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Specific Capital and Vintage Effects on the Dynamics of Unemployment and Vacancies*

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Abstract

In a reasonably calibrated Mortensen and Pissarides matching model, shocks to average labor productivity can account for only a small portion of the fluctuations in unemployment and vacancies (Shimer (2005a)). In this paper, I argue that if vintage specific shocks rather than aggregate productivity shocks are the driving force of fluctuations, the model does a better job of accounting for the data. I add heterogeneity in jobs (matches) with respect to the time the job is created in the form of different embodied technology levels. I also introduce specific capital that, once adapted for a match, has less value in another match. In the quantitative analysis, I show that shocks to different vintages of entrants are able to account for fluctuations in unemployment and vacancies and that, in this environment, specific capital is important to decreasing the volatility of the destruction rate of existing matches.

JEL Classification: E24, E32, J41, J63, J64

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1 Introduction

The Mortensen and Pissarides (MP) matching model has become the dominant framework for studying a variety of labor market issues. Despite its widespread use, the MP model encounters problems in accounting for the cyclical properties of its two key variables, unemployment and

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vacancies. In most papers, aggregate labor productivity shocks have been used as the source of fluctuations in these variables, which has resulted in two major problems. One problem, as argued by Shimer (2005), is that when the model is reasonably calibrated, shocks to aggregate productivity can account for only a small portion of the volatilities in unemployment and vacancies. The second problem is that an MP model with only aggregate productivity shocks generates a very high correlation between vacancies per unemployed worker and average labor productivity, while in the data the correlation is much smaller (close to zero for the period 1986-2005).

In this paper, I show that the addition of specific capital, which has less value in another match once it is already employed in a match, and shocks to the embodied technology of matches that affect only entrants in any given period successfully resolve both problems. In the model, a production unit (a match) is composed of a firm, a worker, and the capital employed in the match. Every match embodies a technology level determined at the time of its creation. This is a plausible representation if investment opportunities available at the time of creation of any match affect the technology of the match permanently. In this setting, part of economic growth is driven by the replacement of old, low-productivity jobs with new high-productivity ones.[1] The embodied technology framework is consistent with the micro-level evidence that firm level productivity is highly persistent and that around half of aggregate productivity growth happens through reallocation of workers across firms.[2] More important, shocks to new vintages are now a significant source of fluctuations.

The inclusion of shocks to the embodied technology of entrants solves the problem related to volatilities because these shocks have a large impact on vacancy postings (and thus on the unemployment rate) and a small impact on average labor productivity, both of which are consistent with the data. A shock to the embodied technology of new entrants affects the productivity of new entrants permanently, while the effects of an aggregate shock (calibrated to aggregate productivity data) is persistent but not permanent. This implies that shocks to embodied technology have a bigger impact on profits for entrants and so on vacancy postings. In addition, because these shocks affect only new entrants and leave the productivity of existing matches unchanged, their effect on average labor productivity is negligible.

The second important element in the model, specific capital, improves the performance of the model because it controls the rate of destruction of existing matches. This feature is important to matching the strong negative correlation between vacancies and unemployment, also known as the

[1] This structure does not exclude the possibility of growth in the productivity of existing matches. It is reasonable to think that some technological upgrades might necessitate a new match, while some others can be adopted within an existing match. In the quantitative analysis I will assume that only a portion of the growth in the macroeconomy happens through the replacement process.

[2] Bartelsman and Dhrymes (1998) show that there is a large degree of persistence in productivity ranking and that a significant part of the economy's productivity growth accrues by means of resource reallocation. Foster et al. (2001) and Bartelsman and Doms (2000) provide an extensive review of the existing literature on the topic.

Beveridge curve. The model with only aggregate shocks gets this correlation right.[3] In the existing literature, there is concern that a model with sector-specific shocks that affect only a portion of the economy (such as shocks to only the new vintage, as in this paper) would be unable to match this relationship.[4] A positive shock to the new vintage results in an increase in the profits of entrants and in vacancy postings. At the same time, existing low-productivity matches might be terminated, so that workers look for employment within the new vintage; and at the macro level vacancies and unemployment might increase together (weakening the negative correlation between unemployment and vacancies). In this model, this problem does not arise and the strong negative correlation between vacancies and unemployment is preserved because the specific capital of the match makes existing matches more valuable relative to new ones. In the face of a positive shock to new matches, mass destruction of existing matches does not occur as that would result in a loss of valuable match-specific capital.

To examine whether the model can account for the data quantitatively, I determine the shock processes by minimizing the distance between the business cycle frequency moments in the data and the moments generated by the model. In the estimations, I find that the volatility of vintage-specific shocks is close to the volatility of aggregate shocks. The combination of aggregate and vintage-specific shocks can account for the volatilities in unemployment, vacancies, and average labor productivity. I also find low correlation between aggregate shocks and vintage-specific shocks, which suggests the importance of distinguishing between shocks that affect only the new investments and shocks that affect the overall economy. In addition, I show that specific capital is important to keep the correlation between vacancies and unemployment strongly negative as in the data.

Most of the papers that responded to Shimer's (2005a) criticism of the benchmark model continue to rely on aggregate productivity shocks. Farmer (2004), Shimer (2004), and Hall (2005) propose rigid wages as a solution to the volatility puzzle. In the standard model where wages are determined through Nash bargaining, wages comove strongly with the productivity shock, preventing a strong response in profits. Rigid wages set close to output allow profits to respond more strongly to aggregate productivity shocks. Although the assumption of rigid wages is consistent with low cyclicality of aggregate wages in the data, Pissarides (2007) and Haefke et. al. (2007) show for the US economy that wages in new jobs respond strongly to market conditions; and in the model the strong response of profits (and vacancies) requires low cyclicality of wages in the newly created jobs.[5] Hagedorn and Manovskii (2007) calibrate the model to match the elasticity

[3] A positive aggregate shock to the economy increases the profits of firms and the vacancy postings. Increased vacancy postings result in higher job-finding rates, decreasing unemployment. As a result, vacancies and unemployment move in opposite directions in response to aggregate productivity shocks.

[4] See Abraham and Katz (1986) and Blanchard and Diamond (1989).

[5] Rigidity of wages in continuing matches has no impact on profits and vacancies if wages in continuing matches are set at a fixed level determined through Nash bargaining at the first period of the match. Rudanko (2006) has

of aggregate wages in the data, but as discussed above the relevant parameter to match should be the response of wages in new jobs, not aggregate wages.

This paper is also related to the literature of embodied (vintage) technology framework. One of the earliest papers that looked at job creation and destruction in a vintage framework is Caballero and Hammour (1994) in which they analyze how job creation and separation responds to demand shocks. My model endogenizes the creation frictions by embedding the vintage framework in a matching environment. Gilchrist and Williams (2000) analyze a putty-clay technology (where productivity is embodied in the capital unit installed, and the capital labor ratio is irreversible) and show that the addition of embodied technology shocks significantly improves the performance of the RBC model. More recent papers, such as Costain and Reiter (2005) and Michelacci and Lopez-Salido (2007), embed a vintage framework into a model with matching frictions in the labor market. Costain and Reiter (2005) show that embodied technology shocks have more amplifying power than aggregate shocks. However, their analysis lacks some key features of my model, such as allowing for endogenous destruction of matches and match-specific capital; without these elements they are unable to fit the data. Michelacci and Lopez-Salido (2007) look at a setting with a single household in which consumption and work are shared equally among workers (similar to Andolfatto (1996) and den Haan et al. (2000)), and their focus is on the different impulse responses of employment to embodied and disembodied technology shocks. Hornstein et al.(2007) look at a setting where all vintages have identical capital intensity (and embodied productivity), but vintages incur different costs of capital, depending on investment-specific technology shocks at the time of creation of the job.

