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Optimal Management of the Growth Potential of Renewable Resources

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This paper offers an analysis of the optimal harvesting of a renewable resource when human activity other than harvesting plays a role in the growth process of the resource. The activity we have in mind is not related to pollution or exploration but refers to the ability to influence the growth process directly by means of creating favorable conditions. The analysis gives rise to results which differ substantially from the usual outcomes in the economics of renewable resources. For instance, in the examples under study an increase in the price of the harvested commodity will lead to an increase of the steady state stock of the renewable resource.

1. Introduction

Renewable resources are commonly analyzed in the context of models where the growth of the renewable resource under study is affected by two factors: the size of the resource itself and the rate of harvest. This specification does not take into account that human activities other than harvesting can have an impact on the growth of the natural resource. There is, of course, increasing awareness of the possibility that pollution negatively affects the growth potential of the resource (see Tahvonen, 1990; Tahvonen and Kuuluvainen, 1991, on this issue). It is furthermore widely recognized that the available volume of natural resources can be increased by exploration activities (see, e.g., Long, 1977). However, the

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objective of the present paper is to allow for human efforts which have a positive impact on the growth potential itself of the existing resources. There is an abundance of examples illustrating these opportunities. In the case of forestry one could think of fertilizers or cleaning activities of the soil. With aquaculture, efforts can be directed towards optimal feeding schemes, optimal water temperature, optimal oxygen levels, and the like. Fish growth in lakes can benefit from keeping the lake purified. Also in the cases of raising cattle and chickens, human activities (like feeding) play a part.

Hence there is clearly scope for modifying the classical growth equation so as to incorporate these effects. A rather general formulation would be

$$\dot{X} = f(X, V) - H \ ,$$

where X denotes the size of the renewable resource, H is the rate of harvest, and V is the rate of effort, which could be called management effort. This equation needs more structure in order to make it accessible for further investigation. Here, a large variety of possibilities arises, depending on the renewable resource under consideration. We shall focus on three cases.

In the first case, the effect of efforts manifests itself through an increased growth potential as a consequence of efforts per capita: what matters is the effort per unit of the existing resource. In the second case, the increment of the growth potential is independent of the resource size. These cases will be treated in Sect. 3. In Sect. 4, it is assumed that efforts affect the growth potential in a much stricter sense: without efforts the resource will become extinct. This case refers, for instance, to aquaculture where feeding of the fish is necessary. In all cases we shall consider the optimal harvesting under competitive conditions. The optimal trajectories will be derived and a sensitivity analysis of the steady state, if any, will be carried out. A general conclusion of this paper is that, under the assumptions made, an increase of the price of the harvested commodity leads to an increase of the steady-state stock of the natural resource. Furthermore, with additive management efforts the steady-state stock of the renewable resource is larger than with no management efforts.

2. The Simplified Traditional Model

The standard model of the harvesting of a renewable resource in a competitive environment is the following (see, e.g., the textbooks by

Hartwick and Olewiler, 1986; Pearce and Turner, 1990):

$$\max \int_0^\infty \exp(-rt)[pH - cE]\,\mathrm{d}t$$

subject to

$$\dot{X} = g(X) - H, \quad X \geq 0, \quad X(0) = X_0 \text{ given },$$
$$H = h(E, X), \quad H \geq 0, \quad E \geq 0 .$$

Here, r is the given rate of interest, p is the constant market price of the harvested commodity, c is the constant price of effort, H is the harvest rate, X is the size of the renewable resource, E is the rate of effort, g denotes the natural growth function, and h relates the efforts and the resource stock to the harvest. The outcomes of the analysis are well-known and need not be repeated here.

A distinctive feature of the above formulation is that the harvest not only depends on the efforts but also on the stock of the renewable resource. This is obviously a good description of reality in some cases. However, one can think of circumstances where the inclusion of the resource stock is not that relevant. Moreover, if the stock is left out the mathematical analysis gets more easily accessible and the points we wish to make in the sequel can conveniently be illustrated. We therefore analyze here the model in a version simplified in this respect. It can be stated as follows:

$$\max_{H \geq 0} \int_0^\infty \exp(-rt)[pH - c(H)]\,\mathrm{d}t$$

subject to

$$\dot{X} = g(X) - H, \quad X \geq 0, \quad X(0) = X_0 \text{ given }. \qquad (2.1)$$

The following assumptions will be made and maintained throughout the paper:

A1. $c(0) = 0, \quad c'(0) = 0, \quad c'' > 0$

A2. there exist $0 < X_{\min} < X_r < \bar{X} < X_{\max}$ such that
$g(X) < 0$ if $0 \leq X < X_{\min}$ or $X > X_{\max}$,
$g(X_{\min}) = g(X_{\max}) = 0, \quad g'(X_r) = r$,
$g'(X) > 0$ if $X < \bar{X}, \quad g'(X) < 0$ if $X > \bar{X}, \quad g'' < 0$.

