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WORKING PAPER NO. 00-12

CAPITALIZATION OF FEDERAL TAXES, THE RELATIVE PRICE OF HOUSING, AND URBAN FORM: DENSITY AND SORTING EFFECTS

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Abstract

We investigate the impact of the tax treatment of owner-occupied housing on urban form in an economy in which high- and low-income households choose between city and suburban communities. Because housing tax policies differentially affect the relative after-tax price of housing for high- and low-income households, and because the extent of capitalization of housing tax policies can differ across city and suburban communities, our analysis finds that housing tax policies can affect not only the density of the metropolitan area but also can influence where rich and poor households choose to live.

We also show that the impacts of housing tax policies differ depending on whether land use constraints such as suburban large lot zoning exist. If there are no land use constraints present, increasing a subsidy to home ownership that is positively correlated with the income of the owner tends to lead to the decentralization of both rich and poor, although there are conditions under which the rich would choose to concentrate in the central city. The ambiguity of the effect on the choices of high income households suggests that impacts of the federal tax treatment of housing may differ across metropolitan areas.

In the presence of binding large lot zoning in the suburbs, the rich have a greater incentive to decentralize while the poor are constrained to the city. Thus, housing tax policy that affects the relative price of land differentially for the rich and poor could have helped exacerbate the intense residential sorting by income that we see in many parts of the United States. Importantly, our analysis of community choice is not driven by different preferences for city or suburb that may be associated with the income elasticity of housing demand. Rather, it results from changes in relative after-tax housing prices faced by poor and rich households. Determining the empirical relevance of prices versus preferences in this matter should be an urgent task for future research.

1. INTRODUCTION

This paper analyzes how current housing tax policy, which provides benefits to home owners that increase with income, can influence urban form. Mills (1987) and others have shown that the favorable tax status of owner-occupied housing has resulted in more housing investment than otherwise would have occurred. We would expect that because land is an input to housing, this increased housing investment would also result in lower population densities. With the exception of Blackley and Follain (1983), however, analyses of the patterns of metropolitan development generally have not seriously considered the impact of the tax treatment of housing.[1] In this paper, we focus on the differential impacts that the tax treatment of housing can have on the consumption of land and on the location choices of high- versus low-income households, and their associated consequences for urban form.

There are two reasons why the tax treatment of housing can affect patterns of metropolitan development beyond simply reducing density. First, housing tax policies have greater direct effects on the after-tax relative price of housing for high-income households and therefore provide incentives with respect to housing and location choices that differ from the incentives for low-income households. Second, differing supply conditions across communities are likely to yield different capitalization of housing-related tax incentives into market prices across locations that also can influence after-tax relative prices. For example, there should be a different supply response, and hence different capitalization, in communities with limited undeveloped land versus communities on the urban fringe that have plentiful supplies of open

[1]For example, in their widely cited review of the causes of suburbanization, Mieszkowski and Mills (1993) consider the impacts of transportation policies and the like, but not housing-related tax expenditures.

land. In addition, the supply of housing available to high-income households will also depend on the housing decisions of low-income residents so that the extent to which housing tax benefits are capitalized into the market price of housing will depend on the geographic distribution of high- and low-income households across communities.

These two features are incorporated in a model that we use to analyze the impact of housing-related tax incentives on the housing consumption and community choices of high- and low-income households that hold idiosyncratic preferences for cities (bounded, supply-constrained communities) and suburbs (unbounded, supply-unconstrained communities). A key feature of our analysis of community choice is that households' choices are not driven by different preferences between high-and low-income households but by the effects of policy on after-tax housing prices. The impact of the tax treatment of housing is examined under two regimes: one with no land-market constraints and one with a binding large lot zoning requirement in the suburbs.

When no binding zoning rules exist so that rich and poor households are equally mobile within the metropolitan area, our results indicate that a policy lowering the relative price of housing for high-income households unambiguously creates an incentive for low-income residents to leave the city and, under most circumstances, induces more high-income households to choose suburban locations as well. There are conditions, however, under which the tax policy would provide an incentive for high-income households to locate in the central city. This ambiguity in the impact on the community choices of high-income households suggests that the impacts of the federal tax treatment of housing may not be uniform across metropolitan areas. The conditions under which the policy creates an incentive for high-income households to locate

in the city are, however, likely to be uncommon, so the net effect most probably is an increased decentralization of both high- and low-income households with an increase in total land consumption.

In the presence of a binding suburban zoning constraint that limits the ability of the poor to purchase a lot in the suburbs, tax policy that subsidizes the rich increases the degree of spatial sorting by income in the metropolitan area. More specifically, if large lot zoning in the suburbs confines poorer households to the central city, then a proportional subsidy that reduces the relative price of housing for high-income households will, under most circumstances, result in greater decentralization of the rich and hence a greater separation of high- and low-income households than otherwise would occur. Thus, it is only in the presence of a binding large lot zoning constraint that a proportional subsidy to owners can generate residential sorting by income of the type we see in the United States.

