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COSTLY TECHNOLOGY ADOPTION

AND CAPITAL ACCUMULATION

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Costly Technology Adoption and Capital Accumulation

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Abstract

We develop a model of costly technology adoption where the cost is irrecoverable and fixed. Households must decide when to switch from an existing technology to a new, more productive technology. Using a recursive approach, we show that there is a unique threshold level of wealth above which households will adopt the new technology and below which they will not. This threshold is independent of preference parameters and depends only on technology parameters. Prior to adoption, households invest at increasing rates, but consumption growth is constant. We also show that richer households adopt sooner and that income inequality increases over time. Both these results are consistent with the evidence from the Green Revolution.

1 Introduction

We study capital accumulation in an environment where technology adoption is costly. We develop a simple model in which a household faces a fixed cost to switch from a less productive technology to a more productive technology. Given the household's initial level of wealth, we examine how the presence of the fixed adoption cost influences the evolution of the household's wealth and the length of time before the household adopts the higher productivity technology.

Several authors have recognized the important role of technology adoption in the process of economic development. Prominent examples of technology adoption and productivity growth episodes include the Industrial Revolution (Mokyr, 1993), the Green Revolution (van Zanden, 1991 and Alauddin and Tisdell, 1991) and the Information Revolution (David, 1990, and Greenwood, 1996). In studying the role of technology adoption, the literature has followed, essentially, two themes: (i) costs and benefits of adoption in a variety of environments and (ii) impact of adoption on macroeconomic variables such as growth, relative wages etc. In the former theme, the decision to adopt a new technology is cast in an environment with rich details on costs and benefits. For instance, Jovanovic and Nyarko (1996) and Pérez-Sebastián (1996) posit a trade-off between the accumulated knowledge in the old technology through learning-by-doing and the risky, unknown, but higher average productivity of the new technology. In the latter theme, the focus is on the behavior of related variables in the economy using relatively simple trade-offs in the technology adoption decision.

For instance, Parente and Prescott (1994), Parente (1994, 1995) and Easterly, King, Levine, and Rebelo (1994) try to explain persistent differences in incomes across countries.

Our model belongs to the latter theme. We subsume the cost of adopting a new technology entirely into an exogenous fixed cost. The benefit of adoption is higher productivity. The fixed cost in our model is the cost of learning the new method of production. For instance, in the context of the Green Revolution, farmers had to learn how to use the new variety seeds with other inputs. Foster and Rosenzweig (1995) document that the learning cost was non-trivial: knowledge about the new seeds was a significant barrier to adopting the high-yield technology during the Green Revolution. Furthermore, Wozniak (1987) finds that the decision to adopt is a human capital intensive activity.

Capital, interpreted broadly to include both physical and human, is the sole factor of production in our model. Both the old and the new technologies are linear.[1] Our aim is to characterize the path of capital accumulation. For infinitely lived households with time separable homothetic preferences, we show that prior to adopting the high productivity technology, capital is accumulated at increasing rates, i.e., the growth rate of capital is monotonically increasing in the level of capital. Once a household reaches a threshold level of wealth, it pays the fixed cost and adopts the new technology. Consumption growth is constant during this period of increasing

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[1] In our model, each household's wealth is its composite physical and human capital stock, so we will treat the terms 'capital' and 'wealth' as synonymous throughout the paper.

investment.

The threshold level of capital at which a household switches to the new technology is independent of the initial level of wealth. Consequently, the duration with the low productivity technology is decreasing in the initial level of wealth. (This is consistent with the findings in Wozniak, 1987, and Alauddin and Tisdell, 1991.) Furthermore, the threshold is independent of preference parameters and depends only on the productivity of the two technologies and the fixed cost of adoption. These technology parameters influence the date of adoption in our model directly through their effect on the threshold level of wealth and indirectly through their effect on the evolution of wealth. Higher productivity of the old technology tends to postpone the adoption date; higher productivity of the new technology induces households to adopt the new technology sooner; and a higher fixed cost postpones the adoption date.

We show numerically that income inequality increases over time along with average capital across households. In our model, the wealthier households take advantage of the more productive technology by adopting it sooner and, hence, benefit proportionately more than the poorer households. Income of all households eventually grows at the same rate, but the levels remain distinct. Our result on increasing inequality while households are engaged in technology adoption is consistent with evidence from rural India (Gaiha, 1987) and the Philippines (Bautista, 1997).

While we derive these results under the assumption that households may neither borrow nor lend amongst themselves, our results are robust to the introduction of

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consumption loans that may be used by low productivity households to indirectly finance the cost of technology adoption. However, we maintain the assumption that low productivity households may not circumvent costly technology adoption by investing with high productivity households.

We also make a methodological contribution in this paper. We set up a sequence of dynamic programs, one for each possible adoption date. Each dynamic program yields a value function that helps us evaluate whether the associated adoption date is optimal. One may use our dynamic programming technique to study a variety of situations where there is a one-time cost. For instance, consider a household deciding whether to migrate from a developing country to a developed country. If there is a one-time fixed cost at the time of migration, then our technique will be useful in determining the wealth accumulation prior to migration and the period in which the household chooses to migrate. Other examples include opening an economy to international trade or implementing a radical change in policy.

In related work, Greenwood and Jovanovic (1990) examine a growth model where agents must pay a one-time start-up cost to avail themselves of the services of financial intermediaries who provide information on the profitability of a risky asset relative to a safe asset. In contrast to their analysis, we derive a unique threshold level of capital at which households adopt the higher productivity technology and show that this threshold is independent of preference parameters. We also explicitly solve for the path of capital accumulation. However, we abstract from issues involv-

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ing uncertainty in the production technologies or intermediation. Bental and Peled (1996) develop a search-theoretic model where technological progress is endogenous. Improvements in technology depend on costly search, and the search process is one of sequential sampling with a fixed cost associated with each sample. Their problem is to determine the optimal *stopping* time, i.e., the number of samples above which one stops looking for improvements in technology. Our problem is to determine the optimal *starting* time, the period when one starts using the better technology. While growth is endogenous in our framework, technological progress is not.

2   The Model

Time is discrete and is indexed by t. Households initially produce output, Y_t, using a technology characterized by a constant marginal product of capital whose value is $A > 1$, i.e., $Y_t = AK_t$, where K_t is the composite physical and human capital stock (or the sum of physical and human wealth), at the beginning of period t. We will model the fixed cost of adoption as a reduction in wealth at the time of adoption, i.e., it costs any household $\lambda > 0$ units of capital to adopt a new technology. This fixed cost λ allows a household permanent access to a more productive technology characterized by a constant marginal product $B > A$.

Any household may be in one of three stages at any point in time: it may be operating the old technology; it might have paid the fixed cost and just adopted the new technology; or it might be operating the new technology, having adopted it in

the past. Let S denote the time at which a given household pays the cost of adoption. Production for this household is then summarized by

$$Y_t = \begin{cases} AK_t & t < S, \\ & \text{if} \\ BK_t & t \geq S. \end{cases}$$

The household faces the following resource constraints conditional on S.

$$
\begin{aligned}
C_t + K_{t+1} &\leq AK_t \text{ for } t = 0, 1 \ldots, S-1, \\
C_S + K_{S+1} &\leq B\left(K_S - \lambda\right), \\
C_t + K_{t+1} &\leq BK_t \text{ for } t = S+1, S+2, \ldots
\end{aligned}
\tag{1}
$$

While capital is assumed to fully depreciate each period, this is without loss of generality, since a depreciation rate may be subsumed into the coefficients A and B.

