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NONERGODIC ECONOMIC GROWTH

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ABSTRACT

This paper explores the role of complementarities and coordination failure in economic growth. We analyze the evolution composed of a countable set of infinitely-lived heterogenous industries. Individual industries exhibit nonconvexities in production and are linked across time through localized technological complementarities. Each industry employs one of two production techniques. One technique is more efficient in using capital than the other, but requires the payment of a fixed capital cost. Both techniques exhibit technological complementarities in the sense that the productivity of capital invested in a technique is a function of the technique choices made by various industries the previous period. These complementarities, when strong enough, interact with incompleteness of markets to produce multiple Pareto-rankable equilibria in ling run economic activity. The equilibria have a simple probabilistic structure that demonstrates how localized coordination failures can affect the aggregate equilibrium. The model is capable of generating interesting aggregate dynamics as coordination problems become the source of aggregate volatility. Modifications of the model illustrate how leading sectors can cause a takeoff into high growth.

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1. Introduction

The large differences in both per capita output levels and growth rates across countries have recently become the focus of considerable research in macroeconomics. Much of this interest stems from the implications of different national experiences for modelling macroeconomic aggregates as devolving from a dynamic general equilibrium framework. A hallmark of the stochastic growth model pioneered by Brock and Mirman [1972] is the convergence of economies with identical sets of preferences and production to a unique limiting distribution for a wide array of initial conditions. Yet many analyses of long run output movements (see Romer [1986], DeLong [1988], Quah [1990], and Bernard and Durlauf [1990], among others) have concluded that divergence is in fact the norm. One approach to this stylized fact is of course to argue that countries possess sufficiently heterogeneous microeconomic characteristics that long run behavior is heterogeneous as well. Jones and Manuelli [1990], for example, show that if the marginal product of capital is sufficiently large over all possible capital/labor ratios, then different specifications of microeconomic parameters such as tax rates, production functions or rates of time preference can lead to divergence of growth rates in a competitive general equilibrium framework. However, most economists seem confident that differences in per capita output levels and growth rates are too vast to be attributed to differences in microeconomic parameters alone. Reinforced by the view that these disparities are Pareto rankable, the bulk of theoretical work on divergence has looked for deviations from the Arrow-Debreu model.

One approach to explaining divergence has relied on modifications of the standard growth model to account for increasing returns to scale as a source of multiple steady states. Romer [1986], Lucas [1988] and Azariadis and Drazen [1990] have argued that various forms of social increasing returns to scale can lead to a divergence between the social and private marginal products of human and physical capital and lead to multiple growth paths. Romer models the individual firm production function as being positively affected by the level of the aggregate capital stock in the economy. The aggregate stock proxies for a host of production complementarities between individual producers such as unpatentable innovations. As a result, a given marginal product of capital is compatible with two different levels of capital, leading to multiple equilibria.

The possibility that multiple equilibria exist for aggregate activity has been explored in a number of other formulations of market failure. The effects of incomplete markets are analyzed in Diamond [1982], where multiplicity occurs due to the externalities associated with search. Heller [1986, 1990] derives similar results through imperfect competition. In his model, firms act as Cournot oligopolists. Because of the feedback of production into aggregate demand, marginal revenue and marginal cost can intersect at several different levels of output. More generally, models of this type emphasize the role of complementarities across firms and consumers in determining average economic activity. Strong complementarities can lead the aggregate economy to a Pareto inferior equilibrium because of coordination failure.

Increasing returns to scale and imperfect competition have been linked to multiple equilibria, with emphasis on long run development, in recent work by Murphy, Shleifer and Vishny [1989]. These authors show that increasing returns can lead to multiple equilibria when production decisions by monopolistic firms are constrained by the extent of market demand. Low and high output equilibria are both possible due to aggregate demand spillovers across all parts of the economy which are created by production in each sector. These different steady states allow low per capita income levels to be interpreted as a manifestation of coordination failure.

One problem in thinking about most coordination failure models as metaphors for aggregate activity is that they are limited by the absence of any stochastic dynamics. These models typically exhibit several constant steady states. The level of long run activity, however, is entirely determined by the initial conditions of the model. No role exists for transition dynamics in the selection of or movements across steady states. Further, most coordination failure models cannot address the question of cycles in aggregate activity. (See Diamond and Fudenberg [1989] for an exception.)

This paper is designed to construct an explicitly dynamic and stochastic model of coordination failure with a focus on the mechanisms by which growth evolves. We work with a variant of the Brock-Mirman economy. The economy consists of a countable number of infinitely-lived, profit maximizing industries. Consumers are risk neutral. Technological complementarities create intertemporal linkages between the production functions of each sector, in ways similar to social increasing returns models. When these complementarities are strong enough, coordination failure can occur which affects long run growth.

Methodologically, we follow an approach to stochastic coordination problems developed in Durlauf [1990a]. The basic idea is to interpret an aggregate equilibrium as a joint probability measure characterizing many agents. The microeconomic characterization of an individual agent is treated as equal to the specification of a probability measure over the agent's actions conditional upon the rest of the economy. The existence of an equilibrium is correspondingly equivalent to the existence of a joint probability measure consistent with the microeconomically generated conditional measures. Uniqueness of the equilibrium occurs whenever the conditional probability measures can generate only one joint measure. When a class of conditional probability measures is consistent with multiple joint measures, the stochastic process is said to be nonergodic. This characterization of coordination problems as the relationship between conditional and joint probability measures provides a natural framework for discussing endogenous evolution towards steady states, as infinite-dimensional stochastic processes can be modelled as the limits of finite-dimensional stochastic processes with different initial and terminal conditions. At the same time, by modelling the economy as a stochastic process, it is straightforward to generate nontrivial time series properties for aggregate output.

The dynamic behavior of our aggregate economy possesses two interesting properties. First, the complementarities in the model act as a source of volatility in output across both industries and time. Intuitively, complementarities in behavior mean that changes in the decisions of one agent spill over and alter the constraint sets of other agents in ways different from the effects which occur in a competitive equilibrium model. High production in one industry induces high production in other industries through the technological complementarities. This type of stochastic volatility is driven by fundamentals and thus contrasts with results showing how incomplete markets can generate sunspots.

Second, the transition probabilities describing how industries react to complementarities create the potential for multiple long run equilibria. If industries are sufficiently sensitive to the production decisions of others, in the sense that high production by a given industry will occur with very low probability in the absence of high production of others, then the economy will exhibit two long run equilibria. It is standard in the coordination failure models that complementarities can lead to multiplicity. (See Cooper and John [1988] for an excellent discussion of the static case.) The current framework differs from most other models by describing this multiplicity as the outcome of a dynamic process and by permitting transition dynamics between equilibria. In addition, our model permits multiplicity to coexist with heterogeneity in the behavior of individual agents. Multiple equilibria will not imply that all agents are simultaneously producing at the high or low level.

In terms of understanding growth mechanisms, our model gives a primary role to two factors. First, strong local linkages across industries can create sequential complementarities which build up over time to affect aggregate behavior. Strong local linkages mean that expansion in one sector will increase the conditional probability that a finite number of additional sectors will expand. Collectively, these effects can lead to aggregate growth. Second, leading sectors can induce takeoff in economic development. Leading sectors are defined as industries that trade with all other industries and hence whose expansion can cause economy-wide complementarities. Growth in these sectors, due to technical change or other factors, can have far reaching effects. In particular, if leading sectors grow and provide services at lower cost to the rest of the economy, the conditional probability structure of microeconomic production decisions will shift toward choosing high production with greater frequency. This shift can, through the strong local complementarities, induce a takeoff to growth.

Our results contrast with the Murphy, Shleifer, Vishny model of industrialization, whose antecedent is the big push model of Rosenstein-Rodan [1943]. In big push theories, industrialization occurs through the simultaneous movement of many sectors. In our model, higher production in a given sector generates complementarities over a finite subset of other sectors. However, because these ranges of complementarities overlap, all sectors are eventually linked. Hence growth can proceed through the sequential expansion of different parts of the economy. Interestingly, our model roughly corresponds to a major competitor of the big push school of industrialization policy: the Hirschman model of economic development through the promotion of leading sectors with strong linkages to industries throughout the economy. In Hirschman's framework, economic development emerges as growth of some sectors spills over to cause growth in other sectors through many different supply and demand links. Different parts of an economy can thus develop at different rates, eventually leading to aggregate expansion. As in our model, Hirschman argues that leading sectors can stimulate aggregate growth through simultaneous interaction with many industries at once which then stimulate the many intertemporal linkages in production.

Section 2 of this paper sets up a simple economic environment embodying localized complementarities. The economy consists of a countable set of interacting industries, each facing a nonconvexity in production. The solution to individual firm capital accumulation problems and existence of the aggregate equilibrium are analyzed. Section 3 discusses conditions for multiple equilibria. In Section 4, the cyclical behavior of the economy is explored through simulations. Section 5 analyzes the role of leading sectors in generating a takeoff from one equilibrium to another. Section 6 provides summary and conclusions. A Technical Appendix follows which outlines some of the mathematics and contains proofs of the various theorems in the text.

2. A model of interacting industries

We consider a countable set of infinitely lived industries indexed by i. Time is discrete. The economy may be thought of as the limit of an increasing sequence of economies composed of finite numbers of industries. Each industry consists of a continuum of identical firms. This specification permits us to treat the industries as individual agents and at the same time ignore strategic considerations in their production decisions. Specifically, the distinction between firms and industries is made to justify assuming that industries act competitively with respect to complementarities. Each firm takes the behavior of its own and other industries as given when making production decisions. All firms produce a homogeneous good; industries are distinguished by distinct production functions rather than distinct outputs. Following the standard Brock-Mirman formulation, the homogeneous final good may either be consumed or converted to a capital good which fully depreciates after one period. Aggregate output Y_t, consumption C_t and capital K_t obey

$$Y_t = C_t + K_t. \tag{2.1}$$

These aggregates are computed by summing over all firms and consumers at each t.

Given our assumptions on internal industry structure, industry behavior is proportional to the behavior of a representative firm which chooses a capital stock sequence $\{K_{i,t}\}$ to maximize the present discounted value of profits $\Pi_{i,t}$

$$\Pi_{i,t} = \mathrm{E}\left(\sum_{j=0}^{\infty} p_{t+j}(Y_{i,t+j} - K_{i,t+j}) \mid \mathfrak{F}_t\right). \tag{2.2}$$

$Y_{i,t}$ equals the output of the i'th industry's representative firm at t; p_t equals the date zero price of output at t (the price of output available at t in terms of the price of output available at time 0); $\mathfrak{F}_t$ denotes all information available to the economy at the beginning of t. Each industry's representative firm has an initial endowment of output $Y_{i,0}$ which can be used as capital.

Consumers in the economy are risk neutral. Consumer r evaluates the consumption stream $\{C_{r,t}\}$ through the utility function

$$U_{r,t} = E(\sum_{j=0}^{\infty} \beta^j C_{r,t+j} \mid \mathfrak{F}_t). \tag{2.3}$$

All firms are owned by the consumers. All uninvested output is paid out as real dividends. As we shall see, in equilibrium, total dividends payments exactly equal consumption for each consumer.