The practical relevance of these theoretical conclusions derives partially from their implication that policy influencing the spatial pattern of relative land prices, not just preferences for lower density lifestyles combined with rising incomes and better transportation systems, may have helped shape urban form in the U.S. In addition, our findings have potentially important implications for the emerging debate surrounding smart growth initiatives. If enacted, some of those initiatives clearly will affect the relative price of land across communities within individual metropolitan areas. The conclusions of our model suggest that the spatial pattern of development and the degree of residential sorting by income can be influenced by policies affecting relative land prices across space.

2. THE MAGNITUDE AND DISTRIBUTION OF FEDERAL TAX EXPENDITURES ASSOCIATED WITH OWNER-OCCUPIED HOUSING

On an annual basis, tax expenditures associated with mortgage interest and local property tax deductions are very large. Auten and Reschovsky (1998) report that the Department of the Treasury estimates the value of the mortgage interest deduction alone to be $53.7 billion and the property tax deduction to be $18.4 billion, for a total of $72.1 billion in fiscal year 1999. Auten and Reschovsky (1998) also note that the subsidy to home ownership from the nontaxation of imputed rent amounts to about $100 billion.

The benefits of mortgage interest and property tax deductions, as well as the nontaxation of imputed rent, do not accrue equally to all owners. In fact, less than one quarter of all filers and less than 40 percent of all home owners have sufficient income, house value, and leverage to warrant itemizing in lieu of taking the standard deduction.[2] For the predominantly low- and moderate-income owners that do not itemize, there are no benefits related to mortgage interest or local property tax deductions. Cecchetti and Rupert (1996) report that over 70 percent of the tax expenditures associated with the mortgage interest deduction accrue to households with annual earnings in excess of $75,000.[3] Thus, the mortgage and property tax deductions result in high- and low-income households' facing different after-tax prices for housing.

Even though all home owners profit from the nontaxation of imputed rent, this aspect of

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[2]The *Statistics of Income Bulletin* (Internal Revenue Service, Washington D.C.: Fall 1995) reports that 23.2 percent of all filers used the mortgage interest deduction. Although the IRS does not tabulate data separately for owners and renters, if we assume that 65 percent of all filers are home owners, then the fact that 23.2 percent of all filers use the mortgage interest deduction suggests that roughly 36 percent of all owners use the standard deduction.

[3]That this is, indeed, a relatively high income is evident from the fact that the median household income in 1998 was $33,400 (in 1998 dollars; 1998 *Survey of Consumer Finances*).

the housing subsidy also is differentially valued by high-and low-income households. Because high-income households face higher marginal tax rates, the nontaxation of imputed rents effectively lowers the after-tax price of housing more for high-income households than for low-income households. For example, Poterba (1991) estimates that tax code-related benefits to owner-occupied housing lower the costs of high-tax-bracket owners by as much as 15 percent.

Thus, the net impact of the tax treatment of housing is to provide greater benefits for households with greater income and wealth. The remainder of the paper examines how this can affect the density and residential makeup of a metropolitan area.

3. A MODEL OF THE IMPACTS OF RELATIVE HOUSE PRICE CHANGES IN A METROPOLITAN AREA WITH BOUNDED AND UNBOUNDED COMMUNITIES

The structure of our model is characterized by the following features: (a) a metropolitan area in which there is a city with a fixed boundary and a suburban community that always can expand on the margin; (b) two types of workers, high and low skill/income; (c) a housing subsidy that only high-skill households receive; and (d) a system of preferences in which households of a given type have identical preferences over housing services and nonhousing services, but can differ in terms of an idiosyncratic preference for city versus suburban location. The remainder of this section discusses these features and their implications in more detail.

3.1 Characterization of the Metropolitan Area

There is a single metropolitan area consisting of two jurisdictions indexed by $j = (c,s)$, with c denoting the central city and s denoting the suburban jurisdiction. The central city boundary is exogenously given and cannot be changed so that the central city's land area, $\overline{L}_c$, is fixed. We refer to the central city as being a bounded community with its land in perfectly

inelastic supply. In contrast, unimproved land in the suburban region, L_s, is assumed to be perfectly elastically supplied. Hence, the suburbs are the unbounded community.[4] Because there is a potentially infinite supply of land to the suburban community, its price, r_s, is fixed at the value of its alternative use–agriculture. City land price, r_c, adjusts to allocate the scarce resource, city land.

In addition, two types of workers indexed by i=(h,l), with h denoting high skill and l denoting low skill, live in the metropolitan area. Each group is fixed in size, N^i, with high-skill workers earning wage w^h and low-skill workers earning wage w^l.[5] The distribution of these two groups across the metropolitan area depends on systematic and idiosyncratic preferences, equilibrium prices, and housing subsidies.

3.2 Characterization of the Housing Subsidy

The various tax code-related benefits to owner occupancy can lower the after-tax price of owner-occupied housing. Abstracting from the nontaxation of imputed rent, we specify the mortgage interest and property tax deductions as a subsidy, τ^i, defined as the fraction of the price of housing services paid by the government. For simplicity, we assume that the level of the standard deduction and the progressivity of the tax code combine to function so that the mortgage

[4]While the central city is always the bounded community and the suburb the unbounded community in our lexicon, the results derived below generally apply to communities with low versus high supply elasticities.

