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SKILL ACQUISITION, CREDIT CONSTRAINTS, AND TRADE

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ABSTRACT

This paper looks at the effect of credit constraints on skill acquisition when agents have heterogeneous abilities and wealth. We use a two factor general equilibrium model and assume credit markets are absent. We explore the effects of trade on factor earnings as well as the evolution of the distribution of income in small and large economies. Our work suggests that developed countries need to ensure access to education when liberalizing trade to ensure they reap the potential gains from trade.

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1 Introduction

What is the role of credit constraints in transferring skills across generations and what is the role of trade in this? What is the effect of tighter credit constraints? Will welfare rise or fall and why? These are the questions addressed in this paper. They are clearly of immense relevance for policy since human capital is not good collateral for loans and the ability to acquire skills can be severely limited by wealth. In this paper we assume credit markets are absent, but consider two settings with different degrees of credit market imperfections. In the first one, credit markets are completely absent and the only way an agent can acquire skills is by paying for training in full up front. This is clearly an extreme case. In the second setting we allow credit constraints to be weakened through a pay as you go system. This can be interpreted as an apprenticeship contract where training is provided in exchange for services. Alternatively, it can be interpreted as a form of work study. For example, most Ph.D. students in the U.S. have their education paid for and obtain a stipend in return for teaching or research services. Many undergraduates finance at least part of their education through work study programs.

An alternative way of acquiring skills is through firms, rather than workers, incurring these costs. There is a fairly large literature that models such contracts. It deals with issues such as which labor market imperfections would make firms pay for general training that is transferable across firms, the features of such contracts and their rationale, the inefficiency of training levels provided by firms, as well as the success of such apprenticeship programs in providing a skilled labor force.[2]

Such issues are not the subject of this paper. Rather, we focus on another, hitherto unstudied aspect of apprenticeships, namely their ability to help circumvent credit constraints. Those with the skills to impart (masters) enter into a contract with the unskilled (apprentices) to "teach as best they know" their technical skills. In return, the apprentice undertakes the tasks assigned to him by the master for the (specified) period of his apprenticeship. He is paid below market wages during this period, receiving payment in the form

[2] For a review of this research, see Acemoglu and Pischke (1999b) and Smits and Stromback (2001).

of training instead.[3] Contrast this with the alternative where the training fees have to be paid up front. In the absence of credit markets, only those with the wealth to pay the up front fee could afford training. Note however, that even if part of the fee is paid up front, as occurs when the apprentice's wage is negative, the less well off may be able to afford the apprenticeship (work study) route.

Are credit constraints important in the U.S. today? It is well understood that intergenerational income correlation is reasonably high.[4] Empirical work on college attendance has consistently shown that parental income does predict college attendance and that the effect on college attendance of tuition is greater for lower income families. Should these facts be taken as evidence of credit constraints? Recent work by Cameron and Heckman (1998) questions such an interpretation of these facts. They estimate decision rules for college attendance that control for family background measures like parents education, family income at 16, and a measure of the child's skill endowment as proxied for by the Armed Forces Qualifying Test (AFQT). They find that family income is not significant which they interpret as evidence that credit constraints are not binding. They argue that such correlations could arise in the absence of credit constraints at the college level: for example, parental wealth could be correlated to the skill endowment of the children[5] or school could have a consumption value so that higher parental incomes result in more education directly.

Keane and Wolpin (2001) argue that credit constraints do matter. They estimate a

[3]Lane (1996) shows that in late 18th century, apprentices earned 41% of the journeyman (skilled) rate while unskilled workers earned 77%. In some cases, apprentices have even paid for the privilege of learning the trade. In fact, by the 18th century, an up front fee had become the norm. While there was considerable variation in the terms specified between the country and the city as well as across occupations, there were instances of large sums, hundreds of pounds, being paid up front when the trade was particularly well rewarded.

[4]Solon (1992) finds a ballpark figure of .4. Charles and Hurst (2003) find the pre-bequest correlation in log wealth to be .37.

[5]This may well be a reflection of credit constraints or differences in the importance given to schooling by parents long before college!

structural model and argue that though borrowing constraints are tight, they are mitigated by agents adjusting labor supply. This is why changes in tuition affect college attendance by relatively little, and have a greater effect on poorer families who are likely to be more constrained by their labor supply. See Keane (2002) for a simple and clear summary of the issues here.

We develop a simple general equilibrium model where apprenticeships help overcome credit constraints that limit the ability of agents with heterogeneous abilities and wealth to acquire skills. There are two tradable final goods, and two factors, unskilled labor and skilled labor which is produced using skilled labor and unskilled labor.

In the static version of our model,[6] the response of supply to price depends on the number of skilled agents in the economy. If there are relatively few skilled agents, the normal supply response obtains. However, with many skilled agents, supply can be decreasing in price so that multiple equilibria may exist. The intuition is that in addition to the normal supply response, there is an induced Rybczynski effect which could work in either direction. When there are relatively few skilled agents, an increase in the price of the skill-intensive good, keeping as given expected future prices, raises the cost of education. This reduces the number of agents who want to acquire training and thus raises the availability of unskilled labor. The availability of skilled labor for production also rises as less skilled labor is needed in training. Whether relative availability of skilled labor rises or falls depends on the endowment of skilled workers to begin with: if this endowment is large, the relative availability of skilled labor falls, while if it is small, it rises. In the former case, relative supply of the skill-intensive good can fall with price, while in the latter case it must rise.

Simple general equilibrium intuition would suggest that weaker credit constraints would raise the availability of skilled labor shifting the relative supply of the skill-intensive good a la Rybczynski, and lowering its relative price in autarky. However, our induced Rybczynski effect can work in the opposite direction. Relaxing credit constraints increases the number of agents who try to become educated at any given price. This, in turn, reduces both

[6] In a static setting expectations about future prices are taken as given.

the skilled and unskilled labor available for production. If the stock of skilled agents is relatively large, then the percentage fall in skilled labor will be low relative to that of unskilled labor so that the relative availability of skilled labor in production rises and the relative supply of the skill-intensive good shifts out. The opposite occurs when the stock of skilled agents is small. As a result, weaker credit constraints can result in a higher price of the skill-intensive good contrary to what simple intuition might suggest.

In steady state, however, non monotonicity of supply and multiplicity of equilibria occur only in the presence of credit constraints. An increase in the price in steady state raises the return to skilled labor today, and hence, the cost of education, but it also raises the return tomorrow. While the increase in the cost of education reduces the demand for training, the increase in the return tomorrow increases it. Since each agent must be able to train more than one worker for there to be skilled workers present in steady state, the latter effect dominates. Thus, an increase in price raises the number of trainees, reducing the availability of unskilled workers for production. In steady state, and in the absence of credit constraints, as the number of trainees rises, so does the number of skilled workers available for production. As a result, an increase in price raises the relative supply of the skill-intensive good. In the presence of credit constraints, relative supply need not be monotonic in price. For relative supply to be backward bending, the increase in education cost resulting from an increase in price must constrain enough of the potential pool of trainees.

In steady state, weaker credit constraints always raise the relative supply of the skill-intensive good and lower its autarky relative price. Weaker credit constraints raise the demand for training, thus reducing the availability of unskilled labor in production. However, as the number of trainees rise, so does the availability of skilled labor for production and hence relative supply of the skill-intensive good.

There may or may not be multiple equilibria in steady state with credit constraints: a key determinant is the distribution of wealth. With such multiplicity, trade may even reduce welfare in steady state if (a) there is limited substitutability in consumption and

(b) the value of final good output evaluated at traded prices falls.

Related Literature Our work is related to the literature on endogenous skill formation in international trade. In an influential contribution, Findlay and Kierzkowski (1983) extend the standard Heckscher-Ohlin model by endogenizing the formation of human capital. They show that trade amplifies initial differences in factor endowments through the Stolper-Samuelson effect: trade raises the reward of the abundant factor in each country.

Cartiglia (1997) incorporates credit constraints into a Findlay-Kierzkowski type model, but uses a static setting. He shows that, trade should lead to convergence in human capital endowments across countries rather than amplification of initial differences. Trade liberalization in a skill-scarce country while reducing the return to education, also reduces the cost of education which weakens credit constraints, resulting in a higher investment in human capital. This effect, in fact, outweighs the Stolper-Samuelson effect of Findlay-Kierzkowski, reversing their results.[7]

The work of Ranjan (2001) is closest to our work. He also allows for borrowing constraints to affect human capital accumulation in small open economy with two traded final goods. Lenders can lend at the world rate of interest, while borrowers can only borrow up till an incentive compatible repayment level. This level is higher for more able agents, i.e., for agents who obtain more effective units of skilled labor upon becoming educated. He points out a third effect that operates through changes in the distribution of income which influences the accumulation of human capital and shows that under certain conditions, trade may encourage skill acquisition in both countries.

His work differs from ours in a number of ways. First, we derive a new effect, the induced Rybczynski effect that helps understand how price increases and differences in extent to which credit constraints operate affect the relative supply of the skill-intensive good in the static model. The differences in the extent of credit constraints are modelled as being institutional in nature: namely, whether education has to be paid for up front or can be paid for as you go.

[7] A similar result obtains in Eicher (1999) via a domestic credit market.

