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HUMAN CAPITAL, HETEROGENEITY, AND ESTIMATED DEGREES OF INTERGENERATIONAL MOBILITY

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ABSTRACT

Some of the important implications of the parental investment model of intergenerational mobility have been derived under the assumption that parental income is the main source of heterogeneity. We explicitly model the variability and inheritability of "innate" earnings ability and the variability of tastes, showing how they affect observed degrees of intergenerational consumption and earnings mobility. Heterogeneity increases the difficulty of detecting the existence of borrowing constrained families. Conversely, the presence of heterogeneity means that economic and linear statistical models of inheritance generate similar intergenerational data on consumption and earnings. In this sense, our findings offer some support for Goldberger's (1989) criticism of human capital models of inheritance. Finally, we suggest that any cross-country differences in intergenerational earnings mobility are more readily interpreted according to the heterogeneity of inherited ability, rather than optimal family responses to country-specific institutions for accumulating human capital.

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1 Introduction

The relationship between a citizen's economic status and his family background is an important clue for understanding and evaluating a country's economic performance. Some (eg, Bowles 1972) have suggested a close relationship between economic status and family background indicates some failure of labor markets or of institutions fostering investment in human capital. Others (eg., Altonji et al. 1992; Mulligan 1997 Table 4.1) have offered an apparently contradictory suggestion, that "efficient" allocations involve a very close relationship, on some measures, between the economic status of parents and children. Still others find family background effects to be the most offensive sources of inequality, regardless of whether those effects are consistent with "efficiency."

Empirical studies of this relationship date back to the earliest days of statistical social science (e.g., Galton's 1869 British study of the "success" and "eminence" of relatives), and many can be found outside the field of economics (see Goode's 1966 survey for a few examples), but the increasing availability of new data sets for exploring the relationship between earnings, income, wealth, and consumption of parents and, years later, of adult children has appropriately led to new and better econometric estimates of "intergenerational mobility," or the intergenerational (auto)regression of various measures of economic status, and has even permitted some careful comparisons of those estimates across countries. However, fewer results are available on interpreting such estimates in terms of economic models of inheritance or of the labor market. Loury (1981), and Becker and Tomes (1986) are among the exceptions,[1] where they show how access to capital markets or other means of financing human capital investment can increase the degree of intergenerational earnings mobility. This has led many to search for, and interpret, cross-country differences in intergenerational earnings mobility as evidence of differential efficiency in those countries' institutions for human capital investment (e.g., Bjorklund and Jantti 1997; Checchi et al. 1999).

It has been pointed out (e.g., Mulligan 1997 pp. 61-2) that Becker and Tomes' analysis assumes that parental income is the dominant source of heterogeneity across families. The purpose of our paper is to build a mathematical economic model of inheritance that not only has heterogeneity of parental income, but also heterogeneity of "earnings ability" and/or of

[1]Others include Becker and Tomes (1979), Behrman et al. (1995), Benabou (1994), Durlauf (1996), Glomm and Ravikumar (1992), Laitner (1992), and Tamura (1991), many of which also emphasize capital market access.

tastes - and is otherwise similar to Becker and Tomes' model - and to derive quantitative and qualitative implications of our model for intergenerational earnings and consumption mobility and, more generally, the relationship between borrowing constraints and the transmission of economic status.

Section 2 builds the model. We display the various structural parameters dictating degrees of intergenerational mobility, and show how they are related to the presence of borrowing constraints. We briefly address the question of the optimal statistical technique for detecting and measuring the importance of borrowing constraints, but the bulk of our analysis relates our economic model to the various linear statistical models that are so prevalent in the literature dating back to Galton (1877). Section 3 shows how, with parental income as the only source of heterogeneity, the linear regression specifications found in the literature are, for the most part, consistent estimates of the structural parameters, and the magnitude of the linear regression estimates are directly related to the existence and importance of borrowing constraints. The bulk of the paper comprises Sections 4 and 5, which report analytic and simulated results showing how linear regression estimates are inconsistent estimates of the structural parameters when there is ability or taste heterogeneity, and how the magnitude of the linear regression coefficients is only weakly related to the existence and importance of borrowing constraints. These inconsistencies are different from, and in addition to, the effects of measurement errors and inadequate sampling which have been emphasized in some of the recent literature (eg., Solon 1989, 1992).

Several lessons are learned from our analysis. First, it further explores the links between statistical and economic models of intergenerational mobility, showing how those links depend on the amount and types of heterogeneity in the economy. Second, it suggests that heterogeneity substantially increases the difficulty of detecting the existence and importance of borrowing constraints with linear regression estimators, or even with nonlinear estimators tailored to the economic model. To the extent that there are important cross-country differences in degrees of earnings or consumption mobility, we offer an alternative interpretation of such differences - that countries differ in the amount of "ability" heterogeneity. Third, our study highlights the importance of the "intergenerational elasticity of substitution" to the economic approach to intergenerational mobility, about which quantitative empirical knowledge is terribly limited.

2 Model

Two mathematical economic models are drawn from Mulligan (1999a). One is called the perfect capital market or permanent income (PI) model, where parents can make both positive and negative financial transfers to their child. When the financial transfers are negative, parents are borrowing against their child's earnings. The other model is called the imperfect capital market (ICM) model, where borrowing against child's earnings is prohibited.

2.1 ICM Model

We denote consumption of parents and their child by C_t and C_{t+1}, respectively. Parents solve the following intertemporal optimization problem:

$$\max_{C_t, C_{t+1}, h_{t+1}, X_{t+1}} E_t \left(\frac{\eta}{\eta - 1} C_t^{\frac{\eta-1}{\eta}} + \alpha \frac{\eta}{\eta - 1} C_{t+1}^{\frac{\eta-1}{\eta}} \right)$$

subject to:

$$\begin{aligned} C_t + X_{t+1} + h_{t+1} = I_t &= \left((1+r)X_t + A_t h_t^{\gamma}\right) \chi_t \\ C_{t+1} &= \left((1+r)X_{t+1} + A_{t+1} h_{t+1}^{\gamma}\right) \chi_{t+1} \\ X_{t+1} &\geq 0 \end{aligned}$$

Parents begin with resources I_t, which is a combination of earnings and financial transfers that they received from their parents. Parents spend I_t on three items: their own consumption C_t, financial transfers to their child X_{t+1}, and investments in the human capital of their child h_{t+1}. Parents are altruistic–they care about their child's consumption in addition to their own consumption. The degree of altruism is α. Parents do not necessarily place the same weight on their consumption as on the consumption of their child ($\alpha \neq 1$).

Notice that children do not have (or care about) grandchildren in our mathematical model. C_{t+1} therefore equals I_{t+1}. This two-generation model is not as restrictive as it seems; an alternative way to motivate our formulation of the model is to allow for infinitely lived dynasties and then assume that parents care about their own consumption and the income of their child according to the homothetic utility function above. In this case, C_{t+1} would, as an approximation, be proportional to I_{t+1}.[2]

A_t and A_{t+1} are the "innate" earnings ability of parents and child, respectively. It is assumed that parents know their child's ability. The random variable χ_t is a shock whose

[2]This is the motivation of the two period model that appears in Becker and Tomes (1986).

value is unknown to parents at the time they invest in children.[3] We assume that χ_t has the following property.

Assumption 1 χ_t is *i.i.d.* and log-normally distributed, $\ln \chi_t \sim N(\mu_{\ln \chi}, \sigma^2_{\ln \chi})$; Moreover, χ_{t+1} is independent of A_{t+1} and I_t.

Notice that shocks to human capital and financial investments are common, and equal to χ_t. We make this assumption not for realism, but to abstract from risk considerations that would affect human capital investment and intergenerational mobility in a more general model.[4]

The anticipated rate of return on financial investments is r. Human capital, ability, and shocks produce earnings for a member of generation t according to the function $e_t = A_t h_t^\gamma \chi_t$ with $\gamma \in (0, 1)$. The anticipated component of ability, A_{t+1}, is multiplicative in our model, affecting both the level of earnings for a given human capital investment and the rate of return to human capital investments. The importance and validity of this assumption is discussed below.

2.1.1 Group I: The Borrowing Constraints Are Not Binding

Families can be divided into two groups according to whether they hit the borrowing constraints. For those whose borrowing constraints do not bind, the Lagrangian multiplier on the borrowing constraint is zero and $X_{t+1} > 0$. We have

$$h^\star_{t+1} = \left(\frac{1+r}{\gamma A_{t+1}} \right)^{\frac{1}{\gamma - 1}} \tag{1}$$

We call $h^\star_{t+1}$ as efficient human capital investment in the sense that if the access to capital market is perfect, all families will make their optimal human capital investment according to equation (1). Define $\mu = E_t(\chi_{t+1}^{1-\frac{1}{\eta}})$ and $\epsilon_{t+1} = \ln \chi_{t+1} - \mu_{\ln \chi}$. Several manipulations lead to

$$C_t = \frac{I_t + (\frac{1}{\gamma} - 1) h^\star_{t+1}}{1 + (\alpha\mu)^\eta (1+r)^{\eta - 1}} \tag{2}$$

[3]Without changing the mathematics of our model, one could allow some component of the ability of child to be unanticipated by parents. Our variable χ would capture such "ability shocks."

[4]Becker and Tomes (1986, p. 23) have some discussion, but more research on the effect of investment risk on parental decisions to investments in children is needed.

and

$$\ln C_{t+1} = \ln((1+r)\alpha\mu)^{\eta} + \mu_{\ln\chi} + \ln C_t + \epsilon_{t+1} \tag{3}$$

$$\ln e_{t+1} = \gamma \ln\left(\frac{1+r}{\gamma}\right)^{\frac{1}{\gamma-1}} + \mu_{\ln\chi} + \frac{1}{1-\gamma}\ln A_{t+1} + \epsilon_{t+1}. \tag{4}$$

2.1.2 Group II: The Borrowing Constraints Are Binding

When the borrowing constraint binds, $X_{t+1} = 0$ and the Lagrangian multiplier on the borrowing constraint is positive. So the optimal human capital investment is determined by

$$I_t - h_{t+1} = (\alpha\mu\gamma)^{-\eta} A_{t+1}^{1-\eta} h_{t+1}^{\gamma+\eta(1-\gamma)}. \tag{5}$$

The above equation implies an implicit function, $h_{t+1} = h(I_t, A_{t+1})$. Note that h_{t+1} is smaller than the efficient human capital investment $h^{\star}_{t+1}$. Intuitively, those borrowing constrained parents would have invested more on kid's human capital if they were able to borrow against child's earnings.

Given the optimal human capital investment h_{t+1}, parent's optimal consumption is

$$C_t = I_t - h_{t+1}.$$

Define $\beta = \frac{\gamma}{\gamma+\eta(1-\gamma)}$. Then $\beta \in (0,1)$. Several manipulations lead to the evolution equations for consumption and earnings:

$$\ln C_{t+1} = \beta \ln\left((\alpha\mu\gamma)^{\eta} A_{t+1}^{\frac{\eta}{\gamma}}\right) + \mu_{\ln\chi} + \beta \ln C_t + \epsilon_{t+1} \tag{6}$$

$$\ln e_{t+1} = \mu_{\ln\chi} + \ln A_{t+1} + \gamma \ln h(I_t, A_{t+1}) + \epsilon_{t+1} \tag{7}$$

with ϵ_{t+1} defined as before; and $h(I_t, A_{t+1})$ is the solution to equation (5).

2.2 PI Model

In the PI model, there is no non-negativity constraint on X_{t+1}. So parents in all families will invest on the human capital at the efficient level $h^{\star}_{t+1}$ as in (1). The optimal consumption is as in (2). Equation (3) gives the evolution equation of consumption from parents to their child. Child's earnings are given by (4).

The optimal financial transfer for all families in the PI model is:

$$X^{\star}_{t+1} = \frac{I_t - h^{\star}_{t+1}\left(1 + (\alpha\mu)^{-\eta}(1+r)^{1-\eta}\cdot\frac{1}{\gamma}\right)}{1 + (\alpha\mu)^{-\eta}(1+r)^{1-\eta}}. \tag{8}$$

Notice that Group I families in the ICM model are those having $X^{\star}_{t+1} > 0$. In other words, (1) and (8) imply that Group I families are those with $(I_t, A_{t+1}) \in \Delta$, where the set Δ is defined as :

$$\Delta \equiv \left\{ (I_t, A_{t+1}) : \ln I_t - \frac{1}{1-\gamma} \ln A_{t+1} \geq k \right\} \tag{9}$$

with $k = \ln \left(1 + (\alpha\mu)^{-\eta}(1+r)^{1-\eta} \cdot \frac{1}{\gamma}\right) - \frac{1}{1-\gamma} \ln \left(\frac{1+r}{\gamma}\right)$. Let $\overline{\Delta}$ be the complementary set of Δ. Then Group II families in the ICM model are those with $(I_t, A_{t+1}) \in \overline{\Delta}$.