[5]We assume that the metropolitan area is an integrated labor market and that all locations are equally accessible to employment so that there are no commuting-related rent or wage gradients.

interest and property tax deductions are attractive only to high-skill workers.[6]

For low-skill, low-income households, $\tau^l=0$, so that the after-tax price of housing services (r^l_{ja}) equals the observed market price of those services, r_j. Because the after-tax price equals the market price for low income households, we drop the superscript on after-tax rents for the remainder of the discussion. For high-skill, high-income households, the after-tax price of housing services is given by equations (1) and (2),

$$r_{ja} = (1- \tau^h)r_j \quad \text{where } 0 < \tau^h < 1, \quad j = c,s. \qquad (1), (2)$$

Note that the after-tax price of housing for high-income individuals depends not only on the direct effect of τ, but also on the effect of τ on equilibrium prices. At one extreme, τ could be perfectly capitalized into market prices such that after-tax prices were unchanged by the subsidy; at the other extreme, there could be no capitalization and r_j would be unaffected by τ while r_{ja} would fall by the full amount of τ.

3.3 Preferences

Individuals consume a market good, X (whose price is the numeraire), and housing services, H_j. An individual consumer, k, maximizes utility by choosing residential location and optimal quantities of X and H_j given r_j, τ^i, and w^i. More formally,

$$\text{Max } U^{ik}(X, H_j) \quad \text{Subject to:} \quad X + (1-\tau^i)r_j H_j = w^i. \qquad (3)$$

Individuals of a given type are assumed to have identical preferences over X, and H_j, but they

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[6]Thus, we effectively assume that low-skill worker households do not itemize. Depending on the extent of capitalization, the tax advantages of mortgage and property tax deductions affect the after-tax price of housing while the tax benefits associated with the standard deduction do not affect the relative price of housing.

differ in their preferences for city or suburban location. The utility function is defined such that the indirect utility function, V^{ik}, takes the following form

$$\begin{aligned} &\text{High skill:} \quad V^{hk} = V(r_{ja}, w^h) + \varepsilon_j^{hk} \\ &\text{Low skill:} \quad V^{lk} = V(r_j, w^l_j) + \varepsilon_j^{lk}, \end{aligned} \tag{4}$$

where V(.) is the systematic component of utility and ε_j^{ik} is the increment to indirect utility associated with the choice of location j. Note that ε_j^{ik} is normalized such that it represents the incremental utility associated with an individual choosing a suburban location. Specifically, let $\varepsilon^{ik} = \varepsilon_c^{ik} - \varepsilon_s^{ik}$ define the relative idiosyncratic preference for locations c and s.

3.4 Location Choice

Because all consumers have identical tastes except for idiosyncratic preferences for city or suburban living, the marginal consumer is defined (separately for rich and poor individuals) by that ε^k, ε^{i*}, satisfying $\varepsilon^{i*} = V_c^i - V_s^i$.

More formally, for each worker type the marginal consumer is defined such that

$$\varepsilon^{h*} = V(r_{ca}, w^h) - V(r_{sa}, w^h) \tag{5}$$

$$\varepsilon^{l*} = V(r_c, w^l) - V(r_s, w^l). \tag{6}$$

Since $r_c \geq r_s$, $\varepsilon^{i*} \leq 0$ or no households would choose to live in the city.[7]

By specifying a distribution, Ψ^i, for ε^{ik}, the number of rich or poor households choosing

[7]Because suburban rent is fixed at the value of its alternative use, city rents are always greater than or equal to suburban rents. Thus, the systematic component of utility in the city is always less than or equal to the systematic component of suburban utility. Therefore, households choosing to live in the city must have a preference for the city, and hence $\varepsilon^{i*} \leq 0$.

city residences, N_c^i, can be determined as a function of relative rents in the city and suburbs.[8] Thus, the numbers of rich and poor households choosing to live in the city are given by:

$$N_c^i = \Psi^i(\varepsilon^{i*}), \text{ for } i = h, l. \qquad (7), (8)$$

It is straightforward that $\partial\Psi/\partial\varepsilon^{i*} > 0$. For example, if indifference requires a very negative ε^{i*}, few people will choose a city residence. Essentially, city rents are relatively high if ε^{i*} is strongly negative. However, as ε^{i*} increases toward zero, the number of people choosing city residences increases. Hence, the derivative must be positive.

Finally, because the number of high-and low-skill individuals is fixed,

$$N_s^i = N^i - N_c^i, \quad i = h, l. \qquad (9), (10)$$

3.5 Housing Demand

Given an indirect utility function, Roy's identity provides the demand for housing by each individual household. Conditional on the choice of jurisdiction, individual housing demand is a function of r_{ja}, and w^h for high-skill households and r_j, and w^l for low-skill households as shown in equations (11)-(14).

$$H_j^h = H^h(r_{ja}, w^h), \quad \text{for } j=c,s; \qquad (11), (12)$$

$$H_j^l = H^l(r_j, w^l), \quad \text{for } j=c,s \qquad (13), (14)$$

[8]Note that increasing wealth is assumed not to change one's intrinsic preference for a city versus suburban location. Thus, when subsidies increase, they do not favor city or suburbs. Similarly, w^i does not affect the population distribution because wages for individuals of a given type within the single metropolitan area labor market are assumed to be the same in both jurisdictions.