Second, we look at both autarky and trade, static and steady state effects, positive and normative effects, rather than restrict attention to the operation of a small open economy in steady state as in Ranjan (2001). We are able to provide simple conditions under which trade reduces welfare. Our work has the policy implication that unless developed countries ensure access to education when liberalizing trade, they may well lose their comparative advantage in it and end up losing from trade in steady state.

Third, our model is set up slightly differently from his which permits us to do more in terms of analyzing the effects on the distribution of income. We assume that differences in ability are reflected in the probability of becoming skilled, not the quantity of skills acquired.[8]

Ranjan (2003) looks at the effect of trade liberalization on skill acquisition, the skilled-unskilled wage differential, and the distribution of wealth. Multiple steady state equilibria also exist in his model and he shows that trade may induce convergence to the good equilibria. However, his focus in on trade in intermediate goods as his single final good is not traded.

The rest of the paper proceeds as follows. Section 2 lays out the model. Section 3 analyzes the equilibrium in the static model and in steady state. It looks at steady state equilibria and how they differ from the static equilibria in a closed economy. Section 4 studies how trade affects welfare. Section 5 endogenizes the distribution of income. Section 6 provides concluding remarks and directions for future research. Details of the proofs are in the Appendix.

2 The Model

There are two goods, X and Z, and one basic factor, unskilled labor in the economy. Unskilled labor can be transformed into its skilled counterpart by a skilled worker. However, if K unskilled workers are taken on by a skilled worker then only $G(K)$ units of the skilled worker's time remains available to him, where $G(K)$ is a decreasing function of K. We

[8] This simplifies the problem in a number of ways.

assume that

$$G(K) = 1 - AK$$

so that there are constant returns to scale as A is the time required per trainee.

We assume that there are constant returns to scale in production and that good Z is relatively more skill-intensive than good X at all factor prices. Good X is the numeraire. Hence, p denotes the relative price of good Z.

We use an overlapping generations framework. There are L agents born in each period. Each agent lives for two periods and is endowed with one unit of time in each period. An agent is characterized by two parameters: γ, the probability of becoming skilled (a master), upon undertaking the needed education, and y, his initial wealth. It is assumed that γ is distributed uniformly in the unit interval ($\gamma \sim U[0,1]$).

In the first period of life an agent makes career choices. He could remain an unskilled worker, work in both periods at the unskilled wage, w. Alternatively, he could spend part of his time acquiring the skills that give him a chance at becoming a master and allowing him, if he so chooses and is successful in his training, to earn the skilled wage in the second period. Agents who try to become skilled but fail can work only as unskilled workers in the second period. Skilled workers could also choose to work as unskilled workers were it in their interests to do so.

We assume that agents consume only at the end of their lifetimes and have identical Cobb-Douglas preferences given by

$$U = \left(c_X^{\delta} c_z^{1-\delta}\right)^{1-\theta} b^{\theta}$$

where c_X is the consumption of good X, c_Z is the consumption of good Z, and b is bequests which are modelled as a "warm glow" from giving. In the early part of the paper we neglect b, in effect setting $\theta = 0$ as we keep the distribution of wealth fixed. We assume that $y_t = b_{t+1}$ is distributed according to distribution function $F(\cdot)$ in $[0, y_{\max}]$, where $y_{\max}$ is the maximal wealth level. Then, optimal consumption of each good is a linear function of lifetime income, Y :

$$c_X = \delta Y, \quad c_Z = (1-\delta) Y,$$

2.1 The Earnings of the Skilled

We study two training systems. In the first, which we interpret as an apprenticeship system, or "Pay-As-You- Go" (PAYG) system, payment for training is not made completely up front. An apprentice supplies β hours of his time to the master at a wage w^A and spends $1-\beta$ of his time studying. If a master takes on K apprentices he obtains βK units of unskilled labor at cost $w^A\beta K$ but has to spend AK hours of his own time in training them. In the second system, called the "Pay Up Front" (PUF) system, unskilled workers pay the master a fee, w^C, up-front. The training takes $1-\beta$ units of their time and they work for the remaining time as unskilled workers.

There are no credit markets, so agents cannot borrow. Hence, each agent has to finance any up-front costs only from his wealth, which comes from bequests. When fees must be paid up front, agents with high ability but low initial wealth are barred from becoming skilled. In the apprenticeship system, credit constraints are less binding. In this manner we explore the implications of apprenticeship as a way of relaxing credit constraints. We set up the problems under the two systems. In either setting, a master can hire himself out and earn the wage of a skilled worker or set up shop and produce. In this latter event, they could hire unskilled labor and/or train unskilled workers, who, in return, work part of their time at below market wages and/or pay to be trained. Each apprentice spends $(1-\beta)$ of his time studying and works the rest of the time as an unskilled worker. Unskilled workers are paid wage w_t and apprentices are paid w_t^A, which may be positive or negative. If w_t^A is positive then we say that credit constraints do not operate as anyone who wishes to become an apprentice can do so. If w_t^A is negative then unskilled workers must pay masters. Only those who have sufficient initial wealth to do so have the option of becoming apprentices and we say that credit constraints operate. However, in both cases, trainees can pay, in part at least, as they go along.

In each period of time there are M_t masters who are the successful trainees from the last period. Since there are constant returns to scale in training and production, under the PAYG system, each skilled worker (master) must maximize value of profits he can obtain by setting himself up in business. But as long as training occurs, this must be the opportunity cost of his time as a teacher. A skilled worker who chooses only to train workers and sell off their labor at the full market price while paying them w_t^A earns

$$w_t^S = \frac{\beta}{A}(w_t - w_t^A). \tag{1}$$

Under the PUF system, trainees pay the master tuition, w_t^C, and spend $(1-\beta)$ hours of their time learning skills. In addition, they can work as unskilled workers β hours of their time and earn βw_t. Each skilled worker must earn the opportunity cost of his time as a teacher. A master can earn w_t^C from each student he trains so that the skilled wage, w_t^S, must be

$$w_t^S = \frac{w_t^C}{A}. \tag{2}$$

3 Autarky Equilibrium

3.1 Static Autarky Equilibrium

In this section we analyze the autarky equilibrium in each period t. First, we describe equilibrium under the PAYG system and then under the PUF system. Then we argue that if the two coexist, it is equivalent to having the PAYG system as it dominates.

3.1.1 The PAYG System

An equilibrium in period t is characterized by a vector of prices $\left(p_t, w_t^A, w_t^S, w_t\right)$. The proportion of agents who become apprentices is denoted by $(1-\tilde{\gamma}_t)$.

Since both goods are essential in the consumption, both goods must be produced in autarky. Therefore, the cost of making good X equals its price of unity.

$$1 = c^X(w_t, w_t^S) \tag{3}$$

The price of Z equals its cost,

$$p_t = c^Z \left(w_t, w_t^S \right). \tag{4}$$

This pins down w_t and w_t^S, and hence w_t^A, (from equation (1)) for a given p_t. From (3) and (4) we know that since Z is skill-labor intensive, as p_t rises, w_t^S rises and w_t falls. This is just the Stolper Samuelson Theorem.

Now it is obvious that for a given p_t, we will obtain the same w_t^S in both the PAYG and PUF systems. Thus, in equilibrium it must be that $\beta \left(1 - w_t^A\right) = w_t^C$ at a given p_t. From (1) it is easy to see that as p_t rises w_t^A falls more than w_t for w_t^S to rise.

Occupational Choice A young agent in period t, has two options. The first is to work both periods of his life as unskilled worker. This gives a lifetime income of $2w_t$. The second option is to invest in skills hoping to become a master. The expected lifetime income in this case equals $\beta w_t^A + \gamma E_t w_{t+1}^S + (1-\gamma) w_t$. Let $\tilde{\gamma}_t$ denote the agent who is indifferent between these two options so

$$2w_t = \beta w_t^A + \tilde{\gamma}_t^A E_t w_{t+1}^S + (1 - \tilde{\gamma}_t) w_t,$$

or

$$\tilde{\gamma}_t = \min \left\{ \frac{w_t - \beta w_t^A}{E_t w_{t+1}^S - w_t}, 1 \right\} = \min \left\{ \frac{(1-\beta) w_t + A w_t^S}{E_t w_{t+1}^S - w_t}, 1 \right\} \tag{5}$$

where the second equality follows from (1).

Agents with $\gamma \in [0, \tilde{\gamma}_t]$, work both periods of their life as unskilled workers, while agents with $\gamma \in [\tilde{\gamma}_t, 1]$, would like to be apprentices. As expected, higher profits for masters today, i.e., lower wages for apprentices today, raises $\tilde{\gamma}_t$ – *fewer* agents become apprentices. If the expected profit of a master tomorrow $(E_t \left(w_{t+1}^S\right))$ rises, i.e., the expected apprentice's wage tomorrow falls, then $\tilde{\gamma}_t$ falls and more agents become apprentices today. Thus,

$$\frac{\partial \tilde{\gamma}_t}{\partial w_t} > 0, \ \frac{\partial \tilde{\gamma}_t}{\partial w_t^S} > 0, \ \frac{\partial \tilde{\gamma}_t}{\partial E_t w_{t+1}^S} < 0.$$

Equilibrium Since each agent spends a fixed share of his income on the consumption of each good, the relative demand for good Z is equal to

$$RD_t = \frac{Z_t^D}{X_t^D} = \frac{(1-\delta)}{\delta p_t}, \tag{6}$$

where X_t^D and Z_t^D are the aggregate demands for good X and good Z respectively.

