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Arbitrage opportunities between NYSE and XETRA?: A comparison of simulation and high frequency data

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Abstract

This paper investigates both the no-arbitrage condition and the efficiency of financial markets by comparing two stock markets: the New York Stock Exchange (NYSE) and the German Exchange Electronic Trading System (XETRA). We analyze German stocks that are traded simultaneously at both exchanges using high frequency data for XETRA, the NYSE, and the foreign exchange rates. Converting Euro-prices into Dollar-prices reveals possibilities to explore the efficiency as well as arbitrage opportunities of these two stock markets using the phenomenon of stock price clustering. We see the result of differing extents of clustering on both exchanges, thus violating the no-arbitrage condition. We propose a trading strategy that exploits these differences. Furthermore, we compare our empirical findings with the results we obtain from simulating financial markets and conclude that simulations based on the no-arbitrage condition are not consistent with our empirical observations.

Keywords: financial markets; simulation; no-arbitrage condition; stochastic processes

JEL Classification: C00; D40; G12

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1 Introduction

There are a lot of paradigms in finance. One of these is the contention that stock markets are efficient. Eugene F. Fama developed this paradigm, The Efficient Market Hypothesis (EMH), and published this idea in his famous article (Fama, 1970). The genesis for the EMH was the observation that stock prices appeared to follow a random walk, which implies that prices in financial markets evolve randomly and that successive price changes are identically distributed independent random variables. In an efficient market, any new information would be immediately and fully reflected in equity prices (Fama, 1970) and thus the information contained in past prices is fully reflected in current prices. Therefore, the opportunity for any abnormal gain on the basis of the information contained in historical prices is eliminated. Market efficiency would then imply that successive price changes are independent. Most of the early studies supported the random walk behavior of stock market prices: Kendall (1953), Cootner (1962), and Fama (1965, 1970) among many others.

Osborne (1959) showed that the geometric Wiener process (GWP) or geometric Brownian motion may be used as a model to describe the behavior of stock market prices. The (continuous) geometric Wiener process can be considered as the limiting process of the (discrete) random walk and is of utmost importance in the theory of finance. For example the Capital Asset Pricing Model (Ross, 1976), Markowitz's mean-variance portfolio theory (Markowitz, 1952), and the Black-Scholes option pricing model (Black and Scholes, 1973) rely on the assumption that returns are normally distributed, a direct implication of the GWP.

The geometric Wiener process can be considered as a model for the price movement when analyzing one stock market. An interesting aspect is to discuss the GWP when comparing two stock markets, or to be more precise, when comparing stocks that are traded simultaneously at two stock markets. This approach implies the following questions: if a geometric Wiener process is used as a model for the price movement then which stock market is more efficient and are there arbitrage opportunities? According to the EMH, financial markets are "informational efficient" and there are no trading strategies that produce positive, expected, risk-adjusted excess returns (LeRoy, 1973, Lucas, 1978). In other words, there are no arbitrage opportunities in efficient markets.

In this paper, we analyze the intraday trades of selected German stocks (Daimler and Deutsche Bank) that are traded simultaneously at the New York Stock Exchange (NYSE) and at the

German Exchange Electronic Trading System (XETRA). The conversion of the XETRA Euro stock prices into US-Dollar stock prices by the foreign exchange rates and vice versa enables us to discuss the question which stock market is more efficient: XETRA or the NYSE?

To investigate this question, we use the phenomenon of stock price clustering as a benchmark for the efficiency of financial markets.

Stock price clustering describes the tendency of prices to deviate from a uniform distribution, tending instead to cluster at certain prices and avoiding others. The phenomenon of price clustering cannot be explained by any definition of the EMH and can be used for measuring the efficiency of stock markets. In line with the EMH, stock market prices should be uniformly distributed. Therefore, stock markets with a higher degree of stock price clustering are considered as less efficient stock markets because they imply a greater deviation from the uniform distribution of the last digits. Stock price clustering can be observed for different stock markets with different market structures and has been widely discussed in the literature (see for example Osborne, 1962, Niederhoffer, 1965, 1966, Ball *et al.*, 1985, Harris, 1991, Christie *et al.*, 1994, Kahn *et al.*, 1999, Vogt *et al.*, 2001, Huang and Stoll, 2001, Sopranzetti and Datar, 2002, Sonnemans, 2006, Aşçıoğlu *et al.*, 2007, Ikenberry and Weston, 2008, Onnela *et al.*, 2009).

In this study, we approach the research problem of stock price clustering and the efficiency of different stock markets from two different points of view: first, a simulation approach and second, an empirical data approach.

In the simulation approach, we assume a geometric Wiener process as a model for the price movement and the aim is to check whether any qualitative effect can be seen for stocks that are traded at two exchanges. Simulation is well suited as a first approach that enables us to focus on the effect of stock price clustering and efficiency for two different stock markets in isolation from other factors. The analysis of the simulated data yields the result of no stock price clustering. We therefore can conclude that stock markets that are characterized by a GWP are efficient and there are no arbitrage opportunities.