Aggregate behavior is determined by the interactions of many heterogeneous industries employing nonconvex technologies. Production occurs with a one period lag; firms at $t-1$ employ both one of two production techniques and a level of capital to determine output at t. Only one technique may be used at a time. Cooper [1987] and Murphy, Shleifer, and Vishny [1989] exploit similar technologies to analyze multiple equilibria; Milgrom and Roberts [1990] discuss how this type of nonconvexity can arise as firms internally coordinate many complementary activities. The technique-specific production functions produce $Y_{1,i,t}$ and $Y_{2,i,t}$ through

$$Y_{1,i,t} = f_1(K_{i,t-1} - F, \zeta_{i,t-1}) \tag{2.4}$$

$$Y_{2,i,t} = f_2(K_{i,t-1}, \eta_{i,t-1}). \tag{2.5}$$

$\zeta_{i,t}$ and $\eta_{i,t}$ are industry-specific productivity shocks and F is an overhead capital cost. $\zeta_{i,t}$ and $\eta_{i,t}$ are elements of $\mathfrak{F}_t$. Recalling that firms within an industry are identical, we define $\omega_{i,t}$ which equals 1 if technique 1 is used by industry i at t, 0 otherwise;

$\underset{\sim}{\omega}_t = \{...\omega_{i-1,t}, \omega_{i,t}, \omega_{i+1,t}...\}$ which equals the joint set of techniques employed at t; and $\Omega_t = \{...\underset{\sim}{\omega}_{t-1}, \underset{\sim}{\omega}_t\}$ which equals the history of technique choices up to t. The entire history of technique choices can be indexed by $\mathbb{Z}^2$, the two-dimensional lattice of integers.

We place several restrictions on these technologies. First, each technique fulfills standard curvature conditions. Further, we associate technique 1 with high production. Specifically, net capital $NK_{i,t}$, which equals $K_{i,t}-F$ for technique 1 and $K_{i,t}$ for technique 2, has a strictly higher marginal (and by implication total) product when used with technique 1 than technique 2.

Assumption 2.1. Restrictions on technique-specific production functions

For all realizations of $\zeta_{i,t}$, $\eta_{i,t}$, and NK, $f_1(NK,\zeta_{i,t})$ and $f_2(NK,\eta_{i,t})$ are twice-differentiable functions such that

A. $f_1(0,\zeta_{i,t}) = f_2(0,\eta_{i,t}) = 0.$

B. $\dfrac{\partial f_1(NK,\zeta_{i,t})}{\partial NK} \geq 0,\ \dfrac{\partial f_2(NK,\eta_{i,t})}{\partial NK} \geq 0;\ \dfrac{\partial^2 f_1(NK,\zeta_{i,t})}{\partial NK^2} \leq 0,\ \dfrac{\partial^2 f_2(NK,\eta_{i,t})}{\partial NK^2} \leq 0.$

C. $\dfrac{\partial f_1(0,\zeta_{i,t})}{\partial NK} = \dfrac{\partial f_2(0,\eta_{i,t})}{\partial NK} = \infty;\ \dfrac{\partial f_1(\infty,\zeta_{i,t})}{\partial NK} = \dfrac{\partial f_2(\infty,\eta_{i,t})}{\partial NK} = 0.$

D. $\dfrac{\partial f_1(NK,\zeta_{i,t})}{\partial NK} > \dfrac{\partial f_2(NK,\eta_{i,t})}{\partial NK}.$

Both techniques exhibit technological complementarities, as the history of realized activity determines the parameters of the production function at t. Romer's [1986] model of social increasing returns shares this feature. Our complementarities differ from Romer's in two respects. First, all complementarities are local as the production function of each firm is affected by the production decisions of a finite number of industries. The index i orders industries by similarity in technology; spillovers occur only between similar technologies. A large body of research in economic history has described the importance of local complementarities in the evolution of technical innovations.[1] Second, our complementarities are explicitly dynamic. Past production decisions affect current productivity, which captures the idea of learning-by-doing as described by Arrow [1952].

Specifically, we model the complementarities through the dependence of $\zeta_{i,t}$ and $\eta_{i,t}$ on the history of industry technique choices. This means, in a learning-by-doing context, that knowledge accumulation is a function of time spent on an activity, as opposed to scale. This seems appropriate if the efficiency of the fixed capital cost of technique 1, which we treat as devoted to organization and administration of production, is more likely to improve due to accumulation of knowledge than the efficiency of variable capital. Since firms in an industry are identical, the industry technique choice represents a measure of total time devoted to a particular industry-specific technique. Complementarities are assumed to be the only source of dependence across shocks. $\mu(x \mid y)$ denotes the conditional probability measure of x given information y; $x(y)$ denotes the random variable associated with this measure. $\Delta_{k,l} = \{i-k \ldots i \ldots i+l\}$ indexes the industries which affect the productivity of firms in industry i.

Assumption 2.2. Conditional probability structure of productivity shocks

[1]Rosenberg [1982] documents many examples of local technological complementarities leading to intertemporal spillover effects across industry production functions. For example, technical change in the early history of the chemical industry helped trigger innovations in metallurgy and electrical products through the provision of cheap inputs-"such essential items as refractory materials, insulators, lubricants...and metals of a high degree of purity." Similarly, David's [1986] discussion of path dependence shows how the evolution of the typewriter evolved as an element of "a larger, rather complex system of production that was technically interrelated." Typewriter operators and producers of typed products jointly interacted in a decentralized, sequential manner to implement innovations such as keyboard design. As David [1988] has argued, these findings suggest that economies exhibit nonergodic behavior along many dimensions.

A. $\mu(\zeta_{i,t} \mid \Im_{t-1}) = \mu(\zeta_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in \Delta_{k,l})$.

B. $\mu(\eta_{i,t} \mid \Im_{t-1}) = \mu(\eta_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in\ \Delta_{k,l})$.

C. The random pairs $(\zeta_{i,t}-\zeta_{i,t}(\Im_{t-1}),\eta_{i,t}-\eta_{i,t}(\Im_{t-1}))$ *are mutually independent of each other* $\forall\ i$.

No markets exist whereby individual firms and industries can coordinate complementarities. Markets are missing in two senses. First, there is no mechanism whereby one industry can be compensated for choosing technique 1 in order to expand the production sets of other industries. At the same time, since the industries are composed of many small, distinct producers, firms within an industry also cannot write contracts in order to act strategically in technique choice. Second, firms cannot be recombined under under joint management to internalize the complementarities.[2] These violations of the standard Arrow-Debreu assumptions will have an essential impact on the dynamics of the economy.

In equilibrium, the representative firm makes a choice of the high or low efficiency technology based upon the level of activity of the complementary industries in the previous period. For a sequence of date zero prices $\{p_t\}$, each firm possesses an optimal capital choice level conditional on the technology choice 1 or 2. Denoting relative prices as $\rho_{t+1} = \frac{p_{t+1}}{p_t}$, these optimal conditional capital choices, $K_{1,i,t}$ and $K_{2,i,t}$ respectively, are implicitly defined by the two first order conditions

$$1 \ = \ E(\rho_{t+1}\frac{\partial f_1(K_{1,i,t}-F,\zeta_{i,t})}{\partial NK} \mid \Im_t) \tag{2.6}$$

[2]The second form of missing markets can be replaced by an assumption which says that managers cannot completely coordinate the activities of sufficiently large organizations.

$$1 = E(\rho_{t+1}\frac{\partial f_2(K_{2,i,t},\eta_{i,t})}{\partial NK} \mid \mathfrak{F}_t). \tag{2.7}$$

Each firm chooses the maximum of

$$E(\rho_{t+1}f_1(K_{1,i,t}-F,\zeta_{i,t})-K_{1,i,t} \mid \mathfrak{F}_t) \tag{2.8}$$

and

$$E(\rho_{t+1}f_2(K_{2,i,t},\eta_{i,t})-K_{2,i,t} \mid \mathfrak{F}_t). \tag{2.9}$$

It is clear from our assumptions that the production of each firm is higher under technique 1 than technique 2. In equilibrium, choices of technique 1 represent greater capital expenditures in exchange for greater future output.

In order to solve for an equilibrium, we exploit the linear utility specification. Observe that when consumption is nonzero every period, equalization of marginal rates of substitution to relative prices means that p_t is proportional to β^t, i.e. $\rho_t = \beta \ \forall \ t$. Any other solutions can be ruled out by the transversality conditions associated with the individual industry and consumer maximization problems. Along this constant relative price ratio, if firms are maximizing profits, then the marginal rate of substitution will equal the marginal rate of transformation of capital into output and consumers are maximizing utility by setting dividends equal to consumption. Consequently, the existence of an equilibrium can be reformulated as showing the existence of a set of optimal firm (and hence industry) production sequences for $\rho_t = \beta$.

We therefore place an assumption on the relationship between available output and desired capital which ensures the existence of constant relative prices starting at date zero, rather than asymptotically. This assumption, which implicitly places restrictions on both the initial output endowments and the technique-specific production functions, renders the conditional technique choice probability measures stationary.

Assumption 2.3. Lower bounds on available capital[3]

For all realizations of $\zeta_{i,t}$ *and* $\eta_{i,t}$, $\sum_{i=-\infty}^{\infty} Y_{i,t} > \sum_{i=-\infty}^{\infty} K_{1,i,t}(\beta)$, *where* $K_{1,i,t}(\beta)$ *fulfills* $1 = \frac{\partial \beta f_1(K_{1,i,t}(\beta) - F, \zeta_{i,t})}{\partial NK}$.

When relative prices are constant, one can characterize the conditional probability measures over technique choices by all industries at all dates. This occurs because the history of technique choices is a sufficient statistic for the conditional probability measures describing the profit maximization problem of each firm.

Theorem 2.1. Structure of conditional technique choice probability measures

The equilibrium technique choice conditional probability measures for each industry obey

$$\mu(\omega_{i,t} \mid \mathfrak{F}_{t-1}) = \mu(\omega_{i,t} \mid \omega_{j,t-1} \; \forall \; j \in \Delta_{k,l}). \tag{2.10}$$

It is now straightforward to verify the existence of an equilibrium for constant relative prices. Theorem 2.1 describes the conditional probability measures for individual technique choices. From the structure of the economy, if a joint probability measure exists over all technique choices in all periods which is compatible with these conditional measures, then an equilibrium exists for characterizing capital and consumption decisions. To see this, suppose that such a measure exists. In this case, an equilibrium characterization of the capital accumulation decisions for all firms over all periods is implicitly defined since the industry technique choices simply define a sequence of

[3]Both Assumptions 2.2 and 2.3 can be relaxed without affecting the Theorems 3.1 and 3.2, which constitute the main results of the paper. The assumptions make the proofs of the Theorems substantially more straightforward.