Obtaining the supply of X and Z is slightly more complicated. For a given level of expected profits, $E_t w_{t+1}^S$, and a given number of masters, the supply of X and Z at any p_t can be derived as follows. For each price, we get w_t^S, w_t^A, w_t. These determine $\tilde{\gamma}_t$ which gives the set of agents who want to become skilled. The intersection of this set with the set of agents who have wealth above w_t^A, determines how many agents are both willing and able to become apprentices. Removing the skilled labor needed for training them from the stock of masters gives the supply of skilled labor available for production, L_t^S. Adding those who choose to become unskilled workers today to the inherited stock of unskilled workers (which includes both those who chose to become unskilled yesterday and those who failed at becoming skilled) and unskilled labor supplied by apprentices today, gives the supply of unskilled labor available for production, L_t^U.

The availability of skilled and unskilled labor for production then gives the size of the standard Rybczynski box and relative supply. Hence, every price p_t corresponds to a point on the relative supply curve which is illustrated in Figures 2 (a) and 2 (b). Price p_1 corresponds to $w_t^S = w_t$. When price is below p_1, the return to skilled labor is less than the wage of unskilled worker, as a result the option of working as unskilled worker is more profitable than the option of being a master even for those who are skilled. For all prices below p_1 the supply of each good is zero. At price p_1 skilled workers are indifferent between two options so that relative supply is horizontal. Price p_2 corresponds to $w_t^S = \dfrac{\beta}{A} w_t$ and, as we can see from (1), to $w_t^A = 0$. For all prices below p_2 the apprentice's wage is positive, so credit constraints do not operate and the relative supply is denoted by RS_t^{Ancc}. For prices above p_2 the apprentice's wage is negative, i.e., workers pay to become apprentices. In this event, agents are subject to credit constraints. Relative supply in this region is denoted by RS_t^{Acc}. When price exceeds p_3, given the expected skilled wage tomorrow, the apprentice's wage is so low that the option of investing in skills is dominated and there are no apprentices ($\tilde{\gamma}_t = 1$).

Next, we turn to the shape of the relative supply curve and the nature of static equilibrium. An equilibrium in which the apprentice's wage is positive is a non-credit-constrained (NCC) equilibrium. An equilibrium in which the apprentice's wage is negative is a credit-constrained (CC) equilibrium.

Proposition 1 *Under the PAYG system, if the number of masters in period t is small enough, i.e., $M_t \leq \tilde{M} = \dfrac{2A}{(1+A-\beta)}$, then relative supply is increasing in price. If there are enough masters, i.e., $M_t > \tilde{M}$, then relative supply need not be increasing in price and multiple equilibria are possible.*

A formal proof in the Appendix. We focus on the intuition behind this result here. Suppose that the price increases. This results in a higher return to skilled labor and in a lower apprentice's wage.[9] In a static setting, the fall in the apprentice's wage makes investing in skills less profitable and $\tilde{\gamma}_t$ increases. Hence, the supply of unskilled labor in period t rises. As fewer agents wish to be apprentices, masters spend less time training them and the supply of skilled labor available for production increases. Note that both skilled and unskilled labor available for production rise so that their relative availability may rise or fall.

The effects on relative supply of a price change can be decomposed into two parts. First, there is the standard positive supply response. An increase in the relative price of the skill-intensive good raises the relative wage of skilled labor a la the Stolper-Samuelson Theorem. This results in substitution away from skilled labor. For given factor supplies, this raises the relative supply of the skill-intensive good. Second, there is an induced Rybczynski effect: an increase in the availability of skilled relative to unskilled labor for production purposes raises the relative supply of the skill-intensive good.

When there are few masters, $M_t < \tilde{M}$, then both effects work in the same direction. Any given increase in $\tilde{\gamma}_t$ that occurs from the price increase releases what amounts to a large

[9] If the there are credit constraints, this translates into the fee paid by an apprentice going up.

percentage increase in the supply of skilled labor so that skilled labor becomes relatively more abundant, and relative supply of Z rises. In this case, relative demand curve can intersect the relative supply curve at most once: either in its non-credit-constrained part, or in its credit-constrained part.

If there are many masters, $M_t > \tilde{M}$, then the induced Rybczynski effect works in the opposite direction from the normal supply response. With many masters, the supply of skilled labor is relatively large. Any change in $\tilde{\gamma}_t$ translates into a small percentage increase in the supply of skilled labor and, as a result, skilled labor becomes relatively less abundant and the relative supply of Z may fall! In this case relative supply may be downward sloping even without credit constraints making multiple equilibria possible. Multiple equilibria in this static set-up arise from the interaction of credit constraints and prices. When price is low, so is the return to skilled labor. In this case, the apprentice's wage is high, there are no credit constraints and a large fraction of the population becomes apprentices. Since M_t is large, despite this, there is a lot of skilled labor available for production and output is high. Since price is low, demand is high and this can be an equilibrium. On the other hand, if price is high, so is the return to skilled labor and for this, apprentice's wages are negative. Credit constraints operate and many agents cannot become apprentices. While this does free up some skilled labor for production, there is an ample supply of unskilled workers. Hence the relative supply of skilled workers is low, as is the relative supply of Z.[10]

Figures 2(a) and 2(b) depict relative supply for the two cases.

3.1.2 The PUF System

An equilibrium in period t is characterized by a vector of prices $\left(p_t, w_t^S, w_t, w_t^C\right)$. As shown below, agents with abilities above the cutoff level, $\tilde{\gamma}_t$, choose to get trained, so that $(1 - \tilde{\gamma})$ is the proportion of agents who become trainees.

[10]Note that not all static equilibria are consistent with steady state: for example, if the intersection occurred at prices above p_3, $\tilde{\gamma} = 1$ and nobody would invest in skills. If there are no masters in period $t+1$, then the return to skilled labor would be infinite which is not consistent with expectations or possible in steady state.

Occupational Choice $\tilde{\gamma}_t$ is determined from

$$2w_t = \beta w_t - w_t^C + \tilde{\gamma}_t E_t w_{t+1}^S + (1 - \tilde{\gamma}_t) w_t,$$

or

$$\tilde{\gamma}_t = \min \left\{ \frac{w_t - \beta w_t + w_t^C}{E_t w_{t+1}^S - w_t}, w_t \right\} = \min \left\{ \frac{w_t - \beta w_t + A w_t^S}{E_t w_{t+1}^S - w_t}, w_t \right\}$$

where the second equality follows from (2).

In both systems the proportion of agents wanting to invest in skills is given by the same function. As in the PAYG system,

$$\frac{\partial \tilde{\gamma}}{\partial w_t^S} > 0, \ \frac{\partial \tilde{\gamma}_t}{\partial E_t w_{t+1}^S} < 0.$$

Equilibrium We can use the approach used in the analysis of the PAYG system to derive the relative supply here. It is easy to check that when there are few masters, the relative supply under the PUF system is upward-sloping, but if the number of masters is large then the relative supply can be downward-sloping.

Proposition 2 *Under the PUF system, if* $M_t \leq \tilde{M}$ *then relative supply is increasing in price and there is a unique equilibrium. If* $M_t > \tilde{M}$, *then relative supply need not be increasing in price and there may be multiple equilibria.*

What can we say about the relative position of relative supply curves under the PUF system and under the PAYG system? For any price there is a common level of earnings for skilled labor, w_t^S, and corresponding to this, an education cost – the tuition fee, w_t^C, in the PUF system or the implicit price of $\beta(1 - w_t^A)$ under the PAYG system. All of the education cost is paid up front in the PUF system, while only $\max\{0, -\beta w_t^A\}$ is paid up front in the PAYG system, so that more is always paid up front under the PUF system. Hence, at any given p_t (or, at any w_t^S corresponding to this price) a larger proportion of agents are credit-constrained under the PUF system.

At any given price p_t, both systems have the same cutoff level for ability. However, since credit constraints are stricter in the PUF system, fewer agents can actually become trainees, the rest of those who want to do so cannot, and remain unskilled so that the supply of unskilled workers is larger at any given p_t under the PUF system. The smaller number of trainees under the PUF system requires less skilled labor to train them. As a result, the skilled labor available for production is also larger under the PUF system. What can we say about relative availability? As before, when there are few masters the relative availability of skilled labor is higher under the PUF system, while when there are many masters the relative availability of skilled labor is lower under the PUF system. Hence, the effect on relative supply follows from the relative factor availability under the two systems.

If there are few masters, $M_t \leq \tilde{M}$, then at any given p_t, the relative supply of the skill-intensive good is more than that under the PAYG system. If there are many masters, $M_t > \tilde{M}$, it is less than that in the PAYG system. This is depicted in Figures 2(a) and 2(b). In Figure 2(a) the number of masters is small, and, as a result, the relative supply curves for both systems are upward-sloping. At any price below p_3 (so that the proportion of agents who invest in skills is positive) the relative supply curve under the PUF system is to the right of the relative supply curve under the PAYG system. When price is above p_3 there are no agents who invest in skills and the two curves coincide. Therefore, there is a unique equilibrium in each system, and the equilibrium price under the PUF system is lower than the equilibrium price under the PAYG system. Since the equilibrium price is lower under the PUF system, we cannot conclude that the tuition in the PUF system is higher than the full fee under the PAYG system.

