NBER WORKING PAPER SERIES

THE FORWARD EXCHANGE MARKET, SPECULATION, AND EXCHANGE MARKET INTERVENTION

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Working Paper No. 1138

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

June 1983

We wish to thank the referees for helpful comments on an earlier draft of this paper. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #1138
June 1983

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ABSTRACT

This paper develops a stochastic equilibrium model of an open economy incorporating speculation in the forward exchange market. The model is used to examine two issues. The first is the role of speculation in stabilizing the economy against stochastic disturbances. Much risk averse speculation stabilizes domestic income against disturbances in the domestic bond market and forward exchange market but exacerbates the effect of foreign disturbances. Speculation may dampen or augment the effect of money market and output supply disturbances depending upon the share of foreign bonds in total wealth and the interest elasticity of bond demand. The second issue that the model addresses is the role of the forward market in stabilization policy. Forward market intervention (or its equivalent in this model, sterilized spot market intervention) does not provide monetary authorities additional leverage in stabilizing income beyond unsterilized spot market intervention. Intervention rules based on reactions to both the forward and the spot exchange rates, however, can outperform intervention policies responding to the spot rate alone, regardless of the market in which intervention occurs.

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I. INTRODUCTION

On several occasions economists have argued that intervention in the forward market for foreign exchange can provide central banks with an additional means of achieving their policy objectives. In a Treatise on Money, Keynes (1930) proposed that intervention take place on three fronts: "I conceive of (Central Banks) as fixing week by week not only their official rate of discount, but also the terms on which they are prepared to buy or sell forward exchange on one or two leading foreign exchange centres and the terms on which they are prepared to buy or sell gold points." (Page 327). Much more recently, Spraos (1959) argued that "the forward rate should not only be supported, as a defense against speculative attack, but should be actually pegged." (Page 21).

Both Keynes and Spraos formulated their proposals for central bank intervention in the forward market under the assumption that the spot rate is fixed, or at least maintained within bands. With the advent of increased flexibility of the spot exchange rate after the demise of the gold standard and Bretton Woods, forward exchange markets have become a much more important phenomenon, with markets existing between most major currencies. But despite this development and Keynes' early recognition of the role of forward market intervention, and despite some experiments with forward market intervention by the Bank of England and the Deutsches Bundesbank (see Spraos (1959) and Day (1976)), there has been little subsequent discussion about the desirability of forward market intervention or the formulation of optimal intervention rules in the forward market.[1/]

By contrast, there has been considerable discussion about the optimal management of the spot exchange rate via intervention in the spot market. Most of the literature has addressed this issue in terms of the choice between perfectly fixed and perfectly flexible exchange rates. However, recently, several authors have treated the degree of intervention as a policy parameter and have derived optimal intervention rules in stochastic open economy models; see, e.g., Boyer (1978), Buiter (1979), Bilson (1978), Roper and Turnovsky (1980), Turnovsky (1983), Buiter and Eaton (1980).[2/]

In this paper we develop a stochastic macroeconomic model of a small, open economy which incorporates both a spot and a forward market in foreign exchange.[3/] An important parameter linking the forward and spot rates is the elasticity of the aggregative speculative position with respect to the differential between the forward and expected future spot rate. The common procedure adopted in the literature of equating the forward rate to the expected future spot rate is equivalent to assuming that this elasticity is infinite and hence emerges as a special case of our analysis.

We use the model to focus on two issues. First, we consider the effects of private speculation through the forward market on the stability of the domestic economy in the absence of any intervention. This issue of whether or not speculative capital flows are stabilizing is an old one and was central to the early debates on fixed versus flexible rates. It has, however, been neglected in the more recent formal analyses of exchange markets. We find that the effect of more elastic speculation is to provide greater insulation for the domestic price level from speculative disturbances, but to increase its sensitivity to foreign disturbances. The effect of more elastic speculation on the sensitivity of the price level to domestic output supply and money demand shocks depends upon the share of domestic bonds in domestic wealth and the interest elasticity of bond demand with respect to the real interest rate. When domestic bonds are a small component of total bond demand, and bond demand is interest inelastic, then speculation stabilizes the domestic economy against these shocks. In the opposite case, speculation is destabilizing.

Second, we consider how central bank intervention can be used to stabilize domestic income. Introducing the forward market increases the scope for intervention in two ways. First, the central bank may intervene directly in the forward market by taking a position in that market. Second, it may use the forward rate in determining its position in either the spot or forward market. Indeed, as we show below, the response to the forward rate is a crucial part of the optimal intervention policy in stabilizing the economy against foreign disturbances.

While we focus on forward exchange market intervention as a policy instrument, our analysis is open to another interpretation. Elsewhere (Eaton and Turnovsky (1983)) we show that when covered interest parity obtains, forward market intervention is equivalent in its effects to sterilized spot market intervention. Since our analysis is in fact based upon this interest parity assumption, our results on optimal forward market intervention can be interpreted equally as applying to optimal sterilized spot market intervention, while the spot market intervention we consider in this paper is unsterilized.

Section II sets forth the basic model. In Section III the model is solved for the case where there is no intervention, so that the exchange rate is perfectly flexible. Here we consider the effects of the elasticity of speculation on exchange rate, price, and output stability. Section IV introduces central bank intervention into the model, while Section V discusses the effects of intervention on the stability of output.

II. THE FORWARD EXCHANGE MARKET IN A MODEL OF FLEXIBLE EXCHANGE RATES

In this section we shall assume that the exchange rate is perfectly flexible, so that the domestic monetary authorities abstain from any form of intervention. The model we develop is a simple one, enabling us to focus on the main issues without undue complication. Specifically, we assume that there is a single traded commodity, whose price in terms of foreign currency is given. Also, we shall assume that the domestic bond is a perfect substitute for a traded world bond when fully covered against exchange rate risk. Thus purchasing power parity (PPP) and covered interest parity (CIP) are assumed to hold. In the concluding section we note how our results extend to the more general case where domestic and foreign bonds are imperfect substitutes. Our analysis focuses on a small economy which takes the foreign price level and interest rate as given.