The paper is organized as follows. In Section 2, I introduce the model environment for the stationary economy and derive the equilibrium conditions. Section 3 does the same for the growth economy. Section 4 presents the calibration and estimation of the parameters and displays the results. Section 5 concludes.

2 Stationary Economy

2.1 Model

The model is a stochastic discrete time version of the Mortensen-Pissarides matching model (Pissarides (1985), Mortensen and Pissarides (1994)). I add embodied technology shocks and specific capital. First, the stationary version of the model is introduced because of its simplicity. In the following section, growth in vintages is incorporated into the model.

a slightly different setting where workers are risk averse and firms are risk-neutral, and the optimal contract results in wages that are less responsive to productivity shocks in the continuing matches. Both settings imply less cyclical wages at the aggregate level, while the response of wages in new matches is strongly procyclical.

2.1.1 Workers, Firms, and Capital

There is a measure one of infinitely lived workers maximizing their lifetime utility. Workers are risk-neutral and have a discount rate of β.

At any point in time, a worker is either working or is unemployed and looking for a job. Employed workers cannot search for jobs. When the worker is unemployed, he receives an income equivalent utility flow of $b > 0$.

Firms are also risk-neutral with the same discount factor as workers and there is an infinite supply of them.

A production unit is composed of a firm, a worker, and $\overline{K}$ units of capital. The rental rate of capital is assumed to be fixed and is equal to r in each period.

The production of a match depends on both the period aggregate shock A_t that affects all matches in the same way and the embodied technology of the match z, which is fixed for the lifetime of the match. The time t output of a match with specific productivity z is equal to:

$$\begin{array}{ll} A_t z & \text{if } K \geq \overline{K} \\ 0 & \text{else} \end{array}$$

To produce positive output, $\overline{K}$ units of capital need to be employed inelastically in the match.

The distribution of specific shocks for new entrants depends on the investment opportunities available at the time of the entry, which are summarized by the state variable Z_t. In each period, match-specific shocks for new entrants are drawn according to some probability distribution function $H(z|Z_t)$. The couplet of shocks (A_t, Z_t) follows a first order Markov process according to a bivariate probability distribution $G(A_{t+1}, Z_{t+1}|A_t, Z_t)$. This specification allows the two shocks to be correlated with each other.

To model specific capital, I assume that once capital is employed in a match, it becomes partially match specific. When the match is dissolved and the match capital is employed for another job, a proportion $\eta \in (0, 1)$ of its value depreciates.

I assume that capital is rented from a risk-neutral intermediary at the rental rate $r = 1/\beta - 1$. If the match ends, the intermediary is compensated for the loss of value in capital by the worker and the firm. The worker pays a proportion $\chi \in (0, 1)$ of the loss in value and the firm pays for the remaining portion $(1 - \chi)$.

Given the above contract, the flow cost of capital to the match is equal to:

$$C_K = r\overline{K} \tag{1}$$

where $\overline{K}$ is the capital that is inelastically needed for production. How the flow cost of capital is shared between the worker and the firm does not affect the results, so I assume without loss of generality that C_K is paid by the firm.

When the match ends, the compensation to the intermediary is equal to C_E:

$$C_E = \frac{\eta r \overline{K}}{(1-\beta)}, \ 1 \geq \eta \geq 0 \qquad (2)$$

C_E is the discounted value of the loss in capital. If the match ends, the worker pays χC_E and the firm pays $(1-\chi)C_E$ to the intermediary.

2.1.2 The Labor Market

There are frictions in the labor market such that firms and workers do not meet immediately. A firm needs to post a vacancy at a fixed cost C_V per period to meet with a worker. In addition, posting a vacancy does not ensure a meeting. The total number of meetings in each period follows the function $m(u_t, v_t)$, where u_t denotes the unemployment rate and v_t denotes the number of vacancies posted in that period. The function $m(u_t, v_t)$ is assumed to be concave in each argument and constant returns to scale in both arguments.

$\theta_t = v_t/u_t$ denotes the market tightness at time t. Because the meeting function is constant returns to scale, the probability of a firm meeting a worker after posting a vacancy and the probability of an unemployed worker meeting a firm both depend only on market tightness θ_t and not on vacancies or unemployment separately.

The probability of an unemployed worker meeting a firm is:

$$f(\theta_t) = \frac{m(u_t, v_t)}{u_t} = m(1, \theta_t)$$

The probability that a firm that has posted a vacancy meets an unemployed worker is:

$$q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = m(1/\theta_t, 1)$$

Since there are match-specific shocks, every meeting does not necessarily turn into employment. For a meeting to turn into employment, both the firm and the worker need to agree about it. Let Ψ_t denote the set of match-specific productivity draws z for which a meeting turns into employment.

Specific shocks may also lead to the endogenous termination of matches. The set Ω_t denotes those specific shocks z for which both the firm and the worker prefer to continue with employment. In addition to endogenous termination of employment, workers and firms separate exogenously with probability δ each period. In the period in which the match is formed, the probability of exogenous termination is zero.

2.1.3 Determination of Wages

Wages are determined according to a Nash bargaining game between the worker and the firm over the total surplus of the match and are renegotiated each period. The bargaining power of the worker is $\lambda \in [0,1]$ and that of the firm is $1-\lambda$.[6]

Another assumption used is that the termination cost C_E is shared in the same proportion to the bargaining weight of each party, that is, $\chi = \lambda$. A motivation for the assumption is that whoever is getting more of the surplus will have more incentive to undertake the cost of termination as the surplus of the match becomes negative.[7]

The formulation that wages are renegotiated in each period of an existing match might seem unrealistic given that in the quantitative analysis the time period is taken to be a week. As referred to in Shimer (2005a), because agents are risk-neutral, the analysis does not necessarily pin down the timing of wage payments. The surplus is not affected by the frequency of wage-negotiation and the analysis below holds as long as the worker gets the value determined by Nash bargaining at the start of the match. Wages can be paid in any manner following employment as long as a commitment technology on promises exist.

2.2 Characterization of Recursive Equilibrium

I look for an equilibrium where the market tightness θ depends only on aggregate state variables (A, Z).

The following section expresses the equilibrium equations for the wage contract that depend on embodied productivity z of the match and aggregate state variables (A, Z). Section 2.2.1 specifies wages under Nash bargaining. Section 2.2.2 simplifies the equilibrium equations using the Nash bargaining wages specified in Section 2.2.1.

The value of the match to the firm is:

$$\pi(A,Z,z) = \begin{array}{c} zA - C_K - w(A,Z,z) - \beta\delta(1-\chi)C_E \\ +\beta\,(1-\delta)E\left[\mathbf{1}_{\{z\in\Omega(A',Z')\}}\pi(A',Z',z) - \mathbf{1}_{\{z\notin\Omega(A',Z')\}}(1-\chi)C_E\right] \end{array} \tag{3}$$

where $\mathbf{1}_{[.]}$ is the indicator function, which takes a value of 1 if the statement inside is true; otherwise, it takes a value of zero. All expectations in the paper are conditioned on current values of (A, Z). $w(A, Z, z)$ is the flow wage rate. Period profit is output less the wage and the flow cost of capital. In the next period, the decision to terminate or continue with the match happens after all agents observe the aggregate productivity shocks (A', Z'). If $z \notin \Omega(A', Z')$ or if there is an exogenous termination (which happens with probability δ) the match ends and the cost to the firm is $(1-\chi)C_E$; otherwise the match continues. Match specific productivity z stays the same next period, so the expectation of profits is taken only over the possible aggregate states of next period.

