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ABSTRACT

While much research in political economy points out the benefits of "limited government," political scientists have long emphasized the problems created in many less developed nations by "weak states," which lack the power to tax and regulate the economy and to withstand the political and social challenges from non-state actors. I construct a model in which the state apparatus is controlled by a self-interested ruler, who tries to divert resources for his own consumption, but who can also invest in socially productive public goods. Both weak and strong states create distortions. When the state is excessively strong, the ruler imposes such high taxes that economic activity is stifled. When the state is excessively weak, the ruler anticipates that he will not be able to extract rents in the future and underinvests in public goods. I show that the same conclusion applies in the analysis of both the economic power of the state (i.e., its ability to raise taxes) and its political power (i.e., its ability to remain entrenched from the citizens). I also discuss how under certain circumstances, a different type of equilibrium, which I refer to as "consensually-strong state equilibrium," can emerge whereby the state is politically weak but is allowed to impose high taxes as long as a sufficient fraction of the proceeds are invested in public goods. The consensually-strong state might best correspond to the state in OECD countries where taxes are high despite significant control by the society over the government.

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1 Introduction

There is now a large and growing literature on the influence of politics on economic outcomes.[1] Much of this literature builds on a central insight of Douglass North's work: the politically-determined structure of property rights need not maximize the efficiency or the growth potential of the economy; instead, it strives to maximize the returns to the rulers or politically strong groups. For example, in his famous book, *Structure and Change in Economic History*, North stresses the "persistent tension between the ownership structure which maximized the rents to the ruler (and his group) and an efficient system that reduced transaction costs and encouraged economic growth" (1981, p. 25). While a structure of property rights that limits potential expropriation encourages investment by the citizens and is generally good for economic growth, rulers will typically attempt to increase their share of the revenues by taxation or expropriation. Therefore, this view suggests that "limited government" and constraints on the power of the state, especially on its power to tax, will generally stimulate growth.

Although there are numerous examples of disastrous economic performance under self-interested political elites and rulers with little check on their powers, many successful growth experiences, most notably those in East Asia, have also taken place under the auspices of strong states. For example, in South Korea General Park ran a highly authoritarian regime, with few formal checks on state power, and used the resources of the state to help industrialization in alliance with the large chaebols (as long as they did not pose a threat to his political power).[2]

Moreover, in contrast to the implications of the simple form of this "limited government" view, government revenues as a fraction of GDP appear to be higher in richer countries and in societies that are generally considered to have more "constrained" governments. This is illustrated in Figures 1 and 2, which plot central government revenue as a fraction of GDP against GDP per capita and the score of constraint on the executive from Polity IV dataset in the same year (all variables averaged over the 1990s).[3] In both cases, a strong increasing relationship is visible. Although in some of the poorest countries, such as Zaire, the illicit rents captured by political elites may be larger than the tax revenues, these patterns suggest that governments in the relatively advanced economies are able to raise higher tax revenues and play a more important role in the economy.

Consistent with this, a large body of work in political science, especially in the context of African politics, views the main barrier to economic development not as the strength of

[1] See, among others, the general discussions in North (1981), Olson (1982), Jones (1981), and North and Weingast (1989), and the empirical evidence in Knack and Keefer (1995), Barro (1999), Hall and Jones (1999), and Acemoglu, Johnson and Robinson (2001, 2002), and the theoretical models in Meltzer and Richard (1981), Persson and Tabellini (1994), Alesina and Rodrik (1994), Krusell and Rios-Rull (1996), Persson, Roland and Tabellini (1997), Acemoglu and Robinson (2002), Besley and Coate (1998), and Dixit (2004).

[2] See, for example, Huer (1989), Wade (1990) and Kang (2002).

[3] The data on government tax revenue are from the World Development Indicators (2003), and only include the central government revenue. The pattern is similar if federal countries are excluded from the sample.

the state, but as lack of state "capacity," state power or monopoly over violence. Political scientists and sociologists have coined the term "weak state" to describe such situations in which the state has a limited capacity to tax and regulate, and consequently, they argue, to play a developmental role.[4] Migdal (1988, p. 33), for example, remarks: "In parts of the Third World, the inability of state leaders to achieve predominance in large areas of their countries has been striking... The ineffectiveness of state leaders who have faced impenetrable barriers to state predominance has stemmed from the nature of the societies they have confronted—from the resistance posed by chiefs, landlords, bosses, rich peasants, clan leaders,..." Herbst (2000) suggests that the economic failure of African nations is directly related to their states' inability to dominate and extract resources from the rest of society, and contrasts this with the success in South Korea and Taiwan, where the state has been both politically and economically more powerful. He argues (p. 115) "the South Korean and Taiwanese states have been able to extract so many resources from their societies in part because the demands to be constantly vigilant provoked the state into developing efficient mechanisms for collecting resources and controlling dissidents groups." North himself was aware that the problems created by the excessive power of rulers was only one side of the coin, and argued that the state, with its monopoly of legitimate violence in society, has an important role to enforce contracts and reduce transaction costs. The traditional theory of public finance similarly recognizes the potential role of the state in public good provision and regulation, though it generally ignores the self-interested motivations of those controlling the state (e.g., Atkinson and Stiglitz, 1980). These considerations suggest that, perhaps as argued by many political scientists, severe constraints on the ability of the state to tax, to regulate and to coerce citizens could be detrimental for economic performance. Nevertheless, an analysis of the trade-off between the distortions that emerge from the taxation power of the state run by self-interested elites and the inefficiencies due to limits on the state power has not been undertaken. Nor does there exist a political economy framework to enable a systematic analysis of the patterns shown in Figures 1 and 2. This paper is an attempt in this direction.

The main argument of the paper is that both weak and strong states create distortions in the allocation of resources, and consequently, both excessively weak and excessively strong states are likely to act as impediments to economic development. While strong states tend to impose high taxes, discouraging investment and entrepreneurial effort by the citizens, weak states fail to invest in public goods such as infrastructure, roads, legal rules for contract enforcement, etc. Weak states underinvest in public goods because self-interested political elites undertake investments only when they expect future private rewards, and when the state is weak, they can appropriate fewer rewards in the future. The key for this result is that the state also takes

[4] See, among others, Migdal (1988), Tilly (1990), Wade (1990), Evans (1995), Reno (1999), Herbst (2000), La Ferrara and Bates (2001), and Bates (2001).

actions that are important for the efficient functioning of the economy; this necessitates an organization of society that provides the right incentives to the self-interested agents controlling the state.

To develop these ideas more systematically, I consider an economy in which a self-interested ruler (or social group/elite) controls the state apparatus and can impose taxes on citizens.[5] Production is carried out by the citizens, and depends on their investment as well as on the quality of the infrastructure, which is determined by the public good investments of the ruler. I first illustrate the main argument using a model focusing on the *economic power of the state,* parameterized as an economic exit option for the citizens, placing a constraint on taxation.[6] If taxes are anticipated to be high, there will be little private investment. However, if taxes are constrained to be very low, the ruler has no incentive to invest in public goods, since he will not be able to appropriate (part of) the future revenues generated by these investments. Intermediate levels of taxes that both encourage investment by the citizens and leave enough surplus for the ruler to entice him to invest in public goods are necessary for good economic outcomes. Because both states that are very weak and very strong will lead to poor economic outcomes, the structure of power has to be "balanced"—at an intermediate level between weak and strong states—to encourage all parties to undertake investments.

This result has a clear parallel to the insights in the theory of the firm where the organization of the firm determines which groups have power, and via this channel, which groups will undertake investments.[7] For example, a structure of ownership/organization between an upstream and a downstream producer that gives all the power to the upstream firm will discourage investment by the downstream firm, whereas one that limits the negotiation power of the upstream firm will cause the reverse underinvestment problem. A more balanced structure of power is necessary for this venture to function. In the same way, a balanced structure of power in the aggregate is necessary for both the state and the citizens to participate productively in economic activity.

While the contrast between economically weak and economically strong states is a useful starting point and highlights the parallel between the results here and the theory of the firm, the economic power of the state is not typically constrained by some technological exit option,

[5] A framework in which policy maximizes the utility of a self-interested ruler subject to constraints imposed by citizens is in the tradition of principal-agent approaches to politics, for example, Ferejohn (1986) or Persson, Roland and Tabellini (1997). Moreover, it provides a useful framework for the analysis of policy in many less-developed countries where political institutions are relatively weak (see Acemoglu, Robinson and Verdier, 2003, for a discussion). Similar qualitative results apply if policies are chosen to maximize a weighted average of the utilities of the ruler and the citizens subject to the same set of constraints.

[6] Citizens' exit options may originate from their ability to shift to informal production, to hide their revenues, or simply to disobey tax laws. In many less developed countries, raising sufficient tax revenue and ensuring compliance with the tax code are major problems, which sometimes induces governments to use inefficient tariffs to raise revenue.

[7] See, among others, Williamson (1975), Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995).

but instead originates from and is constrained by its political power. The second part of the paper shows that the same trade-offs arise when we contrast *politically weak* and *politically strong* states. States are politically weak when rulers can be replaced easily. Politically weak states will choose low taxes because of the constraints that they face, but will also invest little in public goods, while politically strong states will tend to choose excessively high taxes. Consequently, as with the economic power of the state, a balanced distribution of political power between state and society is also necessary for the economy to function efficiently.

In the last part of the paper, I briefly discuss the possibility of a "consensually-strong state," which arises in a situation where the state is politically weak (in the sense that rulers can be replaced easily), but because of a state-society consensus, the government is allowed to impose high taxes as long as a sufficient fraction of the proceeds are invested in public goods (see Acemoglu, 2005a, for a more detailed analysis). An equilibrium with a consensually-strong state best corresponds to the situation in most OECD economies, where the share of taxes in GDP is high, but much of the proceeds are redistributed back to citizens or invested in public goods.[8] Technically, the difference between politically weak states and consensually-strong states corresponds to the difference between Markov perfect and subgame perfect equilibria (supported by trigger strategies). An equilibrium with a consensually-strong state emerges when both the ruler and the citizens deviate from their myopic best responses. This requires an environment in which there is "trust" in politicians and in the functioning of political replacement mechanisms, and sufficient patience on the side of both parties. The equilibrium with the consensually-strong state is quite different in nature than equilibria with weak and strong states, and leads to richer comparative static results. In particular, a reduction in the political power of the state increases investment in public goods. Moreover, tax rates in the consensually-strong state equilibrium may be higher than in the Markov perfect equilibrium, because in return for the higher spending in public goods, citizens tolerate tax levels that they would not otherwise accept.

