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INVENTORIES IN THE KEYNESIAN MACRO MODEL

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ABSTRACT

An otherwise conventional Keynesian macro model is modified to include inventories of final goods by (1) drawing a distinction between production and final sales, and (2) allowing for a negative effect of the level of inventories on production. Two models are presented: one in which the labor market clears and one in which it does not. Both models are stable only if the negative effect of inventories on production is "large enough." Both models also imply that real wages move countercyclically - in direct contrast to the usual implication of Keynesian models. Detailed analysis of the market-clearing model show that there should be negative correlation between the levels of inventories and output, and between changes in inventories and changes in output, over the business cycle. However, inventory change should be positively correlated with the level of output.

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I. Motivation and Relation to Other Literature

If a man from Mars visited this planet and spent a year or so reading all the macroeconomic literature of the past 15-20 years, he would not come away feeling that inventories are of much importance. If we then gave him five minutes with the National Income and Product Accounts of the United States, he would quickly conclude that there was something lacking in his education. Inventories are important. Indeed, as a rough generalization, changes in the rate of real inventory investment have accounted for approximately 70 percent of the decline in real GNP during a typical postwar recession (see Table 1). It would seem likely, therefore, that inventories play a crucial role in the propagation of business cycles.

If our Martian read some of our leading elementary textbooks, he would again find a prominent role assigned to inventories as the principal force driving national income to its "equilibrium" level--the level at which there is no undesired inventory accumulation or decumulation.[1] But if he tried to pursue this line of reasoning in the more advanced textbooks, he would find little more.[2] And if he sought after discussions of inventories in the theoretical literature, he would come up nearly empty-handed.[3] Inventories, in a word, have been neglected by macroeconomic theorists.

In the period immediately following the publication of _The General Theory_, there was a flurry of theoretical work on inventories, culminating in METZLER's (1941) classic paper. Working with the simplest possible difference equation system, METZLER pointed out that inventory investment could conceivably destabilize an otherwise stable system. This paper extends METZLER's line of reasoning by showing that, in a modern Keynesian model, including inventories:

(a) a well-known condition for stability of a monetary economy due to CAGAN (1956) becomes stricter on account of inventories;

(b) inventories add an additional stability condition to the model--a condition that is not innocuous since it could be violated under plausible parameter values.

Table 1

CHANGES IN GNP AND IN INVENTORY INVESTMENT
IN THE POSTWAR RECESSIONS

(1) Dates of Contraction Peak	Trough	(2) Decline in Real GNP[a]	(3) Decline in Inventory Investment[a]	(4) Column (3) As a Percentage of Column (2)
1948:4	1949:4	\$ 6.7	\$ 13.0	194%
1953:2	1954:2	20.6	10.2	50
1957:3	1958:1	22.2	10.5	47
1960:1	1960:4	8.8	10.5	119
1969:3	1970:4	12.0	10.1	84
1973:4	1975:1	71.0	44.8	63

[a] In billions of 1972 dollars.

Source: The National Income and Product Accounts of the United States, 1929-74, and Survey of Current Business.

But while METZLER stressed the destabilizing role of inventories, many other authors have stressed their stabilizing role. Obviously, inventories of finished goods give firms flexibility either to meet abnormally high demand by selling more than they produce, or to cope with abnormally low demand by producing more than they sell. Thus production and employment can be stabilized relative to demand when output is storable. It seems particularly important to recall this role of inventories in light of the recent work on "spillovers" by BARRO and GROSSMAN (1971, 1976) and others.

One of the bases of the BARRO-GROSSMAN analysis is that, when sticky wages and prices prevent the attainment of the full Walrasian general equilibrium in the short run, the actually quantity transacted in a market normally will be the minimum of supply and demand. This so called "min condition" is based on the principle of voluntary exchange, but retains its plausibility only if output is non-storable.[4] Consider, for example, the BARRO-GROSSMAN generalized excess supply scenario. If firms cannot sell all the output they would like to, they react by reducing production and firing workers. In this way, excess supply in the goods market "spills over" into the labor market as well. But what if output is storable at moderate costs? It seems unlikely, under these circumstances, that production cutbacks and layoffs would be a rational reaction to moderate short-run gluts in the product market. Instead, firms can--and apparently do--maintain production and store their excess output for subsequent sale. Only if poor sales performance persist for some time, or are extremely large, do firms reduce their work forces. In this way, inventories limit the spillover of excess supply from the product market to the labor markets to instances of extreme drops in demand.

Or consider the BARRO-GROSSMAN scenario of generalized excess demand. In this case, workers who are unable to purchase the commodities they desire (because these commodities are in excess demand) react by reducing their supply of labor. Why work when (at the margin) there are no goods to buy? Thus excess demand in the goods market spills over into the labor market. But once again, barring generalized stock-outs, this will not happen in an economy in which there are inventories of goods. There may well be a flow excess demand for goods; but, at least for a while, firms can meet this excess demand out of inventories. Thus it seems unlikely that excess demand for goods would lead to a drop in labor supply, except in extreme circumstances.[5] It is worth pointing out that this particular spillover mechanism accounts for what may be the most empirically distressing implication of the BARRO-GROSSMAN model: that positive shocks to aggregate commodity demand, starting from a position of equilibrium, will <u>reduce</u> real output.

The model considered here is very different from the BARRO-GROSSMAN model, though at least one of its basic aims is identical: to explain the link between aggregate demand and real output. For example, the "min condition" for the goods market does not appear here because it makes little sense in the presence of inventories. Instead, I assume that consumers always purchase their quantity demanded. Stockouts at the aggregate level are ignored. When there is excess supply, firms add the excess to their inventories; and when there is excess demand, firms meet this demand by drawing down inventories. In either case, the resulting inventory imbalance induces firms to adjust their production and employment decisions; but the adjustments are gradual, so the sharp corners of BARRO and GROSSMAN are smoothed out.[6]

Finally, since the model proposed here offers an alternative way to forge the link between aggregate demand and real output, mention should be made of the currently most popular way of doing so. In standard Keynesian analysis, money wages are assumed fixed in the short run, so higher prices (caused by higher aggregate demand) encourage employment and output by <u>lowering real wages</u>.[7] By contrast, in the main model presented here, money wages move promptly to clear the labor market, and real wages actually move <u>procyclically</u>. I make the assumption that money wages are fully flexible not for its empirical validity, but to illustrate that inventories provide a link between demand and production that does not rely on wage rigidities. Later in the paper, the assumption of instantly flexible wages is replaced by an expectations-augmented Phillips curve and a "min condition" for the labor market. It is shown that the analysis, while greatly complicated, is not altered in any essential way by these changes. In particular, the conclusion that wages move procyclically is maintained.

The plan of the paper is as follows. The next section offers a general discussion of the motives for holding inventories, noting how each motive bears on the specification of macro models. Then, choosing one particular rationale for inventories, I develop and analyze in Section III a complete macro model in which all markets "clear." Section IV discusses the modifications required when the labor market does not clear, and sketches how the analysis is affected. Section V offers some brief concluding remarks.