A3. $g(\bar{X}) < \bar{H}$, where $\bar{H}$ is defined by $c'(\bar{H}) = p$.

Assumptions A1 and A2 are self-evident and need no further comment. Assumption A3 says that the myopic profit maximizing harvest rate cannot be maintained indefinitely.

The problem stated above is a simple optimal control problem with one state variable. Due to the continuity of the functions involved, the boundedness of the state variable ($X \leq \max[X_0, X_{\max}]$) and the positive discount rate, the problem has a solution (see, e.g., Toman, 1985). Furthermore, the solution is unique in view of the strict concavity/convexity of the functions involved.

The Hamiltonian reads

$$\mathbf{H} = pH - c(H) + \mu[g(X) - H] .$$

The necessary conditions are that (2.1) holds and that there exists a piece-wise differentiable function μ such that

$$p - c'(H) \leq \mu, \quad p - c'(H) = \mu \text{ if } H > 0 , \tag{2.2}$$

$$-\dot{\mu} = \mu[g'(X) - r] . \tag{2.3}$$

Here, μ is to be interpreted as the shadow price of the renewable resource. Hence (2.2) says that along an optimum the marginal revenues of the harvested commodity should equal its marginal cost consisting of the direct harvest costs and the opportunity costs incurred by reducing the size of the resource. (2.3) is an arbitrage rule saying that the rate of change in net present value of the resource stock should equal the interest rate.

It is clear from (2.3) that if $\mu(t) = 0$ for some t it is equal to zero for all t and $H(t) = \bar{H}$ for all t as long as $X > 0$.

Next consider the loci for which X and μ are constant.
μ is constant if $X = X_r$ and if $\mu = 0$. If $X > X_r$ then μ increases.
X is constant if $g(X) = H$. This occurs if $X = X_{\min}$ and $H = 0$, if $X = X_{\max}$ and $H = 0$, and if $X_{\min} < X < X_{\max}$ with $g(X) = H > 0$. In the latter interval, it is easily seen that $\mathrm{d}\mu/\mathrm{d}X = -c''g'(X)$. So $\mathrm{d}\mu/\mathrm{d}X > 0$ if and only if $X > \tilde{X}$.
These observations lead to the phase diagram depicted in Fig. 1.

The optimal trajectory can now be sketched as follows. If X_0 is larger than $X_{\min}$ the initial μ is to be chosen on the stable branch leading to the steady state denoted by E. If $X_{\min} < X_0$ but X_0 is close to $X_{\min}$ then initially $H = 0$ until μ reaches p. If X_0 is smaller than $X_{\min}$ the resource will be exhausted within finite time along a trajectory where $H = \bar{H}$ with $\mu = 0$.

The comparative statics are straightforward. A higher interest rate will decrease X_r and hence the steady state will correspond to a smaller

