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A REAPPRAISAL OF RECENT TESTS OF THE
PERMANENT INCOME HYPOTHESIS

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A Reappraisal of Recent Tests of the
Permanent Income Hypothesis

ABSTRACT

Hall (1978) showed that the permanent income hypothesis implies that consumption (1) follows a random walk, and (2) cannot be predicted by past income. Reexamination of Hall's data results in rejection of the random walk hypothesis in favor of the alternative hypothesis of positively autocorrelated changes. Evidently this is due to Hall's choice of a quadratic utility function. A logarithmic utility function implies a random walk in the log of consumption which is supported by the data.

Hall reported that past income had a negative but insignificant relation to consumption. Changes in the log of income, however, do have a positive predictive relation to changes in the log of consumption. The adjustment of consumption to income seems to be spread over two quarters.

Flavin's (1981) test of the theory is formally equivalent to Hall's except for assuming stationarity around a time trend. Mankiw and Shapiro (1984) have pointed out that the effect of detrending may be to tend to rejection of the theory when it is in fact correct. For Hall's data the effect of detrending is to reverse the sign of the coefficient on past income. Its magnitude is what the Mankiw-Shapiro analysis predicts under the permanent income hypothesis.

1. Introduction

Recent tests of the permanent income/life-cycle hypothesis exploit the implications of the theory for the time series properties of consumption and its relationship to income. In particular, Hall (1978) showed that (1) consumption should follow a random walk with drift, and (2) changes in consumption should be uncorrelated with lagged values of other variables, including income. Hall concluded from tests based on quarterly U.S. per capita data that consumption does indeed behave like a random walk and that past values of income are only marginally useful in predicting consumption in the presence of past consumption. This encouraging assessment seems to be dashed by the contradictory conclusion of a subsequent paper by Flavin (1981) who claims to find evidence implying decisive rejection of the permanent income hypothesis in the form of an 'excess sensitivity' of consumption to income. The objective of this paper is to take another look at the evidence presented in these two papers and examine the sources of their differing conclusions. Briefly, I conclude that Hall's data are more strongly at variance with his hypothesis than his tests suggest, but that Flavin's negative results are due at least in part to inappropriate detrending of consumption and income as suggested in a recent paper by Mankiw and Shapiro (1984).

2. Hall's Model

The basic theoretical result from which Hall proceeds is that consumers maximizing the expectation of the utility of future consumption subject to the present value of uncertain future income will adjust current expenditures so that

$$E_t[u'(c_{t+1})] = [(1+\delta)/(1+r)]u'(c_t)$$

where $E_t[]$ denotes expectation as of time t , $u(c)$ is the contribution of current consumption to utility, δ is the rate of time preference, and r is the real interest rate. This is the expected value version of the condition that marginal utilities of consumption, appropriately discounted, will be equated across time periods. In the case of a quadratic utility function the conditional expectation of consumption next period is

$$E_t(c_{t+1}) = [\bar{c}(r-\delta)/(1+r)] + [(1+\delta)/(1+r)]c_t$$

because marginal utility is linear in consumption with \bar{c} being the bliss level of consumption. Since r must be greater than δ , the intercept is positive and since δ and r are both small the slope is close to but less than unity.

The realized value of c_{t+1} will be this conditional expectation plus a random error, that is a random walk with drift or very nearly so. Estimating the model over Hall's sample period (quarterly, 1948:1-1977:1) using his variable definitions but revised data from Business Statistics 1982 I obtained results using SHAZAM (White, 1978) which are nearly identical to those reported by Hall,

$$c_t = -13.9 + 1.011 c_{t-1} + e_t$$

(7.7) (.003)

$$SE = 13.9; R^2 = .9989; D.W. = 1.70.$$

Standard errors are in parentheses below the coefficients. Hall interprets these results as supporting the random walk hypothesis. I find these results less encouraging. The theory predicts a positive intercept, not a

negative one. It also predicts a slope close to but less than unity, but the estimated slope is larger than unity. In addition, we know that the least squares slope in these models is downward biased; see Fuller (1976) and Evans and Savin (1984). The Durbin-Watson statistic is not appropriate for testing the hypothesis of serially random errors in a model with a lagged dependent variable. The h-statistic suggested by Durbin (1970) for these situations is standard normal under the null hypothesis. Its computed value is 1.58 in this instance which is significant at the .05 level against the alternative of positive autocorrelation which would correspond to the usual notion of a business cycle.