3.6 Aggregate Demand and Supply of Land

Land consumption is assumed to be proportional to the demand for housing services. Assuming purely for convenience that the proportionality factor equals one, land consumption, which cannot exceed $\overline{L}_c$ in the city, is given by

$$\overline{L}_c = H_c^h N_c^h + H_c^l N_c^l. \tag{15}$$

Suburban land, which is not exogenously given, is defined by

$$L_s = H_s^h N_s^h + H_s^l N_s^l. \tag{16}$$

While the assumption of proportionality is a useful one because it simplifies the algebra in the comparative static analysis below by doing away with the need to explicitly model the production of housing with capital and land, it obviously is not realistic. Hence, its potential impact on the results needs to be examined up front. Clearly, if land prices were to increase, capital could be substituted for land in the production of housing services. That is, more people could be accommodated in the rising land price area by increasing the use of capital (i.e., by building up). By assuming proportionality, we abstract from this margin for substitution. However, all our key comparative statics results hold as long as there are diminishing returns to using capital in the production of housing. Because land is a fixed factor in the city, the unit cost of housing will rise with an increasing number of households in the city, even as capital is substituted for land. Rents in the city still adjust to allocate the scarce resource.[9] This highlights that the bounded-unbounded distinction between communities in the metropolitan area is much more important to

[9]This does imply that, when population falls in the city, the remaining city residents consume more city housing services. Implicitly, we are assuming that the housing stock adjusts in terms of size of housing to match demand. This also is unrealistic in the short run, as housing that does not match consumer demands often is left vacant. It is noteworthy that this sort of fixity of city housing stock only reinforces the results below associated with sorting by income.

the model than is our simplified housing supply side.

4. COMPARATIVE STATICS

4.1 Case 1: No Lot Size Constraints

The first case considered is one in which no land use constraints of any type are present. With w^i exogenously fixed, equations (1), (2), (5)-(16) form a system of 14 equations (six of which are identities) in 14 variables r_c, r_{ca}, r_{sa}, ε^{h*}, ε^{l*}, N_c^h , N_c^l, N_s^h, N_s^l, H_c^h, H_c^l, H_s^h, H_s^l, and L_s. Note that r_s does not adjust in this case because agricultural land is perfectly elastically supplied.

To examine the effects of changing the mortgage interest and local property tax deductions on location choices and housing consumption by high- and low-skill workers, these equations can be totally differentiated as follows:

$$d\varepsilon^{h*} = v^h_{r_{ca}} dr_{ca} - v^h_{r_{sa}} dr_{sa} \tag{17}$$

$$d\varepsilon^{l*} = v^l_{r_c} dr_c \tag{18}$$

$$dr_{ca} = (1-\tau)dr_c - r_c d\tau \tag{19}$$

$$dr_{sa} = -r_s d\tau \tag{20}$$

$$dN_c^h = \psi^h_{\varepsilon^{h*}} d\varepsilon^{h*} \tag{21}$$

$$dN_c^l = \psi^l_{\varepsilon^{l*}} d\varepsilon^{l*} \tag{22}$$

$$dH_c^h = H_{r_{ca}}^h dr_{ca} \tag{23}$$

$$dH_c^l = H_{r_c}^l dr_c \tag{24}$$

$$dH_s^h = H_{r_{sa}}^h dr_{sa} \tag{25}$$

$$dH_s^l = H_{r_s}^l dr_s = 0 \tag{26}$$

$$H_c^h dN_c^h + N_c^h dH_c^h = H_c^l dN_c^l + N_c^l dH_c^l \tag{27}$$

$$dL_s = H_s^h dN_s^h + N_s^h dH_s^h + H_s^l dN_s^l + N_s^l dH_s^l \tag{28}$$

$$dN_c^h = -dN_s^h \tag{29}$$

$$dN_c^l = -dN_s^l \tag{30}$$

Equations (17)-(30) then can be solved for $dr_c/d\tau$, $dN_c^h/d\tau$, $dN_c^l/d\tau$, $dH_c^h/d\tau$, $dH_s^h/d\tau$, $dH_c^l/d\tau$, and $dL_s/d\tau$.

4.1.1 Rents

Consider the effects of a change in subsidies on the price of housing services in the city, which is given by:

$$\frac{dr_c}{d\tau} = \frac{\{-H_c^h \psi^h_{\varepsilon^{h*}} (V^h_{r_{sa}} r_s - V^h_{r_{ca}} r_c) + N_c^h H^h_{c\, r_{ca}} r_c\}}{H_c^h \psi^h_{\varepsilon^{h*}} V^h_{r_{ca}} (1-\tau) + N_c^h H^h_{c\, r_{ca}} (1-\tau) + H_c^l \psi^l_{\varepsilon^{lh*}} V^l_{r_c} + N_c^l H^l_{c\, r_c}} > 0 \qquad (31)$$

In the denominator, the first and third terms are negative because the demand for housing by each skill type falls as price increases (i.e., $\partial V^h/\partial r_{ca}<0$ and $\partial V^l/\partial r_c<0$) and because the number of people choosing city residences increases with ε^{h*} (i.e., $\partial \Psi^h/\partial \varepsilon^h>0$). The second and fourth terms are also negative because housing demand decreases as after-tax rents increase (i.e., $\partial H_c^l/\partial r_c<0$ and $\partial H_c^h/\partial r_{ca}<0$). Thus, the denominator clearly is negative.