In the empirical part of this paper, the same aspects are studied quantitatively for stocks that are traded simultaneously at two exchanges. The advantage of our empirical approach is that all security-specific characteristics are held constant. Our empirical data yield the result of different extents of stock price clustering for stocks (Daimler and Deutsche Bank) that are traded simultaneously at XETRA and the NYSE. We use two different methods to decide whether

XETRA or the NYSE is more clustered (or less efficient) which yields different results. The first method directly compares stock price clustering on the two exchanges (without using the exchange rate) while for the second method the foreign exchange rate is taken into account. The latter approach indicates that converted stock prices and actually observed stock prices reveal different degrees of stock price clustering. But according to both approaches, we observe a difference in the clustering structure between XETRA and the NYSE. Although we consider only two stocks, this result should not be observable if both stock markets were efficient. Therefore, the observed difference indicates a violation of the Efficient Market Hypothesis and therefore an inefficiency of both analyzed stock markets.

In addition, the phenomenon of stock price clustering puts some question on the no-arbitrage condition. The no-arbitrage condition of financial markets implies that the Dollar-prices at the NYSE should be obtained by converting the Euro-prices at XETRA and vice versa (for companies that are traded simultaneously at both stock markets). In a last step, we propose a trading strategy that exploits the differences in the observed clustering structure between converted and actually observed stock market prices (quasi-arbitrage opportunities from the behavioral finance point of view) and then we verify this strategy empirically.

Our empirical findings strongly contradict the simulation results that are in line with the no-arbitrage condition. In addition, the underlying stochastic processes of the price movement seem to be different for both stock markets even though considering the same companies.

The remainder of this paper is organized as follows: Section 2 offers a short overview of measures of stock price clustering and provides a short characterization of the geometric Wiener process as a model for the price movement. Section 3 describes the empirical data and Section 4 presents some empirical findings as preliminary results. Section 5 presents the results for the simulated and empirical data and Section 6 concludes.

2 Theory

2.1 Measures of Stock Price Clustering

In order to compare the effect of stock price clustering on different stock markets (and therefore making statements about the efficiency of these stock markets), one has to think about how to measure the degree of stock price clustering. Previous studies provide several methods and variations to determine the degree of stock price clustering (see for example Christie *et al.*, 1994, Grossman *et al.*, 1997, Huang and Stoll, 2001, Ikenberry and Weston, 2008). In this study, we use the approach of Ikenberry and Weston (2008).

A standard Chi-squared 'goodness of fit' statistic D is constructed, which under the null hypothesis of a uniform distribution, should be below some critical value:

$$D = \sum_i \frac{(F_{obs,i} - F_{exp,i})^2}{F_{exp,i}} \tag{1}$$

where $F_{obs,i}$ is the observed frequency of the last digit i ($i = 1, \ldots, n$), $F_{exp,i}$ is the expected frequency of the last digit i under the null hypothesis of a uniform distribution (that means $F_{exp,i}$ equals the number of observations divided by n), and n is the number of possible last digits (for a tick size of 1 Cent n=10). Under standard regularity conditions, the statistic D is Chi-squared distributed with n-1 degrees of freedom where large values of D imply a significant deviation from the expected distribution (uniform distribution). The test statistic D can be calculated for both XETRA (D_{XETRA}) and the NYSE (D_{NYSE}) and one can decide whether the null hypothesis of a uniform distribution can be rejected or not. However, this test does not address whether XETRA is more or less clustered compared to the NYSE. For this purpose, we compare D_{XETRA} and D_{NYSE} by examining the ratio between both (it is assumed that the numerator has to be greater [or equal] than the denominator, otherwise the inverse has to be calculated):

$$\widetilde{D} = \frac{D_{XETRA}}{D_{NYSE}} \tag{2}$$

where $\widetilde{D}$ is F-distributed with M and N degrees of freedom (M – degrees of freedom of D_{XETRA}, N – degrees of freedom of D_{NYSE}). The statistic $\widetilde{D}$ enables us to test whether the degree of stock price clustering is the same for XETRA and the NYSE. Large values of $\widetilde{D}$ imply that price clustering at XETRA is greater compared to the NYSE.

2.1 The Geometric Wiener Process

A general procedure to model the distribution of stock prices is to model the assumed underlying stochastic process that generates the stock prices. The genesis for the EMH was the observation that stock prices appeared to follow a random walk. This implies that stock price changes (of successive prices) have the same distribution and are independent of each other, so the past movement or trend of a stock price or market cannot be used to predict its future movement. A random walk involving price changes that vary according to a normal distribution (Gaussian random walk) is often used as a model for time series data such as financial markets. In this case, the price differences of successive prices are independent and follow a normal distribution.

Stock prices are stochastic processes in discrete time that take only discrete values due to the limited measurement scale. Nevertheless, stochastic processes in continuous time are used as models since they are analytically easier to handle than discrete models.

The limiting process of a random walk is the arithmetic Wiener process that is characterized by stationary and independent increments. Furthermore, the increments are normally distributed. But two features of the arithmetic Wiener process make it an unsuitable model for stock prices. First, it allows for negative values of stock prices, and second, the local variability is higher for high stock prices.

Hence, stock prices are modeled by a geometric Wiener process S_t where the logarithmic price $\ln S_t$ follows an arithmetic Wiener process as a solution of the stochastic differential equation:

$$d \ln S_t = \mu dt + \sigma dW_t \tag{3}$$

where $\ln S_t$ is the logarithmic price at time t, W_t is a standard Wiener process, the parameter μ is the drift and σ the volatility. We use S_t for the (simulated) price in order to distinguish the simulated price clearly from the actual trading price P_t at time t.

