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ABSTRACT

This paper presents a new explanation for the negative correlation between *ex post* real interest rates and inflation found in earlier empirical studies. We begin by showing that there is a strong negative correlation between the permanent movements in *ex post* real interest rates and inflation. We argue that such a correlation can arise when people incorporate anticipated shifts in inflation policy into their expectations. Under these circumstances, a shift to lower (higher) inflation will lead to systematically higher (lower) *ex post* real rates. Using new time series techniques we are able to reject the hypothesis that nominal interest rates were unaffected by anticipated switches in inflation policy in the post-war era. To evaluate the impact of these switches, we then calculate the effects of inflationary expectations upon real rates using a Markov switching model of inflation. Inflation forecasts based upon the estimates of this rational model behave similarly to inflation forecasts from the Livingston survey. When *ex ante* real interest rates are identified with the Markov models of inflation, we find that *ex ante* real interest rate does not contain permanent shocks, nor is it related to permanent shocks in inflation.

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The real interest rate is an economic variable that affects all intertemporal decisions and as such influences both aggregate consumption and investment spending decisions. For this reason, an understanding of real rate behavior is vitally important to understanding the economy. Although a rather large literature has examined the behavior of real interest rates together with its link to inflation, the inflation process itself has changed over time.[1] Despite this changing relationship, previous studies have not considered how expectations of these changes would affect the real rate behavior. In this paper we directly address this issue by examining whether real rates were systematically influenced by anticipated shifts in inflation policy over some periods during the post-war era.

We begin by estimating the long run relationship between inflation and the nominal interest rate implicit in the Fisher equation identity. Using recently developed time series techniques, we show that there is a strong negative correlation between the permanent movements in *ex post* real interest rates and inflation. Under the conventional assumption that agents do not expect shifts in inflation policy, this finding implies, first, that the *ex ante* real rate must contain permanent shocks and, second, that these shocks are negatively correlated with the permanent shocks to inflation.

Although these results can arise in various theoretical ways, we propose an alternative (but not mutually exclusive) explanation for the result.[2] We argue that the *ex ante* real rate can significantly differ from the observed *ex post* real rate if people systematically mispredict the inflation process for some periods. We show that these mispredictions can induce both very persistent shocks and can generate negative correlation between *ex post* real rates and inflation. As a result, shocks to the *ex ante* real rates may be completely transitory, and independent of permanent inflation shocks, even though shocks to the *ex post* real rate are not.

The intuition behind our explanation is best illustrated by the simple example shown in Figure 1. Suppose that the *ex ante* real rate were constant at 2% and that the Livingston survey measured true inflationary expectations exactly as shown in the top panel of the figure. According to the Fisher identity, the nominal interest rate would be the *ex ante* real rate of 2% plus the Livingston survey measure of expected inflation. Since the *ex post* measured real rate is the nominal rate minus actual realized inflation, the *ex post* real rate would be the real rate of 2% plus the Livingston survey measured expectations minus actual inflation. The lower panel of the figure plots the actual inflation together with the *ex post* measured real rate. The *ex post* real rate clearly deviates from the true *ex ante* real rate since the *ex post* realized inflation differs systematically from expected inflation. We refer to this difference between the actual realized inflation and the expected inflation as the forecast residual. As the figure makes clear, the *ex post* real rate and inflation are negatively correlated even though the *ex ante* real rate is constant because the forecast residual and inflation are positively correlated.

Importantly, we show below that the inflation forecast residual will systematically differ from zero during periods when expectations incorporate anticipated future switches or learning about past switches in the inflation process. Thus, systematic deviations between *ex post* and *ex ante* real rates can in fact be consistent with rational behavior.

Our analysis also reveals that inflation forecast residuals may appear to contain permanent shocks during periods when expectations incorporate anticipated future switches or learning about past switches in inflation policy. In this case, the observed high persistence of shocks in *ex post* real rates could arise from persistence in the inflation forecast residual, not necessarily from the *ex ante* real rate. To investigate this possibility directly, we use information about monthly interest rate forecasts, and hence inflation forecasts, that are contained in forward interest rates. In particular, we test a condition that must hold under conventional rational expectations. Interestingly, we reject this hypothesis for the interest rate forecasts implicit in forward rates contracted ahead for one month, three months, six months, nine months, one year, and two year horizons. This evidence indicates that persistence in the inflation forecast residuals indeed contributes to the observed persistence in the *ex post* real rate.

We then examine the behavior of inflation in the post-war period to determine whether the presence of systematic inflation forecast residuals could be rational. We find evidence that the inflation process shifted within the sample. These shifts appear to be captured by a Markov process of shifts in inflation regimes. When *ex ante* real rates are identified with the Markov forecasts of inflation, we find that the *ex ante* real interest rate does not contain permanent shocks, nor is it related to permanent shocks in inflation.

The structure of the paper is as follows. Section I presents the results from estimating the long run relationship between inflation and the nominal interest rate implicit in the Fisher equation identity. Section II illustrates how systematic forecast residuals of inflation can generate these results and tests for the presence of these systematic residuals. In Section III, we present evidence of structural breaks in the inflation process and estimate a Markov switching process that appears to characterize these shifts. Section IV re-evaluates the long run relationship between inflation and the nominal interest rate when economic agents rationally anticipate a shift in the inflation regime. Concluding remarks follow.

I. The Real Interest Rate without Expected Shifts in Inflation Policy

Research on real rates has frequently used the "Fisher equation" as an identity to uncover the time-varying behavior of the unobserved *ex ante* real rate and its term structure.[3] This equation originated with Fisher (1930) who posited that the nominal interest rate equals the real rate of interest plus expected inflation. Based upon this identity, empirical studies have typically found that real rates are negatively correlated with inflation. To motivate our investigation, we begin by ignoring the possible effects of anticipated inflation switches and, in concert with previous empirical studies, assume that inflationary expectations are identified in the standard way. Under this assumption, we examine the implied behavior of the *ex ante* real rate using new time series techniques.

A. Unit Root Disturbances and Implications from the Fisher Equation Identity

The recent recognition that short term nominal rates and inflation rates contain unit roots has introduced a new dimension into the empirical study of real rates. To corroborate results found elsewhere in the literature, Table 1 reports unit root tests for nominal interest rates and inflation.[4] We use interest rates sampled monthly from the McCullough data set from January 1947 to February 1987. The inflation rates are calculated from the CPI-X series published by the Bureau of Labor Statistics constructed by the Congressional Budget Office. This series adjusts for mortgage payments and has been used in other studies of real interest rates such as Huizinga and Mishkin (1986). Since we will be interested in measures of expectations below, we also report unit root tests based upon bi-annual observations using the Livingston survey of inflation, denoted $E(\pi \mid L)$.

Column 2 in the table reports the augmented Dickey Fuller (ADF) test statistics. The bottom panel of the table shows the critical values at 1% and 5% marginal significance levels. As the ADF statistics show, the hypothesis of a unit root is not rejected at the 1% marginal significance level for any of the variables. Furthermore, it is rejected at the 5% level only for the nine month inflation rate.

Column 3 presents the minimum t-statistics developed in Zivot and Andrews (1990). These statistics allow us to ask whether a structural shift in the mean of the variables can make the process appear to contain a unit root.[5] The results are quite similar to the ADF results. For example, while most of the variables do not reject a unit root, the hypothesis is rejected for nine month inflation only at the 1% and not the 5% marginal significance level. On the other hand, the Livingston survey data now appears to reject a unit root after allowing for a shift. It should be noted, however, that the Livingston data are bi-annual and may be subject to small sample problems.

The results in Table 1 confirm the typical finding that inflation and nominal interest rates appear to contain unit root disturbances, generating long run trends in these variables. We therefore begin our analysis by directly testing the implicit long run relationship between the nominal rate and inflation using recent techniques for parameter inference in time-series processes containing unit roots.

As in standard studies of the real rate, it will prove convenient to use the Fisher equation to make inferences about the behavior of the unobserved *ex ante* real rate:[6]

$$R^k(t) = r^k(t) + E(\pi^k(t) \mid t) \tag{1}$$

where $R^k(t)$ is the yield on a k-period discount bond purchased at time t, $r^k(t)$ is the *ex ante* real rate of return over k periods, $E(. \mid t)$ denotes the market's expectations conditional upon information available at time t, and $\pi^k(t)$ is the inflation rate over the maturity of the bond defined as $(\ln[p(t+k)] - \ln[p(t)])/k$, where $p(t)$ is the price level at time t.

The presence of the unobservable inflationary expectation in (1) implies that the *ex ante* real rate is unobservable as well. To address this problem, the standard approach in the literature is to assume that economic forecasts are unbiased so that: $\pi^k(t) = E(\pi^k(t) \mid t) + \epsilon(t+k)$, where ϵ is a forecast residual uncorrelated with all current information. Thus, substituting actual for expected inflation, equation (1) may be written as:

$$R^k(t) = \pi^k(t) + r^k(t) - \epsilon(t+k). \tag{1'}$$

In (1'), only the nominal rate, R^k, and inflation, π^k, are observable. The difference between these two variables is identically the *ex post* real rate comprised of the *ex ante* real rate, r^k, and the inflation forecast residual, ϵ.

Since inflation and nominal interest rates contain unit root disturbances, we can use recent time series techniques to infer whether the *ex post* real rate contains shocks with the same degree of persistence as these variables. In particular, the relationship between the unit root components in these two variables can be found by regressing one of the variables on the other, through a cointegrating regression.[7] The intuition may be described by using the example of the following cointegrating regression of R^k on π^k,

$$R^k(t) = \alpha_0 + \alpha_1 \pi^k(t) + v(t+k). \tag{2}$$

Parameter estimates from cointegrating regressions have different interpretations than ordinary regressions on stationary variables.[8] For a set of variables with unit roots, these regressions provide parameter estimates of the particular linear combination of the variables that will be stationary. In other words, the cointegrating regression of R^k on π^k will provide an estimate of α_1 such that $R^k - \alpha_1 \pi^k$ is a stationary variable without any unit root components.[9] We will follow the literature and denote such a stationary process as I(0).

Comparing the identity in (1') with the cointegrating regression in (2) reveals that we must find $\alpha_1 = 1$ if the *ex post* real rate is I(0) stationary since, by construction, $R^k - \alpha_1 \pi^k$ is an I(0) process. Therefore, the *ex post* real rate may be written in terms of the cointegrating regression (2) as:

$$r^k(t) - \epsilon(t+k) \equiv R^k(t) - \pi^k(t) = -(1 - \alpha_1)\, \pi^k(t) + \text{I(0) process.} \tag{2'}$$

When α_1 equals unity, the *ex post* real rate is simply a stationary process. On the other hand, if $\alpha_1 \neq 1$, then the *ex post* real rate contains the same unit root component as inflation. Note that under standard rational expectations assumptions, non-overlapping forecast errors follow white-noise, a stationary process. Hence, conventional assumptions about expectations would imply that rejecting $\alpha_1 = 1$ must mean that the *ex ante* real rate is subject to permanent unit root disturbances.

B. The Empirical Results

Table 2 reports results from estimating the cointegrating regressions in (2) for maturities of *k* equal to 1, 3, 6 and 9 months and for one and two years. We first tested for cointegration between the nominal interest rates and inflation. The second column labeled λ_{max} presents the maximum eigen value tests developed by Johansen (1988) for the hypothesis that the variables are not cointegrated. Comparing these statistics with the critical values makes clear that this hypothesis is strongly rejected for interest rate maturities up to one year. Although the hypothesis cannot be rejected for longer maturities, the power of the tests are lower since we have fewer independent observations at these horizons.

The third column labeled α_1 reports the estimates and the standard errors of the cointegrating regression in (2). To obtain parameter estimates and standard errors that correct for the problem of finite sample bias present in cointegrating equations, we used the method developed in Stock and Watson (1989).[10] As these estimates show, the hypothesis that $\alpha_1 = 1$ is strongly rejected. The fourth column reports the sum of the coefficients on the leads and lags of the first differences of inflation, used in the Stock-Watson correction.