Signing the numerator requires a bit more analysis. The second term of the numerator is always negative because demand curves slope down. However, the sign of the first term depends on the quantity $(V^h_{r_{sa}} r_s - V^h_{r_{ca}} r_c)$. Invoking Euler's theorem for functions that are homogeneous of degree zero (which is the case for the indirect utility function), it is straightforward to show that $(V^h_{r_{sa}} r_s - V^h_{r_{ca}} r_c) = -\frac{1}{(1-\tau)} \varepsilon^{h*} > 0$ so that equation (31) can be rewritten as:

$$\frac{dr_c}{d\tau} = \frac{H_c^h \psi_{\varepsilon^{h*}}^h \frac{1}{(1-\tau)} \varepsilon^{h*} + N_c^h H_{c_{r_{ca}}}^h r_c}{H_c^h \psi_{\varepsilon^{h*}}^h V_{r_{ca}}^h (1-\tau) + N_c^h H_{c_{r_{ca}}}^h (1-\tau) + H_c^l \psi_{\varepsilon^{lh*}}^l V_{r_c}^l + N_c^l H_{c_{r_c}}^l} > 0 \qquad (31')$$

The numerator now clearly is negative, making the derivative unambiguously positive. While algebraically cumbersome, this is not surprising economically, since an increase in housing subsidies increases the overall demand for housing. This, in turn, increases city prices because housing in this part of the metropolitan area is inelastically supplied.

Equations (31) and (31') basically show the extent to which tax subsidies for housing are capitalized into market prices. Two aspects of these equations warrant further discussion. First, the degree of capitalization will be lower when there are larger numbers of low-income households living in the city. It will also be lower if these households' city housing consumption is highly sensitive to price. To see this more clearly, note that the last two terms in the denominator represent the responses of low-income households to changes in prices, and the larger these terms are, the less capitalization of subsidies into market prices there is. The economic intuition for this result is simply that low-income residents are the suppliers of land to the high-income households receiving the subsidies. And the more elastic that supply, the lower the capitalization of the subsidies to the rich owners.

Second, housing subsidies are never completely capitalized into market prices even if there are no low-income households in the city in which land supply is fixed. Incomplete capitalization arises from the fact that high-income households can opt for a suburban residence where subsidies always lower after-tax market prices (because housing there is in perfectly elastic supply). This can be seen more formally by noting that complete capitalization would leave after-

tax prices unchanged so that $dr_{ca}/d\tau = (1-\tau)dr_c/d\tau - r_c = 0$, or $dr_c/d\tau = r_c/(1-\tau)$. Manipulation of (31) reveals that $dr_c/d\tau < r_c/(1-\tau)$ even when there are no low-income households in the city. These findings with respect to capitalization will be important for our analysis below of the impact of housing subsidies on the community choices of high- and low-income households.

4.1.2 Number of Households Choosing City Residences

Increases in subsidies always reduce the number of low-income households choosing to live in the city as is evident by

$$\frac{dN_c^l}{d\tau} = \psi^l_{\varepsilon^{l*}} V^l_{r_c} \frac{dr_c}{d\tau} < 0 . \tag{32}$$

As subsidies rise, rents in the city increase, so fewer low-income households, which pay the full market price, choose to live in the city.

The situation is more ambiguous for high-income people. The effect of subsidies on the number of high-income people choosing city residences is given by:

$$\frac{dN_c^h}{d\tau} = \psi^h_{\varepsilon^{h*}} (V^h_{r_{ca}} (1-\tau) \frac{dr_c}{d\tau} + (V^h_{r_{sa}} r_s - V^h_{r_{ca}} r_c)) \tag{33}$$

or rewritten in slightly different form,

$$\frac{dN_c^h}{d\tau} = \psi^h_{\varepsilon^{h*}} (V^h_{r_{ca}} (1-\tau) \frac{dr_c}{d\tau} - \frac{\varepsilon^{h*}}{(1-\tau)}) . \tag{33'}$$

This comparative static result cannot be signed unambiguously. The first term within the parenthesis of equation (33') is always negative while the second term is greater than or equal to

zero (when the fact that it is being subtracted is considered). Economically, the change in the number of high-income households living in the city depends on how the utility differential responds to the change in after-tax rents in the city and suburb.