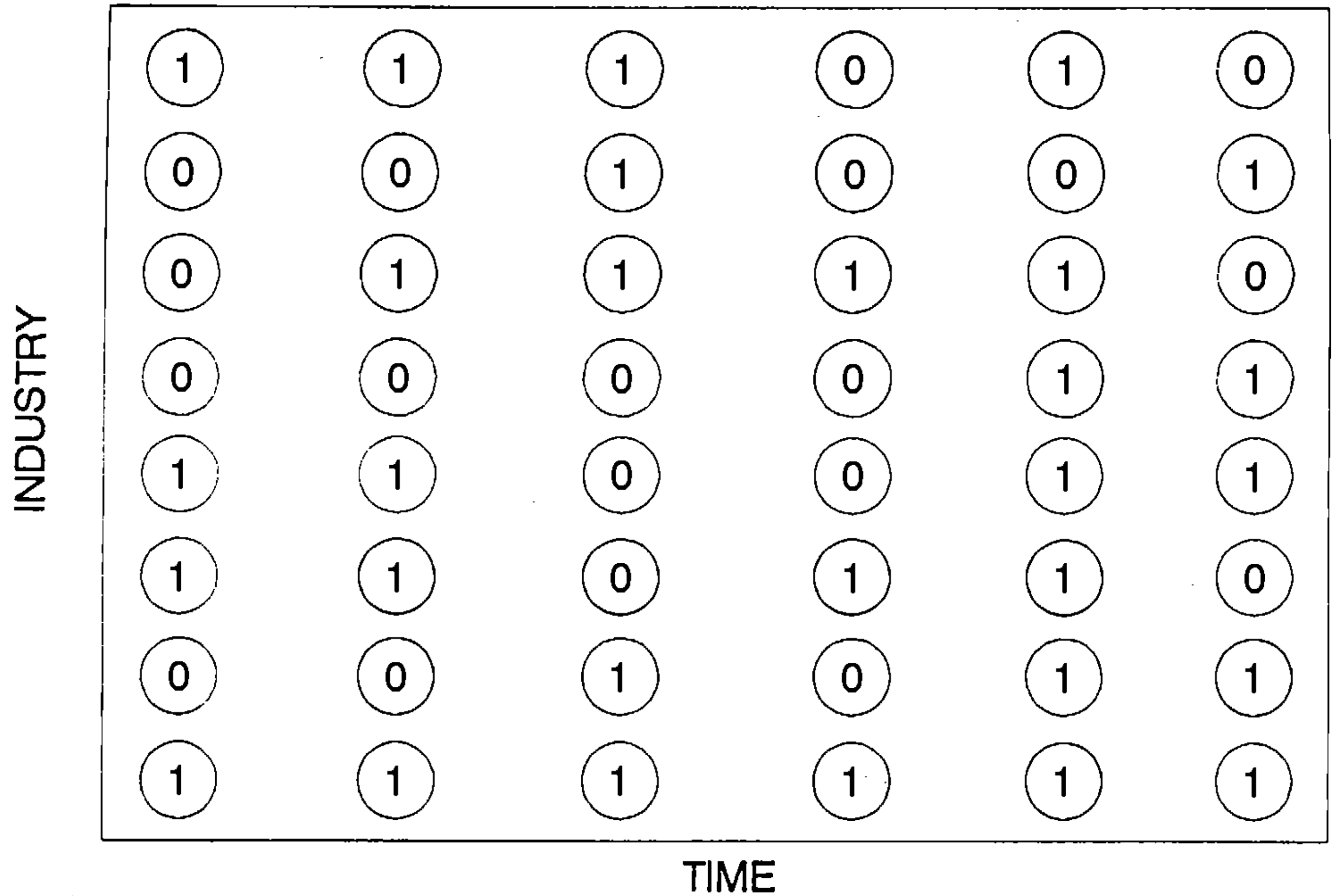
production sets for each firm, resulting in capital choices based on (2.9) and (2.10). As argued earlier, the resulting consumption sequence from these capital choices is also an equilibrium for consumers since relative prices equal the marginal rate of substitution.[4]

Proving the existence of equilibrium is thus equivalent to showing that a set of conditional probabilities over finite subsets of a stochastic process generates a probability measure over an entire process. As shown by Theorem 2.1, our assumptions on localized complementarities ensure that these conditional measures also possess a localized structure in the sense that conditioning the technique choice on the entire history of the economy is equivalent to conditioning on a finite number of other elements in the stochastic process. This implied localized probability structure is generally sufficient to ensure that a joint measure exists. In one dimension, where the index is normally thought of as time, the localized structure ensures existence through standard results in the theory of Markov chains which show how specification of a probability transition matrix generates a joint measure (see discussions in Çinlar [1975] or Rosenblatt [1971]). For our model, the set of technique choices $\omega_{i,t}$ are indexed by $\mathbb{Z}^2$, the two-dimensional lattice of integers. For stochastic processes whose indices run over several dimensions, also known as *random fields*, conditions for the existence of a joint probability measure have been derived by Dobrushin [1968].[5] Dobrushin's criteria, when applied to the industry technique choice probabilities, imply

[4]Put differently, since the conditional probability measure characterizing $\omega_{i,t}$, $K_{i,t}$ and $Y_{i,t}$ obeys $\mu(\omega_{i,t}, Y_{i,t}, K_{i,t} \mid \mathfrak{F}_{t-1}) = \mu(\omega_{i,t}, Y_{i,t}, K_{i,t} \mid \omega_{j,t-1}\ \forall\ j \in \Delta_{k,l})$, the existence of a joint probability measure over the technique choices implies there exists a joint probability measure over capital and output decisions such that all industries are maximizing (2.2).

[5]Our use of Dobrushin's Theorem allows us to show that an equilibrium exists for arbitrary initial conditions. When the initial conditions are specified, existence may be proven using the Kolmogorov Extension Theorem, as the unconditional probability measures of all finite dimensional sets of elements are now defined. It is natural in this model to take the initial conditions for the technique choices as $\underset{\sim}{\omega}_0 = \underset{\sim}{0}$.

Figure 1



Sample Realization of Production Technique Choices

Theorem 2.2. Existence of equilibrium

There exists at least one joint probability measure over all technique choices at all dates, $\mu(\Omega_\infty)$, whose associated conditional probabilities characterizing time- and industry-specific technique choices possess the form given by Theorem 2.1.

Figure 1 illustrates a typical realization of a cross section-time series of Ω_∞.

3. Multiple equilibria and long run behavior

We now restrict the conditional probabilities in order to discuss multiplicity and dynamics. Past choices of technique 1 are assumed to improve the current relative productivity of the technique. As a result, technique 1 choices will propagate over time. Further, we assume that $\underline{\omega}_t=\underline{1}$ is a steady state, which means that when all productivity spillovers are active, the effects are so strong that high production is always optimal.

Assumption 3.1. Impact of past technique choices on current technique probabilities[6]

Let ω and ω' denote two realizations of $\underline{\omega}_{t-1}$. If $\omega_j \geq \omega'_j\ \forall\ j \in \Delta_{k,l}$, then

A. $Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = \omega_j\ \forall\ j \in \Delta_{k,l}) \geq Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = \omega'_j\ \forall\ j \in \Delta_{k,l})$.

B. $Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = 1\ \forall\ j \in \Delta_{k,l}) = 1$.

Whenever some industry chooses $\omega_{i,t} = 0$, a positive productivity feedback is lost. Different configurations of choices at $t-1$ determine different production sets and

[6]This assumption can be reformulated in terms of restrictions on the technique-specific production functions.

conditional technique choice probabilities for each industry. We bound the technique choice probabilities from below and above by $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$ respectively.

$$\Theta_{k,l}^{min} \leq Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = 0 \text{ for some } j \in \Delta_{k,l}) \leq \Theta_{k,l}^{max} \tag{3.1}$$

Since $\underline{\omega}_t=\underline{1}$ is an equilibrium, multiple equilibria exist if for some initial conditions, $\underline{\omega}_t=\underline{1}$ fails to emerge as t grows. Notice that even if $\underline{\omega}_0 = \underline{0}$, favorable productivity shocks will periodically induce industries to produce using technique 1. The choice of technique 1 by one industry, through the complementarities, increases the probability that the technique is subsequently chosen in several industries. With strong spillovers, these effects may build up, allowing $\underline{\omega}_t=\underline{1}$ to emerge from any initial conditions. The model therefore allows us to analyze the stability of a high aggregate output equilibrium from arbitrary initial conditions.

The stability of $\underline{\omega}_t=\underline{1}$ is in fact a function of the transition probabilities which describe how the economy evolves outside of the high production equilibrium. Intuitively, this equilibrium is stable from any initial condition if the probability of high production by an industry independent of all complementarities is sufficiently large. In this case, the spillover effects induced by spontaneous production will cause the economy to iterate towards the high production limit. This can be seen in the extreme case where $\Theta_{k,l}^{min}=1$. Alternatively, if the production probabilities are too low in the absence of active complementarities, the spillover effects from spontaneous production will be insufficient to generate momentum towards $\underline{\omega}_t = \underline{1}$. This can be seen in the extreme case where $\Theta_{k,l}^{max}=0$.[7] For this parameter value, the economy consisting of all low production technique industries is clearly an equilibrium. The interesting cases for uniqueness and multiplicity occur when $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$ are not elements of {0,1}, as these cases will

[7]When $\Theta_{k,l}^{max}=0$, our model reduces to the typical static coordination failure model where the two equilibria impose homogeneous behavior on all agents and imply trivial aggregate time series.

result in nontrivial dynamics. The values of $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$ place bounds on the degree of complementarity in the economy. Small values of $\Theta_{k,l}^{max}$ suggest that complementarities are powerful, as the occurrence of high production is extremely dependent upon the choices of others. Conversely, large values of $\Theta_{k,l}^{min}$ mean that complementarities are weak in the sense that high production frequently occurs even in the absence of high production by relevant industries in the past. The precise relationship between the transition probabilities and multiple equilibria is summarized by

Theorem 3.1. Uniqueness of long run equilibrium as a function of degree of complementarity

For each index set $\Delta_{k,l}$, *with at least one of k or l nonzero, there exist numbers* $\overline{\Theta}_{k,l}$ *and* $\underline{\Theta}_{k,l}$, $0 < \underline{\Theta}_{k,l} < \overline{\Theta}_{k,l} < 1$ *such that*

A. If $\Theta_{k,l}^{min} \geq \overline{\Theta}_{k,l}$, *then* $\lim_{t \Rightarrow \infty} \mu(\omega_{i,t} = 1 \mid \underline{\omega}_0 = \underline{0}) = 1$.

In an economy starting with all low production technique industries, any individual industry will almost surely converge to the high production technology.

B. If $\Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}$, *then*

i. $\lim_{t \Rightarrow \infty} \mu(\omega_{i,t} = 1 \mid \underline{\omega}_0 = \underline{0}) < 1$.

ii. $\lim_{t \Rightarrow \infty} \mu(\underline{\omega}_t = \underline{1} \mid \underline{\omega}_0 = \underline{0}) = 0$.

In an economy starting with all low production technique industries, the probability that any individual industry achieves high production will be strictly bounded below 1. Further, the economy will almost surely fail to converge to the high production *equilibrium.*

Further, the range of values of $\underline{\Theta}_{k,l}$ which generates multiple equilibria is nondecreasing in k and l. The greater the number of industries which must act in unison to ensure that high production is employed with certainty by a given industry, the smaller the range of parameter values for conditional high production probabilities where the high production equilibrium emerges. This may be interpreted as saying that high production outcomes are less likely to emerge as the range of coordination failures expands.

Theorem 3.2. Relationship between conditional probability bounds and range of complementarities

Let $I(\underline{\Theta}_{k,l})$ *denote the set of values of* $\underline{\Theta}_{k,l}$ *which generate multiple equilibria, in the sense of Theorem* 3.1. *If* $\Delta_{k,l} \subseteq \Delta_{m,n}$, *then* $I(\underline{\Theta}_{k,l}) \subseteq I(\underline{\Theta}_{m,n})$.

Consequently, for an arbitrary range of local complementarities, the aggregate dynamics of the economy are jointly determined by initial conditions and transition probabilities. These two factors collectively select a long run equilibrium. When multiple equilibria exist, the transition probabilities do not determine a unique invariant measure for the system. This property is far more common for random fields than one-dimensional time series, as discussed in the Technical Appendix. Intuitively, when economic agents interact along several dimensions, the degree of interdependence between the agents increases sufficiently to generate multiplicity.

Theorem 3.1 illustrates a dynamic path for economic development. Unlike static models of industrialization, our framework demonstrates how high levels of production can emerge endogenously from low production initial conditions. Complementarities can build up across time, leading to a high production long run steady state. This idea suggests a resolution of the paradox posed by Scitovsky [1954], on how a railroad that requires the existence of a steel industry to provide inputs of production and a steel industry that requires the existence of a railroad to transport materials can ever jointly develop. When complementarities occur sequentially and when the probability of high production is still nonzero for an industry even when other industries are inactive, evolution towards the high production equilibrium can result from the buildup of complementarities across time.