Since the estimates of α_1 are less than one, they indicate that the negative correlation between the short term movements of the real rate and inflation found in the literature carries over to their long term movements as well. From (2'), the unit root component of the *ex post* real rate, $R^k - \pi^k$, can be written as: $-(1 - \alpha_1)\,\pi^k$. Thus, positive permanent shocks to inflation translate into permanent shocks in the *ex post* real rate according to the magnitude, $-(1 - \alpha_1)$. Since $\alpha_1 < 1$, positive unit root shocks in inflation correspond to negative unit root shocks in the *ex post* real rate. Under standard rational expectations assumptions, (2') also shows that the *ex post* real rate is equal to the *ex ante* real rate and a stationary I(0) term. From this perspective, the results in Table 2 imply that *ex ante* real rates contain permanent unit root shocks and that these shocks are negatively correlated with the unit root shocks to inflation.

II. Rational Expectations with Systematic Inflation Forecast Residuals

The results above indicate that *ex post* real rates contain shocks with the same degree of persistence as nominal rates and inflation and that *ex post* rates are negatively related with inflation in the long run. While standard rational expectations assumptions imply *ex ante* rates are responsible for this behavior, an alternative explanation is easily apparent by recognizing that *ex post* real rates are equal to *ex ante* real rates less the inflation forecast residual. The alternative may be that these residuals display considerable persistence and are positively correlated with inflation. Interestingly, Figure 1 shows that the forecast residuals calculated from the Livingston survey have these characteristics, providing suggestive evidence of our alternative explanation. We now turn to examine this possibility in detail.

We begin by considering how expectations are affected by anticipated shifts in inflation. This analysis reveals that inflation forecast residuals may appear to contain permanent shocks during periods when expectations incorporate anticipated shifts in inflation. We then use the information about inflationary expectations contained in forward rates to develop tests for the presence of forecast residuals with these characteristics.

A. Switches in the Inflation Process and Expectations

How anticipated shifts in inflation affect expectations is best explained with an example. Suppose people believed that inflation over the next k months could follow one of two processes, the current regime labeled "C" that is currently generating inflation, or an alternative regime labeled "A". For example, Americans who experienced high inflation of the 1970s may believe that episodes of high inflation could happen again, even after the decline in inflation during the early 1980s. Under these circumstances, we can write the expected k month inflation rate beginning n periods ahead as:

$$E(\pi^k(t+n) \mid t) = (1 - \lambda^n(t))\, E(\pi^k(t+n) \mid t,C) + \lambda^n(t)\, E(\pi^k(t+n) \mid t,A) \tag{3}$$

where $\lambda^n(t)$ is the probability that the inflation process will shift between time t to $t+n$. $E(. \mid t,C)$ and $E(. \mid t,A)$ are expectations conditional upon time t information as well as the current regime, C, or the alternative regime, A, respectively.[11]

To examine the implications of these expectations, we require a link between the expected inflation conditional upon each process. For this reason, we define both the ratio of the expected future inflation conditional upon the alternative regime to the same expectation conditional upon the current regime as,

$$\mu^{k,n}(t) \equiv E(\pi^k(t+n) \mid t,A)/E(\pi^k(t+n) \mid t,C),$$

and a term measuring the deviation of expectations from the current regime as, $\delta^{k,n}(t) \equiv \lambda^n(t)(1 - \mu^{k,n}(t))$.

With these definitions, we can rewrite (3) as,

$$E(\pi^k(t+n) \mid t) = (1 - \delta^{k,n}(t))\, E(\pi^k(t+n) \mid t,C). \tag{4}$$

If economic agents do not expect any switch in inflation, then $\delta^{k,n}(t) = 0$ for all t, k, and n.

Equation (4) allows us to examine the implications of anticipated switches on the inflation forecast residuals. For the purposes of illustration, suppose that realizations of the inflation process during the sample period arise from the current regime, denoted as $\pi_C(t)$.[12] In this case, the forecast residuals are:

$$\begin{aligned} \pi_C^k(t+n) - E(\pi^k(t+n) \mid t) &\equiv \epsilon(t+n+k) \\ &= [\pi_C^k(t+n) - E(\pi^k(t+n) \mid t,C)] + \delta^{k,n}(t)\, E(\pi^k(t+n) \mid t,C) \end{aligned} \tag{5}$$

The first component on the right hand side is the forecast residual conditioned upon the current process. When expectations are formed rationally, this component will be a stationary I(0) process with mean zero. The second component will be non-zero during periods when expectations incorporate anticipated shifts in inflation ($\delta^{k,n}(t) \neq 0$). In general, this term will contain permanent shocks since inflation contains a unit root.[13] Thus, unlike the standard rational expectations assumption, equation (5) demonstrates that inflation forecast residuals may contain permanent shocks during periods when expectations incorporate anticipated switches in the inflation process.

Although this example makes clear why the presence of anticipated shifts in inflation may induce persistence in forecast residuals and hence explain the results in Table 2, the cointegrating regressions reported there do not provide sufficient information to determine the validity of this explanation. Instead we require a direct test for the presence of serially correlated forecast residuals with unit root persistence. For this test, we need the information in forward rates.

B. Forward Rate Forecasts

Forward rates contain information about expected future nominal rates. They therefore contain information through the Fisher equation about both expected future real rates and expected future inflation. To see this, define $F^{k,n}(t)$ as the rate of return on a forward contract bought at time t for a k period bond at time $t+n$ and $\Theta(t)$ as the difference between the expected future spot rate and the forward rate:[14]

$$E(R^k(t+n) \mid t) = F^{k,n}(t) + \Theta(t). \tag{6}$$

Then, leading the Fisher identity in (1) n periods forward, taking conditional expectations, and combining the resulting expression with (6) yields:

$$F^{k,n}(t) + \Theta(t) = E(R^k(t+n) \mid t) = E(r^k(t+n) \mid t) + E(\pi^k(t+n) \mid t) \tag{7}$$

In standard theoretical models and empirical studies of the risk premium, $\Theta(t)$ is assumed to be a stationary I(0) process.[15] Therefore, the forward rate can be viewed as a noisy measure of the expected future interest rate where the noise is a stationary term. As (7) makes clear, the forward rate also contains expected future future inflation.

We can use this relationship to investigate whether forecasts of inflation imbedded in forward rates are affected by anticipated shifts in inflation. For this purpose we first substitute the expected inflation from (4) into the right hand side of (7) and rewrite the nominal rate as:

$$\begin{aligned} E(R^k(t+n) \mid t) &= E(r^k(t+n) \mid t) + (1 - \delta^{k,n}(t))\, E(\pi^k(t+n) \mid t,C) \\ &= E(R^k(t+n) \mid t,C) - \delta^{k,n}(t)\, E(\pi^k(t+n) \mid t,C) \end{aligned} \tag{8}$$

where $E(R^k(t+n) \mid t,C) = E(r^k(t+n) \mid t,C) + E(\pi^k(t+n) \mid t,C)$.[16] Next, we equate (8) with the left hand side of (7), and subtract the actual nominal rate conditioned on the current process, R^k_C, from both sides. Rearranging the resulting expression gives

$$R^k_C(t+n) = F^{k,n}(t) + \delta^{k,n}(t)E(\pi^k(t+n) \mid t,C) + \Theta(t) + e(t+n), \tag{9}$$

where $e(t+n) = (R^k_C(t+n) - E(R^k(t+n) \mid t, C))$.

Equation (9) shows why the relationship between nominal interest rates and forward rates can be used to examine whether inflationary expectations are affected by anticipated switches in inflation. In the absence of anticipated inflation switches ($\delta^{k,n}(t) = 0$), the excess returns, $R^k(t+n) - F^{k,n}(t)$, are equal to the sum of the risk premia, Θ, and the error term, e, which are both stationary I(0) processes. On the other hand, if expectations are affected by anticipated inflationary policy switches, excess returns will also depend on $\delta^{k,n}(t)E[\pi^k(t+n) \mid t,C]$. These expectations are captured by the second component in the inflation forecast residuals shown in (5). Hence, if inflation forecast residuals contain permanent components because expectations incorporate anticipated switches in the inflation process, this same permanent component should be detectable in excess returns.

C Testing for the Effects of Anticipated Inflation Switches

We use equation (9) to construct two tests for the presence of anticipated switches in inflation. Since the results in Table 1 indicate that both nominal rates and forward rates contain unit root components, these tests are constructed in terms of restrictions on the parameters of cointegrating regressions.

<u>Test 1</u>: For the first test we consider the cointegrating regression:

$$R^k(t+n) = d_o + d_1 F^{k,n}(t) + u(t+n). \tag{10}$$

As in equation (2), this regression will estimate the parameter d_1 such that $R^k(t+n) - d_1 F^{k,n}(t)$ is an I(0) stationary process by construction.

Combining (9) and (10), excess returns may be written as:

$$\delta^{k,n}(t)E(\pi^k(t+n) \mid t,C) + e(t+n) + \Theta(t) = R^k(t+n) - F^{k,n}(t) = -(1\text{-}d_1)F^{k,n}(t) + I(0) \text{ terms.}$$

Under the null hypothesis of no anticipated switches ($\delta^{k,n}(t) = 0$), the terms on the left hand side are stationary I(0) processes. On the other hand, the right hand side will only be stationary when $d_1 = 1$, because the forward rate contains a unit root. Thus, the null hypothesis of no unit roots in forecast residuals amounts to the hypothesis: $d_1 = 1$. Note that this hypothesis test is a necessary but not sufficient condition for no anticipated shifts in inflation, and therefore a rejection provides strong implications. Under the alternative hypothesis that the inflation forecast residual contains a unit root component, the left hand side will also contain a unit root. In this case, we would find $d_1 \neq 1$ since inflationary expectations $E(\pi^k(t+n) \mid t,C)$ are likely to share a unit root component in common with the forward rate with magnitude $(1-d_1)$.

Table 3 reports the results from estimating the cointegrating regression (10) with the Stock-Watson procedure. The table reports the results for various maturities (k) and for forecast horizons (n) of one, three, six, nine, twelve, and 24 months. The second column reports the Johansen test for cointegration between spot and forward interest rates. As these statistics show, the hypothesis of no cointegration is strongly rejected. The third column reports the estimates of d_1 along with their standard errors. The coefficient estimates for d_1 are less than one in all cases. Furthermore, the hypothesis that these estimates equal one, given in the last column, are rejected at standard significance levels except for the nine month ($k=9$) bill at the three month forecasting horizon ($n=3$).[17]

The results in Table 3 demonstrate that excess returns contain a unit root component in common with the forward rate, a component that is empirically small since d_1 is close to one. Our example of expectation formation above suggests that this component could arise from systematic forecast residuals in inflation. For more direct evidence on this point we consider a second cointegration test.

Test 2: To construct our second test we rewrite equation (9) as

$$R^k_C(t+n) = F^{k,n}(t) + \delta^{k,n}(t)\pi^k_C(t+n) + \Theta(t) + \bar{e}(t+n+k), \tag{9'}$$

where $\bar{e}(t+n+k) = (R^k_C(t+n) - E(R^k(t+n) \mid t,C)) + \delta^{k,n}(t)(E(\pi^k(t+n) \mid t,C) - \pi^k_C(t+n))$ and $\pi_C(t)$ is the realized rate of inflation based upon the current process.

We can test whether the systematic deviation between actual and expected nominal rates is related to permanent shocks to inflation by writing (9') in terms of the cointegrating relationship:

$$R^k(t+n) = a_o + a_1 F^{k,n}(t) + a_2 \pi^k(t+n) + w(t+n+k). \tag{11}$$

The cointegrating regression estimates the parameter vector such that: $R^k - a_1 F^{k,n} - a_2 \pi^k$ is a stationary I(0) process. Under the null hypothesis of no anticipated shifts in inflation, $a_1 = 1$, $a_2 = 0$, and $w(t+n+k)$ is the sum of stationary risk premia and forecast errors; i.e., $w(t+n+k) = \Theta(t) + (R^k_C(t+n) - E(R^k(t+n) \mid t, C))$. If we reject this hypothesis, it implies that some component of $w(t+n+k)$ contains a unit root in common with the forward rate and/or inflation.

