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THE INTERNATIONAL TRANSMISSION OF FISCAL EXPENDITURES AND BUDGET DEFICITS IN THE WORLD ECONOMY

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ABSTRACT

This paper analyses the effects of fiscal policies on rates of interest and wealth in the world economy. Uncertainty concerning the length of life yields an equilibrium in which private and social rates of discount differ and budget deficits exert real effects. It is shown that a current budget deficit (resulting from a tax cut) raises world rates of interest. On the other hand the direction of the effect of an expected future deficit on the short-term rate of interest depends on whether the country is having a surplus or a deficit in its current account of the balance of payments. If it runs a deficit in the current account then the short-term rate of interest rises and vice versa; the future rate of interest, however, must rise. It is also shown that budget deficits raise domestic wealth and lower foreign wealth and thus result in a negative transmission. In the long run, a higher steady-state value of government debt raises the steady-state world rate of interest but its effect on the long-run value of foreign wealth is ambiguous.

The effects of changes in government spending depend on both the timing and the patterns of spending. A transitory (balanced-budget) rise in current government spending raises the current rate of interest and lowers domestic and foreign wealth while a transitory future rise in government spending lowers the current rate of interest, lowers domestic wealth and raises foreign wealth. A permanent rise in government spending lowers the rate of interest if the current account of the balance of payments is in deficit, and vice versa. Finally, the model is generalized to a multi-commodity world and the impact of policies are shown to depend on comparison among various spending and saving propensities of private sectors and of governments.

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I. INTRODUCTION

This paper deals with the international transmission of the effects of fiscal expenditures and budget deficits. One of the major sources of recent friction between Europeans and Americans has been the interpretation of the economic implications of U.S. budget deficits. Theorists and policymakers on both sides of the Atlantic have differed in the analysis of the role of budget deficits in affecting key macroeconomic aggregates. Specifically, some have argued that large budget deficits are responsible for the recently observed high real rates of interest while others have claimed that budget deficits cannot be blamed for these real rates. The latter group claimed that theory does not predict a clear-cut relation between budget deficits and rates of interest and that the empirical record itself is very weak.

The increased integration of the world economy has resulted in increased concern in each country over policy measures taken in the rest of the world. A major topic of recent analysis concerns the impact of government spending and budget deficits on key macroeconomic aggregates. The complex pattern of the economic linkages within the interdependent world economy resulted in a variety of models of the international transmission mechanisms [for surveys see Fair (1979) and Mussa (1979)]. These models include those that highlight the implications of foreign trade multipliers (e.g., LINK models which build on a Keynesian structure) as well as those that highlight the role of the terms of trade along the lines of the elasticity approach to the balance-of-payments. In addision, some analyses have examined whether disturbances can be transmitted negatively to the rest of the world. For example, Laursen and Metzler (1950) showed that in a model without capital flows, domestic autonomous government expenditures which raise domestic output, lower the level of ouput abroad; i.e. domestic spendings are transmitted negatively to the rest of the world. Parallel to these developments there were examinations of the transmission mechanism along the lines of the Mundell-Fleming models [see Mundell (1968) and Fleming (1962)], where it was shown that an expansionary monetary policy is transmitted negatively to the rest of the world whereas fiscal policy is transmitted positively. More recently modelling of exchange-rate dynamics focused on the role of expectations and relied on the real rate of interest as a key factor in the transmission mechanism [see Dornbusch (1976) and Frenkel and Rodriguez (1982)].

The question of the effects of budget deficits on rates of interest, on consumption, and on the nature of the real equilibrium, has been the subject of analysis in both closed and open economy contexts. Much of the traditional analysis demonstrating that budget deficits exert real effects have built on the presumption that government bonds constitute net wealth to individual asset-holders. The key issue of debate has been whether asset-holders take full account of the prospective tax liabilities that are associated with the need to service the debt created by current deficits. Recent research on the effects of fiscal policies has embodied the Ricardian proposition according to which individuals take full account of future tax liabilities and, therefore, as long as government spending remains unchanged, the path of taxes and thereby the path of budget deficits do not affect the real equilibrium of the system. [For expositions see Bailey (1962) and Barro (1974)].

As is well known there are several ways by which a "fully rational" model can incorporate the feature that budget deficits influence the nature of the real equilibrium. For example, models which allow for distortionary taxes, [e.g. Barro (1979), Kydland and Prescott (1980), Lucas and Stokey (1983), and Razin and Svensson (1983), price rigidities or incomplete markets for intergenerational risk sharing [e.g. Stiglitz (1983)], capital controls [e.g. Greenwood and Kimbrough (1984)], or individuals with finite horizons, [e.g. Diamond (1965) and Samuelson (1958)], will all yield the result that fiscal deficits matter.

In this paper we develop a model that is suitable for the analysis of (i) the linkage between fiscal policies and rates of interest, and (ii) the international transmission mechanism of fiscal policies. The key characteristics of our model are: (i) a fully integrated world capital market; (ii) full rationality of all economic agents whose decisions are based on self-fulfilling expectations and are subjected to temporal and intertemporal budget constraints, and (iii) government behavior that is constrained by an intertemporal solvency requirement. The model is of a general equilibrium nature, and the various economies have access to, and are governed by, world markets. These markets determine both temporal prices (commodity terms of trade) and intertemporal prices (rates of interest).

In order to deal with the role of budget deficits in effecting world rates of interest, consumption and international indebtedness, and to study the resulting nature of the international transmission mechanism, our model includes individuals with the possibility of finite horizons. We adopt the formulation developed in Blanchard (1984) according to which the finiteness of the horizon is introduced through the assumption that at each point in time, individuals, who are assumed to have no bequest motive, face a given probability of death. The uncertainty concerning the length of life results in an equilibrium in which private and social rates of discount differ. This difference between discount rates (which is responsible for the departure from the Ricardian Proposition on the irrelevancy of budget deficits), permits a meaninful analysis of the impact of budget deficits.

Section II presents the key features of the model and section III characterizes the properties of equilibrium in the two-country world economy. In section IV we analyse the effects of a current budget deficit that is brought about through a reduction in the home-country's taxes. It is shown that a current budget deficit raises the rate of interest. In addition, the budget deficit raises the value domestic wealth and lowers the value of foreign wealth. Thus, the deficit is transmitted negatively to the rest of the world. Similar qualitative results apply to the analysis of the effects of expected future budget deficits. As shown in section V, an expected future deficit also raises domestic wealth and lowers foreign wealth but its impact on the current period's rate of interest is ambiguous as it depends on whether the country introducing the budgetary deficit runs a surplus or a deficit in the current account of its balance of payments. In section VI we apply the model to an analysis of the steady-state consequences of budget deficits.

In sections VII and VIII we turn to examine the impact of changes in the level of government spending. In order to separate the analysis of the level of fiscal spending from the analysis of budget deficits, we focus on the implications of a balanced-budget change in government spending. It is shown that a transitory rise in government spending raises interest rates and lowers domestic and foreign wealth. On the other hand, an expected future rise in government spending lowers interest rates and reduces the value of domestic wealth and consumption, while it raises the value of foreign wealth and consumption. We then analyse the effect of a permanent rise in government spending. It is shown that its impact on the rate of interest depends on whether the domestic economy is a net saver or dissaver in the world economy or, equivalently, whether it has a current account surplus or deficit. If the home country runs a current account surplus then a rise in government spending raises world interest rates and lowers domestic and foreign wealth and consumption. On the other hand if the home country runs a current account deficit then a permanent balanced-budget rise in government spending lowers interest rates and domestic wealth and raises foreign wealth. We thus, show the critical importance of specifying the details of fiscal policy in terms of the level and timing of government spending and taxes.

In sections IX and X we generalize the model to a multi-commodity world. The extended model is then applied to examine the effects of government spending on the intertemporal terms of trade (as measured by the rates of interest) and on the temporal terms of trade (as measured by the relative price of importables in terms of exportables). We show that the impact of government spending as well as the dynamics of debt accumulation depend on a comparison among (i) domestic and foreign patterns of private sector spending, (ii) domestic and foreign private sectors' saving propensities, and (iii) domestic and foreign patterns of government spending. These comparisons generate a multitude of "transfer problem criteria" that are familiar from the theory of transfers in international trade [see Mussa (1969)]. Finally, section XI contains concluding remarks followed by six Appendices that provide technical material underlying some of the statements made in the text.

II. The Model

The model has two economies consisting of consumers and governments. The path of output in each economy is assumed given. Individuals are assumed to face a given probability of death which is independent of age. We denote the probability of survival between two consecutive periods by γ. Because of uncertain lifetime all loans require in addition to regular interest payments a purchase of life insurance as in Yarri (1965) and Blanchard (1984). In case of death, the estate is transfered to the life insurance company which, in turn, guarantees to cover outstanding debts. It is assumed that there are a large number of individuals and, therefore, competition among insurance companies implies that the percentage insurance premium equals the probability of death. The present-value factor, which is composed of one-period rates of interest compounded from period zero up to period t, is denoted by α_t and, therefore, α_{t-1}/α_t equals one plus the market rate of interest in period t-1. Analogously, γ^{t-1}/γ^t equals one plus the life-insurance premium. It follows that the <u>effective</u> interest rate faced by individuals is $(\gamma^{t-1}/\gamma^t)(\alpha_{t-1}/\alpha_t) - 1$. This is the effective cost of borrowing relevant for individual decision making.

Under the assumptions that the utility function is logarithmic and that the subjective discount factor, δ, is constant, it can be shown[1] that the aggregate consumption function (which is derived from individuals' maximization of expected utility) is

$$c_t = (1-\gamma\delta)W_t \tag{1}$$

where W_t denotes aggregate wealth. Aggregate wealth in turn equals the difference between human wealth, H_t, and private debt B_{pt}. The value of human wealth is the discounted sum of disposable income, y_t, which is computed by using the <u>effective</u> rates of interest.

Government spending can be financed by taxes or by debt issue. The requirement that over time government spending must obey the intertemporal solvency constraint can be expressed as

$$\sum_{v=0}^{\infty} \alpha_v (T_v - G_v) = B_{go} \tag{2}$$

where T_v, G_v denote taxes and government spending in period v and where B_{go} denotes the value of government debt at t=0. Equation (2) states that the value of government debt must equal the sums of the present values of current and future budget surpluses. The sum of Private debt, B_p, and government debt, B_g, equals the value of the economy's external debt, B.

The behavioral equations for the foreign economy can be derived in a similar way and, in what follows, we denote variables pertaining to the foreign economy by an asterisk(*).

III. World Equilibrium

In this section we analyse the determination of the equilibrium path of world rates of interest in the two-country world economy. We will assume that world capital markets are fully integrated and, therefore, that individuals and governments in both countries face the same market rates of interest. This feature provides for the key channel through which policies undertaken in one country impact on economic conditions in the rest of the world. The structure of the model embodies the assumptions that the behavior of individuals is rational, that it is based upon self-fullfilling expectations, and that governments and individuals are constrained by intertemporal budget constraints. These assumptions imply that economic policies have an impact on the entire path of interest rates and, thereby, on the paths of the key economic variables relevant for current and future generations in both countries.