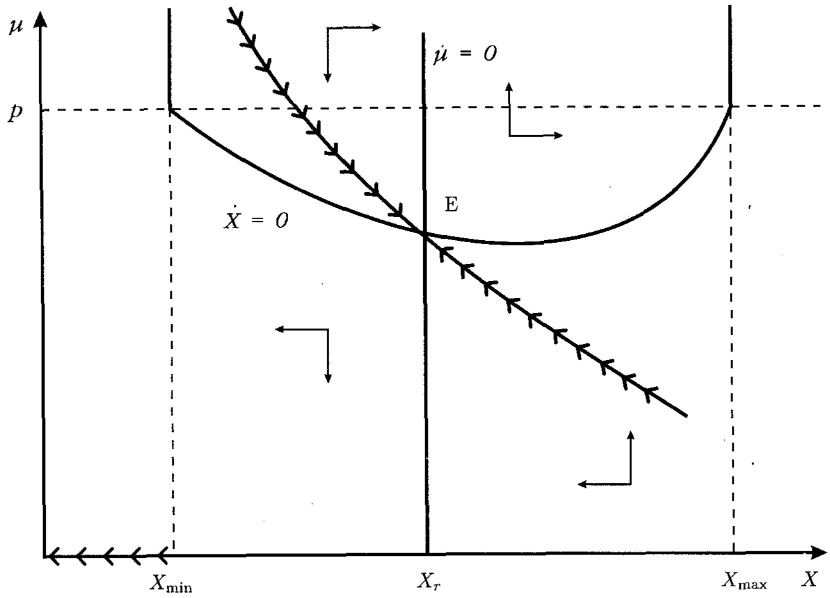

Fig. 1

size of the renewable resource. An increase in the market price of the harvested commodity does not affect the steady-state value of the resource.

3. Additive Management Efforts

In this section, we consider the effect on optimal harvesting when human activities can positively influence the resource stock in an additive fashion. This can be modeled in a number of ways. The appropriateness depends on the type of resource under consideration. For some resources the important thing is how much effort is made in total, for others what matters is the effort per unit of the existing resource stock. We shall consider the latter possibility first. In order to avoid misunderstanding "harvest efforts" will be termed "harvest" for short and the "managing efforts" will just be called "efforts."

The growth equation is now modified as follows:

$$\dot{X} = g(X) + Xf(V/X) - H, \quad X \geq 0, \quad X(0) = X_0 \text{ given}. \qquad (3.1)$$

Here, V is the total effort. About f we shall assume

A4. $f(0) = 0, \quad f' > 0, \quad f'(0) = \infty, \quad f'(\infty) = 0, \quad f'' < 0 .$

So, f is a strictly increasing concave function.

Per unit effort costs are assumed to be given and constant. They are denoted by q.

The Hamiltonian is

$$\mathbf{H} = pH - c(H) - qV + \mu[g(X) + Xf(V/X) - H] .$$

Again there exists a unique solution.

Defining Y as the per unit effort V/X we find as necessary conditions (3.1) and

$$p - c'(H) \leq \mu, \quad p - c'(H) = \mu \text{ if } H > 0 , \tag{3.2}$$

$$q = \mu f'(Y) , \tag{3.3}$$

$$-\dot{\mu} = \mu[g'(X) + f(Y) - Yf'(Y) - r] . \tag{3.4}$$

The interpretation of (3.3) is that the marginal cost (q) of additional effort should equal its marginal benefit consisting of the value of the additional increase of the size of the renewable resource.

We next analyze the loci for which the shadow price and the resource size are constant. This is only slightly more complicated than in the previous case.

Consider first $\dot{\mu} = 0$ for $\mu > 0$. We have the following system of equations:

$$g'(X) + f(Y) - Yf'(Y) = r$$

and (3.3). If μ tends to zero, so does Y according to (3.3). Then $f(Y) - Yf'(Y)$ also tends to zero and X tends to X_r. Furthermore, the locus is easily seen to be increasing in the (μ, X)-space. It cannot be ruled out that for μ large there is no X for which the above conditions are satisfied. If μ goes to infinity, Y goes to infinity, according to (3.3). If $g'(X)$ is bounded from below then the locus $\dot{\mu}=0$ is more likely to be undefined for large X the smaller is the lower bound on $g'(X)$. However, this poses no problem for the phase diagram analysis, because then μ decreases for every sufficiently large μ and X.

The locus $\dot{X}=0$ is determined by (3.2), (3.3), and $g(X) + Xf(Y) = H$. Since we have not fully specified the behavior of g and f for extreme X and Y values several possibilities occur. For example, if

$f(Y) \to \infty$ as $Y \to \infty$ and $g(X)/X \to -\infty$ as $X \to \infty$, the $\dot{X}=0$ locus has the μ-axis as an asymptote, and if (μ, X) is on the locus then $\mu \to \infty$ as $X \to \infty$. And if $g(X)/X$ is bounded from below for all large X with the assumption on f maintained, the $\dot{X}=0$-locus will have a positive μ as an asymptote when $X \to \infty$. On the other hand, if $f(Y)$ is bounded the $\dot{X}=0$ curve may have a positive $X < X_{\min}$ as an asymptote as μ goes to infinity. Since our interest is mainly focused on the steady state, these observations do not cause too much problems in view of the following.