That means the following equation holds for the logarithmic price $\ln S_t$:

$$\ln S_t = \mu \cdot t + \sigma \cdot W_t + \ln S_0 \tag{4}$$

For the purpose of simulation, we take a small basic period length of $\Delta t = 1\text{min}$. It follows from the definition of the arithmetic Wiener process that the logarithmic returns (of successive prices) of the price time series are independent and normally distributed with expectation $\mu \cdot \Delta t$ and variance $\sigma^2 \cdot \Delta t$:

$$\ln S_{t+1} - \ln S_t = \ln \frac{S_{t+1}}{S_t} \sim N(\mu \Delta t, \sigma^2 \Delta t) \tag{5}$$

This expression can be written as an iterative formula for logarithmic prices:

$$\ln S_{t+1} = \ln S_t + \mu \cdot \Delta t + \sigma \cdot \sqrt{\Delta t} \cdot Z_t \tag{6}$$

where the random variables Z_t are independent and follow a standard normal distribution. Setting the basic period length as $\Delta t = 1$min, the above equation simplifies to

$$\ln S_{t+1} = \ln S_t + \mu + \sigma \cdot Z_t \tag{7}$$

The parameters μ and σ are estimated for empirical data for some selected stocks. The initial price S_0 is chosen randomly from the set of possible prices observed for the empirical data. The logarithmic price can be simulated using equation (7), the exponential of the logarithmic price yields the simulated trading price that is rounded to two decimal places.

3 Data description

Our analysis is based on a case study for selected German stocks that are traded simultaneously at XETRA and at the NYSE in November and December 2004 (from November, 15^{th} to December, 29^{th}). We focus our study on the trading days and the time period for which both stock markets are open simultaneously. To be more precise, we analyze the trades of 30 trading days and a time period between 3:30 pm and 5:30 pm for XETRA and between 9:30 am and 11:30 am for the NYSE. Our sample stocks are namely Deutsche Bank and Daimler. We consider only these two companies because these are the only German stocks with a sufficient number of trades in 2004 at the NYSE. The data is obtained from the Trade and Quote (TAQ) database of the NYSE and the XETRA stock market, and includes each trade for the analyzed time period.

In 2004, stock prices at XETRA were listed in Euros with a tick size (smallest trading unit or minimum price variation) of 1 Euro-cent while at the NYSE prices were listed in Dollar with a tick size of 1 Dollar-cent. The number of trades and the mean stock prices of Daimler and Deutsche Bank for the analyzed time periods are presented in Table 3.1.

Table 3.1: Number of trades and mean stock prices of Daimler and Deutsche Bank

	Number of trades		Mean stock price	
	XETRA	NYSE	XETRA	NYSE
Daimler	50178	8934	34.42€	45.81$
Deutsche Bank	63965	4180	64.65€	85.77$

For the purpose of simulation equally spaced prices are needed from the original tick-by-tick data. We have chosen a time interval of $\Delta t = 1$min, thus guaranteeing the best description of our data. For a time interval less than 1 min, we have too many zero price changes which disturb the distribution of successive price changes; for a time interval greater than 1 min, we lose a certain amount of information.

4 Empirical Findings and Preliminary Results

Before presenting the results for the simulated and empirical data, we take a first glance at some empirical findings as preliminary results to motivate our research. The results are discussed more detailed in Section 5.

1. Stock Price Clustering

Figure 4.1 presents a first impression of the frequency distribution of the last digits of the stock prices of the German company Deutsche Bank at XETRA. The phenomenon of stock price clustering is highly observable and cannot be neglected.

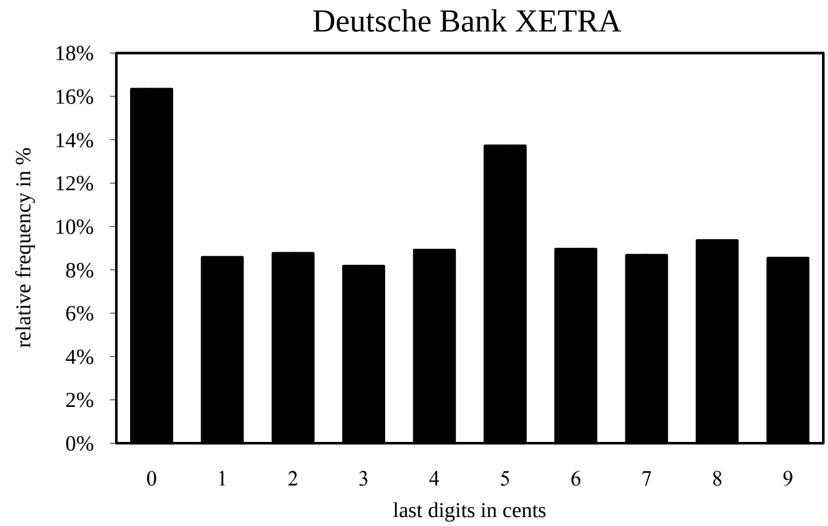

Figure 4.1: Frequency distribution of the last digits of the stock prices of Deutsche Bank at XETRA.