World equilibrium requires that in each period the given supply of world output equals the demand. To facilitate the exposition we start by dividing the horizon into two periods: the present, which is denoted by $t=0$, and the future $(t=1,2,...)$, and we suppose that outputs, government spending and taxes do not vary across future periods $(t=1,2,...)$. This procedure of time-aggregation is modified in section V below. In aggregating the future into a composite single period we need to compute the present values of the various flows. For that purpose we define an average-interest factor

$$R = \frac{1}{1+r}$$

where r denotes the rate of interest. This average-interest factor represents the entire path of the rates of interest that actually do change over time. For further reference R may be termed a "constancy equivalent" interest factor.

The equality between world demands and world output in period $t=0$ is written as

$$(3) \qquad (1-\gamma\delta)W_0 + (1-\gamma\delta^*)W_0^* = (Y_0-G_0) + (Y_0^*-G_0^*) \quad .$$

where the left-hand side of equation (3) describes the sum of domestic and foreign private sectors' consumption and the right-hand side describes the sum of domestic and foreign outputs $(Y_0$ and $Y_0^*)$ net of governments' spending.[2] For expositional simplicity we have also assumed in equation (3) that $\gamma=\gamma^*$. Two additional equilibrium conditions specify the initial values of domestic and foreign wealth:

$$(4) \qquad W_o = (Y_o - T_o) + \frac{\gamma R}{1-\gamma R}(Y - T) + B_{go} - B_o \quad .$$

$$(5) \qquad W_o^* = (Y_o^* - T_o^*) + \frac{\gamma R}{1-\gamma R}(Y^* - T^*) + B_{go}^* + B_o \quad .$$

In equations (4)-(5) the term $\gamma R/(1-\gamma R)$ denotes the present value of an annuity evaluated by using the effective constancy equivalent interest factor. Equation (5) embodies the requirement that the sum of the balance of external indebtedness of both countries must be zero and, therefore, the foreign country's external debt is $-B_o$. It is noteworthy that by Walras' Law we have excluded from the equilibrium conditions the equation stating that the sum of the present values of world outputs net of government spending in the future equals the corresponding sum of world demand.

The system of equations (3)-(5) can be solved for the values of W_o and W_o^* and R for any given value of the parameters. The solutions obtained for the initial values of wealth W_o and W_o^*, are the same as those which may be obtained from the complete system for which the values of the rates of interest within the future may vary. The use of the constancy-equivalent interest factor simplifies the analysis considerably, and it provides complete information about the impact of policies on the precise current values of all key variables including wealth, consumption, and debt accumulation, as well as on the average value of the rate of interest.

IV. The Impact of Budget Deficits

In this section we analyse the impacts of budget deficits on world rates of interest and on the levels of domestic and foreign wealth and consumption. The analysis of the effects of budget deficits on the levels of foreign wealth and consumption serves to clarify the nature of the international transmission mechanism of fiscal policies. In order to focus on the impact of deficits rather than the impact of government spending, we assume that changes in budget deficits are brought about through changes in taxes for a given path of government spending. Since government spending remains unchanged, solvency requires that current changes in taxes be accompanied by offsetting changes in future taxes. The present values of these tax changes must be equal to each other. Thus, starting with an initial balanced budget, a change in current taxes, dT_o, must be related to the future change, dT, according to

$$(6) \qquad dT_o = -\frac{R}{1-R}dT \quad .$$

In equation (6) the term $R/(1-R)$ equals $1/r$, which is the annuity value of a unit tax change.

Throughout the analysis in this section we assume that the foreign government follows a balanced-budget policy. This assumption ensures that changes in world rates of interest which result from domestic fiscal deficits, do not impact on the solvency of the foreign government and, therefore, do not necessitate secondary changes in foreign fiscal policies.

Using equations (3)-(5) the impacts of changes in domestic taxes on R, W_o, and W_o^* (evaluated around an initial balanced budget) are:

$$(7) \qquad \frac{dR}{dT_o} = \frac{(1-\gamma)}{\Delta} \frac{(1-R)(1-\gamma\delta)}{R \quad (1-\gamma R)} > 0$$

$$(8) \qquad \frac{dW_o}{dT_o} = -\frac{(1-\gamma)}{\Delta} \frac{(1-\gamma\delta^*)}{R(1-R)(1-\gamma R)} [\theta(Y-G) + \omega (Y^* - G^*)] < 0$$

$$(9) \qquad \frac{dW_o^*}{dT_o} = \frac{(1-\gamma)}{\Delta} \frac{(1-\gamma\delta)}{R(1-R)(1-\gamma R)} [\theta(Y-G) + \omega(Y^* - G^*)] > 0$$

where, as shown in Appendix II, Δ, θ, and ω are positive.

Equation (7) shows that a budget deficit arising from a reduction in domestic taxes lowers R thus and raises the value of the world interest rate. The economic interpretation of this result is as follows. From equation (4) we note that, at the prevailing rate of interest, the fall in current taxes (which is accompanied by a corresponding rise in future taxes according to equation (6)), raises the value of domestic wealth by $(1-\gamma)(1-R)/R(1-\gamma R)$ and, thereby, raises spending by the value of the marginal propensity to spend, $1-\gamma\delta$, times the change in wealth. The resulting excess demand for current goods raises their relative price in terms of future goods, and thus raises the world interest rate. As shown in Appendix II, $(1/\Delta)$ serves to translate excess demands for current goods into equilibrating changes in the rate of interest.

Equations (8)-(9) show that the domestic budget deficit raises the equilibrium value of domestic wealth, W_o, and lowers the corresponding value of foreign wealth, W_o^*. Thus, domestic budget deficits are transmitted negatively to the rest of the world. The international transmission mechanism is effected through the rate of interest. The rise in the world interest rate lowers the value of foreign wealth and mitigates the initial rise in the value of domestic wealth. These changes in wealth raise domestic spending, lower foreign spending and worsen the domestic current account of the balance of payments.

As may be seen from equations (7)-(9), when the probability of survival, γ, is unity budget deficits do not alter interest rates and wealth. In that case the model yields the familiar Ricardian Proposition according to which the timing of taxes and thereby the timing of deficits do not influence the real equilibrium of the system as long as the path of government spending remains intact. In the general case, however, with $\gamma<1$, budget deficits exert real effects.

The explanation for these results can be given as follows. If the probability of survival, γ, is unity, then the rise in future taxes which is equal in present value to the reduction in current taxes leaves wealth unchanged. On the other hand, the same change in the pattern of taxes raises wealth if each individual knows that there is a positive probability that he will not survive to pay these higher future taxes. Under such circumstances, therefore, the current reduction in taxes constitutes net wealth. Equivalently, the explanation can be stated in terms of the difference between the market and the effective interest factors. While the government solvency requirement implies that changes in current taxes must be made up for by $R/(1-R)$ times the offsetting change in future taxes, individuals discount these future taxes by $\gamma R/(1-\gamma R)$. Therefore, as long as $\gamma<1$, the current budget deficit raises wealth. Yet another interpretation may be given in terms of a "transfer problem criterion" familiar from the theory of international transfers. Accordingly, the budget deficit exerts real effects because it redistributes wealth from those who have not yet been born,and whose marginal propensity to consume current goods is obviously zero, to those who are currently alive, and whose marginal propensity to consume current goods is positive. The clear presumption concerning the results of this redistribution is analogous to the presumption concerning the effects of an international transfer on relative prices in the presence of non-traded goods.

V. The Impact of Future Budget Deficits

The previous analysis of the impact of budget deficits was conducted within a framework which consolidated the entire future into a single period. Consequently, in order to ensure solvency, a current budget deficit had to imply a corresponding rise in future taxes. This procedure of time-aggregation did not permit an analysis of the impact of future budget deficits which are not accompanied by current changes in taxes. In order to examine the effects of future deficits we modify in this section the aggregation of the periods. Specifically, we divide the future into two parts -- the near future and the distant future. As before, the distant future is represented by a consolidation of the entire period subsequent to that of the near future into a single period. In what follows a subscript 0 designates variables pertaining to the present period, a subscript 1 designates variables pertaining to the near future and, finally, variables which appear without a time subscript pertain to the distant future.

Consider an expected change in taxes occurring in the near future. Without changes in present taxes government solvency requires that such a change must be accompanied by further offsetting changes in the distant future. Thus, solvency requires that

$$dT_1 = -\frac{R_1}{1-R_1}\, dT \qquad (6')$$

where R_1 is constancy-equivalent interest factor in the distant future; i.e., it is the average one-period interest factor linking the near future with the distant future. In order to compute the effects of such future policies we note that the equilibrium system consists of the following requirements: (i) world output in the present period must be demanded (analogously to equation (3)), (ii) world output in the near future must be demanded, (iii) the equilibrium values of domestic and foreign wealth consist of the sums of the present values of disposable incomes in the present, the near future and the distant future, adjusted for the initial debt (analogously to equations (4)-(5)). This equilibrium system is described in Appendix III.

The impacts of the near-future budget deficit on domestic and foreign wealth are:

$$(10) \qquad \frac{dW_o}{dT_1} = -(1-\gamma)\frac{(Y_1-G_1+Y_1^*-G_1^*)}{\gamma(1-\gamma\delta)J} < 0$$

$$(11) \qquad \frac{dW_o^*}{dT_1} = (1-\gamma)\frac{(Y_1-G_1+Y_1^*-G_1^*)}{\gamma(1-\gamma\delta^*)J} > 0 \quad ,$$

where

$$J = \frac{(Y-G)(Y_1-G_1+Y_1^*-G_1^*)(1-\gamma R_1)}{\gamma_2 R_1 R_o}\left(\frac{1}{(1-\gamma\delta)(Y-G)} + \frac{1}{(1-\gamma\delta^*)(Y^*-G^*)}\right) > 0 \ .$$

As seen, analogously to the analysis of the effects of current budget deficits, future deficits (that are brought about through a reduction in taxes) also raise the equilibrium value of domestic wealth, W_o , and lower the corresponding value of foreign wealth W_o^*. Thus, domestic <u>future</u> budget deficits are also transmitted <u>negatively</u> to the rest of the world. As before, these changes in wealth do not occur when $\gamma=1$.

The impact of the future budget deficit on the current and future one-period interest factors, R_o and R_1 , respectively, are:

$$(12) \qquad \frac{dR_o}{dT_1} = \frac{(1-\gamma)(\delta^*-\delta)}{J}$$

$$(13) \qquad \frac{dR_1}{dT_1} = \frac{(1-\gamma)(1-\gamma R_1)(Y-G)[(1-\gamma R_1)(Y-G) + \nu(Y^*-G^*)]}{R_1R_0\gamma^3(1-\gamma\delta^*)(Y^*-G^*)J} > 0$$

where

$$\nu = 1-\gamma R_1 + (\delta-\delta^*)\gamma^2(1-\gamma\delta^*) > 0 \quad .$$

Equation (12) shows that the effect of a future budget deficit on the current one-period rate of interest depends on the relation between the domestic and the foreign saving propensities and, therefore, one may not argue that a future budget deficit per se raises short term interest rates. If $\delta < \delta^*$ the future budget deficit lowers R_0 and raises the current one-period rate of interest and conversely if $\delta > \delta^*$. Equation (13) shows that a future budget deficit raises the future rate of interest.