One can associate $\underline{\omega}_t=\underline{1}$ with the equilibrium which would emerge if all firms chose their production levels cooperatively. If production through technique 1 is sufficiently large for $\underline{\omega}_t=\underline{1}$ versus any other configuration, then $\underline{\omega}_t=\underline{1}$ emerges as the cooperative (and efficient) equilibrium after one period. Consequently, incompleteness of markets lowers the mean and increases the variance of industry and aggregate output along the inefficient equilibrium path, as technique choices fluctuate over time. When industries fail to coordinate, production decisions become dependent on idiosyncratic productivity shocks. Observe that the volatility associated with the inefficient equilibrium is caused by fundamentals and is quite distinct from the case where market incompleteness leads to the emergence of sunspots in aggregate activity.

4. Aggregate dynamics

Our model of interacting industries is capable of producing rich individual industry and aggregate dynamics.[8,9] To better understand the behavior of the aggregate economy, we consider some simulations for various ranges of industry interaction. In each simulation, we construct a finite approximation to the infinite economy consisting of 500 industries over 2000 time periods. In all cases, $\underline{\omega}_0 = \underline{0}$. Output per period by each industry is normalized to equal 0 or 1.[10]

We initially consider the case where $\Delta_{1,0}=\{i-1,i\}$, which implies the conditional probability structure

$$\mu(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 1, \omega_{i,t-1} = 1) = 1$$

$$\mu(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 0, \omega_{i,t-1} = 1) = \Theta_1$$

$$\mu(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 1, \omega_{i,t-1} = 0) = \Theta_2$$

$$\mu(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 0, \omega_{i,t-1} = 0) = \Theta_3. \tag{4.1}$$

This model was originally analyzed by Stavskaya and Pyatetskii-Shapiro [1968]. It is known from simulations reported in that paper, as well as subsequent analytical work, that the greatest upper bound on $\Theta_{k,l}^{max}$ which permits the inefficient equilibrium to exist, is slightly greater than .3, when $\Theta_1 = \Theta_2 = \Theta_3$.

---

[8]The results in the previous section imply that each realization of the economy fails to converge to the invariant measure $\underline{\omega}_t = \underline{1}$. When $\Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}$, $\underline{\omega}_0 = \underline{0}$ and $\Theta_{k,l}^{min} > 0$, it follows that each industry i possesses nondegenerate dynamics in the sense that *lim sup* $\omega_{i,t} = 1$ and *lim inf* $\omega_{i,t} = 0$ almost surely. The first limit is obvious since there is a positive probability each period that $\omega_{i,t} = 1$. The latter holds, because the alternative, *lim inf* $\omega_{i,t} = 1$, can hold iff ω_i=1 after a finite number of periods, which requires that all elements $\omega_j = 1$, $j \in \Delta_{k,l}$ after a finite number of periods, which would require that all industries which interact with this previous set equal 1 after a finite number of periods, and so on. This recursion implies that $\lim_{t \Rightarrow \infty} \mu(\underline{\omega}_t = \underline{1} \mid \underline{\omega}_0 = \underline{0}) \neq 0$, which cannot hold by Theorem 2.3.B. See Shnirman [1968] and Vasilyev [1970] for a more formal argument.

[9]When there are a finite number of industries, then as $t \Rightarrow \infty$, the high production equilibrium will almost surely emerge. This holds since each period there is a nonzero probability that all industries will spontaneously choose technique 1. However, a sufficiently large finite economy can replicate the behavior of the infinite economy for an arbitrarily large number of periods with arbitrarily high probability. In this sense, a large finite economy will, for a relevant observable history, appear nonergodic.

[10]A {0,1} support for industry output can be justified by generalizing our assumptions on technology so that $Y_{1,i,t} = \overline{Y}$ if $K_{i,t-1} \geq \overline{K} - \zeta_{i,t-1}$ and $Y_{2,i,t} = f_2(K_{i,t-1})$ and then normalizing output.

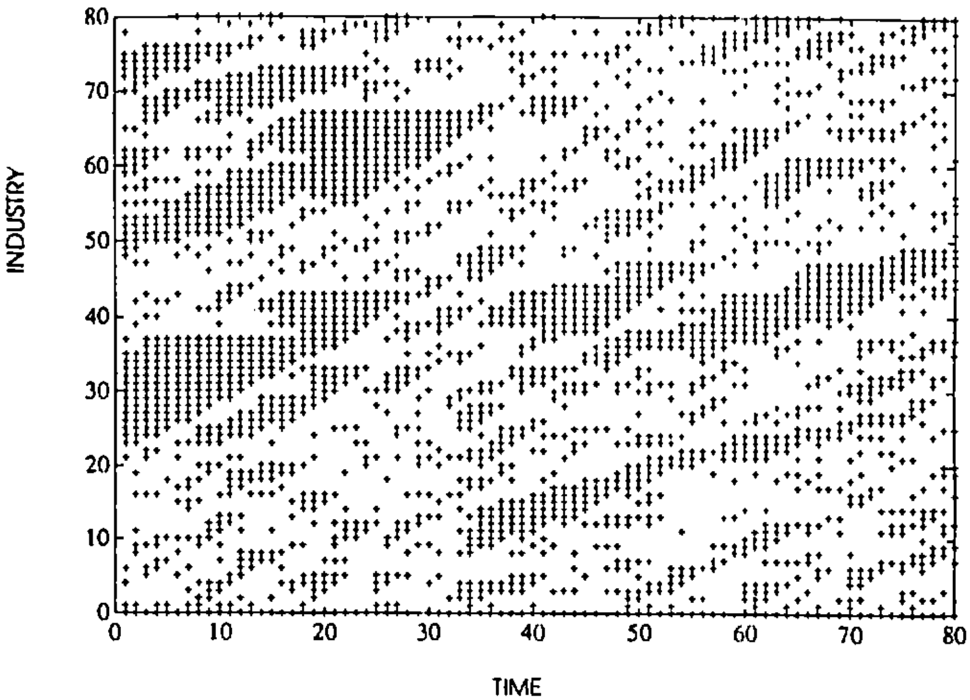


FIGURE 2

80-Period Sample Realization - 80 Industry Economy

$\Delta_{1,0}$

$\theta_1 = .30$
$\theta_2 = .25$
$\theta_3 = .20$

+ = high production by industry i at t

Aggregate output equation

$$Y_t = .09 + .67\, Y_{t-1} + .07\, Y_{t-2}$$

Figure 2 displays a sample path realization over the last 80 periods of an 80 industry cross section when the conditional probabilities are $\Theta_1 = .3$, $\Theta_2 = .25$ and $\Theta_3 = .2$. As the Figure indicates, the economy exhibits considerable cross-section and intertemporal persistence. Different parts of the economy exhibit disparate behavior. The average output level over all 500 industries, Y_t, obeys the AR(2)

$$Y_t = .09 + .67 Y_{t-1} + .07 Y_{t-2} + \epsilon_t.^{11} \tag{4.2}$$

The mean value of Y_t associated with this equation is .35. Recalling that $\Theta_3 = .2$, the mean activity level is nearly double that which is predicted by the probability of production in the absence of active complementarities. The difference between the mean and the lowest production probability shows how high production choices can spill over across sectors, building up over time to raise aggregate output.

Figure 3 contains a realization of the behavior of an 80 industry cross section when the conditional probabilities are $\Theta_1 = .15$, $\Theta_2 = .10$ and $\Theta_3 = .05$. As the Figure suggests, the degree of high production bunching is relatively low when assessed either over a cross-section or over time. This occurs because the probability of production is so low in the absence of active complementarities that spillover effects do not build up to a great extent. The average output equation is

[11] Each regression was computed using the last 1000 observations for all 500 industries of a 2000 period simulation of the economy. Multiple runs produced essentially the same AR coefficients, suggesting that the low equilibrium measure is unique. (Results in Vasilyev [1980] show that if the conditional probabilities Θ_i are small enough, there is a unique low equilibrium joint measure starting from $\omega_0=\underline{0}$. However, his proof did not show that any set of Θ_i's which generate multiple measures will only generate one measure from $\omega_0=\underline{0}$, so we cannot apply his result to our Θ_i choices.) By using a central limit theorem argument, one can show that the demeaned Y_t's can be normalized to possess nondegenerate limits as the number of industries becomes unbounded. This will ensure that the limiting AR representation for the sequence of (normalized) finite economies is also nondegenerate. See the discussion in Pickard [1976,1977].

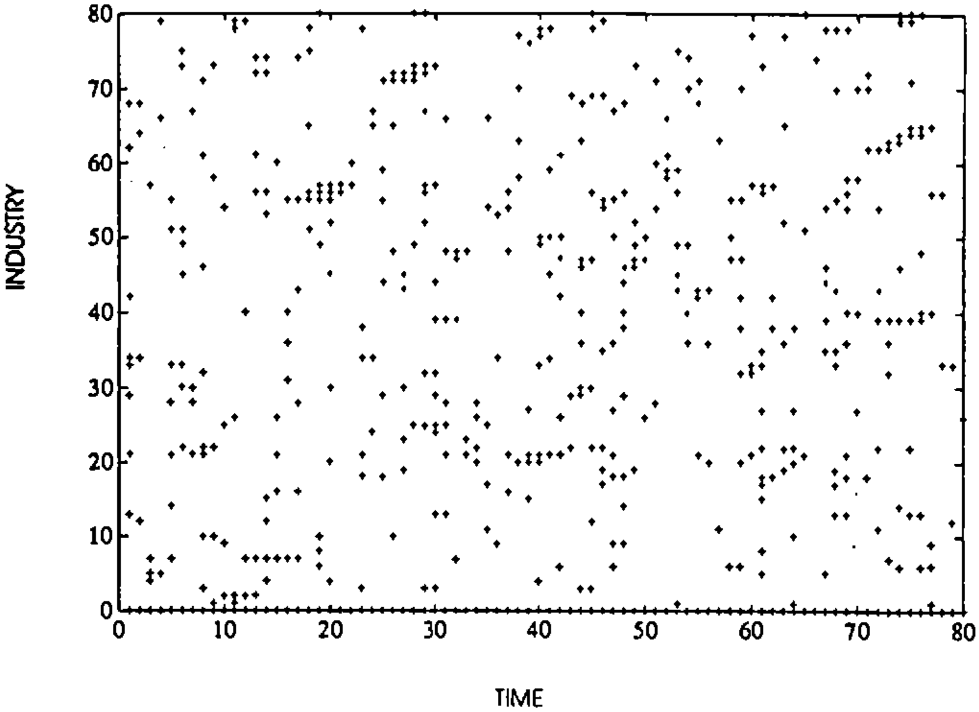


FIGURE 3

80-Period Sample Realization - 80 Industry Economy

$\Delta_{1,0}$

$\theta_1 = .15$
$\theta_2 = .10$
$\theta_3 = .05$

+ = high production by industry i at t

Aggregate output equation

$Y_t = .04 + .25\ Y_{t-1} + .06\ Y_{t-2}$

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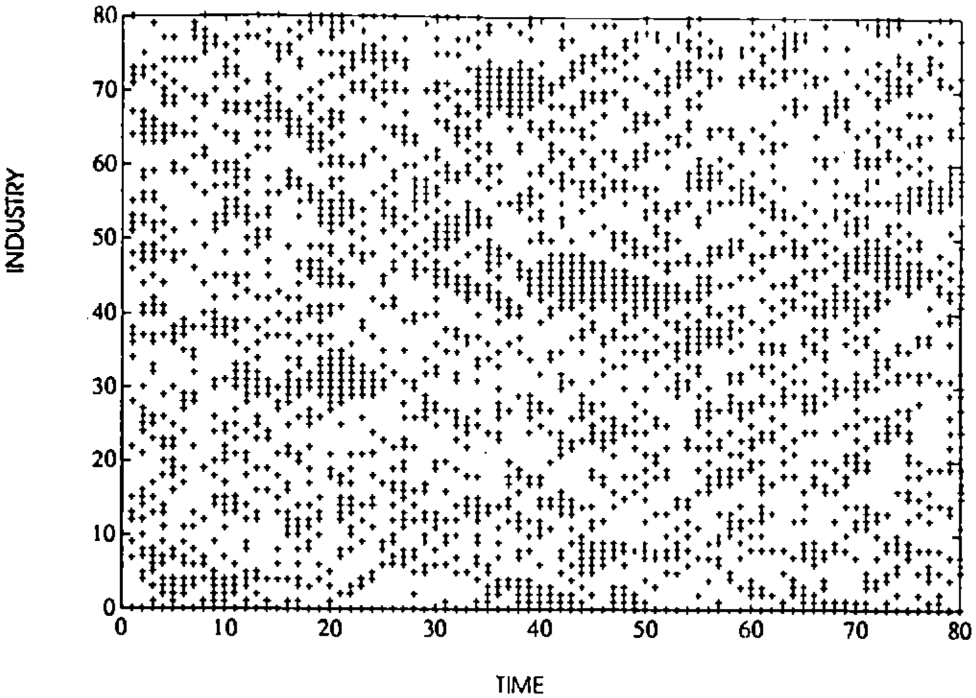