Table 4 reports the results from the cointegrating regression in equation (10). The column under λ_{max} reports the Johansen tests for cointegration. The test rejects the hypothesis of no cointegration at the 5% critical values up to the nine month ahead forecast of the three month interest rate. As before, the hypothesis of no cointegration cannot be rejected for the one year and two year ahead horizons, although the fewer number of independent observations at these horizons make the power of the test suspect.

The columns marked a_1 and a_2 report the point estimates of the coefficients on the forward rate and inflation, respectively. As in Table 3, all of the coefficients on the forward rate have point estimates less than one. Furthermore, all of the a_2 coefficients on inflation have point estimates that are positive, suggesting that inflation and its forecast errors are positively related, as the intuition in Figure 1 indicates.

The last four columns report the marginal significance levels for Wald tests of the null hypothesis. The first set of hypothesis tests are based upon an estimate of the asymptotic variance-covariance matrix of the parameters. Since our data sample may be too small to produce reliable estimates of this matrix, our test statistics may be contaminated by small sample bias. Therefore, the second set of hypothesis tests are based upon a bootstrap empirical distribution of the coefficient estimates from our sample. Details of the bootstrapping procedure are provided in the Appendix C.

We examined two different hypotheses. The first hypothesis tests whether $a_1 = 1$. The second hypothesis is the joint hypothesis that $a_1 = 1$ and $a_2 = 0$. Both hypotheses are strongly rejected at marginal significance levels less than 3% at all maturities using either the asymptotic or bootstrap distributions to calculate the significance levels.

Overall, the results in this section show that the joint behavior of forward rates, nominal rates, and inflation strongly reject the conventional wisdom about the relationship between these variables. The results of our first test, in Table 3, show that excess returns contain an empirically small unit root component. The results of our second test, in Table 4, indicate that this component is related to realized inflation. As our example of expectation formation shows, we would expect to find both sets of results if people incorporated the effects of anticipated switches in the inflation process into their expectations.

III. Switches in the Inflation Process

The results in Table 3 and 4 are easily interpretable if people's expectations incorporated the effects of anticipate shifts in the inflation process. However, for these expectations to be rational, the true process of inflation must in fact experience periodic shifts. In this section, we present evidence that the inflation process shifted over the sample and estimate a model that incorporates these shifts. The model is then used to analyze the effects of anticipated switches upon inflation forecast residuals.

A. Are There Structural Breaks in the Inflation Process?

Two methods were used to investigate whether the inflation rate over the post-war period contained structural breaks. We examined the Brown, Durbin, and Evans (1975) CUSUM statistics and conducted formal tests of structural instability developed by Hansen (1991).

The CUSUM statistics were calculated as the cumulated sum of recursive residuals from a rolling regression AR(3) model for the first difference of quarterly inflation. This model allows for the unit root in the level of inflation and appears to capture all the serial correlation in the first difference when estimated over the whole sample. If there are no structural breaks in the inflation process, the CUSUM statistics plotted in Figure 2 should remain fairly stable. The figure shows that this is not the case. The statistic shifts upward sharply with realized inflation around the first oil price shock in 1974. This signifies that forecasts of inflation from the rolling regressions using data to this point tended to systematically underestimate realized inflation. Between 1974 and 1985 the CUSUM statistics appear relatively more volatile. The statistic is characterized by a strong upward swing in 1981 and a sharp downward jump in 1983. This graphical evidence suggests that the inflation process shifted upward in 1974 and perhaps in 1981, became more variable from 1974 through about 1985, and shifted down around 1983.

To test parameter constancy more formally, we used the L statistic proposed in Hansen (1991) to test for parameter stability against the null hypothesis that the parameters follow a martingale process (with unknown breakpoints). The test statistics were based on the estimates of the AR(3) model for the first difference of quarterly inflation (estimated over the whole sample) shown in Table 5. The lower portion of the table presents the L statistics for the constancy of different combinations of parameters.

These statistics reveal that we can reject the null hypothesis of no structural instability in at least the variance and the autoregressive parameters at the 5% significance level.[18]

B. A Markov Model of Inflation Regimes

To this point our findings suggest that any successful time series characterization of inflation should allow for both the presence of a unit root and for discrete shifts in the process. In addition, the graphical evidence from the CUSUM statistics suggest that an upward shift in the process takes place around 1974, followed by higher variability in the process, and a downward shift around 1983.

To capture these features, we estimated a number of Markov switching processes for quarterly inflation. After conducting a variety of specification tests, the following specification appeared the most accurate representation:

$$\pi_t = \pi_{1,t} S_t + \pi_{0,t} (1 - S_t) \tag{12}$$

where $$\pi_{1,t} = \pi_{1,t-1} + v_{1,t} \qquad v_{1,t} \sim N(0,\sigma^2_{v,1}),$$

and $$\pi_{0,t} = m_0 + m_1 \pi_{0,t-1} + v_{0t} \qquad v_{0,t} \sim N(0,\sigma^2_{v,0}).$$

Switches between the two process, $\pi_{1,t}$ and $\pi_{0,t}$, are governed by the state variable, S_t, that follows a Markov process and takes on the value of one or zero. Inflation follows a driftless random walk process when $S_t = 1$, and an autoregressive process (with $|m_1| < 1$) when $S_t = 0$.

Table 6 presents the maximum likelihood estimates of the model together with some of the specification tests.[19] The table shows that all of the parameters are precisely estimated. The variance of shocks to $\pi_{1,t}$, is considerably higher than the variance of shocks to $\pi_{0,t}$. The unconditional mean of $\pi_{0,t}$, the stationary process, is 1.6% measured at annual rates. The estimates of the transition probabilities that govern the dynamics of S_t indicate that both states are characterized by a great deal of persistence. The probability of remaining in the unit root state from one quarter to the next is about 94% while the probability of remaining in the stationary state is about 96%.

The lower panel of Table 6 reports some specification tests. The first two rows test whether there is serial correlation in the v_{1t} and v_{0t} residuals. These chi-squared statistics are Lagrange Multiplier (LM) tests of a zero restriction on the first-order autocorrelation of the residuals. As the table shows, this restriction is not rejected for either state. The second two rows report statistics for conditional heteroskedasticity in the residuals. The reported statistics are LM tests for the hypothesis that the ARCH coefficient is zero. Again, this restriction is not rejected at standard confidence levels.

Our estimated model allows the process of inflation to shift discretely rather than restricting only the dynamics to change. This fact can be seen easily if we rewrite the model as:

$$\pi_t = S_t \pi_{t-1} + (1 - S_t)(m_0 + m_1 \pi_{t-1}) + S_t v_{1t} + (1 - S_t) v_{0t} \\ + \phi [S_t(1 - S_{t-1}) - m_1 (1 - S_t)S_{t-1}] (\pi_{1t-1} - \pi_{0t-1})$$

where $\phi = 1$.[20] In the more familiar Markov models developed by Hamilton (1990), $\phi = 0$. Although a model with $\phi = 1$ appears to better capture the nature of the structural shifts, we also estimated a restricted version of the model in which $\phi = 0$ to examine the sensitivity of our results. The results from estimating the restricted version of the model were similar to those reported in Table 6 except that the estimated probability of remaining in the unit root state was insignificantly different from one. We therefore incorporate both forms of the Markov transition probabilities in our analysis below.

Figure 3 shows some of the implications of the model estimates in Table 6. The top panel depicts the probability of being in the unit root state from the 1950s until the late 1980s. As the figure shows, the probability of being in state 1 is very low up until 1970 with the probability never exceeding about 1.5%. The inflation process appears to change sharply in 1974 with the first oil price shock and the probability of the unit root state increases dramatically to above 80%. The probability remains high until after the drop in inflation in 1982. Interestingly, the sharp movements in the probability of being in the unit root state correspond roughly to the points of structural change in inflation suggested by the CUSUM statistics in Figure 2.

The middle panel of Figure 3 plots the Markov forecasts of quarterly inflation one year ahead and realized inflation. During the run up in inflation during the late 1960s, the inflation forecasts persistently lag behind realized inflation. This relationship is exacerbated by the oil price shocks in the 1970s. On the other hand, when inflation falls in 1982, the high probability of remaining in the unit root state keeps inflation forecasts above realized inflation. Overall, the figure shows that the forecast residual, identified by the vertical difference between realized inflation and the forecast, is highly serially correlated, even though forecasts are rational by construction with respect to the sample.

To see if the model was capturing the general pattern of survey expectations, the bottom panel of Figure 3 compares the Markov forecasts with bi-annual forecasts of the six month inflation rate from the Livingston survey. As the picture shows, the Markov forecasts follow the general movements in survey expectations. However, since the Markov model forces the forecasts to be rational with respect to the data, they tend to be above the Livingston forecasts early in the sample, with the relationship reversed at the end of the sample.

The Markov model estimates can also be used with nominal interest rates to construct an estimate of the implied *ex ante* real interest rate. We calculated the *ex ante* real rate expected one year ahead by subtracting the Markov forecasts shown in the middle panel of Figure 3 from the one year forward rate on a three month bond. These expected *ex ante* real rates and the *ex post* real interest rates based upon actual inflation are plotted in Figure 4. As the figure shows, the *ex ante* real interest rate remains fairly close to the *ex post* real rate until the early 1970s. The sharp decline in the *ex post* real rate in 1973-74 does not appear in the *ex ante* real rate as the oil price shock to inflation was largely unanticipated. There is also a wide divergence between the rates in the late 1970's when the *ex post* real rates again turned sharply negative while the *ex ante* rate remained positive reaching 2% on occasion. During the early 1980's the sharp rise in *ex post* real interest rates are matched by the *ex ante* real interest rates, although the peaks in 1981 and 1984 are attenuated.

C. A Monte Carlo Investigation of Rational Forecast Residuals

We have shown above how rationally anticipated switches in the inflation process can induce serial correlation in inflation forecast residuals, and how this correlation, in turn, makes *ex post* real rates systematically deviate from *ex ante* real rates. To explain the results in Tables 2 and 4, however, inflation forecast residuals must not only be serially correlated, they must also contain a unit root component in common with realized inflation.

To evaluate this possibility, we conducted a series of Monte Carlo experiments based upon the Markov switching model of inflation. These experiments examined the cointegrating relationship between actual inflation and expected inflation based upon the Markov model. The first cointegrating regression considered was:

$$E(\pi^k(t) \mid t,M) = \gamma_0 + \gamma_1 \, \pi^k(t,M) + \eta(t+k) \tag{13}$$

where $E(\pi^k(t) \mid t,M)$ is the expected inflation rate, and $\pi^k(t,M)$ is the realized inflation process both based upon the estimated Markov model in Table 6. The estimates of γ_1 from this regression allow us to examine whether the forecast residuals from the Markov model contained a unit root component in common with realized inflation. In particular, since $E(\pi^k(t) \mid t,M) - \pi^k(t,M) = (\gamma_1\text{-}1)\, \pi^k(t,M) + I(0)$ terms, if no unit root component is present in the forecast residual, we should find γ_1 equal to one.

In the experiments we used the estimated Markov model to generate samples for realized and expected inflation of 25 years, as in our data set, and also for samples of 100 years and 1000 years. We considered the sensitivity of the unit root component to the sample size in order to evaluate how many observations would be required for this component to vanish.[21] For each sample we then estimated the cointegrating regression in (13) using the Stock-Watson method. The empirical distributions of the estimates for γ_1 were produced by replicating this procedure 1000 times. Appendix D contains a detailed description of these Monte Carlo experiments.

In the first set of experiments we assumed that the entire inflation process was given by the Markov model estimated in Table 6, which we call Model A. These experiments treat the post-war data sample as representative of the history of inflation.

