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ABSTRACT

This paper analyzes the implications of tax policy for the accumulation of human and physical capital and for the overall productivity level of the economy. A comprehensive income tax, applying to both labour income and capital income, discriminates against investments in human capital relative to investments in physical capital. Hence, it has an adverse impact on human capital accumulation. Taking into account a positive external effect of investments in human capital on overall productivity, the adverse effect of income taxation on human capital investments is significantly magnified.

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I. Introduction

In a world with only unskilled labour and physical capital, a comprehensive income tax generates two distortionary effects on resource allocations. It distorts the intratemporal leisure-consumption tradeoffs and the intertemporal saving-consumption tradeoffs. The intratemporal distortion is created by the labour income component of the comprehensive income tax, while the intertemporal distortion is caused by the capital income component. In the presence of investments in human capital the income tax generates additional distortions. First, since the capital income tax component does not apply to investments in human capital, it discriminates against investments in physical capital. On the other hand, since investments in human capital improve the earning capacity of labour, then the wage tax component reduces the net-of-tax rate of return on investments in human capital and thus discriminates against this kind of investment relative to investments in physical capital. Therefore, it may look as if the two components of the comprehensive income tax (when applied with equal rates) offset each other with respect to the choice between the two forms of investment. However, this conclusion is valid only when human capital lasts forever (i.e., people are infinitely lived). When people are *finitely* lived, human capital depreciates to zero at death or even earlier (at retirement). Unlike the case with physical capital, this depreciation of human capital is not commonly tax-deductible. In other words, the wage tax applies to both the yield on *and* the principal of an investment in human capital, while the capital income tax applies only to the yield on an investment in physical capital.

In this paper we analyze the effects of these distortions on overall savings, the composition of savings between investments in human capital and physical capital, and the pattern of economic growth. Section II deals with the asymmetric treatment of human and physical capital by the income tax. Section III develops a stylized overlapping generations model which is used for analyzing the long-run effects of income taxation on investments in human and physical capital. Section IV extends the analysis to include a productivity effect, external to the individual, associated with investments in human capital. Section V provides some concluding remarks.

II. The Income Tax: Differential Treatment of Investments in Human and Physical Capital

Assume an individual who is considering an investment in *human* capital or in *physical* capital. Suppose that by investing H shekels in human capital (education, training, etc.) an individual increases her earning capacity in each period in the future by a factor $g(H)$, where $g(0) = 1$, $g' > 0$, and $g'' < 0$ (namely, there is a positive, but diminishing marginal return to investments in human capital). That is, a unit of her labour time is worth $g(H)$ in units of a standard labour input, priced at w shekels per unit. Alternatively, if she invests in physical capital she earns a return of r shekels per unit at each period in the future.[1] As she allocates her total investment between physical and human capital so as to maximize future returns, it must be the case that she earns at the margin the same return on both forms of investment.

If she lives *indefinitely*, then the net present value of investing an additional unit in human capital is $wg'(H)/r$. The net present value of one shekel invested in physical capital is, by definition, one shekel. Thus, rate-of-return equalization implies that:

$$wg'(H)/r = 1. \tag{1}$$

In this case, if labour income is taxed at the rate θ and capital is taxed at the rate τ, equation (1) becomes:

$$\frac{(1-\theta)w}{(1-\tau)r} g'(H) = 1. \tag{2}$$

Thus, the tax τ on capital income discriminates against investments in physical capital, while the tax θ on labour income discriminates against investments in human capital. In the presence of a comprehensive income tax levied on

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[1] It should be noted that in a case where a firm borrows from the individual in order to invest in physical capital, r will be equal to the marginal product of physical capital less depreciation.

all forms of income (i.e., $\theta = \tau$), the two distortions *offset* each other and the tradeoffs between human and physical capital investments are not affected. (Of course, there still remain the intratemporal leisure-consumption and the intertemporal saving-consumption distortions.)

However, when people are *finitely* lived, then their human capital depreciates to zero at death or even earlier (at retirement). Since this depreciation is not tax-deductible, the income tax is levied essentially both on the investment (principal) and on its return. A person who invests an additional unit in her human capital $T+1$ years before retirement enjoys an additional net-of-tax return in present value of:

$$(1-\theta)wg'(H)\sum_{t=1}^{T}\left[\frac{1}{1+(1-\tau)r}\right]^t = \frac{(1-\theta)w}{(1-\tau)r}g'(H)\left[1-\left(\frac{1}{1+(1-\tau)r}\right)^T\right].$$

The present value of the returns on an additional unit of investment in physical capital is, by definition, one. Hence, maximization of the present value of the net-of-tax returns from all kinds of investments requires:

$$\text{(3)} \qquad \frac{(1-\theta)w}{(1-\tau)r}g'(H)\left[1-\left(\frac{1}{1+(1-\tau)r}\right)^T\right] = 1.$$

In this case, a comprehensive income tax with equal rates applied to labour income and capital income (i.e., $\theta = \tau$) still *distorts* the trade-offs between the two kinds of investments. Since the term in the square brackets of (3) is smaller than one, it follows that the distortion created by the income tax works *against* investments in *human* capital relative to investments in physical capital. Notice that the distortion diminishes the longer the life horizon of the individual [because the term in the square brackets of (3) approaches one]. It vanishes altogether [as we have seen earlier in equation (2)], at the limit when T approaches infinity.