FIGURE 4

80-Period Sample Realization - 80 Industry Economy

$\Delta_{1,1}$

$\theta_1 = .40$
$\theta_2 = .35$
$\theta_3 = .30$

+ = high production by industry i at t

Aggregate output equation

$$Y_t = .18 + .49\, Y_{t-1} + .07\, Y_{t-2}$$

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June 21, 1990

$$Y_t \;=\; .04 + .25\,Y_{t-1} + .06\,Y_{t-2} + \epsilon_t. \tag{4.3}$$

This equation possesses an unconditional mean of .06, which indicates that the spillover effects are relatively weak compared to Figure 2.

Our second set of simulations are constructed based on the interaction range $\Delta_{1,1}=\{i-1,i,i+1\}$. In this case, there are eight different conditional probabilities. We reduce the number of relevant parameters to 3 by assuming the probabilities obey

$$\mu(\omega_{i,t}=1\,|\sum_{j=-1}^{1}\omega_{i-j,t-1}=3)=1$$

$$\mu(\omega_{i,t}=1\,|\sum_{j=-1}^{1}\omega_{i-j,t-1}=2)=\Theta_1$$

$$\mu(\omega_{i,t}=1\,|\sum_{j=-1}^{1}\omega_{i-j,t-1}=1)=\Theta_2$$

$$\mu(\omega_{i,t}=1\,|\sum_{j=-1}^{1}\omega_{i-j,t-1}=0)=\Theta_3. \tag{4.4}$$

This structure can be interpreted as saying each complementarity has the same effect on the production function. Simulations of this structure have shown that the model is nonergodic when all transition probabilities are below .45.

Figure 4 was generated for the case $\Theta_1=.4$, $\Theta_2=.35$, $\Theta_3=.3$. The model exhibits some cross-sectional and intertemporal persistence in fluctuations, but less than that observed in Figure 2. The associated average output equation is

$$Y_t \;=\; .18 + .49\,Y_{t-1} + .07\,Y_{t-2} + \epsilon_t. \tag{4.5}$$

Interestingly, the mean of average output, .41, is not substantially greater than $\Theta_3=.3$. Certain one-period ahead production occurs only when three industries are simultaneously active, which is a low probability event when each was inactive the previous period.

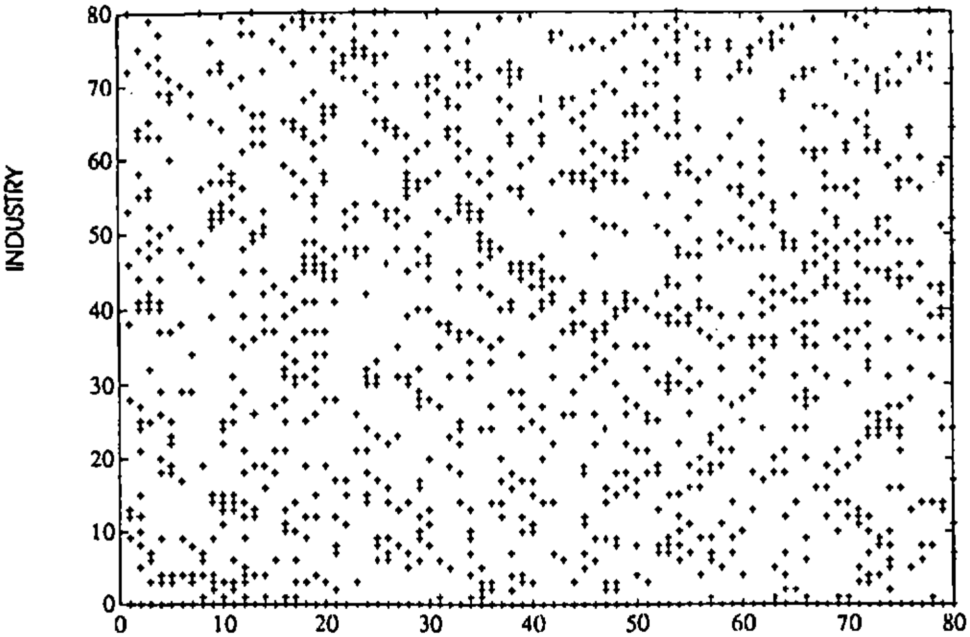

FIGURE 5

80-Period Sample Realization - 80 Industry Economy

$\Delta_{1,1}$

$\theta_1 = .25$
$\theta_2 = .20$
$\theta_3 = .15$

+ = high production by industry i at t

Aggregate output equation

$$Y_t = .13 + .29\ Y_{t-1} + .02\ Y_{t-2}$$

C:\DURLAUF\FIGS\FIG.B
June 13, 1990

Hence, this economy has difficulty in building up active complementarities over time.

Finally we report the behavior of this $\Delta_{1,1}$ economy for $\Theta_1 = .25$, $\Theta_2 = .2$, $\Theta_3 = .15$. The associated output equation is

$$Y_t \;=\; .13 + .29\,Y_{t-1} + .02\,Y_{t-2} + \epsilon_t. \tag{4.6}$$

which possesses a mean of .19. Again, reduction of the transition probabilities reduces both the mean and degree of persistence due to the low probabilities of production in the absence of active complementarities, as evidenced in Figure 5.

These simulations demonstrate how our interacting economy can generate nondegenerate macroeconomic time series with widely differing degrees of persistence, depending upon the transition probabilities. The model cannot, however, generate exact unit roots in output, which have been the subject of so much recent empirical work. This inability occurs because normalized aggregate output behaves as a stationary L^2 process. Unit roots can emerge in our framework, however, through the introduction of deterministic technical change which causes the aggregate production set to become asymptotically unbounded. As described in Durlauf [1990a], multiple equilibria can interact with the evolution of the production set to convert a linear deterministic trend in potential output into a random walk in realized output.

5. Movement across equilibria and takeoff

The existence of a low output equilibrium hinges critically on the probabilities of high production in the absence of potential complementarities. When these probabilities are sufficiently high, then the economy will endogenously evolve towards the high production state. This feature suggests an interpretation of takeoff to industrialization as the consequence of increasing the microeconomic decision probabilities of choosing high production.[12]

Technical change is a natural mechanism for introducing evolving probabilities in technique choice. Two different interpretations exist which can relate technical change to the simultaneous evolution of conditional technique choices across many industries. One possibility is that technical change is highly correlated across technologies. If one thinks of technical change as a multiplicative random walk with drift appended to individual industry production functions, this would require a great deal of cointegration across technologies. An alternative possibility is that there exists some common factor which simultaneously affects many industries. This idea underlies Hirschman's [1958] argument that the growth of leading sectors such as transportation or steel can stimulate production throughout the economy due to various demand and supply links between the leading sectors and other industries.

In order to see how leading sectors can cause the economy to take off, we modify the model as follows. Let the high production technology take the form

$$Y_{1,i,t} = f_1(K_{i,t-1}-F,\zeta_{i,t-1},R^{ls}_{i,t}) \tag{5.1}$$

where $R^{ls}_{i,t}$ is representative firm i's input of a good produced by a single leading sector. When industry i employs technique 1, it starts production at $t-1$ with a capital investment and then employs the leading sector input at t to produce output at t.[13]

The leading sector is assumed to be monopolistically competitive. There is free entry into the industry. At $t-1$, the j'th leading sector firm chooses its level of output at t, $R^{ls}_{j,t}$, by choosing a level of capital $K^{ls}_{j,t-1}$. The firm's production function exhibits increasing returns due to an overhead capital cost F^{ls}_{t-1} combined with an exogenous capacity constraint $\overline{R}^{ls}_{t-1}$. The leading sector uses the final good as a capital input.

[12]See Kelly [1990] for an interesting analysis of takeoff as resulting from the formation of economy-wide markets out of an initial localized market structure.

[13]Under this timing specification, it is natural to think of the leading sector as replacing some phase of the production process which would otherwise require a part of $K_{i,t-1}$. Alternative specifications do not affect the results.

Without loss of generality, we assume that the production function is linear in capital and a multiplicative shift factor ξ_{t-1}, after subtracting overhead capital.

$$R^{ls}_{j,t} = \xi_{t-1}g(K^{ls}_{j,t-1}-F^{ls}_{t-1}) \quad if \quad R^{ls}_{j,t} \le \overline{R}^{ls}_{t-1}$$

$$R^{ls}_{j,t} = \overline{R}^{ls}_{t-1} \quad otherwise. \tag{5.2}$$

We assume that the random vector $(\xi_t-\xi_t(\mathfrak{F}_{t-1}),\overline{R}^{ls}_t-\overline{R}^{ls}_t(\mathfrak{F}_{t-1}),F^{ls}_t-F^{ls}_t(\mathfrak{F}_{t-1}))$ is independent of $(\zeta_{i,t}-\zeta_{i,t}(\mathfrak{F}_{t-1}),\eta_{i,t}-\eta_{i,t}(\mathfrak{F}_{t-1}))$ $\forall$ i and that ξ_t, $\overline{R}^{ls}_t$ and F^{ls}_t are elements of $\mathfrak{F}_t$.

In equilibrium, this sector will produce subject to a no profit condition. In other words, the date zero price p^{ls}_t for leading sector output will equal average cost for each firm in the sector. When demand is large enough, leading sector firms produce at capacity, which means that the price of leading sector output equals

$$p^{ls}_t = \frac{\beta^{t-1}(\overline{R}^{ls}_{t-1}+\xi_{t-1}gF^{ls}_{t-1})}{\xi_{t-1}g\overline{R}^{ls}_{t-1}}. \tag{5.3}$$

The first order conditions for profit maximization ensure that the relative profitability of the first technique is increasing in ξ_t, increasing in $\overline{R}^{ls}_t$ and decreasing in F^{ls}_t.