Many of the implications of the ICM and PI models depend on whether a family is in the set Δ. Becker (1989, p. 514) defines Δ as we do, referring to those families "richer families": "Richer families, defined as families that give bequests (or gifts) to children, can readily and costlessly self-finance investments in the human capital of children by drawing down bequests. They invest in human capital until the marginal rate of return equals the rate on assets." Becker refers to the complement of the set of "richer families" as "poorer families" (p. 515), which we denote $\overline{\Delta}$.

2.3 Structural Parameters of Interest

There are four structural parameters, or combinations of structural parameters, that are of primary interest. The first is the "partial family income elasticity of earnings" among non-borrowing constrained families which measures the effect of an exogenous change in parental income (i.e., a change that holds the ability of children constant) on the earnings of an adult child. As we see in (4), the partial family income elasticity is zero both in the PI model and for Group I in the ICM model because all of these families participate in the same capital market. The second parameter is the partial family income elasticity of earnings among borrowing constrained families, which we denote by Φ. Φ can be computed from (5) and (7):

$$\Phi \equiv \gamma\tau_{h,I}, \quad \text{with } \tau_{h,I} = \frac{\partial \ln h_{t+1}}{\partial \ln I_t}, \text{ evaluated at some } (I_t, A_{t+1}) \in \overline{\Delta}. \tag{10}$$

Although $\tau_{h,I}$ is not a constant - it depends on I_t and A_{t+1} - we define Φ according to some $(I_t, A_{t+1}) \in \overline{\Delta}$. For our purposes, it will not matter which (I_t, A_{t+1}) is used to define Φ, so long as it is in the range of our data. Using (5), it is straightforward to show that

$$\Phi \in \begin{cases} (\gamma, \beta) & \text{if } 0 < \eta < 1, \\ (\beta, \gamma) & \text{if } \eta > 1. \end{cases} \tag{11}$$

The third structural parameter of interest is the persistence of log consumption among non-borrowing constrained families, which measures the effect of an exogenous change in parental consumption (say, from a change in parental income that holds ability and preferences constant) on the consumption of the adult child. According to (3), this parameter is 1. The final structural parameter of interest is the persistence of log consumption among borrowing constrained families which, according to (6), is β.

Notice that the partial family income elasticity is greater among borrowing constrained families while the persistence of log consumption is greater among non-borrowing constrained families. Among non-borrowing constrained families, the partial family income elasticity is zero, which is less than the persistence of log consumption. The two corresponding parameters for borrowing constrained families cannot be ordered.

Becker and Tomes (1979,1986), Becker (1989) and others have discussed how estimated degrees of intergenerational consumption and earnings mobility are related to these four structural parameters. Our paper explores this question, with a special emphasis on how heterogeneous ability and preferences affect these relationships.

In next section, we study a benchmark model with no heterogeneity. Then in Sections 4 and 5, respectively, we introduce two types of heterogeneity: First, we assume that all families are homogeneous in altruism rates, i.e., all parents have the same α, but that earnings ability is variable across families and inheritable across generations. Then we drop the homogeneity assumption on preference so that parents across families have different altruism rates. We study the property of OLS estimators of the degrees of earnings and consumption mobility. In particular, because ability and altruism rates are unobservable and we divide the entire sample into two subsamples, OLS estimates may be inconsistent because of both omitted variables and sample selection. We ask the following questions: What are the direction and magnitude of the inconsistency? How is the inconsistency related to the amount and type of heterogeneity and to the model's structural parameters? Is it possible to empirically distinguish PI and ICM models? When and when not?

3 Intergenerational Mobility in the Absence of "Heterogeneity"

We first derive some benchmark results to which our simulations, and some of the previous literature, refer. In our benchmark case, parental income and the shocks (χ_t, χ_{t+1}) are the only sources of heterogeneity in the model–all children have the same ability and all parents have the same degree of intergenerational altruism. That is,

$$\textbf{Benchmark: } \ln A_t \text{ and } \alpha \text{ are the same for all families, and } \ln A_{t+1} = \theta \ln A_t. \tag{12}$$

The parameter $\theta \in (0, 1)$ indicates the inheritability of innate earnings ability. Note that, although ability is homogeneous, the shocks χ_t and χ_{t+1} introduce earnings heterogeneity among parents and children.

The benchmark case permits analytic results for intergenerational consumption and earnings mobility in both PI and ICM models, results which we report as Propositions 1 and 2 below. As we shall see, the benchmark case is also interesting because its analytical results agree with Becker's (1989) account of the nature of intergenerational mobility in the Becker-Tomes model.

3.1 PI Model

In PI model, the evolution of consumption from parents to kids follows (3), and both parents' and children's earnings have the same function forms as (4). We consider estimating the degree of consumption persistence by running ordinary least squared (OLS) regressions of log child's consumption on log parents' consumption, and estimating the degree of earnings persistence by running OLS regressions of log child's earnings on log patents' earnings. Assumption 1 and specification (12) imply the following.

Proposition 1 In PI model, the OLS estimator of the degree of consumption persistence is consistent (i.e., its probability limit equals to 1). The OLS estimator of the degree of earnings persistence is a consistent estimator of the partial family income elasticity (0), but an inconsistent estimator of the inheritability of ability, θ.

Throughout the paper, probability limits are taken with respect to the number of families in the sample. The proof of the proposition is straightforward. The OLS estimator of the degree of consumption persistence is consistent because ϵ_{t+1} is uncorrelated with C_t and α is constant. So by (3), the probability limit of the OLS estimator is 1. For earnings persistence, because A_{t+1} is constant, both parents' and children's earnings are white noises by (4). So, the probability limit of the OLS estimator of the degree of earnings persistence is zero, and thus, less than θ.

3.2 ICM Model

For ICM model, we split the whole sample into two subsamples: One contains only Group I families and the other Group II families. For each group, we consider estimating the degrees of intergenerational mobility by running OLS regressions of log child's consumption (earnings) on log parents' consumption (earnings). Denote the estimates of the degrees of consumption persistence for Groups I and II by $\hat{\beta_{c1}}$ and $\hat{\beta_{c2}}$, respectively, and the estimates of the degrees of earnings persistence for Groups I and II by $\hat{\beta_{e1}}$ and $\hat{\beta_{e2}}$, respectively. The estimates have the following properties.

Proposition 2 For the benchmark case, the OLS estimate of the degree of consumption persistence for Group I is consistent (i.e., the probability limit equals to 1); the OLS estimate of the degree of earnings persistence for Group I has a probability limit of 0. For Group II, the OLS estimate of the degree of consumption persistence is consistent (i.e., the probability limit equals to β); the OLS estimate of the degree of earnings persistence is a consistent estimator of Φ if grandparents are borrowing constrained, an inconsistent estimator of Φ if grandparents are non-borrowing constrained, and an inconsistent estimator of θ whether grandparents are borrowing constrained.

Proof: See Appendix.

Note that as is well known, when we split the entire sample into two subsamples according to whether families hit borrowing constraints and run OLS regressions for each subsample separately, we potentially introduce selection biases. But with the benchmark assumption of homogeneous earning ability and altruism rate, selection only contaminates the OLS estimate of Group II's earnings persistence. Other group-specific OLS estimates are immune to selection bias.

Proposition 2 also shows that in the benchmark case, the observable differences in the degrees of consumption persistence between richer (Group I) and poorer (Group II) families are $1-\beta$. If grandparents are borrowing constrained, the lower bound of the observable differences of log earnings persistence between poorer (Group II) and richer (Group I) families equals γ if $0<\sigma<1$, β if $\eta>1$.

3.3 Comparison with Results in the Literature

Although we have made some additional functional form assumptions, our Propositions 1 and 2 basically agree with several results derived by Becker and Tomes (1979,1986):

1. there is less consumption mobility in the PI model than in the ICM model;
2. in the ICM model, there is less consumption mobility in Group I;
3. in the ICM model, there is more earnings mobility in Group I.

We quote Becker (1989), "Since endowments regress to the mean at a rate determined by the 'inheritability' of endowments, the earnings of children in richer families tend to regress to the mean at this same rate.... Earnings of poorer children regress upward to the mean more slowly than the rate of inheritability of endowments." (p. 515) Also, "If fertility is unrelated to family resources, children's consumption regresses more rapidly to the mean in poorer families than in richer families...." (p. 515)

4 Variable and Inheritable Earning Ability

Now we assume that ability is transmitted across generations according to

$$\ln A_{t+1} = \theta \ln A_t + \nu_{t+1}. \tag{13}$$

Random variables A_t and ν_{t+1} are assumed to have the following properties.

Assumption 2 A_t, ν_{t+1} and χ_{t+1} are mutually independent and distributed as: $\ln A_t \sim N(\mu_{\ln A}, \sigma^2_{\ln A})$; $\nu_{t+1} \sim N(\mu_\nu, \sigma^2_\nu)$.

Our benchmark is the special case with $\sigma^2_{\ln A} = \sigma^2_\nu = 0$ and $\mu_\nu = 0$, which we now generalize. The altruism rate, α, is still assumed to be the same for all families and constant over time in this section. To simplify notations, we use cst and $error$ to represent intercepts (constants) and error terms, respectively, in all regression equations.

4.1 Analytical Results and Intuitive Discussion

All children have the same ability and efficient human capital investment in our benchmark case. There are three ways to relax this assumption, the effects of which are summarized in Table 1. First, we could introduce heterogeneity among children that is uncorrelated with parental ability or income (our "Case 1"). Second, heterogeneity among children could be perfectly correlated with parental ability (our "Case 2"). Third, heterogeneity among children could be imperfectly correlated with parental ability (our "Case 3"). We begin the Table with some analytical results and complete it with an intuitive discussion of the simulated results.

4.1.1 Analytical Results for the PI Model

In PI model, consumption evolves according to (3). Since the error term is uncorrelated with parents' consumption, i.e., $COV(\epsilon_{t+1}, \ln C_t) = 0$, the OLS estimator of log consumption persistence is consistent. For earnings, both parents and kids have the same formula as (4). Use (13) to get

$$\ln e_{t+1} = cst + \theta \ln e_t + error \tag{14}$$

with $error = \frac{1}{1-\gamma}\nu_{t+1} - \theta\epsilon_t + \epsilon_{t+1}$.

Although log earnings persistence is 0, the theory predicts that log earnings persistence is related to that of ability, i.e., θ. If we run an OLS regression of $\ln e_{t+1}$ on $\ln e_t$ and denote the estimator by β_e, then we have

$$\begin{aligned} \text{plim}(\beta_e) &= \theta + \frac{COV(\ln e_t, error)}{V(\ln e_t)} \\ &= \theta \left(\frac{\frac{1}{(1-\gamma)^2}\sigma^2_{\ln A_t}}{\frac{1}{(1-\gamma)^2}\sigma^2_{\ln A_t} + \sigma^2_{\ln \chi}} \right). \end{aligned} \tag{15}$$

Therefore, although earnings is exclusively determined by "innate" ability in the PI model with *i.i.d.* market lucks, the OLS estimate of the log earnings persistence β_e is, asymptotically, less than the inheritability of ability θ. The inconsistency is minor if the variation of market luck is small relative to that of ability. Because $\theta\sigma^2_{\ln A_t} > 0$, the probability limit of β_e is greater than the partial family income elasticity of earnings for unconstrained families, 0.

The following summarizes the properties of OLS estimators of consumption and earnings mobility in PI model.

Proposition 3 In PI model, with Assumption 2, the OLS estimator of the degree of consumption persistence is consistent (i.e., the probability limit equals to 1). The OLS estimator β_e of the degree of earnings persistence is an inconsistent estimate of the inheritability of ability, θ, with plim$\beta_e < \theta$, and plim$\beta_e \to \theta$ if $\sigma^2_{\ln \chi} \ll \sigma^2_{\ln A_t}$ or $\gamma \to 1$. Because $\theta\sigma^2_{\ln A_t} > 0$, plim β_e is greater than the partial family income elasticity of earnings for unconstrained families, 0.