Consider the locus $\dot{X}=0$ for $\mu < p$. Then we have

$$d\mu/dX = [g'(X) + f(Y)]/[(-1/c'') + Xf'f'/f''\mu]$$

from the implicit function theorem. Therefore the $\dot{X}=0$ locus is decreasing in the (μ, X)-space if and only if $g'(X) + f(Y) > 0$. It follows from (3.4) that $\dot{\mu} < 0$ (for $\mu \neq 0$) if and only if

$$g'(X) + f(Y) > Yf'(Y) + r > 0 \ .$$

Consequently, the $\dot{X}=0$ curve is decreasing in points (μ, X) where $\dot{\mu} < 0$, so that there exists a unique steady state.

Piecing everything together we find the phase diagram as depicted in Fig. 2.

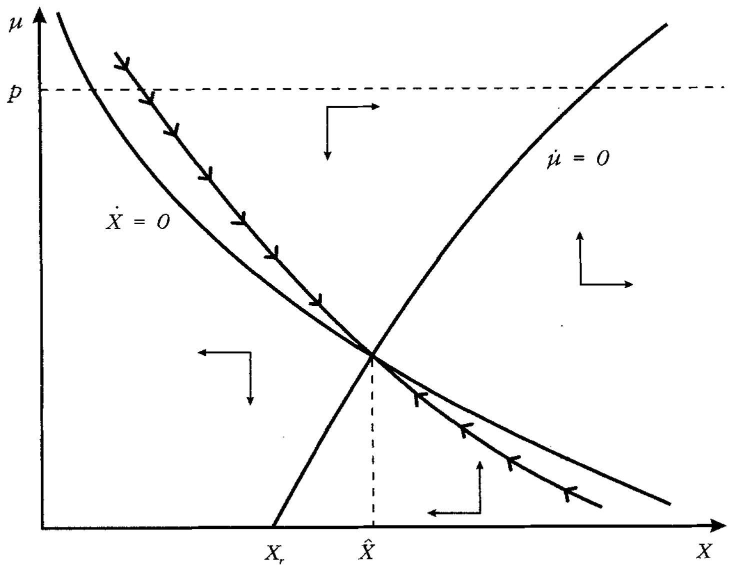

Fig. 2

As in the previous case there exists a unique steady state which is asymptotically approached for X_0 sufficiently large (possibly just $X_0 > 0$); otherwise the renewable resouce is exhausted within finite time. The steady-state size of the resource is larger than in the case where efforts do not play a part. The intuition behind this result is that the net rate of change in the value of the resource is augmented now by the marginal productivity of efforts.

With regard to the comparative statics of the steady state the following results emerge.

A higher market price of the harvested commodity calls for a larger steady-state resource stock. This is in sharp contrast to the result in the previous case. The proof is by contradiction. Suppose that following a rise in p the steady-state μ decreases. Then H increases and Y decreases. As a consequence $g'(X)$ increases and X decreases. On the other hand, $g(X)$ decreases because the steady-state X is smaller than $\bar{X}$. But then (3.4) cannot hold. Therefore μ increases and hence X increases. The intuition behind this result is that the higher market price of the harvested commodity not only makes harvesting more profitable but increases the value of the existing stock as well. The latter effect dominates the former. A higher market price of efforts decreases the efforts and the steady-state stock of the resource. This can be seen as follows. Suppose that a higher q gives a larger Y. Then we have a larger μ. Therefore the steady-state harvest must be smaller. This can only be obtained through a smaller stock of the resource. But this contradicts $g'(X) = r - f(Y) + Yf'(Y)$. Finally, an increase in the interest rate will, as before, yield a smaller stock of the resource. It is worth pointing out what happens on the trajectory towards the steady state as, unexpectedly, the market price of the harvest increases. The $\dot{\mu}=0$ locus is not affected and the $\dot{X}=0$ locus shifts upward. As a consequence, μ jumps upward and the rate of efforts increases.

An alternative growth specification in the class of additive efforts is the following

$$\dot{X} = g(X) + f(V) - H \ . \tag{3.6}$$

Here, one can think of V as for example the water temperature, with V properly scaled. With this example in mind, a plausible assumption with respect to f is

A5. There exist $V_{\max} > \bar{V} > 0$ such that

$$f(0) = f(V_{\max}) = 0 \ ,$$
$$f'(V) > 0 \ (< 0) \text{ for } V < \bar{V} \ (V > \bar{V}) \ ,$$
$$f'(0) = a \text{ for some } a > 0, \quad f'' < 0 \ .$$

Here, $V = 0$ corresponds to the natural temperature. So, the temperature should not be too high to have a positive influence on growth. In order to have an interesting problem it will be assumed that $ap > q$. If this inequality would not hold, it is never worthwhile to make any effort because the marginal costs exceed even the myopic revenues.