The top panel of Figure 5 shows the cumulative distribution function for the γ_1 coefficients generated by Model A. The left hand side shows the distribution for the estimates based upon quarterly inflation while the right hand side shows the same distribution based upon annual inflation. These figures show that the probability of finding estimates of γ_1 less than one is quite high in a sample size of 25 years. The probability of finding estimates less than about .8 declines dramatically with sample sizes of 100 years. However, even as the sample size increases to 1000 years, the probability of finding a coefficient less than one remains quite high. These results indicate that unit root components are quite likely to appear in the inflation forecast residuals.

In a second set of experiments we treated the post War data sample as unrepresentative of the overall process. We assumed that the entire inflation process was given by the Markov model estimated in Table 6 except that the probability of remaining in the unit root state was now set equal to one. In this specification, which we call Model B, any switch into the unit root state lasts for ever. As before, the cointegrating regression in (13) was repeatedly estimated to generate the empirical distribution of γ_1.

The second panel of Figure 5 depicts the cumulative distributions for these coefficient estimates. As in the Model A distribution, the probability of observing an estimate of γ_1 less than one is quite high in data samples of 25 years. However, as the sample size lengthens, this probability declines dramatically. For samples of 1000 years, the probability of finding an estimate for α_1 less than one is essentially zero.

A second way in which we can examine whether the forecast residuals from the Markov model contained a unit root component is to run the cointegrating regression in (13) the other way around:

$$\pi^k(t, M) = \psi_o + \psi_1 E(\pi^k(t) \mid t,M) + \omega(t+k). \tag{14}$$

As before, the forecast residual will be stationary only if the cointegrating coefficient, ψ_1, is equal to one. On the other hand, if there are expected shifts in inflation that generate serial correlation with unit root persistence, the estimates of ψ_1 may differ from one.

As above, the empirical distributions for ψ_1 were calculated by repeatedly estimating (14) using data generated by Models A and B. The third panel of Figure 5 presents the empirical distributions of ψ_1 based upon Model A, which allows inflation to continually switch between processes. The cumulative distribution of ψ_1 for samples of 25 years implies that the probability of observing a coefficient less than one remains quite high, especially for the one year ahead forecasts. In the 1000 year samples, the probability of observing an estimate of ψ_1 less than one is close to zero for the four quarter ahead forecasts, and estimates of ψ_1 appear to converge to a value greater than one for the quarter ahead forecasts.

The bottom panel of Figure 5 presents the empirical distributions for ψ_1 generated by Model B, in which any switch into the unit root state lasts for ever. In this case the cumulative distributions show that the coefficient estimates become heavily concentrated around one as the sample size increases to 1000 years. This concentration accords with the distributions of γ_1 based on Model B.

These Monte Carlo experiments show that the inflation forecast residuals will appear to contain unit root components in data samples as short as 25 years with high probability. Even if inflation forecasts are rational, the estimates of γ_1 and ψ_1 indicate that the forecast residuals would contain unit roots if agents were rationally anticipating shifts in inflation.

IV. A Re-Evaluation

We now draw on the results of the previous section to re-evaluate the long run relationship between inflation and the nominal interest rate when economic agents rationally anticipate a shift in the inflation regime. The results in Table 2 imply that *ex post* real rates share a common unit root component with realized inflation. We have argued that this result could arise from persistence in the inflation forecast residuals created by rationally anticipated switches in inflation. Below we present further evidence in favor of this explanation.

The implications of our Monte Carlo experiments for estimates of the long-run relationship between nominal interest rates and realized inflation are easily seen. Assuming that the Markov model accurately represents the entire inflation process, we can substitute (13) into the Fisher identity (1) to obtain,

$$R^k(t) - \pi^k(t) \;=\; r^k(t) + \gamma_0 - (1-\gamma_1)\pi^k(t) + \eta(t+k). \tag{15}$$

The Monte Carlo experiments shown in Figure 5 indicate that in standard sample sizes, γ_1 is biased downward and would be less than one with high probability. Thus, even if the *ex ante* real rate follows a stationary I(0) process, (15) shows that it is quite likely that the *ex post* real rate will appear to share a unit root component in common with realized inflation.

We can examine the validity of this explanation by comparing the estimates of α_1 obtained from the cointegrating regression,

$$R^k(t) = \alpha_o + \alpha_1\, \pi^k(t) + v(t+k), \tag{2}$$

against an empirical distribution for α_1 generated by Monte Carlo experiments that assume inflation follows the estimated Markov process in Table 6. These experiments are similar to those described above except that they generate nominal interest rates from the Markov forecasts of inflation using the Fisher identity. For this purpose we assume that the *ex ante* real rate follows a stationary I(0) process. The process was parameterized so that once the *ex ante* real rates were combined with the Markov forecasts, they matched the behavior of the nominal interest rate in the data.[22] Appendix D provides a complete description of these experiments.

The second column of Table 7 reports coefficient estimates for α_1 in equation (2). The fourth column reports the p-values based upon standard asymptotic inference for the hypothesis that these coefficients are different than one. These estimates are comparable to the results found in Table 2.[23] The hypothesis that the coefficients equal one are strongly rejected with p-values less than 5%.

Columns 5 through 7 report diagnostics from Monte Carlo experiments based upon Model A in which the entire process generating inflation is the model estimated in Table 6. Columns 5 and 6 report the median and the standard deviation of the empirical distribution of α_1. All of these median values are less than one, indicating the downward bias in these estimates. Column 7 reports the probability of observing the estimated coefficient in column 2 under the hypothesis that the *ex ante* real rate is truly stationary and unrelated to inflation but agents anticipate shifts in the inflation process. As these p-values show, the hypothesis cannot be rejected at standard marginal significance levels.

Columns 8 through 10 report diagnostics from Monte Carlo experiments based upon Model B where any switch in inflation to the unit root state lasts forever. Again the median values of α_1 in column 8 are all less than one and the standard errors in column 9 are relatively high. Column 10 reports the probability of observing the α_1 estimates in column one when the *ex ante* real rate is truly stationary. Although the distributions in Figure 5 did not contain the real interest rate, they showed that coefficients less than one are not as likely in this case. For similar reasons, the probabilities of observing the estimates of α_1 in column 2 are lower than those reported in column 7. Nevertheless, we would not reject the hypothesis that the *ex ante* real rate is stationary at marginal significance levels of 5%, except for the four quarter ahead inflation forecasts.

The long run relationship between inflation and the nominal rate can also be examined with the cointegrating regression of inflation run on the nominal interest rate:

$$\pi^k(t) = \beta_0 + \beta_1 R^k(t) + w(t+k) \tag{16}$$

This equation puts the Fama (1975) regression into a cointegrating framework. As with α_1 in equation (2), the coefficient β_1 must be equal to one if the *ex post* real rate does not contain a permanent disturbances.

The lower panel of Table 7 examines the long run relationship between inflation and nominal interest rates in the context of (16). The second column shows that the estimates of β_1 calculated with the Stock-Watson procedure are all less than one. The fourth column reports the p-values for the hypothesis that these coefficients are equal to one based upon the asymptotic distribution of the estimates. These p-values show that the hypothesis is strongly rejected at standard significance levels. Under standard assumptions about expectations, these results imply that the *ex ante* real rate was subject to permanent shocks.

Alternatively, we can interpret these results allowing for the effects of anticipated switches in inflation. In particular, if the Fisher identity is used to substitute for expected inflation in (14), we obtain:

$$\pi^k(t) = \psi_0 - \psi_1 r^k(t) + \psi_1 R^k(t) + \omega(t+k). \tag{17}$$

The results of the Monte Carlo experiments shown in Figure 5 indicate that ψ_1 is biased downward and would be less than one with high probability in the available sample sizes. Clearly, the cointegrating relationship between inflation and nominal interest rates are the same in (16) and (17) when the *ex ante* real rate follows a stationary I(0) process. Therefore, these Monte Carlo results suggest that is quite likely that we would estimate β_1 to be less than one if people rationally anticipated the inflation switches identified by the Markov model.

Columns 7 and 10 of Table 7 report the probability of observing the estimates of β_1 when inflation shifts between processes estimated by the Markov model. As before, these calculations assume that the *ex ante* real rate follows a stationary I(0) process. The p-values based upon Model A are never less than 10%. Thus, one would not reject the hypothesis that the *ex ante* real rate is stationary at standard confidence levels. The p-values based on Model B, in which a switch into the unit root state lasts for ever, are somewhat smaller. The hypothesis that the *ex ante* real rate is stationary would not be rejected at 95% confidence levels for one and two quarter ahead forecasts, but would be rejected at the three and four quarter ahead horizons.

The last row in each panel of Table 7 provide estimates of the cointegrating relationship between nominal interest rates and the Livingston survey. When the nominal rate is regressed upon the inflation forecast, we cannot reject the hypothesis that the estimated coefficient on the Livingston forecasts is equal to one at high confidence levels. On the other hand, when the inflation forecast is regressed on the nominal rate, we can reject the hypothesis that the coefficient on the nominal rate is equal to one at the 10% level. However, these results need to be interpreted with some caution because the unit root tests in Table 1 suggest that the Livingston data may in fact be stationary. If this is true, no long run relationship exists between the Livingston forecasts and nominal interest rates, and these results may simply reflect spurious regression problems.

Overall, the evidence in Table 7 shows that in typical sample sizes based upon post-war data, we find cointegrating coefficients in the Fisher identity that differ from one. Under standard assumptions, these results would necessarily imply that the *ex ante* real rate contains unit roots. However, the table also shows we are very likely to find these estimates when the *ex ante* real rate does not contain unit roots but investors rationally anticipate shifts in inflation policy. In fact, if the number of shifts in the post-war data sample is representative of the future inflation process, our estimates indicate that we cannot reject the hypothesis that the *ex ante* real rate is stationary.

V. Concluding Remarks

In this paper, we re-examined the long run relationship between inflation and nominal interest rates using recent time series techniques. We showed that there is a negative correlation between the permanent movements in *ex post* real interest rates and inflation. We then argued that this correlation could arise when people incorporate anticipated shifts in inflation policy into their expectations because inflation forecasts tend to be highly persistent under these circumstances. We rejected a necessary condition for the absence of systematic forecast residuals at horizons from 3 month to 2 years. In light of these test results, we then examined the stability of the inflation process, and estimated a Markov switching model of inflation. This model appeared to capture structural shifts in the inflation process and to characterize the behavior of inflation well. Based upon forecasts from the Markov model, we re-examined the long run relationship between nominal interest rates and inflation. When incorporating anticipated shifts in inflation in this way, we were unable to reject the hypothesis that long term movements in nominal interest rates reflect one-for-one long run movements in expected inflation.

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Endnotes

1.For example, Barsky (1987) argues that changes in the inflation process over the last century have affected estimates of the Fisher equation relationship and Bonser-Neal (1990) shows that real rate processes in different countries contain structural breaks around monetary regime changes.

Early studies of the real rate include Fama (1975), Mishkin (1981), and Huizinga and Mishkin (1984). More recently, Mishkin (1988, 1990a, 1990b) and Fama (1990) have examined how the term structure of rates depend upon *ex post* inflation.

2.For example, several explanations have been provided for the negative correlation between inflation and the real interest rate. Mundell (1976) and Tobin (1969) argue that higher inflation leads to a portfolio shift into real assets, thereby pushing down the return on these assets. Fama (1981) and Fama and Gibbons (1982) present another explanation: if increases in output push up money demand without accommodating money supply, these output shocks will push down inflation but will be correlated with higher real rates. A third channel arises in general equilibrium monetary economies through the liquidity

3.For instance, Mishkin (1981) and Huizinga and Mishkin (1986) use nominal rates minus *ex post* realized inflation together with the assumption that inflation forecast residuals are white noise to study the implicit *ex ante* real rate. Other studies include Fama (1975), Carlson (1977), Garbade and Wachtel (1978), and Nelson and Schwert (1977). Most of this research ignores the effects of taxes on nominal interest payments pointed out by Darby (1975) and Feldstein (1976). Unfortunately, it is extremely difficult to know what the appropriate tax rate for the whole economy is because the effective tax rate

4.Mishkin (1991) studies nominal interest rates and inflation rates over different maturities and finds that they contain unit roots. Other studies that find a unit root in inflation include Evans (1991), Ball and Cecchetti (1989), and Shapiro and Watson (1987).

