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Collateral, Credit History, and the Financial Decelerator[1]

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Abstract

We develop a simple model in which financial imperfections can serve to stabilize aggregate fluctuations and not merely aggravate them as in much of the previous literature; we term this a *financial decelerator*.

In our model agents borrow to purchase housing and secure their loans with this long-lived asset. There are two financial imperfections in this model. First, agents are unable to commit to repay their loans — that is, they can strategically default. This limits the amount that lenders are willing to offer. In addition, however, lenders are also imperfectly informed as to a borrower's propensity to default; that is, there is adverse selection. The latter imperfection implies that default may actually occur in equilibrium, unlike in much of the previous literature.

For relatively high house prices the commitment problem ensures that the equilibrium is typically characterized by a standard financial accelerator; that is, the borrowing constraints which prevent default become tighter as falling prices reduce the wealth with which agents can collateralize future loans, thereby exacerbating aggregate fluctuations. However, we show that when prices are low, agents will default, which serves as a stabilizing force.

Keywords: Financial accelerator, default, collateral, credit history
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1 Introduction

Much recent economic research develops the idea that financial factors can aggravate real fluctuations. The common theme of this work is that informational asymmetries may introduce inefficiencies into financial markets that are particularly acute in times of economic downturn. A prominent example is Kiyotaki and Moore (1997).[1] In this work, it is typically the case that a shock to the economy lowers the value of some asset that is used to secure firms' borrowing, thereby aggravating asymmetric information problems and making external financing more difficult to obtain; this, in turn, lowers aggregate output and, hence, asset prices, even further. Similarly, in the context of housing markets, Stein (1995) constructs a model in which drops in housing prices mean that after paying their loan, households have less money available to use as a down payment for a new house and so are less likely to be able to move, further depressing housing prices.[2]

It is the aim of this paper, by contrast, to develop a simple model that has the feature that some financial imperfections may actually serve as a *stabilizing* force. The key mechanism whereby we obtain stabilization in our model is that default may actually occur in equilibrium; by contrast, in most of the existing literature, the borrowing constraints ensure that agents make their promised payments in every eventuality.

In our model agents borrow to purchase housing and secure their loans with this long-lived asset. There are two financial imperfections in this model. First, in common with the previous literature, agents are unable to commit to repay their loans — that is, they can strategically default. This limits the

[1] See also Bernanke and Gertler (1989), Kashyap, Scharfstein, and Weil (1990), Gertler (1992), and Carlstrom and Fuerst (1997).

[2] See also Ortalo-Magné and Rady (2005), who construct a life-cycle model of housing markets in which agents face credit constraints; among other conclusions, they show that the magnitude of housing price fluctuations can exceed those of GDP.

amount that lenders are willing to offer. In addition, however, lenders are also imperfectly informed as to a borrower's propensity to default; that is, there is adverse selection. The latter imperfection implies that default may actually occur in equilibrium.

For relatively high house prices the commitment problem ensures that the equilibrium is typically characterized by a standard financial accelerator; that is, the borrowing constraints that prevent default become tighter as falling prices reduce the wealth with which agents can collateralize future loans, thereby exacerbating aggregate fluctuations. However, we show that when prices are low, agents will default, which serves as a stabilizing force; we term this a *financial decelerator*.

The key reason for this is the fact that under adverse selection what appear as borrowing constraints are actually *endogenous* and in fact result from the best agents' optimizing behavior. When collateral values are low, however, agents may find it too costly to respect these constraints. This then leads to two possible outcomes. Either the bad types will default on their current loans, despite the cost this entails to their reputation or credit history; this will leave them with more income precisely in those states in which house prices are low. Alternatively, the good types may increase their borrowing, which will make the bad types willing to repay now, but at the cost of allowing them to default in the future (because they have borrowed more). In either case current spending on housing will be higher, which will mitigate the decline in its price.

From this discussion one can see that the important feature of this story is the interaction between adverse selection and strategic default. Default is *strategic* in our model in the following sense. First of all, repayment is always an option in that agents do indeed have sufficient funds to cover their debts even when house prices fall. In addition, agents act strategically in weighing the costs and benefits of defaulting against those of repaying. Some of these costs are direct, in the form of the value of the collateral (house) that is surrendered upon default, as well as a personal cost of bankruptcy that each agent incurs; but they are also indirect, in the impact of default on a borrower's reputation.

We note that the type of strategic default we have in mind — waves of default that are correlated with declines in asset prices — is seen by many to have featured in the Texas housing crash of the mid-1980's. This is illustrated in the following statement by Judith Dedmon, head of Fannie Mae's Dallas office in 1987: "[i]n some neighborhoods, the homeowners would walk the house and go down the street and buy the same house at half the price." (*The Dallas Morning News,* March 18, 1996). The message of this paper is that strategic default can actually have the effect of *stabilizing* the housing market and that the impact on the economy might have been even more severe had borrowers been forced to repay all that they owed.

The plan of the paper is as follows. The following section provides further discussion of the model. The model itself is presented in section 3; we first introduce the various types of agents, define our notion of equilibrium and then derive agents' optimal responses under various scenarios. Section 4 is devoted to the equilibrium. We first discuss the equilibrium of the borrowing game. We then show under which circumstances a financial accelerator or decelerator would arise in our model. In section 5 we compute some examples that illustrate the general results of the paper. Section 6 concludes.

2 Discussion

In this section we discuss several aspects of the model, provide further references to the literature, and present some empirical evidence that is supportive of our model and its assumptions.

First of all, we are not the only ones to develop a model in which financial imperfections can stabilize fluctuations. Bacchetta and Caminal (2000) show that the agency costs facing credit-constrained firms can be aggravated by certain *positive* aggregate shocks (in particular a decline in the cost of funds), which thereby allows these agency costs to serve as a dampening force. This mechanism is rather different from that in our paper. In particular, neither price fluctuations (aside from interest rate changes) nor their effects on collateral values play a role in their paper. House (2002) shows that in models where financial imperfections lead to an over-investment problem, aggregate fluctuations that generate a decline in entrepreneurs' net worth may actually be more than offset by this over-investment.