II. The Specification of Inventory Behavior

Where should inventories be brought into conventional macroeconomic models? The answer depends on whether inventories are inputs or outputs, and on why firms hold them; but a wide variety of micro models suggest that higher inventory stocks lead to lower current output.

Among the major motives for holding inventories that appear in the literature is improved production scheduling (see, for example, HOLT, MODIGLIANI, MUTH and SIMON (1960)). The idea is that multi-product firms can operate more efficiently if inventories give them flexibility in scheduling production runs. So this suggests that the stock of inventories, N, should enter the production function as another factor of production in addition to employment, E: $y = f(E,N)$, $f_N > 0$. In a model like this, inventories could either be inputs (i.e., raw materials and intermediate goods) or outputs (i.e., finished goods). The crucial question is whether and how N affects the marginal productivity of labor. The basic rationale seems to suggest that inventories raise labor's productivity; and, if so, this would stimulate employment demand. But it should also be true that, when the stock of inventories rises, the incentive to raise it further by producing diminishes (as long as there are diminishing returns to inventories). Thus, both the costs (via higher productivity) and the benefits of production are reduced by rising N , with consequently ambiguous effects on output.

Other models of inventory behavior seem less ambiguous regarding the effect of inventories on production. For example, it is commonly hypothesized that firms hold inventories as a buffer stock in the face of fluctuating demand (see, for example, MILLS (1962)). In that case, inventories are probably outputs which do not directly effect the production function. Instead higher N reduces the probability of having a stock-out. Given diminishing returns to inventory-holding, this presumably leads to lower production.

A closely related motive for holding inventories is speculation on future price movements. Indeed, this is almost indistinguishable from the buffer stock motive in that expectations of high future prices relative to costs (the speculative motive) and large future sales (the buffer-stock motive) amount to more or less the same thing.

While these last two motives amount to using inventories to smooth production relative to demand, some firms may wish to do just the opposite: to bunch production relative to sales. This could happen where dramatically increasing returns to scale dictate that production be done in large "production runs," which are then put into inventory and gradually sold. In this production run model, it seems fairly clear that excessively high inventories will induce a postponement of the next production run, and hence a reduction in "average" output and employment over an interval of time.

Still another motive for holding output inventories, suggested by MACCINI (1977) among others, is that high inventory stocks may stimulate a single firm's demand by reducing delivery lags. In this case, it seems appropriate to omit N from the production function, but include it in the firm's demand function. But it is not clear that N should have a similar stimulative effect on aggregate demand.

Others have suggested that input inventories are held in order to economize on purchasing costs. This could be either because there is a fixed cost to purchasing inputs--the assumption that underlies "optimal lot size" models, or because firms face a rising supply price of inputs which makes it economical to smooth input purchases relative to input usage.[8] In such a case, the rate at which inventories are <u>used</u> should appear in the production function, while the rate at which they are <u>purchased</u> should depend on the existing stock.

What effects, then, should inventories have on aggregate demand? Presumably, all the models agree that <u>desired inventory investment</u> should be a decreasing function of N. But there is no persuasive reason to think that N has any direct effects on the other components of aggregate demand--what are called <u>final sales</u>.

What about aggregate supply? Considering first output inventories, the production-scheduling motive suggests that $\partial y/\partial N$ might conceivably be positive. But the other motives seem strongly to suggest the opposite: the higher the level of inventory stocks, the less the firm will be inclined to produce. One would also expect high inventories to lead to price cuts.

But input inventories would have different effects. Imagine a firm whose inputs are storable, but whose outputs are not. If it finds itself with too many inventories, it will have an incentive to raise production (and employment) and cut prices. If it can store both inputs and outputs, the implication for production decisions becomes unclear. It depends on the relative costs and benefits of inventorying inputs versus outputs.

Finally, the nature of inventories has implications for the accounting identity governing inventory accumulation or decumulation, $\dot{N}$. For example, if inventories are outputs, then $\dot{N}$ is the difference between production and sales, so a rise in production (other things equal) raises $\dot{N}$. But if inventories are inputs, then $\dot{N}$ is the difference between input purchases and input usage, which will fall when production rises.

The macroeconomic model presented here is based on a very specific micro model of inventory behavior which I have presented in another paper (BLINDER (1978)). The model is one of <u>output</u>

inventories held for reasons of <u>anticipated price appreciation</u> or, what amounts to the same thing, as a buffer stock held because of anticipated fluctuations in demand. As I show in that paper, the level of inventories affects employment demand (and hence output) negatively:

$$E^d = E^d(w,N) \; E^d_w, \; E^d_N < 0 \; .$$

This function has the property that $E^d(w,N*)$, where $N*$ is the desired or optimal level of inventories and w is the real wage, is equal to the inverse of the marginal productivity schedule: $f'^{-1}(w)$. For higher inventories, the labor demand schedule lies below the marginal productivity schedule; and for lower inventories, it lies above.

While I have derived it in a very specific context, let me try to explain why I believe that such a labor demand function would arise under quite general circumstances. Consider any of a family of models where the firm maximizes the discounted present value of its profits subject to (among other things) a constraint that inventory change equals production minus sales:

$$\dot{N} = f(E,\ldots) - x \; .$$

Any of a variety of variables could enter $f(\cdot)$ without affecting the argument; similarly, sales or price could be endogenous or exogenous for present purposes. The Hamiltonian for such a problem would look like this:

$$H(E,\ldots) = \text{sales revenues} - wE - \text{other costs} + \lambda[f(E,\ldots) - x],$$

where again I need not specify the nature of sales revenues or non-wage costs. Here λ is the <u>shadow value of inventories</u>, and a well-known result of optimal control theory is that:

$$\frac{\partial J}{\partial N_0} = \lambda_0 \; ,$$

where J is maximized (with respect to E and other variables) profits, and N_0 is the initial stock of inventories.

The first order condition for optimal employment is $f_E(E,\ldots) = w/\lambda$ at every instant, which has two important implications:

(a) in making <u>production</u> (as opposed to sales) decisions, the firm compares its costs with λ, not with the market price.

This is because it is deciding whether to turn inputs into <u>inventories</u>. Then, in deciding whether to <u>sell</u> out of inventories, it will compare λ with the market price.[9]

(b) optimal E is a decreasing function of w and an increasing function of λ. But this means that we can derive the abovementioned labor demand function by showing that λ_O is a decreasing function of N_O. Now note that:

$$\frac{\partial \lambda_O}{\partial N_O} = \frac{\partial^2 J}{\partial N_O^2} ,$$

and that $\partial^2 J/\partial N_O^2$ must be negative in a wide variety of problems. (This is just a statement of diminishing returns, and is, in fact, a sufficient condition for a maximizing program to exist.)