Still, some useful first inferences about the effects of changing subsidies on the community choices of high-income households can be made by considering alternative values of ε^{h*}. First, suppose $\varepsilon^{h*} = 0$, which is its maximum value and implies that city and suburban rents are identical. Increasing subsidies would unambiguously increase rents and, because the second term of (32') vanishes, reduce the number of rich households choosing city residences.[10] Second, even as ε^{h*} becomes increasingly negative (and r_c increases relative to r_s), there is a range in which $dN_c^h/d\tau$ still is negative because $dN_c^h/d\tau < 0$ as long as $-V_{r_{ca}}^h(1-\tau)\frac{dr_c}{d\tau} > -\frac{\varepsilon^{h*}}{(1-\tau)}$. This range is larger than first may appear because as ε^{h*} becomes more negative, $dr_c/d\tau$ becomes increasingly positive to at least partially offset the effect of ε^{h*}.

The two competing factors determining which effect dominates are as follows: (a) the extent to which housing subsidies are capitalized into city rents, with substantial capitalization tending to reduce the incentive for high-income households to locate in the city because after-tax house prices do not fall by as much in this case; and (b) the size of the city-suburban after-tax rent differential (which also is reflected in the size of ε^{h*}, as it measures the intensity of preference for city living necessary to maintain a wide rent differential); a larger differential increases the incentive for high-income households to live in the city.

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[10]Note that $dr_c/d\tau > 0$ even when $\varepsilon^{h*} = 0$.

Although it is not possible to evaluate mathematically which effect dominates without additional structure on the model, further examination can shed some light on the issue. First, consider the impact of capitalization. Because the city is bounded, if only high-income people lived in the city, increases in subsidies would be capitalized into rents such that aggregate land consumption remains unchanged. The subsidy also would be fully capitalized if there were only one community with a fixed, fully utilized supply of land, with market rents increasing to completely offset the subsidy. In our framework, however, households can choose to move to the suburbs where land is elastically supplied. Because suburban after-tax rents must fall with an increase in subsidy, some high-income households will be attracted to the suburbs on the margin. This implies that if only high-income households live in the city, $dN_c^h/d\tau < 0$.[11] Equilibrium requires that all land in the city be used so that after-tax city rent must fall with an increase in subsidies, thereby confirming that housing subsidies cannot be completely capitalized into rent even though the city's boundary is fixed.

If both high- and low-income households live in the city, the impact of subsidies on the location choice of high-income households is more complex because the supply of city land for high-income households can increase by bidding it away from low-income households. Since subsidies unambiguously increase the market price in the city, low-income residents adjust by lowering housing consumption and choosing suburban locations. Thus, the supply of land available for high-income households increases when subsidies increase. The extent to which city

[11]This can be seen by setting the H_c^l and $N_c^l = 0$ in equation (31) and substituting $dr_c/d\tau$ into equation (33), which after some manipulation yields $\frac{dN_c^h}{d\tau} = \frac{\Psi_{\varepsilon^{h*}}^h V_{r_{sa}}^h r_s N_c^h H_{r_{ca}}^h}{H_c \Psi_{\varepsilon^{h*}}^h V_{r_{ca}}^h + N_c^h H_{r_{ca}}^h} < 0$.

land is elastically supplied to high- income households will depend on the extent of land holdings by low-income households in the city and the sensitivity of city land consumption by low-income land holders to changes in the price of land. This sensitivity is a function of both the elasticity of community choice with respect to price and the elasticity of housing demand with respect to price, given a community choice. High-income households are more likely to choose city residences if there is a very large reduction in low-income consumption of city housing in response to an increase in price. Highly price elastic responses by low-income households limit the extent to which the subsidy is capitalized, which in turn results in a larger reduction of after-tax rent for high-income households.

Limited capitalization alone, however, does not guarantee that $dN_c^h/d\tau > 0$. As was shown earlier, when there is little difference in city and suburban rents, increasing subsidies result in fewer high-income people choosing city residences. For $dN_c^h/d\tau > 0$, there must also be a large enough differential between r_c and r_s so that the proportional subsidy has a sufficiently large absolute impact on after-tax rents in the city and, hence, provides an incentive for high-income households to locate in the city. Of course, the extent by which r_c exceeds r_s and the extent of capitalization often are not unrelated. Generally, we would expect greater capitalization in areas with large rent differentials because the high relative rent presumably reflects some fixity to land supply, and capitalization should be more complete, the more inelastic the effective supply of land to the rich.

It would take rather unusual preferences for there to be both low capitalization of the subsidy and a large city-suburban rent differential. In particular, it would require the distribution of idiosyncratic preferences among low-income households to be such that a larger number of

low-income households would choose city residences despite relatively high prices, yet these same households would have to adjust their consumption of city housing (either by moving or lowering housing consumption) significantly in response to changes in price. One case that might generate such behavior is an area with a large immigrant population that, for networking and language reasons, is willing to pay a premium to live in the central city. Marginal adjustments to housing consumption by a large number of these immigrant households could keep housing supply to the rich elastic enough that capitalization of the subsidy is relatively low so that increases in the subsidy result in lower after-tax rents for the rich and an increase in demand for city residences by these high-income households.[12]

In sum, the impact of the subsidy on the spatial distribution of high-income households could differ by metropolitan area. For regions in which city rents are not high relative to suburban rents, we would expect an increase in the subsidy to foster decentralization of the rich. For regions in which land costs are very high in the city, this need not be the case. However, even in these areas, the rich are more likely to decentralize the greater the extent to which the subsidy is capitalized into city prices. In general, it seems that the cases in which there are both large differentials in city and suburban rents and low capitalization would be rare.