Our model requires that for default to play a role in mitigating business cycles, consumers must be able to retain (non-housing) wealth even after defaulting. We note that "deficiency judgments," which provide for a recovery of the difference between the unpaid loan balance and the property's liquidation value through attachment of the mortgagor's other assets, are either prohibited or restricted by quite a few states and, in any case are, limited by the mortgagor's ability to declare bankruptcy.[3] In addition, the FHA has a policy of not pursuing deficiencies on its loans.

This paper also builds on game-theoretic models of credit markets and reputation, such as Diamond (1989). As in Diamond's paper, default is strategic, and lenders update their beliefs concerning borrowers' creditworthiness based on whether or not they have defaulted in the past.

We are also not the first to model default in general equilibrium. More formal general equilibrium models of default and collateral can be found in Kehoe and Levine (1993), Dubey, Geanakoplos and Shubik (1989, 2005),[4] while credit history is carefully modeled in general equilibrium by Bose (1996).

[3] C.f. Clauretie and Herzog (1990).

[4] For an application of default costs to housing markets with asymmetric information, see Brueckner (2000).

Finally, one component of the stabilizing effect of default is an endogenous transfer of income from the lending sector to strategic defaulters when housing prices fall, which is made up for by a compensating transfer when prices are high and there is no default (in the form of a higher interest rate). In our model, the resulting losses incurred by this sector do not impact either its demand for housing or its ability to lend further. Although this is certainly a restrictive feature of our model, we have in mind a situation in which the negative impact of these losses is smaller than the positive effect of the additional funds available to the strategic defaulters. This is in fact a natural outcome of most models in which agents have a choice between consumption and lending — those with the lower marginal propensity to consume out of current income will naturally emerge as the lenders.[5] Likewise, although we assume here that the supply of funds is perfectly elastic so long as lenders break-even in expectation, our results would continue to hold, albeit more weakly, as long as the lending sector was not perfectly inelastic. Finally, we do not model the possibility that reserve requirements could constrain banks in such a manner that an increase in default would leave them with fewer funds to lend. While such an effect would again mitigate our financial decelerator, it would likely not eliminate it completely.

3 The Model

3.1 Introduction

We work with a two-period model of the economy, in which the periods are denoted 1 and 2. There are two goods, a perishable numeraire and housing, the latter a long-lived asset from which agents accrue utility in each period they consume it. Since it is long-lived, housing is available for use as collateral

[5] On this see Tobin (1975).

on loans. The supply of housing will be fixed at 2 units.

The price of the perishable numeraire good will be normalized to 1 in every state. In period 1, the housing price will have two possible values: p_{1u} and p_{1d}, each of which occurs with probability 1/2. In order to make the interpretation of the model cleaner, we will restrict attention to parameter values that yield equilibria for which $p_{1u} > 1 > p_{1d}$. Conditional on being in a particular node of period 1, there will be a single period-2 value for the housing price, either p_{2u}, if the current price is p_{1u}, or p_{2d}, if it is p_{1d}. That is, all uncertainty is resolved in period 1. To generate a source of price fluctuations in the cleanest manner possible we assume that there is a measure $1 - \sigma$ of non-strategic agents who serve as an outside source of demand for housing in period 1 (only). We return to these agents below when we discuss the determination of the equilibrium house price.

There is a competitive banking sector in this model that is always willing to lend the numeraire on terms that provide it an expected return of 1, and which has income-inelastic demand for housing. That is, we work in a small open economy. This is discussed further below.

To simplify the contractual environment, we also make the following additional assumptions regarding lending contracts. First of all, we restrict attention to standard debt contracts, in which the gross interest rate is r for each unit borrowed and in which the house serves as collateral for the loan. We do not justify this assumption, although we note that nearly all mortgage contracts have this form.

In addition, we assume that there are no *deficiency judgments* — that is, should a borrower refuse to pay, the lender does not have recourse to any assets other than the housing purchased with the proceeds of this loan. Finally, we assume that the collateral can always be costlessly transferred from the borrower to the lender.

We will primarily be interested in the *consumers* of housing, who we will sometimes also refer to as "strategic agents." We assume that they have a measure σ and that they share the following characteristics:

- Each agent enters period 1 with two units of housing.
- We also assume that they have financed the purchase of this housing by borrowing two units of the numeraire (as in Stein, 1995), and they have used the housing to secure this debt (although for low values of the house price it may not suffice to cover the entire loan). Note that the gross interest rate r_0 on this debt, which is due (in period 1), must be determined endogenously in equilibrium, since it will depend on the fraction of agents who choose to default in period 1.

The timing of repayment and consumption is as follows. At the start of period 1 the exogenous shock to housing demand is realized, which leads to a market clearing price of either p_{1d} (the bottom node) or p_{1u} (top node). An agent first makes a repayment on his initial loan of 1; this repayment may be partial (for example, if he defaults and surenders his collateral). He then has an opportunity to borrow further; the rate lenders charge him depends on whether or not he repaid his loan in full. Finally, he is able to consume from his wealth, which is the sum of whatever he retains after repaying his loan and his borrowing. In period 2, there is no further realization of uncertainty. He first makes a repayment and then consumes; there is no further borrowing. This timing is illustrated in Figure 1.

3.2 Consumers

Consumers consume only housing in period 1, and both housing and the numeraire in period 2.[6] Conditional on the realization of uncertainty in

[6] By restricting consumption to housing in period 1 we simplify the model by avoiding having to distinguish between secured and unsecured loans. In any case, for the parameter

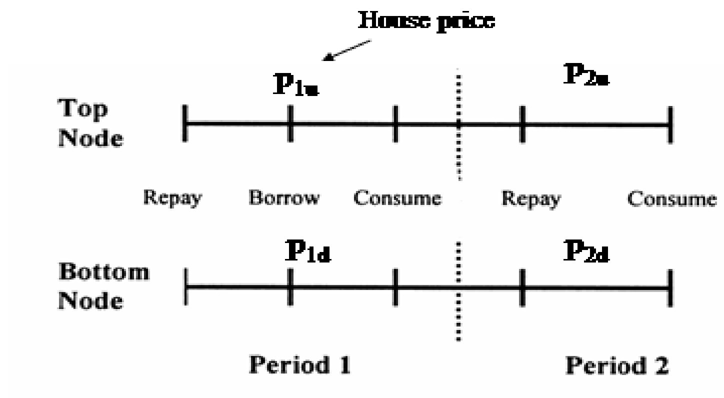

Figure 1: Timing of the Model

period 1, an agent consuming (h_{1i}, h_{2i}) units of housing and x_{2i} units of the consumption good (in period 2) accrues the following utility:

$$v(h_i, x_i) = \log[h_{1i}] + (x_{2i} + \log[h_{2i}]),$$

where $i \in \{u, d\}$.