While the labor demand function used here is therefore quite general, note that the production-scheduling model raises the possibility that $\partial y/\partial N > 0$. It should come as no surprise that models with otuput inventories normally will be stable only if a rise in N reduces $\dot{N} = y - x$, where x is final sales. Thus the stability of the model hinges precariously on the effects of inventories on production decisions--a question that cannot be answered by microeconomic theory, and that has barely been investigated in empirical macroeconomics.[10]

<u>III. A Macroeconomic Model with Inventories</u>

<u>3.1 Specification of the Model</u>

The demand side of the model is quite standard, except that recognition of inventories requires that a distinction be drawn between output and final sales. Thus, instead of an IS curve, the following expression describes real final demand:

$$x = c(y-t(y),r) + g , \tag{1}$$

where y is GNP (or national income), $t(y)$ is real taxes, r is the real interest rate, c is real private demand, and g is real government demand. The capital stock is ignored on the grounds that it can be treated (roughly) as constant. Since the period of time that I am concerned with is quite short, this seems more legitimate than it is in many other contexts.

The demand side is completed by an LM curve based on the strict transactionist view of the demand for money:[11]

$$M/P = L(r + \pi, y) \tag{2}$$

where M is the nominal money stock, P is the price level, and π is the expected rate of inflation. Making the distinction between x and y raises interesting questions about which is the appropriate transaction variable in the demand function for money. But since this is not the subject of this paper I sweep these issues under the rug and adopt the conventional variable: gross national product. Notice that (2) embodies the assumption that r adjusts instantly to maintain money-market equilibrium, but (1) implies no such assumption about the goods market. When inventories are changing $(x \neq y)$, the system is off the IS curve, which is

$$y = c(y - t(y), r) + g \ . \tag{1'}$$

What I call an <u>aggregate demand curve</u> can be derived from (1) and (2). First invert (2) to obtain:

$$r = R(y, m) - \pi \quad R_y = -\frac{L_y}{L_r} > 0 \ , \quad R_m = \frac{1}{L_r} < 0 \ , \tag{3}$$

where $m = M/P$ is the real money stock. Then substitute (3) into (1) to obtain:

$$x = c(y - t(y), R(y,m) - \pi) + g \ .$$

This can be written:

$$x = D(y; m, \pi, g), \tag{4}$$

where the function $D(\cdot)$ has the following derivatives:

$$D_y = c_y(1-t') + c_r R_y$$

$$D_m = c_r R_m > 0$$

$$D_\pi = -c_r > 0$$

$$D_g = 1$$

The conventional assumption in IS-LM analysis that D_y is a positive number less than unity will be reflected in what follows. Aggregate demand is identified with sales by assuming that generalized stockouts do not occur.

The supply side of the model consists of an equation that says that the labor market clears given the (possibly disequilibrium) state of inventories:

$$E^d(w,\ N) = E^s(w),\ E^s_w \geq 0\ , \qquad (5)$$

and a production function:[12]

$$y = f(E^s(w)). \qquad (6)$$

These two equations are solved very simply for an *aggregate supply function*:

$$y = Y(N), \qquad (7)$$

where $Y_N \equiv f'(E)E^d_N \dfrac{E^s_w}{E^s_w - E^d_w} < 0$.

Given predetermined values for the three state variables: N, m, and π, equations (4) and (7) determine the values of x and y for any given g . Figure 1 depicts one such solution on a standard "Keynesian cross" diagram. Equation (7) already tells us how y depends on the state variables. To obtain a similar solution function for x , substitute (7) into (4) to get

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$$x = X(N,\ m,\ \pi;g) \qquad (8)$$

where:

$$X_N = D_y\ Y_N < 0$$

$$X_z = D_z,\ z = m,\pi,g\ .$$

Thus, *initially* monetary or fiscal policy effects x but not y .

The position of the economy defined by (7) and (8) will not in general be an equilibrium because one or more of the state variables will change. Changes in the stock of inventories are governed by a straightforward accounting identity:

$$\dot{N} = y - x\ . \qquad (9)$$

Changes in the expected rate of inflation are assumed to be adaptive:[13]

$$\dot{\pi} = \beta(\dot{P}/P - \pi) \qquad \beta > 0\ . \qquad (10)$$

Finally, since I assume that budget deficits are bond-financed, changes in real balances happen either (a) abruptly due to an open-market operation or (b) smoothly due to changes in the price level. Thus, *except at instants when there are open-market operations,*

$$\dot{m} = -m(\dot{P}/P).$$

This requires an equation for price dynamics, for which the following seems suitable:

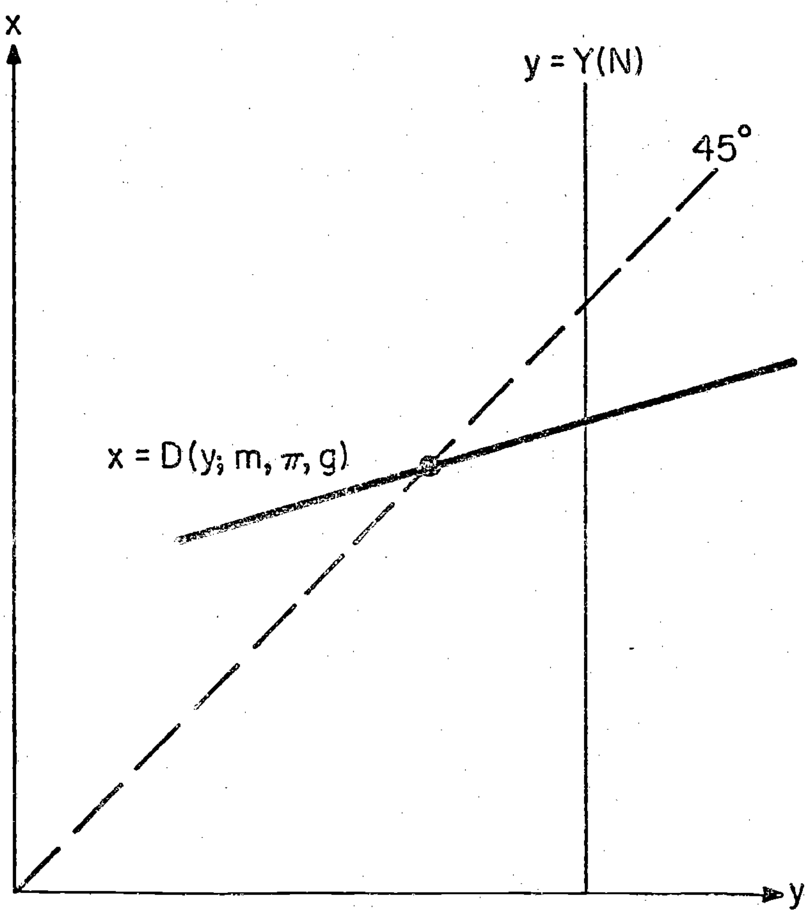

Figure 1

$$\dot{P}/P = \pi + \Theta(N^* - N), \quad \Theta > 0 , \tag{11}$$

where N^* is the specific (optimal) level of inventories that makes $E^d(w, N^*)$ coincide with the marginal productivity schedule. In general, as shown in BLINDER (1978), N^* would depend on the production function, the nature of inventory holding costs, the entire future path of expected prices (and sales constraints, if there are any), and the real rate of interest. However, I ignore all this and treat N^* as a constant in the short run.[14] Equation (11) has empirical support in that unfilled orders are the typical indicator of excess demand in product markets in recent empirical price equations (see, for example, GORDON (1975)). And for firms that produce to order, unfilled orders play the same role as inventories play for firms that produce to stock. Indeed, unfilled orders can be viewed usefully as negative inventories (see MACCINI (1976)). From (11), the equation for changes in the real money supply follows immediately. Except at moments of open-market operations:

$$\dot{m} = -\pi m - \Theta m(N^* - N) . \tag{12}$$

Equilibrium occurs only when (9), (10) and (12) are all equal to zero. That is, when GNP equals final sales, expectations are correct, and inflation is zero.[15]