In Figure 2(b) the number of masters is large. The relative supply curve under the PUF system is to the left of the relative supply curve under the PAYG system. And, as a result, the equilibrium price in the PUF system is higher. Moreover, as the price is higher and so is the full fee and hence the up-front education fee under the PUF system.

Proposition 3 summarizes these results.

Proposition 3 *At any given p_t, relative supply of the skill-intensive good is higher under the PUF system than that under the PAYG system as long as $M_t \leq \tilde{M}$. As a result, the equilibrium price is lower than the equilibrium price under the PAYG system. If $M_t > \tilde{M}$,* *relative supply of the skill-intensive good is lower under the PUF system than that under the PAYG system. Consequently, the equilibrium price under the PUF system is higher than the equilibrium price under the PAYG system.*

It is worth noting that if both systems coexist, the outcome is exactly what it would have been under the PAYG system. The cutoff levels for the two systems cannot differ.[11] Suppose both systems coexisted and the cutoff levels (the $\tilde{\gamma}'$s) for the two differed. Then the payoff from teaching in the two systems would differ and only the system with the higher payoff would operate.

3.2 Steady State Autarky Equilibrium

In this section we solve for autarky steady state equilibrium.

3.2.1 The PAYG System

Occupational Choice $\tilde{\gamma}$ is, as usual, determined from

$$2w = \beta w^A + \tilde{\gamma}^A w^S + (1-\tilde{\gamma})\, w$$

However, in steady state, expected profits tomorrow equal actual profits today so that $w^S = \dfrac{\beta}{A}(w - w^A)$ and

$$\tilde{\gamma} = \min\left\{\frac{(1-\beta)\,w + Aw^S}{w^S - w}, 1\right\} = \begin{cases} 1, \text{ if } w^S \leq \dfrac{(2-\beta)\,w}{1-A} \\ \dfrac{(1-\beta)\,w + Aw^S}{w^S - w}, \text{ if } w^S > \dfrac{(2-\beta)\,w}{1-A} \end{cases} \quad (7)$$

Note that a higher return to skilled labor means a lower apprentice's wage w^A, hence there is a trade-off between a higher earnings tomorrow and a lower wage today. But since a fall in w^A raises the earnings of a master in the next period by more than it reduces the wage of an apprentice in the current period, a higher w^S leads to a lower $\tilde{\gamma}$: more agents become apprentices. Thus,

$$\frac{\partial \tilde{\gamma}}{\partial w^S} < 0.$$

[11] If they are the same, it is equivalent to the PAYG system.

Note that the relationship between $\tilde{\gamma}$ and the return to skilled labor has the opposite sign from that in the static set-up. In steady state the return to the option of investing in skills is increasing in w^S as explained above. In the static set-up, the master's earnings expected in the next period are fixed and agents take into account only the apprentice's wage in the current period, and as a higher w^S means a lower apprentice's wage, the return to the option of investing in skills is decreasing in w^S.

Lemma 1 *In steady state, $\tilde{\gamma} > A$.*

Proof. Using $w^S = \dfrac{\beta}{A}(w - w^A)$ we can rewrite the return to the option of investing in skills as $\beta w + (1 - \gamma) w + w^S (\gamma - A)$. Then, for $\gamma < A$ this return is clearly less than the lifetime income of unskilled worker. As profit goes to infinity ($w^S \to \infty$) the proportion A of agents work as unskilled labor and $(1 - A)$ become apprentices, i.e., $\tilde{\gamma} \to A$. Therefore, in steady state the proportion of agents who become apprentices never exceeds $1 - A$. ■

Equilibrium Relative demand is standard and depends only on relative price. We can use the approach used in the static set-up to derive the relative supply here as well which is depicted in Figure 3. Price p_1 corresponds to w^S such that $\gamma^A = 1$. For all prices below p_1 nobody invests in skills, so production of each good is zero. Price p_2 corresponds to $w^S = \dfrac{\beta}{A}$ and, as in the static set-up, to $w^A = 0$. For all prices below p_2 the apprentice's wage is positive, so that credit constraints are not binding and relative supply is denoted by RS^{Ancc}. For prices above p_2 the apprentice's wage is negative, and agents are subject to credit constraints. Relative supply in this region is denoted by RS^{Acc}.

Next, we turn to the shape of the relative supply curve and the nature of steady state equilibrium. First, we prove that when credit constraints are not operating the relative supply curve is upward-sloping. Suppose that price rises. Then the return to skilled labor rises as well, and this results in lower $\tilde{\gamma}$: more agents choose to invest in skills at a higher price. Since more agents become apprentices, the supply of unskilled labor falls. What can we say about the supply of skilled labor? Subtracting the skilled labor needed for training from the stock of masters gives the supply of skilled labor available for production:

$$\begin{aligned} L^S &= M - AK \\ &= \frac{1}{2}\left(1 - (\tilde{\gamma})^2\right) - A\left(1 - \tilde{\gamma}\right), \end{aligned}$$

where $M = \frac{1}{2}\left(1 - (\tilde{\gamma})^2\right)$ is the number of masters and $K = 1 - \tilde{\gamma}$ is the number of apprentices. As price rises, the number of apprentices increases, as does the number of masters. The total effect on the supply of skilled labor is positive – L^S increases with the price since:

$$\frac{dL^S}{dp} = \frac{d\tilde{\gamma}}{dp}\left(A - \tilde{\gamma}\right) > 0$$

where the inequality follows from Lemma 1. As a result, via standard Rybczynski arguments, the relative supply of the skill-intensive good rises with p.

When agents are subject to credit constraints, the effects of an increase in price on the supply of unskilled and skilled labor are ambiguous and depend on the distribution of wealth. If a higher price results in unskilled labor becoming relatively more abundant, then the relative supply of the skill-intensive good may fall. If, for example, there are many agents who become credit-constrained at this higher price, then a large proportion of agents who invested in skills at lower price cannot afford to do so. Hence, the supply of unskilled labor rises, and the supply of skilled labor available for production of good Z falls. As a result, the relative supply of good Z may be lower at this higher price. Thus, when credit constraints operate, the shape of the relative supply curve depends on the distribution of wealth and can be either upward-sloping or downward-sloping.

Proposition 4 summarizes these results.

Proposition 4 *Under the PAYG system, if $p \leq p_2$ then credit constraints are not binding and steady state relative supply is increasing in price. If $p > p_2$ then credit constraints are binding, steady state relative supply need not be increasing in price, and multiple steady state equilibria may exist.*

3.2.2 The PUF System

Occupational Choice Following the same procedure as above we can see that $\tilde{\gamma}$ is determined from

$$2w = -w^C + \beta w + \tilde{\gamma} w^S + (1 - \tilde{\gamma}) w.$$

Using $w^S = \frac{w^C}{A}$ we have

$$\tilde{\gamma} = \min\left\{\frac{w - \beta w + w^C}{w^S - w}, 1\right\} = \begin{cases} 1, \text{ if } w^S \leq \frac{(2-\beta) w}{1-A} \\ \frac{(1-\beta) w + A w^S}{w^S - w}, \text{ if } w^S > \frac{(2-\beta) w}{1-A} \end{cases} \tag{8}$$

As in the PAYG system, a higher return to skilled labor decreases $\tilde{\gamma}$ – more agents want to become trainees.

Equilibrium Since under the PUF system, credit constraints operate at all prices, we can show the following:

Proposition 5 *Under the PUF system steady state relative supply need not be increasing in price. Multiple steady state equilibria may exist: a key determinant is the distribution of wealth.*

What can we say about the relative position of steady state relative supply curves under the PUF system and under the PAYG system? At any given price, both systems have the same cutoff level of ability. However, since credit constraints are stricter in the PUF system, fewer agents can become trainees, the rest of those who want to do so cannot, and remain unskilled so that the supply of unskilled workers is larger at any given price under the PUF system. The smaller number of agents acquiring skills under the PUF system results in a lower supply of skilled labor available for production in the steady state. As a result, at any price the supply of good Z relative to good X is lower under the PUF system. Figure 3 depicts relative supply curves in both system, where relative supply under the PUF system is denoted by RS^C, relative supply under the PAYG system when credit constraints are not binding is denoted by RS^{Ancc} and when credit constraints are binding by RS^{Acc}. As depicted, relative supply curve under the PUF system is to the left of the relative supply curve under the PAYG system. And, as a result, the equilibrium price under the PUF system is higher. Moreover, as the price is higher, so is the full fee and hence the up-front education fee under the PUF system. This shift out in relative supply of the skill-intensive good when a country has better credit markets, in combination with better credit markets raising the mean wealth, drives the result in Ranjan (2001) that at a given price, the country with better credit markets will export the skill-intensive good.

Proposition 6 summarizes these results. These are in the same spirit as Lemma 1 in Ranjan (2001) except that here we separate out the effects via the distribution of income.