The necessary conditions for an interior solution are

$$p - c'(H) = \mu \ , \tag{3.7}$$

$$q = \mu f'(V) \ , \tag{3.8}$$

$$-\dot{\mu} = \mu[g'(X) - r] \ , \tag{3.9}$$

$$\dot{X} = g(X) + f(V) - H \ . \tag{3.10}$$

The locus of points for which $\dot{\mu} = 0$ is given by $X = X_r$. It is easily verified that the locus of points for which $\dot{X} = 0$ is upward sloping for $X > \bar{X}$ and downward sloping elsewhere. For μ equal to p the corresponding X's are well defined provided that $g(X)$ can be made sufficiently negative. Hence, essentially the same phase diagram results as in the case with no efforts. (Mathematically, the present model strongly resembles Long's, 1977, which deals with exploration.) The effect of the presence of efforts manifests itself only on the trajectory to the steady state, but not in the steady state itself. Also, changes in the market price of efforts have no impact on the steady state itself. A higher input price just reduces the efforts in the same magnitude as the harvest rate. One difference with the no-effort case is that if the effort price is sufficiently low, the resource may be preserved now whereas in the first case it would be exhausted. The differences between this and the previous model need not to be stressed.

4. Multiplicative Management Efforts

It has been assumed in the previous section that efforts contribute to the growth process in an additive fashion. In the first case dealt with the marginal effect of efforts on the growth process is increasing in the resource stock, whereas in the second case the marginal effect of efforts is independent of the resource stock. In those settings, it was assumed away that there can be circumstances where efforts have increasing as well as decreasing returns according to the resource size. It is the aim of the present section to take this into account. There are several ways to do so. We shall consider here the following growth process:

$$\left.\begin{array}{ll} \dot{X} = g(X)f(V) - H & X_{\min} < X < X_{\max} \\ \dot{X} = g(X) - H & \text{elsewhere} \\ X \geq 0,\ X(0) = X_0 & \text{given} \end{array}\right\} , \tag{4.1}$$

where g and f satisfy A2 and A4, respectively. So, there exist bounds on the resource stock within which growth is feasible; however, even within this range efforts are necessary and for stocks close to the boundary the marginal product of efforts is small, whereas in the "middle" the marginal product is large. For stocks outside the favorable range the stock is necessarily decreasing. One of the implications is that the resource gets exhausted once a critical low level is reached, irrespective of efforts. The Hamiltonian reads

$$\mathbf{H} = [pH - c(H) - qV] + \mu[g(X)f(V) - H] .$$

Along a solution trajectory the necessary conditions are (4.1) and

$$p - c'(H) \leq \mu, \quad p - c'(H) = \mu \text{ of } H > 0 , \tag{4.2}$$

$$q = \mu g(X)f'(V) , \tag{4.3}$$

$$-\dot{\mu} = \mu[g'(X)f(V) - r] . \tag{4.4}$$

In the (μ, X)-space the locus of points where $\dot{\mu} = 0$ and $\mu > 0$ can be depicted as in Fig. 3. For X tending to $\bar{X}$, $g'(X)$ tends to 0 so that V goes to infinity, which, according to (4.3), is possible only if μ goes to infinity. If X approaches $X_{\min}$, we must have $g'(X_{\min})f(V) = r$, so that V and hence $f'(V)$ are bounded; this can only happen if μ tends to infinity again. In Fig. 3 is also depicted the locus of points for which $\dot{X} = 0$. For $\mu \geq p$ we must have $X = X_{\min}$ or $X = X_{\max}$. The locus is decreasing for $X < \bar{X}$ and increasing for $X > \bar{X}$.

The figure displays two steady states, namely A and B. It is worth stressing, however, that this need not be the case in general. It could well be that the loci do not intersect, for example when the output price p is small. Also, the possibility of a single steady state cannot be excluded. Finally, without further knowledge of the properties of f and g it could even be that there is an arbitrary number of steady states. It will be assumed in the sequel that there are just two steady states, as in Fig. 3. The first thing to note is that only point B is locally asymptotically stable. This is easily seen from the arrows in the figure, but it can also be derived in a more formal way.