3.2 Comparative Statics of Equilibrium Positions

There seem to be three interesting questions to ask about what policy variables (M or g) do to endogenous variables like y or w. First, what are the instantaneous effects? Second, what are the equilibrium effects? Third, what do the paths look like in the interim period? The first question has already been answered: in the first instant, a rise in g or M increases x, but has no effect on either y or w (or on the inflation rate). I turn next to the second question.

Using (7) and (8), and imposing the requirements for equilibrium, the following equations define steady states of the model:

$$Y(N) = X(N, m, \pi; g) \qquad (\dot{N} = 0)$$
$$\dot{P}/P = \pi \qquad (\dot{\pi} = 0)$$
$$\pi = \Theta(N - N^*) \qquad (\dot{m} = 0)$$

But the last two, in conjunction with the price equation (11), require that $\pi = 0$ and $N = N^*$ in equilibrium.

Thus the equilibrium version of the model can be represented by the standard IS curve, (1'); the LM curve with $\pi = 0$:

$$M/P = L(r,y); \qquad (2')$$

and a classical labor market:

$$w = f'(E^d),\ E^s = E^s(w),\ E^d = E^s .$$

In its most compact form, equilibrium is defined by the single equation:

$$Y(N^*) = X(N^*, m, 0; g) . \qquad (13)$$

Thus, $y^* = Y(N^*)$ is the "natural rate" of output, and $w^* = w(N^*)$ is the equilibrium real wage. Neither of them can be <u>permanently</u> affected by policy. Nor can x, since $x = y$ in equilibrium.

<u>3.3 Dynamic Adjustment Paths</u>

I turn next to the dynamic paths of the important macroeconomic variables, deferring for the moment the issue of whether the dynamic system is stable. Since I ignore many variables that change in the long run, it is these short-run responses--not the steady states--that are of greatest interest.

Figure 2 about here

Figure 2 shows the model in an initial position of equilibrium at point A. Here $x=y$, so N is unchanging; inflationary expectations are correct and equal to zero; and real balances are constant. Now suppose there is a dose of expansionary monetary ($dM > 0$) or fiscal ($dg > 0$) policy, shifting the demand curve upwards from D_0 to D_1 .

Initially, the economy's position shifts upwards to B: sales are raised, but GNP is not. But at B, inventories are disappearing. Consequently, the supply curve starts moving to the right (see equation (7)). At the same time, two effects start working on the demand curve. In a stable system, the more important of these is that (11) implies that inflation begins, eroding real balances, and causing the demand curve to shift downwards towards D_2 . The second effect is that inflation raises inflationary expectations (by (10)), and this reduces the real interest rate (by (3)), which stimulates

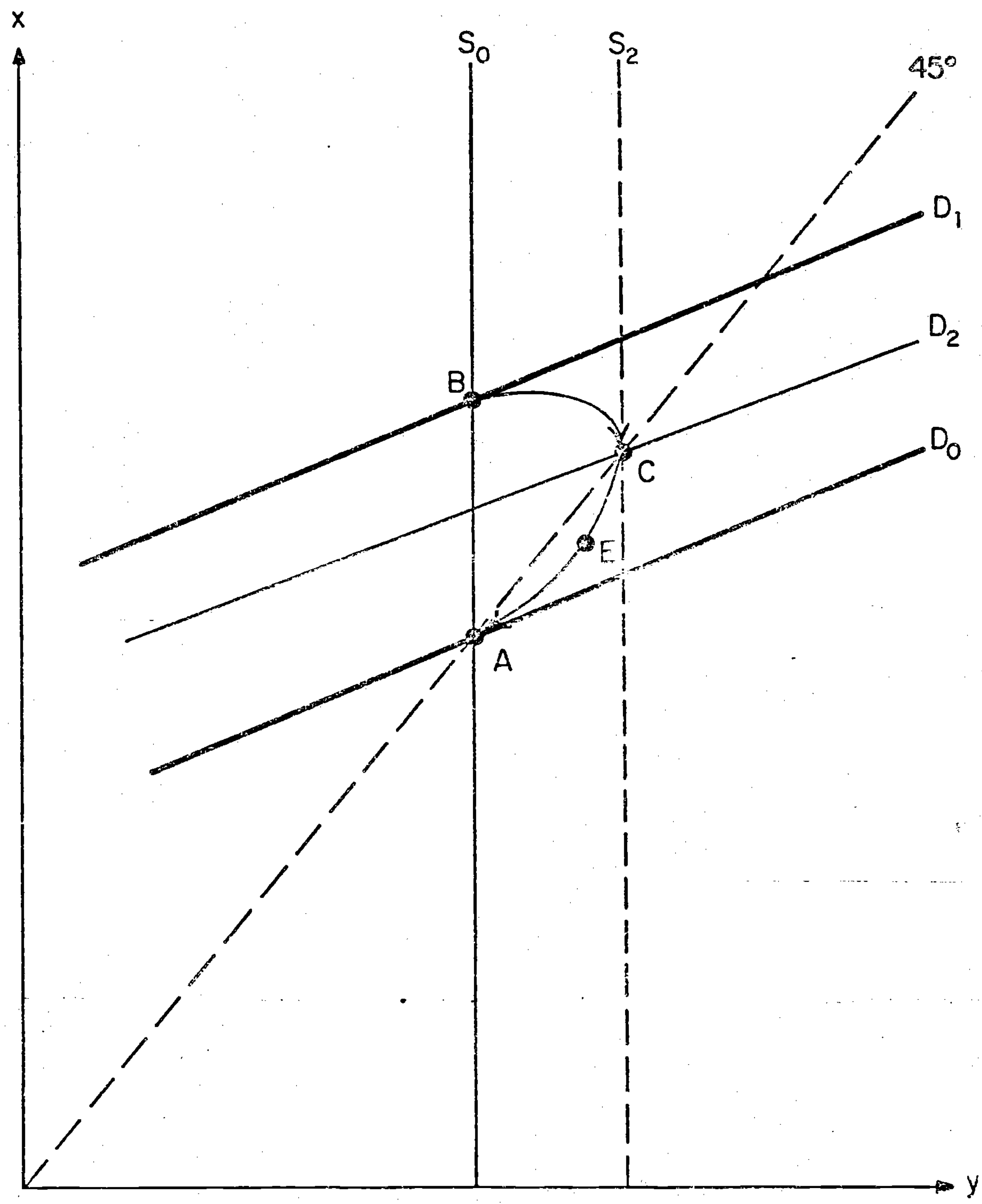

Figure 2

spending. The diagram assumes that the former effect dominates, so that the position of the economy moves towards the south east, as indicated by the arrow emanating from point B .