5.This issue was raised by Perron (1989). He found that for several macroeconomic series, the null hypothesis of a unit root process with drift and an exogenous break point could be rejected in favor of the alternative of a stationary process about a deterministic trend with an exogenous change in trend function. The Zivot and Andrews statistic tests the null hypothesis of a unit root against the alternative that the process is trend stationary with a break in the trend occurring at an unknown point in time.

6.In principle, this equation may incorporate an inflation risk premium for holding nominal bonds relative to real bonds. See, for example, Benninga and Protopapadakis (1985). Also, equation (1) measures the expected depreciation on holdings of nominal money, and therefore differs from the inflation rate due to Jensen's inequality. Evans and Wachtel (1992) find that these components are quantitatively unimportant. However, none of our results below would be affected by the presence of a covariance stationary inflation risk premium.

7.This relationship holds only if the variables with unit roots are cointegrated. Below we present evidence that the variables are indeed cointegrated and so our discussion here proceeds under this assumption.

8.For more discussion of cointegrating regressions, see Stock (1987). Campbell and Perron (1991) provide a survey and a more detailed discussion of the necessary conditions for equations like (2) to be a well-defined cointegrating regression.

9.Since both inflation and nominal interest rates are endogenous variables, it may appear that instrumental variables should be used to address the simultaneous equation problem. However, as discussed in Stock (1987) and Stock and Watson (1989), the level of the right-hand side variable is, in effect, an instrument for its own non-stationary component. Therefore, the estimate of α_1 is asymptotically consistent. The simultaneous equation problem does introduce a bias in finite samples, however. The estimates reported below adjust for this bias using the methods proposed by Stock and

10.An explanation of the Stock-Watson method and how it was implemented in the current situation is given in Appendix A.

11.Appendix B describes how the expectations in (3) can arise when people are learning about a past switch and/or anticipating a future switch in the inflation process.

12.We make this assumption only for the purposes of illustration. In the empirical analysis below, actual switches are easily incorporated.

13.To see this relationship, note that:

$$\delta^{k,n}(t)\ E(\pi^k(t+n) \mid t,C) \equiv \lambda^n(t)\{E(\pi^k(t+n) \mid t,C) - E(\pi^k(t+n) \mid t,A)\}.$$

This term will be stationary only if the difference between the expectations of the current and alternative regimes are stationary. More general alternatives that induce permanent disturbances in this term are: (a) one process is stationary and the other contains a unit root; (b) both processes contain unit roots with different disturbances; (c) both processes contain the same unit root disturbance but with different

14.It is straightforward to show that $F^{k,n}(t) = [(k+n)R^{(k+n)}(t) - nR^n(t)]/k$ for the case of pure discount bonds using the linearized term structure relationship as in Campbell and Shiller (1991).

15.Standard models of time-varying risk premia imply that these are stationary since they depend upon the time-series properties of the change in consumption. See, for example, Campbell (1987) and Grossman and Shiller (1981).

16.Here we implicitly assume that people know the process for the *ex ante* real rate and it is not expected to switch. This assumption only serves to simplify the notation. We have also examined the case where both the real rate and inflation processes may shift. The basic arguments that follow remain true although the algebra is much more tedious.

17.These results indicate that excess returns contain a non-stationary component. Since excess returns are typically treated as stationary, this result may seem to contradict conventional wisdom and empirical practice. However, Evans and Lewis (1990) present Monte Carlo evidence showing that standard unit root tests and time series analysis would not detect a small non-stationary component in excess returns. In the present case, the magnitude of this component would likely be small in the sample since the variance of the non-stationary component depends upon $(1 - a_1)^2$.

18.Even though there is no direct evidence of shifts in the constant, Hansen (1991) finds in Monte Carlo experiments that it is difficult to test the stability of one set of parameters if another subset of parameters is shifting over time. Therefore, shifts in autoregressive parameters or constants may show up as apparent shifts in the variances.

19.The estimates were obtained using a modified version of the filtering algorithm developed by Hamilton (1990).

20.Using the parameter estimates in Table 6 (estimated with $\phi = 1$) we calculated the standard error of ϕ to be 0.68. From this we computed the p-value of a one-sided test that $\phi = 0$ to be 0.095, suggesting the discrete shifts were an important characteristic of the data.

21.This issue mirrors the discussion of the low power of serial correlation tests in Fama and French (1988), and Poterba and Summers (1988) who argue that serial correlation tests often fail to reject the null when it is false.

22.Specifically, the *ex ante* real rates were generated by subtracting the actual nominal rates from the Markov forecasts of inflation: $R^k(t) - E(\pi^k \mid t,M) = r^k(t \mid M)$. An autoregressive model was fit to these estimated real rates. The *ex ante* real rates used in the experiments were then generated from this model.

23.Note that these are based upon quarterly inflation and therefore do not match exactly the same estimates based upon monthly inflation in Table 2.

Appendix A: Cointegrating Regression Methods

1. Stock-Watson Procedure

In this appendix, we describe the methods used to estimate the cointegrating regressions (2), (10), (11), (13), (14), and (16). For more detailed and thorough discussions, see Stock and Watson (1989). For notational simplicity, note that equation (10), for example, may be written as,

$$y_t = \gamma x_t + u_{1t} \tag{A1}$$

$$\Delta x_t = u_{2t}, \tag{A2}$$

where u_{1t} and u_{2t} are stationary, $y_t \equiv R^k(t+n)$, $x_t \equiv F^{k,n}(t)$, and the constant term is omitted for simplicity. We are interested in testing $\gamma = 1$. Since x_t is endogenous, it is likely that the sample covariance between x_t and u_{1t} is not equal to zero. In this case, γ will be biased in any finite sample. Therefore, test statistics on γ must take account of this bias. This bias arises even though the estimate of γ is consistent. Below, we give the steps for estimating (A1) and (A2) using the Stock-Watson method.

Rewrite equation (A1) according to,

$$y_t = \gamma_0 + \gamma x_t + \beta(L) \Delta x_t + u_{1t} \tag{A3}$$

where $\beta(L)$ is a polynomial in the lag operator, L, i.e., $\beta(L) = (L^n + L^{n-1} + \cdots + L + 1 + L^{-1} + \cdots + L^{-n+1} + L^{-n})$. The idea to rewriting (A1) in this form is to include as many leads and lags of Δx on the right-hand side of the equation to make u_{1t} independent of x. This implies that the asymptotic distribution of the OLS estimator of γ is normal. Intuitively, including the leads and lags of Δx_t on the right-hand side gets rid of the simultaneous equation bias problem since the β's soak up the correlation between u_{1t} and u_{2t}. Note that since u_{1t} will be serially correlated in general, the Wald test of $\gamma = 1$ from (A3) should also use the Newey-West estimator for the covariance matrix.

2. Implementing the Stock-Watson Procedure

Implementing this procedure requires a truncation level (n) for the leads and lags of first-differenced right-hand side variables. Furthermore, since the residual in equation (A3) may be serially correlated and conditionally heteroscedastic, we also allowed for autocorrelation by estimating the variance-covariance matrix with the Newey-West (1987) estimator. Estimating this covariance matrix requires a lag for the maximum number of autocovariances.

Therefore, estimation of equation (A3) requires knowledge of both (a) the number of leads and lags for the Stock-Watson correction and (b) the number of lags in the Newey-West estimator. To make sure that our results were not sensitive to these numbers, we estimated cointegrating regressions using a range of Stock-Watson leads/lags of 2 to 12, and of Newey-West lags of 3 to 12. Tables 2 through 4 report the results when Stock-Watson leads/lags were 6 and Newey-West lags were 6. These results are completely representative of the findings for the range of leads and lags.[1]

Appendix B. Learning and Anticipated Future Shifts in Processes

Equation (3) in the text states that the expected future inflation rate is a probability-weighted average of the expectations conditional upon the current process, C, and an alternative process, A. These expectations may be motivated in two alternative but not mutually exclusive ways: (1) learning about a past change in the inflation process, and (2) anticipation of a future shift in inflation.[2]

Case (a): Learning. After a change in the process of inflation, economic agents may require time to learn about the new process of inflation. For example, Americans living with the relatively low inflation of the early 1960's may have required time to learn about the higher inflation levels of the 1970's. To illustrate how learning would affect forecasts, suppose that the process of

[1]Generally, the efficiency of estimation increases as the number of both sets of lags decrease. For this reason, we report relatively conservative representative estimates. The results for the range of these lags are available upon request from the authors.

[2]The impact of future discrete changes on expectational errors was first pointed out by Rogoff (1980), and was recently empirically investigated in nominal interest rates by Lewis (1991). The effects of learning on forecast errors after changes in policy are described in Lewis (1989).

inflation changed from the old process to a new process. Suppose further that agents are not sure if the process has changed. Therefore, they would observe the inflation process following the suspected change to try to learn whether the process has indeed changed. If they learn in a Bayesian way, they begin with a prior probability of a change and then update this probability based upon subsequent observations of inflation. As economic agents learn, they weight forecasts conditional upon each regime by their assessed probability that the old or new process is currently generating inflation. Thus, the expected inflation takes the form of:

$$E(\pi^k(t+n) \mid t) = (1 - \lambda^n(t))\, E(\pi^k(t+n) \mid t,\text{Old}) + \lambda^n(t)\, E(\pi^k(t+n) \mid t,\text{New}) \qquad \text{(B1)}$$

where $\lambda(t)$ denotes the probability of the old process and the expectations conditional upon the old regime and new regime are denoted "Old" and "New", respectively. With sufficient observations from the "New" process, the market will learn the true process is indeed "New", and $\lambda(t)$ will converge to one. Note that letting "C" refer to the "New" regime and "A" refer to the "Old" regime in equation (B1) yields equation (3).

Case (b): Anticipate Future Shifts If agents believe that the inflation process may change in the future, their expectations of future inflation will incorporate expectations conditional upon the alternative inflation process. For instance, Americans who experienced the high inflation of the 1970's may believe that episodes of high inflation could happen again even after the decline in inflation in the 1980's. They then assess positive probability to a switch back to the previous policy regime. In this case, expected future inflation also takes the form of:

$$\begin{aligned} E(\pi^k(t+n) \mid t) = {} & (1 - \lambda^n(t))\, E(\pi^k(t+n) \mid t, \text{Current}) \\ & + \lambda^n(t)\, E(\pi^k(t+n) \mid t, \text{Alternative}) \qquad \text{(B2)} \end{aligned}$$

where $\lambda(t)$ is now the probability of a switch to another regime. Thus, the expected future inflation rate is the probability-weighted average of the expected inflation conditional upon the current regime and the regime anticipated if a switch occurs. In this case, agents understand fully the current regime of the inflation process (and hence do not need to learn), but believe that this process may change in the future.

Note that letting "C" refer to the "Current" regime and "A" refer to the "Alternative" regime in equation (B2) yields equation (3).

In summary, both when (a) the market is learning and when (b) the market anticipates a shift in future policy, expected inflation will weight forecasts based upon an inflation process other than the current process. Either case will imply expected inflation in the form of equation (3).

Appendix C: Bootstrapping Procedure for Table 4 Tests

The cointegrating regressions in Table 4 are based upon an estimate of the asymptotic variance-covariance matrix of the parameters. Since our small data sample may produce relatively poor estimates of this matrix, our test statistics may in turn be contaminated by small sample bias. We therefore used bootstrapping techniques to generate the empirical distribution of the coefficient estimates, a_1 and a_2 and used this distribution to test the null hypothesis.