[6] In the period that the firm and the worker meet, if they decide to turn the match into employment, it is assumed that the wage negotiation happens after capital is employed. As a result, wages are determined in the same manner in each period regardless of whether it is the first or a subsequent period.

[7] There is an equilibrium even without this assumption. The condition just ensures that employment is undertaken and terminated whenever it is efficient to do so.

Let $\Omega_\pi(A,Z)$ denote the set of match-specific productivities for which the firm is willing to continue with employment and let $\Psi_\pi(A,Z)$ be the set of match-specific productivities for which it is willing to turn a meeting into employment. The sets are determined by:

$$\Omega_\pi(A,Z) = \{z : \pi(A,Z,z) \geq -(1-\chi)C_E\} \tag{4}$$

$$\Psi_\pi(A,Z) = \{z : \pi(A,Z,z) \geq 0\} \tag{5}$$

Once capital has been adopted for a match, the cost of terminating the match for the firm is $(1-\chi)C_E$. The firm is willing to continue with employment unless its value drops below $-(1-\chi)C_E$. If capital is not yet employed, the outside value to the firm of rejecting the match is 0.

The value of the match to the worker is:

$$\begin{aligned} W(A,Z,z) &= w(A,Z,z) + \beta\delta\left(E\left[U(A',Z') - \chi C_E\right]\right) \\ &\quad + \beta\,(1-\delta)E\left[\mathbf{1}_{\{z\in\Omega(A',Z')\}}W\left(A',Z',z\right) - \mathbf{1}_{\{z\notin\Omega(A',Z')\}}\left(U(A',Z') - \chi C_E\right)\right] \end{aligned} \tag{6}$$

where $U(A,Z)$ is the value of unemployment in aggregate state (A,Z). The flow value to the worker is the period wage rate. In the following period, if the match is not subject to an exogenous termination shock and $z \in \Omega(A',Z')$ the worker receives next period's value of employment in the same match. Otherwise, he receives his outside option, which is unemployment utility minus the cost of termination of the match χC_E.

The value of unemployment is:

$$\begin{aligned} U(A,Z) &= b + \beta\left(1 - f\left(\theta(A,Z)\right)\right)E\left[U\left(A',Z'\right)\right] \\ &\quad + \beta f\left(\theta(A,Z)\right)\left(E\left[\mathbf{1}_{\{z'\in\Psi(A',Z')\}}W\left(A',Z',z'\right) + \mathbf{1}_{\{z'\notin\Psi(A',Z')\}}U\left(A',Z'\right)\right]\right) \end{aligned} \tag{7}$$

When the worker is unemployed he gets the flow utility of b. With probability $f(\theta(A,Z))$ he gets a meeting with an employer next period, which turns into employment if $z' \in \Psi(A',Z')$; otherwise, he gets unemployment utility in the next period as well. If a match does occur, the worker gets the employment value of the match $W(A',Z',z')$.

Similar to the firm's case, $\Omega_w(A,Z)$ denotes the set of match-specific productivities for which the worker is willing to continue with employment, and $\Psi_w(A,Z)$ denotes the set of match-specific productivities for which he is willing to turn a meeting into employment:

$$\Omega_w(A,Z) = \{z : W(A,Z,z) \geq U(A,Z) - \chi C_E\} \tag{8}$$

$$\Psi_w(A,Z) = \{z : W(A,Z,z) \geq U(A,Z)\} \tag{9}$$

The logic is similar to the one applied to the firm. In an already existing match, the worker's outside opportunity is the unemployment value minus the cost of terminating the match to the worker, χC_E. For a new meeting to be accepted, the expected value within the match needs to be higher than his outside value of unemployment.

The sets $\Omega(A,Z)$ and $\Psi(A,Z)$ can be found by:

$$\Omega(A,Z) = \Omega_w(A,Z) \cap \Omega_\pi(A,Z)$$
$$\Psi(A,Z) = \Psi_w(A,Z) \cap \Psi_\pi(A,Z)$$

The above condition is a result of the assumption that for employment to continue or for a meeting to turn into employment, it should be acceptable to both parties.

As there is free entry to posting vacancies, the expected value to posting a vacancy is equal to zero, which gives the condition:

$$C_V = \beta q(\theta(A,Z)) E\left[\mathbf{1}_{\{z' \in \Psi(A',Z')\}} \pi\left(A', Z', z'\right)\right] \tag{10}$$

The firm pays the cost of posting a vacancy in the current period and meets with a worker next period with probability $q(\theta(A,Z))$, which turns into a match only if $z' \in \Psi(A',Z')$.

Let $\phi(z)$ denote the mass of matches with productivity shock z. $\phi'(z)$ and u' denote, respectively, the mass of people with productivity shock z and the mass of unemployed people in the following period. The law of motion for the unemployment rate u is:

$$u' - u = -uf(\theta(A,Z)) \int \mathbf{1}_{\{z' \in \Psi(A',Z')\}} H\left(z'|Z'\right) dz' + \delta(1-u) + (1-\delta) \int \mathbf{1}_{\{z \notin \Omega(A',Z')\}} \phi(z)\, dz$$

The right-hand side of the above equation can be understood as the result of three types of flows. Of the workers unemployed, $uf(\theta(A,Z))$ of them gets a meeting with a firm, and if the draw of the meeting z' is an element of $\Omega(A',Z')$ the worker exits the unemployment pool. A fraction δ of employed workers' employment ends exogenously. The final term is the addition to the unemployment pool of the workers whose employment is endogenously terminated.

The law of motion for $\phi(z)$ is:

$$\phi'(z) = \mathbf{1}_{\{z \in \Omega(A',Z')\}} \left[(1-\delta)\phi(z) \ \ + \mathbf{1}_{\{z \in \Psi(A',Z')\}} uf(\theta(A,Z)) H\left(z|Z'\right)\right]$$

If $z \notin \Omega(A',Z')$ then the mass of matches with productivity z is zero, as all such matches are endogenously terminated. If $z \in \Omega(A',Z')$, $(1-\delta)$ of existing matches of productivity z continues with employment, and if $z \in \Psi(A',Z')$, a mass of $uf(\theta(A,Z)) H(z|Z')$ new matches of productivity z are created.

The average labor productivity of the economy is:

$$L = \frac{A \displaystyle\int z\phi(z)\, dz}{\displaystyle\int \phi(z)\, dz}$$

The mass of firms $\phi(z)$ is multiplied by their specific productivity and the weighted sum is divided by the total mass of firms.

In the next section I specify the wage rate determination in Nash bargaining to solve for the equilibrium.

2.2.1 Determination of wages (Nash Bargaining)

The surplus of an activated match is:

$$S(A,Z,z) = \pi(A,Z,z) + W(A,Z,z) - U(A,Z) + C_E \tag{11}$$

The term C_E comes from the fact that the match-specific part of capital has a value only as long as the match continues. According to Nash bargaining, the wages and profits are determined as:

$$\pi(A,Z,z) = (1-\lambda)\, S(A,Z,z) - (1-\chi)\, C_E \tag{12}$$

$$W(A,Z,z) = \lambda S(A,Z,z) + U(A,Z) - \chi C_E \tag{13}$$

The firm gets a share $(1-\lambda)$ of the surplus above its outside value of $-(1-\chi)\, C_E$. The worker gets a proportion λ of the surplus above its outside value of $U(A,Z) - \chi C_E$.

2.2.2 Equilibrium Conditions Rewritten with a Nash Bargaining Assumption

In this section, equilibrium conditions are simplified using the surplus allocation rules specified in the previous section. The main aim is to obtain a simplified version of the surplus equation.