III. The Long-Run Effects of Income Taxation on Investments in Human and Physical Capital

The long-run effects of income taxation on investments in human and physical capital are analyzed in a stylized overlapping generations model.

A. *The Model*

Suppose that each generation lives for two periods. Consider the generation which is born in period t. In order to simplify the notation, assume that there is just one individual in each generation (i.e., zero population growth). She consumes c_{1t} and c_{2t} in the first and the second periods of her life, respectively. Her preferences are given by the conventional utility function $u(c_{1t}, c_{2t})$. She possesses an initial endowment of one unit of a standard labour input in each period of her life. However, by investing H_t units of the consumption good in the first period, she augments her effective labour supply in the second period to $g(H_t)$ units of a standard labour input, where:

$$(4) \qquad g(0) = 1, \; g'(0) = \infty;$$

$$g'(H) > 0, \; g''(H) < 0 \text{ for all } H > 0.$$

(Equivalently, we may assume that investments in human capital are done by foregoing a fraction of the labour supply in the first period.) She can also invest I_t units of the consumption good in the first period and receive $I_t[1 + (1 - \tau_{t+1})r_{t+1}]$ in the second period, where r_{t+1} is the return to physical capital (i.e., the real rate of interest) and τ_{t+1} is the tax rate on capital income in period $t + 1$. Thus, the budget constraints in the first and second periods are, respectively:

$$(5) \qquad c_{1t} + H_t + I_t = (1 - \theta_t)w_t + T_{1t}$$

and

$$(6) \qquad c_{2t} = (1 - \theta_{t+1})w_{t+1}g(H_t) + I_t[1 + (1 - \tau_{t+1})r_{t+1}] + T_{2t},$$

where:

w_t = wage per efficiency unit of labour in period t,

θ_t = tax rate on labour income in period t, and

T_{it} = lump-sum transfer to generation t in the ith period of its life ($i = 1, 2$).

Ignoring corner solutions, constraints (5) and (6) can be consolidated into a single present value budget constraint:

$$(7) \quad c_{1t} + c_{2t}\left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1} = (1 - \theta_t)\, w_t + (1 - \theta_{t+1})\, w_{t+1} \cdot \left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1} g(H_t) - H_t + T_{1t} + T_{2t}\left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1}.$$

Maximizing the utility function $u(c_{1t}, c_{2t})$ with respect to H_t, c_{1t}, and c_{2t} yields the rule for an optimal human capital investment:

$$(8) \qquad (1 - \theta_{t+1})\, w_{t+1} g'(H_t) = 1 + (1 - \tau_{t+1})\, r_{t+1},$$

[the $T = 1$ version of equation (3)], and the consumption demand functions:

$$(9) \qquad c_{it} = C_i\left\{\left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1},\ M_t\right\}, \quad i = 1, 2,$$

where M_t is life-time net-income of generation t, given by:

$$(10) \qquad M_t = (1 - \theta_t)\, w_t + (1 - \theta_{t+1})\, w_{t+1}\left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1} g(H_t) - H_t + T_{1t} + T_{2t}\left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1}.$$

H_t is solved implicitly from (8) to yield:

$$(11) \qquad H_t = H\left\{(1 - \theta_{t+1})\, w_{t+1}\left[1 + (1 - \tau_{t+1})\, r_{t+1}\right]^{-1}\right\}.$$

Notice that equation (8) can be rewritten as:

$$(8a) \qquad (1-\theta_{t+1})\,w_{t+1} + (1-\theta_{t+1})\,w_{t+1}\,[g'(H_t)-1] = 1 + (1-\tau_{t+1})\,r_{t+1}.$$

This version of the rule for optimal investment in human capital clearly demonstrates the *discrimination* against human capital investment that is caused by a comprehensive income tax: while the capital income tax τ applies only to the yield (r) on the investment in physical capital, the labour income tax θ applies to both the principal ($w \cdot 1$) of and the yield [$w \cdot 1 \cdot (g'-1)$] on the investment in human capital.

Turning to the production side of the economy we assume a standard constant-returns-to-scale production function:

$$(12) \qquad Y_t = F(K_t, A_t),$$

where:

Y_t = output in period t,

K_t = physical capital stock in period t, and

A_t = effective labour in period t.

The amount of effective labour in period t consists of the sum of effective skilled labour of generation $t-1$ and the unskilled labour of generation t:

$$(13) \qquad A_t = g(H_{t-1}) + 1.$$

Denoting the capital-effective labour ratio by:

$$(14) \qquad k_t = K_t / A_t,$$

and the output per an efficiency unit of labour by:

$$f(k_t) = F(k_t, 1), \tag{15}$$

the marginal productivity (profit maximization) conditions are:

$$r_t = f'(k_t) \tag{16}$$

and

$$w_t = f(k_t) - r_t k_t. \tag{17}$$

Solving (16) for k_t as a function of r_t,

$$k_t = k(r_t), \tag{18}$$

and substituting (18) into (17) yields the factor-price frontier:

$$w_t = \varphi(r_t), \tag{19}$$

where

$$\varphi'(r_t) = -k(r_t) < 0. \tag{20}$$

(Notice also that $\varphi'' = -k' = -1/f'' > 0$.)