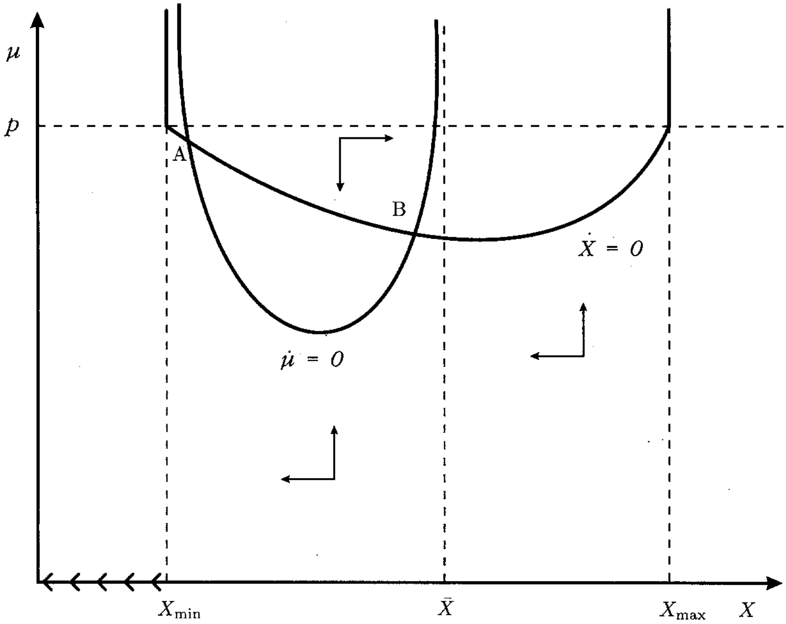

Fig. 3

For X_0 smaller than $X_{\min}$ it is optimal to put μ equal to zero and $H = \bar{H}$ until $X = 0$. If $X_0 > X_{\min}$ it is optimal to converge to the stable steady state by properly choosing μ on the stable branche.

The comparative statics analysis, departing from the stable steady state, yields the following results. An increase in the output price p shifts the $\dot{X}=0$ locus upward. Thereby, the steady-state value of the resource stock increases as well as the steady-state value of the shadow price μ. This implies that both the steady-state rate of harvest and the effort rate are increased. So higher output prices will lead to conservation. A higher price of efforts leads to a decrease in the effort rate. In spite of the fact that this statement is intuitively clear, its proof is not straightforward; at least it cannot proceed as easily as the previous ones. Since in a neighborhood of the steady state $H > 0$ and $V > 0$ and due to the properties of c, f, and g, we can write $H = H(\mu)$ [from (4.2)] and $V = V(X, \mu; q)$ [from (4.3)]. Upon insertion into (4.4) and (4.5) we find

$$\dot{\mu} = -\mu\big[g'(X)f\big(V(X,\mu;q)\big) - r\big] =: F_1(X,\mu;q) \ ,$$

$$\dot{X} = g(X)f\big(V(X,\mu;q)\big) - H(\mu) =: F_2(X,\mu;q) \ .$$

Putting $\dot{\mu} = \dot{X} = 0$ and using the implicit function theorem, we obtain

$$(F_{2x}F_{1\mu} - F_{2\mu}F_{1x})\,\mathrm{d}X = (F_{2\mu}F_{1q} - F_{1\mu}F_{2q})\,\mathrm{d}q \ .$$

Since we are in a stable steady state the first factor on the left hand side is negative. The first factor on the right hand side equals

$$\mu g' f' \frac{\partial V}{\partial \mu}\frac{\partial H}{\partial \mu} = -\mu g' f' / c'' \mu g f'' > 0$$

because $c'' > 0$, $f'' < 0$, $g' > 0$, $f' > 0$. Therefore an increase in the input price will lead to a decrease in the steady-state value of the stock. It follows from (4.4) that effort will decrease, as well as the steady-state harvest rate.

5. Conclusions

We have investigated the effect of the introduction of efforts to manage renewable natural resources so as to influence the growth potential. It is found, among other things, that an increase of the price of the harvested commodity entails an increase of the steady-state stock of the natural resource. This result is obtained in a partial equilibrium setting and for specific functional forms representing the impact of efforts. Nonetheless, it is clearly not the case that price increases will certainly cause overexploitation, as false intuition would suggest. Future research could be directed to a general equilibrium analysis. It could also be interesting to reincorporate the existing size of the resource into the cost of harvesting.

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