Households are assumed to discount future utility by the constant $\beta \in (0,1)$, and their preferences are assumed to be time-separable. We assume $\beta A \geq 1$.[2] Utility over consumption in period t, C_t, is given by $(1-\beta)\frac{C_t^{1-\sigma}}{1-\sigma}, \sigma > 0$. (We interpret the $\sigma = 1$ case as logarithmic utility.) The lifetime utility maximization problem for a household identified by an initial level of wealth of K_0 is presented below.

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[2]If $\beta A < 1$, then, as we will see, it is possible that an initially poor household may never choose to accumulate enough capital to pay the fixed cost of adoption.

$$\max_{S,\{C_t,K_{t+1}\}_{t=0}^{\infty}} (1-\beta) \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

$$\text{subject to } (1),$$

$$C_t \geq 0, \ t = 0, 1, ...,$$

$$K_0 \text{ given.}$$

Two remarks are in order. First, our formulation allows for immediate adoption; households may choose to adopt the new technology in period 0. Indeed, we will show that a household with sufficiently large initial wealth will follow exactly this strategy. Second, the fixed cost of technology adoption is closely related to the time interval between periods in our model. For example, suppose it takes five years of time and material expenditures to bring the new technology on line. Then the fixed cost is the present value of these resources, and the time interval between model-periods is five years.

A straightforward but cumbersome two-step approach to solving the above problem is as follows. First, for each $S \geq 0$ where adoption is feasible (in the sense that $K_S > \lambda$), determine the path of capital accumulation given S, both before and after technology adoption. Given the path of capital, determine lifetime utility as a function of S and initial capital. Next, choose the value of S that yields the highest level of lifetime utility. While the first step is a relatively straightforward utility maximization problem, the introduction of the second step requires solving the first

problem an infinite number of times, since, in the absence of any characterization of the properties of the solution, we have to evaluate lifetime utility for each feasible value of S. We circumvent this problem below by following a recursive approach.

3   Households' Decision Rules

3.1   Technology adoption: a recursive characterization

We formulate a set of indirect lifetime utility functions defined over capital, each associated with a particular period for adoption. This allows us to complete the first step of the solution approach described above and determine the path of capital accumulation given a time for, or more accurately a time until, technology adoption. Moreover, we are able to essentially replace the second step by mapping the original problem of locating the optimal S into an equivalent problem of determining a threshold level of wealth at which the household will adopt the new technology.[3]

To characterize the technology adoption problem, it is necessary to know the value of each household's wealth in three distinct scenarios: the value after adoption, the value assuming it never adopts the new technology, and the value given it adopts the new technology in some arbitrary period T. The results we derive here hold for all $\sigma > 0$ (see Appendix 6.2); however, for expositional convenience, we concentrate on the case where $U(C_t) = (1 - \beta) \log C_t$ (i.e., $\sigma = 1$). We assume that households can

---

[3]Horowitz (1993), in a *finite* horizon model of moving from a traditional sector to a modern sector, solves for the optimal switching time directly (instead of deriving the critical level of capital above which it is optimal to switch).

neither borrow nor lend. This assumption is partially relaxed below.

We start our characterization of the technology adoption decision by characteriz-ing households that have already adopted the more productive technology. Let $H(K)$ denote lifetime utility for a household currently using the high productivity technol-ogy that has K units of capital. For such a household, the constraint correspondence

$$\Gamma_H(K) = \{K' \in \mathbf{R}_+ \mid K' \leq BK\}$$

is the set of feasible values for next period's capital, K'. For households not using the more productivity technology, let the constraint set for K' be defined by

$$\Gamma_L(K) = \{K' \in \mathbf{R}_+ \mid K' \leq AK\}.$$

As $B > A$, it follows that for any $K \geq 0$, $\Gamma_L(K) \subseteq \Gamma_H(K)$. Hence, a household currently using the higher productivity technology will never use the original, less productive technology in future. Consequently, H will solve the following Bellman equation:

$$H(K) = \max_{K' \in \Gamma_H(K)} U(BK - K') + \beta H(K'). \tag{2}$$

The existence of an unique H that solves the functional equation (2) follows from Alvarez and Stokey (1998). We denote the optimal policy for next period's capital associated with this dynamic programming problem as $K' = g_H(K)$.

We define the value of capital for a household under the assumption that it never adopts the more productive technology as $L(K)$:

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$$L(K) = \max_{K' \in \Gamma_L(K)} U(AK - K') + \beta L(K'). \tag{3}$$

The policy function for this benchmark is denoted $g_L(K)$. Equations (2) and (3) may be used to verify that

$$H(K) = \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \frac{\log B}{1 - \beta} + \log K, \tag{4}$$

$$L(K) = \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \frac{\log A}{1 - \beta} + \log K. \tag{5}$$

These solutions yield associated policies: $g_H(K) = \beta B K$ and $g_L(K) = \beta A K$.

Using our solution for the lifetime utility function post-adoption, H, allows us to solve for lifetime utility functions pre-adoption, Z, given some number of periods until adoption, T. Specifically, for $T = 0, 1, \ldots$, $Z(K, T)$ describes lifetime utility as a function of capital when technology adoption will occur in T periods; $T = 0$ represents immediate adoption. Since the constraint set during the adoption period is given by $\{K' \in \mathbf{R}_+ \mid K' \leq B(K - \lambda)\} = \Gamma_H(K - \lambda)$, $Z(K, 0)$ must satisfy (6).

$$Z(K, 0) = \max_{K' \in \Gamma_H(K - \lambda)} U(B(K - \lambda) - K') + \beta H(K'). \tag{6}$$

An examination of equation (2) reveals $Z(K, 0) = H(K - \lambda)$. Iterating backward, the period prior to technology adoption we have

$$Z(K, 1) = \max_{K' \in \Gamma_L(K)} U(AK - K') + \beta Z(K', 0). \tag{7}$$

We note that before the adoption period the constraint set is given by $\Gamma_L(K)$. Generally, $Z(K, T+1)$ is defined recursively using $Z(K, T)$,

$$Z(K, T+1) = \max_{K' \in \Gamma_L(K)} U(AK - K') + \beta Z(K', T). \tag{8}$$

We now solve the time-until-adoption value functions $Z(K, T)$. The function $Z(K, 0)$ is easily determined using the solution for H in (4),

$$Z(K, 0) = \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \frac{\log B}{1 - \beta} + \log(K - \lambda), \tag{9}$$

and implies that $g_Z(K, 0) = \beta B(K - \lambda)$. Next, backward induction using (8) for $T = 1, 2, \ldots$, yields

$$
\begin{aligned}
Z(K, T+1) &= \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \frac{\beta^{T+1} \log B}{1 - \beta} \\
&\quad + \frac{\beta(1 - \beta^T)}{1 - \beta} \log A + \log\left(AK - \lambda A^{-T}\right)
\end{aligned} \tag{10}
$$

and the associated optimal policies

$$g_Z(K, T+1) = \beta AK + \frac{1 - \beta}{A^T} \lambda. \tag{11}$$

Thus far, we have essentially restated the approach to determining S, described in the previous section, in terms of an equivalent problem of determining T. Our determination of lifetime utility prior to adoption as a function of both K and T, $Z(K, T)$, has completed the first step of the approach to characterizing the technology

adoption problem. If we were to now implement the second step and determine the optimal period of adoption, S^*, we would have to find the optimal number of periods to wait until technology adoption, T^*, that satisfies $Z(K, T^*) \geq Z(K, T)$, for $T = 0, 1, 2, ...,$ given K. As we have already noted, this approach is cumbersome, requiring an infinite number of function evaluations. Instead, we determine the threshold level of wealth, K^*, at which each household will adopt the new technology and use the accumulation behavior implied by (11) to determine T^* as the time taken to reach the threshold level of capital starting from an initial capital stock of K_0.