Savings of generation t that are not channelled to investments in human capital form the supply of physical capital in period $t+1$, which at equilibrium must be equal to the demand by the firm (K_{t+1}). Using (5), (13), and (14), this yields:

$$(1-\theta_t)w_t + T_{1t} - c_{1t} - H_t = k_{t+1}[1 + g(H_t)]. \tag{21}$$

To focus on the distortionary effects of income taxation, we make the conventional assumption (in works on excess burden of taxes) that the government does not redistribute income across generations but rather returns in a lump-sum fashion to each person the taxes paid by her in each period, i.e.:

$$(22) \qquad T_{1t} = \theta_t w_t,$$

and

$$(23) \qquad T_{2t} = \theta_{t+1} w_{t+1} g(H_t) + \tau_{t+1} r_{t+1} k_{t+1}[1 + g(H_t)].$$

Employing (9), (10), (11), (18), (19), (21), (22), and (23), the dynamics of the model reduces to just one first-order difference equation in the rate of interest:[2]

$$
\begin{aligned}
(24)\quad & \varphi(r_t) - C_1\Big[[1 + (1 - \tau_{t+1})r_{t+1}]^{-1}, \\
& \qquad \varphi(r_t) + \varphi(r_{t+1})[1 + (1 - \tau_{t+1})r_{t+1}]^{-1} \\
& \qquad \cdot g\big(H\{(1 - \theta_{t+1})\varphi(r_{t+1})[1 + (1 - \tau_{t+1})r_{t+1}]^{-1}\}\big) \\
& \qquad - H\{(1 - \theta_{t+1})\varphi(r_{t+1})[1 + (1 - \tau_{t+1})r_{t+1}]^{-1}\} \\
& \qquad + \tau_{t+1} r_{t+1}[1 + (1 - \tau_{t+1})r_{t+1}]^{-1} k(r_{t+1}) \\
& \qquad \cdot \big[1 + g\big(H\{(1 - \theta_{t+1})\varphi(r_{t+1})[1 + (1 - \tau_{t+1})r_{t+1}]^{-1}\}\big)\big]\Big] \\
& - H\{(1 - \theta_{t+1})\varphi(r_{t+1})[1 + (1 - \tau_{t+1})r_{t+1}]^{-1}\} \\
& \qquad = k(r_{t+1})\big[1 + g\big(H\{(1 - \theta_{t+1})\varphi(r_{t+1}) \\
& \qquad\quad \cdot [1 + (1 - \tau_{t+1})r_{t+1}]^{-1}\}\big)\big].
\end{aligned}
$$

B. *Steady-State Analysis*

Suppose now that the tax rates do not change over time, i.e., $\theta_t = \theta$ and $\tau_t = \tau$ for all t. (Notice, however, that the lump-sum transfers, T_{1t} and T_{2t},

[2] This method was first used by Diamond (1965).

will generally vary over time outside the steady state because the tax bases do.) The long-run steady-state equilibrium of the economy is defined by the dynamic equation (24) when $r_t = r_{t+1} \equiv r^*$. Having solved for the steady-state rate of interest, one can solve for the steady-state values of all other variables: equation (18) defines the capital-effective labour ratio k^*; equation (19) defines the wage rate w^*; equation (11) defines the stock of human capital H^*; equation (9) defines young-age and old-age consumption, C_1^* and C_2^*, respectively; and so on.

Now, let us examine the effect of income taxation on the long-run (steady-state) equilibrium of the economy. We assume that initially no taxes exist, i.e., $\theta = \tau = 0$.

We first employ a small *labour* income tax. In order to find the effect of this change on the steady state, substitute $r_t = r_{t+1} = r^*$ into (24), then totally differentiating with respect to θ and evaluating the derivatives at $\theta = \tau = 0$ yields:

$$\frac{dr^*}{d\theta} = -\frac{m}{a+b}, \tag{25}$$

where $m > 0$, $a < 0$, $b > 0$, and $a + b > 0$ (see the Appendix). Hence, $dr^*/d\theta < 0$. Thus, we conclude that a small wage tax decreases the rate of interest. Hence, such a tax increases the capital-effective labour ratio. This implies that the wage tax discourages investments in human capital relative to investments in physical capital.

Next, consider a small tax on *capital* income. In the Appendix we show that:

$$\frac{dr^*}{d\tau} = -\frac{n}{a+b}, \tag{26}$$

where $n < 0$. Hence, $dr^*/d\tau > 0$. We therefore conclude that a small tax on capital income increases the rate of interest. This means that such a tax lowers the capital-effective labour ratio. That is, the capital income tax discourages investments in physical capital relative to human capital. Observe that the tax on capital income affects the capital-effective labour ratio through two channels.