Agents will have income $i_1 \equiv i = 1/2$ in either node of period 1 (this is high enough so that default is truly strategic for the parameters we consider), and we will assume that their income in period 2 is sufficiently high so that their period-2 housing consumption is income-inelastic. ($i_2 > 2$ would suffice; note that they have quasi-linear utility). As a result, the equilibrium price will be $p_2 = 1/2$ in either node of period 2.

In addition agents also have a fixed personal cost of default, denoted by $k \in [0, 1]$, which is subtracted from their utility in any period in which they default. So for example, conditional on being in the bottom node, an agent

values we consider, even with quasi-linear utility in both periods, agents would not be able to borrow enough to consume the numeraire in period 1.

with cost k who defaults in period 2 (but not period 1) would obtain utility $v(h_i, x_i) - k$.

Agents are distinguished by this cost of default. We will assume that a fraction $1 - \beta$ of the consumers are "good" in that they have a high cost of default $k = 1$, while the other β are "bad" and have a lower cost k, with $k \in (0, 1/2]$; later we will characterize the equilibrium as we vary this cost k. We also assume that this cost is private information — lenders do not observe it directly, although borrowers' behavior can be used to make inferences about it.

In employing default costs, we follow Dubey, Geanakoplos and Shubik (2005), although for simplicity we take these costs as fixed, rather than as increasing in the default. These costs can be interpreted as a household's "stigma" from a bankruptcy filing or else as summarizing the cost associated with bankruptcy, such as time wasted in court. Since we do not want to arbitrarily exclude defaulters from credit markets (unlike Allen (1981) or Kehoe and Levine (1993) and many others), we must discourage default through these costs. In addition, since it is natural to interpret these costs as being private information, this gives us a way to introduce adverse selection into our model.

3.3 Consumer Demand

3.3.1 Introduction

We now specify how consumers optimize in this model.

There are four prices and four interest rates in this model.

As far as the prices, they are p_{1u} in the top node of period 1 and p_{1d} in the bottom node, and p_{2u} and p_{2d} in period 2. For the interest rates, we have r_0, which is the rate for the loan the agents must repay at the beginning of period 1 (regardless of the state of nature). For simplicity we will restrict attention to parameter values such that there is only default in the bottom node of period 1 (when the house price goes down). In particular, we will have $p_{1u} > r_0$ in equilibrium.

Next there is r_u, which is the interest rate paid in the top node of period 2 on loans taken out in the top node of period 1. However, we will focus attention primarily on the bottom node because that is where default occurs (since $p_{1d} < 1$); in the appendix we briefly discuss what occurs in the top node in equilibrium.

In the bottom node the interest rate charged is denoted r_d (it is paid in period 2 on account of borrowing in period 1). In addition, as we will see below, it will also be conditional on an agent's repayment behavior in period 1, as well as on how much is borrowed.

Given the quasi-linear structure of preferences, agents' decisions are:

- Whether to default on their original loan in a given node of period 1.
- Given their default decision, how much to borrow
- Whether or not to repay in the final period (depending on the node and on how much they borrowed in the previous period)

3.3.2 Borrowing Constraints

Given that agents are free to default in this model, lenders of course impose borrowing constraints. The natural constraint in our model is that lenders will not offer more than the *best* agent in a given pool would repay. That is, while it may be the case that the "bad" agents default in a pooling equilibrium, the good agents must repay, since otherwise lenders would not be able to break even.[7] This is an important characteristic of our model; as we

[7] This is actually slightly stronger than is needed in the initial period; because of the uncertainty, it would be possible for all agents to default in the bottom node, but for simplicity we maintain the same constraint throughout the model.

have already noted, classic financial accelerator models typically rule out any default whatsoever in equilibrium.

In the initial period (period 0), since the debt is fixed at 2 units, and agents' income in the following period is $i_1 = 1/2$, this constraint leads rather to a restriction on parameter values so as to ensure that the good agents are able to repay in the low state, that is, for which $2 \times (r_0 - p_{1d}) \geq i_1 = 1/2$ in equilibrium, where r_0 will be determined below.

The constraints are more complicated in period 1. In general, for an agent whose default cost is known by lenders to be κ and with period-1 income i, the maximal borrowing $b_{\max}(i, \kappa)$ that is consistent with this agent repaying in the following period satisfies:

$$[i + b_{\max}(i, \kappa)] \times \frac{p_2}{p_1} - b_{\max}(i, \kappa) = -\kappa.$$

The left-hand side is derived by noting that his spending on housing in period 1 will be $i + b_{\max}$, and so his wealth in period 2 will be $(i + b_{\max}) \times \frac{p_{2j}}{p_{1j}}$; in addition he owes $b_{\max}$ from his period-1 borrowing (the gross interest rate will be $r_1 = 1$ because the constraint ensures that he will repay). The right-hand side is simply the disutility of defaulting in period 2.

We now focus attention on the bottom node (the top node is discussed in the appendix).

The simplest constraint arises for those agents who default in period 1. We will show below that only the bad types will do so in equilibrium. Since their default will allow them to retain all of their income $i_1 = 1/2$ (and surrender only their collateral), their borrowing constraint will be determined by

$$[1/2 + b_{\max}(1/2, k)] \times \frac{p_2}{p_1} - b_{\max}(1/2, k) = -k. \tag{1}$$

The solution to this is $b_{\max}(1/2, k) = \frac{4kp_{1d}+1}{4p_{1d}-2}$.

By contrast, for those agents who do not default in equilibrium, we must have

$$[1/2 - 2(p_{1j} - r_0) + b_{\max}(1/2 - 2(p_{1j} - r_0), 1)] \frac{p_{2j}}{p_{1j}} - b_{\max}(1/2 - 2(p_{1j} - r_0), 1) \times r_{1j} = -1. \tag{2}$$

This differs from the above constraint because (i) the agents' period-2 wealth also depends on what they have left over after repaying in period 1, (ii) the interest rate r_{1j} is not necessarily 1, and finally (iii) the best agents in the pool (who determine the constraint) have a default cost of 1. A messy expression for $b_{\max}(1/2 - 2(p_{1j} - r_0), 1)$ can be derived, although in the interests of space we will not do so here.