There are obvious bounds on the threshold level of capital. For instance, if a household's capital is less than λ, the household cannot adopt the new technology. This does not imply, however, that the household will switch as soon as it reaches a level of capital above λ. To see this, consider a household with λ units of capital. If this household never adopts the new technology, its lifetime utility is $L(\lambda)$. If it decides to adopt the new technology, then from (4) and (5) it is clear that $\lim_{K \to \lambda} Z(K, 0) < L(\lambda)$. By continuity, the household will not adopt even if it has capital close to but greater than λ.

An upper bound on the threshold level of wealth may be found as follows. Let $\overline{K}$ be such that the output, net of fixed cost, under the new technology is the same as the output in the old technology, i.e., $B(\overline{K} - \lambda) = A\overline{K}$ or $\overline{K} = B\lambda/(B - A)$. (See Figure 1.) A household with capital $K \geq \overline{K}$ will immediately adopt the new technology. This follows from a simple observation: any consumption sequence that

is feasible when the household operates the old technology for $T \geq 1$ periods and then switches to the new technology is also feasible if the household adopts the new technology immediately. However, the converse is not true. Hence, the household is better off switching to the new technology if its capital is $K \geq \overline{K}$.

So far we know that the threshold level of capital lies in $[\lambda, \overline{K}]$. It turns out that there is a unique threshold level of wealth above which all households will adopt the higher productivity technology and below which they will not.

Proposition 1 *Let $K^* = \frac{B-1}{B-A}\lambda$. A household will adopt the new technology if and only if $K \geq K^*$.*

The proof is in the Appendix.[4] Briefly, our proof has three steps. First, we show that a household with capital K^* or higher will immediately adopt rather than wait one more period. Second, we prove that at this threshold, the household will adopt immediately rather than wait any *finite* number of periods. Finally, we establish that the household would rather adopt the new technology than never adopt, when its wealth exceeds the threshold.

Clearly, $K^* \in (\lambda, \overline{K})$ since $A > 1$. Furthermore, the threshold level of wealth is decreasing in B, increasing in A, and increasing in λ. Note that, in contrast to Greenwood and Jovanovic (1990), the threshold level of capital in our model is unique. Below, we examine the implication of these properties for the length of time until adoption and the evolution of each household's wealth.

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[4]Section 6.2 of the Appendix proves that this threshold level of capital K^* is independent of σ.

3.2 Evolution of households' wealth

The evolution of wealth after adoption is completely described by the policy function $g_H(K) = \beta BK$. The path of wealth prior to adoption depends on when the new technology is adopted (see equation (11)). However, since we know the threshold level of wealth at which households adopt the new technology, we are able to compute the time taken to reach K^*, given an initial wealth K_0, using (11).

Suppose that a household plans to adopt the new technology in period S and has K_0 units of initial capital. Then, setting $T = S - t$ in (11), we have

$$K_t = \frac{(1-\beta)\lambda}{A^{S-t}} + \beta A K_{t-1}, \quad t = 1, \ldots, S. \tag{12}$$

Iterating backward to period 0, we can derive the path of capital from period 0 to period S:

$$K_t = \frac{(1-\beta^t)\lambda}{A^{S-t}} + (\beta A)^t K_0, \quad t = 1, \ldots, S. \tag{13}$$

Define this household's growth rate of wealth pre-adoption to be $\gamma_t \equiv \frac{K_{t+1}}{K_t}$, $t = 0, 1, \ldots, S - 1$. Using (12) and (13), we have

$$\gamma_t = \frac{(1-\beta)\lambda A}{\lambda + \beta^t (A^S K_0 - \lambda)} + \beta A.$$

It is easy to see that γ_t is rising over time. First, starting from any K_0, the highest level of capital that a household can possibly have in period S is $A^S K_0$; hence, $K_S \leq A^S K_0$. For S to be the adoption period, K_S must be greater than λ; so it

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follows that $A^S K_0 - \lambda > 0$ is a necessary condition. Second, since $\beta < 1$, β^t is a decreasing function of t. Hence, $\gamma_t > \gamma_{t-1} > \gamma_{t-2} > ... > \gamma_0$.

The increasing growth rate of capital implies that the investment rate, $\frac{K_{t+1} - K_t}{AK_t}$, is also rising over time. This may be explained as follows. While a household would like to access the high productivity technology soon, it would also rather have more current consumption than more future consumption. Consequently, the investment rate is low in the earlier periods and high in the later periods. A household that begins with K_0 units of capital invests at an increasing rate until it reaches the threshold K^*, pays the fixed cost of λ units of capital and switches to the new technology. We thus have the following proposition:

Proposition 2 *Prior to technology adoption by a household, the growth rate of its capital and the investment rate are both monotonically increasing over time.*

We can use (13) to determine the adoption period. Assuming that the switching period is S, the wealth at the beginning of period S is given by

$$K_S = (1 - \beta^S)\lambda + (\beta A)^S K_0.$$

For S to be the switching period, we must have $K_S \geq K^*$. By replacing K_S with K^* on the left-hand side of the above expression, we may solve for $\widehat{S}$ and set S as the next largest integer. Informally, graphing the two sides of

$$K^* = (1 - \beta^S)\lambda + (\beta A)^S K_0 \tag{14}$$

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as functions of S, the intersection point gives us the switching period (subject to an integer constraint). This is illustrated in Figure 2. While S cannot be solved analytically, a numerical solution to the nonlinear equation (14) is easy to obtain. This numerical solution is critical for computing the evolution of capital for each household. Once we solve for S, the entire path of capital accumulation in the pre-adoption state can be determined by using (13). Our recursive approach relies on the result that we have a unique threshold level of capital above which households adopt the higher productivity technology.

An implication of (14) is that the switching period depends on the initial wealth. Since the threshold does not depend on the initial level of capital, a lower K_0 implies a higher S for equation (14) to be satisfied. The threshold level of capital is determined by the technology parameters, A, B, and λ. Poorer households, starting with a lower level of initial capital, will take longer to reach K^* even if they accumulate capital at the same rate as wealthier households. In fact, in our setup, poorer households save a smaller fraction of their income relative to the rich households (see Proposition 2). Hence, the new technology is adopted at a later date by households that are relatively poor. This is in contrast to the two-period model of Eswaran and Kotwal (1989) where lack of access to loans is the key reason why poor farmers adopt later. (For more details on the effect of loans in our model, see section 3.3.)

Proposition 3 *The lower is the level of initial household capital, the is later the adoption date.*

Proposition 3 is consistent with the findings in Wozniak (1987), given our interpretation of K as a mix of physical and human capital. He finds that the technology adoption decision is a human capital intensive activity and that early adopters tend to be those with high levels of human capital.