A significant conceptual problem in models with self-interested government behavior concerns the distinction between taxation and expropriation. It can be argued that taxation, which is much more institutionalized and thus predictable, is in essence different from expropriation, which could be arbitrary and uncertain. While this distinction may be important in practice, in most theoretical models taxation by the self-interested ruler, like expropriation, creates a distortionary transfer of resources. How can we then think of the governments of most developed countries, which, as Figures 1 and 2 illustrate, impose significant taxes on producers, as functioning better than the weaker states in sub-Saharan Africa? The idea of

[8] This is simply a comparative statement that the degree of control is considerably higher in these societies than in less developed nations, and does not suggest that voters and citizens can perfectly control the politicians and bureaucrats in the OECD societies.

a consensually-strong state suggests a possible interpretation; a consensually-strong state imposes higher taxes than weak states, but a large fraction of the proceeds are used for the provision of public goods. This interpretation reiterates the intuitive notion that in order to understand whether the state is playing a developmental role in society, we should look not only at the level of taxation, but also at how the proceeds are spent.

There is considerable research in political economy modeling the impact of various electoral rules and political institutions on the behavior of politicians. A number of papers analyze the efficiency of public goods provision and government expenditure under a variety of different political institutions.[9] In these papers, state-society relations are not the focus, and politicians act as the agent of the majority or some other politically-powerful group. A different literature in political economy deals with the problem of modeling dictatorial regimes and discusses various issues related to how the society controls politicians.[10] The perspective in this paper departs from this work by focusing on an environment where the ruler both invests in public goods and also imposes taxes on citizens to redistribute income to himself. I am not aware of any other contribution developing the insight that both (economically or politically) weak and strong states create distortions, and that a balanced structure of power between state and society is necessary for the efficient functioning of the economy.

The rest of the paper is organized as follows. Section 2 outlines the basic environment, characterizes the (Markov Perfect) equilibrium, and compares the costs and benefits of economically weak and strong states. Section 3 extends the model to an environment where citizens can replace the ruler, and discusses the trade-offs introduced by politically weak and politically strong states. Section 4 analyzes non-Markovian subgame perfect equilibria of the model in Section 3, and develops the concept of consensually-strong states. Section 5 concludes.

2 Model

2.1 Description

Consider the following infinite horizon economy. Time is discrete and indexed by t. There is a set of citizens, with mass normalized to 1, and a ruler. All agents discount the future with the discount factor β, and have the utility function

$$u_t = \sum_{j=0}^{\infty} \beta^j \left[c_{t+j} - e_{t+j} \right], \tag{1}$$

[9] See, among others, Buchanan and Tullock (1962), Aldrich (1983), Meltzer and Richard (1981), Baron and Ferejohn (1989), Dixit and Londregan (1995), Myerson (1995), and Lizzeri and Persico (2001).

[10] See, among others, Grossman (1991), Grossman and Noh (1994), McGuire and Olson (1996), Wintrobe (1998), Acemoglu and Robinson (2002), and Bueno de Mesquita et al. (2003). Most closely related is the recent paper by Aghion, Alesina and Trebbi (2004), which discusses optimal constitutional design to balance the costs of delegation of power to and ex post control of politicians.

where c_{t+j} is consumption and e_{t+j} is investment (effort), and I assume that the ruler incurs no effort cost.

Each citizen i has access to the following Cobb-Douglas production technology to produce the unique final good in this economy:

$$y_t^i = \frac{1}{1-\alpha} A_t^{\alpha} \left(e_t^i\right)^{1-\alpha}, \tag{2}$$

where A_t denotes the level of public goods (e.g., the state of the infrastructure, or the degree of law and contract enforcement between private citizens), at time t. The level of A_t will be determined by the investment of the ruler as described below. The important point captured by the specification in (2) is that a certain degree of state investment in public goods is necessary for private citizens to be able to function productively, and in fact, investment by the state is complementary to the investments of the citizens (see Barro, 1990, Barro and Sala-i-Martin, 1992, Benhabib, Rustichini and Velasco, 2001, for similar formulations of the role of public goods in economic growth).

The ruler sets a (linear) tax rate τ_t on income at time t. Also, each citizen can decide to hide a fraction z_t^i of his output, which is not taxable, but hiding output is costly, so a fraction δ of it is lost in the process. This formulation with an economic exit option for the citizens is a convenient, though reduced-form, starting point. In Section 3, I present a model where citizens' option to replace the ruler places an equilibrium constraint on taxation.

Given a tax rate τ_t, the consumption of agent i is:

$$c_t^i \leq \left[(1-\tau_t)\left(1-z_t^i\right) + (1-\delta)\, z_t^i\right] y_t^i, \tag{3}$$

where tax revenues are

$$T_t = \tau_t \int \left(1-z_t^i\right) y_t^i di. \tag{4}$$

The ruler at time t decides how much to spend on A_{t+1}. I assume that

$$A_{t+1} = \left[\frac{(1-\alpha)\,\phi}{\alpha} G_t\right]^{1/\phi} \tag{5}$$

where G_t denotes government spending on public goods, and $\phi > 1$, so that there are decreasing returns in the investment technology of the ruler (a greater ϕ corresponds to greater decreasing returns).[11] The term $\left[(1-\alpha)\,\phi/\alpha\right]^{1/\phi}$ is included as a convenient normalization. In addition, (5) implies full depreciation of A_t, which simplifies the analysis below. The consumption of the ruler is whatever is left over from tax revenues after his expenditure and transfers,

$$c_t^R = T_t - G_t.$$

[11] If $\phi = 1$, there are constant returns to scale, and the equilibrium is not well defined because the preferences in (1) are risk neutral. Similar results hold with $\phi = 1$ and risk-averse preferences, though the analysis is more involved.

The timing of events within every period is as follows:

- The economy inherits A_t from government spending at time $t-1$.
- Citizens choose their investments, $\left\{e_t^i\right\}$.
- The ruler decides how much to spend on next period's public goods, G_t, and sets the tax rate τ_t.
- Citizens decide how much of their output to hide, $\left\{z_t^i\right\}$.

2.2 The First-Best Allocation

In the first-best allocation, the planner takes A_0 as given and chooses $\left\{e_t^i\right\}_{t=0,1\ldots}$, $\left\{z_t^i\right\}_{t=0,1\ldots}$ and $\left\{A_t\right\}_{t=0,1\ldots}$ to maximize the net output (total surplus) in the economy:

$$NY_0 = \sum_{t=0}^{\infty} \beta^t \left[\int \left(\left(1 - z_t^i\right) + (1-\delta)\, z_t^i \right) \left(\frac{1}{1-\alpha} A_t^{\alpha} \left(e_t^i\right)^{1-\alpha} - e_t^i \right) di - \frac{\alpha}{(1-\alpha)\,\phi} A_{t+1}^{\phi} \right].$$

This expression subtracts both the investment costs of citizens and of the ruler from total output, which is computed using (2) and (5). Net output is invariant to the distribution of output and consumption, so taxes do not feature in this expression.

Straightforward differentiation of NY_0 establishes that the first-best allocation involves $z_t^i = 0$ for all i and t (i.e., no output is hidden) and $e_t^i = e_t^{fb} = A_t$. Substituting this into (2) gives output as: $y_t^{fb} = \frac{1}{1-\alpha} A_t$. The optimal level of public goods is obtained as:

$$A_t = A^{fb} \equiv \beta^{1/(\phi-1)}. \tag{6}$$

Consequently, the first-best allocation is characterized by $e_0^{fb} = A_0$, and for all $t > 0$:

$$e_t^{fb} = \beta^{1/(\phi-1)} \text{ and } y_t^{fb} = \frac{1}{1-\alpha} \beta^{1/(\phi-1)}.$$

2.3 Markov Perfect Equilibrium

I now characterize the Markov Perfect Equilibrium (MPE) of this game. An MPE is defined as a set of strategies at each date t, $\left(\left\{e_t^i\right\}, \tau_t, \left\{z_t^i\right\}, G_t\right)$, such that these strategies only depend on the current (payoff-relevant) state of the economy, A_t, and on prior actions within the same date according to the timing of events above. Thus, an MPE is given by a set of strategies $\left(\left\{e^i\left(A_t\right)\right\}, \tau\left(A_t\right), \left\{z^i\left(A_t\right)\right\}, G\left(A_t\right)\right)$.[12]

The convenient feature of the MPE is that we can determine the equilibrium allocation and strategies within each period by backward induction, taking the state of the economy from the previous period, A_t, as given.

[12] To simplify notation I do not introduce the dependence on the actions already taken in the same stage game explicitly (otherwise, I would have to write $\tau_t\left(\left\{e_t^i\right\} \mid A_t\right)$, etc.).

Let us start with the decisions to hide. Given the structure of the game and the focus on MPE, individuals simply maximize their current income, so

$$z_t^i \begin{cases} = 1 & \text{if } \tau_t > \delta \\ \in [0,1] & \text{if } \tau_t = \delta \\ = 0 & \text{if } \tau_t < \delta \end{cases}. \tag{7}$$

Given (7), the optimal tax rate for the ruler is

$$\tau_t = \delta. \tag{8}$$

Next, investment decisions will maximize the utility of citizens, (1) subject to (3). The Markov structure implies that this is equivalent to maximizing the current period returns, $(1-\tau_t)\, y_t^i - e_t^i$, thus: $e_t^i = (1-\tau_t)^{1/\alpha} A_t$. Individual investments are therefore decreasing in the tax rate τ_t because higher taxes reduce their net returns, and are increasing in the level of public goods, A_t, since this raises the marginal productivity of the producers.