To see whether prior values of consumption have predictive power, which would contradict the theory, Hall adds three more lags of consumption to the regression. My near-replication is

$$c_t = \begin{matrix} -8.6 & 1.132 & -.065 & .065 & -.125 & + e_t \\ (8.0) & (.091) & (.140) & (.138) & (.092) \end{matrix} c_{t-1} c_{t-2} c_{t-3} c_{t-4}$$

$$S.E. = 13.8; R^2 = .9989; D.W. = 1.96.$$

Hall reports a positive intercept which must be a typographical error. The F-statistic for testing the hypothesis that lags 2, 3, and 4 do not enter the equation is 1.60 which is acceptable. This may not, however, be a very powerful test if changes in consumption are autocorrelated. If that were the case we would have:

$$c_t - c_{t-1} = a_1(c_{t-1} - c_{t-2}) + a_2(c_{t-2} - c_{t-3}) + a_3(c_{t-3} - c_{t-4}) + e_t$$

$$c_t = (1+a_1)c_{t-1} + (a_2-a_1)c_{t-2} + (a_3-a_2)c_{t-3} - a_3c_{t-4} + e_t.$$

Thus the coefficients of c_{t-2} and c_{t-3} may well be small under the alternative hypothesis and the coefficient of c_{t-1} , ignored in Hall's F-test, may contain information about autocorrelation in the changes. In particular, rearranging the coefficients in Hall's regression we obtain $a_1 = .13$, $a_2 = .07$, $a_3 = .13$ or $.125$ depending on how one uses the four coefficients to solve for the three a's. Estimating the a's directly I obtained $a_1 = .173$, $a_2 = .104$, $a_3 = .169$ with t-ratios of 1.92, 1.13 and 1.86 respectively. The F-statistic is 4.10 which is significant at the 0.01 level.

It would seem then that past changes in consumption do predict future changes, seemingly contrary to the permanent income hypothesis. Rejection is assumed, however, because of the linear form arising from the assumption that utility is quadratic. In Hall's equation, all growth in consumption is attributed to the constant term involving \bar{c} . Actual growth in consumption is exponential rather than linear. The arithmetic of growth ensures that the coefficient of past consumption will exceed unity and that changes in consumption (imposing a unit coefficient) will be positively autoregressive. The quadratic utility function is valid as a local approximation with $(c-\bar{c})$ reflecting the local value of marginal utility. In the long run there is no bliss level of consumption and \bar{c} rises over time. In effect, \bar{c} is a missing variable in the Hall regression and lagged consumption gains additional weight in the regression as its proxy.

3. Tests Based on a Logarithmic Utility Function

A more reasonable form of the theory comes out of assuming a logarithmic utility function for consumption each period. This implies

$$E_t(1/c_{t+1}) = [(1+\delta)/(1+r)](1/c_t)$$

since $u'(c) = \theta/c$, θ a constant. The ratio (c_t/c_{t+1}) is always positive and might reasonably be assumed to be distributed log-normally, in which case we would have

$$E_t(c_t/c_{t+1}) = [(1+\delta)/(1+r)] = \exp(\mu + 1/2 \sigma^2)$$

where μ is the expected value of $\ln(c_t/c_{t+1})$ and σ^2 its variance. The evolution of consumption is then given by

$$\ln(c_t) = \ln(c_{t-1}) - \mu - \varepsilon_t$$

where ε_t is i.i.d. $N(0, \sigma^2)$. Sample values of μ and σ^2 in Hall's data are $-.0047$ and $.36E - 04$ respectively, implying

$$[(1+\delta)/(1+r)] \cong .9953$$

$$\delta \cong -.0046 + .9953 r.$$

These estimates satisfy the condition that the rate of time preference is less than the real interest rate.

The permanent income/life cycle hypothesis predicts that changes in $\ln(c)$ are serially uncorrelated. The sample autocorrelation at lag one is 0.14 with a standard error of 0.09. The sample autocorrelations at lags two through twelve are all smaller than this and their Q-statistic is only 9.6 which is less than its expected value. In contrast, for the unlogged data the corresponding statistics are 0.24 at lag one with standard error 0.09 and a Q-statistic of 21.5 which is significant at the 0.05 level. Regression of the change in $\ln(c)$ on the past three changes yields an F-statistic of 1.2, close to its expected value under the null

hypothesis that $\ln(c)$ is a random walk. Therefore we have little evidence that changes in $\ln(c)$ can be predicted from past changes.