This modification of the technique 1 production functions gives the necessary structure for providing a role for leading sectors in generating a transition to the high production equilibrium. Suppose that technical change leads to monotonic rightward shifts in the distribution functions of ξ_t and $\overline{R}^{ls}_t$ and a monotonic leftward shift in the distribution function for F^{ls}_t. In this case, the conditional probabilities

$$\mu(\omega_{i,t}=1 \mid \omega_{j,t-1} \ \forall\ j \in \Delta_{k,l},\ \xi_t,\ \overline{R}^{ls}_t,\ F^{ls}_t) \tag{5.4}$$

will increase across time for all production histories as technical change occurs.[14] If the

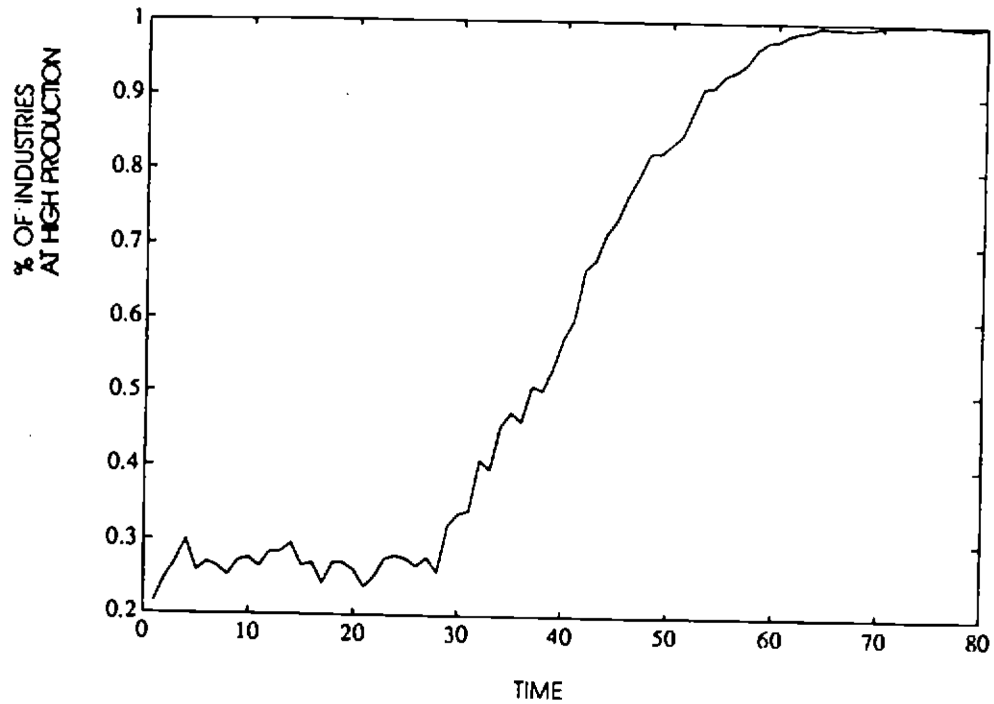

FIGURE 6

Take Off to High-Production Equilibrium: Average Output

$\Delta_{1,0}$

$\theta_{1,t} = \theta_{2,t} = \theta_{3,t} = \theta_t$

$\theta_t = .20 \quad t = 1...25$

$\theta_t = .20 + .01\ (t\text{-}25) \quad t = 26...55$

$\theta_t = .50 \quad t = 56...80$

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conditional probabilities are sufficiently sensitive to the price of output from the leading sector, then takeoff can be induced by innovation in the leading sector production function.

Figures 6 and 7 exhibit how technical change can lead to rapid growth, by showing how a simulated economy will expand as the microeconomic probabilities of high production increase over time. Each Figure contains a sample path realization for average production over 500 industries, employing a zero-one normalization for individual industry output. In Figure 6, all transition probabilities for a $\Delta_{1,0}$ economy are started at $\Theta_i = .2 \ \forall \ i$. After 25 periods, the probabilities Θ_i each grow by .01 per period. This continues for 30 periods, after which the economy evolves with the new conditional probabilities $\Theta_i = .5$. Figure 7 repeats the experiment for a $\Delta_{1,1}$ economy. In this case, the initial probabilities are $\Theta_i = .3$ and the terminal probabilities are $\Theta_i = .6$. Both experiments result in roughly the same S-shaped pattern of growth. In both cases, growth in aggregate output exceeds .01 per period, the probability parameter growth rate; both economies converge to the efficient steady state.

This sort of explanation has been applied to the development of the American economy in the period 1820-1850. North [1966] describes how the early growth of the American economy led to increased demand for transportation, which in turn stimulated the construction of canals. Cheaper transportation costs due to canal construction then stimulated expanded activity in many sectors, including greater agricultural production and land development. Increased sectoral activity in turn stimulated transportation demand so that the cycle repeated itself, leading to substantial increases in aggregate output.

Technical change arguments cannot easily address the question of movements from the high to the low production equilibrium. One possibility for explaining shifts to

[14]For a wide range of market games, decreasing marginal costs in the leading sector due to technical change will lead to greater use of the leading sector input at lower prices by each industry, thereby stimulating high production.

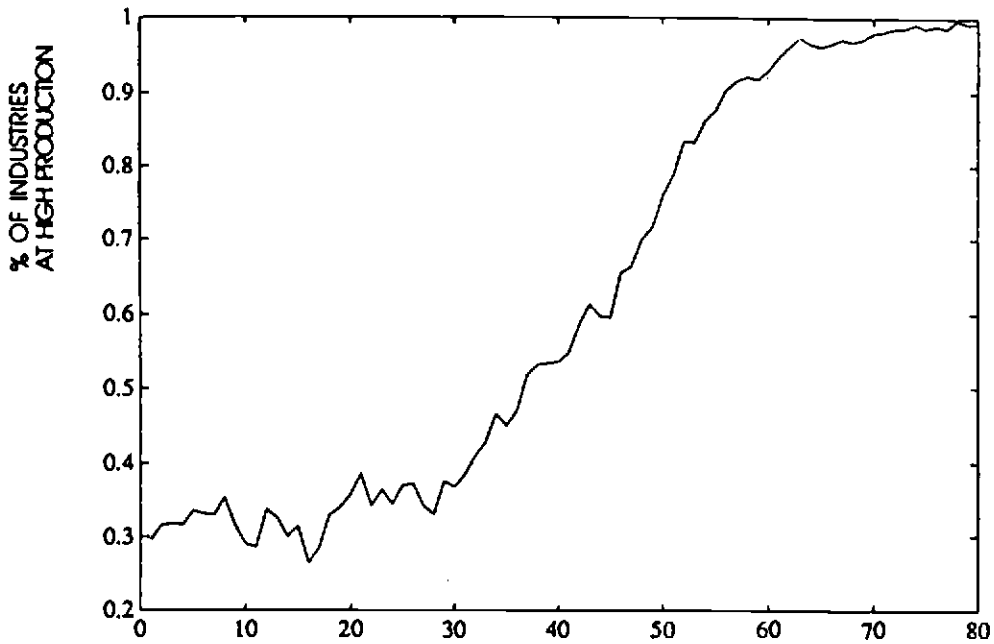


FIGURE 7

Take Off to High Production Equilibrium: A\`.erage Output $\Delta_{1,t}$

$\theta_{1,t} = \theta_{2,t} = \theta_{3,t} = \theta_t$

$\theta_t = .30 \quad t = 1...25$

$\theta_t = .30 + .01\ (t\text{-}25) \quad t = 26...55$

$\theta_t = .60 \quad t = 56...80$

low production is to interpret the leading sector as providing financial services and then to characterize a financial collapse as creating effects analogous to a negative shock to technology. Durlauf [1990b] shows how several of the models of Greenwald and Stiglitz [1988] on interactions between credit market imperfections and investment can be applied in this way. In particular fluctuations in the real sector feed into the financial services sector and thus trigger movements between equilibria.

6. Summary and conclusions

This paper is designed to illustrate how incomplete markets and complementarities can combine to affect aggregate behavior. We have analyzed these issues in the context of a modified Brock-Mirman stochastic growth model. The basic results we have shown are three-fold. First, we have shown how long run capital accumulation and growth may be affected by the interactions of market incompleteness and complementarities. Technological complementarities combine with incomplete markets to generate multiple equilibria. These equilibrium paths generate very different aggregate dynamics from one another. Further, market incompleteness can enhance the role of idiosyncratic productivity shocks in affecting output decisions and lead to the propagation of aggregate shocks across time. Second, by proposing a precise way of measuring the magnitude of technical complementarities, through their effect on conditional production probabilities, it is possible to link the degree of complementarity in the economy to the presence of multiple aggregate equilibria. Third, characterizing the interaction of local and joint complementarities, one may describe how economies shift across equilibria. This view suggests that leading sectors can act as the trigger for aggregate development. Methodologically, the paper has employed a probabilistic characterization of aggregate equilibrium which appears to be a useful language for describing the dynamics of certain types of coordination failure.

A major limitation of coordination failure and complementarities-based models as paradigms for macroeconomic behavior has been their inability to address data questions. Conversely, a great strength of the representative agent/real business cycle paradigms has been the direct mapping of these models into aggregate time series. An important goal of this paper has been to show how complementarities can produce nondegenerate aggregate fluctuations. One suspects that further research will show how models driven by complementarities can generate time series which can be successfully calibrated to US aggregate data. Unless one is satisfied with establishing an observational equivalence between the real business cycle and coordination failure approaches, it is clear that an essential extension of the current paper is the development of empirical tests for complementarities. Our interacting industries model, like most of the multiple equilibria literature, embeds complementarities and market incompleteness at a very disaggregated microeconomic level. It therefore appears that the identification of strong complementarities as a source of aggregate fluctuations will require that macroeconometrics develops firmer microeconomic foundations.

Technical Appendix

1. Random field theory

The mathematics in this paper, based on random field theory, is new to macroeconomics. In this section, we try to provide some intuition into this branch of probability theory. Spitzer [1971] and Griffeath [1976] provide very clear introductions to the general theory. A ***random field*** is any stochastic process made up of individual elements ω_a of some set Ω, where $a \in A$, an index set. Representative agent economies can be expressed as random fields by letting the index set equal time, T. If the index set is one of the integer lattices $\mathbb{Z}^i$, then one can define a ***local random field*** as a random field where the conditional probabilities based upon the rest of the system depend only on elements distance D away.

$$\mu(\omega_a \mid \Omega - \omega_a) = \mu(\omega_a \mid \ \omega_b, \ |b-a| \leq D) \tag{A.1.1}$$

A ***Markov random field*** obeys this structure for $D = 1$. When one of the dimensions indexing random variables is time, this definition treats the past, present and future symmetrically. One can also build up local random fields by conditioning only on the past. For example, letting $A = \mathbb{Z}^2$, $a = (i,t)$ and employing the partitions Ω_t and $\underset{\sim}{\omega}_{t-1}$ described in the text, a temporally Markov random field over elements $\omega_{i,t}$ may be defined by

$$\mu(\omega_{i,t} \mid \Omega_{t-1}) = \mu(\omega_{i,t} \mid \underset{\sim}{\omega}_{t-1}) \tag{A.1.2}$$

whereas a local, temporally Markov random field may be defined by

$$\mu(\omega_{i,t} \mid \Omega_{t-1}) = \mu(\omega_{i,t} \mid \omega_b \in \underset{\sim}{\omega}_{t-1}, \ |b-(i,t)| \leq D). \tag{A.1.3}$$

The economy described in the text generalizes this form by allowing an asymmetric range of interactions.

An important feature of random fields with multidimensional index parameters is that these processes exhibit interesting forms of nonergodicity which are absent from one-dimensional processes. To see this, consider the Markov chain for the time series x_t with state space $\{0,1\}$ and probability transition matrix Q.