Given the subgame perfect equilibrium tax rate implied by (8), we have

$$e_t^i = (1-\delta)^{1/\alpha} A_t. \tag{9}$$

Substituting (8) and (9) into (4), we obtain equilibrium tax revenue as a function of the level of public goods as:

$$T(A_t) = \delta y_t = \frac{(1-\delta)^{(1-\alpha)/\alpha}\, \delta A_t}{1-\alpha}. \tag{10}$$

Finally, the ruler will choose public investment, G_t to maximize his consumption. To characterize this, it is useful to write the Bellman equation for the discounted net present value of the ruler, denoted by $V(A_t)$. This takes the standard form:

$$V(A_t) = \max_{A_{t+1}} \left\{ T(A_t) - \frac{\alpha}{(1-\alpha)\phi} A_{t+1}^{\phi} + \beta V(A_{t+1}) \right\}, \tag{11}$$

which simply follows from writing the discounted payoff of the ruler recursively, after substituting for his consumption, c_t^R, as equal to taxes given by (10) minus his spending on the public goods from equation (5).

Since, for $\phi > 1$, the instantaneous payoff of the ruler is bounded, continuously differentiable and concave in A, by standard arguments (e.g., Stokey, Lucas and Prescott, 1989), the value function $V(\cdot)$ is concave and continuously differentiable. Hence, the first-order condition of the ruler in choosing A_{t+1} can be written as:

$$\frac{\alpha}{1-\alpha} A_{t+1}^{\phi-1} = \beta V'(A_{t+1}), \tag{12}$$

which links the marginal cost of greater investment in public goods to the greater value that will follow from this. To make further progress, I use the standard envelope condition, which is obtained by differentiating (11) with respect to A_t:

$$V'(A_t) = T'(A_t) = \frac{(1-\delta)^{(1-\alpha)/\alpha}\,\delta}{1-\alpha}. \tag{13}$$

The value of greater public goods for the ruler is the additional tax revenue that this will generate, which is given by the expression in (13).

Combining these conditions, we obtain the unique Markov Perfect Equilibrium choice of the ruler as:

$$A_{t+1} = A[\delta] \equiv \left(\beta\alpha^{-1}(1-\delta)^{\frac{1-\alpha}{\alpha}}\,\delta\right)^{\frac{1}{\phi-1}} \text{ and } G_t = G[\delta] \equiv \frac{\beta(1-\delta)^{(1-\alpha)/\alpha}\,\delta A[\delta]}{(1-\alpha)\,\phi}, \tag{14}$$

which also defines $A[\delta]$ and $G[\delta]$, two expressions that will feature prominently in what follows.[13] Substituting (14) into (11) yields a simple form of the ruler's value function:

$$V^*(A_t) = \frac{(1-\delta)^{(1-\alpha)/\alpha}\,\delta A_t}{1-\alpha} + \frac{\beta(\phi-1)(1-\delta)^{(1-\alpha)/\alpha}\,\delta}{(1-\beta)\,\phi(1-\alpha)}A[\delta]. \tag{15}$$

The second term in (15) follows since $G[\delta]$ in (14) is equal to a fraction $1/\phi$ of tax revenue. Note that the value of the ruler depends on the current state of public goods, A_t, which he inherits from the previous period, and from this point on, the equilibrium involves investment levels given by (9) and (14).

The following proposition summarizes the main results (proof in the text):[14]

Proposition 1 There exists a unique MPE where, for all t, $\tau_t(A_t) = \delta$, $G(A_t)$ is given by (14), and, for all i and t, $z^i(A_t) = 0$ and $e^i(A_t)$ is given by (9). The equilibrium level of aggregate output is:

$$Y_t = Y[\delta] \equiv \frac{1}{1-\alpha}(1-\delta)^{(1-\alpha)/\alpha} A[\delta], \tag{16}$$

for all $t > 0$ and

$$Y_0(A_0) = \frac{1}{1-\alpha}(1-\delta)^{(1-\alpha)/\alpha} A_0.$$

Note that because there is full depreciation of public goods and all agents are risk neutral, the economy reaches the steady-state level of output in one period. This feature simplifies the analysis significantly, especially in the next two sections.

[13] Compared to (6), there are three differences in $A[\delta]$. First, because of the distortion in the effort of citizens, it includes the term $(1-\delta)^{\frac{1-\alpha}{\alpha}}$; second, because of the self-interested perspective of the ruler, it includes δ; finally, because the ruler does not internalize the effort cost incurred by the citizens, it includes $\alpha^{-1} > 1$. It can be verified that the first two effects always dominate and $A[\delta] < A^{fb}$.

[14] In addition, it can be shown that the equilibrium in Proposition 1 is the unique subgame perfect equilibrium. This follows since citizens always take individually rational economic decisions and have no credible punishments they can use against the ruler.

2.4 Weak States Versus Strong States

The first result from the above analysis is a parallel between this model and the literature on the theory of the firm. In the incomplete-contract theories of the firm, various stakeholders, such as the owner, suppliers, managers and workers, have a tendency to underinvest because of the ex post bargaining over the output of the firm, which gives them less than their full contribution to firm value.[15] The structure of the firm (in particular, the ownership of assets) determines the ex post bargaining power of the parties, and hence their ex ante investment incentives. The optimal, and sometimes the equilibrium, structure of the firm is the one that balances these incentives. The current framework gives similar insights on the effects of the distribution of power on investment incentives in society.

So far, the main parameter that is treated as exogenous is δ, the exit options of citizens. When δ is high, the state is "economically powerful"—citizens have little recourse against high rates of taxes. In contrast, when δ is low, the state is "economically weak" (and there is "limited government"), since it is unable to raise taxes. With this interpretation, we can now ask whether greater economic strength of the state leads to worse economic outcomes. The answer is ambiguous, as it can be seen from the fact that when $\delta = 0$, i.e., when the state is extremely weak, the ruler will choose $G_t = 0$, while with $\delta = 1$, the citizens will choose zero investments. In both cases, output will be equal to zero.

It is straightforward to determine the level of δ that maximizes output in the society at all dates after the initial one, i.e., Y_t for $t > 0$. It is given by $\max_\delta Y[\delta]$, where $Y[\delta]$ is given by (16).[16] The solution to this program, denoted δ^*, is

$$\delta^* \equiv \frac{\alpha}{\phi(1-\alpha)+\alpha}. \tag{17}$$

If the economic power of the state is greater than δ^*, then the state is too powerful, and taxes are too high relative to the output-maximizing benchmark. This corresponds to the standard case that the political economy literature has focused on. In contrast, if the economic power of the state is less than δ^*, then the state is not powerful enough for there to be sufficient rents in the future to entice the ruler to invest in public goods. This corresponds to the case that the political science literature has identified as "the problem of weak states". Notice an important difference from the arguments in the political science literature, however. The problem here arises because with only limited power to raise taxes in the future, the self-interested ruler has no interest in increasing the future productive capacity of the economy.

The expression for δ^* is intuitive. For example, δ^* is an increasing function of α. This is because, from the production function (2), a greater α implies that the investment of the ruler

[15] See, for example, Williamson (1975), Grossman and Hart (1986) and Hart and Moore (1990).

[16] At the initial date $t = 0$, where A_0 is inherited from prior investments, $\delta_0 = 0$ would maximize output by reducing investment distortions. If we were to look for a value of δ such that $\delta_t = \delta$ for all $t \geq 0$, then this value would depend on A_0 to take advantage of this first-period effect.

is more important relative to the investments of the citizens. Thus the ruler should receive a greater fraction of the ex post rents to encourage him to invest further. δ^* is also decreasing in ϕ, which corresponds to the degree of decreasing returns in the public good technology. Greater decreasing returns imply that the investment of the ruler is less sensitive to his ex post share of the revenues, and thus the optimal division of economic strength in society will give more weight to the citizens.

The parallel to the theory of the firm is apparent here: there, the optimal structure of ownership and control gives ex post bargaining power to the parties that have more important investments. The same principle applies to the allocation of economic strength as captured by the parameter δ; greater power for citizens is beneficial when their investments matter more. When it is the state's investment that is more important for economic development, a higher δ is required (justified). This might also give a possible interpretation of the divide between economic and political scientists; perhaps the emphasis of political scientists on the importance of state capacity and the cost of weak states stems from their belief, not always shared by economists, that state's actions are central for economic development.

The above discussion focused on the output-maximizing value of the parameter δ. Equally relevant is the level of δ, say δ^r, which will maximize the beginning-of-period payoff to the ruler. In particular, consider an economy that starts with a level of public goods at A_0 and δ is fixed at δ_0 at date $t = 0$, and the ruler determines the economic power of the state that will apply from then on, i.e., $\delta_t = \delta^r$, for all $t > 0$.[17] This is the solution to the following maximization problem:

$$\delta^r = \arg\max_{\delta} V^*(A_0) = \arg\max_{\delta} \frac{(1-\delta_0)^{(1-\alpha)/\alpha}\,\delta_0 A_0}{1-\alpha} + \frac{\beta(\phi-1)\,(1-\delta)^{(1-\alpha)/\alpha}\,\delta}{(1-\beta)\,\phi(1-\alpha)} A[\delta],$$

where I rewrote V^* from (15), imposing the assumption that δ is being chosen for all future periods (and is fixed at δ_0 in the initial period). Straightforward maximization gives:

$$\delta^r = \alpha > \delta^*.$$

It is not surprising that the level of δ, thus the division of the surplus, preferred by the ruler is different from the output-maximizing one. At δ^*, the output cost of increasing δ a little beyond δ^* is second-order, whereas the gain to the ruler is first-order.

By analogy, we can also look at the level of δ most preferred by the citizens. Using (1), we obtain the citizens' utility starting with public goods A_0 at date $t = 0$ as:[18]

$$U_0(A_0) = \frac{\alpha}{1-\alpha}(1-\delta_t)^{1/\alpha} A_0 + \frac{\beta}{1-\beta}\frac{\alpha}{1-\alpha}(1-\delta)^{1/\alpha} A[\delta],$$

[17] This could be, for example, by choosing the size of the bureaucracy, but in this case the costs of diverting agents from production to bureaucracy have to be incorporated; see Appendix B.

[18] To obtain this expression, note that in each period citizen utility is given by $u_t^i = c_t^i - e_t^i = \alpha(1-\delta)^{1/\alpha} A_t/(1-\alpha)$. After date $t = 0$, the MPE is followed, so $A_t = A[\delta]$ for $t > 0$ is given by (14).

with $A[\delta]$ given by (14). It is straightforward to show that $\delta^c = \arg\max_\delta U_0(A_0) = \alpha/\phi < \delta^* < \delta^r$, so that citizens prefer an organization of society that gives them a greater share of the surplus than the one that maximizes output or the one that's preferred by the ruler. The intuition is the same as for $\delta^* < \delta^r$: at δ^*, the output loss due to a marginal decline in δ is second-order, whereas the gain to the citizens is first order, so $\delta^c < \delta^*$.