The logarithmic utility version of the permanent income hypothesis also implies that the coefficient of $\ln(c_{t-1})$ is properly taken to be unity. As mentioned earlier, the least squares coefficient in the regression of a random walk on its lagged value is biased towards zero and tests based on standard classical regression theory can be highly misleading. Further, Evans and Savin (1984) have shown that the distribution of the least squares coefficient depends on the unknown value of the intercept. A procedure which does provide an operational test has been developed by Dickey (1976) and Fuller (1976). The obvious competitor to the unit coefficient model is one with stationary fluctuations around a trend since a trend also would account for long term growth. The basic regression is

$$\ln(c_t) = .301 + .960 \ln(c_{t-1}) + .231 \text{ E-03*TIME} + e_t.$$

(.021)

Although the coefficient of $\ln(c_{t-1})$ is about two conventional 'standard errors' less than unity it is well within the body of the sampling distribution under the null hypothesis as tabulated by Dickey. Similarly the 't-ratio' of -1.90 is also well within sampling bounds. Nelson and Plosser (1982) reported a mean t-ratio of -2.22 in a Monte Carlo study based on sample size 100. Since the distribution of the t-ratio was shown to be insensitive to sample size by Dickey, this can be taken as an indication that the t-ratio reported above is actually a bit above the value expected for a random walk. The coefficient and t-ratio are unchanged (to three decimal places) when one or three lags of $\Delta \ln(c)$ are

included in the regression to allow for possible autoregression in first differences.

4. Do Changes in Income Predict Changes in Consumption?

If consumers adjust fully to information about their future incomes, past income and income changes should be uncorrelated with the change in consumption. Hall's test was to regress current consumption on lagged consumption and lagged income (Hall, 1978, Table 3). Rerunning his first regression I obtained

$$c_t = -39.0 + 1.083 c_{t-1} - .049 y_{t-1} + e_t$$

(17.2) (.044) (.031)

$$SE = 13.8; R^2 = .9989; DW = 1.82; h = 1.12;$$

which gives a stronger indication of an influence for lagged income ($t = -1.63$) than the results reported by Hall, presumably due to data revisions. The effect is nevertheless not significant and it is negative, as Hall reported. If the coefficient of c_{t-1} is close to unity, however, then the equation describes first differences in consumption and it is not obvious why the level of income would predict changes in consumption. As also reported by Hall, adding three more lags of income results in a partial F-statistic for income of about 2, below the critical .05 level. However, if one imposes a unit coefficient on c_{t-1} the F-statistic for the four lags of income jumps to 3.58 which is significant at the 0.01 level.

This income effect appears to be largely spurious, however, due to lagged income proxying for lagged consumption changes. This can be seen when both lagged income changes and lagged consumption changes are included:

$$\Delta c_t = 9.07 + .162 \Delta c_{t-1} + .054 \Delta y_{t-1} + e_t$$

(1.65) (.100) (.041)

$$SE = 14.16; R^2 = .0662; DW = 2.02; h = N.A.;$$

Note that the t-ratio on Δy_{t-1} is only 1.29 while that for Δc_{t-1} is 1.62. Adding two more lags of Δy results in a partial F-statistic for income of only 0.61 but Δc_{t-1} has a t-statistic of 1.52. Thus the positive conclusion reached by Hall on the irrelevance of past income is supported by first difference regressions.

The corresponding regression for logarithms gives quite different results, however. The estimated relation is

$$\Delta \ln(c_t) = .004 + .014 \Delta \ln(c_{t-1}) + .129 \Delta \ln(y_{t-1}) + e_t$$

(.001) (.096) (.051)

$$SE = .0058; R^2 = .0662; DW = 2.00; h = N.A.$$

which indicates a significant predictive content for past income ($t = 2.5$). As expected, the lagged consumption term is not significant since rates of change in consumption display little autocorrelation. Adding two more lags in the income variable we have

$$\Delta \ln(c_t) = .003 + .011 \Delta \ln(c_{t-1}) + .130 \Delta \ln(y_{t-1})$$

(.001) (.100) (.052)

$$+ .026 \Delta \ln(y_{t-2}) - .037 \Delta \ln(y_{t-3}) + e_t$$

(.048) (.046)

$$SE = .0058; R^2 = .0746; DW = 2.00; h = N.A.$$

The partial F-statistics for these additional lagged income terms is only 0.34. Evidently, the predictive value of income is confined to a one

quarter lag. This is further clarified by regression on the contemporaneous income change and past income changes:

$$\begin{aligned} \Delta \ln(c_t) = & .003 + .207 \Delta \ln(y_t) + .130 \Delta \ln(y_{t-1}) \\ & (.001) \quad (.044) \quad \quad (.043) \\ & + .038 \Delta \ln(y_{t-2}) - .044 \Delta \ln(y_{t-3}) + e_t \\ & (.043) \quad \quad \quad (.041) \end{aligned}$$

$$SE = .0053; R^2 = .2295; DW = 2.23.$$

Changes in the log of income display little autocorrelation: -0.03, -0.01, and 0.04 at lags one through three. Thus the changes are essentially innovations in the income process which is akin to a random walk. The response of consumption to the current innovation in income is consistent with the permanent income hypothesis, but the response to the previous innovation is not. The lagged effect of income implies positive autocorrelation in $\Delta \ln(c)$ of about .08 at lag one quarter compared with a sample value of .14.