The Proposition shows that if the true model is PI model, then OLS estimates of log earnings persistence by mechanic statistic models are downward inconsistent. On the other hand, the problem of misspecification may be quantitatively unimportant if the inequality of market luck is small relative to the inequality of ability.

The results of Proposition 3 are shown in the second, third, and fourth columns of the PI section of Table 1. The second column (for the case $\theta\sigma^2_{\ln A_t} = 0$) displays zeros in the first two rows to show that β_c and β_e are consistent estimates of 1 and 0, respectively. In the third and fourth columns (for the case $\sigma^2_{\ln A_t} > 0$), an upward omitted variable bias (namely the omission of $\ln A_{t+1}$) affects β_e. In all cases, plim β_e is less than θ.

4.1.2 One Analytical Result for the ICM Model

Even though ability is heterogeneous and inheritable, we can still analytically calculate the probability limit of the OLS estimator of log consumption persistence for Group I

$$\begin{aligned}
\text{plim}(\hat{\beta_{c1}}) &= \frac{COV(\ln C_{t+1}, \ln C_t|\Delta)}{V(\ln C_t|\Delta)} \\
&= 1 + \frac{COV(\epsilon_{t+1}, \ln C_t|\Delta)}{V(\ln C_t|\Delta)} \\
&= 1
\end{aligned} \tag{16}$$

The last inequality is true because ϵ_{t+1} is independent of $(\ln C_t, I_t, A_{t+1})$. We restate this analytical result as Proposition 4.

Proposition 4 Assuming homogeneous preferences and that ability is transmitted across generations according to equation (13), with Assumption 2, the OLS estimate of the degree of consumption persistence using Group I sample is consistent (i.e., the probability limit equals to 1).

The results of Proposition 4 are shown in the fourth row of Table 1 (the row marked "β_{c1} vs 1") as zeros for the Benchmark and three other Cases.

4.1.3 Intuitive Discussion of Simulation Results

As shown in Section 2.1, the presence of a borrowing constraint divides the population into two groups. The mechanical statistical models are typically estimated by pooling the two groups of families together, and are therefore misspecified. Furthermore, compared to PI model, separate group OLS estimates of intergenerational mobility may be subject to selection bias in addition to the effects of intergenerational transmission of ability discussed above. Theoretically, the size of the bias can be assessed by computing the probability limit of the OLS coefficients. However, in order to do that, we must use conditional distributions (conditional on $X^{\star}_{t+1} <$ (or $>$)0). Since it is difficult to obtain conditional expectation for nonlinear functions for the generalized model, we use simulation method to assess the possibilities of directions and magnitudes of the biases.

Before we present the simulated results, consider some of the qualitative properties of OLS estimates of the degrees of consumption and earnings mobility in the presence of heterogeneous and inheritable ability. In Case 1, we assume that A_{t+1} is independent of I_t but, because of the selection rule, the two variables fail to be independent in any one of the two groups. Since the selection rule for Group I is $\ln A_{t+1} \leq (1-\gamma)[\ln I_t - (constant)]$, $\ln A_{t+1}$ and $\ln I_t$ are positively correlated in both groups. Since $\ln A_{t+1}$ enters the earnings (4) and (7) for both groups and the Group II consumption equation (6) with a positive sign, the estimators β_{c2}, β_{e1}, and β_{e2} all suffer from an upward selection bias. This same selection bias carries over to Cases 2 and 3.

In Cases 2 and 3, $\ln A_{t+1}$ and $\ln I_t$ are positively correlated even in a random sample because able parents tend to have more income and tend to have more able children. This positive correlation means that plim β_{e1} and plim β_{e2} are greater than the respective partial family income elasticities of earnings, 0 and Φ, because the corresponding regression equations have $\ln A_{t+1}$ as an omitted variable. Since $\ln A_{t+1}$ also enters the consumption equation for Group 2, β_{c2} is also subject to an upward omitted variable bias.

Table 1: Biases Induced by Heterogeneous Ability (α constant)					
		Benchmark	**Case 1**	**Case 2**	**Case 3**
		$\ln A_{t+1}$ constant	$\ln A_{t+1} = \nu_{t+1}$	$\ln A_{t+1} = \theta \ln A_t$	$\ln A_{t+1} = \theta \ln A_t + \nu_{t+1}$
	β_c vs 1	0	0	0	0
PI	β_e vs 0	0	0	OVB(+)	OVB(+)
	β_e vs θ	RB(-)	0*	RB(-)	RB(-)
	β_{c1} vs 1	0	0	0	0
	β_{e1} vs 0	0	SB(+)	SB(+)	SB(+)
				OVB(+)	OVB(+)
	β_{e1} vs θ	RB(-)	SB(+)*	RB(-)	RB(-)
ICM				SB(+)	SB(+)
	β_{c2} vs β	0	SB(+)	SB(+)	SB(+)
			OVB(+)	OVB(+)	OVB(+)
	β_{e2} vs Φ	0†	SB(+)†	SB(+)	SB(+)
				OVB(+)†	OVB(+)†

RB="regression" or "measurement error" bias — 0 = no bias
SB="selection bias" — (+)=upward bias
OVB="omitted variable bias" — (-)=downward bias

$\Phi = \gamma \cdot$ elasticity of $h(I_t, A_{t+1})$ with respect to I_t evaluated at some (I_t, A_{t+1}) within the range of the data

* If Case 1 is generated by $var(\ln A_t) = 0$, rather than $\theta = 0$, then β_e and β_{e1} are regression biased downward as estimates of θ (as in Cases 2 and 3).

† If grandparents are not borrowing constrained, then there is a regression bias. The bias can be upward or downward. See, e.g., Proposition 2.

Notice that an important part of our argument requires that "anticipated ability" affects the efficient human capital investment in a multiplicative way. If instead ability affected earnings in an additive way, then the efficient human capital investment would not depend on ability and there would not be a "selection bias" resulting from the definition of Groups I and II. Nor would there be an "omitted variable bias" in the OLS estimation of earnings equations for Group II. Several other authors have invoked the multiplicative assumption (Becker (1991), p. 189; Loury (1981), p. 854), but is it realistic? Multiplicative ability seems plausible, but the point of our analysis is that multiplicative ability is hard to distinguish from borrowing constraints as a source of strong intergenerational earnings correlations, and hence difficult to test. In the class of models we consider, human capital investments are uncorrelated with ability and parental income among the unconstrained when ability fails to act multiplicatively, so a finding that human capital investments (such as schooling) were correlated with components of ability (such as IQ) even among rich, and clearly unconstrained, families would suggest that ability multiplicatively affects earnings. Behrman and Taubman (1989) study of twins also suggests that "ability" is an important determinant of schooling, apart from parental income, but it is still difficult to offer a definitive answer to this question.

4.2 Calibration

The simulation starts with an initial condition assuming that, first, grandparents for all families are not borrowing constrained.[5] Hence h_t is determined as in (1) (i.e., $h_t = \left(\frac{1+r}{\gamma A_t}\right)^{\frac{1}{\gamma-1}}$, and $e_t = A_t h_t^{\gamma} \chi_t$). Second, according to the evolution of earnings ability specified in (13), we assume that

$$\ln A_t \sim N\left(0, \frac{\sigma_\nu^2}{1-\theta^2}\right).$$

Then with $\mu_\nu = 0$, $\ln A_{t+1}$ has the same distribution as $\ln A_t$.

Although ability is in its steady state, other variables are not. We do not attempt to choose steady state initial conditions for other variables because (i) steady states would be different for different parameterizations and for different models and (ii) a steady state does not exist for PI model. What is important for our results is not that we do or do not mimic a "steady state" but that we have a model that "fits the data" and illustrates the potential magnitude of the biases of various OLS estimators of intergenerational mobility.

So the parameters we need to calibrate (assuming that $\mu_{\ln\chi} = 0$) are γ, θ, η, $\sigma_{\ln\chi}$, σ_ν, σ_ς, α, r and X_t. We assume that a generation takes 25 years. We choose r and α to satisfy the following two conditions.

(a) The anticipated component of the net rate of return on financial assets $r = 2.39$ so that the annual rate of return is about 5%.

(b) We require that

$$\alpha(1+r) = 1$$

[5] We have also experimented with the assumption that grandparents may be borrowing constrained. The results reported here are robust to those different specifications.

because we only consider steady state of the model.[6] Hence $\alpha = 0.295$ with the annual altruism rate being about 0.952.

The rest of parameters are calibrated so that the simulated data from Case 5 specified in Section 5.3 match the following empirical facts.

(c) The standard deviations of logarithmic earnings for both parents and kids are about 0.4, which matches a conservative estimate from real data. That implies

$$0.4^2 = \frac{1}{(1-\gamma)^2} \cdot \frac{\sigma_\nu^2}{1-\theta^2} + \sigma_{\ln\chi}^2.$$

(d) The OLS estimate of the degree of log earnings persistence by pooling Groups I and II is about 0.4. This is the number reported in Solon(1992) and verified by Mulligan (1997) using a variety of data sources and estimation techniques.

(e) The correlation between child's earnings (logarithmic) and inheritances (logarithmic) is about 0.06. This essentially requires that the variance of parental income should be large enough relative to the variance of ability (see Mulligan 1997).

(f) We also control the fraction of Group I (non-borrowing constrained) families in the whole sample to be around 50%. There are certain arbitrariness on what numbers we should use for the fraction. Mulligan (1997, Table 8.10) uses the 1989 Survey of Consumer Finances to calculate that 32% of U.S. ("adult child") households with male head aged 25-35 received or expected to receive an inheritance of at least \$25,000, which he interprets as a lower bound on the fraction of Group I.[7] We have also tried 30% Group I and 70% Group I and the results (not reported here) for Group I-II comparisons are similar to the 50% case.

We don't have a good prior about the magnitude of η, the elasticity of substitution between parent's and child's consumption. But according to (5) and (11), we know that η affects the elasticity of human capital investment with respect to child's ability and parents' income for Group II. So our sensitivity analysis varies $\eta \in (0, 4)$. In the light of (18), we fix the product $\eta \cdot \sigma_\zeta$ while we vary η, where σ_ζ is the standard deviation of ζ (see (17)). We choose $\eta \cdot \sigma_\zeta = 0.31$ to match the above facts.

The following table summarizes the calibration.

[6] Laitner (1992) and Navarro-Zermeno (1993) suggest that consumption does not grow across generations among the non-borrowing constrained families.

[7] Another 9% received or expected to receive an inheritance, but less than \$25,000.

Calibrated parameters		
$r = 0.05$	$\theta = 0.6$	$\ln \chi_t \sim N(0, 0.3^2)$
$\alpha = 0.295$	$X_t = 0.003$	$\nu_{t+1} \sim N(0, 0.0847^2)$
$\gamma = 0.6$	$\eta \cdot \sigma_\zeta = 0.31$	$\zeta_i \sim N(0, \sigma_\zeta^2)$

Random variables $A_t, \nu_{t+1}, \zeta_i, \chi_t$ and χ_{t+1} are assumed to be mutually independent.

4.3 Simulation Results

Heterogeneity in ability as described in (13) can be decomposed into two components. First, ability may vary cross families within a generation ($\ln A_t$ varies); Second, the inheritability of ability cross generations is imperfect (ν_{t+1} varies). To see the effects of the different forces, we consider three kinds of ability heterogeneity.

Case 1: $\ln A_{t+1} = \nu_{t+1}$; i.e. $\theta = 0$ and $\ln A_t \sim N(0, \sigma_\nu^2)$;

Case 2: $\ln A_{t+1} = \theta \ln A_t$ with $\ln A_t \sim N\left(0, \frac{\sigma_\nu^2}{1-\theta^2}\right)$;

Case 3: $\ln A_{t+1} = \theta \ln A_t + \nu_{t+1}$ with $\ln A_t \sim N\left(0, \frac{\sigma_\nu^2}{1-\theta^2}\right)$ and $\nu_{t+1} \sim N(0, \sigma_\nu^2)$.

So in Case 1, ability is not inheritable. In Case 2, the inheritability of ability cross generations is perfect, but there are variations in ability cross families within a generation. Both forces present in Case 3. From the benchmark to either Case 1 or Case 2, then to Case 3, we add more heterogeneity into the model in the sense of increasing standard deviations of $\ln A_t$.

For each case, we simulate a large random sample for each value of η using both PI and ICM models. For the sample generated from ICM models, we separate it into two subsamples according to whether a family is borrowing constrained. We run OLS regressions using the subsamples as well as the pooled samples.