Next, we study the effect of changes in the underlying parameters upon the switching period. Combining equation (14) with Proposition 1, we have

$$\frac{B-1}{B-A}\lambda = (1 - \beta^S)\lambda + (\beta A)^S K_0. \tag{15}$$

where the left-hand side is, of course, the threshold level of capital, K^*. A higher level of post-adoption productivity $B' > B$ accelerates the pace of technology adoption. First, as noted in the previous section, B' implies a lower threshold than B. Second, the path of wealth accumulation in the pre-adoption state does not depend on the productivity of the new technology. That is, the left-hand side of (15) is lower but the right-hand side remains the same. Hence, the time until adoption is reduced.

Now consider the effect of a higher fixed cost of adoption, $\lambda' > \lambda$. For each unit increase in λ, the left-hand side of (15) increases by $\frac{B-1}{B-A} > 1$ while the right hand side increases by $1 - \beta^S < 1$. Thus, the switching period has to increase to satisfy equation (15). The intuition for this is fairly straightforward. A higher fixed cost raises the barrier to adoption, so starting from the same initial condition households adopt later. Clearly, earlier adoption in this case would suggest that households' behavior was not rational when the fixed cost was lower.

We summarize our results in the following proposition.

Proposition 4 *A more productive new technology implies faster technology adoption while higher adoption costs delay technology adoption.*

Finally, consider the effect of higher productivity in the initial technology, i.e., $B > A' > A$. The reduced net return to adoption suggests a delay in the time of adoption. However, the rise in pre-adoption productivity implies that while the threshold wealth for adoption rises, so does the savings rate prior to technology adoption. Consequently, the path of wealth accumulation shifts up. Overall, it is unclear whether the switching period falls or rises. Figure 2 illustrates the switching time for a household using a simple numerical example. It is clear from the figure that households adopt the new technology later under A' than under A. We have found this result to be robust to changes in parameters.

A notable feature of our model is that household consumption and wealth do not grow at the same rate prior to adoption. To see this, consider a household that plans to adopt the new technology in period S and has K_0 units of initial capital. From the resource constraint (1) we know that the gain to giving up a unit of consumption in period 0 is A units of consumption in period 1. In utility terms, we have the following Euler equation:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} A, \ \ t = 0, \ldots, S - 2.$$

During these periods, consumption growth is constant and equal to βA. Proposition 2, however, showed that the rate of growth of wealth is increasing over time during this period.[5] The contrast in growth rates is especially stark when one considers the case $\beta A = 1$: the *level* of consumption stays constant, but wealth exhibits growth. The reason for saving despite the low rate of return is that even though the 'current' return to savings is low, the 'anticipated rate of return' to savings is high. Households take this into account when making their intertemporal decisions.

Post-adoption our model implies that the rates of growth of capital and consumption are equal. When the household reaches period $S-1$, the gain to giving up a unit of current consumption is B units of future consumption. Thus, the Euler equation is

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} B, \ \ t = S-1, S, ...$$

Once technology adoption is completed, consumption and investment are constant proportions of the current stock of capital.

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[5]Models with minimum consumption requirements (such as Chatterjee and Ravikumar, 1999) imply an increasing rate of consumption growth and an increasing grwoth rate of wealth. For instance, suppose that $U(C_t) = (1-\beta)\log(C_t - \underline{C})$, $\underline{C} > 0$, and that the rate of return to saving is constant. Then, as households accumulate capital they move away from $\underline{C}$ and the growth rates of both consumption and wealth increase.

3.3 Loans

As we have stressed, the form of technology adoption we are examining is a human capital intensive activity. Hence, access to loans may not directly help a household adopt the high productivity technology. However, households may be able to adopt the technology sooner if they have other means of financing *consumption*. Our analysis above has not allowed for this possibility. We relax this assumption now.

Suppose the households that have not yet adopted the high productivity technology could obtain consumption loans from the households that have adopted the high productivity technology. Since the gross rate of return on the new technology is B, the lowest possible interest rate acceptable to the high productivity households is $r = B - 1$.[6] If a household decides to borrow, the only reason for doing so is to switch to the new technology immediately since $r > A - 1$. Consider a household with $K_0 > \lambda$ units of capital borrowing $B\lambda$ units of goods and repaying the loan in constant amounts starting next period. At the interest rate r, the per period payment must equal $rB\lambda$. In period 0, the household uses the borrowed resources $B\lambda$ to finance consumption. From period 1 on, the household repays the loan by the constant amount $rB\lambda$ each period. Thus, the household's budget constraint is

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[6]The presence of fixed cost prevents us from aggregating heterogenous households into a representative household. Hence, in contrast to Becker (1981, 1982), we cannot exploit the equivalence between optimality and competitive equilibrium in order to derive the implications for *equilibrium* interest rates.

$$C_0 + K_1 \leq B(K_0 - \lambda) + B\lambda,$$

$$C_t + K_{t+1} \leq BK_t - rB\lambda \text{ for } t = 1, 2, \ldots.$$

Define $y \equiv \frac{rB\lambda}{B-1}$, so at the interest rate $B - 1$, $y = B\lambda$. We can then modify the household's budget constraint as follows:

$$C_0 + K_1 - y \leq B(K_0 - \lambda),$$

$$C_t + K_{t+1} - y \leq B(K_t - y) \text{ for } t = 1, 2, \ldots.$$

Define $\widetilde{K}_t \equiv K_t - y$ for $t = 1, 2, \ldots$. The above constraints, restated using $\widetilde{K}_t$, allow us to reformulate this household's problem: it begins with $\widetilde{K}_0 = K_0 - \lambda$ units of capital and chooses a sequence $\widetilde{K}_1, \widetilde{K}_2, \ldots$ subject to $\widetilde{K}_{t+1} \in \Gamma_H\left(\widetilde{K}_t\right)$. It follows that the household values this at $H(\widetilde{K}_0) = H(K_0 - \lambda) = Z(K_0, 0)$ using (6). That is, the access to loans potentially helps the household immediately adopt the new technology and realize the value $Z(K_0, 0)$.

The surprising result is that access to such loans does not make the borrowing household better off. To see this, suppose that the initial level of capital is less than the threshold, i.e., $K_0 < K^*$. In this case, the household would be better off by ignoring the loan markets and waiting one or more periods before switching, since $Z(K_0, 0) < Z(K_0, 1)$ for $K_0 < K^*$ (see Lemma 5 in the Appendix). If $K_0 \geq K^*$, the household would indeed switch immediately, but it would have done so even without

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access to loans (see Proposition 1). Thus, the path of wealth and the threshold level of wealth in our model are robust to the introduction of loans to help low productivity households accelerate their technology adoption. To understand this result, note that the per period return on investment for a household operating the low productivity technology exceeds $A - 1$ but is less than $B - 1$. Clearly, such a household will not be willing to borrow at the interest rate $B - 1$.[7]

While we have shown that allowing for consumption loans by poor households does not affect their timing of technology adoption or accumulation of capital, we have not considered the possibility of lending by these households. In the absence of capital market imperfections, households without direct access to the high productivity technology would prefer to invest in the production of those already operating the high productivity technology instead of engaging in costly technology adoption. This in turn would imply that technology adoption occurs only at the wealthiest households, and all remaining households abandon production to become outside lenders. However, such a pattern of adoption would clearly contradict the empirical evidence that during the Green Revolution, poor households, alongside wealthier households, engaged in technology adoption, though they adopted later. (See, for example, Alauddin and Tisdell, 1991). Other evidence includes Jovanovic and Mac-Donald (1994), who study the diffusion of Diesel locomotives and Dekimpe, Parker

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[7]If relatively wealthy households faced any uncertainty regarding loan repayments from poorer households, the interest rate would rise above $B - 1$. Wealthier households would demand a premium to compensate for the default risk. Clearly, the poor households will not borrow in such circumstances as they chose not to do so at an interest rate of $B - 1$.

and Sarvary (1997), who study adoption of cellular phone technology.