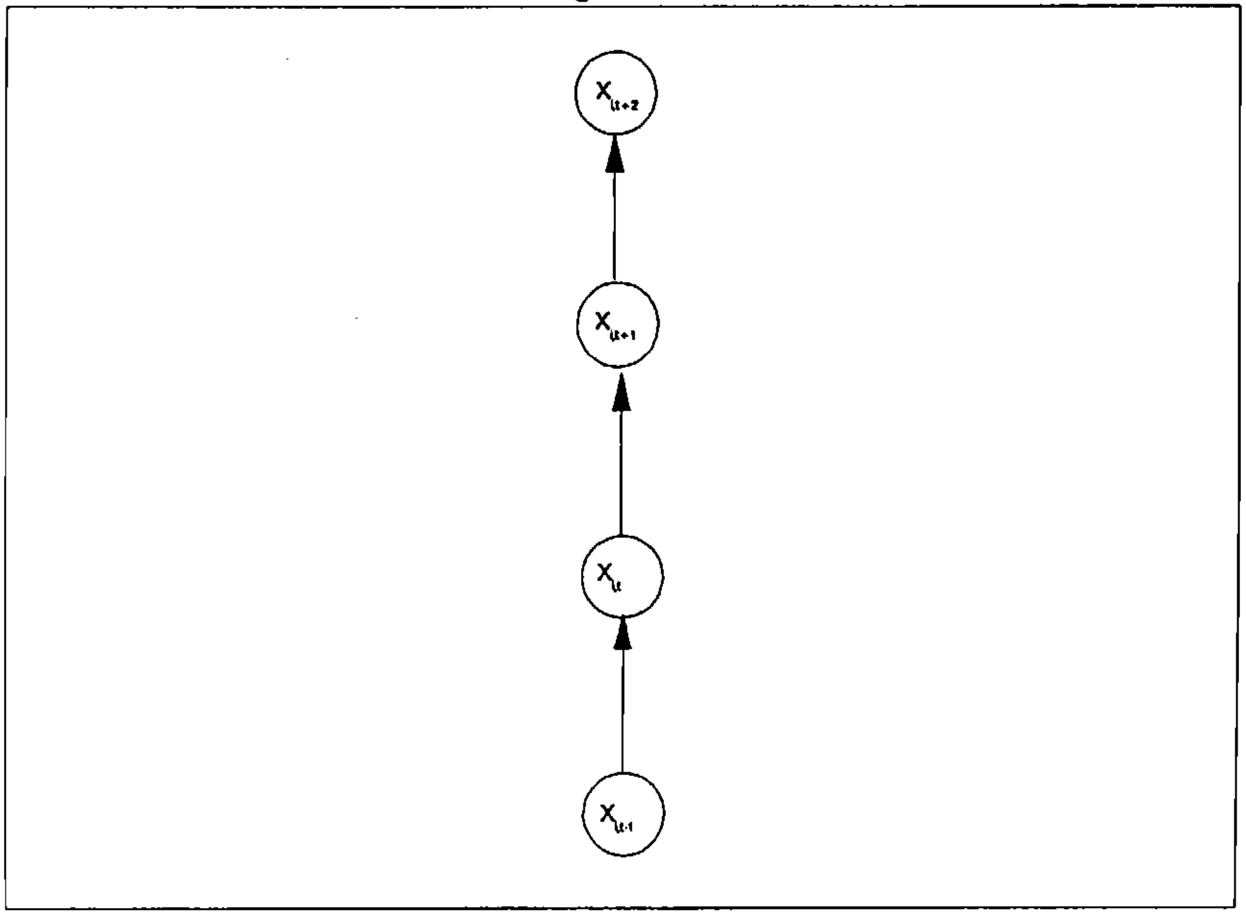
$$Q = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{bmatrix} \tag{A.1.4}$$

The only case where this transition matrix is consistent with more than one invariant limiting measure for x_t is when $\alpha = \beta = 1$, i.e. Q equals the identity matrix. This would mean that the time series generated by the process is degenerate.[15] However, for the multidimensional economies explored above, multiple invariant measures are consistent with nondegenerate time series.

The intuition behind this difference is that multidimensional random fields exhibit a qualitatively different degree of dependence across individual elements than their one-dimensional counterparts. The only way for the one-dimensional Markov system to exhibit multiplicity is to force the transition probabilities to equal one, thereby inducing sufficient dependence across random variables to affect the long run behavior of the system. This skewing of the transition probabilities is necessary because of the relatively sparse number of interactions across elements of the stochastic process. We can see this by contrasting the interactions of x_{t-1} and x_{t+2} in a Markov chain with the

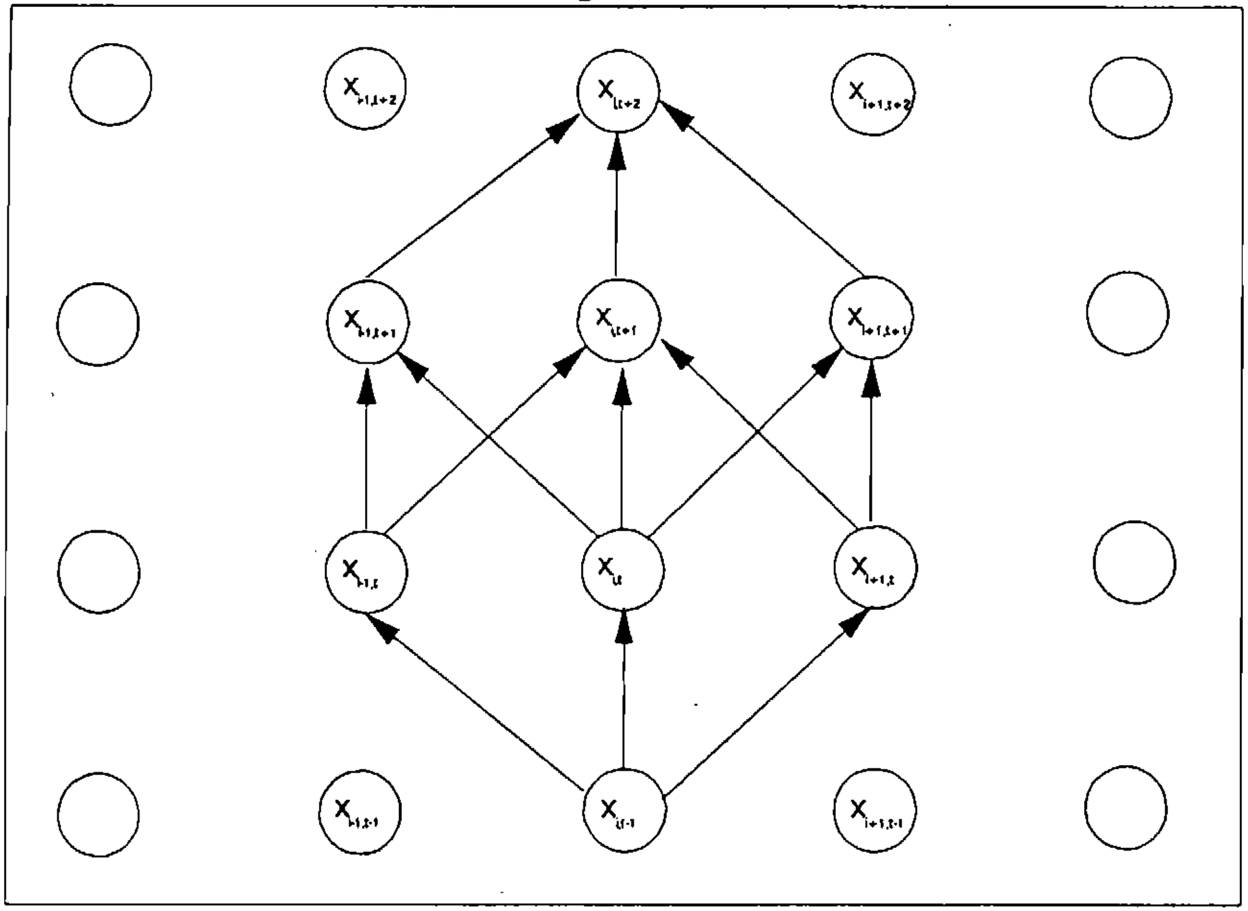
[15] If the state space possesses more than two elements, then it is possible to construct finite Markov chains which are nonergodic yet produce nondegenerate time series. However, this still requires that the system becomes permanently excluded from certain states.

Figure 8



Interactions in One Dimensional Random Field

Figure 9



Interactions in Two Dimensional Random Field interactions of $x_{i,t-1}$ and $x_{i,t+2}$, in a random field corresponding to $\Delta_{1,1}$ in the text. As Figure 8 illustrates, there is only one path linking the realization of x_{t-1} to the conditional distribution of x_{t+2}. The effect of x_{t-1} on x_{t+2} occurs only through the intermediaries x_t and x_{t+1}; generally, the effect of any individual element on future values of the process decays rapidly. For the random field corresponding to $\Delta_{1,1}$, many (7) different paths lead to an effect of the realization of $x_{i,t-1}$ on the distribution of $x_{i,t+2}$. These paths are drawn in Figure 9. The number of interconnections grows as the distance between the elements increases. (The growth in the number of paths is qualitatively different from what occurs if the dimension i assumes only a finite number of values.) This large number of interconnections makes the impact of individual elements on the subsequent history of the stochastic process more persistent. The many interconnections in multidimensional systems lead to a substantially greater degree of dependence across observations than in one-dimensional systems, leading to nonergodicity and hence multiple equilibria in a rich class of models.

2. Proofs of Theorems

Proof of Theorem 2.1.

This Theorem is based upon standard arguments showing that the equilibrium in the Brock-Mirman model can be supported by a representative agent economy with price taking firms. When $\zeta_{i,t}$ and $\eta_{i,t}$ are elements of $\mathfrak{F}_t$ we can define the one-period profit function, $\pi_{i,t}(\zeta_{i,t},\eta_{i,t})$ as the value of

$$\max_{K_{i,t}} \sup(\beta f_1(K_{i,t}-F,\zeta_{i,t})-K_{i,t},\ \beta f_2(K_{i,t},\eta_{i,t})-K_{i,t}). \tag{A.2.1}$$

Observe that $\pi_{i,t}(\zeta_{i,t},\eta_{i,t})$ is well-defined. Given our assumptions on the production function, the timing of information and the size of firms (firms treat the technique choices of all industries in the future as invariant to their choices), it is straightforward to show that maximization of (2.2) in the text with respect to the entire $\{K_{i,t}\}$ sequence is equivalent to maximizing (A.2.1) with respect to $K_{i,t}$ period by period, so long as the solution to (2.2) is bounded. Boundedness holds for any summable date zero price sequence by Assumption 2.1. Further, concavity of the conditional production functions in capital renders the maximum to the profit maximization problem unique. Hence the profit maximization problem has a solution and an optimal sequence of technique choices exists.

Clearly, if $\zeta_{i,t}$ and $\eta_{i,t}$ are known, then the profitability of each technique is known as well, which determines the technique choice unless the two techniques are equally profitable. Assume that all ties are broken by a time invariant rule which is a function of $\zeta_{i,t}$ and $\eta_{i,t}$. Recalling the equivalence of the firm and industry technique choices, by the Continuous Mapping Theorem, $\omega_{i,t}$ is a measurable function of these variables under perfect information. Application of the law of iterated expectations to the probability measure characterizing $\omega_{i,t}$, conditioning for any $\mathfrak{F}_{t-1}$, makes the optimal capital choice a function of the joint conditional probability measure for $\zeta_{i,t}$ and $\eta_{i,t}$, which given Assumption 2.2 yields Theorem 2.1.

Finally, observe that Assumption 2.3 means that the conditional probability measures characterizing $\omega_{i,t}$ are identical across i and t, $\forall\ t \geq 1$.

Proof of Theorem 2.2.