The interpretation of these results is as follows. The future budget deficit creates an excess demand for goods in the near future (i.e., in the period in which the deficit occurs). In order to eliminate this excess demand the future rate of interest must rise according to equation (13). In interpreting the impact of the future deficit on the current short-term interest rate, we note that in the present period no government action takes place and, therefore, the change in the current rate of interest results only from changes in world savings which, in turn, stem from changes in domestic and foreign wealth. At the prevailing short term interest rate foreign wealth falls because of the rise in the future rate of interest while the rise in domestic wealth consequent on the future budget deficit is mitigated by the rise in the future rate of interest. These changes in wealth lower the foreign demand for current goods and raise the domestic demand for these goods. World demand for current goods rises or falls depending on the difference between the two spending propensities. Therefore, in order to restore world equilibrium with zero savings the current one-period interest rate must fall if $\delta > \delta^*$ and must rise if $\delta < \delta^*$. Again, when $\gamma=1$ the future deficit does not alter the rate of interest.

Our analysis of the effects of budget deficits on the values of domestic and foreign wealth has demonstrated that the international transmission of the deficit must always be negative independent of whether the budget deficit occurs in the present or in the future. On the other hand we have shown that the impact of a future budget deficit on the current short-term rate of interest depends on whether the country which introduces the budgetary changes runs a surplus or a deficit in its international current account, i.e., if its saving propensity exceeds or falls short of the foreign saving propensity. Since it was shown in equations (10)-(11) that the directions of the changes in wealth do not depend on the direction of the changes in the current short-term rates of interest and since the link between foreign wealth and domestic budget deficits operates only through world capital markets, it follows that a future deficit must raise the overall "appropriate average" of short and long term rates of interest. In the special case with $\delta = \delta^*$ a future budget deficit does not alter the short term interest rate and is reflected only in higher long-term rates; it thus steepens the yield curve. Again, the Ricardian Proposition reemerges and these changes vanish when $\gamma = 1$.

VI. Long-Run Equilibrium

In this section we analyse the long run effects of budget deficits on the steady state levels of domestic and foreign wealth and on world rates of interest.

The steady-state value of domestic wealth is[3]

$$W = (1-\gamma)\frac{R(Y-T)}{(1-\gamma R)(R-\gamma\delta)} \quad . \tag{14}$$

Analogously, the steady-state value of foreign wealth is:

$$W^* = (1-\gamma^*)\frac{R(Y^*-T^*)}{(1-\gamma R)(R-\gamma\delta^*)} \quad . \tag{15}$$

World equilibrium requires that the sum of domestic and foreign consumption equals the level of world output net of governments' spending. Using the steady-state value of wealth in the consumption function (1) along with the analogous expressions for the foreign country yields equation (16) as the condition for world equilibrium:

(16) $$(1-\gamma\delta)\frac{(1-\gamma)(Y-T)R}{(1-\gamma R)(R-\gamma\delta)} + (1-\gamma^*\delta^*)\frac{(1-\gamma^*)(Y^*-T^*)R}{(1-\gamma^* R)(R-\gamma^*\delta^*)} = Q + Q^*$$

where $Q = Y-G$ and $Q^* = Y^*-G^*$ denote the levels of domestic and foreign outputs net of government absorption. Equilibrium also requires that both governments be solvent, and thus that in each country the value of government debt equal the sum of the present values of current and future budget surpluses[4]

(17) $$(T-G) = (1-R)B_g$$

(18) $$(T^* - G^*) = (1-R)B_g^* \quad .$$

For given values of government debt, B_g and B_g^* (which reflect past budget deficits) and for given values of government spending, G and G^*, the system (16)-(18) can be used to obtain the solutions for the equilibrium steady-state values of world interest factor, R, and of domestic and foreign taxes, T and T^*. Positive steady-state consumption in both countries requires that the equilibrium value of R must be larger than $\gamma\delta$ <u>and</u> $\gamma^*\delta^*$. For exogenously given values of the parameters the existence of such a steady-state is not guaranteed. If this steady-state does not exist, then when $\gamma\delta > \gamma^*\delta^*$ the wealth of the foreign country approaches zero while the home country saves until it owns the entire wealth of the world, and vice versa. Finally, it is relevant to note that when $\gamma=\gamma^*=1$ steady-state equilibrium with positive levels of consumption in both countries requires that $\delta=\delta^*$. In that case the equilibrium value of R equals the common value of the discount factors.

We now examine the effects of past cumulative budget deficits on the long run values of the rate of interest and wealth. A higher cumulative value of past budget deficits in the home country is reflected in a higher steady-state value of government debt, B_g. By differentiating equations (16)-(18) it is shown in Appendix III that

(19) $$\frac{dR}{dB_g} < 0 \quad .$$

Thus, a rise in the steady-state value of domestic government debt raises the steady-state rate of interest (i.e., it lowers the interest factor). It follows therefore that past (cumulative) budget deficits also tend to raise world rates of interest.

The effect of a rise in the steady-state value of domestic government debt on the steady-state value of foreign wealth is specified in equation (20):

$$\text{(20)} \qquad \operatorname{sign} \frac{dW^*}{dB_g} = \operatorname{sign}[\delta^*(1-\gamma R) - R^2(R-\gamma\delta^*)] \quad .$$

Thus the direction of the transmission of past domestic budget deficits on the steady-state value of foreign wealth may be positive or negative depending on the relative magnitudes of the parameters. Specifically, if R falls short of δ^* (but still exceeds $\gamma^*\delta^*$ so that the steady state exists), higher cumulative domestic budget deficits raise the steady-state value of foreign wealth. In view of equation (20) it is of interest to note that even though current and future budget deficits must lower the current value of foreign wealth (as shown in sections IV and V), the long run effects of the cumulative past deficits on the (steady-state) value of foreign wealth are still ambiguous. Finally, we also show in Appendix III that the effect of a higher value of B_g on the domestic value of steady-state wealth contains, in addition to the interest-rate effect discussed above, also a direct negative effect associated with the rise in steady-state taxes necessary to service the debt.[5]

VII. Government Spending, Interest rates and Transmission

The previous sections analysed the effects of budget deficits on the equilibrium of the system. In that analysis changes in the deficit were brought about through changes in taxes while holding government spending intact. In the present section we analyse the effects of changes in the level of government spending. In order to focus on the level of fiscal spending rather than on the consequent changes in deficits, we suppose that all changes in government spending are accompanied by corresponding changes in taxes so as to yield balanced budgets. In what follows we return to the time-aggregate of sections III and IV where we divided the periods into two the present and the consolidated future. We also distinguish between transitory and permanent changes. Transitory changes alter only current spending by dG_o whereas permanent changes also alter future spending by dG.

Using equations (3)-(5) with the aid of Appendix II, the impact of a transitory balanced-budget change in domestic fiscal spending on the interest factor is:

$$\frac{dR}{dG_o} = - \frac{\gamma\delta(1-R)}{\Delta \quad R} < 0 \qquad . \tag{21}$$

Equation (21) shows that a <u>transitory</u> rise in government spending lowers R and, therefore, raises the interest rate. This change in the interest rate is necessary in order to eliminate the excess demand for current goods consequent on the rise in government spending. This balanced-budget rise in government spending results in and excess demand for current goods because the rise in taxes lowers wealth by an amount equal to the rise in government spending and, since the private sector's marginal spending propensity is smaller than unity, private expenditure is reduced by only a fraction of the rise in government spending.

The rise in the rate of interest lowers both domestic and foreign wealth and, thereby, results in a reduction of domestic and foreign consumption. Thus, the current transitory balanced-budget rise in domestic fiscal spending is paid for in part by the foreign country.

The same argument applies to the analysis of the effect of an expected future change in government spending. Formally, using equations (3)-(5) we obtain

$$\frac{dR}{dG} = \frac{\gamma(1-R)(1-\gamma\delta)}{\Delta \quad (1-\gamma R)} > 0 \qquad . \tag{22}$$

As seen in equation (22) a <u>future</u> balanced-budget rise in government spending raises R (namely, it lowers the rate of interest). The fall in the rate of interest is necessary since, at the prevailing rate of interest, the expected rise in future taxes lowers wealth and reduces consumption demand and, thereby, results in an excess supply of current output. This excess supply is eliminated by the fall in the rate of interest which raises foreign wealth and which mitigates the initial fall in domestic wealth.

The results in equations (21)-(22) provide the ingredients necessary for the analysis of the effects of a permanent balanced-budget rise in government spending. In that case $dG_0 = dG = d\bar{G}$, and the impact of this fiscal expansion on the rate of interest can be obtained by adding the expressions in equations (21)-(22). It follows, therefore, that

$$(23) \qquad \frac{dR}{d\bar{G}} = \frac{\gamma(1-R)}{\Delta R(1-\gamma R)}\,(R-\delta) \quad .$$

Thus, the effects of a permanent rise in government spending on the rate of interest depends on whether the interest factor, R , exceeds or falls short of the discount factor, δ . The interpretation of this result can be given in terms of the effect of fiscal spending on the excess demand for current goods or, equivalently, in terms of its effect on world savings. As a result of the permanent rise in fiscal spending, at the prevailing interest rate, both the supply of output net of government absorption, as well as the demand for consumption, are reduced by the same proportion. Therefore, the excess supply of current goods or, alternatively, the level of savings, also changes by the same proportion. If the initial level of domestic savings was negative, then the rise in fiscal spending raises the value of savings (by making it less negative) and,thereby, induces a fall in the relative price of present goods, that is, a fall in the rate of interest. On the other hand, if the initial value of domestic savings was positive, the permanent rise in government spending lowers domestic savings and results in a higher rate of interest. These results are analogous to those obtained for the effect of a future budget deficit on the current one-period rate of interest. The link between this explanation and the expression in equation (23) is completed by noting that if $R > \delta$ the initial value of the home country's savings is negative and the economy runs a current account deficit, whereas the opposite holds if $R < \delta$.

The change in the rate of interest alters the values of domestic and foreign wealth and consumption. If the rate of interest rises, then foreign wealth falls and if the rate of interest falls foreign wealth rises. As is obvious, the permanent rise in domestic fiscal spending lowers domestic wealth; the fall in wealth occurring at the initial rate of interest is reinforced or mitigated depending on whether the equilibrium rate of interest rises or falls.