4.3.1 Degrees of Consumption Mobility

For a sample containing only Group I families, consumption evolves according to (3) and, according to Proposition 4, OLS estimates of the degrees of consumption persistence are consistent. For samples containing only Group II families, (6) can be rewritten as

$$\ln C_{t+1} = cst + \beta \ln C_t + error$$

with $error = \frac{\beta\sigma}{\gamma} \ln A_{t+1} + \epsilon_{t+1}$.

The model predicts that β is between zero and one.[8] Moreover, the magnitude of β is linked to the magnitude of the structural parameters of the model, so it is desirable to estimate β both to learn about the magnitudes of the structural parameters and to distinguish ICM model from PI model. However, the estimated coefficient from an OLS regression of $\ln C_{t+1}$ on $\ln C_t$ may be an inconsistent estimator of β because of the selection issues. Even if there were not selection bias problem, OLS estimates would still be inconsistent because $\ln C_t$ and $error$ are correlated.

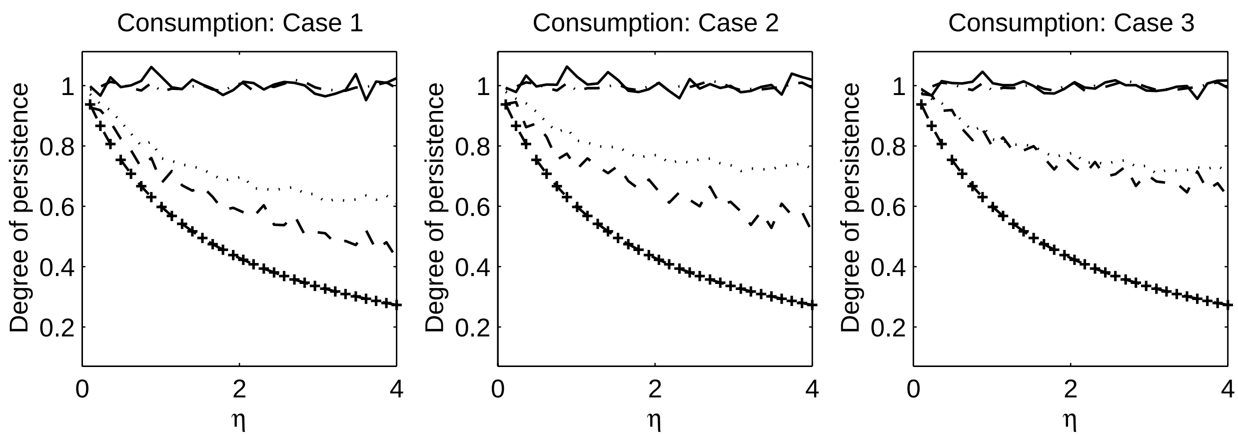

Figure 1: The estimated degrees of consumption persistence in Cases 1,2,3. Legend: $- \cdot -$ PI model, $\cdots\cdots$ Pooled sample, — Group I, $- - -$ Group II, $- + -$ β.

In Figure 1, the three windows show, in order, the estimated degrees of log consumption persistence for Cases 1,2,3. For each case, we plot the OLS estimates obtained from different samples (i.e., pooled sample and Groups I and II). We also plot the parameter β, which, by Proposition 2, is the degree of log consumption persistence for Group II in the benchmark case. In all cases, introducing heterogeneity in ability does not affect the estimated log consumption persistence for both the PI model and Group I in the ICM model: All of them are consistent estimates of (3) (i.e., the probability limit equals to 1). Hence, we have to rely on Group II to distinguish ICM model from PI model.

In all three cases, the estimated degrees of consumption persistence for Group II samples are, asymptotically, greater than the degrees of consumption persistence in the benchmark case (i.e., all estimates are greater than β) for all η's. As we move from the benchmark to either Case 1 or Case 2, then to Case 3, more heterogeneity in earnings ability is introduced into the model. It appears that the estimated degrees of Group II's consumption persistence

[8] Recall that $\beta = \gamma/(\gamma + \sigma(1-\gamma))$.

increase as we add more heterogeneity.

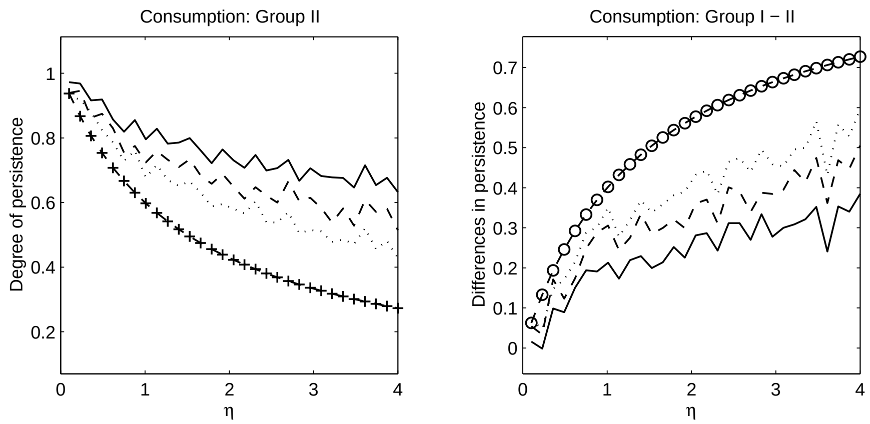

Figure 2: Left: The estimated degree of consumption persistence of Group II in Cases 1,2,3. Right: the differences in the estimated degrees of consumption persistence between Groups I and II in Cases 1,2,3. Legend: $\cdots\cdots$ Case 1, $---$ Case 2, — Case 3, $-+-$ β, $-\text{o}-$ $(1-\beta)$.

This can be seen more clearly in the left window of Figure 2, where in a single graph, we plot the estimated degrees of consumption persistence of Group II from all three cases. The graph shows that from the benchmark to Cases 1, 2 and 3, the estimated degrees of consumption persistence for Group II are getting larger. As a result, since the estimated degrees of Group I's consumption persistence equal to 1 in all cases, the observed differences in the degrees of consumption persistence between Groups I and II are reduced with more heterogeneity introduced into the model. This is shown clearly in the right window of Figure 2, where we plot the differences of the estimated degrees of consumption persistence between Groups I and II.

Notice that η plays an important role in determining the size of observed difference in the degrees of consumption persistence between Groups I and II. As seen in the right window of Figure 2, for all cases, the smaller η is, the smaller the difference. In Case 3, when $\eta < 1$, we observe that consumption regresses toward mean at roughly the same speeds for both Groups I and II since the intergroup differences in the estimated degrees of consumption persistence are small (less than 0.2). This implies that when $\eta < 1$, it is empirically difficult to distinguish PI models from ICM models on the basis of the relative consumption persistence of Groups I and II.[9]

[9]The difference in the degrees of consumption persistence between Groups I and II predicted by theory is $(1-\beta)$, which, as shown in Figure 2 (right window), is increasing in η. So in theory, the difference may be already small when η is small. What heterogeneity does is to make the differences even smaller.

If $\eta > 1$, the differences in the estimated degrees of log consumption persistence between Groups I and II become increasingly apparent. Consumption regresses toward mean more rapidly for Group II than Group I.

4.3.2 Degrees of Earnings Mobility

In Figure 3, the three windows show, in order, the estimated degrees of earnings persistence for the benchmark case and Cases 1,2,3. For each case, we plot the OLS estimates obtained from different samples. Let us first look at PI model. Proposition 1 states that the estimated degrees of earnings persistence should be zeros in the benchmark. The specification of Case 1 implies that $COV(\ln e_{t+1}, \ln e_t) = 0$. So the estimated degrees of earnings persistence are also expected to be zeros. In Cases 2 and 3, according to Proposition 3, the OLS estimates of log earnings persistence are, asymptotically, less than the inheritability of ability θ and greater than the partial family income elasticity of earnings, 0. In fact, under our calibration, (15) implies that the probability limit of the OLS estimates should be 0.26. These are verified by the simulations: In both the benchmark and Case 1, the estimates are zeros; In both Cases 2 and 3, the estimates are about 0.26.

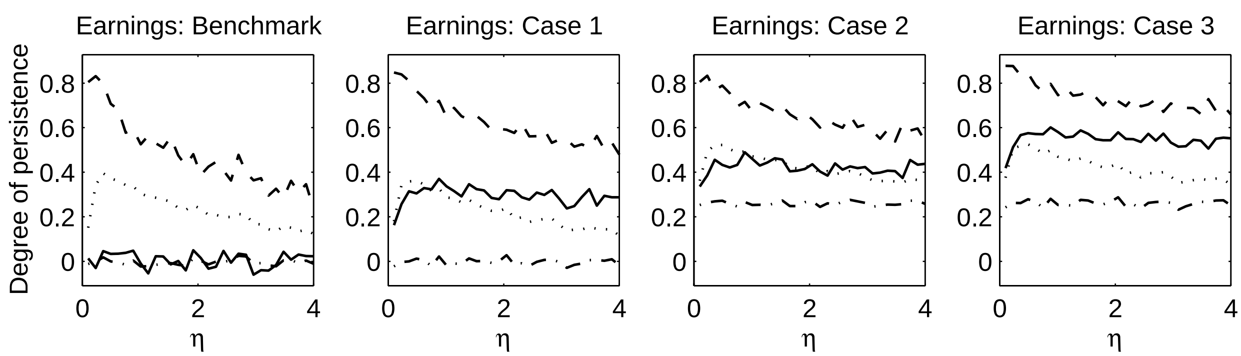

Figure 3: The estimated degrees of earnings persistence in the benchmark and Cases 1,2,3. Legend: $- \cdot -$ PI model, $\cdots\cdots$ Pooled sample, — Group I, $- - -$ Group II.

Note that from the benchmark to Cases 2 and 3, all underlying model's parameters, including the true degree of intergenerational transmission of earnings ability θ, are kept constant, except that the standard deviations of earnings ability (i.e, $\sigma_{\ln A_t}$) are increased. Hence, our finding that more heterogeneity in earnings ability leads to larger observed degrees of earnings persistence suggests an alternative interpretation of some observed differences in the degrees of intergenerational earnings mobility among different countries or regions. For example, Bjorklund and Jantti (1997) found that the observed degree of earnings persistence in Sweden was smaller than that in the US. One possible explanation of this, according to our result, could be that there are more heterogeneity in earnings ability in the US than in Sweden, even though the true degrees of intergenerational transmission of earnings ability θ are the same for the two countries. More discussions on this point are provided in Section 7.

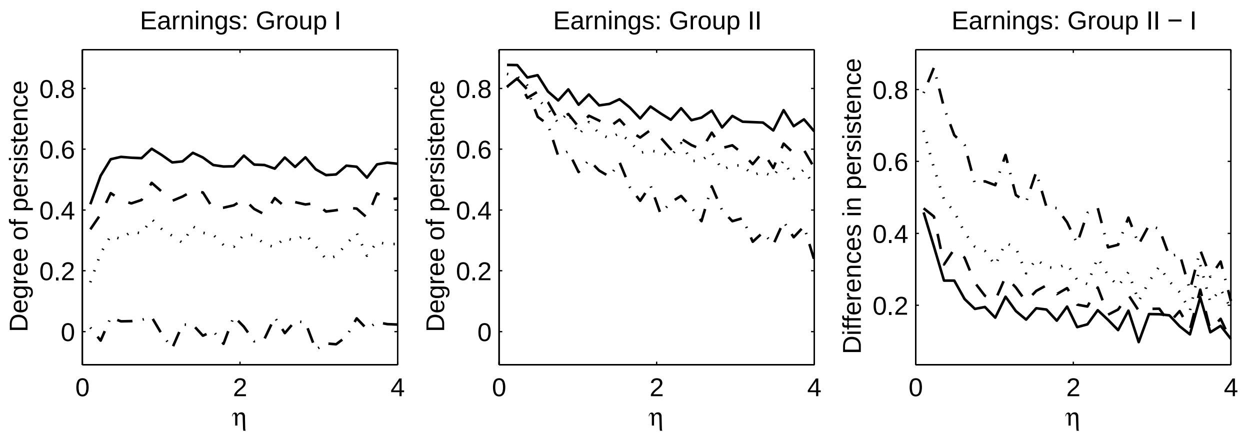

Figure 4: Left and middle: the estimated degrees of earnings persistence of Groups I and II, respectively, in Cases 1,2,3. Right: the differences in the estimated degrees of earnings persistence between Groups II and I in Cases 1,2,3. Legend: $- \cdot -$ Benchmark, $\cdots\cdots$ Case 1, $- - -$ Case 2, — Case 3.