Our model can be summarized by the following set of equations:

(1a) $$p_t = p_t^* + e_t^s$$

(1b) $$m_t - p_t = \alpha_1 y_t - \alpha_2 r_t + u_t^m \qquad \alpha_1 \geqq 0\ , \quad \alpha_2 \geqq 0$$

(1c) $$r_t = r_t^* + e_t^f - e_t^s$$

(1d) $$h_t = \beta_1 y_t + \beta_2 r_t - \beta_3 (p_{t+1,t} - p_t) + u_t^b$$
$$\beta_2 \geqq 0,\ \beta_3 \geqq 0,\ \beta_2 - \beta_3 \geqq 0$$

(1e) $$s_t = \gamma(e_{t+1,t}^s - e_t^f) + u_t^s \qquad \gamma \geqq 0$$

(1f) $$h_t - (1 - \mu)(b_t - p_t) = \mu s_t \qquad 0 < \mu < 1$$

(1g) $$y_t = \theta(p_t - p_{t,t-1}) + u_t^y \qquad \theta \geqq 0$$

where

p = domestic price level

p^* = foreign price level

e^s = current spot exchange rate (measured in terms of units of domestic currency per unit of foreign currency)

e^f = forward exchange rate (measured in terms of units of domestic currency per unit of foreign currency)

m = domestic nominal money supply

y = domestic real output

r = domestic nominal interest rate

r^* = foreign nominal interest rate

h = domestic demand for bonds

b = domestic nominal supply of bonds

s = speculative demand for foreign exchange forward

u^m = stochastic disturbance in demand for money

u^b = stochastic disturbance in bond market

u^s = stochastic disturbance in speculators' demand for foreign exchange forward

u^y = stochastic disturbance in output supply

$x_{t+s,t}$ = expectation of x_{t+s} conditional on information available at time t, $x = p, e^s$.

All variables except r and r* are expressed as logarithmic deviations from steady state levels; r and r* are deviations in natural units. The subscript refers to the time dimension.

Equation (1a) describes purchasing power parity in logarithmic terms; the domestic price of a freely traded commodity equals the price abroad multiplied by the exchange rate. The domestic demand for money is of the usual form and equilibrium in the domestic money market is described by (1b). We assume that all domestic money is held by domestic residents, who in turn hold no foreign money. Equation (1c) specifies covered interest parity and embodies the assumption that domestic bonds and covered foreign bonds are perfect substitutes.

Condition (1d) specifies the domestic demand for bonds. Because bond market interacts in a crucial way with the forward market we depart from the standard practice of specifying a commodity market equilibrium, a savings or absorption equation, and suppressing the bond market equilibrium condition. Instead, we explicitly include the bond market and leave savings to be defined residually from conditions for money market and bond market equilibrium. This specification is also the most convenient in a log-linear framework and is frequently invoked in such models; see, e.g., Lucas (1975). We postulate, in general, that the demand for bonds depends upon real income, the domestic nominal interest rate, and the anticipated rate of inflation. While an increase in income is likely to raise the total demand for financial assets, it will shift the composition of demand toward money. The net effect on bond demand, and hence the sign of β_1, is ambiguous. The positive coefficient on the interest rate, β_2, and the negative coefficient on the expected rate of inflation, $-\beta_3$, together with the additional restriction $\beta_2 - \beta_3 \geq 0$ are readily derived if one assumes that (i) money and bonds are gross substitutes and (ii) the demand for bonds is more sensitive to its own real rate of return $(r_t - p_{t+1,t} - p_t))$ than to the real rate of return on money $-(p_{t+1,t} - p_t)$.

The specification of the forward market is given in equations (1e) and (1f) and can be derived from intertemporal portfolio maximization; see, e.g., Solnik (1973), Kouri (1976), Fama and Farber (1979), Eaton and Turnovsky (1981). In our model, this market has two functions. First, it provides holders of foreign bonds a means of eliminating exchange risk by selling the foreign currency proceeds of their bonds forward. Such sales constitute <u>arbitrage</u> activity on this market. Second, the forward market provides a means of <u>speculating</u> on exchange rate movements.[4/] A first order approximation to the rate of return on a forward purchase of one unit of foreign currency is given by $e^s_{t+1} - e^f_t$. In equation (1e) we postulate that the supply of foreign exchange for speculation, s_t, is an increasing function of the expected difference $(e^s_{t+1,t} - e^f_t)$. When speculators are risk neutral, or when exchange risk is absent, the speculation coefficient will tend to infinity. In the absence of any official intervention in the forward exchange market, equilibrium in the forward exchange market requires that the excess demands for forward exchange for these two purposes sum to zero. This condition is described by (1f). Because of the assumption that domestic and foreign bonds are perfect substitutes, there is in fact only a single demand function for total bonds, h_t; the demand for foreign bonds is simply the difference between the aggregate national demand and the supply from domestic sources; that is[5/]

$$H = \frac{B}{P} + H^f \tag{2}$$

where H is the total real bond demand, B is the domestic nominal bond supply, P is the domestic price, and H^f denotes the level of the real demand for foreign bonds, all expressed in levels. To express this relationship using variables defined as logarithmic deviations from steady state, we approximate (2) by

$$h_t = (1 - \mu)(b_t - p_t) + \mu h^f_t \qquad 0 < \mu < 1 \tag{2'}$$

where lower case letters denote the logarithmic deviations from steady state of the corresponding upper case variables and the parameter $\mu = \bar{H}^f/\bar{H}$ is the average holdings of foreign bonds divided by total bond demand.[6/] Since foreign bonds are covered, forward market equilibrium requires that $s_t = h^f_t$ and equation (1f) follows.

The supply of domestic output is specified by (1g). This relationship postulates that the deviation in output from some fixed capacity level depends upon the unanticipated component of the current price of domestic output. This formulation resembles a Lucas (1973) supply function, although as Flood (1979) has argued, with both international and intranational trading, this rationale is inappropriate. Rather, it may be justified in terms of the wage determination model of Gray (1976) and Fischer (1977a).

Finally, the disturbances u_t^m, u_t^b, u_t^s and u_t^y are assumed to have zero means and finite second moments and to be identically and independently distributed. The same assumptions are made about the disturbances in the foreign price level and foreign interest rate, p_t^* and r_t^*, respectively.[7/]

III. THE ROLE OF SPECULATION UNDER PERFECTLY FLEXIBLE EXCHANGE RATES

We now solve the model outlined in equations (1a)-(1g) under the assumption that there is no government intervention. We therefore set $m_t = b_t = 0$ for all t, so that domestic nominal money and bond supplies remain at their constant, steady state levels. Taking conditional expectations of the system (1) at time t for time t+i ($i \geq 1$), we can easily establish that the rational expectations of future spot exchange rates satisfy the first order difference equation

$$(3) \qquad e^s_{t+i+1,t} = \left[\frac{\beta_2 + \mu\gamma(1+\alpha_2) + \alpha_2(1-\mu+\beta_3)}{\alpha_2(1-\mu+\beta_3)}\right] e^s_{t+i,t} \equiv \phi e^s_{t+i,t} \qquad i = 1, 2, \ldots$$

Given the above parameter restrictions, the coefficient ϕ exceeds unity and accordingly, the expected exchange rate at time t+i remains bounded as $i \to \infty$ if and only if

$$(4) \qquad e^s_{t+i,t} = 0 \qquad \text{for } i = 1, 2, \ldots, \text{ and for all } t$$