3.3.3 Equilibrium Concept

Since this is a model of adverse selection in which pooling and separation play a major role, we choose to work with the Wilson-Miyazaki equilibrium, as in Miyazaki (1977) and Wilson (1977). In this equilibrium the allocation is the one that maximizes the utility of the best types. This is an attractive solution concept because it allows for pooling of types in equilibrium, which can then lead to equilibrium default.

By contrast, the Nash equilibrium outcome would have the feature that the best types restrict their borrowing so much that there is never any default in period 2. While this would still deliver our main results in the model presented in this paper (since the initial loan is specified exogenously here), Nash equilibrium has the unattractive feature that were one to extend the model and endogenize the borrowing in period 0, it would be difficult to generate default in period 1. The reason is that "cream-skimming" would give the good types an incentive to reduce their borrowing to the fully collateralized level.

3.3.4 Equilibrium Allocations

Given these borrowing constraints and this equilibrium concept, we now derive the optimal allocations in the bottom node. We will show that the equilibrium of the borrowing game is characterized by one of three possible "regimes." First, there is a "safe" regime where the bad type's borrowing constraints are respected and there is no default — neither in period 1 nor period 2; in this case both types are pooled together. Conversely, there is a "default" regime where the bad types default in period 1 and are thereby separated from the good types. Finally, there is a "risky" regime in which there is no default in period 1, yet the level of borrowing in that period is sufficiently high that default occurs in period 2. We will demonstrate in the sections below that the choice of regime depends both on the parameters β and k, as well as the market-clearing housing price p_{1d}.[8]

We begin by considering the interest rate r_0, which is due at the start of period 1 (on account of borrowing in period 0). As discussed earlier, in equilibrium none of the good agents will default. In addition, suppose that a fraction $q \in [0, 1]$ of the bad types also repay in period 1.

Then the initial interest rate r_0 that those agents who do not default must pay is determined as follows:

$$1/2 \times r_0 + 1/2 \times [(1 - \beta) \times r_0 + \beta q \times r_0 + \beta(1 - q) \times p_{1d}] = 1$$

This simply states that the total payment per unit must equal the amount borrowed. The agents all repay in the top node (which occurs with probability 1/2). In addition, in the bottom node all of the good types (measure $1-\beta$) and a fraction q of the bad types also repay. The remaining bad types

[8] In this sense our model shares some similarities with Mester (1994), who develops a model to explain why interest rates on credit cards are "sticky." As in our paper, a change in a macroeconomic variable can affect the degree of pooling of types in equilibrium.

(of measure $\beta \times [1-q]$) default and surrender their collateral, which is worth p_{1d} per unit. Observe that lenders always break even in expectation in every period (i.e. over the two nodes). If there is default in period 1, for example, so that the bad types do not all repay in the bottom node, this raises the interest rate r_0, which must be paid by all agents in the top node, and by the good types in the bottom node as well (so it can potentially affect their consumption).

As far as the borrowing in period 1, recall that under the Wilson-Miyazaki solution concept, the equilibrium level of borrowing will be determined by the best allocation for the good type. Before analyzing the good type's decisions more formally, it is useful to first study the behavior of the bad types when they default in the bottom node of period 1. In this case they are identified as bad (since the good types never default) and so the borrowing constraint (1) derived above binds; since they will therefore never be able to borrow so much that they default in period 2, they thus face an interest rate of $r_{1d} = 1$.

Given their quasi-linear preferences, they would like to borrow b_{1d} so as to maximize

$$\log[(i_1 + b)/p_{1d}] + (i_1 + b)\frac{p_{2d}}{p_{1d}} - b,$$

subject to their borrowing constraint. Given that $p_{2d} = 1/2$ and $i_1 = 1/2$, it is easy to see that the optimal value is $b_{1d} = 2p_1 + 14p_1 - 2$. This never less than the constrained level $b_{\max}(1/2, k) = \frac{4kp_{1d}+1}{4p_{1d}-2}$ (which we derived above) whenever $k \leq 1/2$, so this constraint will in fact always bind when the bad types default and they will borrow $b_{\max}(1/2, k)$.

The resulting utility for these agents will be

$$\log\left[\frac{1/2 + b_{\max}(1/2, k)}{p_{1d}}\right] - 2k = \log\left[\frac{1 + 2k}{2p_{1d} - 1}\right] - 2k; \qquad (3)$$

the term $-2k$ results from the fact that (i) the agents are defaulting in period 1, and then (ii) borrowing their maximum into period 2 (the maximal borrowing leaves them indifferent to defaulting or not in period 2, hence the extra $-k$).

We now turn our attention to the good types; these agents essentially determine the equilibrium allocations. There are three possibilities.

1. *Safe Regime*

   First of all, the good type may choose to restrict his borrowing so that the bad type does not default in period 2 (after repaying in period 1); we term this "safe borrowing," since there will be no default in period 2. That is, he could restrict his borrowing to be no more than $b_{\max}(i_1 - 2(p_{1d} - r_0), k)$, which is defined by:

   $$[i_1 - 2(p_{1d} - r_0) + b_{\max}(i_1 - 2(p_{1d} - r_0), k)] \frac{p_{2d}}{p_{1d}} - b_{\max}(i_1 - 2(p_{1d} - r_0), k) \times r_{1d} = -k,$$

   where now $r_{1d} = 1$. It is not difficult to see that the good type will always want to borrow up to this maximum; to see this, recall that the borrowing constraint (1) was binding for the bad type when he defaults, and when agents repay they are poorer, which means both that their constraint is tighter and that they wish to borrow even more.

   Furthermore, notice the best allocation of this type for both the good and bad agents will occur when everyone repays in period 1 — i.e., when $q = 1$. This is because the initial interest rate r_0 is decreasing in q (reaching a minimum of 1 at $q = 1$) while $r_{1d} = 1$ is constant in q when the borrowing is restricted to ensure that there is no default. This implies that any such equilibrium will have $q = 1$, in which case

$r_0 = 1$ and the borrowing simplifies to

$$b_{\max}(1/2 - 2(p_{1d} - 1), k) = \frac{4(1+k)p_{1d} - 3}{4p_{1d} - 2}.$$

Observe that this is increasing in the bad type's default cost k, since a higher k means that they are more reluctant to default (in period 2) and hence can be trusted to repay more.