VIII. Government Spending and Interest Rates: A Further Elaboration

The previous analysis employed the constancy-equivalent interest factor in order to collapse the entire future into a single period. This procedure permitted a complete specification of the impact of policies on the current values of the key variables. However, it did not permit a detailed examination of the impact of policies on the paths of the various variables *within* the consolidated future. Since the central channel of the transmission mechanism operates through the rates of interest, it is instructive to analyse in further detail the impact of government spending on the entire path of these rates. For that purpose we need to return to the complete disaggregated system. Since the qualitative results in equations (21)-(23) of the impact of government spending do not depend on whether γ is smaller than or equal to unity, we assume henceforth that $\gamma = \gamma^* = 1$. The complete system is described in Appendix IV, where the precise solution for the entire path of the present value factors, α, is given in equation (IV-6). We turn to an analysis of the role of the timing of government spending on the path of the rates of interest and we distinguish between transitory and permanent changes in fiscal spending.[6] Suppose that a *transitory* change is expected to occur in period s the *future*. The impact of a future transitory change in government spending on the path of the present-value factors is obtained by differenting equation (IV-6) with respect to Q_s for $t = s \neq 0$ and for $t \neq s \neq 0$. These changes in the present-value factors can be translated into corresponding changes in rates of interest by recalling that $\alpha_t = [(1+r_o)(1+r_1)\dots(1+r_t)]^{-1}$. Specifically, when the paths of outputs are stationary the effect of a transitory reduction in future government spending on the rates of interest prevailing in the adjacent period $s - 1$, i.e., on r_{s-1} (where $r_{s-1} = (\alpha_{s-1}/\alpha_s - 1)$) is

$$(24)\quad \frac{dr_{s-1}}{dQ_s} = \frac{(1-\delta^*)\alpha_{s-1}}{\bar{Q}\,\alpha_s\Delta_1}\left[\frac{\alpha_s}{\alpha_{s-1}}\lambda\,(\delta^{s-1}-\delta^{*s-1}) + \frac{1-\delta^{*s}}{1-\delta^*} + \lambda\frac{\delta^{*s}}{1-\delta^*} + \lambda^*\frac{\delta^*}{1-\delta}\right] > 0 .$$

An analogous expression pertains to dr_s/dQ_s.[7] The effect of the transitory future reduction in government spending on interest rates prevailing in all other periods $t-1$ (excluding $t=1$, $t=s$, $t=s+1$) depends on the relation between the saving propensities. Tnus,

$$\text{(25)} \qquad \frac{dr_{t-1}}{dQ_s} = \frac{\alpha_s \lambda (1-\delta^*)}{\bar{Q}\, \alpha_t^2 \Delta_1} \delta^{t-1} \delta^{*t-1} (\delta^{*t-1} (\delta^* - \delta) \quad .$$

The results reported in equations (24)-(25) specify the effects of future changes in government spending on the various rates of interest. Figure 1 illustrates the effect of a transitory future reduction in government spending occurring in period $s=4$ on the pattern of current and future short (one-period) interest rates for the case $\delta > \delta^*$. In that figure the solid curve (for which $\lambda = \lambda^* = 0.5$) describes the initial pattern of interest rates, and the dashed curve describes the pattern corresponding to a situation with an expected future transitory fall in fiscal spending. As is seen, the expected transitory reduction in future government spending tilts the entire term structure of interest rates.

We turn now to examine the effect of a <u>permanent</u> fall in domestic fiscal spending (i.e., a permanent rise in net output) on the rates of interest. This effect is best illustrated for the case in which outputs are stationary. Differentiating equation (IV-6) yields:

$$\text{(26)} \qquad \frac{dr_{t-1}}{dQ} = \frac{\lambda^* \delta^{*t-1} \delta^t (1-\delta)(1-\delta^*)}{\alpha_t^2 \bar{Q}} (\delta^* - \delta) \quad .$$

In terms of Figure 1, the solid schedules illustrate the effect of a permanent change in government spending (as parameterized by the values of λ and λ^*) for the case in which $\delta > \delta^*$. As seen, with $\delta > \delta^*$, a permant fall in domestic government spending (i.e. a rise in λ and a fall in λ^*), lowers the entire structure of interest rates. This qualitative result is reversed if $\delta < \delta^*$.

The interpretation of this result can be given in terms of the effect of the change in government spending on world savings at the prevailing rates of interest. The rise in net domestic output changes current domestic savings by $[dQ_0 - (1-\delta)dW_0]$. In the stationary case, with the prevailing rates of interest, the percentage change in domestic output, $dQ_0/Q_0 = \mu$, equals the percentage change in domestic wealth dW_0/W_0. In that case the incipient change in domestic savings is $\mu[Q_0 - (1-\delta)W_0]$, where the term in the brackets measures the initial value of domestic savings. Clearly, at the prevailing rates of interest, foreign savings do not change. As is evident, the initial value of domestic savings is positive if the domestic marginal propensity to save exceeds the foreign propensity, i.e., if $\delta > \delta^*$ (in that case equilibrium requires that initially foreign savings were negative). Conversely, if $\delta < \delta^*$, domestic savings were negative. Thus, the permanent fall in domestic fiscal spending (the rise in Q), raises world savings and induces a fall in the rates of interest if $\delta > \delta^*$, and vice versa.

IX. The Model With Variable Terms of Trade

The analysis in the previous sections was confined to a world with a single composite commodity. In that world the only relevant relative price was that of consumption in different periods, i.e., the rate of interest. In this section we extend the model to allow for two different commodities. Therefore, in addition to the intertemporal terms of trade, the extended model also incorporates the role of the more conventional terms of trade, i.e., the relative price of importables in terms of exportables. The two-good world contains additional channels of interdependence that were not present in the one-good world. Since the relative prices of goods reflect the pattern of spending in both countries, the analysis of fiscal policies needs to specify the spending patterns of the government. In general it will be seen that the impact of policies depends on relations between various behavioral propensities. Specifically, the key factors determining the outcomes of policies are differences among the spending patterns and the saving propensities of four groups: foreign and domestic private sectors as well as foreign and domestic governments. These differences govern the evolution of relative prices and rates of interest following fiscal changes. In order to highlight the key economic factors that affect the equilibrium we abstract in this section from issues concerning the timing of government spending. We thus assume that all changes are permanent and that the paths of outputs (net of government spending) are stationary. We also continue to assume that $\gamma = \gamma^* = 1$.

Let the home country exportable good be denoted by x and its importable good by m. To simplify the analysis assume that each country is completely specialized in production. Thus, good x is only produced in the home country at the level X and good m is only produced in the foreign country at the level M. Consumers on the other hand are assumed to consume both goods but, since tastes may differ across countries, consumption patterns may differ. More formally, the expanded menu of goods is now incorporated into the utility function by noting that c_t is a composite good which is defined as a Cobb-Douglas function of its components, i.e., $\log c_t = \beta \log c_{xt} + (1-\beta) \log c_{mt} - \gamma$.[8] Private spending is $z_t = c_{xt} + p_t c_{mt}$, measured in units of good x . Government spending also falls on both goods; G_x denotes government spending on x and G_m denotes government spending on m . Intertemporal utility maximization yields the spending function $z_t = (1-\delta)W_t$. The temporal utility maximization yields the demand functions $c_{xt} = \beta z_t$ and $c_{mt} = (1-\beta)\frac{z_t}{p_t}$. It is relevant to emphasize that δ measures the marginal propensity to save out of <u>wealth</u> whereas β measures the marginal propensity to consume good x out of <u>spending</u>. The foreign country is modelled in an analogous fashion.

In order to facilitate the analysis of comparative statics we define henceforth x as domestic product net of the home country's government spending on domestic product, and m as the foreign product net of the foreign country's government spending on its product. Thus, $x = X - G_x$ and $m = M - G_m^*$.

The equilibrium conditions are outlined in Appendix V. In addition to the previous requirements, equilibrium now also requires that the world market for <u>each</u> good clear. The solution of the system for the equilibrium values of W_o^*, W_o and p_o is given in equations (27)-(29):[9]

$$(27) \qquad W_o^* = \frac{(x-G_x^*)[-\beta(xm-G_m G_x^*) + m(x-G_x^*) + \beta(1-\delta)(m-G_m)B]}{(x-G_x^*)[\beta^*(1-\delta^*)m-\beta(1-\delta)G_m] + \beta\beta^*(mx-G_x^* G_m)(\delta^*-\delta)}$$

$$(28) \qquad W_o = \frac{(x-G_x^*)[\beta^*(xm-G_mG_x^*) - G_m(x-G_x^*) - \beta^*(1-\delta^*)(m-G_m)B]}{(x-G_x^*)[\beta^*(1-\delta^*)\, m-\beta(1-\delta)G_m] + \beta\beta^*(mx-G_x^*G_m)(\delta^*-\delta)}$$

$$(29) \qquad P_o = \frac{(1-\beta)(x-G_x^*) + (1-\delta^*)(\beta-\beta^*)W_o^*}{\beta(m - G_m)} \quad .$$

The explicit solution for P_o can be obtained by substituting (27) for W_o^* into (29).[10]

The equilibrium solutions of the present-value factors, in terms of goods x and m are given by equations (30)-(31):

$$(30) \qquad p_t = \delta^t + \frac{(1-\delta^*)[(1-\beta)x + (1-\delta)\beta B](\delta^{*t} - \delta^t)}{x[1-(1-\beta)\delta^* - \beta\delta]}$$

$$(31) \qquad \alpha_t p_t = \frac{(1-\beta)x[\beta^*(1-\delta)\delta^t+(1-\beta^*)(1-\delta^*)\delta^{*t}]+(1-\delta^*)(1-\delta)[\beta(1-\beta^*)(\delta^{*t}-\beta^*(1-\beta)\delta^t)]B}{\beta^* m[1 - (1-\beta)\delta^* - \beta\delta]}$$

X. Government Spending, The Terms of Trade, Interest Rates and Transmission

In this section we analyze the effects of fiscal spending on domestic and foreign real wealth and on the rates of interest. In general it will be seen that the key factors determining the outcomes of government spending are differences among the spending patterns and the saving propensities of four groups: foreign and domestic private sectors as well as foreign and domestic governments. These differences govern the evolution of relative prices and rates of interest following fiscal changes.

Since real consumption levels are proportional to real wealth, we examine the nature of the transmission mechanism by focusing on the impact of fiscal spending on real consumption levels. Recalling that the values of real consumption at home and abroad are

$$c_t = (1-\delta)W_t/p_t^{1-\beta} \quad \text{and} \quad c_t^* = (1-\delta^*)W_t^*/p_t^{1-\beta^*} ,$$

it can be shown from equations (27)-(29) that the logarithmic derivatives of real consumption with respect to changes in government spending on domestic goods, G_x , and on foreign goods, G_m , evaluated around an initial equilibrium with zero government spending and zero initial debt are:

$$\text{(32)} \qquad \frac{d \log c_o^*}{dG_x} = -\frac{\beta^*}{x} < 0$$

$$\text{(33)} \qquad \frac{d \log c_o^*}{dG_m} = \frac{1}{m}\left[\frac{\beta(1-\delta)}{\beta^* K} - (1-\beta^*)\,\frac{1+JS}{1+J}\right]$$

$$\text{(34)} \qquad \frac{d \log c_o}{dG_x} = -\frac{\beta}{x} < 0$$

$$\text{(35)} \qquad \frac{d \log c_o}{dG_m} = \frac{1}{m}\left[\frac{\beta(1-\delta)-K}{\beta^* K} - (1-\beta)\,\frac{1+JS}{1+J}\right]$$

where

$$K = 1 - (1-\beta)\delta^* - \beta\delta > 0$$

$$J = \frac{(\beta-\beta^*)(1-\delta^*)}{\beta^* K}$$

$$S = \frac{\beta(1-\delta)-\beta^*(1-\delta^*) - \beta\beta^*(\delta^*-\delta)}{\beta^* K}$$

As may be seen, a rise in fiscal spending on <u>domestic</u> goods lowers the values of real consumption at home and abroad. The reduction in foreign consumption thus "finances" part of the increased government spending. The extent of the reduction in the values of real consumption of the private sectors in both countries is proportional to β^* and β -- the importance of that good in private sectors' budgets. The precise effects of a rise in government spending on <u>foreign</u> goods depends on the magnitudes of the various propensities as may be seen from equations (33) and (35).