Finally, we can also investigate what level of δ would maximize net output (total surplus) when all agents are pursuing their equilibrium strategies. Net output differs from total output, because the costs of investment by both the citizens and the ruler need to be subtracted (and it differs from the utility of the citizens because it takes into account the utility of the ruler). With a similar argument to before, the discounted value of net output starting with A_0 is:

$$\begin{aligned} NY_0(A_0) &= \frac{\alpha+(1-\alpha)\delta_0}{1-\alpha}(1-\delta_0)^{(1-\alpha)/\alpha}A_0 - \frac{1}{1-\beta}\frac{\alpha}{(1-\alpha)\phi}(A[\delta])^\phi \\ &\quad + \frac{\beta}{1-\beta}\left(\frac{\alpha+(1-\alpha)\delta}{1-\alpha}A[\delta](1-\delta)^{(1-\alpha)/\alpha}\right), \end{aligned}$$

which includes the consumption of both the ruler and the citizens (the α term for the citizens and the $(1-\alpha)\delta$ for the ruler). Defining $\delta^{wm} = \arg\max_\delta NY_0(A_0)$, it can be shown that $\delta^c < \delta^{wm} < \delta^r$ for the same reasons as $\delta^c < \delta^* < \delta^r$ (i.e., welfare maximization takes into account the returns both to citizens and the ruler).[19] Summarizing this (proof in the text):[20]

Proposition 2 Let δ^*, δ^{wm}, δ^r and δ^c be the values of δ that respectively maximize output, social welfare, ruler's utility and citizens' utility. We then have:

$$0 < \delta^c < \delta^* < \delta^r < 1 \text{ and } 0 < \delta^c < \delta^{wm} < \delta^r < 1.$$

The main conclusion from this analysis is that when both the state and the citizens make productive investments, it is no longer true that limiting the rents that accrue to the state is always good for economic performance. Instead, there needs to be a certain degree of *balance of powers* between the state and the citizens. When self-interested rulers expect too few rents in the future, they have no incentive to invest in public goods. Consequently, excessively weak states are likely to be as disastrous for economic development as the unchecked power and expropriation by excessively strong states.

A number of shortcomings of the analysis in this section should be noted at this point. The first is that it relied on exit options of the citizens as the source of their control over the state,

[19] For example, to see that $\delta^{wm} < \delta^r$, recall that equilibrium public good spending is $G[\delta]$ given by (14), which satisfies $G'[\delta = \delta^r] = 0$ and is concave in δ. Net output can be written as

$$NY[\delta] = \text{constant } + \frac{G[\delta]}{1-\beta}\left[\phi\frac{\alpha}{\delta} + \phi(1-\alpha) - 1\right].$$

The term in square brackets is positive and its derivative with respect to δ is negative, so $G'[\delta = \delta^{wm}] > 0$. Since $G[\delta]$ is concave, we have $\delta^{wm} < \delta^r$.

[20] In addition, it can be proved that $\delta^{wm} \lesseqgtr \delta^*$ depending on whether $(1-\alpha) \lesseqgtr 1/\phi$.

whereas, in practice, political controls may be more important. The second is that changes in the power of the state affect taxes and investments in public goods in the same direction; weak states do not tax their citizens, but neither do they invest in public goods. Relatedly, the pattern in Figure 2 suggests that more constrained governments collect higher tax revenues, which is also difficult to reconcile with the model here. These issues will be discussed in the next two sections.

3 Political Power

So far, the analysis focused on the distribution of economic power in the economy, and modeled the main constraint on the taxation power of the state as the technological exit options of the citizens. Although exit options, including access to the informal sector and tax evasion, place limits on the level of taxes that the state can impose, the taxation capability of the state and the constraints on it often emanate from its "political power". The political power of the state depends on how easily citizens can replace a ruler who is pursuing policies detrimental to their interests. In this section, I extend the model to allow for citizens to replace the ruler at the beginning of each period with a new identical ruler. The costs of replacing the ruler determine the political power of the state. The main result of this section is to show that the same trade-offs that were highlighted in the previous section also apply to the political power of the state. In particular, both excessively strong and weak states lead to poor economic performance.

I modify the baseline model of Section 2 as follows: now there is a large set of identical potential rulers, and citizens decide whether to replace the current ruler, denoted by $R_t \in \{0, 1\}$. After replacement, citizens can reclaim part of the tax revenue and redistribute it to themselves as a lump sum transfer, S_t. At the time of replacement, the public goods spending, G_t, is already committed and ruler replacement does not affect next period's level of public goods. Instead, citizens take back a fraction $\eta \in (0, 1]$ of the tax revenue, T_t, and the rest of the revenue is lost in the process, so the consumption of the ruler is equal to zero, and the ruler is also assumed to receive zero continuation utility after replacement.[21] Replacement is costly, however, and at time t citizens face a cost of replacing the current ruler with a new ruler equal to $\theta_t A_t$, where θ_t is a nonnegative random variable with a continuous distribution function $\tilde{F}_\lambda$, with (finite) density $\tilde{f}_\lambda$. This is a tractable formulation for introducing stochastic replacements of the ruler along the equilibrium path. The cost is multiplied by A_t to ensure that the level of public goods does not have a mechanical effect on replacement. Finally, I

[21] The results are similar if, after losing power, rulers work as citizen-producers rather than obtain zero utility, and/or if the ruler's consumption, when replaced, is equal to $c_t^R = (1 - \eta) T_t - G_t$ rather than zero. Both of these assumptions are adopted to simplify the analysis and the exposition. The second assumption implies that we should think of $\eta < 1/\phi$, so that (in steady state), what citizens take is less than government spending on public goods (see equation (14) in the previous section).

impose:

$$\frac{\tilde{f}_\lambda(x)}{1-\tilde{F}_\lambda(x)} \text{ is nondecreasing in } x \text{ and } \tilde{F}_\lambda(0) < 1. \tag{A1}$$

The first part is the standard monotone likelihood ratio (MLR) assumption, which is also equivalent to $1-\tilde{F}_\lambda$ being log concave (see Burdett, 1979), and the second part rules out the degenerate case where there is always replacement.

The timing of events in this endogenous replacement game can be summarized as:

- The economy inherits A_t from government spending at time $t-1$.
- Citizens choose their investments, $\{e_t^i\}$.
- The ruler commits the spending on next period's public goods, G_t, and sets the tax rate τ_t.
- Citizens decide how much of their output to hide, $\{z_t^i\}$.
- θ_t is realized.
- Citizens choose R_t. If $R_t = 1$, the current ruler is replaced and a fraction η of the tax revenue is redistributed to the citizens as a lump-sum subsidy $S_t = \eta T_t$.

I assume that although citizens make their economic decisions independently, the political decision, the choice of R_t, is made to maximize group utility.[22]

An MPE is defined similarly to before, as a set of strategies at each date t, $\left(\{e_t^i\}, \tau_t, \{z_t^i\}, R_t, G_t\right)$, such that these strategies only depend on the current state of the economy, A_t, and on prior actions within the same date according to the timing of events above. Thus, it can be summarized by a set of strategies $\left(\{e^i(A_t)\}, \tau(A_t), \{z^i(A_t)\}, R(A_t), G(A_t)\right)$. In addition, it is convenient to focus on a steady-state MPE where $A_t = A_{t+\ell}$ for all $\ell \geq 0$.

To simplify the analysis, I also assume that

$$\delta \in (\delta^*, \alpha), \tag{A2}$$

where δ^* is given by (17). This assumption ensures that taxes are always less than the value $\delta^r = \alpha$ that maximizes ruler utility (see Section 2.4), and also allows for taxes that are potentially higher than the output-maximizing level, δ^*.

[22] This is without loss of any generality; since all citizens have identical preferences regarding replacement, this outcome would result from various ways of solving the political collective action problem among the citizens. For example, if each citizen votes between $R_t = 0$ and $R_t = 1$ to maximize their own utility, voting for the choice that maximizes group utility is a weakly dominant strategy for each.

Citizens' hiding decisions are still given by the privately optimal rule, (7). Moreover, in the MPE, they will replace the ruler, i.e., $R_t = 1$, whenever

$$\theta_t < \frac{\eta T_t}{A_t}. \tag{18}$$

Intuitively, in the MPE replacing the ruler has no future costs or benefits (since all future rulers condition their strategies only on the payoff-relevant state variable, A_t), so it is in the citizens' interest to replace the ruler when the immediate benefit, ηT_t, exceeds the cost, $\theta_t A_t$. The important substantive implication of (18) is that greater taxes will lead to a higher likelihood of ruler replacement.

Condition (18) immediately implies that the probability that the ruler will be replaced is $\tilde{F}_\lambda\left(\eta T_t / A_t\right)$. To simplify the notation, define

$$\mathcal{T}\left(\tau_t\right) \equiv \frac{\left(1-\tau_t\right)^{(1-\alpha)/\alpha} \tau_t}{1-\alpha}, \tag{19}$$

so that $T_t = \mathcal{T}\left(\tau_t\right) A_t$. Let us also parameterize the distribution function as $\tilde{F}_\lambda\left(x\right) = \lambda F\left(x/\eta\right)$ for some continuous distribution function $F\left(x\right)$ with (finite) density $f\left(x\right)$ for all $x > 0$, which will be useful both to simplify notation and for comparative static exercises below.[23] This assumption implies that the probability that the ruler will be replaced is $\lambda F\left(T_t/A_t\right) = \lambda F\left(\mathcal{T}\left(\tau_t\right)\right)$. The monotone likelihood ratio property of $\tilde{F}_\lambda$ in (A1) clearly carries over to F.