Using the wage setting rule of Nash bargaining, equation (4), and the assumption that $\chi = \lambda$, it is seen that the firm is willing to continue employment whenever the surplus of the match is positive $S \geq 0$. Likewise, the worker is willing to continue employment under the same condition, $S \geq 0$, which is found by using equation (8). As a result, the set $\Omega(A,Z)$ is:

$$\Omega(A,Z) = \{z : S(A,Z,z) \geq 0\} \tag{14}$$

To specify the set $\Psi(A,Z)$, it is notationally more convenient to define the surplus of a meeting in the first period before capital is activated. I denote this as S_B:

$$S_B(A,Z,z) = S(A,Z,z) - C_E \tag{15}$$

The difference between S_B and S is that, as capital is not yet activated, C_E is not part of the surplus. Using the wage setting rule of Nash bargaining, equation (5) and the assumption that $\chi = \lambda$, it is found that the firm is willing to turn the meeting into employment whenever the surplus of the meeting (before capital is employed) is positive $S_B \geq 0$. Likewise, using equation (9) the worker is also willing to turn the meeting into employment whenever $S_B \geq 0$. Following these, the set $\Psi(A, Z)$ is:

$$\Psi(A, Z) = \{z : S_B(A, Z, z) \geq 0\} \tag{16}$$

Because $\lambda = \chi$, using equations (12) and (15), I get $\pi = \lambda S_B$. Substituting this and equation (18) into equation (10), the free-entry condition becomes:

$$C_V = \beta q(\theta(A, Z))(1 - \lambda) E \max\left\{S_B\left(A', Z', z'\right), 0\right\} \tag{17}$$

The derivation of the surplus expression below is given in Appendix 1.

$$S(A, Z, z) = \begin{array}{c} zA - (1-\eta)C_K - b - \frac{\theta(A,Z)\lambda C_V}{(1-\lambda)} \\ +\beta\ (1-\delta)E \max\{S(A', Z', z); 0\} \end{array} \tag{18}$$

The surplus equation is quite intuitive. The first term is the period output of the firm. The flow cost of capital is subtracted, but the cost of capital is $(1-\eta)C_K$ rather than C_K, as the value of capital is only $(1-\eta)K$ in other matches. The next two terms are related to the unemployment utility of the worker that is subtracted from the surplus. The next period's surplus is added only for the states in which it is above zero.

The equilibrium for $\theta(A, Z)$ can be solved numerically by using equations (17) and (18). The solution algorithm is given in Appendix 3. In the model, market tightness θ (and thus vacancies) adjust immediately. Once $\theta(A, Z)$ and sets $\Omega(A, Z)$ and $\Psi(A, Z)$ are solved for, the mass of people of different productivities and unemployment can be simulated using their laws of motion.

3 Growth economy

3.1 Augmented Model

In this section, I add growth to the model economy presented in the preceding section. The growth in the economy can be driven by either growth in aggregate productivity A or growth in new vintages. The growth in A, since it does not affect the relative productivity between matches, does not change the endogenous mechanisms. In the following section, growth in the productivity of vintages is considered.

The productivity parameters A, Z and z are stationary in this section also, but there is a trend growth of $1 + g$ in the embodied productivity of new meetings. The time t output of a match created at time $\tau \leq t$, with specific productivity z is equal to:

$$\begin{cases} (1+g)^{\tau} z A_t & \text{if } K \geq \overline{K}(1+g)^{\tau} \\ 0 & \text{else} \end{cases} \quad ; \ \tau \leq t$$

For a balanced growth path, I require that the minimum capital needed for positive production grows at the same rate as the trend productivity growth for new cohorts $(1+g)$, and once the match is formed, the required capital is fixed for that cohort. With this assumption, the flow cost of capital of a match formed at time $\tau \leq t$ becomes:

$$C_{K,\tau} = r\overline{K}\,(1+g)^{\tau} = (1+g)^{\tau} C_K; \quad \tau \leq t$$

where $C_K = r\overline{K}$.

As in the stationary economy, proportion η of employed capital becomes match specific. The cost of exit for a match that is created at time $\tau \leq t$ is:

$$C_{E,\tau} = \frac{\eta r\overline{K}}{1-\beta}\,(1+g)^{\tau} = (1+g)^{\tau}\,C_E; \quad 0 \leq \eta \leq 1,\ \tau \leq t$$

where $C_E = \frac{\eta r\overline{K}}{1-\beta} = \frac{\eta C_K}{1-\beta}$

For a balanced growth path, it is possible to keep the cost of posting a vacancy C_V constant, and let $\theta(A,Z)$ grow at a constant rate; or keep $\theta(A,Z)$ stationary, and let the cost of posting a vacancy grow at rate $1+g$. The two assumptions will result in the same dynamics in other variables. I use the second approach and assume:

$$C_{V,t} = (1+g)^t C_V$$

For a balanced growth path, the flow utility of unemployment needs to grow at rate $(1+g)$ as well:

$$b_t = (1+g)^t b$$

The rest of the assumptions are the same as in the stationary economy.

3.2 Characterization of Equilibrium

In this section, only stationary versions of the equilibrium equations are presented. Non-stationary equilibrium equations for the growth economy are given in Appendix 2. To make the model stationary, the surplus equation is detrended by $(1+g)^t$. A variable x is denoted as $\widehat{x}$ in the detrended economy.

I again look for an equilibrium where the market tightness θ depend on the aggregate state variables (A,Z).

In the stationary economy, the surplus $\widehat{S}$ and set $\widehat{\Omega}$, in addition to aggregate and match specific shocks (A,Z,z), depends on the age of the match, which is the difference between the current period and the period in which the match is formed $d = t-\tau$, $\tau \leq t$.

The surplus of a match is:

$$\widehat{S}(A,Z,z,d)=\begin{array}{l}(1+g)^{-d}\left[zA-(1-\eta)\,C_K\right]-\left(b+\frac{\lambda C_V}{(1-\lambda)}\theta(A,Z)\right)\\+\beta(1+g)(1-\delta)E\left[\max\left\{\widehat{S}(A',Z',z,d+1);0\right\}\right]\end{array} \tag{19}$$

The surplus in the growth economy is very similar to the surplus found in the no-growth economy. The only difference is that the output minus the flow cost of capital that is attached to the vintage $(zA-(1-\eta)\,C_K)$ is shrinking relative to the terms related to the unemployment utility of the worker $\left(b+\frac{\lambda C_V}{(1-\lambda)}\theta(A,Z)\right)$. This comes from the fact that the output zA and the flow cost of capital C_K of an existing match are shrinking at rate $(1+g)$ relative to the productivity of the newest vintage, as the match ages. As the unemployment utility of the worker is increasing relative to staying in the match as the match gets older, the surplus must become negative as the match ages.