Now let us turn to ICM model. Our theory implies that if there were no selection problems, the OLS estimates for Group I in the ICM model should have the same probability limits as those in the PI model. Proposition 2 states that selection does not matter for the benchmark. But the figure shows that selection does matter for Cases 1,2,3. In all three cases, the OLS estimates for Group I in the ICM model are larger than those of the PI model. Those upward biases are solely due to sample selection.

As before, in order to see the effects of adding more heterogeneity into the model, we plot in a single graph the estimated degrees of earnings persistence for each group of families from all cases. The first two windows of Figure 4 correspond to Groups I and II, respectively. As we add more heterogeneity from the benchmark to either Case 1 or Case 2, then to Case 3, the observed degrees of earnings persistence increase for both groups of families. The differences of the observed degrees of earnings persistence between Groups II and I are plotted in the right window of Figure 4. It shows that the differences in the observed degrees of earnings persistence between Groups II and I are smaller in Case 3 than those in the benchmark and Cases 1 and 2. In other words, more heterogeneity makes it more difficult to detect the effects of capital market imperfection. This is especially true when η is large, because the differences decrease in η and are less than 0.15 for $\eta > 1$.

4.3.3 Mobility in Pooled Samples

What happens if the data are generated by an ICM model and we run a regression with all families pooled together, as is often done in the studies built on a mechanical statistical model of intergenerational mobility? Figure 1 shows that in all cases, pooling Groups I and II tends to overestimate the degrees of consumption persistence for Group II (borrowing constrained) families (the degree of overestimation is small in Case 3) and underestimate for Group I (non-borrowing constrained) families (the degree of underestimation is large in Case 3). Figure 3 shows that in all cases, a mechanical model tends to underestimate the degrees of earnings persistence for Group II. It overestimates the degrees of earnings persistence for Group I in the benchmark; and underestimates those in Cases 1 and 2 for $\eta > 1$, and in Case 3 for all possible η's.

However, it seems that for estimating the degrees of consumption mobility, the misspecification of a mechanical model may not be important if the observed differences of the degrees of consumption persistence between Groups I and II are already small in the ICM model. This is true when $\eta < 1$. For estimating the degrees of earnings mobility, the misspecification of the mechanical model presents more serious problems.

5 Heterogeneous Altruism Rates

In this section, we examine the possible specification bias caused by heterogeneous altruism rates. We model the heterogeneity according to

$$\ln \alpha_i = \ln \alpha + \zeta_i, \tag{17}$$

where ζ_i is assumed to be *i.i.d.* across all periods and families and normally distributed as $N(0, \sigma_\zeta^2)$. Note that $\ln \alpha_i$ has a constant mean of $\ln \alpha$.

5.1 PI Model: Some Analytical Results

Use (3) to get the evolution equation of consumption

$$\ln C_{t+1} = cst + \ln C_t + \eta \zeta_t + \epsilon_{t+1}. \tag{18}$$

Hence,

$$\text{plim}(\hat{\beta}) = 1 + \eta \frac{COV(\ln C_t, \zeta_t)}{V(\ln C_t)}. \tag{19}$$

Since (2) implies that $\frac{\partial \ln C_t}{\partial \zeta_t} < 0$, the covariance term in equation (19) is negative.[10] So the OLS estimator of consumption mobility is downward inconsistent with respect to the coefficient of $\ln C_t$ in equation (3) (i.e., less than 1).

To see the economics of this result, consider a group of families that are identical in every way except their degrees of intergenerational altruism α. More altruistic parents would choose higher consumption for children and lower consumption for parents, while less altruistic parents would choose lower consumption for children and higher consumption for parents. A cross-sectional regression of $\ln C_{t+1}$ on $\ln C_t$ would actually yield a negative coefficient, which is obviously a downward inconsistent estimator of the true coefficient, 1.

Heterogeneity in altruism rates does not add any other new biases to the estimation of earnings mobility in PI model. That is because earnings in PI model only depend on the efficient human capital investment, which is the same for all families with the same A_{t+1}, regardless how much they love their children, or how rich or poor parents are. Hence, adding heterogeneity in altruism rates does not change the evolution of earnings across generations (which is (14)).

In summary, the above analysis has proved the following.

Proposition 5 In PI model, with assumptions (13) and (17), the OLS estimate of the degree of consumption mobility is downward inconsistent with respect to (3) (i.e., less than 1). The OLS estimate of the degree of earnings mobility has the same property as in Proposition 3.

The results of Proposition 5 are displayed in the first two rows of Table 2. To isolate the effects of heterogeneous altruism, Cases 4 and 5, as defined below, are juxtaposed with their corresponding homogeneous altruism cases. Heterogeneous altruism adds a downward omitted variable bias to β_c and no additional bias to β_e.

Because the bias of OLS estimates of consumption mobility in above proposition is merely caused by the correlation between C_t and ζ_t, an instrumental variable (uncorrelated with parental altruism but correlated with parental consumption), say parental income, could be used in the usual way to eliminate the bias.

[10] It can be shown that if random variables x and y are independent, and $f(x, y)$ is increasing (decreasing) in x, then $COV(f(x, y), x)$ is greater (less) than 0.

Table 2: Biases Induced by Heterogeneous Altruism					
		Benchmark	**Case 4**	**Case 3**	**Case 5**
		α constant	α varies	α constant	α varies
		$\ln A_{t+1}$ constant		$\ln A_{t+1} = \theta \ln A_t + \nu_{t+1}$	
	β_c vs 1	0	OVB(-)	0	OVB(-)
PI	β_e vs 0	0	0	OVB(+)	OVB(+)
	β_e vs θ	RB(-)	RB(-)	RB(-)	RB(-)
	β_{c1} vs 1	0	OVB(-)	0	OVB(-)
			SB(-)		SB(-)
	β_{e1} vs 0	0	0	SB(+)	SB(+)
				OVB(+)	OVB(+)
	β_{e1} vs θ	RB(-)	RB(-)	RB(-)	RB(-)
ICM				SB(+)	SB(+)
	β_{c2} vs β	0	SB(-)	SB(+)	SB(?)
			OVB(-)	OVB(+)	OVB(?)
	β_{e2} vs Φ	$0^{\dagger}$	SB(+)†	SB(+)	SB(?)
				OVB(+)†	OVB(+)†

RB="regression" or "measurement error" bias; 0 = no bias; (?) = bias ambiguous
SB="selection bias"; (+)=upward bias
OVB="omitted variable bias"; (-)=downward bias

$\Phi = \gamma \cdot$ elasticity of $h(I_t, A_{t+1})$ with respect to I_t
evaluated at some (I_t, A_{t+1}) within the range of the data

† If grandparents are not borrowing constrained, then there is a regression bias. The bias can be upward or downward. See, e.g., Proposition 2.

5.2 ICM Model: One Analytical Result

Introducing heterogeneous altruism rates into the benchmark does not affect the property of the estimated degrees of earnings persistence for Group I families in ICM models. This is because

$$\begin{aligned}
\text{plim}(\hat{\beta_{e1}}) &= \frac{COV(\ln e_t, \ln e_{t+1}|\Delta)}{V(\ln e_t|\Delta)} \\
&= \frac{COV(\epsilon_t, \epsilon_{t+1}|\Delta)}{V(\ln e_t|\Delta)} \\
&= 0
\end{aligned} \tag{20}$$

The last equality is true because ϵ_{t+1} is independent of $(\epsilon_t, I_t, \alpha)$ and $\ln A_{t+1}$ is constant. This leads to the following.

Proposition 6 In the model with the benchmark assumption (12) and assumption (17), the OLS estimator of the degree of Group I's earnings mobility is a consistent estimate of the degree of unequal earnings opportunity, 0, and a downward inconsistent estimate of inheritability of ability, θ.

The result of Proposition 6 is displayed in the second column of Table 2: β_{e1} remains an unbiased estimator of 0 when we deviate from the benchmark case by introducing heterogeneous altruism. As shown in the fourth column, we do not in general expect heterogeneous altruism to introduce any new biases to β_{e1}.

Table 2 reports that heterogeneous altruism does create selection and omitted variable biases in the estimation of consumption persistence in either group. The selection biases occur because α enters the consumption equations with a positive sign and because the selection introduces a negative correlation between α and I_t. The omitted variable biases are negative because, even in the absence of selection, α and C_t are negatively correlated.

Heterogeneous altruism also creates downward selection biases in the estimation of earnings persistence in Group II because α enters the earnings equation with a positive sign and because the selection introduces a negative correlation between α and I_t.

5.3 Simulation Results for Both Models

To see how heterogeneity in altruism rates affects the OLS estimates in ICM model, we conduct two kinds of experiments:[11]

Case 4: Benchmark plus (17) ;

Case 5: Case 3 plus (17).

That is, in Case 4 we introduce heterogeneity in altruism rates into the benchmark model, while in Case 5 there exists heterogeneity in both earnings ability and altruism rates. So Cases 3 and 4 show separately the effects of heterogeneity in ability and altruism rate, and Case 5 shows the aggregate effects of the two types of heterogeneity.

[11] Recall that $\alpha = 0.295$ and $\zeta_t \sim N(0, \sigma_\zeta^2)$.

5.3.1 Degrees of Consumption Mobility

In Figure 5, the last two windows show, in order, the estimated degrees of consumption persistence for Cases 4 and 5. For each case, we plot the OLS estimates obtained from different samples. As before, we also plot the parameter β, and for comparison, we replicate the plot of consumption persistence in Case 3. We first compare Group I families of the ICM model with those in the PI model. The common feature of Cases 4 and 5 is that the estimated degrees of consumption mobility for both the PI model and Group I sample in the ICM model are downward biased with respect to the coefficient of $\ln C_t$ in (3) (i.e., less than 1). For the PI model, this is what our theory predicts. Also note that in Case 4, the observed differences between the PI model and Group I sample of the ICM model are solely due to selection biases (The selection rule $X^*_{t+1} > 0$ is correlated with both C_t and α even if A_{t+1} is constant) since without selection issues there should be no difference between them. When heterogeneity in both ability and preference presents in Case 5, there is also omitted variable bias for the OLS estimate of Group I samples of ICM model. From the last two windows of Figure 5, it is apparent that the estimated degrees of consumption persistence of Group I are higher in Case 5 than those in Case 4. This is clearly shown in the left window in Figure 6, where in a single graph, we plot the estimated degrees of consumption persistence of Group I families from all three cases.

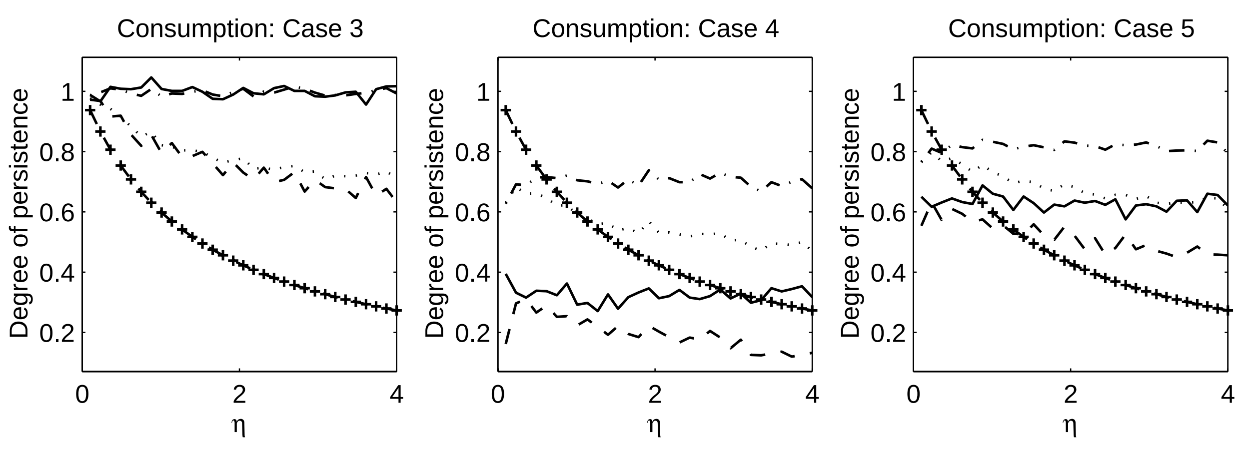

Figure 5: The estimated degrees of consumption persistence in Cases 3,4,5. Legend: $- \cdot -$ PI model, $\cdots\cdots$ Pooled sample, — Group I, $- - -$ Group II, $- + -$ β.