[12]Adding more housing by building up (i.e., increasing density) also would make for a more elastic effective supply limiting capitalization, and, thereby, narrowing the range over which the rich will wish to choose the suburbs. This is an empirical issue that suggests that the ease with which density can be increased could affect observed behavior across metropolitan areas. As noted above, the theoretical results hold as long as there are diminishing returns to the fixed factor in the model.

4.1.3 Household Land Consumption

Household land consumption depends on the after-tax rent for high-income households and market rents for low-income households. For high-income households, the effect of subsidies on city land consumption is given by

$$\frac{dH_c^h}{d\tau} = H_{cr_{ca}}^h \frac{dr_{ca}}{d\tau} \geq 0. \tag{34}$$

As discussed in section 4.1.1, the subsidy cannot be fully capitalized, so subsidies must lower after-tax rents in the city for high-income households. Thus, housing consumption by high-income households rises with an increase in subsidy.

In the suburbs, because land is elastically supplied, there is no capitalization and, as is evident from equation (20), $dr_{sa} / d\tau = - r_s < 0$. Thus,

$$\frac{dH_s^h}{d\tau} = H_{sr_c}^h \frac{dr_s}{d\tau} > 0. \tag{35}$$

Increased subsidies lower after-tax suburban prices and increase suburban land consumption by high-income households.

Since low-income households do not receive any subsidy and market rents in the city increase with subsidies, increased subsidies must reduce low-income households' land consumption in the city as shown in equation (36):

$$\frac{dH_c^l}{d\tau} = H_{cr_c}^l \frac{dr_c}{d\tau} < 0. \tag{36}$$

Finally, because subsidies do not affect suburban market rents, $dH_s^l / d\tau = 0$.

4.1.4 Aggregate Land Consumption

Aggregate land consumption in the city is fixed at $\overline{L}_c$ and, therefore, is unaffected by subsidies.[13] The effects of subsidies on the amount of suburban land consumed is given by

$$\frac{dL_s}{d\tau} = H_s^h \frac{dN_s^h}{d\tau} + N_s^h \frac{dH_s^h}{d\tau} + H_s^l \frac{dN_s^l}{d\tau} \tag{37}$$

The sign of equation (37) is ambiguous in general.[14] However, if the number of high-income households living in the suburbs increases with subsidies, then total land consumption in the suburbs increases with an increase in subsidy.

4.1.5 Summary

With no constraints in the land market, introducing a subsidy to ownership that is positively correlated with income has the following impacts. First, it unambiguously increases market rents in the city. Second, low-income households that do not benefit from the housing subsidy program and therefore confront higher market rents in the city have an incentive to decentralize to the suburbs. Third, the implication for the location choice of high-income households is ambiguous. Upon closer examination, however, it appears that in most

[13]We assume that there are at least enough households that wish to live in the city at rents equal to their alternative use (r_s) to occupy all available city land.

[14]The ambiguity here derives from the fact that the proportional subsidy reduces the relative price of housing more in absolute terms in areas where the rent is higher. Thus, there are potentially (but as we have argued, unlikely) circumstances in which the subsidy could make the bounded community more attractive if it has a significantly large downward impact on the after-tax price of housing in that community. This result is in the spirit of Blackely and Follain (1983), who, in a monocentric city context, argue that it is possible for the tax treatment of housing to result in greater equilibrium density.

circumstances the tax subsidy will result in greater numbers of high-income households' also choosing suburban locations. In particular, decentralization of richer households is likely in metropolitan areas with the following traits: (a) city rents are not especially high relative to those in the suburbs; and/or (b) capitalization of the subsidy into land prices is extensive. Only in areas with relatively high rents in the city and in which the subsidy is not capitalized would we expect the rich to have a greater incentive to locate in the city. Thus, with no constraints in the land market, it does not appear likely that the subsidy will generate residential sorting by income in this model. If it did, it would leave the rich concentrated in the city, not the suburbs.

If decentralization of the rich does occur, aggregate land consumption in the suburbs increases unambiguously. On a per household basis, an increase in the subsidy clearly leads a rich household to consume more land in the city and the suburbs. For the typical low-skill household, land consumption clearly falls in the city and is unchanged in the suburbs.

Finally, it is noteworthy that the impacts described by this model are symmetric. If low-income households rather than high income households received the subsidy, low income-households' choices would be affected in the same way that high-income households would be if they received the subsidy. That is, if a special tax credit were instituted that was available only to low-income households, high income households unambiguously would have an incentive to suburbanize, and low-income households would, in most circumstances, have an incentive to suburbanize as well. In our model, it is the effect of the subsidy on the relative price of housing that drives behavior, not the incomes of the owners *per se*.