2. Differences in Efficiency and arbitrage opportunities?

To get an idea of the degree of clustering or to determine the efficiency of different stock markets, Figure 4.2 compares the frequency distribution of the last digits of Deutsche Bank observed at XETRA and the NYSE. At first glance, XETRA seems to be more clustered compared to the NYSE: there we observe that 16% of all prices end in zero while for the NYSE only 14% of all prices end in zero. This empirical finding puts some question on the no-arbitrage condition of financial markets.

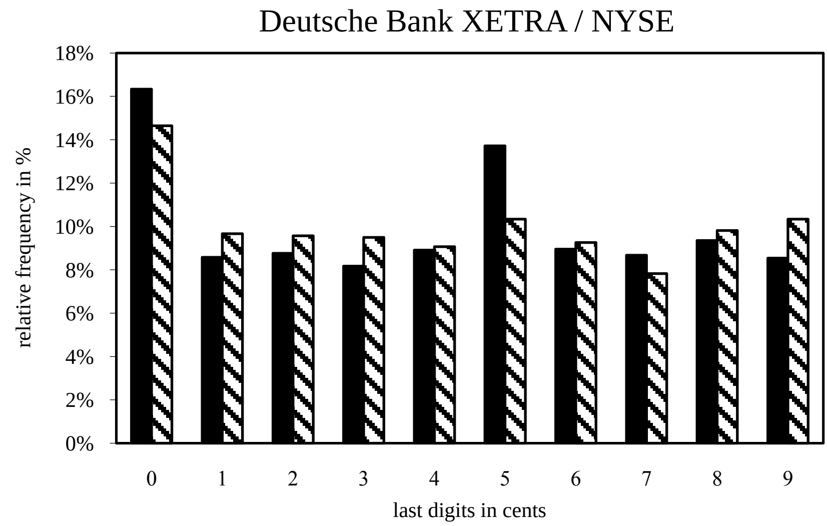

Figure 4.2: Frequency distribution of the last digits of the stock prices of Deutsche Bank at XETRA (solid black bar) and the NYSE (dashed bar).

3. Different Stochastic Processes

The phenomenon of stock price clustering and the differences in the clustering structure for XETRA and the NYSE imply that the underlying stochastic processes on both stock markets seem to be different although analyzing the same companies.

5 Results

5.1 Simulation Results

For simulating financial markets we use the geometric Wiener process as a model for the price movement. Simulation is a well-suited approach for it enables us to discuss the empirical observations of the data (Section 4) in isolation from other factors. For this purpose the simulations are run 1000 times.

The GWP implies that the logarithmic prices follow an arithmetic Wiener process (see Subsection 2.2). For a time interval of $\Delta t = 1\text{min}$ (minute-by-minute data), that is most suitable for our data, the logarithmic prices are simulated by using the iterative formula (7)

$$\ln S_{t+1} = \ln S_t + \mu + \sigma \cdot Z_t \qquad (7)$$

The initial price S_0 is set as the first price entry in the empirical price time series, $\ln S_t$ is the simulated logarithmic price at time t, and the random variables Z_t are independent standard normally distributed. We use S_t for the (simulated) price in order to distinguish the simulated price clearly from the actual trading price P_t at time t. The parameter μ is estimated from the empirical data as the mean of successive logarithmic price changes or logarithmic returns (for minute-to-minute data) and the parameter σ is its corresponding standard deviation or volatility. The exponential of the logarithmic price yields the simulated trading price S_t that is rounded to two decimal digits. To avoid artifacts, we exclude all overnight gaps prior to simulation.

Figures 5.1 and 5.2 show actually observed trading prices P_t (corrected for overnight gaps) for Deutsche Bank and six realizations of the corresponding geometric Wiener process (simulated using parameters from the empirical data) at XETRA and the NYSE[1]. Note that the number of trades in each simulated price time series is equal to those in the empirical ones. The simulated stock prices of Daimler reveal similar results[2].

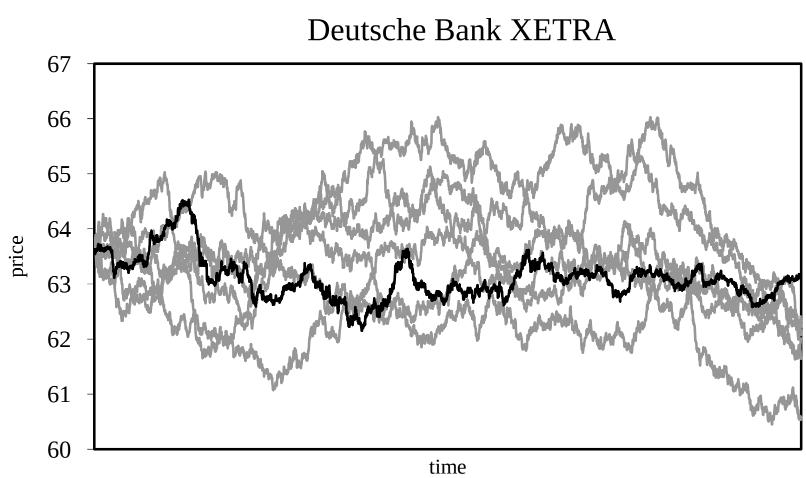

Figure 5.1: Actual trading prices (black line) and six realizations of a corresponding geometric Wiener process (grey lines) for Deutsche Bank at XETRA.