For Group II families, we plot the estimated degrees of consumption persistence from all three cases in the middle window of Figure 6. In Case 4, with only heterogeneity in altruism

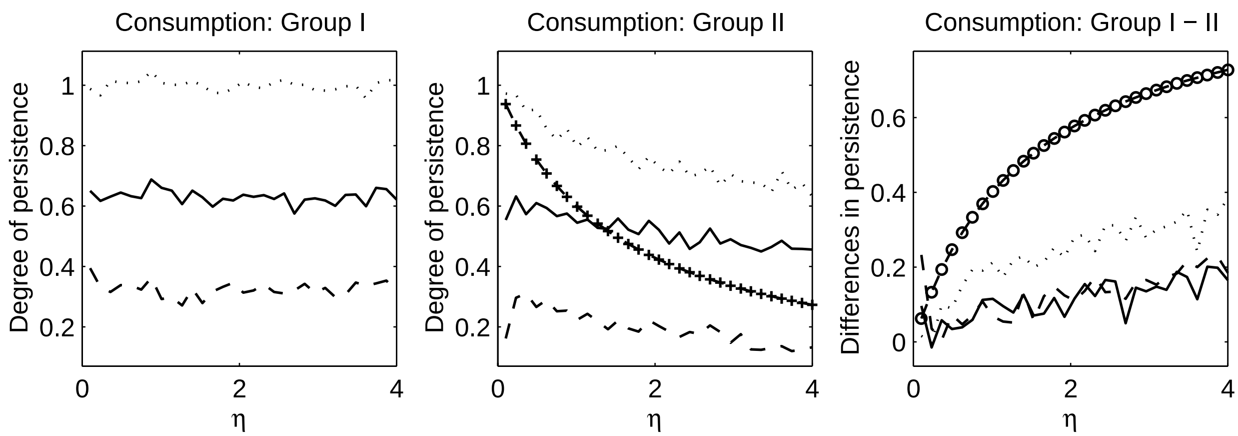

Figure 6: Left and middle: the estimated degrees of consumption persistence of Groups I and II, respectively, in Cases 3,4,5. Right: the differences in the estimated degrees of consumption persistence between Groups I and II in Cases 3,4,5. Legend: ······ Case 3, $---$ Case 4, — Case 5, $-+-\ \beta$, $-\text{o}-\ (1-\beta)$.

rates presented in the economy, the estimated degrees of consumption persistence for Group II are downward biased with respect to the coefficient of $\ln C_t$ in (6) (i.e., less than β) for all values of η. In Case 5, the estimated degrees of consumption persistence for Group II samples are higher than those in Case 4 for all η, and are upward biased (i.e., greater β) when η is large and downward biased when η is small. The observed degrees of consumption persistence in both Cases 4 and 5 are lower than those in Case 3.

We plot the differences of the estimated degrees of consumption persistence between Groups I and II in the right window of Figure 6. Although it is still qualitatively true that Group I families have larger degrees of persistence in consumption than Group II ones, the observed quantitative differences are less than 0.2 for all η, and in particular less than 0.1 for $\eta < 1$. So when $\eta < 1$, it is empirically impossible to distinguish ICM models from PI models using the estimated degrees of consumption mobility.

5.3.2 Degrees of Earnings Mobility

Figure 7 shows the simulation results for earnings mobility in Cases 3,4,5. We first look at the PI model and Group I in the ICM model. In Case 4, with only heterogeneity of altruism rates presented, the estimated degrees of earnings mobility for both the PI model and Group I sample of the ICM model are consistent (i.e., the probability limits equal to 0). In theory, if there were no selection issue, the estimated degree of earnings persistence for Group I in

Case 5 would be the same as those in Case 3. As we note before, selection problem exists for the OLS estimates of earnings persistence in Case 3. What heterogeneity in preference does is to introduce an additional source that can cause selection bias. We find that for Group I, the estimated degrees of earnings persistence in Case 5 are different from and indeed lower than those in Case 3. This is clearly seen in the left window of Figure 8, where in a single graph we plot the estimated degrees of Group I's earnings mobility from all three cases. Finally, Figure 7 also shows that adding heterogeneity in ability (from Case 4 to Case 5) leads to higher observed degrees of earnings persistence for PI models and even more so for Group I in ICM model. Nonetheless, all estimates are still downward inconsistent with respect to $\theta = 0.6$.

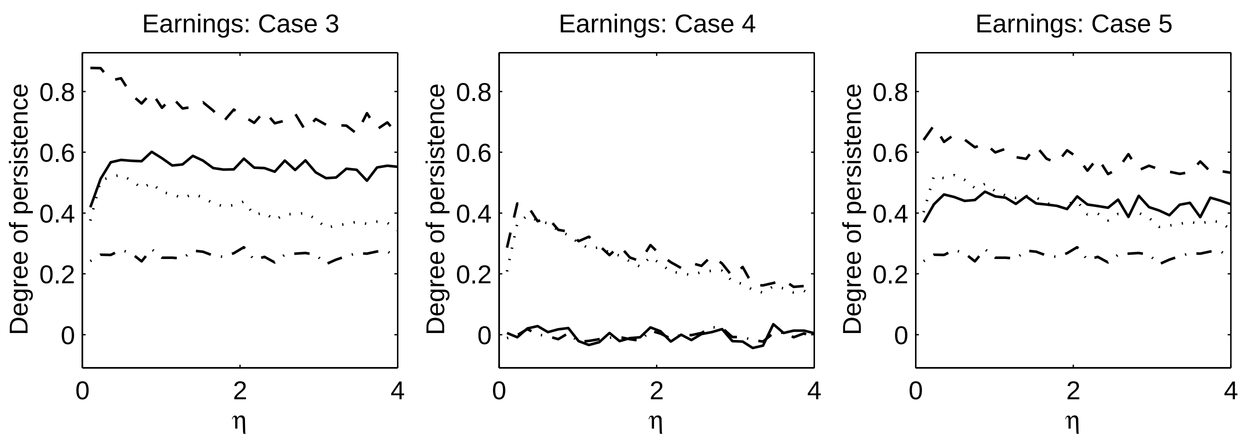

Figure 7: The estimated degrees of earnings persistence in Cases 3,4,5. Legend: $- \cdot -$ PI model, $\cdots\cdots$ Pooled sample, — Group I, $- - -$ Group II

For Group II, Figure 7 shows that their observed degrees of earnings persistence are larger than Group I in all cases. So earnings for non-borrowing constrained families observably regress to mean faster than borrowing constrained families. In the middle window of Figure 8, we plot the estimated degrees of Group II's earnings mobility from all three cases. The plot shows that, unlike Group I families, heterogeneity in altruism rates does have effects on the estimated degrees of earnings persistence for Group II in Case 4 – they are lower than those in the benchmark. But like Group I, heterogeneity in altruism rates reduces estimated degrees of earnings persistence for Group II families in Case 5, comparing to those in Case 3. Finally, adding heterogeneity in ability (from Case 4 to Case 5) increases the observed degrees of earnings persistence for Group II families.

The last window of Figure 8 shows that for both Cases 3 and 5, the differences in the

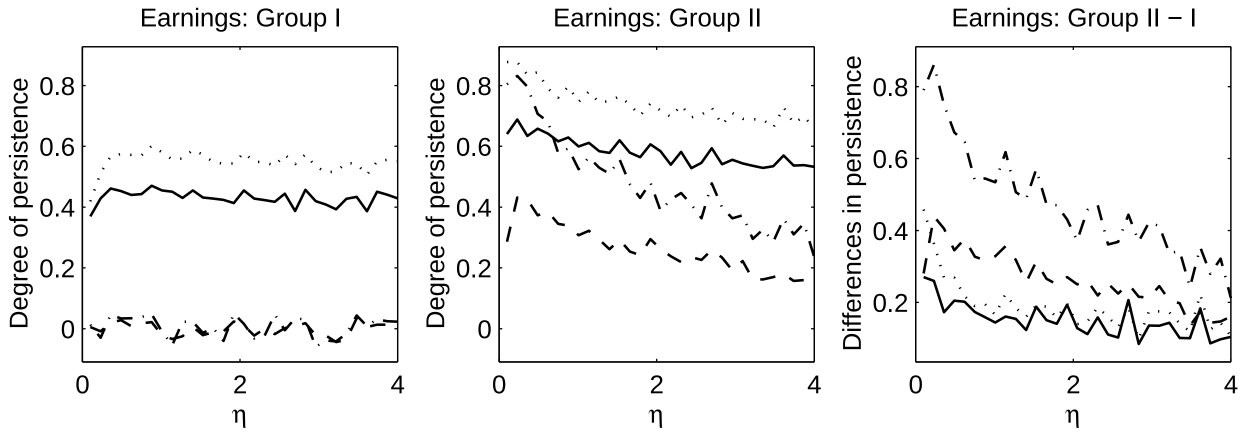

Figure 8: Left and middle: the estimated degrees of earnings persistence of Groups I and II, respectively, in Cases 3,4,5. Right: the differences in the estimated degrees of earnings persistence between Groups II and I in Cases 3,4,5. Legend: $-\cdot-$ Benchmark, $\cdots\cdots$ Case 3, $---$ Case 4, — Case 5.

estimated degrees of earnings persistence between Groups II and I become smaller when η gets larger, and are much lower than the lower bound of the differences in the benchmark. Indeed, when $\eta > 1$, the differences are always less than 0.15. So it becomes more difficult to distinguish PI and ICM models when η are large. Conversely, this implies that ICM model may only be empirically useful for estimating the degrees of earnings mobility when η is in the low range of $(0, 4)$.

5.3.3 Mobility in Pooled Samples

Suppose that the intergenerational data are generated by an ICM model with heterogeneity in both earnings ability and altruism rate, as in Case 5. The third window of Figure 5 shows that if we pool all families together to obtain OLS estimates of intergenerational mobility, the misspecification leads to an overestimate of the degrees of consumption persistence for Group II (by almost 0.2 for all η) and a slight overestimate for Group I. The overestimation for Group I families is particularly small when η is large.

For earnings, Figure 7 (third window) shows that regressions using pooled samples underestimate the degrees of earnings persistence for Group II (by almost 0.2 for all η) . But for all η, the misspecification errors are small in estimating the degrees of earnings persistence for Group I.

6 The Effects of Heterogeneity on Nonlinear Estimates of Intergenerational Mobility

Our analysis so far considers a linear specification of the OLS estimations of the degrees of intergenerational mobility. Studying the linear specification is important because such regressions are the most common in the empirical literature and are the starting point of other estimation methods. Nonetheless, there are studies that appreciates the nonlinearities of the underlying economic models (Cooper et al. 1994; Durlauf 1996). Although a thorough study on the effects of heterogeneity in those methods is clearly a subject of another paper, we report here some exploratory results for two ad hoc nonlinear specifications. The first is a "quadratic specification." That is, we consider a regression of child's outcome (consumption or earnings) on both parents' outcome and its square. The regression is run by using the whole sample. The second is a "rich-poor specification." That is, we split the whole sample into two subsamples according to parental income. "Rich families" are those whose parental incomes are above certain income percentile of the sample and "poor families" are those below that percentile. The rich-poor specification then applies a linear specification to the rich and poor subsamples.

The data are generated by ICM model. For each of the two specifications, we obtain the estimated degrees of intergenerational mobility for three cases: the benchmark, Case 3 and 5. Recall that in the benchmark, families are different in only parental incomes. In Case 3, there is heterogeneity in ability. In Case 5, there is heterogeneity in both ability and altruism rate. As reported below, our simulation results show that heterogeneity also dulls the power of both nonlinear specifications to detect the presence of borrowing constraints.