In order to highlight the role of the government's spending propensities we define the government spending function G as:

$$\text{(36)} \qquad G = G_x + pG_m$$

and we assume that the government spending propensities are β_G on good x, and $(1-\beta_G)$ on good m. Thus, $G_x = \beta_G G$ and $pG_m = (1-\beta_G)G$, which implies that, formally, G_m becomes a function of G_x; $G_m = G_m(G_x)$. It follows that around an initial equilibrium with zero government spending:

$$(37) \qquad dG_m = \frac{\phi}{p} dG_x$$

where $\phi = (1-\beta_G)/\beta_G$. Using this specification of government spending we note that

$$(38) \qquad \frac{d \log c_o^*}{\partial G} = \frac{\partial \log c_o^*}{\partial G_x} + \phi \frac{\partial \log c_o^*}{p\partial G_m} \quad .$$

To obtain insights into the economic factors which are at play, we now turn to examine some special cases. These cases correspond to specific assumptions concerning the marginal propensities. Assume first that the domestic and foreign marginal saving propensities are the same, i.e., $\delta = \delta^*$. Using equations (32)-(33) and the specification of government spending which is embodied in (38) along with the solution for p_o we obtain:

$$(39) \qquad \frac{d \log c_o^*}{dG} = \frac{\beta^*}{\beta_G \beta(1-\beta)x} [\beta(\beta-\beta_G) + 2(1-\beta^*)(1-\beta_G)(\beta-\beta^*)] \quad .$$

Thus, when $\delta = \delta^*$ the effect of domestic fiscal spending on foreign real consumption depends on differences among the patterns of spending of domestic private and public sectors, $\beta - \beta_G$, as well as between domestic and foreign private sectors, $\beta - \beta^*$. The economic interpretation of this result is as follows. When $\delta = \delta^*$ we know from equation (30) that in both countries the interest rate equals the subjective discount rate. Therefore, in each country savings are zero. In that case, neither the rates of interest, nor the total level of world spending are altered in response to government spending since the government, like the private sectors, is a zero saver. It follows that the potential effects of fiscal policies can only operate through changes in relative prices. In conformity with the standard analysis of economic transfers, such changes can occur only if the spending patterns differ among private and public sectors.

To demonstrate the role of the spending patterns consider the following special cases:

(i) When all spending patterns are identical, i.e., when $\beta=\beta^*=\beta_G$, we note from equation (39) that $d \log c_0^*/dG = 0$. Thus, in this case only the domestic private sector is crowded out and the effects of fiscal policy are not transmitted internationally. In that case aggregate behavior in the various markets is not affected by the fiscal policy and, therefore, there are no changes in relative prices.

(ii) When only the spending patterns of the domestic and foreign private sector are identical, i.e., when $\beta=\beta^*$, we note from equation (30) that the direction of the change in foreign real consumption following a rise in domestic fiscal spending depends only on the difference between β and β_G. If $\beta > \beta_G$, a rise in government spending creates an excess supply of good x and an excess demand for good m at the initial relative price. Equilibrium is restored through a rise in p, the relative price of good m. This rise in p raises the real value of foreign wealth and, thereby, raises real consumption. In the extreme case for which $\beta_G = 1$, i.e., when government spending falls entirely on good x, foreign real consumption must fall (as was already shown in equation (32)). At the other extreme, when $\beta_G = 0$, i.e., when government spending falls entirely on good m, (the case which corresponds to equation (33)), the value of foreign real consumption must rise.

The effect of the rise in government spending on the home country's real consumption must always be negative independent of the patterns of government spending.. Thus, in conformity with the traditional results of economic transfers, the secondary gain that might occur through an improvement of the terms of trade cannot offset the primary loss which, in the present case, is the tax levied to finance government spending.

The preceding analysis was confined to the case in which $\delta=\delta^*$ so that each country's income equaled its spending. In that case the international transmission of government spending operated entirely through changes in the relative price of goods without any impact on the rates of interest. We turn now to examine the case in which the saving propensities differ, i.e., $\delta \neq \delta^*$. In order to isolate the effects of differences between private and public spending patterns we will assume that $\beta = \beta^*$. Analogously to the previous derivation it can be shown that if

$$(1-\delta)-(1-\beta)[1-(1-\beta)\delta^*-\beta] > 0$$

then

$$(39') \qquad \text{sign}\,\frac{d\log c_o^*}{dG}\ \text{sign}\ \left\{\frac{(1-\beta_G)}{\beta_G} - \frac{(1-\beta)[1-(1-\beta)\delta^*-\beta\delta]}{(1-\delta)-(1-\beta)[1-(1-\beta)\delta^*-\beta]}\right\} \quad ,$$

and vice versa. Thus, the sign in (39') depends on the relation between the spending patterns of the private sectors and the government, as well as on the relationship between the saving propensities. In the special case for which $\beta=\beta_G$, equation (39') becomes

$$(39'') \qquad \text{sign}\,\frac{d\log c_o^*}{dG} = \text{sign}\ (\delta^*-\delta) \quad .$$

In that case, with $\beta=\beta^*=\beta_G$, the patterns of world spending on goods are independent of fiscal policies, and the two goods can be aggregated into a composite commodity as in Section VII. In that case the key factor determining the effect of fiscal spending on foreign wealth and, thereby, real consumption is the relation between δ^* and δ . Analogously to the analysis of the one commodity world, the interpretation of this result is in terms of the effect of government spending on the rate of interest. If $\delta^* > \delta$, the foreign country saves part of its income whereas the domestic economy dissaves; thus the domestic country's marginal propensity to save out of <u>income</u> is negative. In that case, a rise in the home country's government spending amounts to transferring income from a dissaver (the home country's private sector) to a zero saver (the government) and, thereby, creating (at the prevailing rates of interest) excess world savings. To restore equilibrium the rates of interest must fall. The fall in the rates of interest raises foreign wealth and real consumption. The opposite occurs when $\delta^* < \delta$. The effect of government spending on the home country's real consumption is always negative and the analysis is analogous to that of the one-commodity world. As usual, the secondary gain arising from a fall in the rate of interest (that occurs when $\delta^* > \delta$) cannot outweigh the primary loss from the tax that is levied to finance government spending.

One of the central mechanisms governing the international transmission mechanism operates through the real rates of interest. In what follows we elaborate on the effects of government spending on the real rates of interest. The example underlying equation (39'') assumes that the patterns of spending are identical among domestic and foreign governments and private sectors. The equality between β and β^* implies that the real rates of interest are equal across countries. The additional assumption that $\beta = \beta_G$ implies that the effects of changes in fiscal spending on the real rates of interest depend only on the differences between the domestic and the foreign saving propensities. The example underlying equation (39'), assumes that $\beta_G \neq \beta = \beta^*$. In that case real rates of interest are equal across the world but the impact of government spending on the real rates of interest depends not only on the relation between the two countries saving propensities, but also on the relation between β and β_G. In the general case for which all spending patterns differ, real rates of interest differ across countries and the effect of government spending on these rates is more complex.

To illustrate, consider the special case in which government spending falls entirely on the domestically produced good, i.e., $\beta_G = 1$. The effects of government spending on the real rates of interest can be computed from the effects of government spending on the present-value factors measured in terms of the consumption baskets. The domestic and foreign real present-value factors are $\alpha_t p_t^{1-\beta}$ and $\alpha_t p_t^{1-\beta^*}$, respectively. It can be shown that

$$(40) \qquad \frac{d \log (\alpha_t p_t^{1-\beta})}{d \log x} = -(1-\beta) < 0 \quad ,$$

and

$$(40') \qquad \frac{d \log (\alpha_t p_t^{1-\beta^*})}{d \log x} = -(1-\beta^*) < 0 \quad .$$

Thus, a reduction in government spending G_x (i.e., a rise in x) lowers the real present-value factors in both countries and raises the corresponding real rates of interest. In contrast with the case of equality among private and public spending patterns (in which the direction of changes in real rates of interest depended on the relation between saving propensities), here these propensities play no role. In the present case with $\beta_G = 1$, domestic and foreign real rates of interest move in the same direction. The extent of their response to government spending depends on the relative share of the good which is not consumed by the government (good m) in private sectors' spending.

In the intermediate case for which government spending falls on both goods but the relative share β_G exceeds the corresponding shares of the private sectors, β and β^*, the effect of government spending on the real rates of interest reflects the influence of both spending patterns and savings propensities. For example, if $\delta^* > \delta$ a reduction in government spending lowers world savings (as demonstrated for the one-commodity world) and necessitates a higher rate of interest; this reinforces the effects embodied in equations (40)-(40'). On the other hand, if $\delta > \delta^*$, the reduction in government spending raises world savings. Its impact on the real rates of interest tends to mitigate and may even reverse the effects operating via equations (40)-(40').

XI. Concluding Remarks

In this paper we analysed the effects of budget deficits and government spending, on world interest rates, and wealth. We examined in detail the nature of the international transmission mechanism. The model that was used embodied the assumptions that world capital markets are fully integrated and that individuals behave rationally on the basis of self-fulfilling expectations. Economic behavior of individuals and governments was assumed to be governed by temporal and intertemporal budget constraints. In order to capture the effects of budget deficits on world equilibrium, we assumed that the probability of survival is less than unity and, therefore, individuals behave as if their horizons was finite. The formulation assured that the model was not subject to the Ricardian proposition according to which budget deficits do not matter. Our analysis demonstrated that the dependence of world rates of interest, wealth, and consumption, on fiscal policies is highly sensitive to the detailed specification of the policies in terms of the level and the time path of government spending and taxes.

The departure from the Ricardian proposition in this paper stems from the difference between the <u>effective</u> interest-factor that indiviudals use in discounting future taxes and the corresponding <u>market</u> interest-factor that governs governments' behavior. In the present analysis the specific reason responsible for the difference between the two interest factors was that individuals were assumed to be mortal while governments were implicitely assumed to be immortal. In general, of course, the probability that governments and their committments survive indefinitely is also less than unity. Under such circumstances the impact of budget deficits is governed by the relation between the survival probabilities of individuals and governments. It is relevant to note that while the formulation in this paper has focused on the survival probability as the reason for the departure from the Ricardian proposition, the key argument need not hinge on this specific formulation. In general, independent of the reasons for the differences between effective and market interest factors, such differences imply that, within the present framework, budget deficits exert real effects on the world economy.

The examination of the effects of government spending showed that the impact of fiscal expenditures can be analysed by reference to a multitude of "transfer problem criteria," which are familiar from the theory of international economic tranfers. In the present case the impact of policies depended on relations among the spending propensities of domestic and foreign private sectors and governments as well as on the difference between domestic and foreign saving propensities. Our analysis drew a distinction between permanent and transitory policies as well as between current policies and expected future policies.

The analysis of the impact of government spending on real rates of interest revealed that even when capital markets are highly integrated, real rates of interest may differ if spending patterns differ across countries. With such differences in spending patterns, fiscal policies exert different quantitative effects on real rates of interest in the various countries. An implication of this analysis is that in the presence of non-traded goods, fiscal policies may also exert different qualitative effects on real rates of interest in different countries since, depending on the nature of the transmission mechanism and on the patterns of government and private sectors' spending, the relative prices on non-traded goods, and thereby the price indices, might be negatively correlated between countries [see Dornbusch (1983)].