[1] We randomly chose six realizations of the simulated GWP for the plots in Figure 5.1 and Figure 5.2 and not all paths because it enables a better visualization. For calculations we use all simulated trading prices.

[2] The simulation results for the German company Daimler can be shown by the corresponding author on request.

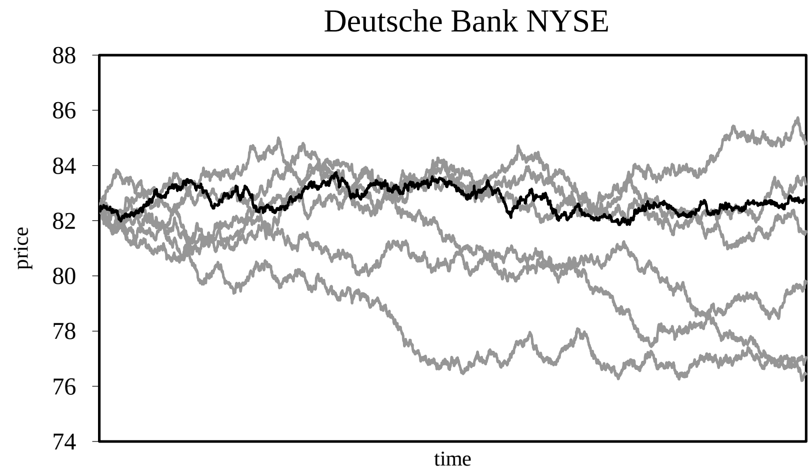

Figure 5.2: Actual trading prices (black line) and six realizations of a corresponding geometric Wiener process (grey lines) for Deutsche Bank at the NYSE.

Figure 5.3 compares the frequency distributions of the last digits of the simulated stock prices of Deutsche Bank[3].

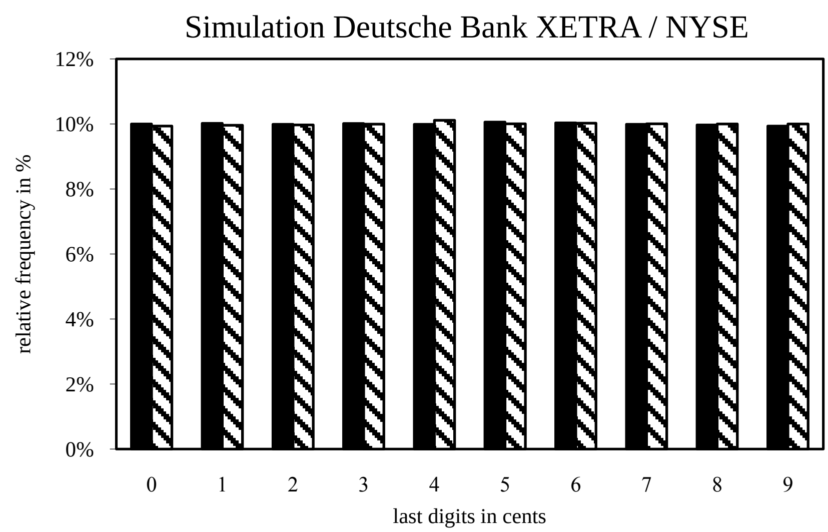

Figure 5.3: Frequency distribution of the last digits of the simulated stock prices of Deutsche Bank at XETRA (solid black bar) and the NYSE (dashed bar).

[3]The simulated stock prices of the German company Daimler show similar patterns concerning the clustering structure and can be shown by the corresponding author on request.

Figure 5.3 implies a uniform distribution of the last digits of simulated stock prices and therefore no significant stock price clustering (Chi-squared goodness of fit test, 1% level of significance), thus contradicting our Empirical Finding 1. Furthermore, the degree of stock price clustering does not differ between the two exchanges implying that there are no differences in the efficiency of these stock markets (*F*-Test, 1% level of significance). We expect these results of the simulated data as we assume a model for the price movement that is in line with the Efficient Market Hypothesis and therefore with the no-arbitrage condition of financial markets. But obviously it contradicts our Empirical Findings 2 and 3. For a further interpretation of our simulation results, we compare them with the empirical results in Section 5.2.

5.2 Empirical Results

5.2.1 Stock Price Clustering, Efficiency, and the No-Arbitrage condition

One possible way to answer the question whether XETRA and/or the NYSE are inefficient and which stock market is more efficient (clustered) is to analyze the last digits of the stock prices of Daimler and Deutsche Bank at both stock markets without using the exchange rate.

For the purpose of directly comparing our empirical results with the simulation results, Figure 5.4 presents the frequency distribution of the last digits of minute-by-minute data. In addition the last digits of the original tick-by-tick data are presented in Figure 5.5.