The relevant value function for the ruler can then be written as:

$$V\left(A_t\right) = \max_{\tau_t \in [0,\delta], A_{t+1}} \left\{ \left(1 - \lambda F\left(\mathcal{T}\left(\tau_t\right)\right)\right) \left(\mathcal{T}\left(\tau_t\right) A_t - \frac{\alpha}{\phi(1-\alpha)} A_{t+1}^{\phi} \right) + \beta \left(1 - \lambda F\left(\mathcal{T}\left(\tau_t\right)\right)\right) V\left(A_{t+1}\right) \right\}. \tag{20}$$

Now the ruler's maximization problem involves two choices, τ_t and A_{t+1}, since taxes are no longer automatically equal to the maximum, δ. In this choice, the ruler takes into account that a higher tax rate will increase the probability of replacement. The first-order condition with respect to τ_t yields:

$$\frac{\partial \mathcal{T}\left(\tau_t\right)}{\partial \tau_t} \left[\left(1 - \lambda F\left(\mathcal{T}\left(\tau_t\right)\right)\right) - \lambda f\left(\mathcal{T}\left(\tau_t\right)\right) \left(\mathcal{T}\left(\tau_t\right) - \frac{G_t}{A_t} + \beta \frac{V\left(A_{t+1}\right)}{A_t} \right) \right] \geq 0 \text{ and } \tau_t \leq \delta, \tag{21}$$

with complementary slackness,[24] where recall that $G_t = \alpha A_{t+1}^{\phi} / \left(\phi(1-\alpha)\right)$. Assumption A2 implies that $\tau < \alpha$, so $\partial \mathcal{T}\left(\tau_t\right) / \partial \tau_t > 0$ and in an interior equilibrium the term in square brackets has to be equal to zero. In other words, the additional expected revenue brought

[23] More formally, this should be $\tilde{F}_\lambda\left(x\right) = \min\left\{\lambda F\left(x/\eta\right), 1\right\}$, but I suppress the min to simplify notation.
This parameterization can be loosely interpreted as follows: with probability $1-\lambda$, $\theta = \infty$, and replacing the ruler is impossible (infinitely costly), while with probability λ, the cost of replacing the ruler is drawn from the distribution F (though, contrary to what is implied by this analogy, λ can be greater than 1). This interpretation will be particularly useful in the next section.

[24] I ignore the possibility $\tau_t = 0$, since this can never be the case in a steady state equilibrium.

by higher taxes (the first term in square brackets) must be balanced by higher probability of losing these taxes and the continuation value (the second term in square brackets).

Assuming that $V(A_{t+1})$ is differentiable in A_{t+1}, the first-order condition for A_{t+1} is still given by (12) in Section 2 (since the term $(1-\lambda F(\mathcal{T}(\tau_t)))$ cancels from both sides).[25] The expression for $V'(A_{t+1})$ again follows from the envelope condition,

$$V'(A_{t+1}) = (1-\lambda F(\mathcal{T}(\tau_{t+1})))\,\mathcal{T}(\tau_{t+1})\,. \tag{22}$$

It only differs from the corresponding condition in Section 2, (13), because with probability $\lambda F(\mathcal{T}(\tau_{t+1}))$, the ruler will be replaced and will not enjoy the increase in future tax revenues.

Using this, the first-order condition with respect to A_{t+1} implies that in an interior equilibrium:

$$A_{t+1} = A[\tau_{t+1}] \equiv \left(\alpha^{-1}\beta(1-\lambda F(\mathcal{T}(\tau_{t+1})))(1-\tau_{t+1})^{\frac{1-\alpha}{\alpha}}\tau_{t+1}\right)^{\frac{1}{\phi-1}}.$$

The optimal value of A_{t+1} for the ruler depends on τ_{t+1} since, from the envelope condition, (22), the benefits from a higher level of public good are related to future taxes.

To make further progress, let us focus on the steady state equilibrium where $\tau_{t+\ell} = \tau^*$ for all $\ell \geq 0$ (which follows since in steady state $A_t = A_{t+\ell}$). Hence

$$A[\tau^*] \equiv \left(\alpha^{-1}\beta(1-\lambda F(\mathcal{T}(\tau^*)))(1-\tau^*)^{\frac{1-\alpha}{\alpha}}\tau^*\right)^{\frac{1}{\phi-1}}. \tag{23}$$

In addition, we have $G[\tau^*] = \beta\mathcal{T}(\tau_t)\,A[\tau^*]/\phi$. Therefore, the value function for the ruler in steady state can be written as $V(A[\tau^*]) = (1-\lambda F(\mathcal{T}(\tau^*)))\,v[\tau^*]\,A[\tau^*]$, where

$$v[\tau^*] \equiv \mathcal{T}(\tau^*) + \frac{(\phi-1)}{\phi}\frac{\beta(1-\lambda F(\mathcal{T}(\tau^*)))\,\mathcal{T}(\tau^*)}{(1-\beta(1-\lambda F(\mathcal{T}(\tau^*))))}. \tag{24}$$

Now using (20), (21), (24), and the fact that in steady state $\tau_{t+\ell} = \tau^*$ for all $\ell \geq 0$, we obtain the following equation for an interior steady-state equilibrium tax rate, τ^*:

$$\lambda f(\mathcal{T}(\tau^*))\,v[\tau^*] - (1-\lambda F(\mathcal{T}(\tau^*))) = 0. \tag{25}$$

This equation is intuitive. The first term is the cost of a unit increase in $T(\tau)$. This increase reduces the probability of staying in power by an amount equal to $\lambda f(\mathcal{T}(\tau))$. This is multiplied by the (normalized) value of staying power, $v[\tau^*]$, (since $v[\tau^*] = \mathcal{T}(\tau^*) - G(A[\tau^*])/A[\tau^*] + \beta V(A[\tau^*])/A[\tau^*]$). The second term is the benefit of a unit increase in tax revenue, which the ruler receives with probability $1-\lambda F(\mathcal{T}(\tau))$. Note that $\tau^* = 0$ can never be a solution to this equation, since $\beta v[0] = \mathcal{T}(0) = 0$. Therefore, there will be an interior solution as long as

$$\lambda f(\mathcal{T}(\delta))\,v[\delta] > 1-\lambda F(\mathcal{T}(\delta))\,. \tag{26}$$

[25] In general, $V(A_t)$ is not always differentiable because the maximization problem in (20) is not necessarily jointly concave in A_{t+1} and τ_{t+1}, and may have multiple solutions. Assumption A3 below ensures uniqueness and differentiability.

If, on the other hand, (26) does not hold, then the equilibrium will be a corner solution with $\tau = \delta$ and $A[\tau = \delta]$ This establishes the existence of an MPE. To establish uniqueness, we need to impose an additional condition:

$$\left(1 - \frac{\beta}{\phi}(1 - \lambda F(0))\right)^2 - (\phi - 1)\frac{\beta}{\phi}(1 - \lambda F(0)) > 0. \tag{A3}$$

This assumption requires $\beta(1 - \lambda F(0))$ not to be too large, and can be satisfied either if β is not too close to 1 or if $\lambda F(0)$ is not equal to zero. Intuitively, if $\beta(1 - \lambda F(\tau^*))$ is close to 1, $v[\tau^*]$ can be very large, creating a non-monotonicity in (25). Assumption (A3) is sufficient to ensure that this is not the case, so the left-hand side of (25) is everywhere increasing and there is a unique equilibrium (see the proof of Proposition 3). This observation, combined with the MLR assumption, (A1), also leads to unambiguous comparative static results:

Proposition 3 Suppose (A1), (A2) and (A3) hold. Then, in the endogenous replacement game of this section, there exists a unique steady-state MPE.

- In this equilibrium, if (26) does not hold all rulers set $\tau = \delta$ and $A[\tau = \delta]$. If (26) holds, then they set $\tau^* < \delta$ given by (25), and $A[\tau^*]$ given by (23). Citizens replace a ruler whenever $\theta_t < \eta \mathcal{T}(\tau^*)$ where $\mathcal{T}(\cdot)$ is given by (19).
- In this equilibrium, $\partial\tau^*/\partial\lambda \le 0$, $\partial\tilde{A}/\partial\lambda \le 0$, $\partial\tau^*/\partial\delta \ge 0$ and $\partial\tilde{A}/\partial\delta \ge 0$.
- There also exists $\lambda^* \in (0, \infty)$ such that output is maximized when $\lambda = \lambda^*$.

Proof: The analysis above establishes that the steady-state MPE tax rate, τ^*, is characterized by (25), or by

$$\mathcal{F}(\tau^*) \equiv \frac{\lambda f(\mathcal{T}(\tau^*))}{1 - \lambda F(\mathcal{T}(\tau^*))} v[\tau^*] - 1 = 0,$$

as long as (26) is satisfied, and by $\tau^* = \delta$ otherwise. To establish existence and uniqueness of equilibrium, first suppose (26) holds. Then note that $\mathcal{F}(\tau^* = 0) = -1$ and given that (26) holds, $\mathcal{F}(\tau^* = \delta) > 0$. Since $\mathcal{F}(\tau^*)$ is continuous in τ^*, an interior solution exists. To prove that this solution is unique, it is sufficient to show that $\mathcal{F}(\tau^*)$ is increasing in τ^* whenever $\mathcal{F}(\tau^*) = 0$ (which implies that τ^* must be unique from the mean value theorem). Note that: (1) since $\tau^* < \alpha$ from (A2), $\mathcal{T}(\tau^*)$ is increasing in τ^*; (2) given the MLR assumption, (A1), $\lambda f(\mathcal{T}(\tau^*)) / (1 - \lambda F(\mathcal{T}(\tau^*)))$ is nondecreasing in $\mathcal{T}(\tau^*)$. Therefore, if $v[\tau^*]$ is indecreasing in $\mathcal{T}(\tau^*)$, $\mathcal{F}(\tau^*)$ would be increasing in τ^* whenever $\mathcal{F}(\tau^*) = 0$. To obtain this last step, note that we can write

$$v'[\tau^*] = 1 + \beta\frac{\phi - 1}{\phi}\frac{(1 - \lambda F(\mathcal{T}(\tau^*)))}{1 - \beta(1 - \lambda F(\mathcal{T}(\tau^*)))} - \beta\frac{\phi - 1}{\phi}\frac{\lambda f(\mathcal{T}(\tau^*))\mathcal{T}(\tau^*)}{[1 - \beta(1 - \lambda F(\mathcal{T}(\tau^*)))]^2}.$$

Then (25) implies that $\lambda f(\mathcal{T}(\tau^*)) = (1 - \lambda F(\mathcal{T}(\tau^*)))/v[\tau^*]$. Substituting this in the preceding expression, using (24) and simplifying, we have:

$$v'[\tau^*] = \frac{1}{1-\beta(1-\lambda F(\mathcal{T}(\tau^*)))}\left[1 - \frac{\beta}{\phi}(1-\lambda F(\mathcal{T}(\tau^*))) - \beta\frac{\phi-1}{\phi}\frac{1-\lambda F(\mathcal{T}(\tau^*))}{1-\frac{\beta}{\phi}(1-\lambda F(\mathcal{T}(\tau^*)))}\right]$$

Assumption (A3) ensures that $v'[\tau^*] \geq 0$ since $F(\mathcal{T}(\tau^*)) \geq F(0)$. This establishes existence and uniqueness for the case where (26) holds. When (26) does not hold, $\tau^* = \delta$ is the unique equilibrium.