Proposition 6 *At any given* p*, steady state relative supply is lower under the PUF system than that under the PAYG system. As a result, the steady state autarky equilibrium price is higher under the PUF system.*

4 Effects of Trade

Having described the closed economy, we turn to the analysis of the effects of opening the economy up to trade. Consider the welfare effects of trade when the country is small and cannot affect the world price of good Z, denoted by p^T.[12] Steady state relative supply need not be increasing in price and multiple equilibria may exist under either system if credit constraints operate. As we show below, non-monotonicity of relative supply may result in trade equilibria where the country ends up importing good Z at world prices higher than its autarky price and, as a result, loses from opening up to trade.

Consider the situation depicted in Figure 4. The relative supply is non-monotonic and there are two stable steady state autarky equilibria: E_1^A and E_2^A. At equilibrium E_1^A the price, p_1^A, is low and relative supply of good Z is high, while at E_2^A the price, p_2^A, is high and supply is low. Suppose that initially the economy is in autarky equilibrium E_1^A. What

[12]Convergence to steady state will be very quick, limited only by number of masters available to train the needed skilled labor.

happens when the country opens up to trade and faces the world price of good Z which is higher than its autarky price? As the price rises, the earnings of a master rise as well, so that the option of investing in skills becomes more attractive. As more agents decide to invest in skills, the up-front education fee goes up. Credit constraints become tighter and more agents are unable to afford education. Consequently, the supply of the skilled labor decreases, resulting in lower output of good Z as well as lower relative supply, denoted by RS^T. As the price goes up, the relative demand for good Z, denoted by RD^T, falls. If a significant proportion of agents becomes credit constrained at this higher price, then the decrease in relative supply is considerable and exceeds the decrease in relative demand: opening up to trade allows only few rich agents to invest in skills and, as a result, the supply of the skill-intensive good falls dramatically and cannot satisfy domestic demand at this price. Thus, the country imports good Z even though the world price is higher than the autarky price! Since the country loses its comparative advantage in good Z and has to import this good at a higher price, the aggregate welfare can be lower with trade.

Proposition 7 provides sufficient conditions for welfare to fall with trade when all agents have identical homothetic preferences so that aggregation is possible.

Proposition 7 *Opening up an economy to trade need not be welfare-improving. Aggregate welfare may fall with trade if a country imports good Z at world prices higher than autarky price. If substitution in consumption is small enough and if the value of autarkic output at trade prices exceeds that of the trade output, welfare must fall due to trade.*

Proof. In the Appendix. ■

The outcome in Figure 4 is depicted in Figure 5 in a way that highlights the condition given above. The production possibility envelope in steady in the presence of credit constraints need not have the usual shape: it can be bowed in.[13] At p^T, and similarly at p^A, there is a given availability of skilled and unskilled labor for production in steady state.

[13]It must also lie inside the steady state PPF envelope in the absence of credit constraints (which has the usual bowed out shape as in Findlay and Kierzkowski (1983)). This makes sense as credit constraints result in inefficiency: the wrong people are trained, which limits production possibilities.

This defines the usual PPF at the given price. These PPFs are tangent to their respective price lines at Q^T and Q^A respectively as depicted in Figure 5. The relative demand for good Z exceeds relative supply at p^T but equals it at p^A as shown. If there is no substitution in consumption, demand always lies along the ray OA and if as depicted, the line connecting Q^T and Q^A is flatter than p^T, then welfare must fall with trade. Note that limited substitution in consumption is essential: if substitution in consumption is large enough, say if the indifference curves are close to being linear so that they are not too far from the autarky price line, then the result does not go through.

The intuition is simple: when the price of the skill-intensive good rises through trade, credit constraints are tightened and its relative supply falls. Since the supply of the skill-intensive good is too low to begin with, the distortion is worsened by trade and welfare falls. This suggests that developed countries, who presumably have a comparative advantage in the skill-intensive good, need to ensure access to education to fully reap the gains from trade.

5 Endogenizing the Distribution of Income

We consider a small open economy. We look first at the situation where credit constraints do not bind and then move on to when they are binding.

5.1 No Credit Constraints

Consider the PAYG system when w^A is positive so that there are no credit constraints. In this case, the income distribution does not affect the equilibrium, but the equilibrium affects the income distribution so that there is only one way causation. An agent's total income, which is allocated across the goods and bequests at the end of his second period of life is the sum of his bequest, b_t, and his earned income. Let $Y_t^j(b_t)$ denote this total income for $j = u, f, s$ since earned income can take only three values, that of unskilled (u), the failed skilled (f), and the skilled (s). The level of these depend on p as this determines w^A.

Thus, bequests of those born in period t who inherit b_t are given by $b(b_t, j) = \theta\left(Y_t^j(b_t)\right)$ or

$$b(b_t, j) = \begin{cases} \theta\left(b_t + \beta w^A + w\right), \text{ for } j = f \\ \theta\left(b_t + 2w\right), \text{ for } j = u \\ \theta\left(b_t + \beta w^A + \dfrac{\beta\left(w - w^A\right)}{A}\right), \text{ for } j = s \end{cases}$$

Note that for a given p, earned income is lowest for the failed skilled and highest for the skilled. Moreover, an increase in p raises w^S and reduces w. Since $\dfrac{\beta\left(w - w^A\right)}{A} = w^S$, this means that w^A falls and that the fall in w^A exceeds the fall in w. Thus, the nominal income of the unskilled falls as does that of the failures, and so does their real income. Income of the skilled may increase or decrease depending on p.[14]

Bequest lines are depicted in Figure 6. Note that there is no way for bequests to exceed $\bar{b}^H$ or fall below $\underline{b}^L$: if all an agent's ancestors were skilled, the bequest he would get would be $\bar{b}^H$ and if all of them were failures, it would be $\underline{b}^L$. The bequests that can be made by agents of type $j = f$ are contained in the interval $\left[\underline{b}^L, \bar{b}^L\right]$ while those made by agents of type $j = u$ are contained in $\left[\underline{b}^M, \bar{b}^M\right]$, while those made by agents of type s are contained in $\left[\underline{b}^H, \bar{b}^H\right]$. Since the bequest line is flatter than the 45^0 line, the Feller condition is met and as the distribution of abilities is common across all agents, the Markov process converges to a unique invariant bequest and associated total income distribution.

To analyze this further, we assume that the intervals defined above are disjoint as in Figure 6.[15] Let p^L, p^M, p^H (for low, middle and high) denote the fraction of agents in steady state who are in each of these intervals. Then we can calculate the steady state levels of these variables for given values of $\tilde{\gamma}$ for the given distribution of talent, which we take to be uniform over $[0, 1]$. Thus, we can obtain the steady state distribution of bequests and wealth at a given p since for each p we get a unique $\tilde{\gamma}$. We can show the following.

Proposition 8 *An increase in p raises inequality in the distribution of income in the absence of credit constraints both because the poor get poorer and the rich richer and because*

[14] Income of skilled decreases if $dw < (1 - A)dw^A$.

[15] This requires that θ be small enough.

there are more of them.

Proof. In the Appendix. ■

Being rich or poor is just a matter of luck with no long term consequences so that one thinks of inequality as being less pernicious in the above setting.

5.2 Credit Constraints

With credit constraints, there is an additional complication: namely that only the subset of agents who have bequests over w^C, the up front costs of education, can be trained. Suppose that w^C lies between $\bar{b}^L$ and $\underline{b}^M$ (Case A). The unskilled are thus all those with ability below $\tilde{\gamma}$ (which have a mass of $\tilde{\gamma}$) as well as those whose parents failed at becoming skilled but whose γ exceeds $\tilde{\gamma}$ (which have a mass of $(1-\tilde{\gamma})p^L$). Note that with credit constraints, there is two way causation: not only does the equilibrium affect the income distribution, but the income distribution affects the equilibrium through the extent of credit constraints. Let $\varsigma(\tilde{\gamma}) = \dfrac{\left(1-\tilde{\gamma}^2\right)}{2}$ denote the expected yield of the successes and let $\phi(\tilde{\gamma}) = \dfrac{(1-\tilde{\gamma})^2}{2}$ denote the expected yield of failures when the cutoff level is $\tilde{\gamma}$. Then in steady state:

$$
\begin{aligned}
p^L &= \frac{(1-\tilde{\gamma})^2}{2}\left(1-p^L\right) = \phi(\tilde{\gamma})\left(1-p^L\right) \\
p^M &= \tilde{\gamma} + (1-\tilde{\gamma})\, p^L \\
p^H &= \frac{\left(1-\tilde{\gamma}^2\right)}{2}(1-p^L) = \varsigma(\tilde{\gamma})(1-p^L)
\end{aligned}
$$

Note that in this instance, the children of the skilled and unskilled are not constrained by bequests, only the children of the unfortunate failures are. However, since their children can become skilled, the mixing condition is still met.

Since ϕ and ς are increasing in p, p^L rises, and p^M falls, though p^H may rise or fall. As the price rises, the incomes of the skilled rise while that of the unskilled and failures fall.

Proposition 9 *An increase in p raises inequality in the distribution of income in the presence of credit constraints because the poor get poorer and the rich richer but can reduce inequality to the extent that there are fewer rich in steady state.*

If w^C lies between $\bar{b}^M$ and $\underline{b}^H$ (Case B) then the children of the unskilled cannot become skilled and the economy falls into a poverty trap. This can be seen since

$$\begin{aligned} p^L &= \phi(\tilde{\gamma})p^H \\ p^M &= 1 - p^H + \tilde{\gamma}p^H \\ p^H &= \varsigma(\tilde{\gamma})p^H \end{aligned}$$

so the only solution is $p^H = p^L = 0$.