Otherwise, expectations become unbounded and this in turn implies that the asymptotic variances of the spot rate (and the forward rate) will become infinite. In order to rule this possibility out, we therefore focus on the bounded solution given by (4).[8/] In particular, setting i = 1 in (4) and noting the PPP condition, we obtain the relevant expectations

$$e^s_{t+1,t} = p_{t+1,t} = 0 \quad \text{for all } t$$

Thus setting $e^s_{t+1,t} = p_{t,t-1} = 0$ in (1) implies the following unique solutions for domestic output, the price level, and the spot and forward prices of foreign exchange, in terms of contemporaneous domestic disturbances and foreign price and interest rate shocks:

$$\text{(5a)} \quad y_t = \{-(\beta_2 + \mu\gamma)\theta u^m_t + [\beta_2 + (1 + \alpha_2)\mu\gamma + \alpha_2(1 - \mu + \beta_3)]u^y_t + \alpha_2\theta u^f_t + \alpha_2\mu\gamma\theta(p^*_t + r^*_t)\}\Delta^{-1}$$

$$\text{(5b)} \quad p_t = \{-(\beta_2 + \mu\gamma)u^m_t - \Gamma_2 u^y_t + \alpha_2 u^f_t + \alpha_2\mu\gamma(p^*_t + r^*_t)\}\Delta^{-1}$$

$$\text{(5c)} \quad e^s_t = \{-(\beta_2 + \mu\gamma)u^m_t - \Gamma_2 u^y_t + \alpha_2 u^f_t - [\beta_2 + \mu\gamma + \alpha_2(1 - \mu + \beta_3) + \Gamma_2\theta]p^*_t + \alpha_2\mu\gamma r^*_t\}\Delta^{-1}$$

$$\text{(5d)} \quad e^f_t = \{(1 - \mu - \beta_2 + \beta_3 + \beta_1\theta)u^m_t - [\beta_1 - \alpha_1(1 - \mu + \beta_3) + \Gamma_1]u^y_t + (1 + \alpha_2 + \alpha_1\theta)u^f_t - [\beta_2 + \alpha_2(1 - \mu + \beta_3) + \Gamma_1\theta](p^*_t + r^*_t)\}\Delta^{-1}$$

where

$$\Gamma_1 \equiv \alpha_1\beta_2 + \alpha_2\beta_1$$

$$\Gamma_2 \equiv \Gamma_1 + \alpha_1\mu\gamma$$

$$\Delta \equiv \beta_2 + (1 + \alpha_2)\mu\gamma + \alpha_2(1 - \mu + \beta_3) + \Gamma_2\theta$$

$$u^f_t \equiv -u^b_t + \mu u^s_t$$

In general, Γ_1, Γ_2, and Δ are ambiguous in sign. The indeterminacy arises if β_1, the income elasticity of the demand for bonds is strongly negative. For simplicity, and not implausibly, we will assume that if β_1 is negative it is sufficiently close to zero to ensure that Γ_1, Γ_2 and Δ all remain positive.

Equations (5a)-(5d) summarize how the various disturbances impinge on the domestic economy. Many of these effects are familiar, and we shall restrict our remarks to those which relate to the existence of the forward market and to the nature of speculative behavior.[9/]

The Effects of Domestic Disturbance

We begin by considering the response of the spot and forward exchange rates to the three domestic disturbances. An increase in money or bond demand, or in output, all act to lower the spot rate, while an increase in the speculative demand for foreign exchange forward has the opposite effect. Bond demand and speculative disturbances (as reflected in the composite disturbance u_t^f) affect the forward rate in the same direction and in much greater magnitude, while money demand and output supply shocks have an ambiguous effect on the forward rate.

The reason why the spot rate reacts to these disturbances as it does is straightforward, as is the reason why the forward rises in response to a forward market disturbance. Less obvious is why the direction of the response of the forward rate to money demand and output supply disturbances is ambiguous. The reason is that these disturbances affect the spot exchange rate and the price level in ways that create opposing effects on the demand for foreign bonds and hence on the forward rate. A positive money demand or output supply shock, by lowering the price level, raises the real supply of domestic bonds. By lowering the spot exchange rate, however, these shocks act, via CIP, to raise the nominal interest rate and hence the total demand for bonds. The net effect on foreign bond demand is therefore ambiguous. When the share of domestic bonds in total bonds held $(1 - \mu)$ is large and the interest rate response of bond demand $(\beta_2 - \beta_3)$ is small, the supply effect is more likely to dominate: positive money demand and output supply shocks, by reducing the demand for foreign bonds, reduce the supply of foreign exchange forward as forward cover. The forward rate is consequently higher. Conversely when the interest response of bond demand $(\beta_2 - \beta_3)$ is large relative to the share of domestic bonds in total bonds held $(1 - \mu)$. In the case $\beta_1 = 0$, the sign of either response is given by $(1 - \mu - \beta)$ where $\beta \equiv (\beta_2 - \beta_3)$ can be interpreted as the semi-elasticity of the demand for bonds with respect to the nominal interest rate, given a constant real interest rate.[10/] An implication of these results is that, if the share of domestic bonds is large and bond demand is interest inelastic (in this sense), the forward rate may react in the opposite direction from the spot rate to a shock in the domestic money demand or output supply. Of course, in the extreme case in which speculation is perfectly elastic $(\gamma \to \infty)$, the forward rate equals the expected future spot rate (shown previously to be zero) and is unaffected by any current, temporary disturbances.

Because of PPP the effect of domestic disturbances on the price level is the same as that on the spot rate. The sign of the effect on output is also the same with respect to money demand and forward market disturbances, while the effect of output supply disturbances is reversed.

The Effects of Foreign Disturbances

As with domestic shocks, when speculation is perfectly elastic the forward rate is unaffected by current foreign shocks. So is domestic output and the domestic price level when the demand for money is interest inelastic $(\alpha_2 = 0)$. In general, an increase in the foreign price level p^*_t will, via PPP, lower the spot rate, thereby increasing the domestic interest rate. If this has no effect on the demand for money $(\alpha_2 = 0)$, e^s_t will move to offset completely the effect of the change in p^*_t on p_t. Otherwise, the demand for money will fall, raising p_t and y_t to maintain money market equilibrium. An increase in the foreign interest rate $r*_t$ will, via CIP, raise the domestic interest rate. If $\alpha_2 = 0$ there are no further effects on e^s_t, y_t, or p_t. Otherwise, the induced increase in the demand for money will cause these three variables to increase.

The Role of Speculation

Another issue we consider is how the stability of the small economy is affected by speculative behavior. To do this we analyze how the magnitudes of the responses of the endogenous variablesto changes in the exogenous random variables are affected by changes in the coefficient γ, which measures the elasticity of speculation with respect to the risk premium. In effect, this involves examining the cross partial derivatives

$$\partial|\partial z_t/\partial x_t|/\partial\gamma, \quad z_t = y_t,\ p_t,\ e^s_t,\ e^f_t;$$
$$x_t = u^y_t,\ u^m_t,\ u^f_t,\ p^*_t,\ r^*_t\ .$$

From (5a)-(5d) several qualitative propositions follow:

Proposition 1: An increase in the elasticity of speculation stabilizes the forward exchange rate against all random disturbances.