Again it is of use to define the surplus of a meeting in the first period before capital is employed, $\widehat{S}_B(A,Z,z)$:

$$\widehat{S}_B(A,Z,z)=\widehat{S}(A,Z,z,0)-C_E$$

As in the stationary economy, whenever the relevant surpluses are positive, a meeting is turned into employment or employment continues into the next period. Sets $\widehat{\Psi}\,(A,Z)$ and $\widehat{\Omega}\,(A,Z,d)$ are determined by:

$$\widehat{\Omega}\,(A,Z,d)=\left\{z:\widehat{S}(A,Z,z,d)\geq 0\right\}$$

$$\widehat{\Psi}\,(A,Z)=\left\{z:\widehat{S}_B(A,Z,z)\geq 0\right\}$$

The no profit condition is:

$$C_V=\beta q\left(\theta\left(A,Z\right)\right)\left(1-\lambda\right)E\left[\max\left\{\widehat{S}_B(A',Z',z');0\right\}\right] \tag{20}$$

The flow of motion for the mass of matches with productivity z belonging to cohort d, $\phi\,(z,d)$, is:

$$\phi'\,(z,d)=\mathbf{1}_{\left\{z\in\widehat{\Omega}(A',Z',d)\right\}}(1-\delta)\phi\,(z,d-1)\ \text{ for } d>0$$

After a period passes, cohort $d-1$, becomes cohort d, since the match ages one more period. If $z\notin\widehat{\Omega}\,(A',Z',d)$, then the mass of matches of $\phi'\,(z,d)$ becomes zero; otherwise, they are exogenously terminated at rate δ.

$$\phi'\,(z,0)=\mathbf{1}_{\left\{z\in\widehat{\Psi}(A',Z')\right\}}uf\left(\theta(A,Z)\right)H\left(z|Z'\right)$$

Cohort $d=0$ consists of only new matches. The total mass of new meetings is $uf\left(\theta(A,Z)\right)$ out of which $H\left(z|Z'\right)$ has productivity z. If $z\in\widehat{\Psi}\,(A',Z')$, then those meetings turn into employment.

It is easy to see that:

$$\lim_{d\to\infty} \phi(z,d) = 0 \text{ for } \forall z, \quad \text{if } \delta > 0 \text{ and } \phi(z,0) < \infty$$

As cohorts age, in the limit their mass approaches zero as they are exogenously terminated at rate δ each period.

The flow of motion for unemployment u is:

$$u' - u = -\phi'(z,0) + \delta(1-u) + (1-\delta)\sum_{d=0}^{\infty}\int \mathbf{1}_{\left\{z\notin\widehat{\Omega}(A',Z',d+1)\right\}}\phi(z,d)\,dz$$

As in the stationary economy, the change in the number of unemployed is composed of new employments that exit the unemployment pool, plus the exogenously and endogenously terminated matches.

Finally, the detrended average labor productivity in the economy is:

$$\widehat{L} = \frac{A\sum_{d=0}^{\infty}(1+g)^{-d}\int z\phi(z,d)\,dz}{\sum_{d=0}^{\infty}\int \phi(z,d)\,dz}$$

For the numerator, the mass of firms $\phi(z,d)$ is multiplied by their productivity level z and as the productivity is expressed relative to the productivity of the newest cohort, it is discounted according to its age $(1+g)^{-d}$. The weighted sum is divided by the total mass of firms to get the average productivity.

4 Quantitative Exercise

The summary statistics of unemployment, vacancy postings, labor productivity, and market tightness for the US economy are displayed in Table 1. The table is taken from Hagedorn and Manovskii (2007). Unemployment and vacancies are 10 times and market tightness is 20 times more volatile than average labor productivity. In Shimer (2005a), the model-generated volatility of market tightness is only twice the volatility of labor productivity (in his paper, changes in average labor productivity are the only driving force of changes in market tightness).

The correlation of unemployment and vacancies is -0.919. This strong negative correlation is also known as the Beveridge curve in the literature. The correlation between labor productivity and market tightness is 0.393. As we will see, with only shocks to average labor productivity, the model generates almost full correlation between these variables. Although this is partially due to the fact that there is no propagation mechanism in the model, it also suggests that there might be other sources of fluctuations in market tightness and unemployment.

The model is solved numerically to evaluate its quantitative implications. Below describes the specification of functional forms, choice of parameter values, and evaluation of results.

4.1 Specification of Functional Forms

For the matching function I use the functional form introduced by Den Haan et al. (2000):

$$m(u, v) = \frac{uv}{(u^l + v^l)^{1/l}}$$

This function ensures that the probability of finding a job and of filling a vacancy always lies between 0 and 1.

Productivity shocks (A_t, Z_t) follow a first order Markov process of the form below.

$$\begin{bmatrix} \log(A_{t+1}) \\ \log(Z_{t+1}) \end{bmatrix} = \begin{bmatrix} \rho_A & 0 \\ 0 & \rho_Z \end{bmatrix} \begin{bmatrix} \log(A_t) \\ \log(Z_t) \end{bmatrix} + \begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{Z,t} \end{bmatrix}$$

Innovations $(\varepsilon_{A,t}, \varepsilon_{Z,t})$ are serially independent, multivariate normal random variables distributed as below:

$$\begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{Z,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_A^2 & \rho_{A,Z}\sigma_A\sigma_Z \\ \rho_{A,Z}\sigma_A\sigma_Z & \sigma_Z^2 \end{bmatrix} \right)$$

For simplicity, I assume that all new matches formed at time t have productivity Z_t.

4.2 Choice of Parameter Values

Standard parameters are calibrated in line with the preceding literature. The parameters governing the shock processes are chosen by minimizing the distance between the moments in the data and the moments of the simulations of the model. For comparative purposes, I perform the same exercises for an economy with only aggregate shocks. In the economy with only aggregate shocks match-specific productivities are normalized to one. The first part describes the calibration of the parameters, which are displayed in Tables 2-3.

The time period in the model is a week. Weekly discount factor β is chosen as $0.99^{1/12}$ which corresponds to a 4% yearly risk-free interest rate.

For the calibration of matching function parameters, I use the estimates in the literature as described in Hagedorn and Manovskii (2007). Shimer (2005a) estimates a monthly job finding rate f of 0.45 and a separation rate of 0.026 (not adjusted for time aggregation). Den Haan et al. (2000) estimates a job filling rate q of 0.71, which gives an average value of $\theta = f/q = 0.634$. At weekly frequencies these estimates imply a job finding rate of 0.139 and a job separation rate of 0.0081. The exogenous separation rate δ is chosen such that the average weekly separation rate in the model (which includes both the endogenous and exogenous separations) is 0.0081. The flow cost of posting a vacancy, C_V, and the matching parameter l are calibrated to fit the average market tightness $\theta = 0.634$ and the average job finding rate $f = 0.139$.

The bargaining weight of the worker λ is determined by the Hosios efficiency condition. With a Cobb-Douglas matching function, the Hosios condition of equating bargaining power to the unemployment elasticity can be satisfied everywhere. This is possible for HRW's matching function only on average because the elasticity is not constant. I set λ equal to the steady state unemployment elasticity of the matching function, $1/(1+\theta^{-l})$.

The flow utility of unemployment b is chosen to be 0.45 which is 70 % of net labor income.[8] This value of flow unemployment utility is above Shimer's (2005a) 40% (of labor income)and below Hagedorn and Manovskii's (2007) 95.5% (of labor income).

The total capital cost consists of the flow cost of capital C_K and the loss of value in capital when it is employed in another match C_E. The flow cost of capital C_K is calibrated to match a capital share of 35%. The calibrated C_K is 0.27 and 0.22 for the no-growth and growth economies respectively. The difference arises from the fact that the cost of exit C_E used is higher in the growth economy and to keep total capital cost fixed at 35% C_K is decreased to 0.22 in the growth economy. Increasing C_E without a corresponding decrease in C_K increases the volatilities of unemployment and vacancies on its own.[9] In that respect, quantifying C_E as part of the physical capital cost works against me when trying to explain the volatilities of unemployment and vacancies.

For the growth economy, I assume that embodied technology grows at rate of 1% per year, which would imply that the rest of the growth in labor productivity comes from growth in aggregate productivity A.

For the parameters of the output process $\left\{\rho_A, \rho_Z, \rho_{A,Z}, \sigma_A, \sigma_Z\right\}$ and the cost of terminating a match C_E, I minimize the distance between the business cycle moments of the data and model-generated simulations. The key business cycle moments matched are as in Table 1.