The first, and standard one, is the negative effect of the tax on savings (the so-called "double taxation of savings"). The second is the positive effect of the tax on investments in human capital through lowering the alternative rate of return.

Now, consider the two components of the *comprehensive* income tax together, so that $\theta = \tau \equiv \beta$. In this case:

$$
(27) \qquad \frac{dr^*}{d\beta} = -\frac{m+n}{a+b}
= \frac{H'w^*(1+k^*g')(1+r^*)^{-1}}{a+b}
- \frac{r^*(1+r^*)^{-2}\{C_{11} + C_{12}[w^*g + k^*(1+g)(1+r^*)]\}}{a+b}.
$$

The first term in the expression for $dr^*/d\beta$ is positive, while the second is negative (assuming gross substitutability). Thus, the sign of $dr^*/d\beta$ cannot a priori be determined. Consequently, the effect of a comprehensive income tax on the capital-effective labour ratio (k^*) cannot be generally established. Even though we observed in the preceding section that an income tax discriminates against investments in human capital vis-à-vis investments in physical capital, nevertheless, since an income tax discourages total savings, its effect on the ratio k^* cannot a priori be determined.

We therefore resort to numerical methods in order to examine the effect of an income tax on the capital-effective labour ratio.

C. *Quantitative Sensitivity Results*

C.1 The Household

To arrive at closed-form analytical solutions for the decision variables entering the microeconomic optimization problem, we employ the following specifications for the human capital production function $g(H_t)$ and the lifetime utility function u:

$$
(28) \qquad g(H_t) = \frac{1}{\gamma}(\gamma + H_t)^\gamma, \quad 0 < \gamma < 1;
$$

and

(29) $$u = \alpha \log c_{1t} + (1-\alpha)\log c_{2t}, \quad 0 < \alpha < 1.$$

These simple functional forms are consistent with all qualitative requirements of the present model and are chosen for expositional convenience only. Maximizing u with respect to H_t, c_{1t}, and c_{2t} subject to the consolidated budget constraint (7) yields the optimal human capital investment and consumption demand functions:

(30) $$H_t = \left[\frac{(1-\theta_{t+1})w_{t+1}}{1+(1-\tau_{t+1})r_{t+1}}\right]^{\frac{1}{1-\gamma}} - \gamma;$$

(31) $$C_{1t} = \alpha M_t,$$

and

(32) $$C_{2t} = (1-\alpha)\left[1+(1-\tau_{t+1})r_{t+1}\right]M_t,$$

where

$$M_t = w_t + w_{t+1}\left[1+(1-\tau_{t+1})r_{t+1}\right]^{-1} g(H_t) - H_t$$
$$+ \tau_{t+1} r_{t+1} k_{t+1}\left[1+g(H_t)\right]\left[1+(1-\tau_{t+1})r_{t+1}\right]^{-1}.$$

M_t is calculated from (10), (22), and (23), and denotes life-time net-income of generation t.

C.2 Production

Utilizing a common specification for the standard constant-returns-to-scale production function Y_t:

(33) $$Y_t = K_t^{\varepsilon} A_t^{1-\varepsilon}, \quad 0 < \varepsilon < 1,$$

entails the following closed-forms for the factor-price frontier $w_t = \varphi(r_t)$ and the capital-effective labour ratio $k_t = k(r_t)$:

$$w_t = (1-\varepsilon)\left(\frac{\varepsilon}{r_t}\right)^{\frac{\varepsilon}{1-\varepsilon}}, \tag{34}$$

and

$$k_t = \left(\frac{\varepsilon}{r_t}\right)^{\frac{1}{1-\varepsilon}}. \tag{35}$$

C.3 Steady-State.Solution

Given the above solutions for w_t, k_t, H_t, $g(H_t)$, and C_{1t}, the basic dynamic equation (24), i.e.:

$$w_t - C_{1t} - H_t - k_{t+1}\left[1 + g(H_t)\right] = 0,$$

can now be solved for the steady-state rate of interest r^*, assuming that $\theta_t = \theta_{t+1} = \theta$, $\tau_t = \tau_{t+1} = \tau$, and $r_t = r_{t+1} = r$ for all t. This is done by numerical methods.[3] Having established the steady-state rate of interest, the long-run equilibrium values of all other endogenous variables of the economy can be determined in a recursive way. Thus, we obtain r^* (implicitly) from (24), the wage rate w^* from (34), the capital-effective labour ratio k^* from (35), human capital investment H^* from (30), physical capital investment I^* from:

$$I_t = k_{t+1}\left[1 + g(H_t)\right], \tag{36}$$

overall savings S^* from $S^* \equiv H^* + I^*$, young-age consumption C_1^* from (31), old-age consumption C_2^* from (32), and, finally, output Y^* from:

$$Y_t = k_t^{\varepsilon}\left[1 + g(H_{t-1})\right]. \tag{33a}$$

---

[3] The algorithm used is based on Broyden's secant method. See Dennis and Schnabel (1983, Chapter 8) for details.