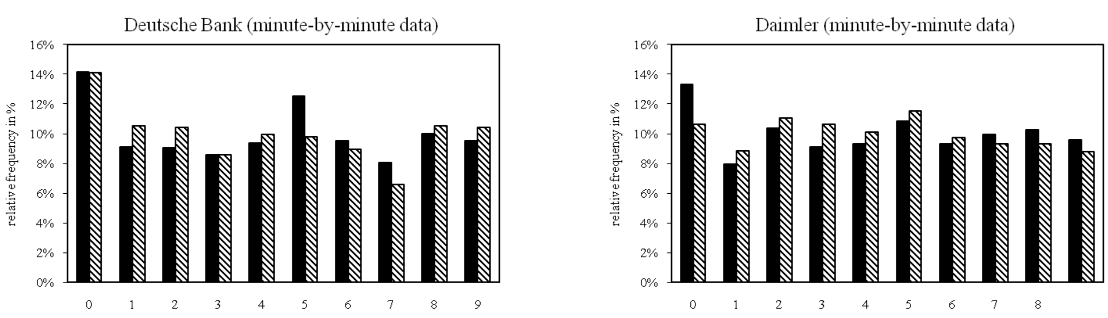

Figure 5.4: Frequency distribution of the last digits of the minute-by-minute stock prices of Deutsche Bank and Daimler at XETRA (solid black bar) and the NYSE (dashed bar).

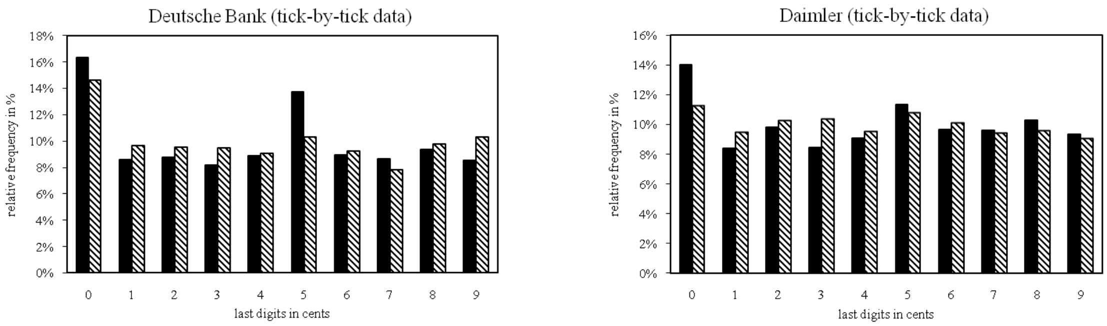

Figure 5.5: Frequency distribution of the last digits of the tick-by-tick stock prices of Deutsche Bank and Daimler at XETRA (solid black bar) and the NYSE (dashed bar).

It is obvious that both data sets describe the same pattern concerning the clustering structure. In the following, therefore we reduce our arguments to tick-by-tick data that contains the most information. As can be seen from both Figures 5.4 and 5.5, stock price clustering exists (supporting Empirical Finding 1 and contradicting our simulation results) and seems to be more pronounced at XETRA. This first impression of different extents of price clustering can more formally be tested by applying the measure D of clustering that is used in Ikenberry and Weston (2008).

Table 5.1 presents the numerical values of D_{XETRA} and D_{NYSE} for Daimler and Deutsche Bank and the corresponding ratio $\widetilde{D}$ (for tick-by-tick data). The latter indicates that XETRA is more clustered compared to the NYSE (F-test, 1% level of significance) and therefore the NYSE would be the more efficient stock market according to this analysis. In addition, the numerical values of D_{XETRA} and D_{NYSE} imply that the last digits of the stock prices of Daimler and Deutsche Bank are not uniformly distributed (Chi-squared goodness of fit test, 1% level of significance).

Table 5.1: Chi-squared test statistics and *F*-test statistics, * denotes significance at the 1% level.

	D_{XETRA}	D_{NYSE}	$\widetilde{D} = \frac{D_{XETRA}}{D_{NYSE}}$
Daimler	1243*	38.82*	32.02*
Deutsche Bank	4314*	119.18*	36.2*

Taking into account the no-arbitrage condition of financial markets, stock prices at the NYSE should be obtained by multiplying the stock prices observed at XETRA with the corresponding exchange rate (that is valid for the observed time point) and perhaps rounding these converted prices to the next possible Dollar-price (and vice versa). To be more precise, we do not expect any difference in the clustering structure between actually observed Dollar-prices and Dollar-prices that result from converting the Euro prices (and between actually observed Euro-prices and Euro-prices that result from converting the Dollar-prices, respectively). The resulting frequency distributions of the last digits of those converted stock prices and of the last digits of the actually observed stock prices are presented in Figure 5.6 and 5.7.

Considering the German company Daimler in Figure 5.6, there seems to be no difference in the clustering structure between actually observed Dollar-prices and into Dollar converted Euro-prices. The opposite way does not reveal the same empirical finding: the observed degree of stock price clustering at XETRA differs from the clustering the converted Dollar-prices reveal. Analyzing the last digits of the transactions data of Deutsche Bank (Figure 5.7), we observe substantial differences between the clustering of converted and actually observed stock prices. In addition, the converted Euro-prices and the converted Dollar-prices seem to be uniformly distributed for both companies. We used a Chi-squared ‘goodness of fit’ test with a test statistic C to check whether the observed distribution of the last digits differs from the expected distribution (that results from converting the stock prices). The numerical values of the test statistic are presented in Table 5.2, indicating statistical significance (at the 1% level) that the actually observed distribution of the last digits differs from the distribution we would expect after converting stock prices.

