^c\right) = \sum_{\omega^j \in S^{dis}}\left(1 - c_j\left(\omega^j\right)\right) > 0$.

In this subgame the displaced bidder knows that if the defector wins the rtd he chooses not to extend so his value for the rtd $V^{dis} = \max\left\{0, v_j\left(B^0\right) - v_j\left(B^{def}, B_j^c\right)\right\}$.

The condition for collusion to be an equilibrium is as in proposition 2 :

$v_i \left(\widehat{B}_i, B_j^c \right) - v_i \left(\mathbf{B}^c \right) \leq V^{dis}$ which is equivalent to $v_i \left(\widehat{B}_i, B_j^c \right) - v_i \left(\mathbf{B}^c \right) \leq v_j \left(B^0 \right) - v_j \left(B^{def}, B_j^c \right). \blacksquare$