It is important to note that this is an admissible candidate equilbrium only when the bad types are indeed willing to repay in period 1 in order to then borrow on these terms. So for the bad types to be willing to repay in period 1, it must be the case that the utility they derive from repaying and pooling with the good types is at least as high as the utility — calculated in (3) above — that they get from defaulting and separating themselves.

That is, the bad types will repay when

$$\log[((i_1 + 2(p_{1d} - 1) + b_{\max}(1/2 - 2(p_{1d} - 1), k)) / p_{1d}] - k \geq \log\left[\frac{1+2k}{2p_{1d} - 1}\right] - 2k$$

i.e.,

$$\log\left[2 + \frac{1-2k}{1-2p_{1d}}\right] - k \geq \log\left[\frac{1+2k}{2p_{1d} - 1}\right] - 2k.$$

Solving this, we can determine that when $q = 1$ and the borrowing is at the maximal safe level, the bad agents are willing to repay when p_{1d} is *above*

$$p_{\text{nodef}} \equiv \frac{1 + e^k\,(3 - 2\,k) + 2\,k}{4\,e^k}.$$

For the extreme case of $k = 0$ we have $p_{\text{nodef}} = 1$, which means that the bad agents would never want to repay under these terms (and so this regime would not exist). In general, however, p_{nodef} will be below 1, since a higher k both makes default more costly and increases the maximal safe borrowing level.

2. *Default Regime*

   Conversely, the good types may want to encourage the bad types to all default in period 1. In this case we would still have $r_{1d} = 1$ (this time because we are in a separating equilibrium), but because $q = 0$ we would have $r_0 = \frac{2-\beta p_{1d}}{2-\beta} > 1$. As before, this is an admissible candidate equilibrium only when the bad types do indeed prefer to default. That is, the utility that the bad types receive from defaulting in period 1 — calculated in (3) above — must exceed that which they would receive from repaying and then pooling with the good types. This implies a restrction both on the good types' borrowing (it must not be too high) as well as on the house price p_{1d} (it must be sufficiently low). In particular, for default to dominate, the good type's borrowing b must be such that

   $$\log\left[\frac{1+2k}{2p_{1d}-1}\right] - 2k \geq \log\left[\frac{i_1 + 2(p_{1d} - r_0) + b}{p_{1d}}\right] - k, \tag{4}$$

   where the right-hand side is the utility the bad types would receive from repaying in period 1.

   When p_{1d} is sufficiently high the solution to (4) is not be an admissible candidate; that is, this regime would not exist. One way to see this is to observe that another alternative that is always available to the good types is to repay but then borrow the fully collateralized level $b_{\max}(1/2 - 2(p_{1d} - r_0), k)$ instead (which they could do even if lenders were aware of their type). When p_{1d} is high the bad types' income after repaying would also be fairly high and so the fully collateralized level would actually exceed the solution to (4). In particular, for the bad types to be willing to default we must have p_{1d} *below*

$$p_{\text{def}} \equiv \frac{6+\beta-4\,k+2\,\beta\,k+\frac{(2-b)\,(1+2\,k)}{e^k}}{8};$$

this is obtained simply by setting the fully collateralized level equal to the solution to (4) and then solving for p_{1d}.

Intuitively, this cutoff is decreasing in the bad types' default cost k (it is equal to $p_{1d} = 1$ when $k = 0$). It is also decreasing in the fraction of bad types β, because the more bad types there are the higher the initial interest rate r_0 and thus the more attractive it is to default. It is not hard to show that $p_{\text{def}} > p_{\text{nodef}}$.[9]

Below this cutoff price, the solution to (4), which is the maximal amount the good types can borrow and still leave the bad types at least indifferent to defaulting, is

$$b_{\text{def}} \equiv \frac{6+\beta-8\,p_{1d}}{4-2\,\beta}+\frac{(1+2\,k)\;p_{1d}}{e^k\;(2\,p_{1d}-1)};$$

it is easy to see that since this is below the good types' optimum, the constraint will in fact bind.

As is evident from this discussion, this regime will arise when the price p_{1d} is sufficiently low, in which case there will be default in equilibrium.

3. *Risky Regime*

The final set of possible allocations occur when there are (at least) some bad types who repay in the initial period, and the borrowing in period 1 by those agents who did repay is greater than the fully collateralized

[9]This reflects the non-convexity that arises in this model; when all of the bad types default, the interest rate is higher, which makes repaying less attractive.

safe level (so that there is default in period 2). In this case we will have $q > 0$ and $r_{1d} > 1$.

The interest rate $r_{1d}(b)$ that is paid in period 2 (on account of period-1 borrowing) is now determined as follows:

$$b = \frac{1-\beta}{(1-\beta)+\beta q} \times r_{1d} \times b + \frac{\beta q}{(1-\beta)+\beta q} \times (i_1 + 2(p_{1d} - r_0) + b)\frac{p_{2d}}{p_{1d}}.$$

The left-hand side is the amount borrowed. On the right we have the amount repaid: the good types repay the entire loan of b (since it will necessarily be less than their borrowing constraint as derived above), while the bad types default and repay only their collateral, which is worth $(i_1 + 2(p_{1d} - r_0) + b)\frac{p_{2d}}{p_{1d}}$ in period 2. It is not hard to derive a (messy) closed-form expression for $r_{1d}(b)$, which is increasing in b, β, and q.

There are two possibilities. Either $0 < q < 1$ (i.e. not all the bad types repay), in which case the bad types must be indifferent between repaying and defaulting in period 1 for this to be an equilibrium, or else $q = 1$ (all repay), in which case the bad types must weakly prefer to repay.

When $0 < q < 1$, then to make the bad types indifferent, the borrowing $b_{\text{indiff}}(q)$ for those who repay in period 1 must satisfy

$$\log\left[\frac{1+2k}{2p_{1d}-1}\right] - 2k = \log\left[\frac{i_1 + 2(p_{1d} - r_0) + b_{\text{indiff}}(q)}{p_{1d}}\right] - k,$$

where now $r_0 = \frac{2-\beta(1-q)p_{1d}}{2-\beta(1-q)}$.