The data are averages for each quarter and temporal aggregation can induce the appearance of lags when in fact there are none in the underlying relationship defined over some shorter time interval as Tiao and Wei (1976) have shown. As a rough check on this possibility I collected third-month-of-quarter data which reduces the degree of temporal aggregation. The consumption variable is nondurables only, and no adjustment is made for population, although population contributes very little variation to the per capita numbers. The monthly data is available only for the second half of Hall's sample period, starting in 1961. The basic regression result is

$$\Delta \ln(c_t) = .006 - .026 \Delta \ln(c_{t-1}) + .126 \Delta \ln(y_{t-1}) + e_t$$

(.144) (.116)

$$SE = .009; R^2 = .0213; DW = 1.93; h = N.A.$$

Evidently the end-of-quarter data is much noisier, but the point estimate for $\Delta \ln(y_{t-1})$ is about the same. The smaller t-ratio is of course reflective of the smaller sample size. Indeed, when two and three lags are added they are more significant; the partial F-statistic for three lags of $\Delta \ln(y)$ is 2.71 which just misses being significant at the 0.05 level.

5. Flavin's Test and the Mankiw-Shapiro Critique

Flavin (1981) adopts the conventional specification of the permanent income hypothesis and shows that it implies the conditions tested by Hall. Permanent income is defined as the annuity value of the individual's discounted future income and consumption is permanent income plus an error. Consumption changes only in response to innovations in income because forecastable movements in income have already been taken into account in the calculation of permanent income. If the income process is

$$y_t = \mu_1 + \rho y_{t-1} + \varepsilon_{1,t}$$

then consumption responds according to

$$\begin{aligned} \Delta c_t &= \mu_2 + \delta(\varepsilon_{1,t}) + \varepsilon_{2,t} \\ &= \mu_2 + \delta(y_t - \mu_1 - \rho y_{t-1}) + \varepsilon_{2,t} \end{aligned}$$

where δ depends on the discount rate and ρ . An additional term in the current change in income Δy_t should be irrelevant, so the coefficient β in

$$\Delta c_t = \mu_2 + \delta(\varepsilon_{1,t}) + \beta \Delta y_t + \varepsilon_{2,t}$$

measures 'excess sensitivity' of Δc_t to Δy_t . Flavin shows that the income process along with the consumption equation constitute a just-identified structural system which may therefore be estimated in its reduced form

$$\Delta c_t = \mu_2' + \beta(\rho-1)y_{t-1} + v_t$$

$$= \mu + \pi y_{t-1} + v_t$$

where y_t has been eliminated by substitution.

This formulation makes sense if income is a stationary process, $\rho < 1$. If income is a random walk then Δy_t is the innovation $\varepsilon_{1,t}$ and we have no way of distinguishing excess sensitivity from appropriate sensitivity unless somehow we know δ . Flavin, however, imposes stationarity (in the sense of mean reversion) on the data by removing a fitted trend. The resulting 'detrended' data will behave like a stationary time series even if the process generating them is a trendless random walk. In a previous paper Kang and I showed that a detrended random walk will tend to exhibit cycles which is reflected in an autocorrelation function that is shaped like a damped sin wave (Nelson and Kang, 1981 and 1985). This introduces a predictability into the detrended data which is purely artifactual. Mankiw and Shapiro (1984) have pointed out recently that this will result in a spurious indication of excess sensitivity even if there is none. They show that in the case where consumption is equal to income detrending leads to an estimate of β equal to one, that is 'complete excess sensitivity.'