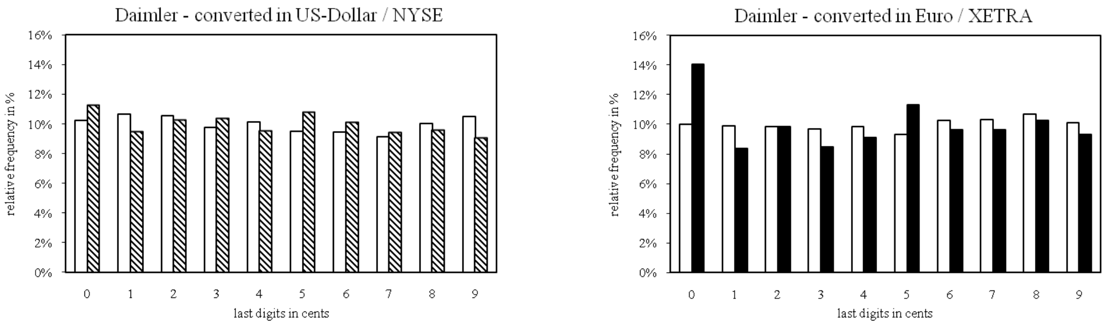

Figure 5.6: Frequency distributions of the last digits of the stock prices of converted Daimler-prices (solid white bars) and of actually observed stock prices of Daimler at the NYSE (solid black bar) and at XETRA (dashed bar).

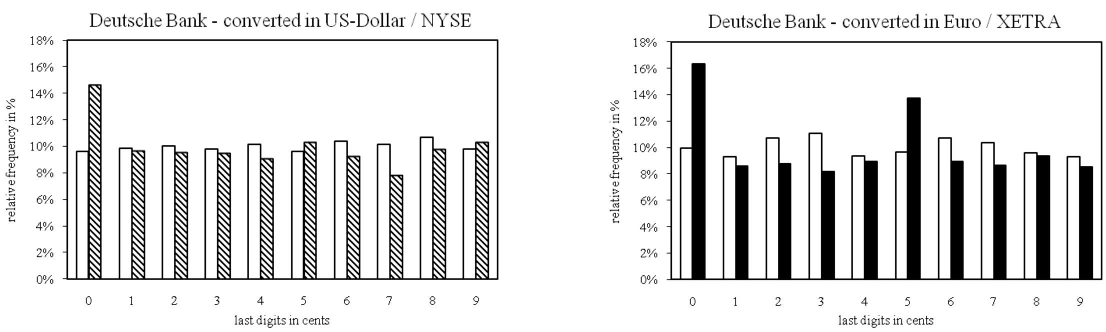

Figure 5.7: Frequency distributions of the last digits of the stock prices of converted Deutsche Bank-prices (solid white bars) and of actually observed stock prices of Deutsche Bank at the NYSE (dashed bar) and at XETRA (solid black bar).

Table 5.2: Chi-squared test statistics, * denotes significance at the 1% level.

	Converted in US-Dollar vs. actually observed US-Dollar C	Converted in Euro vs. actually observed Euro C
Daimler	67.12^{*}	191.96^{*}
Deutsche Bank	149.42^{*}	4891.10^{*}

Considering, for example, Deutsche Bank, it is obvious that the actually observed Dollar-prices at the NYSE reveal a clustering pattern, while there is only a low degree of stock price clustering for the last digits of into US-Dollar converted Euro-prices (Figure 5.7). Furthermore, the latter seem to be uniformly distributed. But the degree of stock price clustering of Dollar-prices that result from converting the Euro-prices at XETRA corresponds to the degree of stock price clustering we observe at XETRA; for these are the same prices, only in different currencies. This leads to the result that the NYSE reveals an additional stock price clustering and therefore we can conclude that the NYSE has a higher degree of stock price clustering compared to XETRA and also that XETRA is a more efficient stock market. That means, when comparing converted XETRA-prices and actually observed NYSE-prices we see that XETRA is more efficient for the example of Deutsche Bank. The opposite direction leads to a contrary result.

Summarizing our empirical findings concerning the phenomenon of stock price clustering with respect to efficiency, we observe different extents of price clustering for the same stocks traded simultaneously on two stock markets. This implies inefficiency between both analyzed stock markets XETRA and the NYSE (for the stock prices of Daimler and Deutsche Bank). But the different approaches on how to compare the degree of stock price clustering (with and without using the exchange rate) indicate that we cannot strictly answer our question of whether XETRA or the NYSE is the more efficient stock market. If the exchange rate is taken into account, we obtain the result that XETRA is more efficient when comparing converted Euro-prices and actually observed Dollar-prices. Nevertheless, we do not expect this observed inefficiency between both stock markets if the no-arbitrage condition of financial markets held, which supports Empirical Finding 2. In addition, the underlying stochastic processes seem to be different for both stock markets although considering the same companies and the same time horizon, which is a contradiction to our simulation results (and emphasizes Empirical Finding 3). In the following, we want to provide a trading strategy on how to benefit from this observed inefficiency and we then calculate a proxy of possible profits.