As speculation becomes more elastic, the forward rate becomes more closely tied to the spot rate expected to prevail in the subsequent period and less responsive to current shocks.

Proposition 2: An increase in the elasticity of speculation stabilizes the spot exchange rate against forward market disturbances and foreign price disturbances. It destabilizes the spot rate against foreign interest rate disturbances. More elastic speculation stabilizes or destabilizes the spot rate against domestic money demand and output supply disturbances according to whether these disturbances have a negative or positive effect on the forward rate. In the case of a monetary disturbance, the relevant condition is

$$1 - \mu - \beta + \beta_1\theta \lesseqgtr 0 \; ; \tag{6}$$

in the case of a supply disturbance, it is given by

$$\alpha_1(1 - \mu - \beta) - (1 + \alpha_2)\beta_1 \lesseqgtr 0 \; . \tag{7}$$

Thus more elastic speculation is more likely to stabilize against domestic monetary and supply disturbances when bond demand is interest elastic and the share of domestic bonds held is small. The converse applies when bond demand is interest inelastic and the share of domestic bonds is large.

To interpret these results we consider first the domestic disturbances. As speculation becomes more elastic, the disturbance u_t^f, which impinges on forward market equilibrium, is accommodated by offsetting speculation. As γ approaches infinity, the condition for forward market equilibrium reduces to $e_t^f = e_{t+1,t}^s$, so that u_t^f becomes irrelevant. The case of monetary and supply disturbances is more complicated. Positive money demand and output supply disturbances initially create an excess demand for money. A lower spot rate restores equilibrium by: (i) lowering the price level via PPP, thereby raising the real money supply; (ii) lowering real output via the supply function; and (iii) raising the nominal interest rate via CIP. To the extent that speculation ties the forward rate to the future spot rate, the forward rate changes by less in response to a current shock. If a positive money demand or output supply shock raises the forward rate then a smaller change in the spot rate is required to restore equilibrium, since the higher forward rate raises, via CIP, the interest rate and lowers money demand. Speculation, by tying the forward rate more to the future, reduces the amount by which the forward rate rises. Consequently a larger change in the spot rate is needed. Speculation therefore destabilizes. Conversely, when these shocks act to lower the forward rate the excess supply of money is exacerbated. A larger spot rate change is needed to equilibrate the money market. Speculation now stabilizes by tying down the forward rate, preventing it from falling as far as before.[11/]

Consider now the case of foreign disturbances. A rise in the foreign price level p_t^* raises the domestic price level via PPP, thereby creating an excess demand for money (both because the real money supply falls and income rises). A fall in the spot rate restores equilibrium for the three reasons given above. The forward rate falls in response to a positive value of p_t^*. The less the forward rate falls in response to the rise in p_t^* the more a given fall in the spot rate raises the interest rate, via CIP. Consequently, the more speculation ties the forward rate to the expected future spot rate the more responsive is the nominal interest rate to the spot rate. A smaller change in e_t^s is thus required to restore money market equilibrium. A rise in the foreign interest rate r_t^* also affects money market equilibrium, in this case through CIP. Now, however, a change in e_t^f acts to dampen the effect on money demand, since it offsets the effects of the change in r_t^* on the domestic interest rate. As γ rises, e_t^f is less sensitive, so that now a larger change in e_t^s is required.

Proposition 3: An increase in the elasticity of speculation stabilizes the domestic price level against forward market disturbances but destabilizes it against foreign interest rate and price level disturbances. More elastic speculation stabilizes or destabilizes the price level against domestic money demand disturbances according to condition (6) and against domestic output supply disturbances according to condition (7).

To the extent that speculation stabilizes the spot rate, through PPP it stabilizes the domestic price level given the foreign price level. Hence, the responses of p_t and e_t^s to all shocks except p_t^* are affected by speculation in the same direction. In the case of foreign price level shocks, however, movements in e_t^s stabilize p_t. By stabilizing e_t^s against p_t^* an increase in the elasticity of speculation destabilizes p_t.

Proposition 4: An increase in the elasticity of speculation stabilizes domestic output against forward market disturbances but destabilizes it against foreign interest rate and foreign price level disturbances. More elastic speculation stabilizes or destabilizes output against money demand disturbances according to condition (6) and against domestic output supply disturbances according to the converse of condition (7).

Except for domestic supply disturbances, any effect which an increased elasticity of speculation has on income is qualitatively the same as its effect on the domestic price level. The exception arises because of an increase in u_t^y acts to raise the demand for money, lowering p_t. The drop in p_t dampens the increase in y_t originating from the disturbance u_t^y. When speculation reduces the responsiveness of p_t to u_t^y, it diminishes this dampening effect, and conversely.

IV. INTERVENTION POLICIES

We now relax the assumption of a perfectly flexible exchange rate and assume instead that the domestic monetary authorities continually intervene in both the spot and forward exchange markets. In specifying the intervention rules it is important to observe that the term b_t can be interpreted either as a percentage change in the nominal bond supply or as government sales of foreign exchange forward as a percentage of government debt. To see this equivalence note that, expressed in terms of levels, forward market equilibrium is described by

$$H - \frac{B}{P} = S + \frac{EG}{P}$$

where G denotes government purchases of foreign exchange forward. Define the parameter g as the nominal value in domestic currency units of these commitments, expressed as a share of outstanding bonds, i.e.,

$$g = EG/B$$

Substituting gB for EG in the forward market equilibrium condition and taking a log linear approximation, as before, gives

$$h_t = \mu s_t + (1 - \mu)(b_t + g_t - p_t) \qquad \text{(1e')}$$

In deriving (1e') we assume that the share g_t is sufficiently small to allow the approximation $\ln(1 + g_t) = g_t$. The terms b_t and g_t enter only equation (1e') additively and nowhere else in the model. Accordingly, variations in the bond supply and in the government's forward market position do not have linearly independent effects on the economy. For concreteness we focus on the forward market position g_t as a policy instrument, setting $b_t = 0$.[12]

To incorporate official intervention into the analysis requires only a modest modification to the basic model (1a)-(1g). Specifically, we replace the forward market equilibrium condition (1e) by (1e') and append policy rules describing the intervention in the spot and forward markets, with all other relationships remaining unchanged. The rules we consider are hypothesized to be of the form

$$m_t = a_1 e_t^s + a_2 e_t^f \tag{1h}$$

$$g_t = b_1 e_t^s + b_2 e_t^f \tag{1i}$$

These are direct generalizations of the types of rules specified in the current intervention literature, which typically postulate policies that make the domestic money supply vary with the current spot exchange rate.[13/] Here we assume that the intensity of intervention in both the spot and forward markets depends upon both the spot and forward exchange rates and are described by the parameters a_1, a_2, b_1, b_2. We may note from these intervention rules that a fixed exchange rate may be attained either by letting $a_1 \to -\infty$ or $b_1 \to -\infty$.