Specifically, we constructed time-series for the future interest rate based upon equation (11) as follows. First, the residuals to the Stock-Watson regressions were saved. Then, a series of interest rates equal to the sample length were generated by sampling from these residuals using equation (11) where we constrained $a_1 = 1$ and $a_2 = 0$. Next, these generated series were regressed upon the forward rates and inflation as in equation (11), saving the coefficient estimates. These steps were repeated one thousand times. This procedure provides an empirical distribution of the coefficient estimates when in fact $a_1 = 1$ and $a_2 = 0$ by construction. Using this empirical distribution, the probability that the point estimates from the cointegrating regression (11) come from this distribution are reported in the last column of Table 4.

To evaluate the robustness of our results, we also generated the series allowing for $a_2 \neq 0$. For this purpose, we created interest rate series as described above except that a_2 was set equal to its point estimate. In the next to last column, Table 4 reports the probabilities that the estimates from the cointegrating coefficients are drawn from this distribution.

As a final check, we also constructed the series allowing for conditional heteroscedasticity, by estimating a separate ARCH model for each equation and saving these residuals. Then, drawing from these residuals, we generated heteroscedastic residuals from the estimated ARCH process and then used these scaled residuals to construct the nominal rates. We considered both cases in order to examine the finite sample sensitivity of our Monte Carlo experiments to assumptions about heteroscedasticity. However, these experiments provided virtually identical p-values as the homoscedastic case. We report these results in Table A.

Appendix D: Monte Carlo Experiments of Markov Switching Model

This appendix details the Monte Carlo experiments described in Section III (C) and Section IV.

Section III (C): The Monte Carlo experiments in this section were used to evaluate the cointegrating regressions, equations (13) and (14), in the text. These regressions involve series of expected inflation and actual inflation based upon the estimated inflationary Markov model. As described in the text, two types of Monte Carlo experiments were conducted based both upon this model and an assumption about the entire history of inflation.

The first, Model A, treats the post-war period as a representative drawing from this distribution. For this reason, actual inflation was generated from the estimated model in Table 6, where the probabilities of remaining in the same state are .936 and .961 for state 1 and for state 0, respectively. Then, we calculated the Markov forecasts at horizons of one, two, three, and four quarters, assuming the current state is known and the transition probabilities are known. After generating the actual and expected series, the cointegrating regressions in equations (13) and (14) were estimated with the Stock-Watson procedure. The coefficient estimates were saved. This procedure was repeated 1000 times, generating an empirical distribution of coefficients. These experiments were conducted for sample lengths of 25, 100, and 1000 years. Figure 5 shows the results for the one quarter and four quarter ahead forecasts, although the two quarter and three quarter ahead forecasts gave similar results.

The second model, Model B, treats the post-war period as unrepresentative of the entire inflation process. The actual inflation was generated from the estimated model in Table 6, where the probability of staying in state 2 is .961, as before, but the probability of staying in state 1 is equal to one. In other words, state 1 is absorbing. We then generated the actual and expected inflation from this process and estimated the cointegrating regressions, (13) and (14), as for Model A. This procedure was repeated

1000 times to generate empirical distributions for the coefficients. Again, the two quarter and three quarter ahead forecasts implied similar distributions to those depicted in Figure 5 for the one quarter and four quarter ahead forecasts.

We also conducted experiments where the transition probabilities were not known by agents, but were estimated over time based upon the number of times a transition was made from one state to another. These experiments gave essentially the same results as those reported in the text and we therefore do not report them.

Section IV: The Monte Carlo experiments in this section were used to evaluate the Fisher equation estimates in equations (2) and (16). These regressions require series on the actual and expected inflation processes from the Markov process, as generated in Section III (C), but also require the *ex ante* real rate to produce a nominal interest rate series. The *ex ante* real rates were generated by subtacting the actual nominal rates from the Markov forecasts of inflation: $R^k(t) - E(\pi^k|\ t,M) = r^k(t,M)$. Then an autoregressive model of the *ex ante* real rate process was estimated. The *ex ante* real rates were then generated from this model. Given these generated *ex ante* real rates, the nominal rates were constructed as: $r^k(t,M) + E(\pi^k(t)\ |\ t, M) = R^k(t, M)$. By construction, these nominal interest rates match the behavior of the nominal interest rates in the data.

The Monte Carlo experiments followed the two forms described above for Model A and Model B. For each set of generated nominal interest rates and inflation, equations (2) and (16) were estimated using the Stock-Watson method. The empirical distributions of the coefficients were calculated. Since the *ex ante* real rate is stationary and the Fisher equation holds by construction, these empirical distributions are used to calculate the p-values in Table 7.

Table 1: Unit Root Tests

variable x_t (1)	τ (2)	$t(\lambda)$ (3)
	(monthly data)	
π^1	-2.982	-3.250
π^3	-2.233	-2.836
π^6	-1.889	-3.117
π^9	-3.918	-5.199
π^{12}	-2.858	-4.293
R^1	-1.209	-1.868
R^3	-1.346	-1.911
R^6	-1.191	-1.801
R^9	-1.189	-1.896
R^{12}	-1.489	-2.256
$F^{1,3}$	-1.428	-1.984
$F^{3,3}$	-1.363	-2.008
$F^{3,6}$	-1.361	-2.114
$F^{6,6}$	-1.774	-2.649
$F^{9,3}$	-1.806	-2.827
	(bi-annual data)	
$E[\pi^6 \mid L]$	-2.102	-6.009
π^6	-1.517	-4.174
critical values 1%	-3.99	-5.34
5%	-3.43	-4.80

Notes: π^k and R^k are the k month inflation and nominal interest rates. $F^{k,n}$ is the rate of return on a forward contract bought at t for a k month bond at $t+n$. $E[\pi \mid L]$ is the Livingston survey forecast of inflation.

τ: Augmented Dickey Filler test statistics for $H_0{:}\alpha = 1$, allowing for a constant and time trend. Monthly tests include 6 lags of the first difference Δx_t in the regression, bi-annual tests include 1 lag:

$$x(t) = \mu + \beta \text{ trend} + \alpha x(t-1) + \Sigma_i a_i \Delta x(t-i) + u(t).$$

$t(\lambda)$: Zivot and Andrews' minimum t-statistics for $H_0{:}\alpha=1$, allowing for shifts in the mean. Regressions include constant and time trend, 6 lags of first difference Δx_t in monthly data, and 1 lag in bi-annual data:

$$x(t) = \mu + \beta \text{ trend} + \alpha x(t-1) + \Sigma_i a_i \Delta x(t-i) + \Theta DU(\lambda) + u(t)$$

where $DU(\lambda) = 1$ if $t > [T\lambda]$ for sample size T, and 0 otherwise, where $1 > \lambda > 0$.

Table 2: Long-Run Fisher Equation Estimates

$$R^k(t) = \alpha_o + \alpha_1\pi^k(t) + \Sigma_{i=-6}^{6} a_i\, \Delta\pi^k(t\text{-}i) + v(t+k)$$

Months	Cointegration Tests	Parameter Estimates		t-statistics
k	λ_{max}	α_1	$\Sigma_i a_i$	$H_o{:}\alpha_1 = 1$
(1)	(2)	(3)	(4)	(5)
1	48.862	0.611	-3.558	4.723
		(0.082)	(0.799)	
3	44.568	0.595	-5.441	3.770
		(0.107)	(1.007)	
6	30.448	0.703	-1.835	3.320
		(0.089)	(1.510)	
9	26.308	0.752	-1.484	2.998
		(0.083)	(1.732)	
12	21.109	0.810	-2.307	2.640
		(0.072)	(1.575)	
24	11.120	0.758	-6.256	3.241
		(0.075)	(2.039)	
36	10.880	0.783	-12.058	3.324
		(0.065)	(3.152)	

Notes: λ_{max} is Johansen's maximum eigen value test for the number of stochastic trends in $\{\pi^k(t), R^k(t)\}$. The 5% and 10% critical values are 14.595 and 12.783. Statistics greater than the critical value indicate that the null of no cointegration (or equivalently 2 stochastic trends) can be rejected. All standard errors are corrected for the presence of conditional heteroskedasticity and moving average errors.

Table 3: Cointegrating Regressions between Spot and Forward Interest Rates

$$R^k(t+n) = d_o + d_1 F^{k,n}(t) + \Sigma_{i=-6}^{6} b_i \Delta F^{k,n}(t-1) + u(t+n)$$

Months	Cointegration Tests	Parameter Estimates		t-statistics
n, k	λ_{max}	d_1	$\Sigma_i b_i$	H_o: $d_1=1$
(1)	(2)	(3)	(4)	(5)
1, 2	74.401	0.971 (0.006)	1.030 (0.096)	4.846
1, 3	53.236	0.978 (0.006)	1.093 (0.071)	3.963
1, 5	54.714	0.983 (0.004)	1.066 (0.048)	4.358
3, 3	33.783	0.952 (0.015)	2.937 (0.191)	3.228
3, 6	26.356	0.976 (0.010)	3.049 (0.130)	2.471
3, 9	22.777	0.986 (0.008)	3.387 (0.113)	1.606
6, 3	45.250	0.911 (0.042)	1.937 (0.520)	2.084
6, 6	38.113	0.935 (0.039)	2.375 (0.524)	1.657
9, 3	34.957	0.875 (0.069)	2.662 (0.634)	1.811
12, 12	24.833	0.879 (0.065)	4.071 (0.723)	1.866
24, 12	30.892	0.795 (0.107)	1.473 (1.446)	1.917

Notes: λ_{max} is Johansen's maximum eigen value test for the number of stochastic trends in $[F^{k,n}(t), R^k(t)]$. The 5% and 10% critical values are 14.595 and 12.783. Statistics greater than the critical value indicate that the null of no cointegration (or equivalently 2 stochastic trends) can be rejected. All standard errors are corrected for the presence of conditional heteroskedasticity and moving average errors.

Table 4: Cointegrating Regressions of Spot Interest Rates on Forward Rates and Inflation

$$R^k(t+n) = a_o + a_1 F^{k,n}(t) + a_2 \pi^k(t) + \Sigma_{i=-6}^{6} b_i \Delta F^{k,n}(t\text{-}i) + \Sigma_{i=-6}^{6} c_i \Delta \pi^k(t\text{-}i) + u(t+n)$$

Months	Cointegration Tests	Parameter Estimates				Hypothesis Tests Asymptotic p-values (%)		Hypothesis Tests Bootstrap p-values (%)	
n k	λ_{max}	a_1	a_2 (x100)	$\Sigma_i b_i$	$\Sigma_i c_i$ (x100)	H_0: $a_1=1$	H_0: $a_1=1$, $a_2=0$	H_0: $a_1=1$	H_0: $a_1=1$ $a_2=0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1 2	22.312	0.968 (0.007)	0.503 (0.578)	1.045 (0.120)	0.955 (2.383)	<0.00	0.01	0.04	0.02
1 3	37.094	0.980 (0.006)	0.437 (0.446)	1.108 (0.069)	3.321 (2.762)	0.96	2.69	<0.00	<0.00
1 5	25.771	0.982 (0.004)	0.449 (0.386)	1.068 (0.059)	-1.497 (2.064)	<0.00	0.02	<0.00	<0.00
3 3	33.390	0.953 (0.019)	3.213 (1.504)	2.908 (0.223)	9.890 (10.800)	1.46	2.84	1.00	0.80
3 6	27.746	0.966 (0.010)	2.670 (0.961)	2.997 (0.159)	-10.622 (6.579)	0.13	0.15	0.40	0.30
3 9	22.118	0.968 (0.010)	3.651 (1.024)	3.313 (0.138)	-15.010 (6.816)	0.18	0.09	0.30	0.10
6 3	32.268	0.836 (0.051)	22.654 (4.832)	1.244 (0.665)	-24.521 (23.755)	0.13	<0.00	<0.00	<0.00
6 6	19.633	0.850 (0.044)	20.190 (4.126)	1.708 (0.644)	-36.865 (27.144)	0.06	<0.00	<0.00	<0.00
9 3	26.311	0.724 (0.057)	37.207 (6.321)	1.445 (0.815)	-61.472 (42.096)	<0.00	<0.00	<0.00	<0.00
12 12	12.271	0.772 (0.057)	30.531 (4.742)	2.653 (0.769)	-130.947 (47.415)	0.01	<0.00	<0.00	<0.00
24 12	13.816	0.729 (0.067)	37.533 (5.854)	1.433 (1.348)	-287.254 (84.621)	0.01	<0.00	<0.00	<0.00

Notes: λ_{max} is Johansen's maximum eigen value test for the number of stochastic trends in $R^k(t+n)$, $F^{k,n}(t)$ and $\pi^k(t)$. The 5% and 10% critical values are 14.595 and 12.783. Statistics greater than the critical value indicate that the null hypothesis of 2 or more stochastic trends (or equivalently no cointegration) can be rejected. All standard errors are corrected for the presence of conditional heteroskedasticity and moving average errors. The Bootstrap p-values are calculated from the Monte Carlo experiments described in Appendix C.