6.1 Ad Hoc Specification 1 – Quadratic

For consumption mobility, we estimate the equation:

$$X_{t+1} = b_0 + b_1^c X_t + b_2^c X_t^2 + \upsilon^c,$$

where X_i is the difference between $\log C_i$ and its cross-section mean for $i = t, t+1$. So b_1^c measures the degree of consumption persistence for families with log parental consumption equal to the mean of log parental consumption in the sample. The estimated b_1^c and b_2^c are plotted in the upper two windows of Figure 9. The figure shows different effects of heterogeneity on b_1^c and b_2^c. First, adding heterogeneity in ability (Case 3) in the otherwise homogeneous model (the benchmark) increases the estimated b_1^c, especially for $\eta > 1$. But with heterogeneity in both ability and altruism rate (Case 5), the estimated b_1^c becomes only slightly smaller than those in the benchmark, except for $\eta < 1$. Second, heterogeneity in both ability and altruism rate reduces the estimates for b_2^c. In particular, with both types of heterogeneity (Case 5), the estimated b_2^c's are less than 0.1 for most η. This implies that heterogeneity reduces the power of the nonlinear specification for estimating the degree of consumption mobility.

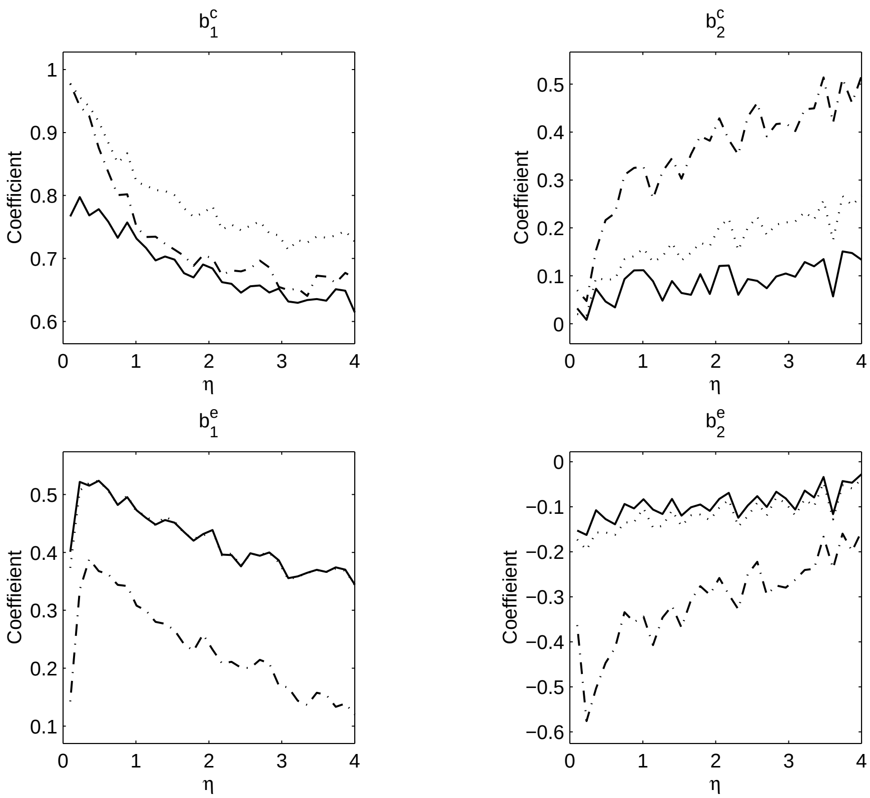

Figure 9: A quadratic specification of the OLS estimation of the degrees of consumption and earnings persistence. Legend: $- \cdot -$ Benchmark, $\cdots\cdots$ Case 3, — Case 5.

For earnings mobility, we estimate the equation:

$$X_{t+1} = b_0 + b_1^e X_t + b_2^e X_t^2 + \upsilon^e,$$

where X_i is the difference between $\log e_i$ and its cross-sectional mean for $i = t, t+1$. So b_1^e measures the degree of earnings persistence for families with log parental earnings equal to the mean of log parental earnings in the sample. The estimated b_1^e and b_2^e are plotted in the bottom two windows of Figure 9. The figure shows that in the benchmark, except for some small η, b_1^e is decreasing in η (from 0.37 to 0.17), and b_2^e is increasing in η (from -0.5 to -0.15). With heterogeneity in ability (Case 3), b_1^e is increased by about 0.2 for all η, and the absolute value of b_2^e is reduced by as much as 0.4 for $\eta < 1$ and about 0.3 for $\eta > 1$. In particular, b_2^e ranges from -0.2 to -0.05. The figure also shows that heterogeneity in altruism rate has no effects on b_1^e, but slightly increases b_2^e, making it even closer to 0. So, heterogeneity also reduces the power of the nonlinear specification for estimating the degree of earnings mobility.

6.2 Ad Hoc Specification 2 – Rich/poor

For the experiment reported here, we define "rich families" as those whose parental incomes are in the top 50 percentile of the sample and "poor families" as those in the lower 50 percentile. We then apply a linear specification to each subsample to obtain OLS estimates of the degrees of consumption and earnings mobility.

Notice that besides the sources of bias discussed in Tables 1 and 2, the rich-poor specification induces a bias from imperfect classification of Group I-II in the underlying ICM model. The classification is imperfect because whether a family is borrowing constrained depends on not only parental income but also child's ability and parents' altruism rate. This misclassification bias is minimized here because in calibrating ICM model, we control the fraction of Group I (non-borrowing constrained) to be around 50%. In particular, in the benchmark, parental income is the only factor that determines whether a family is borrowing constrained. So we expect that there will be essentially no misclassification bias there.[12] This is confirmed by our simulations.

In the top row of Figure 10, we plot the estimated degrees of consumption persistence for both rich and poor families as well as the difference between the rich and the poor. The estimates for earnings persistence are plotted in the low row of the figure. For the benchmark, except for some small η, Figure 10 does not show any difference of the estimated degrees of both consumption and earnings persistence from those in Figures 6 and 8. For Cases 3 and 5, the misclassification does have effects: The estimated degrees of both consumption and earnings persistence for the rich (the poor) in both Cases 3 and 5 are smaller than those for

[12]Some misclassifications still exist in the benchmark because our calibration is based on Case 5. Also the fraction of Group I can be more or less than, but on average, equal to, 50% for different η.

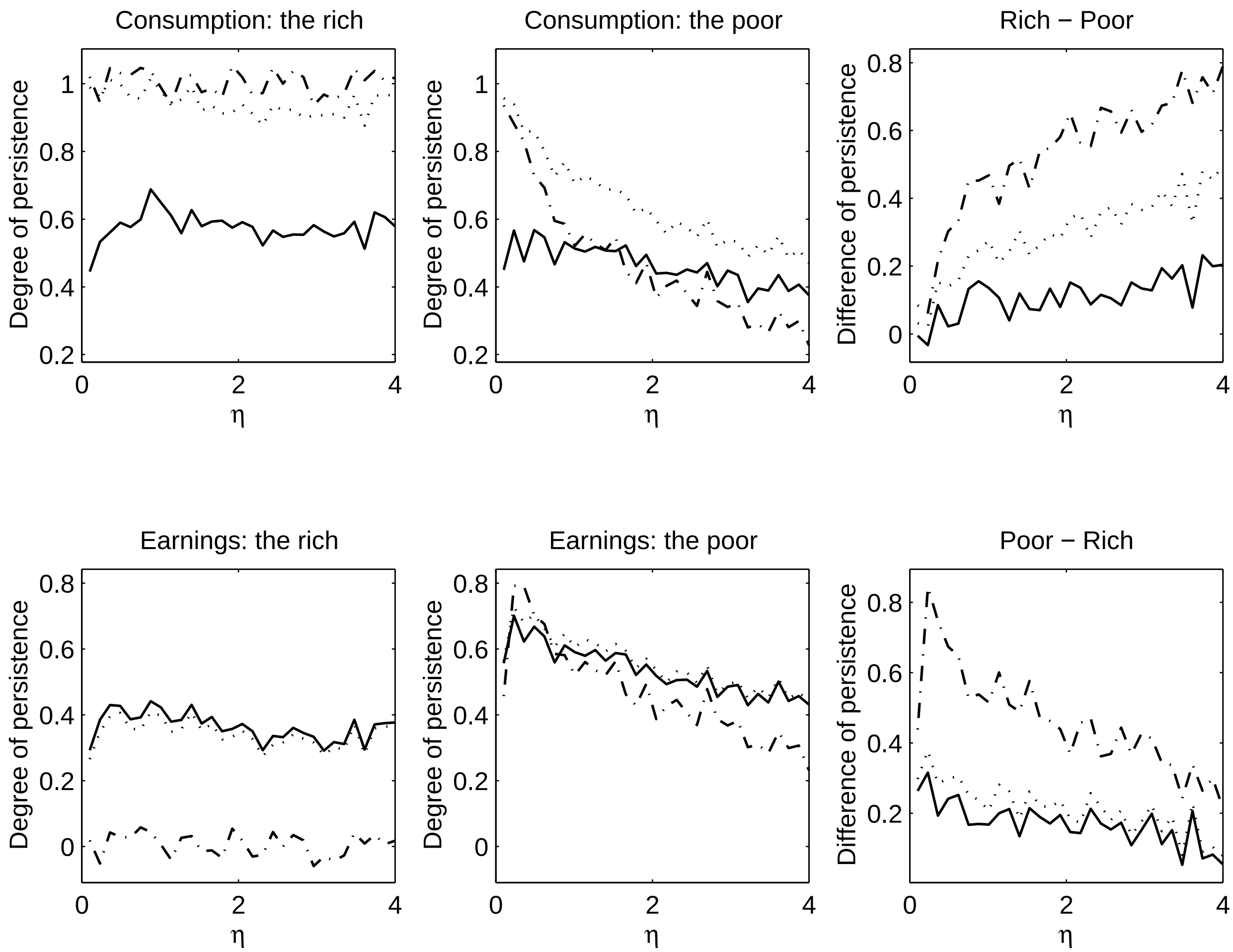

Figure 10: A "rich-poor" specification for the estimation of the degrees of consumption and earnings persistence. Legend: $- \cdot -$ Benchmark, $\cdots\cdots$ Case 3, — Case 5.

Group I (Group II). For example, in Case 3, the estimated degree of consumption persistence for Group I (non-borrowing constrained) is 1, but it is less than 1 for the rich. The estimated degree of earnings persistence for Group I is about 0.55, but it is less than 0.4 for the rich. Similar results hold for Group II and the poor, as well as for Case 5.

However, the effects of heterogeneity on the rich-poor differences in the estimated degrees of both consumption and earnings persistence are still similar to those on the Group I-II differences. As in Figures 6 and 8, Figure 10 shows that with more heterogeneity introduced in the model (from the benchmark to Case 3 to Case 5), the rich-poor differences get smaller. In particular, with heterogeneity in both ability and altruism rate, the rich-poor differences in consumption persistence are less than 0.15 for all but some large η and no more than 0.2 for all η; the rich-poor differences in earnings persistence (in absolute value) are less than

0.2 for all but some $\eta < 1$ and no more than 0.3 for all η. In short, heterogeneity makes it empirically difficult to distinguish the rich from the poor for consumption mobility if η is not large and for earnings mobility if $\eta > 1$.

6.3 Efficient Estimation

We have a mathematical model of intergenerational mobility, and have fully specified functional forms and the distributions of unobservables – it is straightforward to derive a likelihood function from our model. Maximum likelihood would seem to be the preferred estimation method, even though most of the empirical literature explores least squares and two-stage least squares estimators (Mulligan (1997), Appendix E is one exception). Nevertheless, our analysis of linear estimators highlights some of the difficulties inherent in detecting the existence and importance of borrowing constraints even with nonlinear estimators tailored to our economic model. First, there is a "regression bias" in the estimation of the inheritability of ability θ because ability is unobserved, and earnings is an imperfect proxy for ability. This is a familiar measurement error problem and – whether it be with instruments, aggregation, or some independent indicator of the relationship between signal and noise – must be addressed in any consistent maximum likelihood estimate of θ.

Second, there are omitted variable biases. Some variables, such as child's ability and parents' altruism rate, are unobservable, but they are determinants of child's consumption (earnings) even if given other observables such as parents' consumption (earnings). Omitting these variables will result in biased estimate even in a correctly specified maximum likelihood estimation. To reduce omitted variable biases, one has to use instrumental variables or find proxies for those unobservables. For example, in Dearden et al. (1997) and O'Neill and Sweetman (1998), test score is used as a proxy for child's ability.

Third, there are selection biases. The correct likelihood function would explicitly recognize the selection process, and would make some kind of "correction" relative to an OLS estimate. However, this is no substitute for having an instrumental variable that affect the earnings (consumption) of children differently than it affects selection into Group I or Group II (olsen 1980, pp. 1818-19, comments in more detail). Such instrumental variables are not readily available, although perhaps region or country of residence might interpreted as such a variable, and the cross-regional studies of Cooper (1996) and Mulligan (1999) and the cross-country comparisons we review in Section 7 might be interpreted as attempts to identify selection into the constrained or unconstrained group.