It is possible to show that in this case the good types' utility is maximized as $q \to 0$, for all admissible values of p_{1d}, β or k. That is, the good types would like to ensure that in fact all of the bad types default in period 1. The reason is that the increase in the initial interest rate r_0 this would entail is more than compensated for by the fact that, when the bad agents all default, the good types can borrow risklessly from period 1 to 2 (recall that those agents who default are identified as bad and are not in this pool). This is in fact the default regime we have analyzed above.

So without loss of generality we assume $q = 1$. In this case the bad types must weakly prefer to repay over defaulting in period 1, that is, we must have $b \geq b_{\text{indiff}}(1)$ where

$$\log\left[\frac{1+2k}{2p_{1d}-1}\right] - 2k = \log\left[\frac{i_1 + 2(p_{1d}-1) + b_{\text{indiff}}(1)}{p_{1d}}\right] - k;$$

that is, $b_{\text{indiff}}(1)$ is defined as the minimal borrowing needed to make the bad types willing to repay in period 1 when $q = 1$. In this equation the left-hand side is the utility that the bad types would receive were they to default in period 1 and the right-0hand side gives the utility from repaying and then borrowing $b_{\text{indiff}}(1)$.[10]

A candidate Wilson-Miyazaki equilibrium will be the value of b that maximizes the utility of the good types subject to this constraint as well as subject to the borrowing constraint, which in this case is that the good types must be willing to repay $r_{1d}(b) \times b$; this was derived in (2) above.

When $q = 1$ the good types' utility is

$$\log\left[\frac{i_1 + 2(p_{1d}-1) + b}{p_{1d}}\right] + [i_1 + 2(p_{1d}-1) + b]\frac{p_{2d}}{p_{1d}} - r_{1d}(b) \times b \quad (5)$$

It is not hard to show that, for the parameter values we consider,

[10] The $-k$ reflects the fact that the bad types will then default in period 2.

the borrowing constraint (2) never binds, because as b increases the interest rate $r_{1d}(b)$ increases rapidly enough to make further borrowing unattractive to the good types.

As above, the value of b that maximizes (5) will be an admissible candidate allocation only for house prices p_{1d} that are sufficiently low. The reason is that for this "risky" regime to exist, by definition this solution must exceed the safe level of borrowing $b_{\max}(1/2 - 2(p_{1d} - 1), k)$. When $\beta + k \leq 1/2$ this holds for all values of $p_{1d} < 1$ (because when β and k are low, the safe borrowing level is low relative to the risky level, as discussed below). More generally, we must have

$$p_{1d} \leq \frac{5}{4} - \frac{\beta + k}{2}.$$

In this case the value of b that maximizes the good types' utility (5) is

$$b_{\text{risky}} = \frac{3 - 14\,p_{1d} + 4\,\beta\,p_{1d} + 8\,p_{1d}^2}{2 - 4\,p_{1d}}.$$

In addition, recall that we had a further constraint: the bad types must all be willing to repay in period 1 and borrow b_{risky}. That is, we must have $b_{\text{risky}} \geq b_{\text{indiff}}(1)$ as derived above. This occurs if and only if the fraction of bad types β satisfies

$$\beta \leq 1 - \frac{1 + 2\,k}{2\,e^k}.$$

That is, we must have β and k sufficiently small for the bad types to be willing to repay. The reason is that with high k the bad types can borrow a lot when they default; similarly, when β is high then the adverse selection problem is relatively severe when they repay, which means that they cannot borrow enough to make repayment worthwhile.

4 Equilibrium

4.1 Equilibria of the Borrowing Game

To summarize the discussion of the previous sections, the Wilson-Miyazaki equilibrium is determined by choosing the best allocation for the good types among the following candidates, which we term "Safe" (S), "Default" (D), and "Risky" (R).

The candidate regimes are as follows:

1. **Safe:** All agents repay in period 1 and then borrow $b_{\max}(1/2 - 2(p_{1d} - 1), k)$, which is the maximal safe level (so no agent defaults in period 2). This can occur only when $p_{1d} \geq p_{\text{nodef}}$.

2. **Risky:** Everyone repays in period 1, and then borrows b_{risky}. The bad types default in period 2. This can occur only when $p_{1d} \leq \frac{5}{4} - \frac{\beta + k}{2}$ and $\beta \leq 1 - \frac{1+2\,k}{2\,e^k}$.

3. **Default:** The bad types default in period 1 and then borrow $b_{\max}(1/2, k)$. The good types repay and borrow b_{def}, which leaves the bad types indifferent to defaulting. This can occur only when $p_{1d} \leq p_{\text{def}}$.

In general the choice of which regime is best for the good types must be determined via numerical simulation, which we carry out for several examples below. However, we can make several qualitative statements.

Remark: If we trace out the equilibria as a function of the period-1 housing price p_{1d}, then there are five possible combinations of these regimes, depending on the parameters β and k. They are S-D-R (i.e., all regimes), S-D (no risky), S-R (no default), R-D (no safe), and R (only risky). Figure 2 below characterizes the parameter space in terms of these regions, and we provide further discussion and examples below.

- First of all, the arrangement of regimes is hierarchical, in the sense that as we lower the price p_{1d}, we go from safe, to risky, to default, although not all of these will necessarily occur for any given parameter pair (β, k).
- Since $p_{\text{def}} > p_{\text{nodef}}$, there is always an equilibrium.
- If β and k are sufficiently close to 0, then there is only a risky regime.
- If $1 - \frac{1+2k}{2e^k} < \beta$, there is no risky regime, only safe and default.
- The safe regime is always strictly preferred by the good types over the default regime. So when $p_{1d} > p_{\text{nodef}}$, the safe regime is chosen.