Table 5: Tests for Structural Instability in Quarterly Inflation

Model Estimates
(quarterly data)

$$\Delta\pi_t = \underset{(0.161)}{0.095} - \underset{(0.083)}{0.673}\,\Delta\pi_{t-1} - \underset{(0.088)}{0.458}\,\Delta\pi_{t-2} - \underset{(0.041)}{0.255}\,\Delta\pi_{t-3} + \epsilon_t$$

Residual Correlations:	ρ_1	ρ_2	ρ_3	ρ_6
	0.017	0.070	0.064	-0.195

Hansen L Statistics			
Constant alone (1 d.f.)	0.074	Variance alone (1 d.f.)	0.959**
Constant and variance (2 d.f.)	1.043*	Constant, variance, and AR parameters (5 d.f.)	1.649*

Notes: * and ** indicates a rejection of the hypothesis of parameter stability at the 5% and 1% significance levels, respectively. $\pi(t)$ is the quarterly rate of inflation. Time periods are measured in quarters.

Table 6: Estimates of Markov Model For Quarterly Inflation

Model Estimates:
(quarterly data)

$$\pi_t = \pi_{1,t}S_t + \pi_{0,t}(1-S_t)$$

$$\pi_{1,t} = \pi_{1,t-1} + v_{1,t} \qquad \sigma^2_{v,1} = \underset{(1.538)}{6.717}$$

$$\pi_{0,t} = \underset{(0.194)}{0.442} + \underset{(0.089)}{0.726}\,\pi_{0,t-1} + v_{0,t} \qquad \sigma^2_{v,0} = \underset{(0.414)}{2.191}$$

$$pr(S_t=1 \mid S_{t-1}=1) = \underset{(0.053)}{0.936} \qquad pr(S_t=0 \mid S_{t-1}=0) = \underset{(0.027)}{0.961}$$

Specification Tests:	χ^2	Sig.
LM test for 1'st. order serial correlation in state S_t = 1:	2.171	0.141
LM test for 1'st. order serial correlation in state S_t = 0:	0.478	0.489
LM test for 1'st. order ARCH in state S_t=1:	0.250	0.617
LM test for 1'st. order ARCH in state S_t=0:	0.075	0.745

Notes: $\pi(t)$ is the quarterly rate of inflation and time periods are measured in quarters. Asymptotic standard errors are reported in parenthesis below the maximum likelihood parameter estimates.

Table 7: Cointegrating Regressions with Actual Inflation and Livingston Survey Forecasts

$$R^k(t) = \alpha_0 + \alpha_1 \pi^k(t) + \Sigma_i a_i \, \Delta\pi^k(t\text{-}i) + v(t+n)$$

quarters k	α_1	$\Sigma_i a_i$	Asy. P-value (%) H_0: $\alpha_1 = 1$	Monte Carlo Experiment A median α_1	Std. α_1	P-value(%)	Monte Carlo Experiment B median α_1	Std. α_1	P-value (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(quarterly data)									
1	0.692 (0.148)	-2.926 (0.561)	3.71	0.919	0.259	12.90	0.990	0.206	5.60
2	0.762 (0.106)	-3.584 (0.526)	2.48	0.882	0.238	20.80	0.991	0.192	7.20
3	0.732 (0.110)	-3.067 (0.603)	1.45	0.853	0.228	20.40	0.982	0.183	5.50
4	0.619 (0.140)	-4.930 (0.639)	0.60	0.819	0.215	12.20	0.994	0.175	2.70
(bi-annual data: Livingston)									
2	0.927 (0.696)	-1.141 (0.413)	44.66						

$$\pi^k(t) = \beta_0 + \beta_1 R^k(t) + \Sigma_i b_i \, \Delta R^k(t\text{-}i) + w(t+n)$$

quarters k	β_1	$\Sigma_i b_i$	Asy. P-value (%) H_0: $\beta_1 = 1$	Monte Carlo Experiment A median β_1	Std. β_1	P-value(%)	Monte Carlo Experiment B median β_1	Std. β_1	P-value (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(quarterly data)									
1	0.693 (0.101)	2.650 (0.581)	0.24	0.901	0.287	26.70	0.917	0.191	12.40
2	0.636 (0.104)	2.970 (0.550)	0.05	0.957	0.288	18.50	0.933	0.179	6.20
3	0.570 (0.108)	2.742 (0.529)	0.01	1.018	0.291	13.60	0.945	0.170	3.80
4	0.548 (0.113)	3.840 (0.579)	0.01	1.057	0.303	12.20	0.946	0.168	3.40
(bi-annual data Livingston)									
2	0.803 (0.103)	0.401 (0.269)	5.60						

Notes: The reported coefficients are corrected for finite sample bias with the Stock-Watson procedure. 3 leads and lags of the first difference regressor are included in the quarterly regression and 1 lead and lag in the bi-annual regressions. Asymptotic standard errors are reported in parenthesis below estimates. The asymptotic p-values are calculated from the Wald statistics of the null hypothesis $\alpha = 1$. The statistics allow for MA(3) serially correlated errors in the quarterly data and MA(2) serially correlated errors in the bi-annual data. The Monte Carlo p-values report the probability that the ex ante real rate is stationary. Experiments based on Model A assume that actual inflation switches in and out of the unit root state throughout the sample. Experiments based on Model B assume that any switch into the unit root state lasts for ever. The Monte Carlo experiments are described in Appendix D.

Table A: Alternative Bootstrap P-Values

$$R^k(t+n) = a_0 + a_1 F^{k,n}(t) + a_2 \pi^k(t) + \Sigma^6_{i=-6}\, b_i\, \Delta F^{k,n}(t-i) + \Sigma^6_{i=-6}\, c_i\, \Delta \pi^k(t-i) + u(t+n)$$

Months	Arch Parameter Estimates					Hypothesis Tests Bootstrap p-values (%)	
n k	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	H_0: $a_1=1$	H_0: $a_1=1$, $a_0=0$
1 2	0.073 (0.019)	0.028 (0.046)	0.144 (0.080)	0.202 (0.096)	0.211 (0.080)	0.60	0.50
1 3	0.097 (0.025)	0.103 (0.075)	0.109 (0.089)	0.097 (0.072)	0.239 (0.087)	<0.00	<0.00
1 5	0.122 (0.026)	0.166 (0.108)	0.052 (0.063)	0.134 (0.064)	0.149 (0.059)	<0.00	<0.00
3 3	0.537 (0.073)	0.404 (0.137)				3.00	2.50
3 6	0.430 (0.128)	0.268 (0.116)	0.153 (0.029)	0.199 (0.068)		0.60	0.60
3 9	0.932 (0.117)	0.351 (0.135)				0.60	0.30
6 3	1.266 (0.154)	0.343 (0.116)	0.020 (0.020)			<0.00	<0.00
6 6	1.098 (0.341)	0.569 (0.115)	0.051 (0.024)	-0.039 (0.022)	0.275 (0.067)	<0.00	<0.00
9 3	0.632 (0.182)	0.242 (0.111)	0.027 (0.036)	0.089 (0.094)	0.216 (0.077)	<0.00	<0.00
12 12	8.135 (1.329)	0.120 (0.069)	0.055 (0.053)	0.025 (0.045)	0.149 (0.072)	<0.00	<0.00
24 12	38.091 (7.317)	0.093 (0.064)	0.226 (0.097)	0.044 (0.055)	0.102 (0.041)	<0.00	<0.00

Notes: Conditional heteroskedasticity is introduced into the Monte Carlo experiments by estimating ARCH processes for the estimated residuals u(t):

$$\sigma^2_u(t) = \phi_0 + \Sigma_{i=1}\phi_i u(t-i)^2$$

The estimated parameters of the ARCH process are reported above their standard errors. The order of the ARCH models were chosen as follows: First, we used Engle's two-step regression procedure to estimate a 4'th. order process. Since this procedure does not constrain the ϕ's to be non-negative, it is possible that some of the estimated conditional variances from the model are negative. We therefore checked this before "accepting" the model. If any of the estimated variances were negative, a 3'rd. order ARCH model was estimated. This process was repeated until suitable ARCH models were found for each case.