A non degenerate equilibrium can be restored by perturbing the economy a bit. The perturbation can be interpreted as the government giving aid by removing the credit constraint of a fraction σ of those who are constrained, independent of their ability. This makes sense since ability need not be easily verifiable so that targeting the most able, as would be efficient, may not be easy. If we do so then we get

$$\begin{aligned} p^H &= \varsigma(\tilde{\gamma})\left(p^H + \sigma(1 - p^H)\right) \\ p^L &= \phi(\tilde{\gamma})\left(p^H + \sigma(1 - p^H)\right) \\ p^M &= \tilde{\gamma} + (1 - p^H)(1 - \tilde{\gamma})(1 - \sigma) \end{aligned}$$

Solving for p^H gives

$$p^H = \frac{\sigma\varsigma(\tilde{\gamma})}{1 - \sigma\varsigma(\tilde{\gamma})}$$

The untalented ($\tilde{\gamma}$) as well as the credit constrained talented ($(1 - p^H)(1 - \tilde{\gamma})$) who do not get their credit constraints relaxed earn unskilled wages in both periods which explains p^M. The successful able children of parents who earned skilled wages as well as the successful able children of the rest of the population who had their credit constraints relaxed make up p^H. Their failed able children make up p^L. Note that even a small σ moves the economy away from the poverty trap.

Finally, we ask when inequality might be more pernicious. Here we look at the consequences of a more subtle manifestation of credit constraints. Recall that conditioning on ability (test scores), income seems to play no role on college attendance. This suggests that income mainly operates through affecting the ability of children at the time of graduation from high school! The poor, one way or the other, seem to not invest as much in their children as the rich do. We model this by having the skilled draw from a different distribution of talent, say one which is uniform over $[\lambda, 1]$.

Proposition 10 *An improvement in the distributions that the skilled draw from has a multiplier effect in the steady state distribution of income and of skills. The elasticity of* p^H *with respect to* λ *is increasing in* λ*, suggesting that when the school systems accessed by the rich and poor differ greatly, we should see very significant inequality even if there are no credit constraints per se.*

Proof. In the Appendix. ■

6 Conclusions

In this paper we develop a model where apprenticeships help overcome credit constraints that limit the ability of agents with heterogeneous abilities and wealth to acquire skills. We show that in the static version of our model, under either system, the response of supply to price depends on the number of skilled agents in the economy. If there are relatively few skilled agents, the normal supply response obtains. However, with many skilled agents, supply can be decreasing in price so that multiple equilibria may exist. In steady state, however, such non monotonicity of supply and multiplicity of equilibrium obtains only in the presence of credit constraints. Since credit constraints are stricter in the PUF system, relative supply of the skill-intensive good is always higher, at any given price, under the apprenticeship system. There may or may not be multiple equilibria in steady state: a key determinant is the distribution of wealth. Finally, we show that opening the economy to trade could easily reduce welfare.

Appendix

Proof of Proposition 1. For any price p_t we get wages, w_t^S and w_t, and the ratio of unskilled to skilled labor in each sector, $\left(\frac{L_Z^U}{L_Z^S}\right)_t$ and $\left(\frac{L_X^U}{L_X^S}\right)_t$. Since good Z is relatively skill-intensive we have $\left(\frac{L_Z^U}{L_Z^S}\right)_t < \left(\frac{L_X^U}{L_X^S}\right)_t$ for any p_t. This is depicted in Figures 1(a) and 1 (b), where the supply of skilled labor is on horizontal axis and the supply of unskilled labor is on vertical axis. The allocation of skilled and unskilled labor between sectors is at P. Now suppose that price increases. Then the ratio of unskilled to skilled labor in each sector increases. For fixed endowments of skilled and unskilled labor the outcome is at P_1. It is easy to see that at P_1 supply of good Z is higher and supply of good X is lower compared to P. This is the usual supply response for fixed factor endowments. However, the increase in p_t also increases both endowments of skilled and unskilled labor. If the supply of skilled and the supply of unskilled labor increase proportionately, then given the assumption that production functions are homothetic outputs of Z and X also increase proportionately. Hence, from P_1 to P_2 relative supply does not change.

Figure 1 (a) illustrates the case when change in endowment of skilled labor is proportionately more than the change in the endowment of unskilled labor. In this case, the 'Rybczynski' effect works in the same direction as the usual supply effect, moving the equilibrium to P'. Figure 1 (b) illustrates the case when the change in the endowment of skilled labor is proportionately less than the change in the endowment of unskilled labor. Now the 'Rybczynski' effect works in the opposite direction and tends to decrease relative supply of good Z. This is shown by P' being closer to the origin O_Z than P_2.

Next, we show that whether the change in the endowment of skilled labor is proportionately less or proportionately more than the change in the endowment of unskilled labor depends on the number of masters M_t.

When credit constraints do not bind the available supplies of skilled and unskilled labor are

$$S_{t,ncc} = M_t - A\left(1 - \tilde{\gamma}_t\right)$$

$$U_{t,ncc} = \beta\left(1-\tilde{\gamma}_t\right) + \tilde{\gamma}_t + 1 - M_t$$

When credit constraints operate the available supplies of skilled and unskilled labor are

$$\begin{aligned} S_{t,cc} &= M_t - A\left(1-\tilde{\gamma}_t\right)\left(1-F(-w_t^A)\right)) \\ L_{t,cc}^U &= \tilde{\gamma}_t + F(-w_t^A)\left(1-\tilde{\gamma}_t\right) + \beta\left(1-F(-w_t^A)\right)\left(1-\tilde{\gamma}_t\right) + 1 - M_t \end{aligned}$$

If credit constraints are not binding then

$$d\left(\frac{L_t^U}{L_t^S}\right)/dp_t = \left(M_t\left(1+A-\beta\right) - 2A\right)\frac{d\tilde{\gamma}_t}{d\left(w_t^S\right)}\frac{d\left(w_t^S\right)}{dp_t}$$

If credit constraints are binding then

$$\begin{aligned} d\left(\frac{L_t^U}{L_t^S}\right)/dp_t &= \left(\frac{d\tilde{\gamma}_t}{d\left(w_t^S\right)}\left(1-F(-w_t^A)\right) + \frac{d(-w_t^A)}{d\left(w_t^S\right)}f(-w_t^A)\left(1-\tilde{\gamma}_t\right)\right)\times \\ &\quad \times\left(M_t\left(1+A-\beta\right)-2A\right)\frac{d\tilde{\gamma}_t}{d\left(w_t^S\right)}\frac{d\left(w_t^S\right)}{dp_t} \end{aligned}$$

Since

$$\frac{d\tilde{\gamma}_t}{d\left(w_t^S\right)} > 0, \frac{d\left(w_t^S\right)}{dp_t} > 0 \text{ and } \frac{d(-w_t^A)}{d\left(w_t^S\right)} > 0$$

we have that

$$sign\left(d\left(\frac{L_t^U}{L_t^S}\right)/dp_t\right) = sign\left(M_t\left(1+A-\beta\right)-2A\right)$$

Therefore, if $M_t < \tilde{M} = \dfrac{2A}{(1+A-\beta)}$, then $d\left(\frac{L_t^U}{L_t^S}\right)/dp_t < 0$ and the 'Rybczynski' effect works in the same direction as the usual supply response. ■

Proof of Proposition 7. Let $e(P,u)$ be the usual expenditure function. Let p, Q, C, u denote the price, output and consumption vectors while u denotes utility. The superscripts A and T refer to the autarky and trade outcomes. We know that

$$\begin{aligned} e(p^A,u^A) &= p^A Q^A = p^A C^A \text{ and } Q^A = C^A \\ e(p^T,u^T) &= p^T Q^T = p^T C^T \\ e(p^A,u^T) &< p^A C^T, \\ e(p^T,u^A) &< p^T C^A = p^T Q^A. \end{aligned}$$

Thus,

$$e(p^T, u^T) - e(p^T, u^A) > p^T C^T - p^T C^A$$
$$= p^T Q^T - p^T Q^A. \tag{9}$$

If there is no substitution in consumption, the first inequality is an equality. Hence, if

$$p^T Q^A > p^T Q^T \tag{10}$$

then $e(p^T, u^T) - e(p^T, u^A) < 0$ so $u^T < u^A$. By continuity, if substitution in consumption is small enough and (10) holds, $u^T < u^A$. ■

Proof of Proposition 8. p^M equals the mass of all agents who choose to be unskilled, or $\tilde{\gamma}$. Similarly, p^L must equal the mass of agents who are failures, or $\int_{\tilde{\gamma}}^{1} (1-\gamma)\, d\gamma$, and p^H must equal the mass of agents who become skilled, or $\int_{\tilde{\gamma}}^{1} \gamma d\gamma$. Thus,

$$p^L = \frac{(1-\tilde{\gamma})^2}{2}$$
$$p^M = \tilde{\gamma}$$
$$p^H = \frac{(1-\tilde{\gamma}^2)}{2}.$$

It is easy to verify that $\frac{dp^L}{d\tilde{\gamma}} < 0$, $\frac{dp^M}{d\tilde{\gamma}} > 0$, $\frac{dp^H}{d\tilde{\gamma}} < 0$.