At some point--indicated in the diagram by point C--the supply and demand curves (S_2 and D_2) intersect on the 45^o line. At this moment, the inventory decumulation is halted, and inventories begin to be replaced. So the supply curve starts shifting back toward its original position. However, while they are rising, inventories remain low, so the impetus for inflation remains. Prices keep rising while real output falls. In fact, for a period, inflation is accelerating while output is falling.[16] Whether or not this is to be called a phase of "stagflation" or not is a matter of terminological dispute. But it does create an interval of time during which changes in unemployment and changes in inflation are positively correlated--an upwards sloping "Phillips curve" if you will.

Before turning to the conditions under which this stable scenario actually obtains, let me outline some of the observable consequences of the model. Following a stimulus to aggregate demand:

(1) Final sales rise quickly to a peak,[17] and then decline to their original level. GNP rises much more slowly to a peak, and also declines. So the composition of GNP between final sales and inventory change varies dramatically over the cycle.

(2) Both employment and real wages follow the path of GNP, rising to a peak and then returning to their equilibrium levels. Thus, in contrast to the traditional Keynesian and search theoretic models, real wages move procyclically.

(3) The trough in the level of inventories (N) coincides with the peak in output (both occur at point C in Figure 2). As Figure 3 shows, N and y display negative correlation over the cycle.

igure 3 about here

(4) The peak in inventory investment ($\dot{N}$) lags the peak in production. (In terms of Figure 2, $\dot{N}$ peaks at point E, while y peaks at point C.)[18] As Figure 3 indicates, $\dot{N}$ and y are positively correlated, while $\dot{N}$ and $\dot{y}$ are negatively correlated over the cycle.

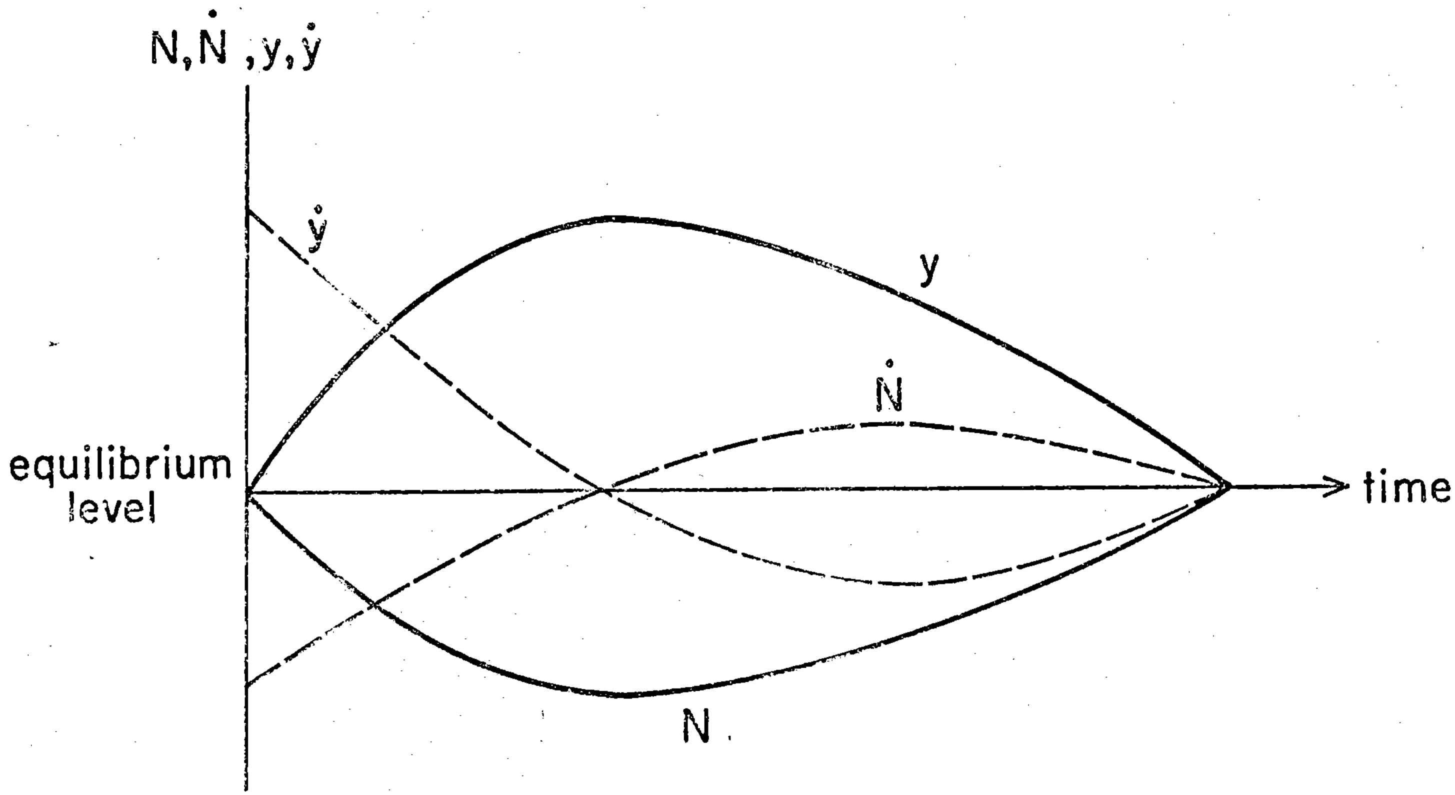

Figure 3

(5) Prices rise throughout the adjustment period, reaching a permanently higher level. The peak in the rate of inflation lags the peak in GNP.

3.4 Stability

The scenario just outlined is, of course, of interest only if the model is dynamically stable. A formal stability analysis of this system is relegated to the Appendix, where it is shown that one of the three necessary and sufficient conditions for stability is:

$$1 + \beta \frac{L_r}{m} > \frac{\beta}{X_N - Y_N} \quad . \qquad (14)$$

The righthand side of (14) is a positive number which is smaller (a) the slower the speed of adjustment of inflationary expectations, and (b) and the more negative is $Y_N - X_N$. The lefthand side is familiar from the work of CAGAN (1956). CAGAN found that his model (a full employment model where the "interest rate" variable in the demand for money was just π) would be stable if and only if:

$$1 + \beta \frac{L_r}{m} > 0 \ . \qquad (14')$$

Here I require instead (14), which is stronger than (14').[19]

Notice the fundamental role played by Y_N . Should $Y_N - X_N = (1-Dy)Y_N$ be zero or positive--a possibility raised by the production-scheduling model--the model is definitely unstable. Even if it is negative, the model will still be unstable unless $Y_N - X_N$ is large enough, where the precise meaning of "large enough" is spelled out in (14).