The solution of the system for given arbitrary intervention parameters can be obtained by first substituting the intervention rules (1h) and (1i) into the system (1a)-(1g) (with (1e') replacing (1e)) and then following the procedure outlined in the previous section. As before, by taking conditional expectations of the modified system we find that the expectations of future spot exchange rates satisfy a first order difference equation analogous to (3), namely

$$e^*_{t+i+1,t} = \phi' e^*_{t+i,t} \tag{3'}$$

The coefficient ϕ' is a function of the intervention parameters a_1, a_2, b_1, b_2 as well as the parameters describing private sector behavior. Indeed in the absence of any intervention $\phi' \equiv \phi$. In the case that $|\phi'| > 1$, then as before, the only bounded solution is for $e^s_{t+i,t} = 0$. However, with active intervention it is now possible for $|\phi'| < 1$. Such a case will arise, for example, if

$a_1 = 1 + \alpha_2$, $b_1 = 1 - \beta_2/(1 - \mu)$. Under these conditions, the requirement that expectations be bounded and the equivalent restriction that the asymptotic variance of the spot rate be finite imposes no restriction on $e^s_{t+i,t}$. The solutions for $e^s_{t+1,t}$ and therefore for e^s_t are non-unique and other, stronger criteria are required to determine the solution.

One procedure, suggested by Taylor (1977), involves imposing the requirement that the arbitrary constantin the solution to (3') be chosen to minimize the asymptotic variance of e^s_t. However, this procedure is essentially arbitrary since there is no obvious mechanism to ensure that the variance will be minimized in this way. Also, the value of the constant that minimizes the asymptotic variance of the spot exchange rate is not the same as the one that minimizes the asymptotic variance of the price level. The question of which variance (if any) rational expectations do actually minimize is not clear.[14/]

In the present analysis we shall adopt a simpler (but equally arbitrary) argument to determine $e^s_{t+1,t}$. We shall simply assume that irrespective of whether $|\phi'| \gtrless 1$, the rational expectations generates a solution for e^s_t which is based on the minimum amount of information. It can be shown that if $e^s_{t+i,t} \neq 0$ and $|\phi'| < 1$ the stochastic process determining e^s_t depends upon an infinite distributed lag of past disturbances; for $|\phi'| > 1$ it depends only upon current disturbances. Applying the minimum information argument requires us to set $e^s_{t+i,t} = 0$ for all ϕ', in which case e^s_t depends only upon current disturbances. This approach also has the related advantage of ensuring the continuity of the optimal policy about the point where $|\phi'| = 1$ and avoiding some of the complications associated with the minimum variance criterion noted in footnote 14. Thus setting all expectations to zero in the modified system, the solutions for the endogenous variables can be obtained in a form directly analogous to (5a)-(5d).

V. OPTIMAL INTERVENTION

We turn now to the question of the optimal degree of intervention. The typical approach to this problem is to minimize some objective function expressed in terms of the asymptotic variances of the endogenous variables in the economy. The objective function most frequently postulated, and the one we shall adopt, is the stabilization of income.

The solution for income derived in the previous section is given by

$$
\begin{aligned}
y_t = \frac{1}{D}\{ & -\theta[\beta_2 + \mu\gamma - b_2(1-\mu)]u_t^m + \theta(\alpha_2+a_2)u_t^f \\
& +[(\alpha_2+a_2)[\beta_3 - \beta_2 + (1-\mu)(1-b_1)] + (1+\alpha_2-a_1)[\beta_2 + \mu\gamma - b_2(1-\mu)]]u_t^y \\
& + \theta[\alpha_2[\mu\gamma - b_2(1-\mu)] - a_2\beta_2]r_t^* \\
& + \theta[(\alpha_2-a_1)[\mu\gamma + \beta_2 - b_2(1-\mu)] - (\alpha_2+a_2)[\beta_2 + b_1(1-\mu)]]p_t^*\}
\end{aligned}
\tag{8}
$$

where
$$
\begin{aligned}
D \equiv\ & (1+\theta\alpha_1+\alpha_2-a_1)[\mu\gamma + \beta_2 - b_2(1-\mu)] \\
& + (\alpha_2+a_2)[\beta_3 - \beta_2 + \theta\beta_1 + (1-\mu)(1-b_1)]
\end{aligned}
$$

In principle, the optimal degrees of intervention a_1, a_2, b_1, b_2, can be obtained by first calculating the variance of y_t, σ_y^2 say, and then minimizing σ_y^2 with respect to these four parameters. This yields four independent values for the intervention parameters which are functions of the random variables and their covariances. Given the complexity of y_t, to determine the optimal policies in this way turns out to be extremely cumbersome and not very enlightening. In addition, tractability would require that all the parameters of the model be treated as constants. We have shown elsewhere (see Eaton and Turnovsky (1981)) that the elasticity of speculation depends, among other things, on the variance of the spot exchange rate. Since different intervention rules imply different asymptotic variances of the exchange rate, γ is implicitly a function of the intervention parameters. Strictly speaking, this dependence needs to be taken into account in the derivation of the optimum.

Thus, rather than pursuing an explicit optimization, we focus on the optimal intervention with respect to the different random variables, taken individually, and also grouped as domestic and foreign disturbances. This approach enables us to determine the extent to which (sets of) disturbances may be eliminated and does not require us to treat γ as a constant.

To facilitate the economic understanding of the optimal policies we shall obtain, it is convenient to summarize the system (7b'), (1a)-(1d), (1e'), (1f)-(1i) in the following way.

$$a_1e_t^s + a_2e_t^f - (p_t^* + e_t^s) = \alpha_1y_t - \alpha_2[r_t^* + e_t^f - e_t^s] + u_t^m \tag{9a}$$

$$\beta_1y_t + \beta_2[r_t^* + e_t^f - e_t^s] + \beta_3(p_t^*+e_t^s) = -\mu\gamma e_t^f + (1-\mu)[(b_1-1)e_t^s + b_2e_t^f - p_t^*] + u_t^f \tag{9b}$$

$$y_t = \theta(q_t+e_t^s) + u_t^y \tag{9c}$$

where we have substituted the intervention rules (1h), (1i) into the appropriate market clearing conditions, and used PPP to eliminate the price level. Following the argument of Section IV, the expectations variables $e_{t+1,t}^s$, $p_{t,t-1}$ have been set to zero. These three equations thus describe money market equilibrium, forward exchange market equilibrium, and the domestic output supply function, respectively.