Since $\tilde{\gamma}$ falls as p rises, $\frac{dp^L}{dp} > 0$, $\frac{dp^M}{dp} < 0$, $\frac{dp^H}{dp} > 0$, so that an increase in p moves mass from the middle to the tails in the bequest distribution. Note that the location of the tails themselves will also change as p also affects incomes, and hence $Y_t^j(b_t)$. ■

Proof of Proposition 10. First, consider the case when credit constraints do not bind. The mass of agents who choose to be unskilled are those with abilities below $\tilde{\gamma}$. The ones who have skilled parents, who are (p^H) of the total in steady state, have a probability $\left(\frac{\tilde{\gamma}-\lambda}{1-\lambda}\right)$ of being in the region $[\lambda, \tilde{\gamma}]$ which is less than $\tilde{\gamma}$, that of agents who do not have skilled parents. Thus, p^M is a convex combination of these two probabilities. Those who choose to try and get trained are those with abilities above $\tilde{\gamma}$. The ones who have skilled parents, who are (p^H) of the total in steady state, have a probability $\left[\frac{(1-\tilde{\gamma})^2}{2}\right]\frac{1}{1-\lambda}$ of being in the region $[\tilde{\gamma}, 1]$ while those who do not have skilled parents, who are $(1-p^H)$ of the total in steady state, have a probability $\left[\frac{(1-\tilde{\gamma})^2}{2}\right]$. Making similar calculations for p^H yields:

$$\begin{aligned}
p^L &= \left[\frac{(1-\tilde{\gamma})^2}{2}\right]\left[(1-p^H)+\frac{p^H}{1-\lambda}\right] \\
p^M &= \tilde{\gamma}(1-p^H)+p^H\left(\frac{\tilde{\gamma}-\lambda}{1-\lambda}\right) \\
p^H &= \left[\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right]\left[(1-p^H)+\frac{p^H}{1-\lambda}\right].
\end{aligned}$$

Solving this gives

$$\begin{aligned}
p^H &= \frac{\left[\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right]}{\left[1-\left(\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right)\left(\frac{\lambda}{1-\lambda}\right)\right]} \\
p^L &= \left[\frac{(1-\tilde{\gamma})^2}{2}\right]\left[(1+\frac{\left(\frac{\lambda}{1-\lambda}\right)\left[\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right]}{\left[1-\left(\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right)\left(\frac{\lambda}{1-\lambda}\right)\right]}\right] \\
p^M &= \tilde{\gamma}-(1-\tilde{\gamma})\frac{\left(\frac{\lambda}{1-\lambda}\right)\left[\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right]}{\left[1-\left(\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right)\left(\frac{\lambda}{1-\lambda}\right)\right]}.
\end{aligned}$$

Similar, if credit constraints operate (Case A) we have

$$\begin{aligned}
p^L &= \left[\frac{(1-\tilde{\gamma})^2}{2}\right]\left[p^M+\frac{p^H}{1-\lambda}\right] \\
p^M &= p^L+\tilde{\gamma}p^M+p^H\left(\frac{\tilde{\gamma}-\lambda}{1-\lambda}\right) \\
p^H &= \left[\frac{\left(1-\tilde{\gamma}^2\right)}{2}\right]\left[p^M+\frac{p^H}{1-\lambda}\right].
\end{aligned}$$

Solving this gives

$$
\begin{aligned}
p^H &= \frac{1}{\left[\frac{1+\frac{(1-\tilde{\gamma})^2}{2}}{\frac{\left(1-\tilde{\gamma}^2\right)}{2}} - \left(\frac{\lambda}{1-\lambda}\right)\right]} \\
p^L &= \frac{\left[\frac{(1-\tilde{\gamma})^2}{2}\right]}{\left[1+\frac{(1-\tilde{\gamma})^2}{2}\right] - \frac{\left(1-\tilde{\gamma}^2\right)}{2}\left(\frac{\lambda}{1-\lambda}\right)}
\end{aligned}
$$

Note that p^L and p^H are higher and, hence, p^M is lower, than when they draw from the same distribution. ■

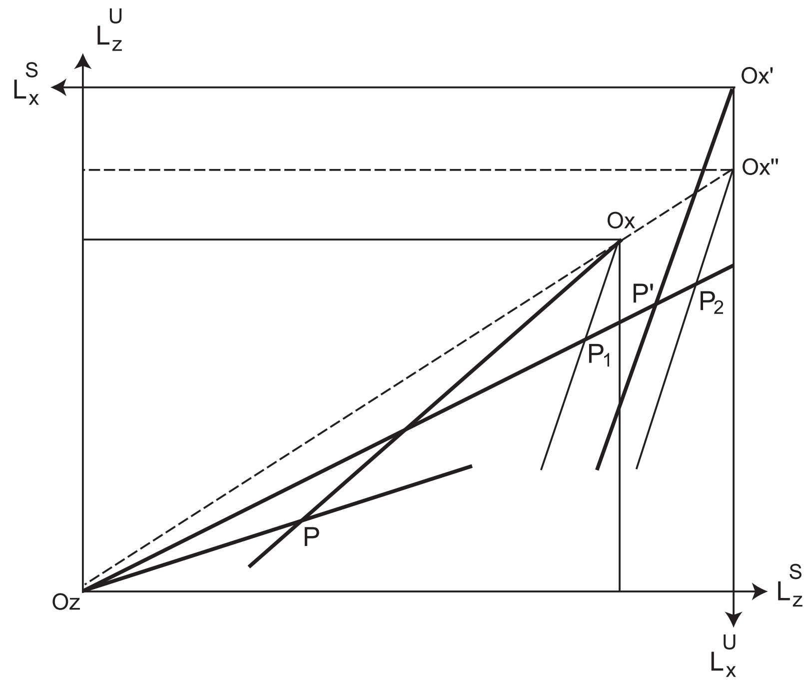

Figure 1 (a).

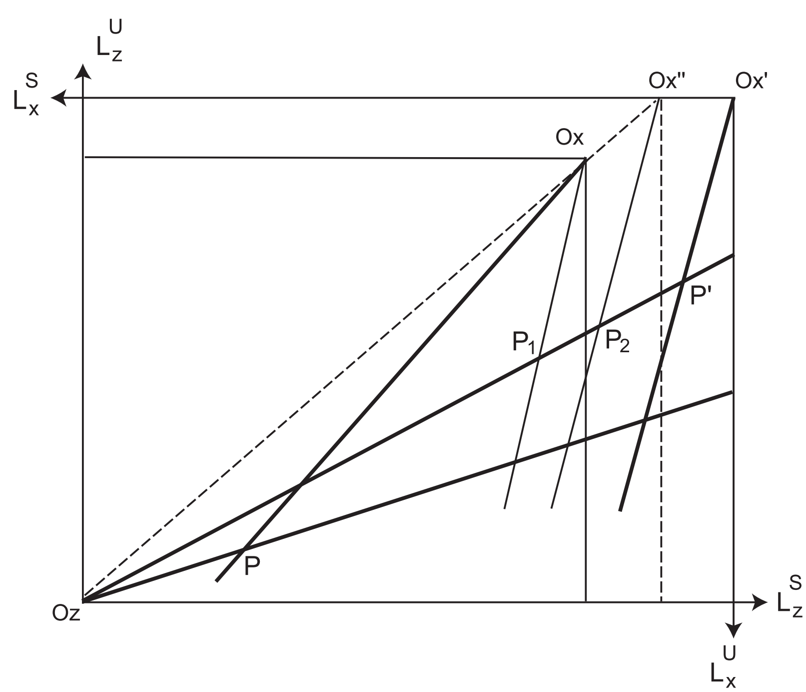

Figure 1 (b).

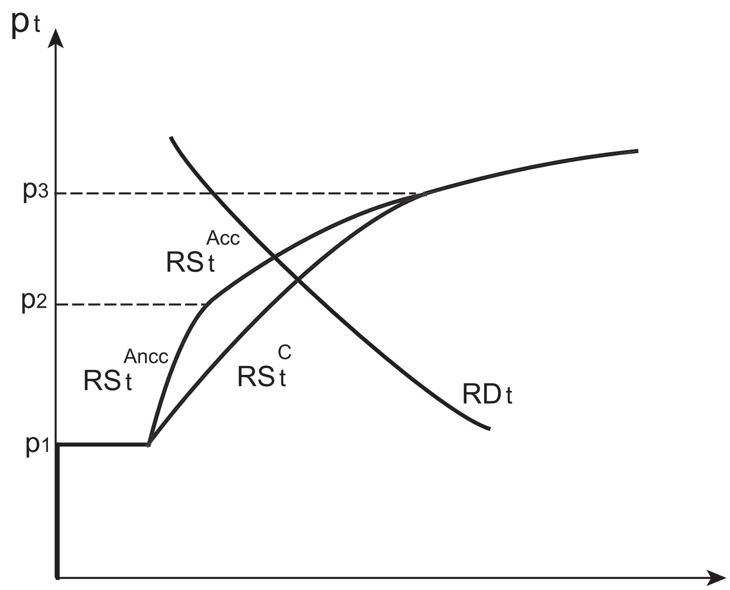

Figure 2 (a): Small M_t.