IV. A Model with a Non-Clearing Labor Market

The model presented in the last section includes two important features that I am unhappy about. First, the assumption that the labor market always clears in the short run means that the labor market adjusts to shocks much faster than the goods market. Second, the "Keynesian" short-run response of output to stabilization policy can occur only if the aggregate supply curve of labor slopes upward. Both assumptions are open to doubt, to say the least. But both can be avoided by assuming instead that the labor market does not clear, and instead wages adjust to the discrepancy between supply and demand for labor. In this section, I outline such a model. Since its formal analysis is quite complicated, involving four differential equations (for $\dot{N}$, $\dot{P}$, $\dot{\pi}$, and $\dot{w}$), interesting qualitative results are obtainable only if I suppress price expectations and assume that π is <u>always</u> at its steady state value of zero.

4.1 Specification

I specify a nonclearing labor market in the usual way. Actual employment is determined by the principle of voluntary exchange:

$$E = \min\,(E^d(w,N),\ E^s(w)), \tag{15}$$

where now the $E^s(w)$ function may well have zero or negligible slope. The production function is written:

$$y = f(E)\ . \tag{6'}$$

The aggregate supply function defined by (15) and (6'),

$$y = f(\min(E^d,E^s)) = y(N,w)\ , \tag{16}$$

depends on which regime we are in. Specifically:

$$y_N = E^d_N f' < 0 \quad \text{if} \quad E^d < E^s$$

$$= 0 \qquad\quad \text{if} \quad E^s < E^d\ ,$$

$$y_w = E^d_w f' < 0 \quad \text{if} \quad E^d < E^s$$

$$= E^s_w f' > 0 \quad \text{if} \quad E^s < E^d\ .$$

I also require a specification of wage dynamics, for which the following Phillips curve model seems appropriate:

$$\dot{W}/W = \pi + \gamma(E^d(w,N) - E^s(w))\ , \tag{17}$$

where W is the <u>money</u> wage and γ is a positive constant. Since I am restricting my attention to cases where π is zero, this reduces to:

$$\dot{W}/W = \gamma(E^d - E^s)\ ,$$

so that by subtracting $\dot{P}/P$ (using equation (11) with $\pi=0$) I arrive at a law of motion for the real wage:

$$\dot{w}/w = \gamma(E^d(w,N) - E^s(w)) + \Theta(N - N^*). \tag{18}$$

Along with equations (9) and (12) (for $\dot{N}$ and $\dot{m}$) of the clearing model, this constitutes the dynamics of the disequilibrium model.

The aggregate demand curve (4) is exactly the same as in the clearing model, except that expected inflation is now constrained to be zero. So the new solution function for x:

$$x = x(N, m, w; g) \tag{19}$$

is defined by:

$$x(N, m, w; g) = D(y(N,w); m, 0, g)$$

so that

$$x_N = D_y y_N$$

$$x_m = D_m$$

$$x_w = D_y \; y_w$$

$$x_g = 1 \; .$$

This completes the specification of the non-clearing version of the model.

4.2 Steady States and Stability

What can we hope to learn from such a complicated model? First consider the steady state properties, which hold also in a more elaborate version of the model in which the adaptive inflationary expectations equation is maintained. As before, (10) implies that actual and expected inflation are equal, so that (11) implies that $N=N^*$. Then (12) implies that the equilibrium inflation rate is zero, and (18) implies that the labor market clears: $E^d(w,N^*) = E^s(w)$. This equation pins down the equilibrium real wage, and hence the equilibrium values of E, y and x, and allows no effect of either policy variable. The rest of the model (the full-employment IS-LM model) determines r and P as usual. Nothing very interesting here.

Of greater interest are the short-run responses of the variables to shocks. But before enquiring into these dynamics, it is important to know what parameter configurations render the non-clearing model stable. The appendix shows that stability requires:

$$\Theta + \gamma E^d_N < 0 \; , \tag{20}$$

which turns out to be critical to the cyclical response of real wages (see below). Hereafter I assume that (20) holds. Notice once again that this is an assumption that inventory effects on production are "strong enough."

4.3 Short Run Dynamic Responses

Given an initial state of disequilibrium in the labor market, what are the effects of stabilization policy on employment and wages? The answer is obtained with the aid of Figure 4. Here $E^s(w)$ is the labor

Figure 4 about here

supply schedule, $E^d(w, N_0)$ is the labor demand schedule, and the initial real wage is assumed to be w_0--which leads to an excess supply of labor (see point B). The initial level of inventories, N_0, could be above or below the optimal level, N^*, and, depending on where we are in the cycle, $\dot{N}$ could be either rising or falling. Irrespective of this, any increase in g or M will reduce $\dot{N}(t)$ for some interval of time, thus pushing N down relative to what it otherwise would have been. This is shown in Figure 4 by an upward shift in the demand function for labor from $E^d(w,N_0)$ to $E^d(w,N_1)$ (where $N_1 < N_0$).

That the expansionary stabilization policy has two distinct effects on the rate of change of real wages can be seen from equation (18). First, a lower N raises $\dot{w}/w$ through the first term in (18). This represents a "tightening" of the labor market (see equation (17)). Second, a lower N reduces $\dot{w}/w$ through the second term in (18). This happens because smaller inventories lead to faster increases in product prices (see equation (11)).

But which effects dominates? The answer follows from stability condition (20): in a stable system, the first effect must be stronger so a reduction in inventories leads to an acceleration in real wage growth in the short run. Figure 4 shows what happens to output. In the absence of policy, wages would have fallen to some level like w_1 at time t_1, and the position of the economy would have been point C, with employment E_1. Expansionary policy pushes the labor demand curve outward and retards the fall in wages. Wages fall only to w_1', and the position of the economy at time t_1 is point D instead. The effect of policy on employment is, therefore, $E_1' - E_1$, a positive number.

Notice that this model generates an unambiguous prediction about the short-run behavior of real wages, whereas in the BARRO-GROSSMAN analysis "it all depends" on whether prices or money wages react more expeditiously to disequilibrium. How have I avoided this indeterminacy and obtained an answer that does not depend on relative adjustment speeds? The answer is that the short-run movement of w does still depend on relative adjustment speeds, but stability condition (20) places a quantitative restriction on γ and Θ that enables