Our analysis is subject to several limitations which stem from some of the simplifying assumptions. We assumed that the output levels were given exogenously. An extension would allow for a process of investment which responds to rates of interest and which changes the paths of outputs [see, for example, Buiter (1984)]. Such an extension would modify the pattern of the current account and debt accumulation [see, for example, Sachs (1981)]. The endogeneity of output could also be introduced through the incorporation of some Keynesian features such as price rigidities. Under such circumstances government spending would alter the level of economic activity and would be transmitted internationally through mechanisms similar to those of the foreign-trade multipliers.

An additional extension would modify the utility function so as to generate a saving function possessing life-cycle characteristics. Specifically, the possibility that savings respond negatively to the rate of interest would influence the impact of budget deficits on interest rates.

Throughout the analysis in this paper we assumed that the foreign government followed a balanced-budget policy. This assumption was adopted in order to ensure that changes in world rates of interest which result from domestic fiscal policies do not impact on the solvency of the foreign government. If the foreign government does not follow a balanced-budget policy, then interest-rate changes would necessitate secondary changes in foreign spending or taxes in order to restore solvency. An extension of the analysis would relax the foreign balanced-budget assumption and would allow for the necessary adjustment of foreign fiscal management. A more major extension would recognize that the interdependencies among the various open economies provide incentives for strategic behavior by individual countries.With such incentivies, countries would attempt to exploit monopoly power in goods and capital markets and to extract maximum seigniorage from debt issue [see, for example, Hamada (1984)]. The analysis would then determine the optimal pattern of government spending along with the optimal trade-cum-capital-flows tax structure as implied by the optimal tariff literature. Such a strategic behavior could then be incorporated into a more elaborate game-theoretic framework. Further, the interdependencies and the strategic behavior could result in inefficient outcomes from a global perspective that may call for harmonization of fiscal policies. In such an extended framework, government spending and the timing of taxes would become endogenous variables that are determined in the context of world equilibrium.

APPENDIX

I. The Formal Model

Let the utility function at period t of an individual who was born in period s be:

$$(I\text{-}1) \qquad U = \sum_{v=t}^{\infty} \delta^{v-t} \log c_{sv}$$

where δ denotes the subjective discount factor and where c_{sv} denotes the rate of consumption in period v of an individual born in period s. The probability, as of period t, that the individual will be alive in period v (and enjoy the utility level $\log c_{sv}$) is γ^{v-t}. Therefore, expected utility is:

$$(I\text{-}2) \qquad E_t \sum_{v=t}^{\infty} \delta^{v-t} \log c_{sv} = \sum_{v=t}^{\infty} (\gamma\delta)^{v-t} \log c_{sv} .$$

The budget constraint in period $t-1$ for an individual born in period s is:

$$(I\text{-}3) \qquad b_{st} = \frac{\gamma^{t-1}\alpha_{t-1}}{\gamma^{t}\,\alpha_t}\left(b_{st-1} + c_{st-1} - \tilde{y}_{t-1}\right)$$

where b_{st} denotes the value of debt at the beginning of period t (which was carried over from the end of period $t-1$), and $\tilde{y}_{t-1}$ denotes disposable income which is assumed to be the same across all individuals regardless of age. The solvency requirement is

$$(I\text{-}4) \qquad \lim_{t\to\infty} \gamma^{t}\alpha_t b_{st} = 0 \quad .$$

Consolidation of the periodical budget constraints together with the solvency requirement yields:

(I-5) $$\sum_{v=t}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^t \alpha_t} c_{sv} = \sum_{v=t}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^t \alpha_t} \tilde{y}_v - b_{st} = w_{st}$$

where w_{st} denotes wealth.

Maximization of (I.2) subject to the consolidated budget constraint (I-5) yields:

(I-6) $$c_{sv} = (1-\gamma\delta) \frac{\gamma^v \alpha_t}{\gamma^t \alpha_v} w_{st}$$

where c_{sv} denotes planned consumption for period v of an individual born in period s and who makes his plan in period t.

Population is normalized so that every cohort born in period s starts with one individual. Due to death, its size in period t becomes γ^{t-s}. The equality between the probability of survival of a given cohort and its frequency relative to its initial size stems from the law of large numbers. Since at each period t there are γ^{t-s} members of each cohort, the (constant) aggregate size of population is:

$$\sum_{s=-\infty}^{t} \gamma^{t-s} = \frac{1}{1-\gamma} ,$$

and, therefore, aggregate disposable income is $y_v = \frac{1}{1-\gamma} \tilde{y}_v$.

Aggregate consumption in period t is the sum of consumption of individuals from all cohorts. Since consumption of the s cohort is $\gamma^{t-s} c_{st}$, aggregate consumption, c_t , is

(I-7) $$c_t = \sum_{s=-\infty}^{t} \gamma^{t-s} c_{st} \quad .$$

Using the individual consumption function (I.6) (for v=t) and the definition of wealth from equation (I-5) yields

(I-8) $$c_t = (1 - \gamma\delta)(H_t + B_{pt}) = (1 - \gamma\delta)W_t$$

where

$$(\text{I-9a}) \qquad H_t = \sum_{v=t}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^t \alpha_t} y_v$$

and

$$(\text{I-9b}) \qquad B_{pt} = \sum_{s=-\infty}^{t} \frac{\gamma^t}{\gamma^s} b_{st} \ .$$

Lagging equation (I-9a) by one period yields an analogous expression for H_{t-1} which, together with equation (I-9a), yields:

$$(\text{I-10}) \qquad H_t = \frac{\gamma^{t-1} \alpha_{t-1}}{\gamma^t \alpha_t} (H_{t-1} - y_{t-1}) \ .$$

The evolution of aggregate private debt, B_{pt}, can be obtained by substituting equation (I-3) into the definition of debt in equation (I-9b) along with the definitions of lagged aggregate consumption, income and debt:

$$(\text{I-11}) \qquad B_{pt} = \frac{\alpha_{t-1}}{\alpha_t} (B_{pt-1} + c_{t-1} - y_{t-1}) \ .$$

Using the aggregate consumption function from equation (I-8), together with the law of motion of human wealth from equation, (I-10) and iterating yields:

$$(\text{I-12}) \qquad B_{pt} = \frac{(\gamma\delta)^t}{\alpha_t} \left[(B_{po} - H_o) - (1-\gamma) \sum_{v=1}^{t-1} (\gamma\delta)^{-v} \alpha_v H_v \right] + H_t$$

where H_v is defined by equation (I-9a) and where B_{po} is the initially given level of debt. The path of total wealth w_t can be obtained by subtracting equation (I-12) from H_t:

$$(\text{I-13}) \qquad W_t = \frac{1}{\alpha_t} \left[(\gamma\delta)^t w_o + (1-\gamma) \sum_{v=1}^{t} (\gamma\delta)^{t-v} \alpha_v H_v \right] \ .$$

Government solvency requirement is:

$$(I\text{-}14) \qquad \sum_{v=t}^{\infty} \frac{\alpha_v}{\alpha_t} (T_v - G_v) = B_{gt}$$

where T_v, G_v denote taxes and government spending in period v and where B_{gt} denotes the value of government debt at the beginning of period t.

Equation (I-14) implies that government debt in the beginning of period t is:

$$(I\text{-}15) \qquad B_{gt} = \frac{\alpha_{t-1}}{\alpha_t} (B_{gt-1} + G_{t-1} - T_{t-1}) .$$

The sum of private debt, B_p, and government debt B_g equals the value of the economy's external debt, B. Thus, by adding equations (I-11) and (I-15) we obtain:

$$(I.16) \qquad B_t = \frac{\alpha_{t-1}}{\alpha_t} [B_{t-1} + (c_{t-1} + G_{t-1}) - Y_{t-1}]$$

where $Y_{t-1} = y_{t-1} + T_{t-1}$ is gross domestic product.

World equilibrium requires that in each period the given supply of world output equal world demand. Equilibrium in period $t=0$ requires that world private spending equal world output net of world governments' spending:

$$(I\text{-}17) \qquad (1-\gamma\delta)(H_o + B_{go} - B_o) + (1-\gamma^*\delta^*)(H_o^* + B_{go}^* + B_o) = \bar{Y} - G_o - G_o^* .$$

In equation (I-17) the left hand side describes the sum of domestic and foreign consumption where domestic consumption reflects equation (I-8) and where foreign variables are indicated by an asterisk (*); $\bar{Y}$ denotes world output, and we have substituted B_o for $-B_o^*$.

For all future periods $(t > 0)$, the equilibrium condition can be written as:

$$(1-\gamma\delta)\left[(\gamma\delta)^t(H_o + B_{go} - B_o) + (1-\gamma)\sum_{v=1}^{t}(\gamma\delta)^{t-v}\alpha_v H_v\right] + \tag{I-18}$$

$$(1-\gamma^*\delta^*)\left[(\gamma^*\delta^*)^t(H_o^* + B_{go}^* + B_o) + \right.$$

$$\left.(1-\gamma^*)\sum_{v=1}^{t}(\gamma^*\delta^*)^{t-v}\alpha_v H_v^*\right] = \alpha_t(\bar{Y}_t - G_t - G_t^*) \quad .$$

The next set of conditions states that the values of human wealth in each period must equal the sum of the present values of disposable incomes. Thus, for period $t=0$, this requirement is:

$$H_o = \sum_{v=0}^{\infty} \gamma^v \alpha_v (Y_v - T_v) \ , \tag{I-19}$$

$$H_o^* = \sum_{v=0}^{\infty} \gamma^{*v} \alpha_v (Y_v^* - T_v^*) \ , \tag{I-20}$$

and for all other period $(t > 0)$:

$$\alpha_t H_t = \frac{1}{\gamma^t} \sum_{v=t}^{\infty} \gamma^v \alpha_v (Y_v - T_v) \tag{I-21}$$

$$\alpha_t H_t^* = \frac{1}{\gamma^{*t}} \sum_{v=t}^{\infty} \gamma^{*v} \alpha_v (Y_v^* - T_v^*) \ . \tag{I-22}$$

The system is closed by the requirements that both governments be intertemporally solvent. Using equation (I.14) and its foreign-country analogue, we obtain:

$$\sum_{t=0}^{\infty} \alpha_t (T_t - G_t) = B_{go} \tag{I-23}$$

$$\sum_{t=0}^{\infty} \alpha_t (T_t^* - G_t^*) = B_{go}^* \quad . \tag{I-24}$$

For any given initial values of the external debt, governments' debt and the paths of governments' spending, the system of equations (I-17) - (I-24) solves for the initial period's values of H_o and H_o^* as well as for α_t , $\alpha_t H_t$ and $\alpha_t H_t^*$ for all other periods (t > 0). In addition, the system also solves for the sum of the present values of taxes in each country. Finally, it is relevant to note that by Walras' Law one of the equations (I-17)-(I-22) can be excluded.