C.4 Quantitative Analysis

For the results reported in Table 1, we choose as a benchmark parameter set:

(37) $$\theta = \tau = 0.2;\ \ \gamma = 0.5,\ \ \alpha = 0.6,\ \ \varepsilon = 0.25.$$

To allow for a direct comparison of the different orders of magnitude, the comparative-static equilibrium effects are presented in elasticity-form.

< *Table* 1 >

Main findings:

- The effects of income taxation on the long-run equilibrium of the economy can be seen from the first three columns of Table 1.
  - As already noted in (25) and (26), a higher labour income tax decreases the steady-state rate of interest, while a higher capital income tax has the opposite effect; thus, the overall impact of a *comprehensive* income tax could not be determined a priori [see (27)]. The sensitivity results of Table 1, however, reveal that the wage-tax elasticity of the steady-state rate of interest, $\eta_{r^*,\theta}$, is much larger (in absolute terms) than the τ-elasticity of the interest rate. In fact, the positive effect of a higher wage tax on the capital-effective labour ratio clearly outperforms the negative effect on k^* of a higher capital income tax. Consequently, a comprehensive income tax *lowers* the equilibrium interest rate, *raises* the capital-effective labour ratio, and hence *discourages* investments in *human* capital relative to investments in physical capital. Note that even though total savings S^* decline with $\beta\,(\equiv \theta = \tau)$, the ratio k^* increases; the negative impact of the comprehensive income tax β on human capital investments H^* is simply too strong.

    Irrespective of the tax considered, the effects of tax changes on human capital investments are always more pronounced than those on physical capital investments. A one percent increase in θ entails a drop in H^* by about 1.3 %, and a rise in I^* by 0.02 %; a one percent increase in τ leads to a rise in H^* by about 0.06 %, and to a fall in I^* by 0.01 %. The direction of these changes might have been expected from the analysis of Section III.B. However, the sheer magnitude of the difference between the respective effects is quite surprising: the relative impact of a change in earnings taxation on H^* is more than 50 times larger than that on I^*; in the case of capital income taxation, the (indirect) effect on H^* is about six times the effect on I^* (in absolute terms). Of course, these ranges bear upon the steady-state impact of a *comprehensive* income tax: a one percent rise in β *diminishes* the steady-state level of human capital investments by 1.2 % and *augments* the steady-state level of physical investments by 0.01 %. Thus, the distortions created by a comprehensive income tax clearly work *against* investments in *human* capital relative to investments in physical capital.

- All tax elasticities of young-age and old-age consumption are weakly positive.
- The steady-state output Y^* declines with θ and increases with τ. A comprehensive income tax *reduces* the equilibrium level of output.

- A rise in the human capital production parameter γ has a strong negative effect on H^* and a slight positive effect on I^*. The steady-state levels of the interest rate, consumption, and output all go down; the capital-effective labour ratio increases.

  A greater preference for young-age consumption affects the long-run equilibrium of the economy in the expected way: human and physical capital investments both decline, first-period consumption increases, second-period consumption falls, so does the level of output. The α-elasticity of the capital-effective labour ratio is negative.

  Finally, enhancing the production elasticity of the capital-input distinctly raises the steady-state rate of interest; nevertheless, since r^* and ε operate on k^* in opposite directions, the ε-elasticity of the long-run equilibrium level of the capital-effective labour ratio is small. Human and physical capital investments react inelastically to a greater relative share of capital in total output. Y^* goes up.

IV. External Productivity Effects

It is often the case that investments in human capital add not only to the individual investor's earning capacity, but also rather add to the *general* state of technological knowledge and know-how. This effect may be formalized as improving the overall productivity level of the economy.[4]

We therefore add a productivity term B_t to the production function (12), which now becomes:

$$Y_t = B_t^{\nu} F(K_t, A_t), \tag{12a}$$

where $0 < \nu < 1$, and F exhibits, as before, constant returns-to-scale. [Notice, however, that the overall production function $B^{\nu}F(K, A)$ exhibits *increasing* returns-to-scale in B, K, and A, the magnitude of which is determined by ν.]

Formally, we assume that the level of productivity at time t is equal to the depreciated level of productivity at time $t-1$ (at rate δ) plus the investment in human capital made in period $t-1$. That is:

$$B_t = (1-\delta)B_{t-1} + H_{t-1}, \tag{38}$$

where $0 < \delta < 1$.[5]

The new modified production function (12a) gives rise to a modified relationship between the capital-effective labour ratio and the rate of interest:[6]

$$k_t = \bar{k}(r_t, B_t), \tag{18a}$$

---

[4] See Hirofumi Uzawa (1965), Assaf Razin (1972), Paul Romer (1986 and 1989), and Robert Lucas (1988). For a recent empirical corroboration see Adams (1990).

[5] Obviously, this spillover effect of the human capital investment introduces an external economy, thereby making the competitive equilibrium pareto-inefficient. Efficiency can be restored by a Pigouvian subsidy scheme; see Lucas (1988). This issue is not dealt with here.