Similar to models in the diffusion literature, our model implicitly assumes a capital market imperfection: low income households are unable to invest with high productivity households. We suggest the following basis for this assumption. If low productivity investors have no ability to ensure that their loans to high productivity households will be repaid, the unenforceability of loan contracts would eliminate borrowing and lending altogether in our model. There is, in fact, considerable evidence that legal protection of investors' rights varies across economies. For example, La Porta, Lopez-de-Silanes, Shleifer and Vishny (1998) examine rules protecting investment across 49 countries and find significant differences in the legal protection of investors. One interpretation of their work is that, all else being equal, poorer economies offer weaker investor protection. In fact, the insecurity of economic rights due to arbitrary decision-making by government and the uncertainty of legal and contractual rights are believed to be particularly acute, with adverse effects on the availability of investment, in many developing economies (World Bank, 1987, p.62).

In the context of our model, the interest rate, r, on loans from low productivity to high productivity households must have the property that $r \geq B - 1$ to ensure finite demand for such loans. However, at these interest rates, high productivity households are always better off defaulting on loans if they are able to do so. For such households, borrowing at interest rates bounded below by $B - 1$ does not offer any positive net benefit. Therefore, given any level of investment, a high productivity household

would retain all output and refuse to repay the investment loan. (This result holds even if we assume that a borrower who defaults on loans will be credibly excluded from subsequent borrowing.) Consequently, low productivity households would be unwilling to lend to high-productivity households in the absence of a commitment technology that ensured repayment of loans. Such commitment devices are likely to be absent in less developed economies, and thus our model is most applicable to such environments.

4   Income Inequality

As discussed in the previous section, a crucial assumption in our model is the lack of institutions that channel the endowments of those operating the low productivity technology to those operating the high productivity technology. While this may be empirically justifiable in the context of a developing country during the Green Revolution, this assumption has implications for the evolution of income inequality that may provide another test of our model. Suppose that the initial level of wealth was different across households. Then, as noted in propositions 2 and 3, wealthier households will grow faster in the pre-adoption stage and adopt the high productivity technology earlier. In the post-adoption stage, all households grow at the same rate. In figure 3, we illustrate path of capital for different levels of initial capital. The technology adoption decision is evident in the ridge that is a result of the fixed cost of technology adoption and the high rates of capital growth displayed by households

as they approach the adoption decision. Examining the period of adoption as a function of the initial level of capital, we see that wealthier households will adopt earlier. After adoption, the rise in the return to savings as a result of the high productivity technology generates faster increases in wealth. Since the rate of growth of wealth is initially increasing in the level of wealth and subsequently independent of the level of wealth, the poor never catch up with the rich in our model. Thus, while all households eventually achieve the same growth rate of income, their *levels* of income remain distinct.

In Figure 4, we illustrate the evolution of income inequality over time. We use the Lorenz Dominance criterion to rank income distributions in different periods. (See Nygard and Sandstrom, 1981, for a definition of Lorenz Dominance.) Over time, the Lorenz curve moves away from the 45-degree line, indicating that income inequality is increasing over time.[8] In Figure 5, we summarize income inequality in each period with a commonly used index, the Gini coefficient, and plot its evolution over time. The top panel contains the log of mean income across households, so the slope at any point in time is the growth rate of average income. The bottom panel contains the Gini coefficient. Note that income inequality is increasing as the average capital increases. This is consistent with several studies on income distribution and agricultural productivity growth. For instance, Gaiha (1987) documents that during

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[8]Since average income (across households) in our economy is changing over time, Generalized Lorenz Dominance (Shorrocks, 1983) may be a more appropriate way to rank income distributions. However, we focus only on positive growth, so Lorenz Dominance and Generalized Lorenz Dominance yield the same ranking of distributions.

the Green Revolution income inequality increased in rural India since large cultivators benefited disproportionately from the new, more productive technology. Bautista (1997) finds that during the period of dramatic growth in rice yield in the Philippines (1965-80) proportionately larger income benefits accrued to large-farm than small-farm households, resulting in an increase in income inequality.

5   Concluding Remarks

We have developed a model of costly technology adoption where the cost is irrecoverable and fixed. The path of wealth in this model has been characterized using a recursive approach. Specifically, we formulated a sequence of value functions, one for each possible adoption date, and showed that there is a unique threshold level of wealth at which households adopt the more productive technology. We find that this threshold is independent of initial wealth and depends only on technology parameters. Hence, wealthier households undertake earlier technology adoption. Prior to adoption, households' savings rates rise until they reach the threshold level of wealth, but the rate of consumption growth remains constant. Our results on technology adoption and capital accumulation hold even if the households that have already adopted the high productivity technology could indirectly finance the adoption cost of other households.

A crucial assumption in our model is the lack of institutions that channel the endowments of those operating the low productivity technology to those operating

the high productivity technology. Given that some farmers had already adopted the high productivity technology during the Green Revolution, an interesting question is why the remaining farmers did not rent out their factors of production. In future research, we plan to follow the approach in Atkeson (1991) and Phelan (1995) and examine whether lack of contract enforcement helps explain why the low productivity households do not lend to the high productivity households.

6 Appendix

6.1 Proof of Proposition 1

Our proof of Proposition 1 proceeds along the three steps outlined in the text.

Lemma 5 *For a household with $K \geq K^*$, it is better to switch immediately rather than wait one more period. Formally, $Z(K,0) \geq Z(K,1) \iff K \geq K^*$.*

Proof. The functional equations (9) and (7) imply that

$$
\begin{aligned}
Z(K,0) &\geq Z(K,1) \Leftrightarrow \log B + \log(K - \lambda) \geq \log(AK - \lambda) \\
&\Leftrightarrow B(K - \lambda) \geq AK - \lambda.
\end{aligned}
$$

The last inequality is true for $K \geq K^*$. ∎

Lemma 6 *If a household has $K \geq K^*$, it is better to switch now than wait a finite number of periods, i.e., for $K \geq K^*$, $Z(K,0) \geq Z(K,T)$, $T = 2,3,....$*

Proof. Suppose that the household prefers to switch T periods hence, instead of switching immediately. Note from the optimal policy (11) that the growth rate of wealth exceeds βA. Since $\beta A \geq 1$ by assumption, the wealth $T - 1$ periods hence will be greater than K^*. By Lemma 5 it is optimal to switch in period $T - 1$ rather than wait until period T. A similar argument applies to $T - 2$ versus $T - 1$. Working backward, it is easy to see that it is optimal to switch immediately. ∎

Lemma 7 *If a household has $K \geq K^*$, it is better to switch to the new technology now than never switch, i.e., for $K \geq K^*$, $Z(K, 0) \geq L(K)$.*

Proof. The functional equation (10) implies that

$$\lim_{T \to \infty} Z(K, T) = \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \frac{1}{1 - \beta} \log A + \log K$$

since $\beta \in (0, 1)$ and $A > 1$. Note that the right-hand side is the same as $L(K)$. Thus, the sequence of functions $Z(K, T)$ converges to the function $L(K)$. Now, suppose for $K \geq K^*$, $Z(K, 0) < L(K)$. Then, there exists a T sufficiently large such that $Z(K, 0) < Z(K, T)$. This contradicts Lemma 6. ∎

It is clear from lemmas 5, 6, and 7 that K^* is the threshold level of wealth, as stated in Proposition 1.