The result that $\partial\tau^*/\partial\lambda \leq 0$ follows from the implicit function theorem, since $\mathcal{F}(\tau^*)$ is nondecreasing in τ^* given MLR and is also increasing in λ. From (23), $A[\tau^*]$ is a decreasing function of λ and Assumption (A2) also implies that it is a decreasing function of τ^*. Since $\partial\tau^*/\partial\lambda \leq 0$, this implies that $\partial A[\tau^*]/\partial\lambda \leq 0$. Finally, when $\tau^* < \delta$, δ has no effect on equilibrium values, and when $\tau^* = \delta$, $\partial\tau^*/\partial\delta \geq 0$ and $\partial A[\tau^*]/\partial\delta \geq 0$.

Finally, aggregate output, $Y[\tau^*]$, is given by equation (16), with $A[\tau^*]$ replacing $A[\delta]$. We have that τ^* and $A[\tau^*]$, and hence (16), are continuous in λ. As $\lambda \to \infty$, $\tau^* \to 0$, and $\lim_{\tau^* \to 0} Y[\tau^*] = 0$. Moreover, as $\lambda \to 0$, $\tau^* \to \delta > \delta^*$ (recall that $\delta \in (\delta^*, \alpha)$ from (A2)), so $Y[\tau^*]$ is decreasing in the neighborhood of $\tau^* = \delta$. This, combined with the continuity of $Y[\tau^*]$ in λ, establishes that aggregate output is maximized at some $\lambda = \lambda^* \in (0, \infty)$. **QED**

The most important result in this proposition is that, similar to the analysis of the economic power of the state in the previous section, there is an optimal level of the political power of the state. Intuitively, when $\lambda < \lambda^*$, the state is excessively powerful, citizens expect high taxes and choose very low levels of investment (effort). When $\lambda > \lambda^*$, the state is excessively weak and there is the reverse holdup problem; high taxes will encourage citizens to replace the ruler, and anticipating this, the ruler has little incentive to invest in public goods, because he will not be able to recoup the costs with future revenues. Therefore, this proposition reiterates the main insight from the previous section: there needs to be a balanced distribution of (both economic and political) power between the state and the citizens to encourage both parties to make investments in the productive resources of the society.

The proposition also establishes the equilibrium tax rate and public good spending are decreasing in λ (and increasing in δ). These are intuitive. A lower value of λ corresponds to a situation in which politicians are more entrenched and more costly to replace, thus politically more powerful. Since taxes are constrained by the political power of the citizens (i.e., their power to replace the ruler when taxes are high), a lower λ implies that the ruler will impose higher taxes and will be willing to invest more in public goods. Consequently, this model, like the one in the previous section, implies that the (economic or political) power of the state affects taxes and investment in the public good in the same direction, and it also suggests that it should be less constrained governments that collect higher taxes and invest more in public goods. We will next see that when non-Markovian subgame perfect equilibria are considered, these implications change significantly.

4 Consensually-Strong States

The analysis so far focused on Markov Perfect Equilibria (MPEs), where the repeated nature of the game between the ruler and the citizens is not exploited. In this framework, weak states are costly because rulers are unable to impose high taxes and do not have sufficient incentives to invest in public goods. However, when the state is politically weak, in the sense that the politician in power can be replaced easily, a consensus between state and society can develop whereby citizens will tolerate high taxes (and will not replace the government because of these high taxes) as long as a sufficient fraction of the proceeds are invested in public goods. I refer to this as a "consensually-strong state," and in this section, I briefly investigate how a consensually-strong state can arise as a subgame perfect equilibrium in the game of the previous section.

An analysis of consensually-strong states is interesting not only to relax the restriction to MPE (which may not be warranted given the repeated interaction between the ruler and the citizens), but also because the concept of a consensually-strong state might be useful in providing us with a simple framework to think about state-society relations in many developed countries. As suggested by Figures 1 and 2, though they appear to be politically constrained, governments in these societies impose relatively high taxes, and then spend a high fraction of the proceeds on public goods. Such an outcome appears difficult in the models of the previous two sections; if δ is high or λ is low, the government imposes high taxes, but consumes a high fraction of the proceeds. However, in the "consensually-strong state" equilibrium of this section, the pattern with high taxes and relatively high investments in public goods will emerge as the equilibrium when both δ and λ are high (also when the discount factor β is high).

A subgame perfect equilibrium is a set of strategies that are best responses to each other given all histories, and the on-the-equilibrium-path behavior in this equilibrium can be described as a set of strategies $\left(\left\{e^i(A)\right\}, \tau(A), \left\{z^i(A)\right\}, R(A), G(A)\right)$. The purpose of this section is not to provide a comprehensive analysis of the set of subgame perfect equilibria, but to have a first look at the the different implications that arise once we consider non-Markovian strategies. I will therefore focus on the "consensually-strong state" equilibrium, which is defined as a stationary strategy profile maximizing the steady-state utility of the citizens.[26] Since all rulers are ex ante identical, the best such equilibrium should keep the ruler in power as long as he follows the implicitly-agreed strategy. Let us think of this equilibrium as a policy vector $(\tau, \tilde{A})$ such that as long as the ruler follows this policy vector, he will never be replaced, and

[26] The major simplification here is to focus on steady states rather than start at some arbitrary level of public good, A_0, and then trace the law of motion of A_t.

his continuation value when he deviates is derived from a credible punishment strategy of the citizens.

To simplify the analysis further, I first discuss the case where $\tilde{F}_\lambda = \tilde{F}^*_\lambda$ with $\tilde{F}^*_\lambda$ taking the following simple form: with probability $1-\lambda$, $\theta = \infty$, and with probability λ, $\theta = 0$. This implies that with probability $1-\lambda$, citizens cannot replace the ruler, and with probability λ, they can do so without any costs. Finally, all of the proofs relevant for this section are contained in Appendix A.

4.1 Analysis When $\tilde{F}_\lambda = \tilde{F}^*_\lambda$

Let $V^c(\tilde{\tau}, \tilde{A} \mid A)$ be the value of the ruler in such an equilibrium where the current state is A, and all future taxes and public good investments are given by $(\tilde{\tau}, \tilde{A})$. We then have:

$$V^c(\tilde{\tau}, \tilde{A} \mid A) = \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \tilde{\tau} A}{1-\alpha} - \frac{1}{1-\beta} \frac{\alpha \tilde{A}^\phi}{(1-\alpha)\phi} + \frac{\beta}{1-\beta} \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \tilde{\tau} \tilde{A}}{1-\alpha}. \tag{27}$$

Here the superscript c denotes "cooperation," and the form of this expression immediately follows from (15), incorporating the fact that future policies are $(\tilde{\tau}, \tilde{A})$ and there is no replacement of the ruler.

In contrast, if the ruler decides to deviate from the implicitly-agreed policy $(\tilde{\tau}, \tilde{A})$, his continuation value will depend on the punishment strategies he expects. Recall that with probability $1-\lambda$, citizens are unable to replace the ruler ($\theta = \infty$), whereas with probability λ, they can replace the ruler without any cost. Since citizens cannot coordinate their economic decisions, replacing the ruler with probability λ and then playing the MPE strategies is the worst (credible) punishment.[27] Anticipating replacement with this probability, the problem of the ruler is similar to that analyzed in Section 2. In particular, he will always tax at the maximum rate, δ, and choose the level of investment in public goods consistent with his own objectives (since following a deviation, the ruler is replaced with probability λ irrespective of the tax rate, he sets the highest possible tax rate, δ). Thus his deviation value as a function of the current state A and the tax expectation of the citizens, $\tilde{\tau}$,[28] is given by

$$V^d(A \mid \tilde{\tau}) = \max_{A^d} \left\{ (1-\lambda) \left(\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \delta A}{1-\alpha} - \frac{\alpha}{(1-\alpha)\phi} \left(A^d\right)^\phi \right) + \beta(1-\lambda)\tilde{V}^d(A^d) \right\}. \tag{28}$$

This expression takes into account that when the ruler deviates, he takes advantage of the fact that citizens invested expecting a tax rate of $\tilde{\tau} < \delta$, and then taxes them at the rate δ. Subsequently, he invests an amount A^d in the public good, consistent with his own maximization

[27] Worse punishments could include citizens reducing their investments below the privately optimal level, thus reducing the ruler's future revenues. Such punishments are not possible/credible, however, given the assumption that individuals take the privately optimal economic decisions.