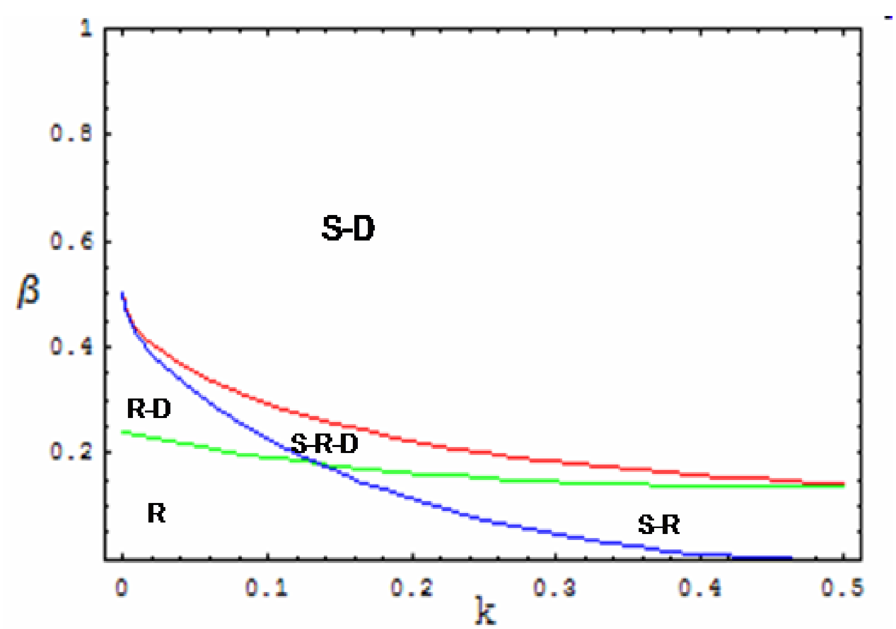

Figure 2: The Regimes

4.2 Accelerators and Decelerators

The various classes of equilibria that we described above lead to very different outcomes in terms of the effect of aggregate fluctuations on house prices.

When we are in the safe regime, total period-1 housing consumption by the strategic consumers is $2 - \frac{1-2k}{2p_{1d}-1}$. Observe that as p_{1d} decreases, consumption also goes down. This is because in the safe regime the equilibrium is determined by the no-default constraint of the risky types which gets tighter as the house price falls (because their remaining income after repaying the initial loan is lower). This regime will generate a classic financial accelerator as in much of the literature.

By contrast, in the risky regime, the consumers' housing demand is $\frac{2(1-\beta)}{2p_{1d}-1}$. No longer is it the case that demand decreases as the price falls. The reason is that in the risky regime the good types allow the bad ones to free-ride and do not try to limit their borrowing to the safe level. While this free-riding reduces the good types' demand (observe the term $1-\beta$ in the numerator), this does not get worse as the price falls.

Finally, in the default regime, total consumption of housing is given by $\frac{1+2k}{e^k(2p_{1d}-1)}$; this is the weighted average of consumption by the good and bad types. As in the risky regime, while adverse selection reduces consumption relative to the first-best (since $k < 1$), this is not exacerbated by declining prices.

In addition to the effect of falling prices within each regime, we also need to compare consumption across regimes. As discussed above, as prices fall we go from the safe, to the risky, and finally the default regime (although not all of these will necessarily occur for all parameter values). It is easy to see that consumption in the risky and default regimes will exceed that of the safe regime when those regimes are chosen by the good types. However, by comparing consumption in the two regimes, it is also not hard to see that consumption in the default regime exceeds that of the risky regime only when

$\beta > 1 - \frac{1+2\,k}{2\,e^k}$, which is precisely when the risky regime does not exist. So going from the risky to the default regime entails a drop in aggregate housing demand.

Thus in our model we can have (i) a "standard" accelerator in the safe regime and (ii) a discrete deceleration and/or acceleration as the equilibrium jumps from one regime to another.

In particular, the combinations of the regimes yield the following as the house price falls from $p_{1d} = 1$:

- R: neither accelerator nor decelerator
- S-D: accelerator for p_{1d} close to 1 and decelerator for lower p_{1d}
- S-R: accelerator for p_{1d} close to 1 and decelerator for lower p_{1d}
- R-D: accelerator for low p_{1d} only
- S-R-D: accelerator for p_{1d} close to 1, then decelerator, then accelerator again for low p_{1d}

The key goal of this paper, developing a model in which equilibrium is characterized by a financial accelerator when house prices are high but by stabilization when prices are low, is characteristic of regimes S-D and S-R, which together make up the majority of the parameter space. Also observe that even when we shift from the risky to the default regime and strategic demand falls, it is still the case that consumption is higher than it would be if the no-default constraint of the safe regime were in effect.

We will give some examples of these regimes in the following sections.

4.3 General Equilibrium

As discussed above, this is a small open economy, in which the cost of funds in this economy is fixed at 1. Thus only the housing prices $p_{1d}, p_{1u}, p_{2d}, p_{2u}$

are determined in equilibrium, with the strategic agents' demand for housing derived from the equilibrium of the borrowing game.

We noted earlier that since utility is quasi-linear in the second period, supply of housing is 2 units, and consumers are assumed to have sufficient funds in period 2, the second-period prices will always be $p_{2u} = 2 = p_{2d}$.

As far as period 1, suppose that the measure of strategic consumers is σ (and $2-\sigma$ for the non-strategic) and that the outside (non-strategic) spending on housing is δ_d and δ_u in the bottom and top nodes. We will focus attention of the bottom node; the top node is discussed in the appendix.

In the safe regime, we have determined that the demand by the strategic agents is $2 - \frac{1-2k}{2p_{1d}-1}$. So the price in the bottom node is determined by:

$$\sigma \times \left(2 - \frac{1-2k}{2p_{1d}-1}\right) + (1-\sigma) \times \frac{\delta_d}{p_{1d}} = 2$$

We would like the law of demand to hold in this economy — that is, as the outside housing demand falls, we would like the price to fall as well. In order for this to be the case, however, it is important that there not be too many strategic consumers in the economy, since in the safe regime their borrowing constraints lead to a financial accelerator in which falling prices decrease their demand. By examining the above market-clearing condition, it can be seen that for the parameter values we consider below, it is sufficient for strategic agents to make up less than two thirds of the population, i.e., $\sigma < 2/3$.

As far as the risky equilibrium, the market-clearing condition is that

$$\sigma \times \left(\frac{2(1-\beta)}{2\,p_{1d}-1}\right) + (1-\sigma) \times \frac{\delta_d}{p_{1d}} = 2$$

Finally, in the default equilibrium we have

$$\sigma \times \left(\frac{1+2\,k}{e^k\ (2\,p_{1d}-1)}\right) + (1-\sigma) \times \frac{\delta_d}{p_{1d}} = 2$$

In addition, since there are non-convexities in this economy (due both to the discrete default decision as well as the discrete default costs) we need to randomize in the standard way when switching between regimes.

Although it is possible to derive (messy) closed-form solutions for the house price in each of these regimes, it is more useful to examine specific numerical examples, which we do in the following section.