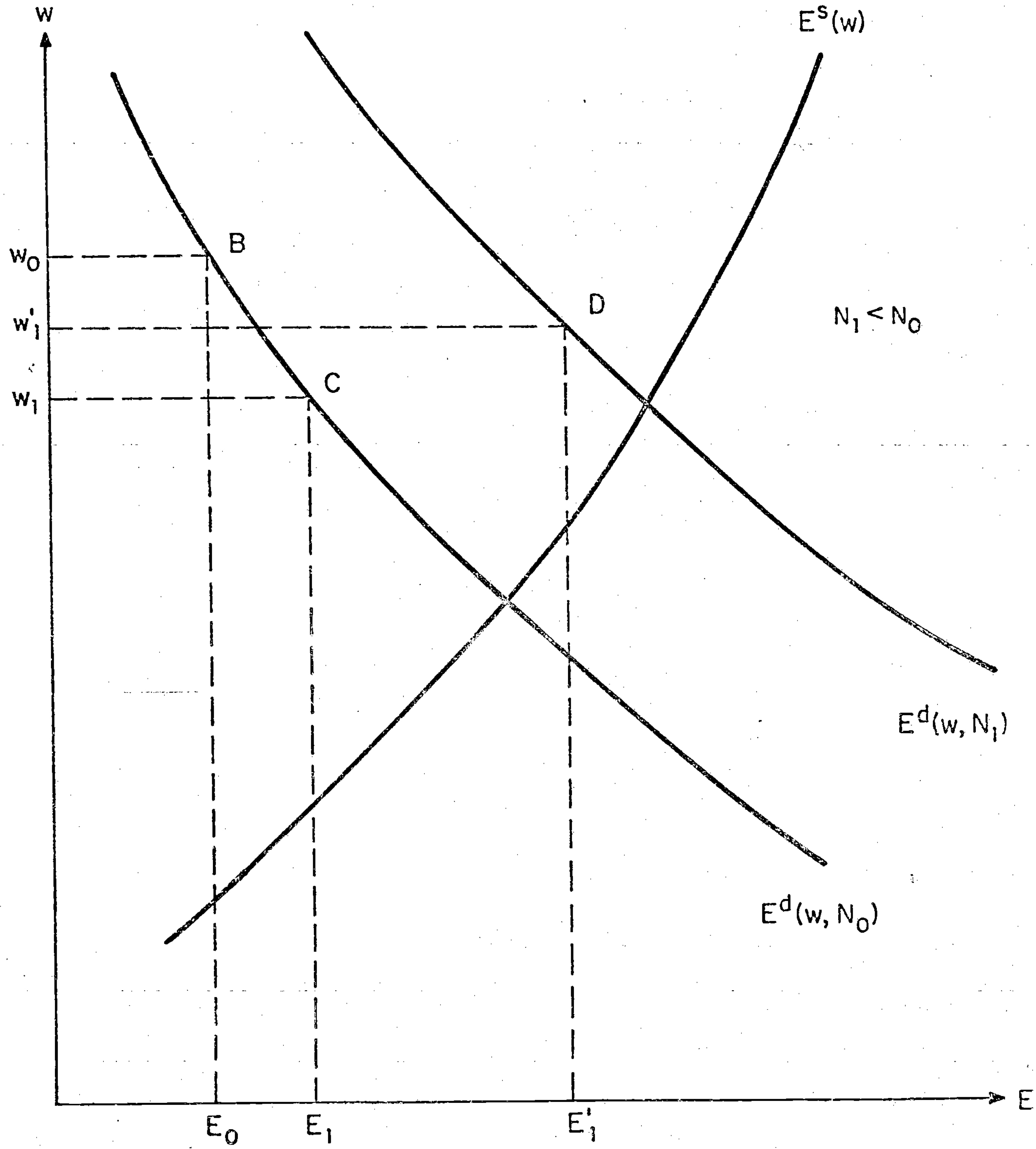

Figure 4

me to determine the sign $\dot{w}$ in the short-run.[20]

Precisely analogous arguments can be used to show that employment and wages also rise when expansionary policies are applied under conditions of excess demand or of equilibrium. In each case, a stimulus to aggregate demand leads to an interval of time in which $\dot{N}$ is more negative and $\dot{w}/w$ is more positive than it otherwise would have been. It can also be shown that output rises. Thus, just as in the clearing model, we conclude that real wages move procyclically. In addition, the present model implies a certain symmetry where BARRO and GROSSMAN found asymmetry. More demand always leads to higher real wages and higher output in the very short run, and less demand leads to lower real wages and output. However, the symmetry is only qualitative, not quantitative. Because employment is demand-determined when there is excess supply, and supply-determined when there is excess demand, the responses of w and y to policy will surely differ in the two cases. In particular, we expect a much greater output response when there is excess supply of labor than we do when there is excess demand.

V. Summary and Concluding Remarks

1. In a sense, the most basic conclusion of this paper may be that inventories really do matter in macroeconomic theory. The presence of storable output apparently can change even basic qualitative aspects of the behavior of macro models.
2. While the great variety of motives for holding inventories suggest a number of ways in which inventories might enter the macro model, many of them seem to suggest that output inventories should have a negative effect on the demand for labor (or supply of output). Input inventories remain an unexplored territory worthy of study.
3. While inventories play an important stabilizing role at the level of the firm, they tend to be destabilizing at the macro level in the sense that models with inventories are stable in a smaller subset of the parameter space than are models without inventories. This message dates back to Metzler (1941); but the mechanisms and precise stability conditions are quite different in this model than they were in Metzler's. In general, stability requires not only that inventories have a negative effect on the demand for labor, but that this effect be "large enough."

4. Because of inventory changes, short run fluctuations in aggregate demand have quicker and more dramatic effects on final sales than they do on production.
5. Real wages respond positively to positive shocks to aggregate demand, because inventory changes shift the demand curve for labor. In the case of a nonclearing labor market, this conclusion hinges upon a stability condition which again states that the inventory-induced shifts in labor demand are "large enough."

As was pointed out in the introduction, this conclusion is the reverse of that reached by standard Keynesian analysis, and also by search-theoretic models. This is because those models consider a cyclically-sensitive labor supply curve shifting along a labor demand curve, while the model developed here has a cyclically sensitive labor demand curve shifting along a fixed labor supply curve. If both curves were allowed to shift simultaneously, demand stimuli would have ambiguous effects on real wages. Which effect dominates in practice is an empirical issue.

It is probably apparent that other mechanisms that shift the demand for labor during the business cycle could be introduced.[21] But putting inventories into the labor demand function is not a contrivance designed to make real wages move procyclically. Quite the contrary, it seems to be an almost inescapable conclusion on both microeconomic and macroeconomic grounds. From the micro perspective, given any kind of imperfection in the market that allows the shadow value of inventories to depart from the market price, optimizing behavior seems to dictate that employment be a decreasing function of inventories in a wide variety of models.[22] From the macro perspective, it is hard to make sense of either the Keynesian cross or the IS curve without explicit consideration of the firm's reaction to inventory imbalances.[23]

6. Finally, the Keynesian model with inventories predicts that real output will move in the same direction as aggregate demand, regardless of whether the demand shock is administered from an initial position of equilibrium, excess supply, or excess demand. In this respect, it contrasts sharply with the implications of the BARRO-GROSSMAN model.

A. Stability Analysis in the Clearing Model

Using the solution functions given in the text for y and x, the dynamic system can be written as a system of three differential equations, the first two of which are nonlinear:

$$\dot{N} = Y(N) - X(N,m,\pi;\ g)$$

$$\dot{m} = -\Theta m(N^* - N) - \pi m$$

$$\dot{\pi} = \beta\Theta(N^* - N)\ .$$

Linearizing the nonlinear equations around equilibrium ($x=y$, $N^*=N$, $\pi = \dot{P}/P = 0$) gives the following stability matrix:

$$\Delta = \begin{pmatrix} Y_N - X_n & -X_m & -X_\pi \\ \Theta m & 0 & -m \\ -\beta\Theta & 0 & 0 \end{pmatrix} .$$

The ROUTH-HURWITZ necessary and sufficient conditions for (local) stability in this case are that:[24]

(i) $tr(\Delta) < 0$

(ii) $det(\Delta) < 0$

(iii) $-\Theta(Y_N-X_N)\ [m\ X_m - \beta\ X_\pi] - \beta\Theta m X_m > 0$.