[28] Expectations matter because citizens choose their investments as a function of the promised tax rate, τ.

problem, and receives the MPE continuation value. An analysis similar to that in Section 2 shows that this value is

$$\tilde{V}^d(A) = \frac{(1-\lambda)(1-\delta)^{\frac{1-\alpha}{\alpha}}\delta A}{1-\alpha} + \frac{\beta(1-\lambda)^2(\phi-1)(1-\delta)^{\frac{1-\alpha}{\alpha}}\delta A[\delta \mid \lambda]}{(1-\beta(1-\lambda))(1-\alpha)\phi},$$

with $A[\delta \mid \lambda]$ defined by:

$$A\left[\delta \mid \lambda\right] \equiv \left(\alpha^{-1}\beta\left(1-\lambda\right)\left(1-\delta\right)^{\frac{1-\alpha}{\alpha}}\delta\right)^{1/(\phi-1)}. \tag{29}$$

Therefore, the deviation value of the ruler is:

$$V^d\left(A \mid \tilde{\tau}\right) = \frac{(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\delta A}{1-\alpha} + \frac{\beta(1-\lambda)^2(\phi-1)(1-\delta)^{\frac{1-\alpha}{\alpha}}\delta A[\delta \mid \lambda]}{(1-\beta(1-\lambda))(1-\alpha)\phi}. \tag{30}$$

In the consensually-strong state equilibrium, the ruler is expected to follow the agreed policy, $(\tilde{\tau}, \tilde{A})$, starting from a level of public goods equal to $\tilde{A}$. Therefore, the ruler incentive compatibility constraint is:

$$V^c(\tilde{\tau}, \tilde{A} \mid \tilde{A}) \geq V^d\left(\tilde{A} \mid \tilde{\tau}\right). \tag{31}$$

This incentive compatibility constraint requires that the ruler prefers the equilibrium strategy to deviating and taxing at the highest possible rate for his own consumption. It must also be in the interest of the citizens not to replace the ruler pursuing the implicitly-agreed policy. When they do so, the analysis in Section 2 implies that their payoff is given by:

$$U^c\left(\tilde{\tau}, \tilde{A} \mid A\right) = \frac{\alpha}{1-\alpha}(1-\tilde{\tau})^{1/\alpha}A + \frac{\beta}{1-\beta}\frac{\alpha}{1-\alpha}(1-\tilde{\tau})^{1/\alpha}\tilde{A}. \tag{32}$$

In contrast, if they deviate, the society reverts back to the MPE (because all future rulers will expect the citizens to play the MPE strategies), where a ruler is replaced with probability λ and taxes at the maximal rate, δ (investing in A only to increase future tax revenues). The payoff to the citizens if they deviate (in some period in which $\theta = 0$) is given by:

$$\begin{aligned} U^d\left(\tilde{\tau}, \tilde{A} \mid A\right) &= \frac{1}{1-\alpha}(1-\tilde{\tau})^{(1-\alpha)/\alpha}\left(\alpha\left(1-\tilde{\tau}\right)+\eta\tilde{\tau}\right)A + \\ &\frac{\beta}{1-\alpha}(1-\delta)^{(1-\alpha)/\alpha}\left(\alpha\left(1-\delta\right)+\lambda\delta\right)\tilde{A} + \frac{\beta^2}{1-\beta}\frac{1}{1-\alpha}(1-\delta)^{(1-\alpha)/\alpha}\left(\alpha\left(1-\delta\right)+\lambda\delta\right)A\left[\delta \mid \lambda\right]. \end{aligned} \tag{33}$$

To understand this expression, note that when they deviate, the citizens obtain a fraction η of the tax revenue, $\mathcal{T}\left(\tilde{\tau}\right)$, which introduces an additional $(1-\tilde{\tau})^{(1-\alpha)/\alpha}\eta\tilde{\tau}A$ in the current period. Moreover, at the time of deviation, the ruler has already undertaken the investment in public good, $\tilde{A}$, so the MPE level of public goods, $A\left[\delta \mid \lambda\right]$, applies only from two periods on. Citizens' incentive compatibility is satisfied when they prefer to maintain a ruler who follows the agreed policy, i.e., when

$$U^c\left(\tilde{\tau}, \tilde{A} \mid \tilde{A}\right) \geq U^d\left(\tilde{\tau}, \tilde{A} \mid \tilde{A}\right). \tag{34}$$

The consensually-strong state (the best steady-state subgame perfect equilibrium from the viewpoint of the citizens) can be characterized as a solution to the following maximization problem:

$$\max_{\tilde{\tau},\tilde{A}} U^c\left(\tilde{\tau}, \tilde{A} \mid \tilde{A}\right) \tag{35}$$

subject to (31) and (34).

To characterize the equilibrium, I start with a solution in which (34) is slack, and then show that for sufficiently high values of β, in particular for $\beta \geq \beta^*$, (34) will indeed be slack. Therefore, the problem is to maximize (35) subject to (31). It is straightforward to see that the constraint (31) has to be binding (otherwise, taxes can be reduced to increase citizen utility) and that $\tilde{A} = 0$ or $\tilde{\tau} = 0$ cannot be solutions (since in the former case citizens would receive zero utility and in the latter case, given $\tilde{A} > 0$, (31) would be violated). Since both the objective function (35) and the boundary of (31) are continuously differentiable in $(\tilde{\tau}, \tilde{A})$, the first-order conditions together with a boundary condition for $\tilde{\tau} \leq \delta$ are necessary for an equilibrium.[29] The first-order conditions are given in Appendix A, and boil down to two conditions, which are shown diagrammatically in Figure 3:

$$\begin{aligned} & \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}\tilde{A}}{(1-\alpha)(1-\beta)} - \frac{1}{1-\beta}\frac{\alpha\tilde{A}^{\phi}}{(1-\alpha)\phi} \\ - & \frac{(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\delta\tilde{A}}{1-\alpha} - \frac{\beta(1-\lambda)^2(\phi-1)(1-\delta)^{\frac{1-\alpha}{\alpha}}\delta A[\delta \mid \lambda]}{(1-\beta(1-\lambda))\phi(1-\alpha)} = 0, \end{aligned} \tag{36}$$

which represents the incentive compatibility constraint of the ruler, and the condition

$$(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}(1-(1-\beta)(1-\lambda)\delta) \geq \tilde{A}^{\phi-1}, \tag{37}$$

and $\tilde{\tau} \leq \delta$ with complementary slackness. This condition captures the trade-off between taxes and public good investments from the citizens' viewpoint.

Equation (37), when holding as equality, defines the locus of combinations of $(\tilde{\tau}, \tilde{A})$ consistent with the optimal trade-off for the citizens when $\tilde{\tau} < \delta$. Since $\phi > 1$, this locus is downward sloping, drawn as the thick broken line in Figure 3. Intuitively, from citizens' viewpoint, high levels of public goods should be associated with low levels of taxes. These combinations also have to satisfy (36), which is drawn an upward-sloping curve; if the ruler is required to invest more in public goods, taxes also need to increase to ensure incentive compatibility. Appendix A shows that this locus is indeed upward sloping, and also establishes that as long as

$$\phi - 1 \geq 1 - \alpha, \tag{A4}$$

an increase in λ, which corresponds to the state becoming politically weaker, leads to higher investments in public good (i.e., $\partial\tilde{A}/\partial\lambda > 0$).

[29] Appendix A also shows that, given Assumption (A4), these first-order conditions characterize a maximum, thus an equilibrium.

Finally, we need to ensure that when policy $(\tilde{\tau}, \tilde{A})$ is followed, the incentive compatibility constraint of the citizens, (34), is satisfied. It can be proved that this will be the case as long as $\beta \geq \beta^* \equiv \left(\sqrt{1+4\lambda}-1\right)/2\lambda < 1$ (proof in Appendix A).

Proposition 4 Consider the endogenous replacement game of Section 3, and suppose that $\tilde{F}_\lambda = \tilde{F}^*_\lambda$ (i.e., $\theta = \infty$ with probability $1-\lambda$, and $\theta = 0$ with probability λ), Assumptions A2 and A4 hold, and let $\beta^* \equiv \left(\sqrt{1+4\lambda}-1\right)/2\lambda$. Then for all $\beta \geq \beta^*$, a consensually-strong state equilibrium exists. In this equilibrium, the ruler always follows the policy $(\tilde{\tau}, \tilde{A})$, and is never replaced, and taxes are lower than in the MPE, i.e., $\tilde{\tau} \leq \delta$. Moreover, as long as $\tilde{\tau} < \delta$, we have that when economic or political power of the state increases, investments in public goods decrease, i.e., $\partial\tilde{A}/\partial\lambda > 0$ and $\partial\tilde{A}/\partial\delta < 0$.

Therefore, the results with the consensually-strong state are very different from those in the previous two sections; in particular, as the economic or political power of the state decreases, investments in public goods increase, while the implications for the equilibrium tax rate are ambiguous. For example, when the state becomes politically less powerful (i.e., λ increases), the incentive compatibility constraint of the ruler, (36) shifts down as shown by the shift to the dashed curve in Figure 3. Simultaneously, the curve for (37) shifts out (again to the dashed curve). Consequently, while $\tilde{A}$ increases, the implications for $\tilde{\tau}$ are ambiguous. Intuitively, when it becomes easier to control the ruler (because deviating from the agreed policy becomes less profitable for him), citizens demand greater investments in public goods, which may necessitate greater taxes to cover the public expenditures and the rents that the ruler needs to be paid to satisfy his incentive compatibility constraint (see Acemoglu, 2005a). Similar results obtain in response to changes in the economic power of the state, δ. Interestingly, however, the comparative static with respect to δ need not hold when $\tilde{\tau} = \delta$; in this case, a decline in δ forces a lower tax rate, and investments in public goods may also need to decrease to satisfy the incentive compatibility constraint of the ruler.

These results enable us to envisage a situation similar to those in OECD countries, where the government imposes high taxes but also invests a high fraction of the proceeds in public goods. This would correspond to a high value of δ (otherwise, $\tilde{\tau} = \delta$ and taxes would be constrained to be low) and also a high value of λ (otherwise, the incentive compatibility constraint of the ruler would be excessively tight, and only low levels of investment in public goods can be supported). Naturally, for all of these outcomes the society also needs to coordinate on the consensually-strong state equilibrium, and the discount factor, β, needs to be sufficiently high.