[6] The functions $k(\cdot)$ in (18) and $\bar{k}(\cdot)$ in (18a) are related to each other by $\bar{k}(r_t, B_t) = k(r_t/B_t^{\nu})$.

and to a modified factor-price frontier:[7]

$$w_t = \bar{\varphi}(r_t, B_t). \tag{19a}$$

With these modifications, the basic dynamic equation of the previous case (24) now becomes:

$$\begin{aligned}
(24a)\quad \bar{\varphi}(r_t, B_t) - C_1\Big[&[1 + (1-\tau)r_{t+1}]^{-1}, \\
&\bar{\varphi}(r_t, B_t) + \bar{\varphi}(r_{t+1}, B_{t+1})[1 + (1-\tau)r_{t+1}]^{-1} \\
&\cdot g\big(H\{(1-\theta)\bar{\varphi}(r_{t+1}, B_{t+1})[1 + (1-\tau)r_{t+1}]^{-1}\}\big) \\
&- H\{(1-\theta)\bar{\varphi}(r_{t+1}, B_{t+1})[1 + (1-\tau)r_{t+1}]^{-1}\} \\
&+ \tau r_{t+1}[1 + (1-\tau)r_{t+1}]^{-1}\bar{k}(r_{t+1}, B_{t+1}) \\
&\cdot \big[1 + g\big(H\{(1-\theta)\bar{\varphi}(r_{t+1}, B_{t+1}) \\
&\cdot [1 + (1-\tau)r_{t+1}]^{-1}\}\big)\big]\Big] \\
-H\{(1-\theta)&\bar{\varphi}(r_{t+1}, B_{t+1})[1 + (1-\tau)r_{t+1}]^{-1}\} \\
&= \bar{k}(r_{t+1}, B_{t+1})\big[1 + g\big(H\{(1-\theta)\bar{\varphi}(r_{t+1}, B_{t+1}) \\
&\quad \cdot [1 + (1-\tau)r_{t+1}]^{-1}\}\big)\big].
\end{aligned}$$

Equations (24a) and (38) describe the dynamics of the new productivity-augmented model. In models of this type a steady state is reached when the endogenous variables of the economy (i.e., r_t, w_t, k_t, H_t, B_t, c_{1t}, c_{2t}) grow at constant rates. In general, some assumptions about preferences (such as homotheticity of the utility function) and about the magnitude of ν (which determines the degree of increasing returns to scale) are necessary for sustaining a steady state. In our case (without any further assumptions) one obvious steady state is reached when the level of productivity stays constant over time. That is, the investment in human capital (H^*) at each period is just sufficient to cover the depreciation of the existing stock of B^*:

$$H^* \equiv H\{(1-\theta)\bar{\varphi}(r^*, B^*)[1 + (1-\tau)r^*]^{-1}\} = \delta B^*. \tag{39}$$

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[7] The functions $\varphi(\cdot)$ in (19) and $\bar{\varphi}(\cdot)$ in (19a) are related to each other by $\bar{\varphi}(r_t, B_t) = B_t^{\nu}\varphi(r_t/B_t^{\nu})$.

The steady state is then determined by (39) and (24a), where $r_t = r_{t+1} = r^*$ and $B_t = B^*$ are substituted in.

We resort to numerical methods in order to examine the effect of an income tax on the steady-state productivity level B^*, the capital-effective labour ratio $k^* \equiv \bar{k}(r^*, B^*)$, and the long-run equilibrium values of all other endogenous variables of interest.

At the household side of our model, nothing has changed; the corresponding analytics of Section III.C remains formally the same.

The production side, however, is affected as follows. Combining the new productivity-augmented production function (12a) with the previous specification (33) gives:

$$Y_t = B_t^\nu K_t^\varepsilon A_t^{1-\varepsilon}, \quad 0 < \nu, \varepsilon < 1. \tag{40}$$

This leads to modified closed-form expressions for the factor-price frontier $w_t = \bar{\varphi}(r_t, B_t)$ and the capital-effective labour ratio $k_t = \bar{k}(r_t, B_t)$:

$$w_t = (1-\varepsilon)B_t^\nu \left(\frac{\varepsilon B_t^\nu}{r_t}\right)^{\frac{\varepsilon}{1-\varepsilon}}, \tag{41}$$

and

$$k_t = \left(\frac{\varepsilon B_t^\nu}{r_t}\right)^{\frac{1}{1-\varepsilon}}. \tag{42}$$