II. The Impact of Current Budget Deficits

In this Appendix we derive the formal solution to the impact of current budget deficits on R, W_o and W_o^* . Differentiating equations (3)-(5) of the text (under the assumption that $\gamma = \gamma^*$) yields:

$$\begin{bmatrix} 1-\gamma\delta & 1-\gamma\delta^* & 0 \\ 1 & 0 & \dfrac{-\gamma(Y-G)}{(1-\gamma R)^2} \\ 0 & 1 & \dfrac{-\gamma(Y^*-G^*)}{(1-\gamma R)^2} \end{bmatrix} \begin{bmatrix} dW_o \\ dW_o^* \\ dR \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \frac{1-\gamma}{1-\gamma R} dT_o$$

The determinant of the matrix is denoted by $-\Delta$. The value of Δ is:

$$\Delta = \frac{(1-\gamma\delta^*)}{R(1-R)} [\psi(Y^*-G^*) + \omega(Y^*-G^*)] > 0$$

where

$$\omega = \frac{(1-\gamma R)^2 - (1-\gamma(1-\gamma R^2)}{(1-\gamma^* R)^2} > 0$$

$$\psi = \gamma \frac{(1-\gamma R^2) - (1-R)^2 (\gamma\delta-\gamma\delta^*)}{(1-\gamma R)^2} > 0 \quad .$$

The solutions of the system are given in equations (7)-(9) in the text where

$$\theta = \frac{(1-\gamma R)^2 - \gamma(1-R)^2 - (1-\gamma)(1-\gamma R^2)}{(1-\gamma R)^2} > 0 \quad .$$

III. The Impact of Future Budget Deficits and Steady-State Equilibrium

In Part A of this Appendix we derive the formal solutions to the impact of future budget deficits on world rates of interest (short-term and long-term) and on W_o and W_o^*, for the case in which $\gamma=\gamma^*$. In Part B we examine the impact of cumulative budget deficits on the steady-state values of interest rates and wealth.

A. The equilibrium conditions are

$$(1-\gamma\delta)W_o + (1-\gamma\delta^*)W_o^* = (Y_o-G_o) + (Y_o^*-G_o^*) \tag{III-1}$$

$$(1-\gamma\delta)[\gamma\delta W_o+(1-\gamma)R_oH_1]+(1-\gamma\delta^*)[\gamma\delta^* W_o^*+(1-\gamma)R_oH_1^* = R_o[(Y_1-G_1)+(Y_1^*-G_1^*)] \quad . \tag{III-2}$$

$$W_o = (Y_o-T_o) + R_o\gamma(Y_1-T_1) + R_o \frac{\gamma R_1}{1-\gamma R_1} (Y-T) + B_{go} - B_o \tag{III-3}$$

$$W_o^* = (Y_o^*-T_o^*) + R_o\gamma (Y_1^*-T_1^*) + R_o \frac{\gamma R_1}{1-\gamma R_1} (Y^*-T^*) + B_{go}^* + B_o \tag{III-4}$$

where

$$H_1 = (Y_1-T_1) + \frac{R_1}{1-\gamma R_1} (Y-T)$$

and

$$H_1^* = (Y_1^*-T_1^*) + \frac{R_1}{1-\gamma R_1} (Y^*-T^*) \quad .$$

Equations (III-1) and (III-2) specify the world goods market clearing conditions for the present and for the near future periods; and equations (III-3) and (III-4) define the values of domestic and foreign wealth in the present period, where H_1 and H_1^* denote the values of human capital in the near future as evaluated at t=1. Differentiating this system with respect to future tax changes and evaluating the solutions around the point of balanced budgets and the stationary future (i.e., $Y_1=Y$, $T_1=T=G_1=G$ and $Y_1^*=Y^*$, $T_1^*=T^*=G_1^*=G^*$), we obtain equations (10)-(13) in the text.

B.

The steady-state equilibrium conditions are shown by equations (16)-(18) in the text. Differentiating these equations around an initial equilibrium with $B_g = B_o^* = 0$, $Q = Q^*$, $\gamma = \gamma^*$ yields

(III-5)

$$\frac{dR}{dB_g} = \frac{R(1-R)\ (1-\gamma\delta\ (1-\gamma R)\ (R-\gamma\delta)\ (R-\gamma\delta^*)^2}{Q\{(R-\gamma\delta)(R-\gamma\delta^*)[(1-\gamma\delta)(R-\gamma\delta^*)+(1-\gamma\delta^*)(R-\gamma\delta)]-R[(1-\gamma\delta)(R-\gamma\delta^*)^2+(1-\gamma\delta^*)(R-\gamma\delta)^2]} < 0 .$$

In order to compute the effect of B_g on the steady-state value of foreign wealth, we first note that $dW^*/dB_g = (dW^*/dR)(dR/dB_g)$ and since, from (III-5), $dR/dB_g < 0$, we only need to determine the effects of R on W^*. Differentiating equation (15) in the text yields

$$\text{(III-6)} \qquad \frac{dW^*}{dR} = (1-\gamma)Q^* \frac{\gamma R^2(R-\gamma\delta^*) - \gamma\delta^*(1-\gamma R)}{(1-\gamma R)^2\ (R-\gamma\delta^*)^2} \gtrless 0 \qquad .$$

As is seen the transmission of domestic past deficits to the steady-state value of foreign wealth may be positive or negative. For example, if $R<\delta^*$ (but still $R > \gamma\delta^*$ so as to ensure the steady-state) $dW^*/dR < 0$ and, therefore, $dW^*/dB_g > 0$ and the transmission of past budget deficits to foreign wealth is positive.

The effect of B_g on the steady-state value of domestic wealth is

$$\text{(III-7)} \qquad \frac{dW}{dB_g} = \frac{\partial W}{\partial R}\frac{dR}{dB_g} + \frac{\partial W}{\partial B_g} \qquad .$$

Differentiating equation (14) in the text yields

$$\text{(III-8)} \qquad \frac{dW}{dB_g} = (1-\gamma)\{Q\frac{\gamma R^2(R-\gamma\delta) - \gamma\delta(1-\gamma R)}{(1-\gamma R)^2\ (R-\gamma\delta)^2}\frac{dR}{dB_g} - \frac{(1-R)R}{(1-\gamma R)(R-\gamma\delta)}\}$$

The second term on the right-hand-side of equation (III-8) does not have a counterpart in the expression for dW^*/dB_g; this negative term represents the direct effect of B_g on domestic wealth which operates through the necessary rise in steady-state tax collections. A sufficient condition for a decline in domestic steady-state wealth consequent on a rise in B_g is that $\gamma R^2(R-\gamma\delta) > \gamma\delta(1-\gamma R)$.

IV. The Solution of the Complete System with $\gamma=1$

In this Appendix we examine the properties of the model for the special case in which the probability of survival is unity. In that case, with $\gamma=1$, the market interest factor equals the effective interest factor, and the equilibrium conditions are:

$$(1-\delta)\, W_o + (1-\delta^*)\, W_o^* = \bar{Q}_o \tag{IV-1}$$

$$(1-\delta)\delta^t\, W_o + (1-\delta^*)\delta^{*t}\, W_o^* = \alpha_t \bar{Q}_t \tag{IV-2}$$

$$W_o = \sum_{t=0}^{\infty} \alpha_t Q_t - B_o \tag{IV-3}$$

where

$$W_o = H_o + B_{go} - B_o\ ,\ W_O^* = H_o^* + B_{go}^* + B_o$$

$$Q_t = Y_t - G_t \quad \text{and} \quad \bar{Q}_t = (Y_t - G_t) + (Y_t^* - G_t^*) \quad .$$

Equation (IV-1) describes the requirement that current world demand equal current world output net of governments' spending, and equation (IV-2) describes the corresponding requirement for $t=1,2,\ldots$. Equation (IV-3) defines the equilibrium value of domestic wealth. This specification substitutes government spending, G_t, for taxes, T_t, by using the government solvency requirement. The solution of this system is

$$W_o = \frac{(1-\delta^*)\,[\bar{Q}_o \sum_{t=0}^{\infty} \delta^{*t}\, \lambda_t - B_o]}{(1-\delta)\Delta_1} \tag{IV-4}$$

$$W_o^* = \frac{\bar{Q}_o \sum_{t=0}^{\infty} \delta^t\, \lambda_t^* + B_o}{\Delta_1} \tag{IV-5}$$

$$\alpha_t = \frac{(1-\delta^*)\{\bar{Q}_o[\delta^{*t} \sum_{\tau=0}^{\infty} \delta^{\tau}\lambda_{\tau}^* + \delta^t \sum_{\tau=0}^{\infty} \delta^{*\tau}\lambda_{\tau}] + (\delta^{*t} - \delta^t)B_o\}}{\bar{y}_t\Delta_1} \tag{IV-6}$$

where $\Delta_1 = \sum_{t=0}^{\infty} \delta^{*t}\lambda_t + \sum_{t=0}^{\infty} \delta^t\lambda_t^* > 0$, and where λ_t^* denotes the share of foreign product net of government spending, i.e., $\lambda_t^* = Q_t^*/\bar{Q}_t$ and, λ_t denotes the corresponding share of the home country, i.e., $\lambda_t = (Q_t/\bar{Q}_t) = 1 - \lambda_t^*$.

As is evident by inspection of equations (IV-4)-(IV-6), the system conforms with the Ricardian proposition. Thus, since in present case we assume that $\gamma = 1$, it follows that as long as the paths of government spending (which govern the paths of Q and Q^*) are given, the time patterns of taxes and government debt issues are irrelevant for the determination of world equilibrium.

V. The Two-Commodity System

The equilibrium conditions for the system can be obtained as follows: equilibrium requires that the world market for each good clear. Using the demand functions, the equilibrium conditions for both goods are:

$$\beta(1-\delta)\frac{\delta^t}{\alpha_t}W_o + \beta^*(1-\delta^*)\frac{\delta^{*t}}{\alpha_t}W_o^* = x_t - G_{xt}^* \tag{V-1}$$

$$(1-\beta)(1-\delta)\frac{\delta^t}{\alpha_t p_t}W_o + (1-\beta^*)(1-\delta^*)\frac{\delta^{*t}}{\alpha_t p_t}W_o^* = m_t - G_{mt} \,. \tag{V-2}$$

In period 0, $\alpha_o = 1$ and, therefore, the market clearing conditions for $t = 0$ become:

$$\beta(1-\delta)W_o + \beta^*(1-\delta^*)W_o^* = x_o - G_{xo}^* \tag{V-3}$$

$$(1-\beta)(1-\delta)\frac{W_o}{p_o} + (1-\beta^*)(1-\delta^*)\frac{W_o^*}{p_o} = m_o - G_{mo} \tag{V-4}$$

where the values of wealth are

$$W_o = x\,\Sigma_{t=o}^{\infty}\,\alpha_t - G_m\,\Sigma_{t=o}^{\infty}\,\alpha_t p_t - B \tag{V-5}$$

$$W_o^* = m\,\Sigma_{t=o}^{\infty}\,\alpha_t p_t - G_x^*\,\Sigma_{t=o}^{\infty}\,\alpha_t + B \,. \tag{V-6}$$

By Walras' Law we may ignore one of the equations in the system.