The following procedure is used to minimize the distance between the moments of the data and the simulations: Let Φ^d denote the vector of moments from the data. For arbitrary values of $\Xi = \left\{\rho_A, \rho_Z, \rho_{A,Z}, \sigma_A, \sigma_Z, C_E\right\}$, market tightness $\theta(A,Z)$, and entry $\Psi(A,Z)\left(\widehat{\Psi}(A,Z)\right)$ and continuation sets $\Omega(A,Z)\left(\widehat{\Omega}(A',Z',d)\right)$ are solved for. (The computational methods used for the stationary and growth economies are explained in Appendix 3.) Using the solution of the model

[8] Net labor income refers to wage income minus the portion of the cost of exit paid by the worker (a portion λ of cost of exit C_E is paid by the worker).

[9] Increasing C_E (without a corresponding decrease in C_K) increases the volatilities of unemployment and vacancies. The mechanism works in the same way as increasing the flow cost of capital C_K or increasing the flow utility of unemployment b. Lowering the portion of output that is shared between the worker and the firm (which is equal to output minus the flow unemployment utility of the worker and all other costs that are paid to third parties for production) increases the volatility of market tightness. In that respect, increasing C_E, C_K, and b has similar impacts on increasing the volatility of market tightness.

and starting with an arbitrary mass of firms, the model is simulated, quarterly series are calculated using averages of weekly series and the quarterly series are HP filtered with a smoothing parameter of 1600 in an identical manner to the empirical data. The vector of simulation moments $\Phi^s(\Xi)$ (which includes the same business cycle moments as in Φ^d) is an average over 50 simulations each 500 quarters in length. The estimate of Ξ minimizes the weighted distance between the actual and simulated moments. Formally I solve the below problem using a search routine:

$$\Gamma(\Xi) = \min_{\Xi} \left\{ \left[\Phi^s(\Xi) - \Phi^d \right]' W \left[\Phi^s(\Xi) - \Phi^d \right] \right\} \tag{21}$$

where W is a positive definite weighting matrix.

For the weighing matrix, I chose to perfectly match the volatility and autocorrelation of average labor productivity. This follows the spirit of previous exercises in the literature, which is to show whether it is possible to match the volatilities of market tightness and unemployment within the standard search and matching model given the low volatility of average labor productivity in the data. The search and matching model creates a low autocorrelation of vacancies relative to the data which might also affect correlations to some degree. To minimize that problem, I put a higher weight on volatility statistics than autocorrelation and correlation statistics. In all experiments, the weighting matrix used is identical.

4.3 Results

4.3.1 Standard Model

For comparison purposes, I first present the results of an economy with only aggregate (disembodied) shocks. The results are displayed in Tables 4-5. The volatility and autocorrelation of the shocks are pinned down only by autocorrelation and volatility of average labor productivity (because a very high weight is put on these moments). In the simulation results, volatilities of vacancies and unemployment are around one-fourth of the data. These volatilities are higher than the ones found by Shimer (2005a), because there is capital in my model, and the flow unemployment utility chosen (as a share of labor income) is higher. Both of these factors increase the response of profits/vacancies to productivity shocks.

The model generates nearly full correlation of 0.988 between the market tightness and average labor productivity, while in the data, the correlation is much lower at 0.393. As I mentioned in the previous section, a third problem is that the autocorrelation of vacancies is lower in the model than the data, and this problem will persist in the augmented setting also.

4.3.2 Model With Embodied Technology Shocks

Role of Embodied Technology Shocks:

The results of the stationary model with both embodied and disembodied technology shocks are presented in Tables 6-7. The estimated conditional and unconditional volatility of embodied shocks are a bit higher than that of disembodied shocks. The correlation between the two shocks is low at 0.11. The volatilities from the model with embodied technology shocks fit the data much better than does the exercise with only aggregate shocks. Volatilities of vacancies, unemployment and market tightness have increased to the level of the data. In addition, because the correlation between embodied and disembodied shocks is low, the correlation of market tightness with labor productivity approaches that observed in the data.

In Table 8, I feed in the estimated parameters for embodied and disembodied shocks separately to decompose the effect of the shocks. The second row feeds in only the embodied technology shocks (normalizing disembodied productivity to one), and the third row feeds in only the disembodied shocks (normalizing embodied productivity to one for all matches). Embodied technology shocks on their own have very little impact on average labor productivity, while they explain almost all of the fluctuations in unemployment and vacancies. The profits and market tightness are much more sensitive to embodied technology shocks because when embodied technology shock is higher than the trend, given that there is mean reversion in the shocks, the firm has a strong incentive to post a vacancy and embody that technology in the new job.[10] Embodied technology shocks have little impact on average labor productivity because they affect only the new cohort of matches, and new employment constitutes only a small portion of total employment. The fact that a big portion of employed workers do not change productivity from one period to the next also implies that the autocorrelation of average labor productivity is high. The third row shows that disembodied technology shocks explain a much bigger portion of the volatility of average labor productivity but explain only a quarter of the movements in unemployment and vacancies.

Role of specific capital, C_E:

With the inclusion of embodied technology shocks, the correlation between unemployment and vacancies increases slightly relative to the model with only disembodied shocks, but there is still a strong negative correlation. The correlation stays strong due to the cost of exit C_E, which is set at 7 weeks of output. In Table 9, I give the results of an exercise where I decrease the cost of exit C_E to 3.5 (in the exercise C_K is adjusted to keep the total cost of capital at 35%). In this case, the volatility of vacancies is higher and the correlation between unemployment and vacancies is weaker at -0.240. The two paragraphs below explain the mechanism for these effects.

[10] With mean reversion in embodied technology shocks, there might be cases where next period's embodied technology shock is lower than current period's shock. Note however that, because these shocks are deviations from trend, occurence of these cases is not frequent. It is also possible to conceptualize the embodied technology shocks as resulting from changing capital intensities in different cohorts of entrants due to changing price of capital at the time of entry in which case a higher price of capital would lead to lower capital intensity. In addition, impediments like technological secrecy, patents, and captured consumers, may make it cosly to replicate existing technologies.

Decreasing C_E increases the correlation between unemployment and vacancies. The mechanism is best explained using a positive embodied technology shock for the new entrants. When there is a positive shock to the new entrants, the outside value of employed workers increases since they have the option to terminate the current match to be employed in the productive new vintage. In addition, with a positive shock more vacancies are posted per unemployed worker making the duration of unemployment shorter, further increasing the outside value of employed workers. As the outside value of employed workers increases, more workers will terminate their current match to go into the unemployment pool to be reemployed in the new vintage. If the cost of terminating the match is low, then a positive shock results in an increase in unemployment (due to the termination of low of productivity jobs), and also an increase in vacancies per unemployed worker, since productivity is higher. This leads to a higher correlation of unemployment and vacancies relative to the case of high termination cost. In that respect, C_E is important to preserve the strong negative correlation between unemployment and vacancies observed in the data.

In addition, when C_E decreases, the volatility of vacancies increases. In response to a positive embodied technology shock market tightness increases (as expected profits increase), and at the same time, with a lower cost of exit, unemployment increases because of endogenous separations. Since vacancies are equal to the multiplication of market tightness with unemployment, the response of vacancies will be stronger when there are more endogenous separations making vacancies more volatile.