We now discuss how intervention can be used to eliminate disturbances of different types:

5.1. Domestic Monetary and Speculative Disturbances

Suppose that the only random disturbance is in the domestic demand for money, u_t^m. Then from (8) it can be seen that output y_t can be stabilized exactly in any one of the following four ways:[15/] (i) $a_1 \to -\infty$; (ii) $a_2 \to -\infty$; (iii) $b_1 \to -\infty$; (iv) $b_2 = (\beta_2+\mu\gamma)/(1-\mu)$. Setting either $a_1 \to -\infty$ or $b_1 \to -\infty$ leads to fixing the spot rate. Given purchasing power parity, the domestic price level is fixed and given that there are no disturbances impinging on supply, domestic output must be fixed as well; see (9c). Suppose on the other hand that the monetary authority intervenes with infinite intensity in the spot market in response to the forward rate, setting $a_2 \rightarrow -\infty$. This will fix the forward rate e_t^f. It then follows that the spot rate e_t^s and output y_t are jointly determined by the forward market (9b) and the domestic stability function (9c). Since both these relationships are independent of the monetary disturbance both e_t^s and y_t are therefore fixed; in particular, output is stabilized perfectly as before. Finally, we may note that if $b_2 = (\beta_2+\mu\gamma)/(1-\mu)$ the effects of the forward rate are eliminated from the forward market equilibrium (9b). Substituting this value of b_2 into (9b), it is seen from (9b) and (9c) that e_t^s is now fixed and again output is stabilized perfectly.

Disturbances in speculation u_t^f can be stabilized analogously by setting either (i) $a_1 \rightarrow -\infty$, (ii) $a_2 \rightarrow -\infty_2$; (iii) $b_1 \rightarrow -\infty$; (iv) $b_2 \rightarrow -\infty$ and a similar explanation can be given.

Note that all policies which eliminate these two disturbances involve, directly or indirectly, fixing e_t^s. Thus the established result that fixed exchange rates isolate the domestic economy from domestic monetary shocks (see, e.g., Boyer (1978), Turnovsky (1983), Buiter and Eaton (1980)) extends to speculative shocks as well. The central bank can, however, stabilize e_t^s by forward market intervention (or sterilized spot market intervention) as well as by direct (unsterilized) intervention in the spot market. Furthermore, when e_t^s is fixed, γ is likely to be large which, from proposition (iv) above, reinforces the stabilizing effects of intervention when shocks are speculative.

5.2. Domestic Supply Disturbances

To stabilize disturbances originating in the supply of domestic ouput it a little more complicated. This requires setting the coefficient of u_t^y in (8) to zero. Any combination of coefficients a_1, a_2, b_1, b_2 satisfying the relationship

$$(\alpha_2+a_2)(\beta_3-\beta_2+(1-\mu)(1-b_1)) = (1+\alpha_2-a_1)(\beta_2+\mu\gamma-b_2(1-\mu))$$

with the exception of $b_2 = (\beta_2+\mu\gamma)/(1-\mu)$ and $a_2 = -\alpha_2$ will do. This special case eliminates the forward rate e_t^f from both the money market and forward market equilibrium conditions. In effect, the spot rate e_t^s is required to equilibrate both markets and the system is overdetermined. Two simple intervention rules which do succeed in stabilizing income exactly against this type of disturbance are: (i) $a_1 = 1 + \alpha_2$, $a_2 = -\alpha_2$; (ii) $a_1 = 1 + \alpha_2$, $b_1 = 1 + (\beta_3-\beta_2)/(1-\mu)$. The first of these policies involves intervening in the spot market alone; by increasing the domestic money supply appropriately in response to a depreciation in the spot rate and decreasing it appropriately in response to a depreciation in the forward rate, e_t^f, e_t^s and the random influences they embody can be eliminated from the money market, thereby stabilizing income exactly. The second policy rules require intervention in both the spot and forward exchange markets; the domestic money supply should again be increased appropriately in response to a depreciation of the spot rate, although this should now be accompanied by an adjustment (which may be either positive or negative) in the domestic monetary authority's holdings of forward exchange. In economic terms, setting $a_1 = 1$ eliminates the spot rate from the money demand function, while setting $b_1 = 1 + (\beta_3-\beta_2)/(1-\mu)$ eliminates the spot rate from the foreign exchange market. Thus output and the forward rate become jointly determined by the money market and forward market equilibrium conditions. Since these are free of stochastic disturbances output and the forward rate remain fixed, pegging output at zero.

In the case of domestic monetary or speculative disturbances, income can be stabilized perfectly by adopting the traditional form of intervention, namely unsterilized intervention through the money market in response to the spot rate alone. With respect to domestic supply shocks this is no longer true. Complete stabilization requires either: (i) an appropriate response to the forward rate either through money market or forward market intervention; or (ii) an appropriate response to the spot rate through forward market intervention.

5.3. Domestic Disturbances as a Group

Next we consider the question of whether or not it is possible to stabilize domestic income exactly against all the domestic disturbances simultaneously. Given that there are only three disturbances and that there are four policy parameters to be chosen this might seem possible. But in fact it is infeasible. Let us consider the alternatives. First, we must eliminate u_t^m and u_t^f. One possibility would appear to be to set $b_2 = (\beta_2+\mu\gamma)/(1-\mu)$, $a_2 = -\alpha_2$. But as we have noted in connection with the supply disturbance u_t^y this combination of intervention parameters involves the elimination of the forward exchange rate from both the money market and forward exchange market and leads to an inconsistency. The only alternatives are therefore to eliminate these random variables by pegging the exchange rate by setting either $a_1 \to -\infty$, or $b_1 \to -\infty$. In either case, this yields

$$y_t = u_t^y$$

so that domestic supply disturbances are fully reflected in domestic output. In effect, the introduction of a fixed exchange rate means that the possibility of stabilization through the forward market is lost. Alternatively, the policies which may eliminate u_t^y will not succeed in eliminating both u_t^m and u_t^f simultaneously. By choosing $a_1 = 1 + \alpha_2$, $a_2 = -\alpha_2$ it is possible to stabilize both u_t^f and u_t^y, but it is not possible to stabilize exactly for u_t^m and u_t^y taken together. In short, there is no way of stabilizing against all three domestic random disturbances simultaneously. It is possible to stabilize against u_t^m, u_t^f and u_t^f, u_t^y pairwise, but this cannot be achieved for the remaining pair u_t^m and u_t^y.

5.4. Foreign Disturbances

We turn now to the foreign disturbances, which impinge on the domestic economy through the foreign price level and interest rate. From (8) it is seen that any combination of intervention parameters satisfying the pair of equations

(10a) $$\alpha_2[\mu\gamma - b_2(1-\mu)] - a_2\beta_2 = 0$$

(10b) $$[\beta_2 + \mu\gamma - b_2(1-\mu)](\alpha_2-a_1) - (\alpha_2+a_2)[\beta_2 + b_1(1-\mu)] = 0$$

will stabilize domestic output exactly. There are an infinite number of combinations which will achieve this, although in all cases the intervention rule must respond to the forward rate; see (10a). An important result worth highlighting is that perfect stabilization can be attained by intervening in either the forward market or the spot market alone. The optimal intervention parameters in these two cases are respectively

(11a) $$a_1 = a_2 = 0;\ b_1 = 0,\ b_2 = \mu\gamma/(1-\mu)$$

(11b) $$b_1 = b_2 = 0;\ a_1 = 0,\ a_2 = \alpha_2\mu\gamma/\beta_2$$

In either case, the intervention rule involves accommodating the forward rate; the movement of the spot rate is irrelevant.