5.2.2 Trading Strategy

For the purpose of investigating possible arbitrage opportunities (or quasi-arbitrage opportunities to be more precise), it is necessary to know for example, the bid price for a specific stock at XETRA, the ask price at NYSE, and the corresponding exchange rate at a point in time. As these orderbook data are in most cases not available, we use transaction prices as proxies for this procedure. Analyzing those trades provides a strong indication about the existence of quasi-arbitrage opportunities. That means we are noting a stock price at XETRA at one point in time and we are converting this Euro-price into a Dollar-price by using the exchange rate that is valid at the time (and vice versa). In a next step we compare the difference between this converted price and the next possible transaction that occurs at the NYSE (and vice versa). If the no-arbitrage condition is fulfilled, this difference is zero. Our data and analysis provide empirical evidence that the differences are not zero in most cases. Table 5.3 presents the proportions of zero differences and non-zero differences between converted and actually observed stock prices for Daimler and Deutsche Bank. The proportion of non-zero differences exceeds 80% (the results are significant at the 1% level, binomial test).

Table 5.3: Proportions of zero and non-zero differences between converted and actually observed stock prices for Daimler and Deutsche Bank.

	Converted in US-Dollar vs. actually observed US-Dollar		Converted in Euro vs. actually observed Euro	
	Difference<> 0	Difference =0	Difference<> 0	Difference =0
Daimler	83.77%	16.23%	81.15%	18.85%
Deutsche Bank	92.11%	7.89%	91.10%	8.90%

As the proportion of non-zero differences clearly indicates possible quasi-arbitrage opportunities, we want to provide a trading strategy that takes advantage of the observed inefficiency between the two analyzed stock markets and of the observed non-zero differences.

As a signal to buy or sell shares (in this context, a sale also can be a short sale), we consider the most recent observable difference (between converted XETRA-prices and actually observed NYSE-prices or converted NYSE-prices and actually observed XETRA-prices, respectively). If this difference exceeds 0.05 US-Dollar or Euro, we buy the stock on the observed market and sell the same on the other market. We propose a short sale of shares on the main market and a buy (or buy to cover) on the other market, if the observed difference is less than -0.05 US-Dollar or Euro. The following Table 5.4 presents the average profits of this strategy for the period of investigation and one traded share.

Table 5.4: Mean profits for the suggested trading strategy.

	Converted in US-Dollar vs. actually observed US-Dollar		Converted in Euro vs. actually observed Euro	
	Mean profit per one share per trade in US-Dollar	Number of strategy trades	Mean profit per one share per trade in Euro	Number of strategy trades
Daimler	0.012952	227	0.019178	73
Deutsche Bank	0.034326	1172	0.033522	565

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It can be argued that the proposed strategy yields not enough profit to achieve a considerable net profit when taking transaction costs into account. But considering the fee structure of the U.S. broker TradeStation Securities, Inc., a complete transaction can for example be traded for less than 0.00699 US-Dollar per share by using a so called flat fee[4]. For this case, the calculated average profit per trade seems to be quite lucrative. We can conclude that the suggested trading strategy yields a positive profit even when considering transaction costs.

[4] TradeStation Securities, Inc. Flat fee 6.99$ per trade max 5000 shares per trade, minimum 30 Trades per month on account. The example is calculated with 2000 shares per trade.

6 Conclusion

In this paper we investigate the research problem of both the efficiency and the no-arbitrage condition of different stock markets from two different points of view, which are first a simulation approach and second, an empirical data approach. We compare two stock markets, XETRA and the NYSE, by analyzing high frequency data of selected German stocks (Daimler and Deutsche Bank) that are traded simultaneously at both stock markets in November and December 2004. The phenomenon of stock price clustering (the tendency of prices to cluster at certain last digits and avoiding others) contradicts any strict definition of the Efficient Market Hypothesis and can be used for measuring the efficiency of stock markets.

In the simulation approach, we assume a geometric Wiener process as a model for the price movement. The simulation enables us to focus on the effect of stock price clustering, efficiency, and the no-arbitrage condition for two different stock markets in isolation from other factors. One of our results is that stock markets which can be characterized by a geometric Wiener process are efficient in terms of non-existing stock price clustering and there are no arbitrage opportunities.

The emphasis of this paper lies on the empirical approach, where the same aspects are studied quantitatively by analyzing stocks that are traded simultaneously at XETRA and the NYSE. The advantage of our empirical approach is that all security-specific characteristics are held constant. If we take the exchange rate into account, we show that German stocks are traded more efficiently at the German stock exchange XETRA (for the example of Deutsche Bank). This result also hints at arbitrage possibilities. Although analyzing only two selected German stocks in a relatively short time period the results should not be observable if stock markets are efficient and the no-arbitrage condition held. Furthermore, we analyze quasi-arbitrage opportunities by suggesting a trading strategy. We show that simultaneous trading on both stock markets leads to profits after subtracting transaction costs. Even if this can only serve as a proxy, it provides a clear indication. Furthermore, we conclude that stock price clustering and the differences in the market efficiency cannot be explained by assuming a geometric Wiener process (with differing parameters) as a model for the price movement. The underlying stochastic processes seem to be different for both stock markets (simply varying the parameters is not sufficient). To summarize, simulations that are linked by the no-arbitrage condition cannot explain our empirical observations and trading strategies seem to be able to exploit this result.

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