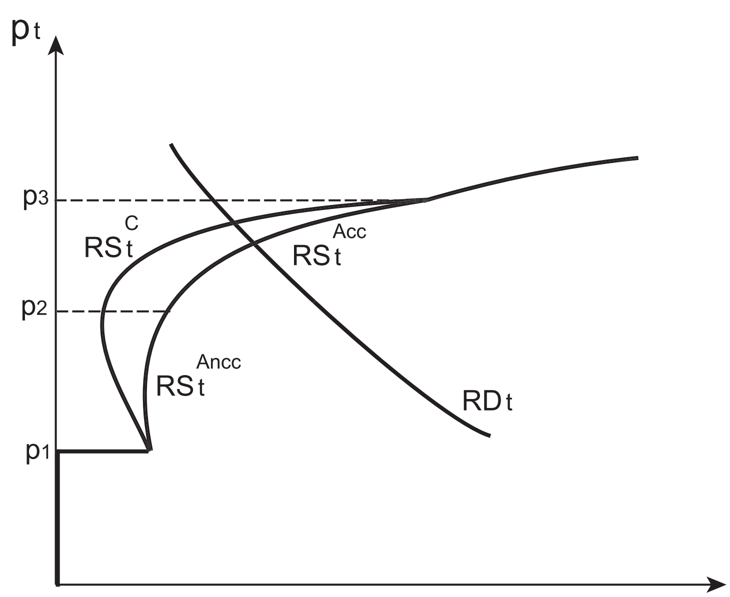

Figure 2 (b): Large M_t.

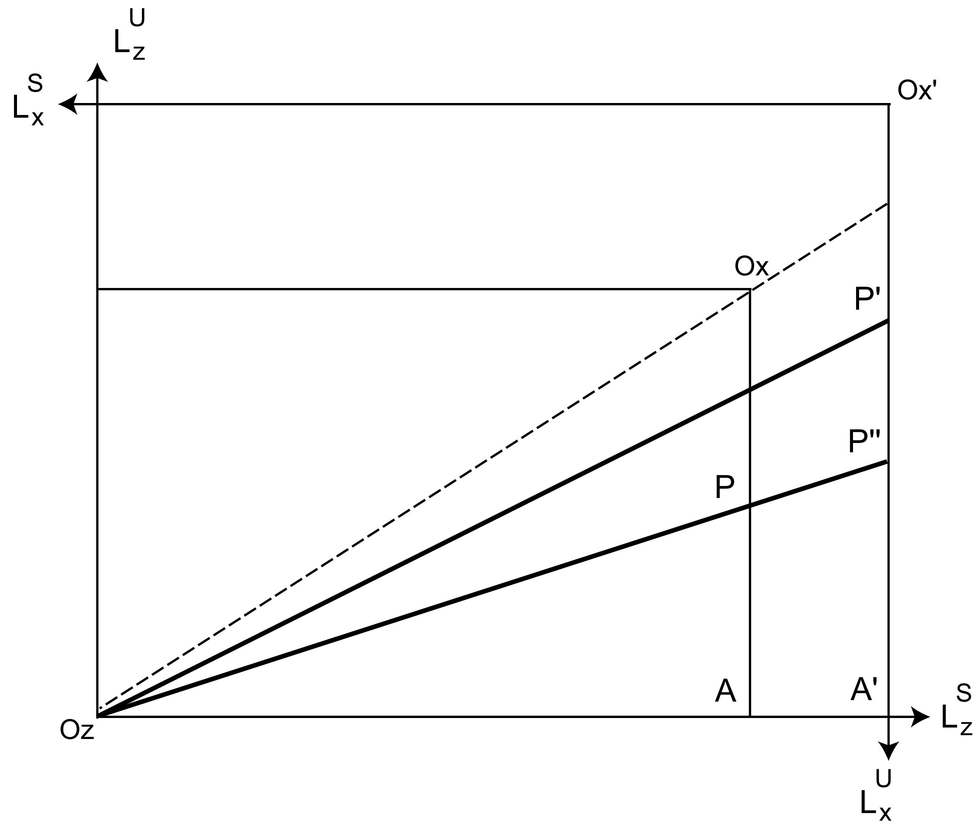

Figure 3.

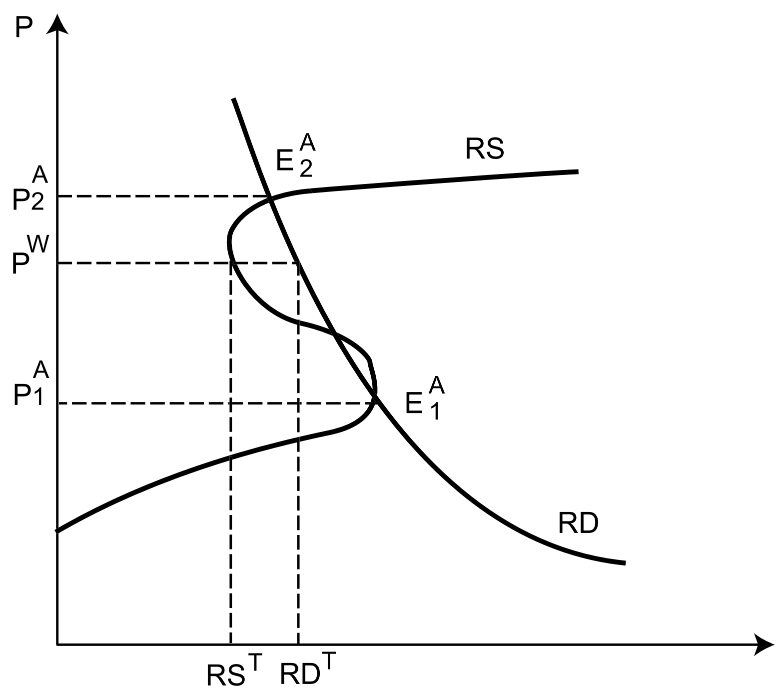

Figure 4.

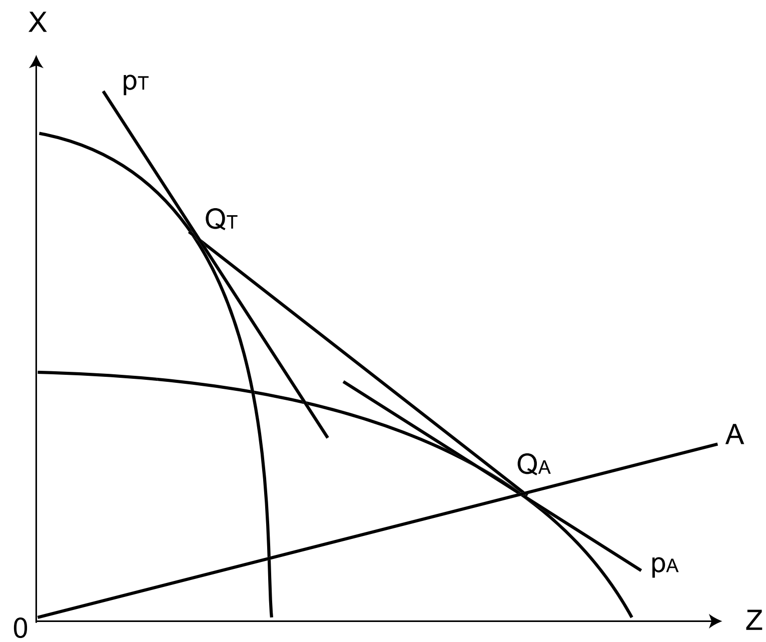

Figure 5.

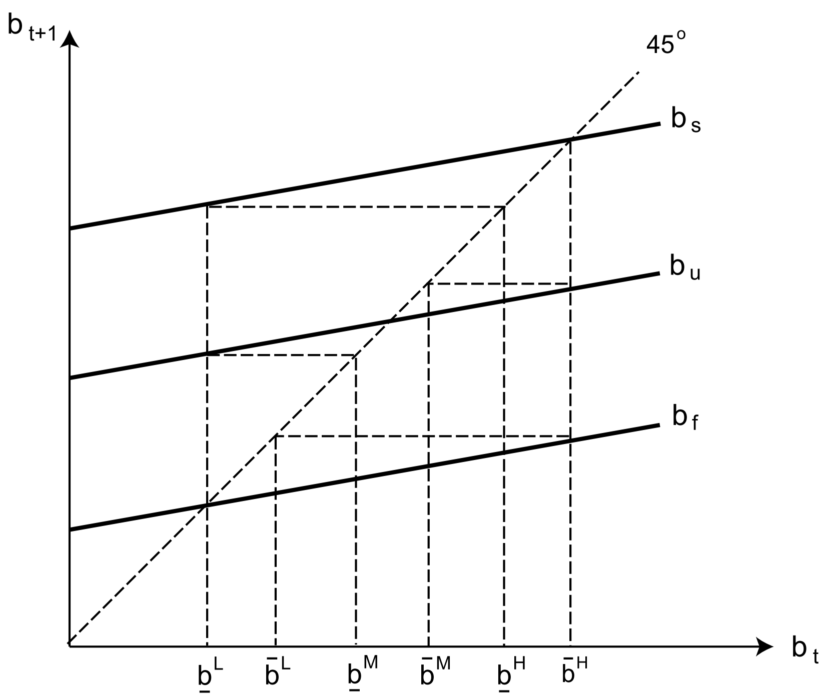

Figure 6.

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