5 Examples

In this section we present several examples that illustrate the possible types of equilibria that can occur in our model. We will take half the population to be strategic consumers ($\sigma = 0.5$) and assume the default cost for the bad types is $k = 0.$; this gives us the widest possible range of equilibria.

The first case we consider is $\beta = 0.4$. Referring to Figure 2 above, we can see that these parameters correspond to the $S - D$ equilibria, that is, the safe regime for high values of p_{1d} and the default equilibrium for low values. This combination occurs because (i) with a relatively high value of k ($k = 0.4$) it is not too costly for the good types to restrict their borrowing to the level at which the bad still repay and (ii) with many bad types ($\beta = 0.4$) the risky equilibrium — in which the bad types default — would be very costly for them. Recall that this is the leading case for the paper — we have a financial accelerator for high values of the house price, but stabilization obtains for low prices.

We choose the per capita spending by the outside sector δ_d so as to calibrate the model at $p_{1d} = 1$. We then lower δ_d; the results are plotted below. As we do so, the price clearly falls. Observe that the slope is relatively steep; this reflects the financial accelerator.

When we hit $p_{1d} = p_{\text{nodef}} = 0.85$, the bad types no longer wish to repay and we switch to the default regime. Observe that when we switch regimes the slope of the price as a function of δ is shallower — this reflects the fact that when the bad types default in period 1, their wealth in this period no longer decreases as the house price falls (since lenders seize their collateral). Although lenders recoup some of their losses in the form of a higher interest rate (paid by the good types, who do not default), the rest is paid in the top node (where the price is much higher). Thus the net effect on consumption is positive — that is, default endogenously serves as a stabilizing force.

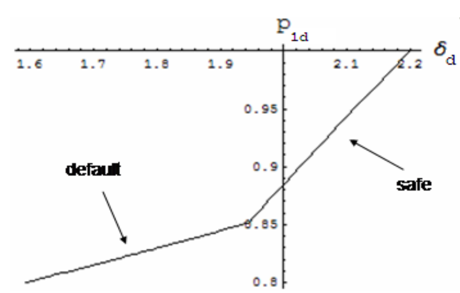

Figure 3: Regimes S-D: $\beta = 0.4$

Now suppose that $\beta = 0.1$. These parameters correspond to the case in which there is no default regime. For high prices we are in the safe regime, with its financial accelerator, and for low prices, we are in the risky regime. Once again default serves as a stabilizing force, although this time it is default in period 2; that is, for low prices the good types find it too costly to restrict their borrowing to the safe level. We perform the same exercise as in the previous example; observe that there is now a flat region between the safe and risky regimes in which we must randomize.

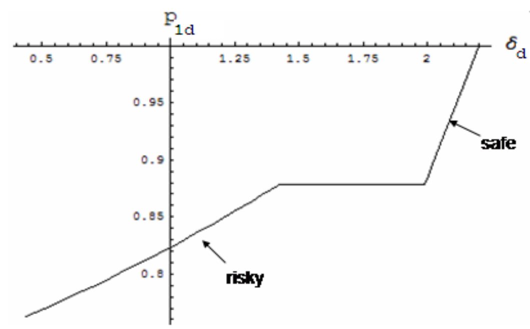

Figure 4: Regimes S-R: $\beta = 0.1$

Finally, let $\beta = 0.15$ and $\sigma = 0.1$.[11] We now have the mixed case S-R-D, in which all of the regimes appear. Notice that there is a discrete jump down when we switch from the risky to the default regime, although both the level of the price is higher than it would have been had we restricted all agents to the safe level regime, and its slope shallower. As discussed above, for parameters in this region we do not have a clean transition from accelerator to decelerator as the price falls.

[11] We needed to lower the fraction of strategic agents in this example so that the non-convexity that occurs when we switch from the risky to the default regime does not force us into the region where the good agents would default as well.

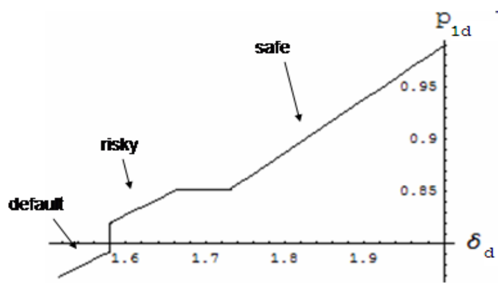

Figure 5: Regimes S-R-D: $\beta = 0.15$

6 Conclusion

In this paper we have developed a model of secured borrowing in which a drop in the value of the underlying collateral can generate strategic default, which in turn can serve to stabilize aggregate fluctuations because it leaves agents with more wealth precisely when the house price is lowest. Strategic default arises in equilibrium because the presence of adverse selection means that default is not always ruled out by binding borrowing constraints.

There are several directions in which this model could be extended. One interesting avenue would be to endogenize the partially collateralized debt contract agents use to borrow. This paper also simplifies the effect of default on the banking sector — obviously a rash of bank failures induced by a sharp increase in borrower default could have serious consequences. Finally, it would be interesting to interact our stabilizer with a simple model of investment in which home equity served to secure business loans, as does indeed seem to be increasingly common. Such a model would also generate interesting tradeoffs and might allow our mechanism to engender positive real effects for the economy as a whole.

7 Appendix - the Top Node

In this section we briefly discuss the top node. Because of the agents' quasi-linear utility functions, the equilibrium housing price in period 2 will be $p_{2u} = 1/2$ (as in the bottom node).

As far as period 1, it is easy to see that the maximal interest rate will be $r_0 = \frac{9}{8}$. In order to keep the analysis focused on the bottom node, we will restrict attention to parameters in which there is no default in the top node, neither in period 1 nor period 2. For this it is sufficient to assume that the outside demand δ_u is chosen so as to ensure that $p_{1u} > \frac{11}{8}$. It is then the case that the optimal borrowing from period 1 to period 2 can always be fully collateralized and the equilibrium in the bottom node would have no effect on the housing price in the top node.

Alternatively, one could allow parameter values that yield a lower equilibrium price p_{1u}. In this case the borrowing would be constrained (as in the analysis of the bottom node above) and then default in the bottom node — which raises r_0 — would reduce wealth in the top node and thereby tighten borrowing constraints in this node and lead to a lower housing price p_{1u}.

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