The trace,

$$tr = Y_N - X_N = (1-D_Y)Y_N\ ,$$

is negative so long as D_Y (the marginal propensity to spend) is less than unity and Y_N is negative. The determinant is simply $-\beta\Theta m\ X_m$, which is negative so long as rising real balances stimulate demand. Only condition (iii) requires further analysis, and by using the definitions of X_m and X_π it can be expressed as equation (14) in the text.

B. Stability Analysis in the Nonclearing Model

In the nonclearing model, w replaces π as the third state variable. Also, the solution functions differ and depend on whether there is excess supply or excess demand (see the text). The dynamic system is:

$$\dot{N} = y(N,w) - x(N,m,w;g)$$
$$\dot{m} = \Theta m(N - N^*)$$
$$\dot{w} = \gamma w(E^d(w,N) - E^s(w)) + \Theta w(N - \dot{N}^*).$$

Linearizing it around equilibrium ($N=N^*$, $E^d = E^s$) gives the stability matrix:

$$\Delta^* = \begin{pmatrix} y_N - x_N & -x_m & y_w - x_w \\ \Theta_m & 0 & 0 \\ \gamma w E^d_N + \Theta w & 0 & \gamma w(E^d_w - E^s_w) \end{pmatrix}.$$

The three ROUTH-HURWITZ necessary and sufficient conditions for local stability are:

(i*) $tr(\Delta^*) = y_N - x_N + \gamma w(E^d_w - E^s_w) < 0$

(ii*) $det(\Delta^*) = \Theta m x_m \gamma w(E^d_w - E^s_w) < 0$

(iii*) $-[y_N - x_N + \gamma w(E^d_w - E^s_w)].[(y_N - x_N)\gamma w(E^d_w - E^s_w) -$

$- (y_w - x_w)(\Theta w + \gamma w\ E^d_N)] - (y_N - x_N)\ x_m\ \Theta m > 0$

The first two are clearly satisfied whether the system has excess demand or excess supply in the labor market, but (iii*) looks different in the two cases. The excess demand case is simpler since here $y_N = x_N = 0$, $y_W - x_w = (1 - D_y)y_w > 0$. The condition reduces to:

$$\gamma w(E^d_w - E^s_w)(y_w - x_w)(\Theta w + \gamma w E^d_N) > 0,$$

which is true if and only if:

(20) $\Theta + \gamma\ E^d_N < 0$,

as stated in the text.

When there is excess supply in the labor market, $y_N - x_N = (1 - D_y)$ $y_N < 0$ and $(y_w - x_w) = (1 - D_y)y_w < 0$, so a sufficient (though not necessary) condition for stability is:

$$y_N \gamma w(E^d_w - E^s_w) > y_w(\Theta + \gamma\ E^d_N).$$

But, looking back at (16), we see that $y_N E^d_w = y_w E^d_N$, so this reduces to:

$$-\gamma E^d_N\ f'\ E^s_w > \Theta f'\ E^d_w,$$

which is true if (20) holds.

1. Among the many examples that could be cited, see Samuelson (1976), pp. 222-225.
2. For example, Branson's (1979) popular text never mentions inventories once it gets past the rehash of freshman-level materials. Even Lovell (1975), himself an inventory expert, fails to give inventories any role in the elaborated IS-LM model.
3. A notable exception is Maccini (1976).
4. Barro and Grossman note this quite explicitly. See, for example (1971, p. 85n) or (1976, p. 41n).
5. The statement applies to the U.S. and other advanced industrial nations. The Barro-Grossman excess demand scenario may be applicable to centrally planned economies where consumer goods are in chronically short supply. (On this, see Howard (1976).) The preceding discussion is in the spirit of Leijonhufvud (1973).
6. This has important implications for econometric specification of macro models. The Barro-Grossman model, with its many cases, would require a complex "switching regressions" approach of the sort discussed e.g., by Goldfeld and Quandt (1976). The model that I shall present has no switches of regimes.
7. For a version of this scenario consistent with rational expectations, see Fischer (1977).
8. Alternatively, a falling supply price (e.g., quantity discounts) will give the firm an incentive to bunch its input purchases.
9. For a full discussion of when λ can or cannot differ from the market price, see Blinder (1978). Suffice it to say that some deviation from perfect markets--for example, some monopoly power--is required.
10. A notable exception is Fair's (1976) model. His equation for output (equation (10) on page 49) can be written (if I ignore lags and dummy variables):

$$y = \text{constant} + 1.2x - .236N ,$$

which certainly shows a rather strong negative effect of inventories on output.

11. See Ando and Shell (1975).
12. Given (5), it does not matter whether I put E^d or E^s into the production function.
13. The expectational mechanism is not critical to any results in this paper, and is needed only to connect nominal and real interest rates. For a model with a similar, though somewhat simpler, structure that includes explicit stochastic terms and utilizes rational expectations, see Blinder and Fischer (1979).
14. Feldstein and Auerbach (1976) have suggested that, as an empirical matter, changes in N^* proceed very sluggishly in U.S. durable manufacturing industry.
15. Had I modelled monetary policy as fixing the growth rate, $\dot{M}/M$, rather than the level, M, inflation would be possible in equilibrium. However, my choice seems the more natural one in the context of an ultimately static model. The whole model can be transformed into a growth model with relatively little difficulty.
16. This conclusion is the only one in the paper that depends on the assumption of adaptive expectations. Because of this, the <u>rate of change</u> of the rate of inflation is:

$$\frac{d}{dt}\left(\frac{\dot{P}}{P}\right) = \dot{\pi} - \Theta\dot{N}$$
$$= \beta\left(\frac{\dot{P}}{P} - \pi\right) > 0 \text{ at the point where } \dot{N} = 0.$$

17. In the model, they reach this peak in the "first instant," but if lags in the consumption and investment function were allowed, the "multiplier" would take some time.
18. Point E is where the slope of the trajectory, $\dot{x}/\dot{y}$, is equal to unity, for at this point $\dot{N} = \dot{x} - \dot{y} = 0$.
19. Assuming (14), of course, does not guarantee monotonic convergence. I depicted this case in Figure 2, but nothing of consequence hinges on it; overshooting is possible.
20. An open question is whether the effect on $\dot{w}$ could be signed by a similar stability analysis of the Barro-Grossman model.
21. For example, the stock of capital or the intensity of its utilization might affect labor demand.
22. On this, see Blinder (1978) and Blinder and Fischer (1979).
23. On this, see Blinder (1977).
24. See, for example, Gandolfo (1971), p. 241.

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