More generally, suppose that the income variable we measure is a random walk so that its first differences are its innovations. Under the

permanent income hypothesis, consumption responds to these innovations as well as to other increments in information. The consumption-income relation would be

$$\Delta c_t = \delta \Delta y_t + \varepsilon_t$$

$$c_t = \delta y_t + \eta_t$$

where η_t is the accumulation of the ε_t through time t . For given sample size the detrending operator is well defined and the same for each variable, hence we have

$$\tilde{c}_t = \delta \tilde{y}_t + \tilde{\eta}_t$$

where tildes indicate detrended variables. Now the least squares coefficient $\hat{\pi}$ in Flavin's regression is easily shown to be

$$\hat{\pi} = \delta(\rho_1 - 1) + \left[\frac{\sum (\tilde{\eta}_t - \tilde{\eta}_{t-1}) \tilde{y}_{t-1}}{\sum \tilde{y}_{t-1}^2} \right]$$

where ρ_1 is the sample autocorrelation of detrended income at lag one. The second term involves only the sample covariance between errors and income which are independent by assumption. The value of ρ_1 tends to be about $(1 - 10/N)$ for a detrended random walk, which is 0.92 in the case of $N=121$ for Hall's data set. Thus π will tend to be about $-.08 \delta$ even when there is no excess sensitivity. Under Flavin's specification this would be interpreted as $\beta(\rho - 1)$ so the effect of detrending is to misinterpret δ , appropriate sensitivity, as β , excess sensitivity. The least squares estimate of δ in the regression of Δc on Δy using the Hall data is 0.20. Thus the predicted value of π is $(-.08)(.21) = -.017$. After detrending the Hall data in logs one finds

$$\Delta \tilde{c}_t = -.0001 - \frac{.024}{(.016)} \tilde{y}_{t-1} + e_t$$

with a t-ratio for π of -1.49. The implied value of β is 0.29 if the deterministic trend specification is correct. Whether consumption is in fact excessively sensitive to income is not clear from this number--it may reflect an entirely appropriate value of δ and inappropriate detrending of the data.

If the random walk specification is correct then $\rho=1$ and π no longer has the interpretation of excess sensitivity to the current income change. It nevertheless is the case that past income should have no predictive value for $\Delta \ln(c)$ if consumption reflects available information. The regression results without detrending are

$$\Delta \ln(c_t) = -.038 + \frac{.0054}{(.0028)} \ln(y_{t-1}) + e_t$$

with a t-ratio for π of 1.92. Note that the sign of π has reversed from negative to positive as a result of not detrending the data, and the statistical significance is substantially greater. This suggests that the mean of $\Delta \ln(c)$ is not constant but drifted upward over the sample period.

6. Summary and Conclusions

Hall (1978) showed that the permanent income theory of consumption implied under certain assumptions that (1) real consumption per capita follows a random walk, and (2) that consumption is not predictable from past income (or any past information) given prior consumption. Although Hall found the random walk hypothesis acceptable, a reexamination of the data using tests designed to detect serial dependence leads to rejection.

This turns out to be due to Hall's choice of a quadratic utility function which is equivalent to assuming that growth in real consumption is linear. A logarithmic utility function implies exponential growth and a random walk in the log of consumption, a hypothesis which is supported by the data.

Past income was reported by Hall to have a negative but insignificant predictive value for consumption. Changes in the log of income, however, do have predictive content for changes in the log of consumption and the correlation is positive. Briefly, the adjustment of consumption to innovations in income seems to take place over the current and following quarter. The coefficient on the prior income innovation while statistically significant is small, about half as large as the response to the contemporaneous innovation.

The test of the permanent income theory proposed by Flavin (1981) is formally equivalent to Hall's test, but the coefficient on lagged income is shown to be a measure of excess sensitivity of consumption to income. This interpretation depends on income being stationary around a time trend and Flavin detrends income and consumption prior to testing. Mankiw and Shapiro (1984) have pointed out that the effect of detrending will be the spurious appearance of excess sensitivity when the theory is in fact correct and both series are random walks. The effect of detrending over Hall's sample period is to produce a negative coefficient on lagged income which is roughly the size predicted by the Mankiw-Shapiro analysis given the contemporaneous sample correlation between changes in the log of consumption and income. It is therefore not clear from the Flavin test that consumption is excessively sensitive to income changes.

One reaction to finding a lag in the response of consumption to income is simply that the theory fails. My own is that the theory holds up remarkably well in view of the severity of its assumptions, its simplicity, and the quality of the data. Numerous studies suggest that the real rate of interest is not constant but varies over time. It would be surprising therefore if the log of consumption were a strict random walk with constant mean rate of change. The lag in the response of consumption to income is short, only one quarter. It is not costless for consumers to reassess their income position and consumption patterns and it would be surprising if they did so instantaneously or if all did each quarter. Finally, the quality of the data is not to be taken too seriously. According to Business Statistics 1982, certain components of personal consumption expenditures are based on interpolation between benchmark surveys and annual data. The effect of interpolation in general would be to create the appearance of slower adjustment than what actually occurs.

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