One clarification is that increasing C_E affects the results only when there are endogenous termination of matches, that is, increasing C_E decreases the endogenous termination of matches, which impacts other moments also. Once endogenous terminations approach zero, increasing the termination cost is neutral for the results. This is because I decrease C_K proportionately when I increase C_E, so that the total cost of capital is the same in all exercises. Otherwise, as mentioned in the calibration section, the increasing cost of C_E leaves less surplus to be shared between the firm and worker, and that increases the volatility of market tightness on its own. So neutralizing the effect of C_E by a proportionate change in C_K works against me when trying to explain the volatilities observed in market tightness and unemployment. Given the neutrality of C_E above a threshold, the choice of C_E does not strictly come from a minimization problem of $\Gamma(\Xi)$ (Eq(21)). It is chosen high enough such that there is almost no endogenous termination of matches and increasing it further does not affect the simulation results.

It is also useful to compare the C_E used in the simulations with the estimates of specific human and physical capital in the literature. The cost of displacement includes the loss of accumulated job/industry/occupation specific human capital, the training cost of the new worker, and the physical capital possibly being discarded, relocated, or staying idle until a new worker is found.

With respect to specific human capital, Jacobson et al. (1993), Kambourov and Manovs kii

(2007), and Topel (1991) find that workers who were displaced in the preceding 5 years suffer around a 15 to 25% reduction in their weekly earnings. In the case where the physical capital is relocated to a new firm, Ramey and Shapiro (2001) estimate that the average market value of equipment is 28 cents per dollar of the replacement cost for the aerospace industry. The referred studies imply a higher value of specific capital than what I used in the quantitative exercise, but as I explained above, increasing specific capital above the value I used does not affect the simulation results (while decreasing it does affect the results).

4.3.3 Model With Growth

The growth economy results are displayed in Tables 10-11. The results are very similar to the case of the stationary economy. The biggest difference concerns the cost of exit C_E. The cost of exit found has now increased to 11 weeks of output. In the growth economy, a higher cost of exit is needed to decrease endogenous separations to almost zero. With growth, there are two different forces at work that might cause matches to be endogenously terminated. One is that, as in the stationary economy, a vintage of entrants might have low detrended productivity shocks. The second is that as the vintage ages, productivity of the vintage relative to that of new entrants decreases. Whatever the cost of exit is, old cohorts choose at some point to end the employment and it is impossible to fully prevent endogenous terminations. What is important is that as vintages age, their proportion in the population approaches zero because of exogenous terminations. If the endogenous termination is late enough in their lifecycle, its effect on unemployment becomes negligible. In this setting, a higher cost of exit is needed to decrease the endogenous termination of matches to negligible levels.

5 Conclusion

In this paper, I give a competing explanation for the source of fluctuations in unemployment and vacancies. A vintage model of embodied technology is attractive because it is consistent with the micro-level evidence that firm-level productivity is highly persistent and that around half of productivity growth happens through reallocation of workers across firms, not by within firm productivity growth. Second, I do not need to resort to unreasonable parameterizations or non-standard wage setting rules to get rigidity in wages to account for the volatilities in unemployment and vacancies. Finally, the relatively low correlation of market tightness with average labor productivity in the data suggests that there are significant sources of fluctuations other than shocks to average labor productivity, and cohort-specific shocks might be one such source.

I find that with the addition of embodied technology shocks, the MP model is able to account for the cyclical fluctuations in unemployment and vacancies. The estimated volatility of embodied technology shocks is close to the volatility of disembodied technology shocks, but they explain almost all of the fluctuations in unemployment and vacancies. A necessary element in order for the model to produce desirable results is the existence of some type of specific human or physical capital.

6 APPENDIX:

A1: Derivation of the surplus equation in the stationary economy

Equation (3) simplifies to:

$$\pi(A,Z,z)=\begin{array}{c} zA-C_K-w(A,Z,z)-\beta(1-\chi)C_E \\ +\beta\ (1-\delta)E\left[\mathbf{1}_{\{z\in\Omega(A',Z')\}}\left(\pi\left(A',Z',z\right)+(1-\chi)\,C_E\right)\right] \end{array} \tag{22}$$

Equation (6) simplifies to:

$$\begin{aligned} W\left(A,Z,z\right) &= w\left(A,Z,z\right)+\beta\left(E\left(U\left(A',Z'\right)-\chi C_E\right)\right) \\ &\quad +\beta(1-\delta)E\left[\mathbf{1}_{\{z\in\Omega(A',Z')\}}\left(W\left(A',Z',z\right)+\chi C_E-U\left(A',Z'\right)\right)\right] \end{aligned} \tag{23}$$

Equation (7) simplifies to:

$$\begin{aligned} U(A,Z) &= b+\beta E\left(U(A',Z')\right) \\ &\quad +\beta f\left(\theta(A,Z)\right)E\left[\mathbf{1}_{\{z'\in\Omega(A',Z')\}}\left(W\left(A',Z',z'\right)-U\left(A',Z'\right)\right)\right] \end{aligned} \tag{24}$$

For the part below this I use the Nash bargaining wage determination rule specified in Section 2.2.1.

Because $\lambda=\chi$, using equations (13) and (15), I get $W-U=\lambda S_B$. Substituting this and equation (9) into equation (24) I get:

$$U(A,Z)=b+\beta E\left(U(A',Z')\right)+\beta f\left(\theta(A,Z)\right)\lambda E\left[\max\left\{S_B\left(A',Z',z\right),0\right\}\right]$$

Finally, substituting for $E\left[\max\left\{S_B\left(A',Z',z'\right),0\right\}\right]$ from the no profit condition (17) unemployment equation simplifies to:

$$U(A,Z)=b+\beta E\left(U(A',Z')\right)+\frac{\theta(A,Z)\lambda C_V}{(1-\lambda)} \tag{25}$$

Plugging equations (22), (23) and (25) into the surplus equation (11), surplus is found to be:

$$S(A,Z,z)=\begin{array}{c} zA-C_K+(1-\beta)\,C_E-b-\frac{\theta(A,Z)\lambda C_V}{(1-\lambda)} \\ +\beta\ (1-\delta)E\left[\mathbf{1}_{\{z\in\Omega(A',Z')\}}\left(\begin{array}{c}\pi\left(A',Z',z\right)+W\left(A',Z',z\right)\\ +C_E-U(A',Z')\end{array}\right)\right] \end{array}$$

Substituting $S = \pi + W - U + C_E$ for the term within the expectation term:

$$S(A,Z,z) = \begin{array}{l} zA - C_K + (1-\beta)\,C_E - b - \frac{\theta(A,Z)\lambda C_V}{(1-\lambda)} \\ +\beta\,(1-\delta)E\left[\mathbf{1}_{\{z\in\Omega(A',Z')\}}S\left(A',Z',z\right)\right] \end{array}$$

Substituting equation (14) into the expectation term and substituting for C_E using equation (2), I get equation (18).

A2: The non-stationary equilibrium equations for the growth economy

This section gives the surplus equation of the growth economy before it is detrended. The derivation of the equilibrium equations from more primitive conditions is very similar to the no-growth economy.

I look for equilibria where θ depends on the aggregate state variables (A, Z). Set Ψ depends on (A, Z). Set Ω depends on (A, Z, τ, t), where τ is the period of entry for the match and t is the current period, $\tau \leq t$. The surplus of the match depends on (A, Z, z, τ, t). The sets $\Omega(A, Z, \tau, t)$ and $\Psi(A, Z)$ are found by the conditions:

$$\Omega\,(A,Z,\tau,t) = \{z : S(A,Z,z,\tau,t) \geq 0\}\,;\ \tau \leq t$$

$$\Psi\,(A,Z) = \{z : S(A,Z,z,t,t) - C_E \geq 0\}$$

Although set $\Psi(A, Z)$ seems to depend on the current period t, t drops out when the variables are detrended.