To understand the economic intuition underlying these policy rules, let us substitute (11a) into the system described by (9a)-(9c) at the same time setting all domestic disturbances to zero, to yield

(9a') $$-(p_t^*+e_t^s) = \alpha_1 y_t - \alpha_2(r_t^*+e_t^f-e_t^s)$$

(9b') $$\beta_1 y_t + \beta_2[r_t^* + e_t^f - e_t^s] + [\beta_3 + (1-\mu)](p_t^*+e_t^s) = 0$$

(9c') $$y = \theta(p_t^*+e_t^s)$$

Inspecting (9a')-(9c'), it is evident that if the intervention is through the forward market in accordance with (11a), the equilibrium conditions for the domestic economy reduce to $r_t^* + e_t^f - e_t^s = 0$, $p_t^* + e_t^s = 0$, $y_t = 0$. In particular, any random increase in the foreign price level must be exactly offset by an equivalent appreciation of the domestic spot rate, leaving real output fixed.

When intervention takes place through the spot market in accordance with (11b), the system becomes

(9a") $$\alpha_2\mu\gamma e_t^f/\beta_2 - (p_t^*+e_t^s) = \alpha_1 y_t - \alpha_2(r_t^*+e_t^f-e_t^s)$$

(9b") $$\beta_1 y_t + \beta_2[r_t^* + e_t^f - e_t^s] + [\beta_3 + (1-\mu)](p_t^*+e_t^s) = -\mu\gamma e_t^f$$

(9c") $$y_t = \theta(p_t^*+e_t^s)$$

In this case the intervention rule offsets the effect of movements in the domestic nominal interest rate on money demand. Again the proportionality of domestic output and the domestic price level ensures that output must remain fixed.

VI. CONCLUDING COMMENTS

In this paper we have developed a stochastic model of a small open economy having both a forward and a spot market for foreign exchange. Two main issues have been addressed. These are (i) to determine the extent to which speculation is stabilizing; (ii) to characterize income stabilizing intervention rules in the spot and forward markets.

Whether or not speculation is stabilizing depends upon the origin of the random disturbances and certain characteristics of the bond market. Speculation tends to increase stability against those disturbances that impinge initially on the bond and forward markets. By contrast, it tends to destabilize domestic income and the domestic price level against those disturbances originating from abroad. Whether or not speculation stabilizes the economy against shocks in domestic money demand or output supply depends largely on the share of foreign bonds in total bond holdings, as well as on the interest sensitivity of aggregate domestic bond demand. If the foreign bond share is large and bond demand is highly elastic, speculation is likely to stabilize the price level against both types of shocks. Conversely, if domestic bonds dominate and bond demand is interest inelastic, speculation is likely to destabilize the price level against these shocks as well.

In analyzing intervention in the foreign exchange market, we have not found it fruitful to derive explicit optimal policies for the general case, although in principle this could be done. Rather we have focused on the separate disturbances, both individually and in groups, and considered policies which may insulate domestic real income against them. We have shown how real income can be stabilized against any single domestic random disturbance in a variety of ways, although despite the fact that we are free to choose four independent intervention parameters, it is impossible to stabilize against all three domestic random variables simultaneously. In some but not all cases, pairs of these random disturbances can be stabilized perfectly. By contrast, an infinite range of intervention policies exist for stabilizing the foreign disturbances. In all cases, the intervention rule must take account of the forward exchange rate, thus highlighting the importance of the forward market in stabilization. However, although the rule must be responsive to the forward rate, the intervention itself may take place either through the spot market or through the forward market. Equivalently, spot market intervention may be sterilized or unsterilized.

Finally, we may note that for analytical convenience we have conducted our analysis for the limiting case where domestic bonds and foreign covered bonds are perfect substitutes. It is straightforward to relax this assumption and allow them to be imperfect substitutes. In that case it can be shown that the general propositions obtained in this paper, and which we have just summarized, continue to hold.

FOOTNOTES

1. Important exceptions are the papers by Tsiang (1959) and Day (1976, 1977). Both authors consider intervention in the partial equilibrium context of the foreign exchange market. They do not consider intervention as a tool of stabilization policy, which is our purpose here.

2. The more recent formal analyses of the relative stability of fixed versus flexible exchange rates (e.g., Turnovsky (1976), Fisher (1977b), Flood (1979)) abstract entirely from speculative capital movements.

3. Bilson (1978) does incorporate a forward market into his analysis, but does not consider intervention in this market. Furthermore, he assumes that speculation in the forward market brings the forward rate into equality with the expectation of the spot rate prevailing when the forward contract matures. Optimal speculative behavior derived from portfolio maximization models implies that this assumption is generally valid only when agents are risk neutral or when there is no exchange risk; see, for example, Solnik (1973), Kouri (1976), Adler and Dumas (1977), Eaton (1978), Eaton and Turnovsky (1980).

4. We find it analytically convenient to separate forward market participation into pure speculation and pure arbitrage. We implicitly treat the acquisition of an amount x of uncovered foreign bonds as combining a covered investment of x in foreign bonds and a speculative purchase of foreign currency forward in amount x. In a portfolio model of foreign investment we identify a third motive for participating in the forward market as hedging against domestic inflation. Forward positions for hedging purposes depend upon the relative variability of the domestic and foreign price levels and do not respond to the variables we are concerned with here. We may thus treat the forward position due to hedging as a constant absorbed in s_t; see Eaton and Turnovsky (1981).

5. The domestic bond market equilibrium condition is based on the assumption that foreigners hold zero stocks of domestic bonds. In Eaton and Turnovsky (1981) we show that this is more likely to arise when the domestic price level is more sensitive to the exchange rate than is the foreign price level. Note we assume (i) that money and bonds are both outside assets and (ii) that the money and bonds of each country are denominated in the currency of that country. Frankel (1979) discusses the relevance of these assumptions for our specification.

6. This approximation is obtained as follows

$$\frac{(H_t-\bar{H})}{\bar{H}} = \frac{\overline{B/P}}{\bar{H}}\left[\frac{B_t}{P_t} - \overline{\left(\frac{B}{P}\right)}\right]\Big/\overline{\left(\frac{B}{P}\right)} + \frac{\bar{H}^f}{\bar{H}}[H_t^f - \bar{H}^f]/\bar{H}^f$$

Now, for any variable X,

$$\frac{X_t-\bar{X}}{\bar{X}} \simeq \ln\left[1 + \frac{X_t-\bar{X}}{\bar{X}}\right] = \ln X_t/\bar{X} = x_t - \bar{x}$$

where $x = \ln X$. Thus we may write

$$h_t = \mu(b_t - p_t) + (1-\mu)h_t^f$$

7. A more complete analysis would derive the foreign disturbances r_t^* and p_t^*, as they impinge upon the small country we consider here, from underlying money market, bond market, and output disturbances abroad. To do so requires that we specify a model of the rest of the world which may be used to relate r_t^* and p_t^* to these underlying disturbances abroad. Since this approach introduces a number of complications which do not illuminate the issues we consider here, we do not pursue it. See Turnovsky (1983) for a model of optimal intervention that does follow this "extended" small country approach.