6.2 Isoelastic Preferences

Before establishing our results for the isoelastic case, we define $s_A = (\beta A^{1-\sigma})^{\frac{1}{\sigma}}$ and $s_B = (\beta B^{1-\sigma})^{\frac{1}{\sigma}}$. When $U(C) = (1 - \beta)\frac{C^{1-\sigma}}{1-\sigma}$ where $\sigma > 0$ and $\sigma \neq 1$, we may repeat the approach of section 3 to obtain

$$H\left(K\right) = \left(\frac{1-\beta}{1-\sigma}\right)\frac{(BK)^{1-\sigma}}{(1-s_B)^{\sigma}}, \tag{17}$$

$$L\left(K\right) = \left(\frac{1-\beta}{1-\sigma}\right)\frac{(AK)^{1-\sigma}}{(1-s_A)^{\sigma}}, \tag{18}$$

with $g_H\left(K\right) = s_B BK$ and $g_L\left(K\right) = s_A AK$. Thus, we see that s_A represents the savings rate under the plan which permanently uses the low productivity technology while s_B is the rate of savings under the high productivity program. We assume that $s_A A > 1$ and $s_B B > 1$. Following the approach taken for the log case, we are able to derive

$$Z\left(K,0\right) = \left(\frac{1-\beta}{1-\sigma}\right)\frac{\{B\left(K-\lambda\right)\}^{1-\sigma}}{(1-s_B)^{\sigma}}, \tag{19}$$

$$Z\left(K,1\right) = \left(\frac{1-\beta}{1-\sigma}\right)\frac{(AK-\lambda)^{1-\sigma}}{(1-s_B)^{\sigma}} \tag{20}$$

and $g_Z\left(K,0\right) = s_B B\left[K-\lambda\right]$ and $g_Z\left(K,1\right) = s_B AK + (1-s_B)\lambda$.

Finally, as in section 3, we now solve for the family of value functions, $Z(K,T+1)$. For notational brevity, define

$$X_{T+1} \equiv \frac{\left(1-s_A^T\right)(1-s_B) + s_A^T\left(1-s_A\right)}{1-s_A}, \quad \text{for } T = 0,1,...$$

Then,

$$Z\left(K,T+1\right) = (1-\beta)\left(\frac{X_{T+1}}{1-s_B}\right)^{\sigma}\frac{\left(AK-\lambda A^{-T}\right)^{1-\sigma}}{1-\sigma} \tag{21}$$

$$g\left(K,T+1\right) = \frac{s_A X_T AK + (1-s_B)\lambda A^{-T}}{X_{T+1}}.$$

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Note that $\lim_{T\to\infty} Z(K, T+1) = L(K)$ and $\lim_{T\to\infty} g(K, T+1) = g_L(K)$. We will show that the threshold value of capital, K^*, is unchanged relative to that which defined the level of wealth associated with technology adoption for the case of logarithmic utility. Before we proceed, it is useful to show that the capital stock is monotonically rising over time in the pre-adoption state.

Lemma 8 *For all $\sigma > 0$, given $s_A A > 1$, $g(K, T+1) > K$, $T = 0, 1, ...$*

Proof. When $\sigma < 1$, as $B > A$, $s_B > s_A$ or $1 - s_B < 1 - s_A$. Since $0 < s_A < 1$, this implies that $\left(1 - s_A^T\right)(1 - s_B) + s_A^T(1 - s_A) < \left(1 - s_A^{T-1}\right)(1 - s_B) + s_A^{T-1}(1 - s_A)$ or $X_{T+1} < X_T$. Therefore $g(K, T+1) > s_A A K > K$.

When $\sigma > 1$, the ratio $\frac{X_T}{X_{T+1}}$ is increasing in T. At its lowest value, when $T = 0$, we have $g(K, T+1) > s_A \frac{X_0}{X_1} A K = s_B A K > K$. $\blacksquare$

Note that our additional assumption $s_A A > 1$ is sufficient, but not necessary, to prove that capital rises over time. However, given our result that the stock of capital is growing over time, independently of T, we are able to apply lemmas 5-7 for the case of isoelastic utility.

Proposition 9 *The high yield technology is adopted if and only if $K \geq K^*$.*

Proof. A direct comparison of $Z(K, 0)$ and $Z(K, 1)$ indicates that the threshold K^* is unchanged. Next lemma 8 shows that capital is growing over time, independently of T. This implies that lemma 6 continues to apply and the household would rather adopt immediately than wait any finite number of periods. Finally, since

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$\lim_{T \to \infty} Z(K, T+1) = L(K)$, we can apply lemma 7 as well, so the household would rather adopt immediately than never adopt. ∎

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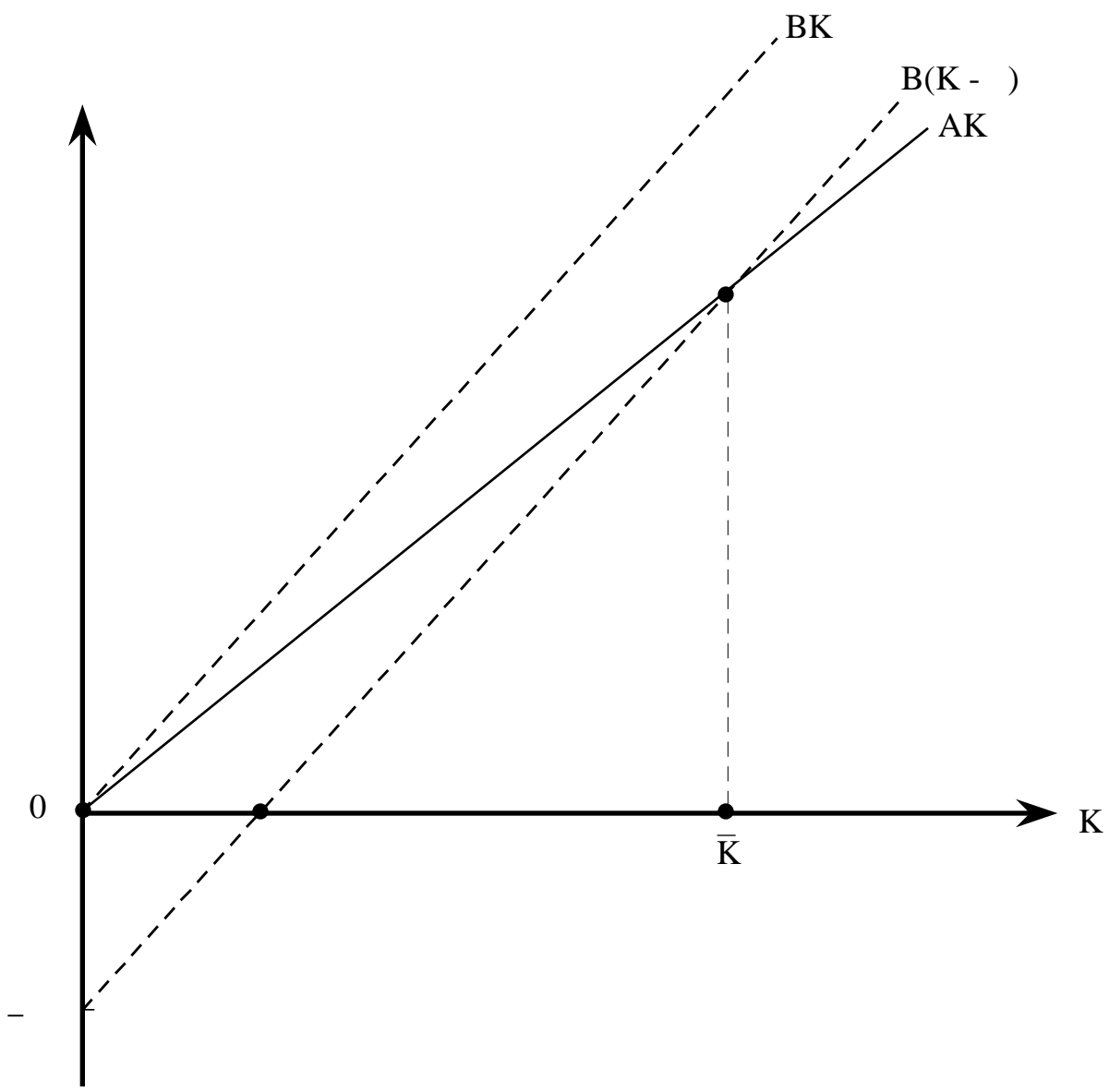
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Figure 1. Bounds on the threshold capital