Substituting (V-5)-(V-6) into (V-3)-(V-4) yields:

(V-7) $$\beta(1-\delta)[x\Sigma_{t=o}^{\infty} \alpha_t - G_m \Sigma_{t=o}^{\infty} \alpha_t p_t - B] +$$

$$\beta^*(1-\delta^*)[m \Sigma_{t=o}^{\infty} \alpha_t p_t - G_x^* \Sigma_{t=o}^{\infty} \alpha_t + B] = x - G_x^*$$

(V-8) $$(1-\beta)(1-\delta)[x \Sigma_{t=o}^{\infty} \alpha_t - G_m \Sigma_{t=o}^{\infty} \alpha_t p_t - B] +$$

$$(1-\beta^*)(1-\delta^*)[m \Sigma_{t=o}^{\infty} \alpha_t p_t - G_x^* \Sigma_{t=o}^{\infty} \alpha_t + B] = p_o(m - G_m).$$

We then multiply both sides of (V-I) by α_t, sum both sides over all t (from 0 to ∞) and substitute (V-5) and (V-6) for W_o and W_o^*. The resulting equation is then:

(V-9)
$$\beta[\Sigma_{t=o}^{\infty} \alpha_t - G_m \Sigma_{t=o}^{\infty} \alpha_t p_t - B] + \beta^*[m \Sigma_{t=o}^{\infty} \alpha_t p_t - G_x^* \Sigma_{t=o}^{\infty} \alpha_t + B] = (x - G_x^*) \Sigma_{t=o}^{\infty} \alpha_t .$$

Equations (V-7), (V-8) and (V-9) constitute the relevant system. The economic interpretation is as follows: Equations (V-7) and (V-8) describe the equilibrium in the markets for the two goods in period 0. It is evident that this is a rational expectations equilibrium since the demand functions are based on the fully-expected realization of future values of rates of interest and prices. Equation (V-9) requires that the sum of the present values of world demand for good x equal the corresponding sum of world supply. As may be noted we have used Walras' Law to ignore the analogous requirement for good m.

This system is used to solve for the three unknowns: The sum of the present value factors in terms of good x -- $\Sigma_{t=o}^{\infty} \alpha_t$; the sum of the present value factors in terms of good m -- $\Sigma_{t=o}^{\infty} \alpha_t p_t$; and the relative price of good m in terms of x in the first period -- p_o. These solutions can then be used in (V-5) and (V-6) to obtain the values of W_o and W_o^*. These are the solutions that are reported in equations (27)-(29) in the text.

VI. Effects of Fiscal Spending in the Two-Commodity System[11]

In this part of the Appendix we derive the effects of fiscal policies on the real values of domestic and foreign consumptions as well as on real rates of interest. These results are computed around an initial equilibrium with zero debt and zero government spending. From equations (27) in the text we obtain

$$\text{(VI-1)} \qquad \frac{dW_o^*}{dx} = \frac{1-\beta}{\beta^* D}$$

$$\text{(VI-2)} \qquad \frac{dW_o^*}{dG_m} = \frac{x}{m}\,\frac{\beta(1-\beta)(1-\delta)}{\beta^{*2} D^2} \quad .$$

From (V-3) in Appendix V and from (VI-1) and (VI-2) we obtain

$$\text{(VI-3)} \qquad \frac{dW_o}{dx} = \frac{1}{D}$$

$$\text{(VI-4)} \qquad \frac{dW_o}{dG_m} = \frac{x[\beta(1-\delta)-D]}{\beta^* m D^2} \quad .$$

Differentiating equation (29) in the text with respect to x and using (VI-1) yields

$$\text{(VI-5)} \qquad \frac{dp_o}{dx} = \frac{(1-\beta)-(1-\delta^*)(\beta-\beta^*)\dfrac{(1-\beta)}{\beta^* D}}{\beta m} \quad .$$

From equations (27)-(29) in the text we obtain

$$\text{(VI-6a)} \qquad W_o^* = \frac{(1-\beta)x}{\beta^* D} \quad ,$$

$$\text{(VI-6b)} \qquad W_o = \frac{x}{D}$$

where $D = 1 - (1-\beta)\delta^* - \beta\delta > 0$. To convert (VI-5) into a logarithmic derivative we first substitute (VI-6a) for W_o^* into equation (29) in the text to obtain:

$$\text{(VI-7)} \qquad p_o = \frac{(1-\beta)x}{\beta m}\left[1 + \frac{(1-\delta^*)(\beta-\beta^*)}{\beta^* D}\right] \quad ;$$

and dividing (VI-5) by (VI-7) yields

$$\text{(VI-8)} \qquad \frac{d \log p_o}{dx} = \frac{1}{x} \quad .$$

Multiplying (VI-8) by $(1-\beta^*)$ and subtracting from the logarithmic derivative of W_o^* with respect to x (obtained from (VI-1) and (VI-6a)) we get $d\log c_o^*/dx$ which is equation (32) in the text, with a minus sign (since $dx = dG_x$).

Analogously, dividing (VI-3) by (VI-6b) yields the logarithmic derivative of W_o with respect to x, from which we subtract the product of $(1-\beta)$ and (VI-7), to obtain $d\log c_o/dx$ which is equation (34) in the text (with a minus sign).

To compute the effects of changes in G_m on real consumption we first note that from (29) and (VI-2):

$$\text{(VI-9)} \qquad \frac{dp_o}{dG_m} = \frac{(1-\beta)}{\beta}\,\frac{x}{m^2}\left\{1 + \frac{(\beta-\beta^*)(1-\delta^*)}{\beta^{*2}D^2}\left[\beta(1-\delta)-\beta^*(1-\delta^*)-\beta\beta^*(\delta^*-\delta)\right]\right\}$$

and dividing by (VI-8) yields

$$\text{(VI-10)} \qquad \frac{d \log p_o}{dG_M} = \frac{1}{m}\,\frac{1+JS}{1+J}$$

where J and S are defined in equation (35) in the text. Multiplying (VI-10) by $(1-\beta^*)$ and subtracting from the ratio of (VI-2) and (VI-6a) yields equation (33) in the text.

Analogously, dividing (VI-4) by (VI-3) and subtracting the product of $(1-\beta)$ and (VI-10) yields equation (35) in the text.

In computing the effects of domestic permanent fiscal spending on domestic and foreign real rates of interest we focus on the case in which government spending falls entirely on good x, i.e., the case for which $\beta_G = 1$.

Equation (V-1) in Appendix V (around an initial equilibrium with zero government spending and initial debt) implies that

$$\alpha_t = 1/x[\beta(1-\delta)\delta^t W_o + \beta^*(1-\delta^*)\delta^{*t} W_o^*] \quad , \tag{VI-11}$$

and its logarithmic derivative is:

$$\frac{d \log \alpha_t}{dx} = \theta \frac{d \log W_o}{dx} + (1-\theta)\frac{d \log W_o^*}{dx} - \frac{1}{x} \tag{VI-12}$$

where, using (VI-5a)-(VI-6b)

$$\theta = \frac{\beta(1-\delta)\delta^t}{\beta(1-\delta)\delta^t + (1-\beta)(1-\delta^*)\delta^{*t}} \quad .$$

Using (VI-3) and (VI-6b) for $d \log W_o/dx$, and using (VI-1) and (VI-6a) for $d \log W_o^*/dx$, we obtain

$$\frac{d \log \alpha_t}{dx} = 0. \tag{VI-13}$$

Analogously, from equation (V-2) in Appendix V we obtain:

$$\frac{d \log p_t}{dx} = \tilde{\theta} \frac{d \log W_o}{dx} + (1-\tilde{\theta})\frac{d \log W_o^*}{dx} - \frac{d \log \alpha_t}{dx} \quad . \tag{VI-14}$$

where

$$\tilde{\theta} = \frac{\beta^*(1-\delta)\delta^t}{\beta^*(1-\delta)\delta^t + (1-\beta^*)(1-\delta^*)\delta^{*t}}$$

Using (VI-3) and (VI-6b) for $d \log W_o/dx$, and using (VI-1) and (VI-6a) for $d \log W_o^*/dx$, we obtain after substituting (VI-13):

$$\frac{d \log p_t}{dx} = \frac{1}{x} \quad . \tag{VI-15}$$

As may be observed by comparing (VI-15) with (VI-8), the effect of x on the initial price p_o is the same as its effect on the entire path of prices, p_t. This result reflects the finding in (VI-13) where it was shown that the change of x does not alter the entire path of interest rates.

Finally, differentiating logarithmically the real present-value factors, $(\alpha_t p_t^{1-\beta})$ and $(\alpha_t p_t^{1-\beta^*})$ with respect to x, and using (VI-13) and (VI-15), yields equations (40)-(40') in the text.

FOOTNOTES

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[1]The complete model and the various derivations are outlined in Appendix I which draws on Frenkel and Razin (1984).

[2]In this formulation government spending reduces the resources available for private sector consumption without yielding utility services. The interaction between public and private goods in the utility function is a separate issue, with which we do not deal in the present paper. It is relevant to note, however, that none of our results are altered by the introduction of "useful" government as a <u>separable</u> argument in the utility function.

[3]In order to compute the long-run values of wealth we use equations (I-10) and (I-13) of Appendix I, we first substitute R^v for α_v (where $R = 1/(1+r)$ is the constant steady-state interest factor), and $(Y-T)$ for (Y_t-T_t); then we take the limit of W_t as $t \to \infty$. To ensure a finite value of long-run wealth we assume that $R > \gamma\delta$ since if $R < \gamma\delta$, wealth is unbounded. As is evident from equation (14) if $\gamma=1$ then, if $R > \delta$ wealth shrinks to zero.

[4]Formally, equation (17) is obtained by substituting B_G for B_{gt} and $(T-G)$ for (T_v-G_v) in equation (I-14) in Appendix I; an analogous procedure underlies the derivation of equation (18).

[5]While it seems likely that a higher value of domestic government debt lowers the steady-state value of domestic wealth, we were unable to formally rule out the opposite outcome.

[6]The subsequent analysis draws on Frenkel and Razin (1985a).

[7]Recall that Q is defined as the value of output net of government spending; thus, a reduction in government spending is equivalent to a rise in Q.

[8]The constant term $\gamma = \beta \log \beta + (1-\beta) \log (1-\beta)$ as is chosen so as to simplify, without loss of generality, the subsequent expressions of real consumption.

[9]This specification assumes that B is denominated in units of good x. This assumption concerning the indexation of debt may be material when there are unexpected changes in the terms of trade. In our subsequent analysis we assume that the initial value of B is zero and, thereby, the specification of the units of B does not alter our results. In general, however, such changes may provide for an additional channel through which government spending impacts on the real equilibrium.

[10]To gain insight into the determinants of p_o we note that the equilibrium price must equate one country's trade balance surplus with the other's trade deficit or equivalently, the equilibrium price must ensure that the value of world spending on goods equal the value of world output. Thus, the equilibrium price p_o must satisfy $[(1-\delta)W_o + G_x + p_o G_m] - X = p_o M - [(1-\delta^*) W_o^* + G_x^* + p_o G_m^*]$ where the left-hand-side measures the home country's trade balance deficit and the right-hand-side measures the foreign country's trade balance surplus. To ensure that the market for each good clears we use the market clearing condition for good x in period $t=0$ and substitute for W_o into the trade-balance condition. The solution for p_o yields equation (29).

[11]This Appendix drawn on Frenkel and Razin (1985b).

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