The steady-state solution of the productivity-augmented model is found by solving the two basic equations (24a) and (39), i.e.:

$$w_t - C_{1t} - H_t - k_{t+1}\left[1 + g(H_t)\right] = 0,$$

and

$$H_t - \delta B_t = 0,$$

simultaneously for r^* and B^*, given that $\theta_t = \theta_{t+1} = \theta$, $\tau_t = \tau_{t+1} = \tau$, $r_t = r_{t+1} = r$, and $B_t = B_{t+1} = B$ for all t. This, again, is done by numerical methods. Having determined in that way the steady-state rate of interest r^* and the steady-state level of productivity B^*, the new long-run equilibrium values of all other endogenous variables of the economy can be established along the lines of Section III.C. Thus, we obtain the wage rate w^* from (41), the capital-effective labour ratio k^* from (42), human capital investment H^* from (30) [taking into account the modified function (41)], physical capital investment I^* from (36) [noticing (41) and (42)], overall savings S^* from $S^* \equiv H^* + I^*$, young-age consumption C_1^* from (31), old-age consumption C_2^* from (32), and, finally, output Y^* from:

$$(40a) \qquad Y_t = B_t^{\nu} k_t^{\epsilon} \left[1 + g(H_{t-1})\right].$$

For the quantitative results reported in Table 2, we select as a benchmark parameter set:

$$(37a) \qquad \theta = \tau = 0.2; \;\; \gamma = 0.5, \;\; \alpha = 0.6, \;\; \varepsilon = 0.25; \;\; \nu = 0.05, \;\; \delta = 0.2.$$

Again, the comparative-static equilibrium effects are presented in elasticity-form.

< *Table 2* >

The main findings of our simulations are:

- Long-run equilibrium effects of income taxation:
  - The interesting new endogenous variable is the steady-state level of productivity B^*. As can be seen from Table 2, the equilibrium level of productivity reacts *elastically* to wage-tax changes, but *inelastically* to variations in the capital income tax: a one percent rise in θ reduces B^* by 1.6 %, a one percent rise in τ increases B^* by only 0.07 %. Thus, a *comprehensive* income tax *lowers* the steady-state level of productivity.
  - The factor-price frontier and the capital-effective labour ratio both are affected by the new productivity-augmented production function. In fact, the wage-tax and comprehensive-tax elasticities of w^* are subject to a *reversal of sign:* whereas an increase of θ or β raised the

steady-state wage rate in the basic model, it now (i.e., including the impact on B^*) diminishes w^*. The tax elasticities of k^* do not change sign, but significantly change in absolute terms; the comprehensive-tax elasticity of the steady-state capital-effective labour ratio, $\eta_{k^*,\theta}$, goes up by 35 %.

- Since in the productivity-augmented model investments in human capital play a double role (i.e. adding to the individual investor's earning capacity *and* increasing the general state of knowledge, thereby improving the overall productivity level of the economy), it comes as no surprise that H^* now responds substantially *stronger* than before to parameter changes. The negative impact of a comprehensive income tax on equilibrium investments in human capital is reinforced by 26 %. As before, comprehensive income taxation severely discriminates against human capital investments relative to physical capital investments.
- The wage-tax elasticities of young- and old-age consumption both *change sign* in the productivity-augmented model: steady-state consumption now *falls* as θ goes up. In the main responsible for this sign reversal is the new sign of $\eta_{w^*,\theta}$: taking into account the tax effect on the overall productivity level B^*, the equilibrium wage w^* declines with θ; so does M^*, the steady-state level of life-time net-income.
- The negative impact of a comprehensive income tax on the long-run equilibrium level of output is magnified by more than 100 % in the productivity-augmented model.

• The modified production function (12a) or (40) exhibits increasing returns-to-scale, quantified by ν. Raising this parameter leads to a higher steady-state level of productivity as well as to a higher interest rate; consequently, the overall impact on the capital-effective labour ratio is weak. Investments in human capital react positively, investments in physical capital negatively. Consumption and output go up.
There is another new parameter: the rate of productivity-depreciation δ. A higher depreciation rate lowers the equilibrium level of productivity, the steady-state rate of interest, the wage rate, human capital investments, consumption, and output; physical capital investments, total savings, and the capital-effective labour ratio slightly increase.

V. Conclusion

This paper analyzes the implications of tax policy for the accumulation of human and physical capital and for the overall productivity level of the economy. A comprehensive income tax, applying to both labour income and capital income, discriminates against investments in human capital relative to investments in physical capital. Hence, it has an adverse impact on human capital accumulation. Taking into account a positive external effect of investments in human capital on overall productivity, the adverse effect of income taxation on human capital investments is significantly magnified.

APPENDIX

In this Appendix we prove (25) and (26). Totally differentiating (24) yields:

$$(A1) \qquad a dr_t + b dr_{t+1} + m d\theta + n d\tau = 0,$$

where

$$(A2) \qquad a = -k^*(1 - C_{12}),$$

$$(A3) \qquad b = (1+r^*)^{-2}[C_{11} + (C_{12}g + H' + kg'H')(f + k^*)] - k'(1+g),$$

$$(A4) \qquad m = H'w^*(1+r^*)^{-1}(1 + k^*g'),$$

and

$$(A5) \quad n = -r^*(1+r^*)^{-2}\{C_{11} + C_{12}[w^*g + k^*(1+g)(1+r^*)] + H'w^*(1+k^*g')\};$$

C_{11} is the gross cross-price derivative of first-period consumption, and C_{12} is the income derivative of first-period consumption.[8] Thus, by setting $dr_t = dr_{t+1} = dr^*$ and $d\tau = 0$, we conclude that:

$$(A6) \qquad \frac{dr^*}{d\theta} = -\frac{m}{a+b},$$

which is (25).