Figure 2. Low productivity technology and the period of adoption
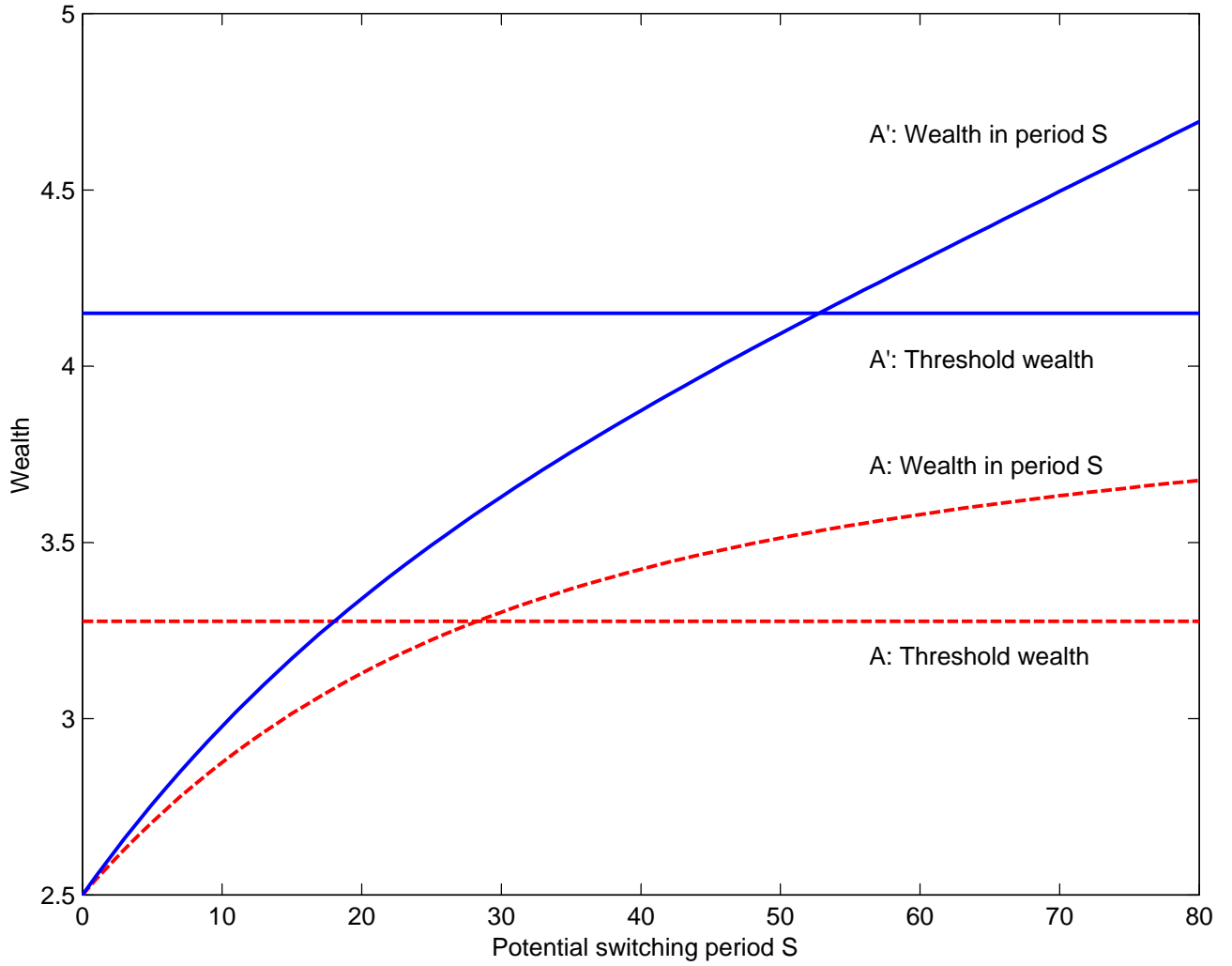
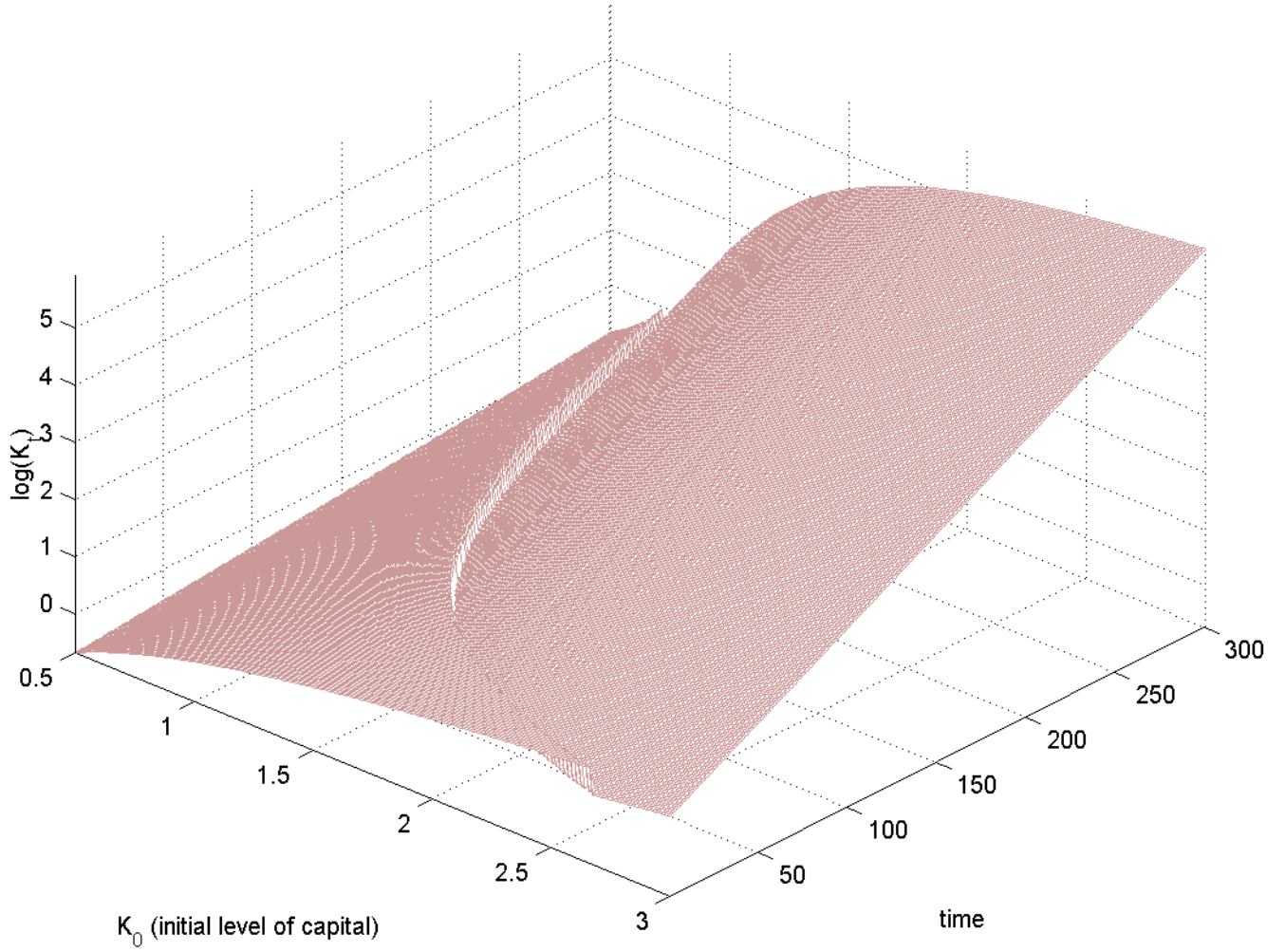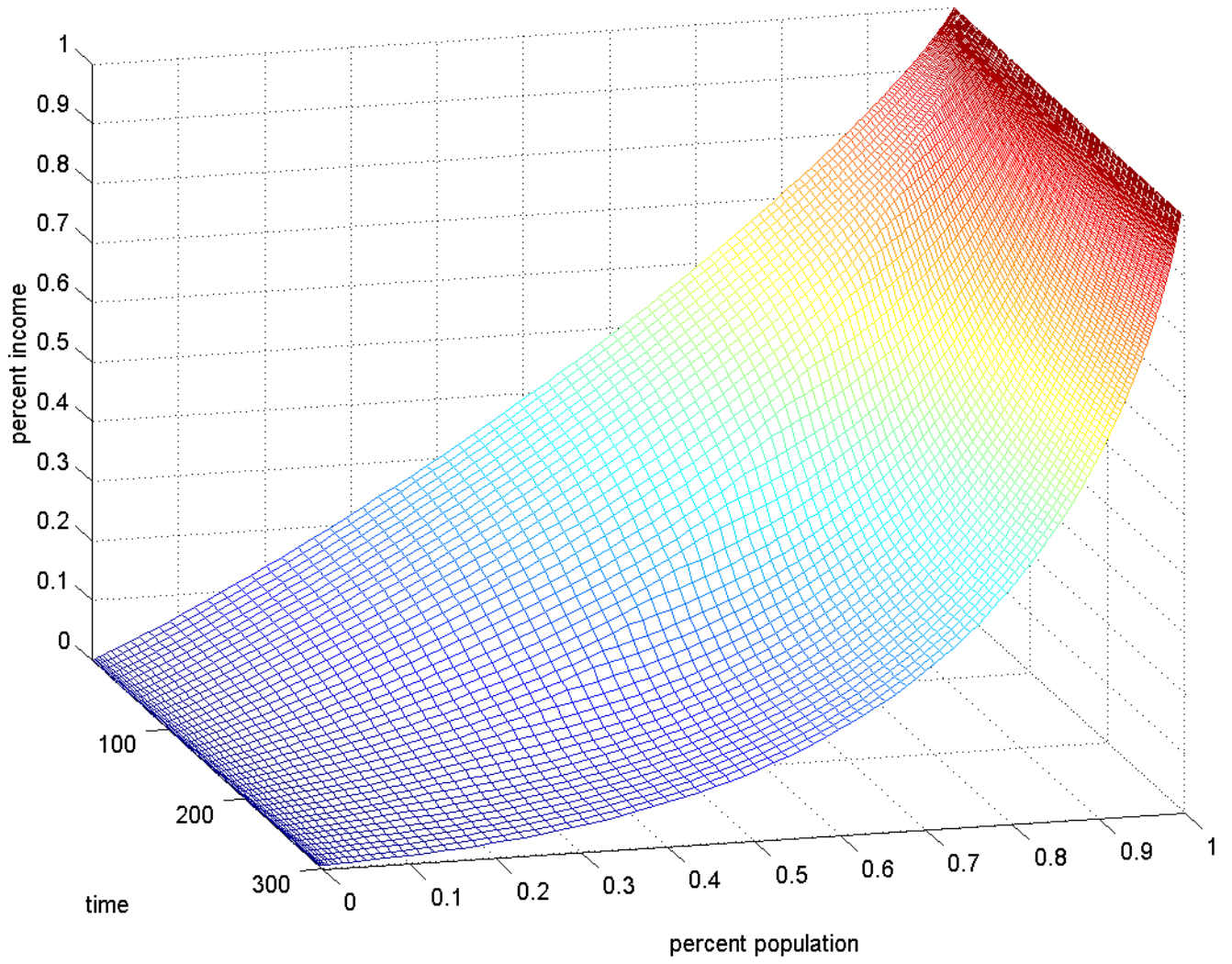