Figure 1: Inflation, Expected Inflation and Estimated Real Rates

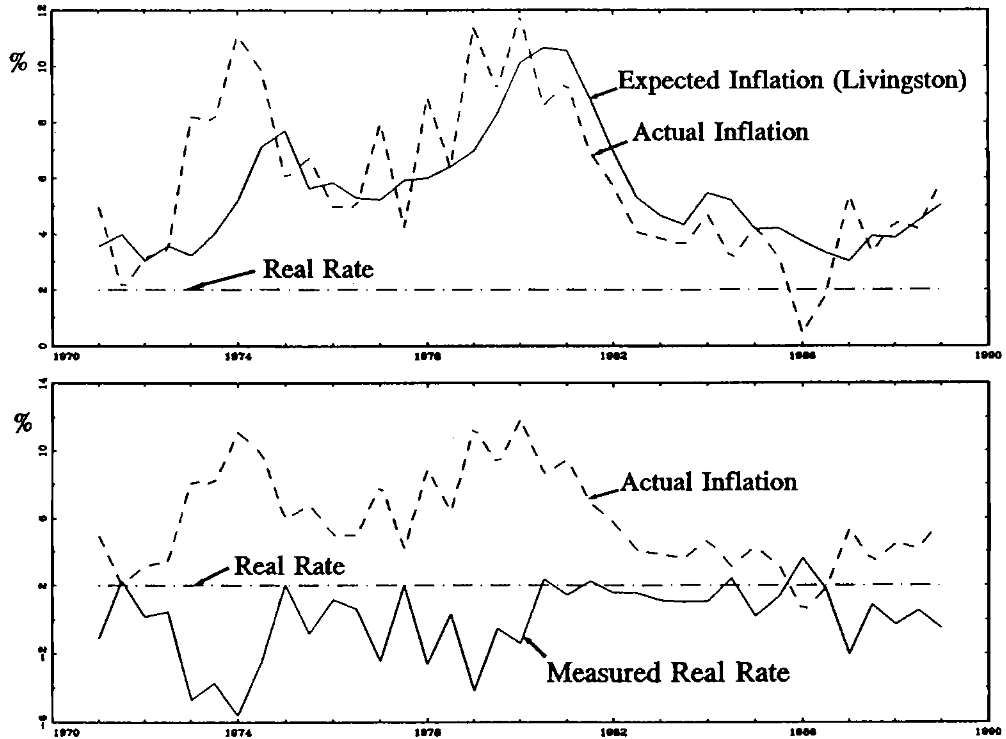


Figure 2: Cusum test for Structural Instability in Quarterly Inflation

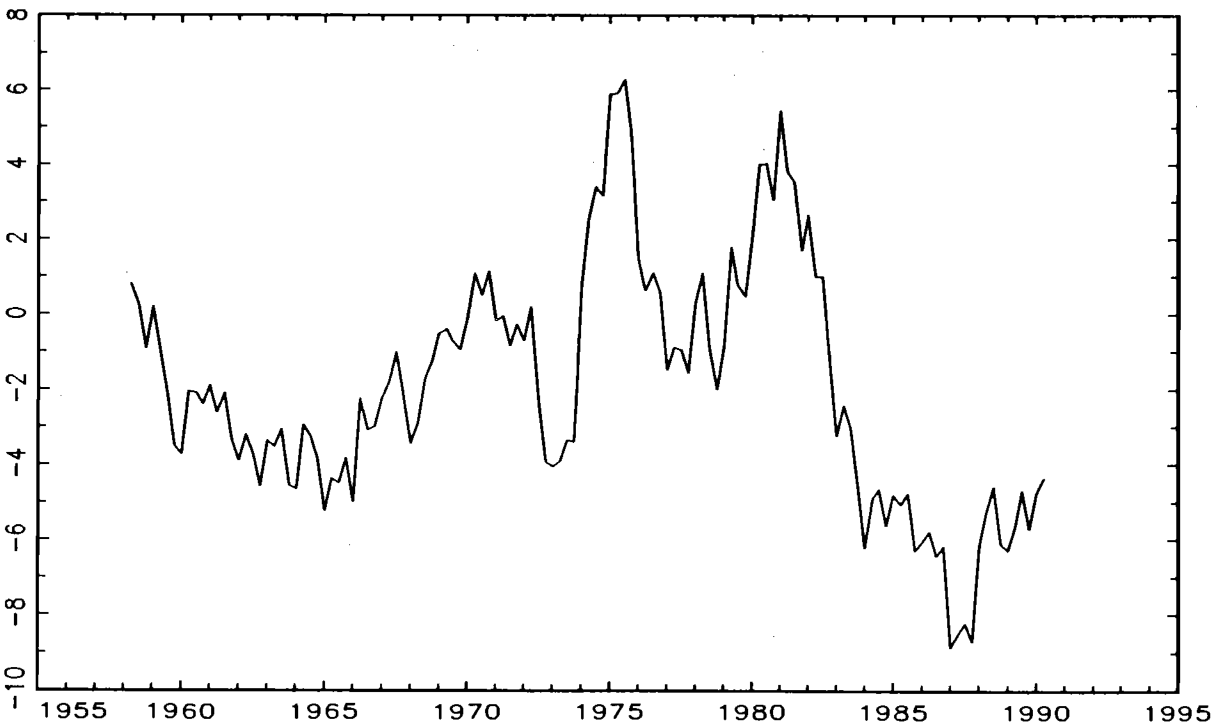
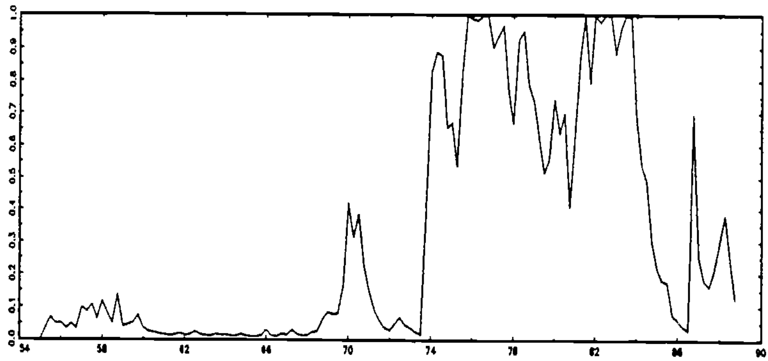


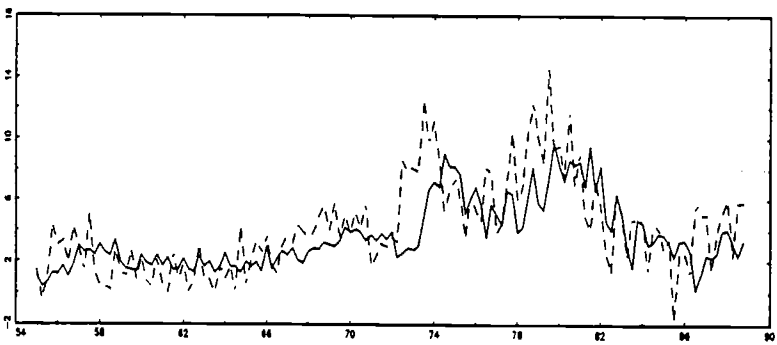
Figure 3: Markov Model Estimates

Filtered probability of being in unit root state: $\Pr(S_t = 1)$



Quarterly Inflation One Year Ahead

key: ——— quarterly markov forecast 1, - - - actual quarterly inflation



Livingston and Markov Forecasts of 6-Month Inflation

key: ——— Livingston, - - - Markov

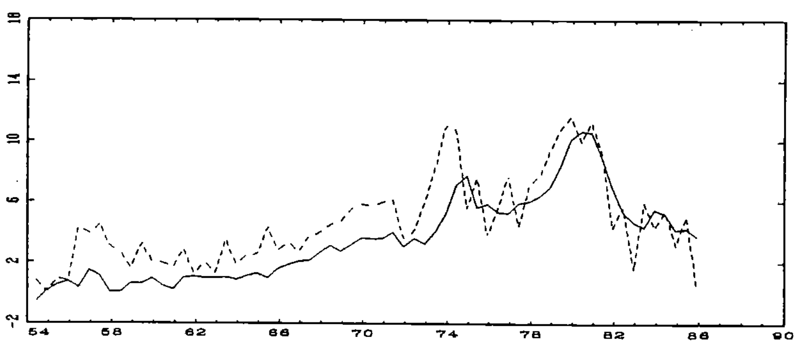


Figure 4: Expected and Actual Three-month Real Rate Forecasts One Year Ahead

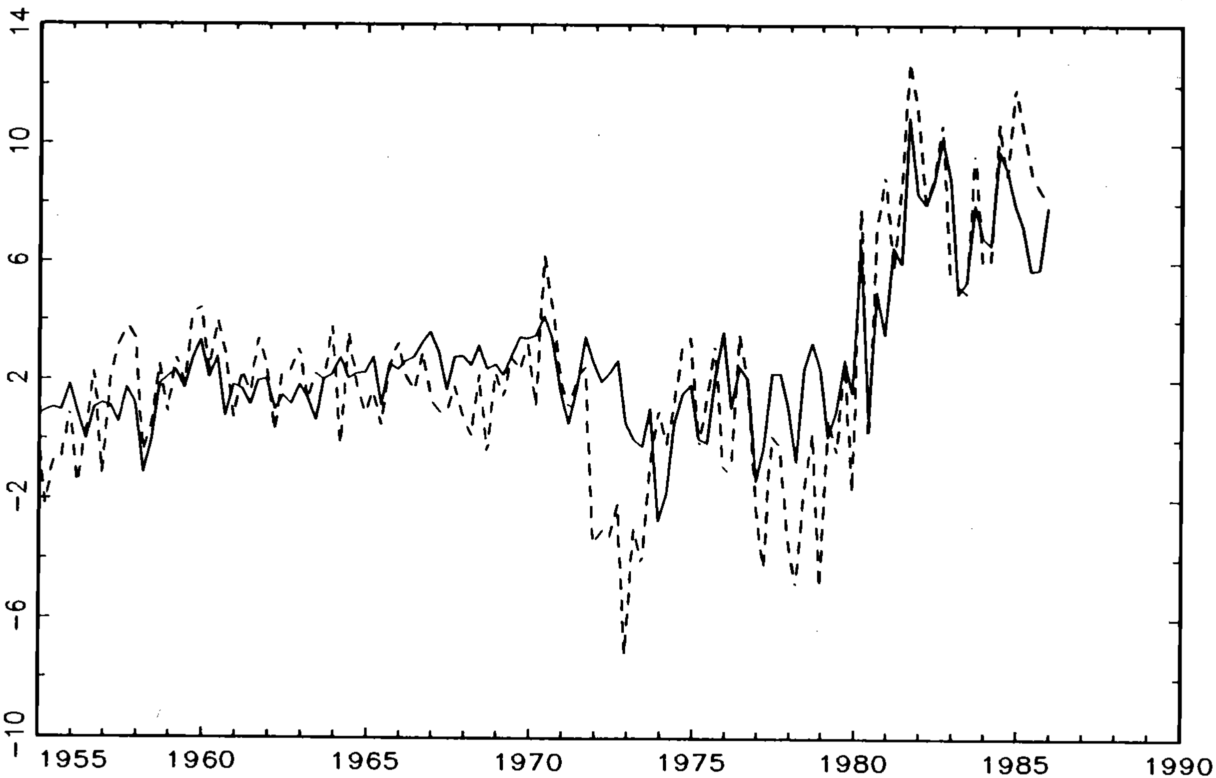
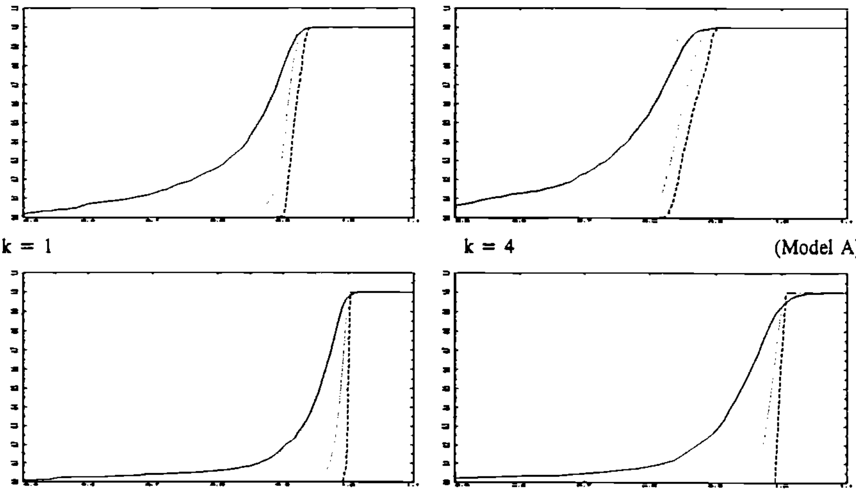


Figure 5: Monte Carlo Distributions of Cointegrating Parameters

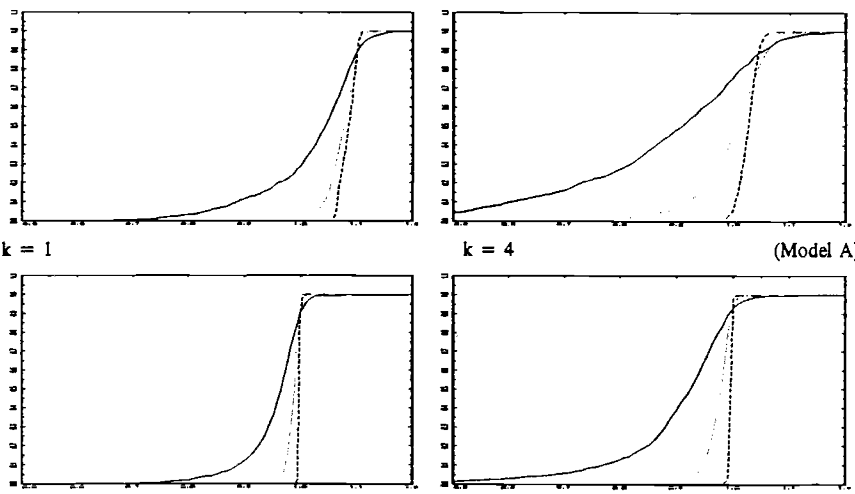
Equation: $E(\pi^k(t) \mid t,M) = \gamma_0 + \gamma_1 \pi^k(t) + \eta(t+k)$



k = 1 k = 4 (Model A)

k = 1 k = 4 (Model B)

Equation: $\pi^k(t) = \psi_0 + \psi_1 E(\pi^k(t) \mid t,M) + \omega(t+k)$



k = 1 k = 4 (Model A)

k = 1 k = 4 (Model B)

Key: samples 25 yrs. ———, 100 yrs. ..... 1000 yrs - - - (quarterly data)