4.2 Case 2: Lot Size Constraints

The second case introduces a common suburban land use restriction in the form of a minimum lot size requirement for residential development. To help simplify the analysis, it is assumed that lot size constraints exist such that no low-skill people choose to live in the suburbs, but the constraints are not binding for high-skill workers.[15] In other words, high-skill workers earn sufficiently high wages that they always choose lots at least as large as the constraint whenever they choose a suburban site. Low-skill workers, on the other hand, have sufficiently low wages that they never choose to purchase a lot as large as the minimum in the suburbs.

These simplifying assumptions imply the following modifications to the comparative statics analyzed in Case 1. Equation (22) is no longer relevant, since low-skill workers never choose to live in the suburbs, and equations (38) and (39) simplify to

$$H_c^h dN_c^h + N_c^h dH_c^h = -N_c^l dH_c^l \tag{38}$$

$$dL_s = H_s^h dN_s^h + N_s^h dH_s^h. \tag{39}$$

The new comparative static result for city rents is given in equation (40):

$$\frac{dr_c}{d\tau} = \frac{H_c^h \psi^h_{\varepsilon^{h*}} \frac{1}{(1-\tau)} \varepsilon^{h*} + N_c^h H^h_{c r_{ca}} r_c}{H_c^h \psi^h_{\varepsilon^{h*}} V^h_{r_{ca}} (1-\tau) + N_c^h H^h_{c r_{ca}} (1-\tau) + N_c^l H^l_{c r_c}} > 0 \tag{40}$$

[15]The model would generate the same qualitative results with a weaker assumption. The only requirement is that zoning precludes some low-skill workers from choosing a suburban location.

Equation (40) is the same as (31') except that it does not include the term representing low-income households' community choice response to a change in city prices. Although it is not possible to directly compare the magnitudes of the rent impact under the two cases because they represent different equilibria, one might be tempted to say that the effect on rents would be greater in the constrained case because low-income households cannot move away from the city and, hence, have only one margin on which to adjust their consumption of city land. This would imply that land would be less elastically supplied to high-income households that received the increased subsidy, and hence, a greater share of the subsidy would be capitalized. This is not the complete story, however, because there also may be more low-income households in the city. Consequently, the aggregate response of households lowering housing demand in response to increasing market prices could exceed the aggregate response in the unconstrained case in which low-income households can adjust both by moving and by reducing housing consumption.

That said, given an existing set of land use restrictions and a corresponding distribution of low-income households, increasing subsidies would result in greater capitalization of the subsidy than would occur conditional on the same distribution of low-income households whose future mobility was unconstrained. Subsidy changes generally take place in the context of land use policies that are long-standing, rather than in the context of shifting from constrained to unconstrained regimes. Therefore, empirically one would expect to see greater capitalization of subsidy changes in areas in which land use policies restrict the mobility of low-income households. Recall from above that greater capitalization implies a higher likelihood that subsidy increases result in fewer high-income households choosing city residences. Further following the logic outlined in the previous section, subsidies to higher income people such as those arising

from the mortgage-interest deduction are more likely to foster the separation of the rich into suburban neighborhoods and poor in city neighborhoods when a land use constraint such as binding large lot zoning exists.

5. DISCUSSION AND CONCLUSION

Our analysis has shown that a public policy subsidizing home ownership differentially along income lines can affect the density of the metropolitan area, as well as influence where rich and poor households choose to live. We have shown that these impacts need not be uniform across metropolitan areas and that the impacts also depend importantly on whether binding land use constraints exist. If none are present, increasing the subsidy definitely leads to decentralization of the poor and probably to the decentralization of the rich. However, there are cases in which the rich would disproportionately choose to live in the city. Such cases can arise when the following two conditions obtain: (a) there are strong idiosyncratic preferences for city life among the low-income residents that can sustain large differentials in city and suburban rents; and (b) aggregate reductions in city land consumption by low-income households in response to market price increases are sufficiently large to prevent significant capitalization of the subsidy. We view this combination of circumstances as relatively unlikely. Consequently, in the absence of land market constraints, tax code-related benefits to home ownership probably lead to the decentralization of both the rich and the poor and, further, seem unlikely to be able to account for the intense residential sorting by income that we see in most metropolitan areas in the United States.

The implications for residential sorting, however, are different when a constraint such as large lot suburban zoning exists. Because city rents tend to increase more with the subsidy in this

case, the rich will tend to have a greater incentive to locate in the suburbs. With the poor stuck in the city because of the large lot zoning limitations, residential sorting by income is likely to be greater because of the subsidy policy as long as there is some elasticity to the demand curve for residential land.

Determining the empirical relevance of this case should be an important issue for future research, as it suggests that policy, not just preferences for less density, may have played a role in shaping America's urban form. Pinning down the influence that price and preference have had not only would be interesting in its own right but certainly would help inform the current debate surrounding urban sprawl and its costs and benefits. Careful measurement of the subsidies themselves, along with estimates of demand elasticities, are needed to determine if policy could have influenced our urban form in a meaningful way. Another useful extension of the model would be to incorporate local amenities that are at least partially determined by the makeup of the local population. Endogenous local amenities could affect prices in a way that encourages sorting by income even more than in our second model. However, we leave that work for the future.

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