To prove the existence of a joint probability measure for $\rho_t = \beta$, we verify the two conditions derived by Dobrushin [1968] for the existence of a joint probability measure generated by a given set of conditional measures. First it is necessary to show that probabilities over all finite sets can be consistently defined. This may be seen through the construction of the conditional probabilities. For any initial condition $\underline{\omega}_0$, it is possible, given the transition probabilities, to compute $\mu(\omega_{i,1})\ \forall\ i$. One can proceed to define probabilities based on the individual elements for any finite set in Ω_1. Repeating this procedure, it is possible to assign probabilities for any finite set in Ω_t. Letting $t \Rightarrow \infty$, this means that all conditional probabilities over finite sets can be consistently defined.

The second condition in Dobrushin is that for any finite set S and any $\delta > 0$, there exists a finite set of elements, $\Gamma(S, \delta)$, $S \not\subseteq \Gamma(S, \delta)$, such that

$$| \mu(S \mid \Gamma) - \mu(S \mid \Omega_\infty - S) | \leq \delta. \tag{A.2.2}$$

This condition immediately holds for the probability structure we have examined. Consider the case $S = \omega_{i,t}$ where the range of interactions is $\Delta_{k,l}$. Choose the surrounding set Γ as

$$\Gamma = \{\omega_{p,q} \textit{ such that } 0 < |p-i| \leq k+l,\ 0 < |q-t| \leq k+l\}.$$

Let Γ' be any set of elements such that $\Gamma' \cap \Gamma = \Gamma' \cap \omega_{i,t} = \phi$. It is clear, given the $\Delta_{k,l}$ structure, that the conditional probability of any Γ', given Γ, is equal to the conditional probability given Γ and $\omega_{i,t}$.

$$\mu(\Gamma' \mid \omega_{i,t}, \Gamma) = \mu(\Gamma' \mid \Gamma) \tag{A.2.3}$$

Since this is true for all sets Γ', it is also true for $\Omega_\infty - \Gamma - \omega_{i,t}$, i.e.

$$\mu(\Omega_\infty - \Gamma - \omega_{i,t} \mid \omega_{i,t}, \Gamma) = \mu(\Omega_\infty - \Gamma - \omega_{i,t} \mid \Gamma) \tag{A.2.4}$$

or

$$\frac{\mu(\omega_{i,t}, \Gamma, \Omega_\infty - \Gamma - \omega_{i,t})}{\mu(\omega_{i,t}, \Gamma)} = \frac{\mu(\Gamma, \Omega_\infty - \Gamma - \omega_{i,t})}{\mu(\Gamma)}, \tag{A.2.5}$$

which implies

$$\frac{\mu(\omega_{i,t}, \Gamma, \Omega_\infty - \Gamma - \omega_{i,t})}{\mu(\Gamma, \Omega_\infty - \Gamma - \omega_{i,t})} = \frac{\mu(\omega_{i,t}, \Gamma)}{\mu(\Gamma)} \tag{A.2.6}$$

or

$$\mu(\omega_{i,t} \mid \Omega_\infty - \omega_{i,t}) - \mu(\omega_{i,t} \mid \Gamma) = 0, \tag{A.2.7}$$

which shows that (A.2.2) holds for $S=\omega_{i,t}$. This argument generalizes to any S, which proves the Theorem.

Proof of Theorem 3.1.

The proof of (A) is based upon a generalization of a standard argument in probability theory describing the limiting behavior of percolation models. (See Kindermann and Snell [1980] and Grimmett [1989] for a description.) To show that the high production equilibrium is attainable from any initial condition, given a sufficiently high value of $\Theta_{k,l}^{min}$, it is sufficient to provide conditions such that

$$\lim_{t \Rightarrow \infty} \mu(\omega_{i,t} = 0 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{0}) = 0. \tag{A.2.8}$$

(This limit is equivalent to the probability that $\omega_{i,t}$ equals 0 if we start the economy far enough back in the past.) The probability that $\omega_{i,t}$ equals 0 is bounded from above by the probability that at least one of the elements in $\omega_{j,t-1}$, $j \in \Delta_{k,l}$, equals 0, since if none of these elements equals 0, $\omega_{i,t}$ equals 1 with certainty. Each of the elements $\omega_{j,t-1}$ is a function of $k+l+1$ elements of time $t-2$. If none of the elements in $\omega_{j,t-2}$, $j \in \{i-2k-1 \ldots i+2l+1\}$ equals 0, then none of the relevant elements $\omega_{j,t-1}$ can equal 0. Hence, in order to have $\mu(\omega_{i,t}=0) \neq 0$, it is necessary for there to be a path of zeroes linking the elements of $\omega_{j,t-2}$, $j \in \{i-2k-1 \ldots i+2l+1\}$ through $\omega_{j,t-1}$, $j \in \Delta_{k,l}$, to $\omega_{i,t}$. Making tbis recursion n times leads to $(k+l+1)^n$ different paths leading from elements at $t-n$ to $\omega_{i,t}$.

The probability that any path of length $t-1$ consists of all 0's is bounded from above by $(1-\Theta_{k,l}^{min})^{t-1}$. Consequently, the probability that at least one path of zeroes leads from $\underset{\sim}{\omega}_0$ to $\omega_{i,t}$ can be bounded from above by the sum of the probabilities of each of the paths, which implies

$$\mu(\omega_{i,t} = 0 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{0}) \;\le\; (k+l+1)^{t-1}(1-\Theta_{k,l}^{min})^{t-1}. \tag{A.2.9}$$

If $\Theta_{k,l}^{min} > \frac{k+l}{k+l+1}$, then the limit of the right hand side is 0 as $n \Rightarrow \infty$, which is the required result.

The proof of (B) is based upon known results concerning the model for $\Delta_{1,0}=\{i-1,i\}$. We prove (B.i); the proof for (B.ii) is identical. Let $\mu_{i,t}^{k,l}(\cdot)$ denote the conditional probability measure of $\omega_{i,t}$ for index set $\Delta_{k,l}$ and $\mu_t^{k,l}(\cdot)$ denote the conditional probability measure for $\underset{\sim}{\omega}_t$ given $\Delta_{k,l}$. Shnirman [1968] proves that if $\underset{\sim}{\omega}_0=\underset{\sim}{0}$, there exists a $\underline{\Theta}_{1,0} > 0$ such that (B.i) and (B.ii) bold. We can thus assume that for some $\underline{\Theta}_{1,0}$,

$$\lim_{t \Rightarrow \infty} \mu_{i,t}^{1,0}(\omega_{i,t} = 1 \mid \underset{\sim}{\omega}_0 = 0) \;<\; 1. \tag{A.2.10}$$

In order to show that these properties hold for an arbitrary index set $\Delta_{k,l}$, it is sufficient to show that there exists a $\underline{\Theta}_{k,l}$ greater than 0 such that

$$\lim_{t \Rightarrow \infty} \mu_{i,t}^{k,l}(\omega_{i,t} = 1 \mid \underset{\sim}{\omega}_0 = 0) \;<\; 1. \tag{A.2.11}$$

We assume that the conditional probabilities of production under $\Delta_{1,0}$ always equal $\underline{\Theta}_{1,0}$ unless all relevant industries chose technique 1 the previous period, i.e.

$$\underline{\Theta}_{1,0} = \mu_{i,t}^{1,0}(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 1, \omega_{i,t-1} = 0)$$

$$= \mu_{i,t}^{1,0}(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 0, \omega_{i,t-1} = 1)$$

$$= \mu_{i,t}^{1,0}(\omega_{i,t} = 1 \mid \omega_{i-1,t-1} = 0, \omega_{i,t-1} = 0). \tag{A.2.12}$$

For any choice of $\underline{\Theta}_{k,l}$ such that

$$\underline{\Theta}_{k,l} \leq \underline{\Theta}_{1,0} \tag{A.2.13}$$

and any sample path realization of $\underline{\omega}_{t-1}$, $k \geq 1$ and $l \geq 0$ imply

$$\mu_{i,t}^{1,0}(\omega_{i,t} = 1 \mid \underline{\omega}_{t-1}) \geq \mu_{i,t}^{k,l}(\omega_{i,t} = 1 \mid \underline{\omega}_{t-1}). \tag{A.2.14}$$

The distribution of $\underline{\omega}_{t-1}$ differs, however, under the two sets of transition probabilities. The proof is complete once it is verified that the distribution of $\underline{\omega}_{t-1}$ is at least as likely to produce a high production industry under $\mu_{t-1}^{1,0}(\cdot)$ than under $\mu_{t-1}^{k,l}(\cdot)$.

For each $\underline{\omega}'$, define $\Xi_{\underline{\omega}'}$ as the set of all configurations $\underline{\omega}$ such that $\omega_i \geq \omega_i'$ $\forall$ i. From the assumption on the transition probabilities, (A.2.13), it is clear that the probability of a draw in $\Xi_{\underline{\omega}'}$ for $\underline{\omega}_1$ is at least as great under $\mu_1^{1,0}(\cdot)$ than under $\mu_1^{k,l}(\cdot)$, for any $\underline{\omega}'$. This implies

$$\mu_1^{1,0}(\underline{\omega}_1 \in \Xi_{\underline{\omega}'} \mid \underline{\omega}_0 = \underline{0}) \geq \mu_1^{k,l}(\underline{\omega}_1 \in \Xi_{\underline{\omega}'} \mid \underline{\omega}_0 = \underline{0}). \tag{A.2.15}$$

Making this argument recursively $t-2$ times yields

$$\mu_{t-1}^{1,0}(\underline{\omega}_{t-1} \in \Xi_{\underline{\omega}'} \mid \underline{\omega}_0 = \underline{0}) \geq \mu_{t-1}^{k,l}(\underline{\omega}_{t-1} \in \Xi_{\underline{\omega}'} \mid \underline{\omega}_0 = \underline{0}). \tag{A.2.16}$$

Choose $\underset{\sim}{\omega}'$ so that $\omega'_k = 1$ if $k = i,\ i{-}1$, 0 otherwise. This means that $\Xi_{\underset{\sim}{\omega}'}$ defines the set of sample path realizations of $\underset{\sim}{\omega}_{t-1}$ such that $\mu^{1,0}_{i,t}(\omega_{i,t} = 1 \mid \underset{\sim}{\omega}_{t-1}) = 1$. A subset of these sample path realizations imply $\mu^{k,l}_{i,t}(\omega_{i,t} = 1 \mid \underset{\sim}{\omega}_{t-1}) = 1$. Hence it is at least as likely that $\underset{\sim}{\omega}_{t-1}$ creates a conditional probability equalling 1 under $\mu^{1,0}_{i,t}(\cdot)$ as under $\mu^{k,l}_{i,t}(\cdot)$.

We now can combine three results. First, (A.2.14) shows that conditional on any given $\underset{\sim}{\omega}_{t-1}$, the probability of $\omega_{i,t}=1$ is at least as large for $\mu^{1,0}_{i,t}(\cdot)$ than any $\mu^{k,l}_{i,t}(\cdot)$. Second, (A.2.16) states that that the probability that the draw of $\underset{\sim}{\omega}_{t-1}$ produces a conditional probability equal to 1 is also at least as large for $\mu^{1,0}_{i,t}(\cdot)$. Third, (A.2.13) shows that any configuration $\underset{\sim}{\omega}_{t-1}$ such that the conditional probability of high production does not equal 1 under $\mu^{1,0}_{i,t}(\cdot)$ generates a conditional probability under $\mu^{1,0}_{i,t}(\cdot)$ that is at least as great as under $\mu^{k,l}_{i,t}(\cdot)$. Therefore,

$$\mu^{1,0}_{i,t}(\omega_{i,t} = 1 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{0}) \ \geq\ \mu^{k,l}_{i,t}(\omega_{i,t} = 0 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{0}). \qquad \text{(A.2.17)}$$

Taking the limit as $t \Rightarrow \infty$ for both sides of this inequality and recalling (A.2.10) gives the desired result.

Proof of Theorem 3.2.

This Theorem immediately follows from the proof of Theorem 3.1.

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