7 Summary and Conclusions

Some of the important implications of the parental investment model of intergenerational mobility have been derived under the assumption that parental income is the main source of heterogeneity. We further explore the links between statistical and economic models of intergenerational mobility, showing how those links depend on the amount and types of heterogeneity in the economy. The presence of heterogeneity substantially increases the difficulty of detecting the existence and importance of borrowing constraints with regression estimators, or even with nonlinear estimators tailored to the economic model. In the following, we summarize our main findings, show how they fit into the previous literature, and point out their implications for future research.

7.1 PI and ICM Models Have Similar Predictions for Intergenerational Mobility

In the benchmark model, families are identical in every aspect except income. From the benchmark to either Cases 1 or 2, then to Cases 3 and 5, we sequentially introduce heterogeneity in earnings ability and altruism rate. Although it is qualitatively true that the observed degree of consumption persistence is larger among non-borrowing constrained families than constrained ones, and that the observed degree of earnings persistence is larger among borrowing constrained families than unconstrained ones, the observed quantitative intergroup differences in the degrees of both consumption and earnings persistence are reduced with more heterogeneity. So heterogeneity makes it more difficult to empirically distinguish ICM model from PI model. Or to put it another way, borrowing constraints have only subtle implications for intergenerational mobility.

OLS regressions using pooled samples, as suggested by mechanical statistical models, tend to overestimate the degrees of consumption persistence and underestimate the degrees of earning persistence for Group II (borrowing constrained) families, but have less adverse effects on estimating degrees of persistence of both consumption and earnings for Group I (non-borrowing constrained) families. In the range of η where PI and ICM models are not distinguishable from each other, the misspecification of mechanical models does not cause very serious problems in estimating the degrees of intergenerational mobility of both consumption and earnings.

We also find that different types of heterogeneity have different effects on the estimations. Both our intuitive discussions (Tables 1 and 2) and simulated results show that heterogeneity in earnings ability tends to make OLS estimates of consumption mobility downward biased and earnings mobility upward biased, while heterogeneity in altruism rate has somewhat opposite effects. The direction and magnitude of the joint effects of the two types of heterogeneity depend on the parameter of preference, η. Next subsection provides more comments on this.

In our calibration process, we try to be conservative regarding the degree of heterogeneity introduced into the model. Our benchmark case shows that less heterogeneity would increase the distinctness of the predictions of the PI and ICM models. Our intuition as well as some additional (unreported) simulated results suggest that more heterogeneity would further reduce observable differences between the PI and ICM models.

7.2 The Importance of η

The problem of heterogeneity presents a bigger challenge to ICM models especially for $\eta < 1$ in term of consumption mobility and for $\eta > 1$ in term of earnings mobility. When $\eta < 1$, the observed differences in the degrees of consumption persistence between Groups I and II are less than 0.2 and 0.1 in Cases 3 and 5, respectively. When $\eta > 1$, the observed differences in the degrees of earnings persistence between Groups II and I are less than 0.2 and 0.15 in Cases 3 and 5, respectively. Hence, η, the elasticity of substitution between parents' and children's consumption, plays an important role in determining whether alternative models can be distinguished empirically from each other and from the mechanical statistical model.

Since η is such an important parameter in the model, we briefly discuss its economic content. Technically, η serves two roles in our model–as an intergenerational elasticity of substitution and as a coefficient of intergenerational risk aversion. As Hall (1988) points out, our functional forms parameterize these economically distinct concepts in a single dimension while most of our implications are really only related to a family's willingness to substitute consumption across generations rather than its willingness to accept risk. Nor is our parameter η related to the life cycle substitution elasticities that have been estimated in the labor and macroeconomic literatures such as Ghez and Becker (1975), MaCurdy (1981), Mulligan (1999b), Runkle (1991) and others.[13] Economic theory alone does not restrict the relationship between the willingness of a family to substitute consumption or leisure over the life cycle of its members and its willingness to substitute consumption across members.[14] Hence, empirical evidence is sorely needed in order to say something about the magnitude of η, but we unaware of a single empirical estimate in the literature to date.

7.3 Earnings Mobility Around the World

Researchers have for decades compared estimates of intergenerational mobility across countries (eg., Goode 1966). The methods for collecting and/or analyzing intergenerational data typically differ across countries, so that estimates may vary across countries mainly for statistical reasons, but significant progress has recently been made in applying uniform sampling, measurement, and estimation procedures across countries in order to mitigate the importance of statistical differences across countries (eg., Bjorklund and Jantti (1997, U.S. vs Sweden), Couch and Dunn (1997, U.S. vs Germany), and Checchi et al (1999, U.S. vs Italy)). To the extent that intergenerational earnings mobility differs across countries - some of the studies suggest that Sweden (and Italy?) is both more equal and more mobile - one interpretation suggested by the human capital models of Loury (1981), Becker and Tomes (1986), and others is that the mobile countries have institutions for human capital accumulation that alleviate borrowing constraints. Our analysis suggests that, even if borrowing constraints are of differential importance across countries, it will be hard to detect earnings mobility differences. To see this, notice from our "PI model" and "pooled sample" simulations that the complete elimination of borrowing constraints decreases earnings persistence by at most 0.2, and often less (except for some very small η). Since even the "best" country will not completely eliminate borrowing constraints, the size of the cross-country earnings persistence gap that can be reasonably be attributed to borrowing constraints is 0.1 or less.[15]

[13] Bénabou (1997) studied a model with Kreps-Porteus preferences, where an agent's risk-aversion is independent of his intertemporal elasticity of substitution. But he did not explicitly study intergenerational mobility.

[14] Mulligan (1993, pp. 56-63) shows that, when the household head is altruistic, the life cycle and intergenerational elasticities of substitution are identical when the household head linearly aggregates utilities of different members. The intergenerational elasticity is smaller when the household head has a preference for equalizing utilities (which Mulligan calls "egalitarian"). Perhaps it is also possible that parents have a preference for unequal utilities (and perhaps primogeniture is evidence revealing such a preference?), in which case the life cycle elasticity would be smaller than the intergenerational elasticity.

[15] Our study identifies "selection bias" as one of the difficulties with dividing a single country's data into groups of "constrained" and "unconstrained" for the purpose of detecting the mobility effects of borrowing

Cross-country differences in the *heterogeneity* of inherited ability can produce differences in earnings inequality and *substantial* earnings mobility differences even when the inheritability of ability and the importance of borrowing constraints are the same across countries. To see this, suppose for the sake of argument that the permanent income model applies in all countries. Suppose that, with the exception of "ability" inequality (namely, σ_ν^2), all countries have the same model parameters, and parameters equal to our benchmark. In particular, the inheritability of ability (θ) is the same in all countries. Let ability inequality be greater in the U.S. ($\sigma^2_{\ln A_t} = \sigma^2_{\ln A_{t+1}} = 0.026$) than in Sweden ($\sigma^2_{\ln A_t} = \sigma^2_{\ln A_{t+1}} = 0.005$), perhaps because the U.S. is geographically, racially, and ethnically more diverse than Sweden. It follows from (4) and (15) that Swedish earnings are more equal (std dev of log earnings = 0.35) and more mobile (earnings persistence = 0.16) than in the U.S. (std dev of log earnings = 0.50, earnings persistence = 0.38). Of course, these calculations do not suggest that borrowing constraints do not operate in either Sweden or the US, only that cross-country differences in the amount of inherited heterogeneity can lead to cross-country differences in earnings inequality and substantial cross-country differences in earnings mobility, even when borrowing constraints are equally prevalent in all countries.

7.4 A Life Cycle Interpretation

Although we are solely concerned with the distribution of resources across generations, our models are mathematically similar to life cycle savings models that have appeared in the literature, including Hall (1978), Zeldes (1989), Runkle (1991), and others. In particular, our results suggests that, in the presence of heterogeneous determinants of the liquidity constraint or heterogeneous rates of time preference, it may be difficult to empirically distinguish the "permanent income" and "liquidity constraint" models by estimating consumption Euler equations as often advocated in the literature. Some work needs to be done to verify our conjecture, because there are a number of obvious differences between a life cycle and a dynastic model. First is the length of a period, and thereby the calibration of the interest rate, the discount rate, and the persistence of unobservables. Second, even after correcting for the time dimension differences, it still seems that unobservables would be more persistence in

constraints. Dividing data from around the world into subsamples by country presumably alleviates selection bias, and this is one of the advantages of the cross-country comparisons. The disadvantage is that country of residence is only weakly correlated with facing a borrowing constraint, because there are certainly some unconstrained Americans and some constrained Swedes.

a life cycle model than in a generational model because the former is about the same person at different points in time while the latter is about different people. Other parameters are also different in a life cycle model, as is the nature of the technologies for substituting consumption over time when borrowing constraints bind.

Appendix: Proof of Proposition 2

1. The OLS estimate of the degree of consumption persistence for Group I is consistent, i.e., the probability limit equals to 1.

$$\begin{aligned} \text{plim}(\hat{\beta_{c1}}) &= \frac{COV(\ln C_{t+1}, \ln C_t|\Delta)}{V((\ln C_t)|\Delta)} \\ &= 1 + \frac{COV(\epsilon_{t+1}, \ln C_t|\Delta)}{V(\ln C_t|\Delta)} \\ &= 1 \end{aligned} \tag{21}$$

The last equality is true because ϵ_{t+1} is independent of $\ln C_t$ and I_t.

2. The OLS estimate of the degree of earnings persistence for Group I has probability limit of 0.

$$\begin{aligned} \text{plim}(\hat{\beta_{e1}}) &= \frac{COV(\ln e_t, \ln e_{t+1}|\Delta)}{V(\ln e_t|\Delta)} \\ &= \frac{COV(\epsilon_t, \epsilon_{t+1}|\Delta)}{V(\ln e_t|\Delta)} \\ &= 0 \end{aligned} \tag{22}$$

3. The OLS estimate of the degree of consumption persistence for Group II is consistent, i.e., the probability limit equals to β.

$$\begin{aligned} \text{plim}(\hat{\beta_{c2}}) &= \frac{COV(\ln C_{t+1}, \ln C_t|\overline{\Delta})}{V(\ln C_t|\overline{\Delta})} \\ &= \beta + \frac{COV(\epsilon_{t+1}, \ln C_t|\overline{\Delta})}{V(\ln C_t|\overline{\Delta})} \\ &= \beta \end{aligned} \tag{23}$$

4. For Group II, the probability limit of the OLS estimate of the degree of earnings

persistence is

$$
\begin{aligned}
\text{plim}(\hat{\beta_{e2}}) &= \frac{COV(\ln e_t, \ln e_{t+1}|\overline{\Delta})}{V(\ln e_t|\overline{\Delta})} \\
&= \frac{COV(\ln e_t, \gamma \ln h(I_t, A_{t+1})|\overline{\Delta})}{V(\ln e_t|\overline{\Delta})} \\
&= \gamma \cdot \frac{COV(\ln e_t, \frac{e_t}{I_t}\tau_{h,I} \ln e_t|\overline{\Delta})}{V(\ln e_t|\overline{\Delta})},
\end{aligned}
\tag{24}
$$

where $\tau_{h,I} = \frac{\partial \ln h(x, A_{t+1})}{\partial \ln x}$, evaluated at some $x = \xi$. The last equality is obtained by using (5) and mean-value theorem. Note that ξ is a random variable correlated with I_t and $(\xi, A_{t+1}) \in \overline{\Delta}$. Using (5), it is straightforward to show that $\epsilon_{h,I} \in (\frac{\beta}{\gamma}, 1)$ if $\eta > 1$, and $\epsilon_{h,I} \in (1, \frac{\beta}{\gamma})$ if $0 < \eta < 1$.

If grandparents are borrowing-constrained, $I_t = e_t$. So $\text{plim}(\hat{\beta_{e2}}) \in (\gamma, \beta)$ if $0 < \eta < 1$; and $\text{plim}(\hat{\beta_{e2}}) \in (\beta, \gamma)$ if $\eta > 1$. By (11), $\text{plim}(\hat{\beta_{e2}}) = \Phi$, evaluated at some $(I_t, A_{t+1}) \in \overline{\Delta}$. If grandparents are non-borrowing-constrained, $I_t \neq e_t$. Then in general, the OLS estimator is an inconsistent estimator of Φ.

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