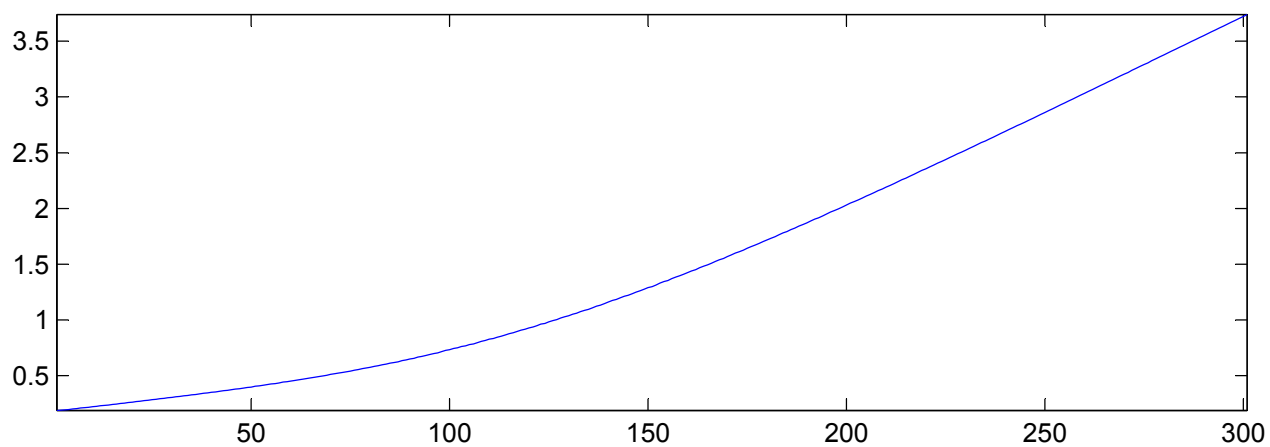Figure 3: Capital Accumulation

Figure 4: Lorenz Curves

Figure 5A: logarithm of mean income

Figure 5B: Gini Coefficients