8. This requirement that the asymptotic variance be finite may be justified by appealing to some appropriate transversality condition for a corresponding model derived from optimizing behavior. Under appropriate conditions this requirement also imposes boundedness on price expectations.

9. The effects of various exogenous stochastic disturbances on the domestic economy and how these are influenced by the degree of speculative behavior are also considered by Driskill and McCafferty (1980, 1982) and Turnovsky and Bhandari (1982). These models differ in many critical respects from the analysis presented here. For example, the Driskill-McCafferty model is much more partial equilibrium, while the Turnovsky-Bhandari analysis does not formulate the forward market explicitly.

10. The reason β_2 and β_3 and hence β are semi-elasticities is that r_t and $(p_{t+1,t} - p_t)$ are in percentage change terms, while the demand for bonds is expressed in logarithms. To see the interpretation of β write the bond demand function in the form

$$h_t = \beta_1 y_t + (\beta_2 - \beta_3) r_t + \beta_3 [r_t - (p_{t+1,t} - p_t)] + u_t^b$$

Note that the function can also be written as

$$h_t = \beta_1 y_t + (\beta_2 - \beta_3)[p_{t+1,t} - p_t] + \beta_2 [r_t - (p_{t+1,t} - p_t)] + u_t^b$$

in which case β has an analogous interpretation with respect to inflationary expectations.

11. The following more formal argument may be given. Substituting for p_t and y_t in the money market equilibrium and noting (4), we can easily show

$$(1 + \alpha_1\theta + \alpha_2)\frac{\partial}{\partial\gamma}\left(\frac{\partial e_t^s}{\partial x_t}\right) = \alpha_2 \frac{\partial}{\partial\gamma}\left(\frac{\partial e_t^f}{\partial x_t}\right) \qquad x = u_t^m,\ u_t^y$$

where $\partial e_t^s/\partial x_t < 0$ and $\partial e_t^f/\partial x_t \gtrless 0$. If $\partial e_t^f/\partial x_t > 0$, then

Proposition 1 implies $\frac{\partial}{\partial\gamma}\left(\frac{\partial e_t^f}{\partial x_t}\right) < 0$ and hence we obtain $\frac{\partial}{\partial\gamma}\left(\frac{e_t^s}{\partial x_t}\right) < 0$. It thus follows that increased speculation causes the fall in the spot rate to increase and hence is destabilizing. The argument is reversed if $\partial e_t^f/\partial x_t > 0$.

12. The consequences of forward market intervention for the consolidated balance sheet of the fiscal authority and central bank are discussed in the Appendix.

13. See, e.g., Cox (1980) and Roper and Turnovsky (1980), for example.

14. Elsewhere, in a related analysis, Turnovsky (1980) has applied this procedure for determining the stochastic process, with the minimization of the one-period variance of the spot rate as the criterion. This turns out to make the subsequent determination of the optimal intervention policy rather complicated. Essentially one has to consider the two cases: (i) $|\phi| > 1$, (ii) $|\phi| < 1$ separately, and ensure that the optimal policy in each case is consistent with the restrictions applicable to that case. The overall optimum is obtained by taking the superior of these two cases and also involves a consideration of the boundary cases.

15. Since as noted above, γ is in principle a function of the intervention parameters, we must have $\gamma = \gamma(a_1, a_2, b_1, b_2)$. Thus in case (iv) the complete specification of the optimum intervention is described by $a_1 = a_2 = b_1 = 0$, $b_2 = \beta = \gamma(0, 0, 0, b_2)$. The same comment applies to other cases where the optimum policy involves relating an optimal intervention parameter to γ.

APPENDIX

BALANCE SHEET CONSTRAINTS ON CENTRAL BANK INTERVENTION IN THE SPOT AND FORWARD EXCHANGE MARKETS

In Sections IV and V of the text we allow the central bank to establish a money supply and a forward market position in response to the spot and forward exchange rates. In this appendix we state the balance sheet constraint incumbent on the central bank in engaging in these activities and then state the assumptions about central bank actions implicit in our equations (1h) and (1i).

The central bank, we assume, holds as assets government bonds and foreign reserves and issues, as liabilities, money. By making forward market commitments the central bank also earns capital gains and losses which it may finance by monetary issue or by varying its holdings of foreign reserves or government bonds. Let G_{t-1} denote central bank purchases (in natural units) of foreign exchange forward in period t-1. This purchase has no implications for the central bank's balance sheet in period t-1. In period t, however, if, say, $E_t^s > E_{t-1}^f$ and $G_{t-1} < 0$, the central bank must deliver an amount G_{t-1} of foreign currency at a cost $-E_t^s G_{t-1}$. In exchange it receives $-E_{t-1}^f G_{t-1}$ of domestic money. If it buys G_{t-1} in the spot market for foreign exchange then there is a net increase in the money supply of $-(E_t^s - E_{t-1}^f)G_{t-1}$. Instead it could buy only $-E_{t-1}^f G_{t-1}$ in the spot market and reduce its reserve holdings by $-(E_t^s - E_{t-1}^f)G_{t-1}$, thereby not affecting the domestic money supply. Alternatively, it could sterilize the effect on the money supply by selling $-(E_t^s - E_{t-1}^f)G_{t-1}$ in government bonds. The overall constraint binding the central bank is that

$$(A.1) \qquad (E_t^s - E_{t-1}^f)G_{t-1} + (M_t^c - M_{t-1}^c) - E_t^s(R_t - R_{t-1}) - (B_t^c - B_{t-1}^c) = 0$$

where R_t denotes the central banks holdings of foreign reserves in period t, B_t^c its holdings of government bonds, and M_t^c its outstanding monetary liabilities, all measured in natural units.

The change in the total money stock held by the public is

$$M_t - M_{t-1} = M_t^c - M_{t-1}^c - (R_t^* - R_{t-1}^*) \qquad \text{(A.2)}$$

where R_t^* denotes the foreign central bank's reserve holdings. In equations (1h) and (1i) we assume that the central bank sets a money stock and establishes a forward market position independently. Implicit is the assumption that the central bank finances money supply changes and forward market gains and losses by varying its reserve holdings or its holdings of government bonds. The effect of changes in the foreign central bank's reserve holdings is offset by the same means.

We assume that the consequences of any central bank action for the bond supply are offset by appropriate taxes and transfers by the domestic fiscal authority. Since we consider a small economy we may ignore the effects of changes in domestic reserve holdings for the foreign money supply. Alternatively we could assume that these changes are offset by appropriate taxes and transfers abroad.

Finally, the behavioral relationships we postulate assume implicitly that private agents, in their own behavior, are not affected by the central bank's portfolio position. For instance, a private individual does not include the central bank's reserve holding as a component of his own wealth.

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