The surplus is found as:

$$S(A,Z,z,\tau,t) = \begin{array}{c} (1+g)^{\tau}\left(zA - (1-\eta)C_K\right) - (1+g)^{t}\left(b + \frac{\lambda C_V}{(1-\lambda)}\theta(A,Z)\right) \\ +\beta\,(1-\delta)E\max\left\{S(A',Z',z,\tau,t+1);0\right\} \end{array}\ ;\ \tau \leq t$$

In the surplus equation, the flow variables attached to the vintage (output zA and flow capital cost C_K) are multiplied by $(1 + g)^\tau$ where τ is the time of the creation of the match. Terms $b + \frac{\lambda C_V}{(1-\lambda)}\theta(A, Z)$ are related to the unemployment utility of the worker and they are multiplied by $(1 + g)^t$. The reason is that the flow unemployment utility b and the value to the worker of new matches grows with the current time t.

The surplus equation is detrended by dividing it by $(1+g)^t$, that is $\widehat{S}(A, Z, z, d) = S(A, Z, z, \tau, t)/(1+g)^t$. The stationary $\widehat{S}$ depends only on (A, Z, z, d) where d is the time difference between current time and the time the match was formed, $d = t - \tau$, $\tau \leq t$. The stationarized versions of the equilibrium conditions are given in Section 3.2.

A3: The Computational Strategy

To solve the model numerically, I use the discrete state space method. Each of the aggregate shocks A and Z is discretized into 25 grids (resulting in 25×25 aggregate states). I approximate the shock process with a discrete Markov chain. For that purpose, I integrate the bivariate normal distribution over each interval to compute the Markov transition matrix.

The solution algorithm for the stationary economy involves the following:

(i) Assume an initial $\theta^0(A, Z)$

(ii) Use $\theta^0(A, Z)$ and an initial guess for $S^0(A, Z, z)$ to iterate over the surplus equation (18) till it converges. (Sets $\Psi(A, Z)$ and $\Omega(A, Z)$ are also found in this step.)

(iii) Using the no profit condition (17), I can update $\theta^1(A, Z)$. Using $\theta^1(A, Z)$, repeat steps (i) and (ii) until $\left|\theta^{i+1}(A, Z) - \theta^i(A, Z)\right| < \epsilon$ where i is the number of iterations and ϵ is a very small number.

For the growth economy, the strategy is the same except for the second step of iterating the surplus equation. In the growth economy, finding the surplus value $S(A, Z, z, d)$ for all $d \in \mathbb{N}$ is an impossible task. Instead, I use the fact that in the growth economy any match's surplus becomes negative as it ages and it is eventually terminated. I set $d_{\max}$ high enough such that all matches are terminated before that age, and use backward induction to solve for the value functions. Specifically, I set $S(A, Z, z, d_{\max}) = 0$, then solve for $S(A, Z, z, d_{\max} - 1)$ using equation (19) and continue iterating backwards till $d = 0$. After $S(A, Z, z, 0)$ is found, $\theta^i(A, Z)$ can be updated from the no profit condition (Eq(18)). As before, I continue iterating until $\left|\theta^{i+1}(A, Z) - \theta^i(A, Z)\right| < \epsilon$.

Table 1: Summary statistics, quarterly US data, 1951:1 to 2004:4

		u	v	v/u	p
Standard Deviation		0.125	0.139	0.259	0.013
Quarterly Autocorrelation		0.870	0.904	0.896	0.765
	u	1	-0.919	-0.977	-0.302
Correlation Matrix	v	-	1	0.982	0.460
	v/u	-	-	1	0.393
	p	-	-	-	1

Notes: The table is from Hagedorn and Manovskii (2007). Seasonally adjusted unemployment data, u, is from the Bureau of Labor Statistics. The seasonally adjusted help-wanted advertising index, v, is from the Conference Board. Both u and v are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, from the BLS. All variables are in logs as deviations from HP trend with smoothing parameter 1600.

Calibrated Parameter Values

Table 2: Shared parameters

Parameter	Definition	Value
β	discount rate	$0.99^{1/12}$
b	value of non-market activity	0.45
l	matching parameter	0.3995
λ	workers' bargaining power	0.4546
δ	separation rate	0.0081
b	value of non-market activity	0.45

Table 3: Parameters Specific to the Economy

Parameter	Definition	Aggregate	Stationary	Growth
C_V	cost of posting vacancy	0.22	0.56	0.33
C_K	flow cost of capital	0.35	0.27	0.22
$1+g$	growth rate	-	-	$1.01^{1/48}$

Notes: Different specifications are: economy with only aggregate shocks (Aggregate), stationary economy with both aggregate and embodied technology shocks (Stationary), growth economy with both aggregate and embodied technology shocks (Growth):

Results for the economy with only aggregate shocks

Table 4: Estimation Results

ρ_A	0.9902
σ_A	0.0032
ρ_Z	-
σ_Z	-
$\rho_{A,Z}$	-
C_E (weekly output)	0

Table 5: Simulations results with the estimated parameters

		u	v	v/u	p
Standard Deviation		0.033	0.038	0.068	0.013
Quarterly Autocorrelation		0.832	0.631	0.769	0.769
	u	1	-0.789	-0.937	-0.935
Correlation Matrix	v	-	1	0.954	0.952
	v/u	-	-	1	0.998
	p	-	-	-	1

Notes: All variables are in logs as deviations from an HP trend with smoothing parameter 1600.

Results for the stationary economy with aggregate shocks and embodied technology shocks

Table 6: Estimation Results

ρ_A	0.9858
σ_A	0.0031
ρ_Z	0.9809
σ_Z	0.0039
$\rho_{A,Z}$	0.1074
C_E (weekly output)	7.0

Table 7: Simulation results with the estimated parameters

		u	v	v/u	p
Standard Deviation		0.118	0.144	0.243	0.013
Quarterly Autocorrelation		0.805	0.535	0.717	0.766
Correlation Matrix	u	1	-0.714	-0.909	-0.414
	v	-	1	0.940	0.307
	v/u	-	-	1	0.383
	p	-	-	-	1

Table 8: Decomposing the Effect of Shocks

	u	v	v/u	p	$corr(u,v)$
Data	0.125	0.139	0.259	0.013	-0.919
Embodied	0.107	0.131	0.221	0.004	-0.718
Disembodied	0.027	0.032	0.056	0.012	-0.781
Total	0.118	0.144	0.243	0.013	-0.714

Table 9: Sensitivity Analysis with respect to C_E

		u	v	v/u	p
Standard Deviation		0.119	0.174	0.231	0.013
Quarterly Autocorrelation		0.632	0.539	0.723	0.772
	u	1	-0.240	-0.692	-0.387
Correlation Matrix	v	-	1	0.865	0.282
	v/u	-	-	1	0.411
	p	-	-	-	1

Notes: The results are for the stationary economy with aggregate shocks and embodied technology shocks, where the cost of exit is reduced to $C_E = 3.5$. C_K is adjusted to keep total capital cost at 35%. For the rest of the parameters, the calibration and estimation results for the stationary economy with both aggregate and embodied technology shocks are used

Results for the growth economy with aggregate shocks and embodied technology shocks

Table 10: Estimation Results

ρ_A	0.9852
σ_A	0.0031
ρ_Z	0.9833
σ_Z	0.0037
$\rho_{A,Z}$	0.0791
C_E (weekly output)	13.0

Table 10: Simulations results with the estimated parameters

		u	v	v/u	p
Standard Deviation		0.118	0.144	0.243	0.013
Quarterly Autocorrelation		0.814	0.545	0.727	0.765
	u	1	-0.712	-0.909	-0.420
Correlation Matrix	v	-	1	0.940	0.306
	v/u	-	-	1	0.386
	p	-	-	-	1

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