Notice that $0 < C_{12} < 1$ is required for both first-period and second-period consumption to be normal goods, an assumption which we maintain. This implies that $a < 0$. Now assuming that first-period and second-period consumption are gross substitutes (i.e., $C_{11} > 0$), we conclude that $b > 0$ (recall that $k' = 1/f'' < 0$). Next, we look at the local stability of the steady state. For this purpose, set $d\theta = d\tau = 0$ to conclude that:

[8] In deriving the above equation we make use of (8).

$$(A7) \qquad \frac{dr_{t+1}}{dr_t} = -\frac{a}{b} > 0.$$

Assuming that the steady state is locally stable requires [in view of (A7)]:

$$(A8) \qquad \frac{dr_{t+1}}{dr_t} = -\frac{a}{b} < 1.$$

The latter implies:

$$(A9) \qquad a + b > 0.$$

To prove (26), substitute $dr_t = dr_{t+1} = dr^*$ and $d\theta = 0$ in (A1).

REFERENCES

Adams, James D. (1990): "Fundamental Stocks of Knowledge and Productivity Growth," *Journal of Political Economy*, 98, 673–702.

Dennis, J.E. and Schnabel, R.B. (1983): *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Englewood Cliffs, NJ: Prentice-Hall.

Diamond, Peter A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150.

Lucas, Robert E. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3–42.

Razin, Assaf (1972): "Optimum Investment in Human Capital," *Review of Economic Studies*, 39, 455–460.

Romer, Paul M. (1986): "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94, 1002–1037.

——— (1989): "Capital Accumulation in the Theory of Long-Run Growth," in Robert J. Barro, ed., *Modern Business Cycle Theory*, Cambridge: Harvard University Press.

Uzawa, Hirofumi (1965): "Optimum Technical Change in an Aggregative Model of Economic Growth," *International Economic Review*, 6, 18–31.

Table 1

Comparative Static Analysis of the Long-Run Equilibrium

BASIC MODEL

(Ceteris-Paribus Elasticities: $\eta_{z,x} \equiv \frac{\partial z}{\partial x}\frac{x}{z}$)

z	$\eta_{z,\theta}$	$\eta_{z,\tau}$	$\eta_{z,\beta}$	$\eta_{z,\gamma}$	$\eta_{z,\alpha}$	$\eta_{z,\varepsilon}$
r^*	-0.115	0.012	-0.103	-0.579	0.075	1.419
w^*	0.038	-0.004	0.034	0.193	-0.025	0.077
k^*	0.153	-0.016	0.137	0.773	-0.100	-0.008
H^*	-1.289	0.057	-1.232	-1.693	-0.183	-0.003
I^*	0.023	-0.010	0.013	0.088	-0.119	-0.008
S^*	0.002	-0.009	-0.008	0.059	-0.120	-0.008
C_1^*	0.011	0.072	0.084	-0.359	0.965	0.118
C_2^*	0.005	0.059	0.064	-0.391	-1.531	0.196
Y^*	-0.092	0.002	-0.090	-0.491	-0.044	0.410

Benchmark Parameter Specification:

$\theta = 0.2,\ \tau = 0.2;$

$\gamma = 0.5,\ \alpha = 0.6,\ \varepsilon = 0.25.$

Table 2

Comparative Static Analysis of the Long-Run Equilibrium

PRODUCTIVITY-AUGMENTED MODEL

(Ceteris-Paribus Elasticities: $\eta_{z,x} \equiv \frac{\partial z}{\partial x}\frac{x}{z}$)

z	$\eta_{z,\theta}$	$\eta_{z,\tau}$	$\eta_{z,\beta}$	$\eta_{z,\gamma}$	$\eta_{z,\alpha}$	$\eta_{z,\varepsilon}$	$\eta_{z,\nu}$	$\eta_{z,\delta}$
r^*	-0.234	0.017	-0.216	-0.724	0.059	1.414	0.020	-0.095
B^*	-1.621	0.072	-1.548	-2.041	-0.234	-0.020	0.062	-1.301
w^*	-0.030	-0.001	-0.031	0.105	-0.035	0.075	0.011	-0.055
k^*	0.204	-0.018	0.185	0.829	-0.094	-0.006	-0.008	0.040
H^*	-1.621	0.072	-1.548	-2.041	-0.234	-0.020	0.062	-0.301
I^*	0.034	-0.011	0.023	0.102	-0.119	-0.008	-0.002	0.009
S^*	0.006	-0.010	-0.003	0.065	-0.121	-0.008	-0.001	0.004
C_1^*	-0.068	0.076	0.008	-0.460	0.953	0.115	0.013	-0.064
C_2^*	-0.081	0.063	-0.018	-0.500	-1.544	0.193	0.014	-0.070
Y^*	-0.199	0.007	-0.193	-0.622	-0.060	0.407	0.018	-0.086

Benchmark Parameter Specification:

$\theta = 0.2,\ \tau = 0.2;$

$\gamma = 0.5,\ \alpha = 0.6,\ \varepsilon = 0.25;$

$\nu = 0.05,\ \delta = 0.2.$