4.2 Analysis for General $\tilde{F}_\lambda$

The analysis so far was simplified with the assumption that $\tilde{F}_\lambda = \tilde{F}^*_\lambda$. Now consider a more general $\tilde{F}_\lambda$, again parameterized as $\tilde{F}_\lambda(x/\eta) = \lambda F(x)$ and satisfying the MLR Assumption A1 above. Let us continue to look for an equilibrium in which along the equilibrium path the ruler is not replaced, and in which, following a deviation by either party, the equilibrium reverts to the MPE path.[30] In this case, the basic equations change in an intuitive way. While equations (27) and (32) still give the payoff to the ruler and the citizens from cooperation, the deviation payoff to the ruler now changes from (30) to

$$V^d(A \mid \tilde{\tau}) = (1-\lambda F(\mathcal{T}(\hat{\tau}(A)))) \left\{ \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \hat{\tau}(A) A}{1-\alpha} + \frac{\beta(1-\lambda F(\mathcal{T}(\bar{\tau})))(\phi-1)(1-\bar{\tau})^{\frac{1-\alpha}{\alpha}} A[\bar{\tau}]}{(1-\beta(1-\lambda F(\mathcal{T}(\bar{\tau}))))(1-\alpha)\phi} \right\}, \tag{38}$$

where $\hat{\tau}(A)$ is the tax rate that the ruler finds optimal upon deviation given the value of the state variable at A, and $\lambda F(\mathcal{T}(\hat{\tau}(A)))$ is the probability of replacement at this tax rate (this follows from the analysis in Section 3, since after a deviation, the economy switches to the MPE). In addition, $\bar{\tau}$, $A[\bar{\tau}]$ and $\lambda F(\mathcal{T}(\bar{\tau}))$ are the optimal continuation tax rate, investment in public goods by the ruler, and the corresponding replacement probability following a deviation. Moreover, as in Section 3, we have

$$\mathcal{T}(\tau) = \frac{(1-\tau)^{(1-\alpha)/\alpha}\tau}{1-\alpha} \text{ and } A[\bar{\tau}] \equiv \left(\alpha^{-1}\beta(1-\lambda F(\mathcal{T}(\bar{\tau})))(1-\bar{\tau})^{\frac{1-\alpha}{\alpha}}\bar{\tau}\right)^{\frac{1}{\phi-1}}.$$

Since $\hat{\tau}(A)$ is optimally chosen by the ruler, it satisfies a condition similar to (25),

$$\lambda f(\mathcal{T}(\hat{\tau}(A)))\bar{v}(\bar{\tau}, A) - (1-\lambda F(\mathcal{T}(\hat{\tau}(A)))) = 0, \tag{39}$$

where $\bar{v}(\bar{\tau}, A)$ is the expression in curly brackets in (38) divided by A.

A similar reasoning to before implies that the consensually-strong state equilibrium is given by the solution to

$$\max_{\tilde{\tau}, \tilde{A}} U^c\left(\tilde{\tau}, \tilde{A} \mid \tilde{A}\right)$$

subject to (31) and (34), with the only difference being that in these constraints $V^d\left(\tilde{A} \mid \tilde{\tau}\right)$ and $U^d\left(\tilde{\tau}, \tilde{A} \mid \tilde{A}\right)$ are now different. The first one is given by (38), while the second one is developed in Appendix A. The rest of the analysis is similar. Appendix A provides the details of the analysis and proves the following proposition:

Proposition 5 *Consider the endogenous replacement game of Section 3, and suppose that Assumptions A1, A2 and A4 hold. Then there exists $\beta^{**} < 1$ such that for $\beta \geq \beta^{**}$, a consensually-strong state equilibrium exists. In this equilibrium, the ruler always follows the policy $(\tilde{\tau}, \tilde{A})$, and is never replaced. Moreover, as long as $\tilde{\tau} < \delta$, we have that when economic or political power of the state increases, investments in public goods decrease, i.e., $\partial\tilde{A}/\partial\lambda > 0$ and $\partial\tilde{A}/\partial\delta < 0$.*

[30] When $\tilde{F}_\lambda = \tilde{F}^*_\lambda$, the MPE is the most severe credible punishment. This is typically no longer the case for general $\tilde{F}_\lambda$. Here I focus on punishment strategies that use the MPE for simplicity.

Therefore, the main thrust of the analysis remains the same when the assumption of $\tilde{F}_\lambda = \tilde{F}^*_\lambda$ is relaxed. There is nevertheless an important difference between this proposition and Proposition 4; this proposition no longer states that the consensually-strong state tax rate is below the MPE tax rate. In fact, a simple example shows that this is no longer true. Take the case where $\lambda \to \infty$; the analysis in Section 3 shows that in the MPE $\tau^* \to 0$ and $A[\tau^*] \to 0$, a very undesirable outcome from the point of view of the citizens. In contrast, with the consensually-strong state, the equilibrium tax rate always satisfies $\tilde{\tau} > 0$ (as long as $\alpha > 0$).[31]

This result is of considerable interest for the interpretation of an otherwise puzzling feature; OECD governments typically tax at higher rates than the governments of many less-developed countries.[32] This analysis shows that this need not be because governments are "politically stronger" in these more developed polities. Instead, it might be the outcome of a consensually-strong state equilibrium where politically weak governments are allowed to impose high taxes as long as a sufficient fraction of the proceeds are invested in public goods. Interestingly, the analysis also highlights that even in the consensually-strong state equilibrium, the delivery of public goods comes with significant rents for the ruler; the incentive compatibility constraint necessitates that, despite its political weakness, the ruler receive sufficient rents so that he is not tempted to use the tax revenues for his own benefit. Therefore, the image of OECD-type governments that emerges from this model is one of politically weak, but economically strong states that are allowed to impose high taxes with the (credible) promise of delivering public services. Naturally, what makes this whole equilibrium possible is sufficient rents for the politicians.

5 Conclusion

While a large body of work in economics highlights the benefits of "limited government," many political scientists view "weak states," which lack the capacity to raise sufficient revenues or regulate the economy, as the culprit in the disappointing economic performance of many less developed nations. This paper constructs a simple model where both weak and strong states create distortions. The costs of strong states are familiar in the political economy literature; the absence of checks on the redistributive power of the ruler or the political elites controlling the state apparatus creates an environment where citizens' investment and effort are discouraged. The cost of weak states are also related to the incentives of those in power; if the state is excessively weak, meaning that it is unable to capture a sufficient fraction of the society's

[31] The reason why this result did not emerge in Section 4.1 is simple: when $\tilde{F}_\lambda = \tilde{F}^*_\lambda$, the MPE tax rate takes the highest possible value, $\tau = \delta$.

[32] This discussion does not necessarily suggest that we should think of the Markov equilibrium concept applying to less-developed countries, while the subgame perfect equilibrium concept applies to OECD countries. Instead, there are various circumstances in which the subgame perfect equilibrium will be similar to the MPE. These include low rates of discounting for rulers, low values of δ (leading to excessively weak states), or very low values of λ (implying that there are no political controls on rulers).

resources, those controlling the state will have little incentive to undertake their side of the investments, for example in public goods, in infrastructure or in law enforcement. A balanced distribution of power between state and society is therefore necessary to encourage investments both by the citizens and those controlling the state apparatus.

In the model economy of the paper, the exit options of citizens (e.g., production in the informal sector) place constraints on the taxes that the government can impose. When these exit options are low, the state is *economically* strong, and citizens face excessively high tax rates stifling investment and effort. When they are high, the state is weak, and the political elites, anticipating only limited future benefits, do not undertake the necessary investments to raise the productive capacity of the economy. The "optimal" strength of the state from a second-best viewpoint depends on whether citizens' or the state's investments are more important for economic development (though there is no presumption that the actual strength of the state, determined by other political economy considerations, will come close to this optimal strength).

While a formulation where the state's strength is parameterized by its ability to raise taxes is tractable, in practice the strength of the state depends on the political constraints placed on it by various groups in society. The second part of the paper analyzes the trade-off between *politically* weak and strong states. Citizens can replace the ruler when he pursues policies that are not in their interest. When the state is politically weak, it cannot impose high taxes, and anticipating this, the ruler invests little in public goods. Consequently, the same trade-off between economically weak and strong states also arises between politically weak and strong states.

The contrast between weak and strong states highlighted by these models does not, however, provide us with a framework for thinking about the role of the state in many OECD nations where the state appears politically weak (in the sense that political elites can be replaced easily), imposes high taxes, but then invests a high fraction of the revenues in public goods. In the last part of the paper, I show how an equilibrium of this sort, which I dub the "consensually-strong state" equilibrium, can emerge when citizens accept high taxes as long as there is a credible promise that a sufficient fraction of these will be invested in public goods. This equilibrium is made possible by the fact that the state is politically weak, so the elites can be replaced easily if the ruler deviates from the prescribed behavior.

This paper is a first attempt to develop a framework for understanding the trade-offs created by weak and strong states. As such, it abstracts from many important aspects of the question at hand. The most important omission relates to the sources of the constraints on states' power; the strength of the state in many less developed nations is not limited by the power of the citizens, but by some other privileged social group, such as tribal chiefs, various strongmen or sometimes groups of wealthy landowners. The costs and benefits of weak states in societies where there are multiple cleavages, for example, between the state and society and/or between rich and poor agents within the society remains an area for future research. Another important area for future study is an empirical investigation of trade-offs between weak and strong states, and whether there are certain types of societies, for example those at the earlier stages of development, where weak states are more costly for economic prosperity.

6 Appendix A: Proofs

First-Order Conditions in Section 4.1: Let us denote the Lagrangian function by $\mathcal{L}$ and the multiplier for (31) by μ^V, and write the first-order conditions as follows:

$$\frac{\partial \mathcal{L}}{\partial \tilde{A}} = \frac{1}{1-\beta}\frac{\alpha}{1-\alpha}(1-\tilde{\tau})^{\frac{1}{\alpha}} + \mu^V \left[\frac{1}{1-\beta}\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}}{1-\alpha} - \frac{1}{1-\beta}\frac{\alpha \tilde{A}^{\phi-1}}{1-\alpha} - \frac{(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\delta}{1-\alpha} \right] = 0. \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\tau}} = -\frac{1}{1-\beta}\frac{1}{1-\alpha}(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{A} + \tag{41}$$

$$\mu^V \left[\frac{1}{1-\beta}\frac{1}{1-\alpha}((1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} - \frac{1-\alpha}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\tilde{\tau})\tilde{A} + \frac{1-\lambda}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\delta\tilde{A} \right] \geq 0 \text{ and } \tilde{\tau} \leq \delta,$$

with complementary slackness. Finally,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mu^V} &= \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}\tilde{A}}{(1-\alpha)(1-\beta)} - \frac{1}{1-\beta}\frac{\alpha\tilde{A}^{\phi}}{(1-\alpha)\phi} \\ &- \frac{(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\delta\tilde{A}}{1-\alpha} - \frac{\beta(1-\lambda)^2(\phi-1)(1-\delta)^{\frac{1-\alpha}{\alpha}}\delta A[\delta \mid \lambda]}{(1-\beta(1-\lambda))\phi(1-\alpha)} = 0. \end{aligned} \tag{42}$$

Combining (40) and (41) gives (37), while (42) gives (36).

Proof of Proposition 4: The objective function is continuous. In addition, from (37), $0 \leq \tilde{A} \leq A^{\max} \equiv ((1-(1-\beta)(1-\lambda)\delta))^{1/(\phi-1)}$, so $\left(\tilde{\tau}, \tilde{A}\right) \in [0,\delta] \times [0, A^{\max}]$. Moreover, (36) and (37) define a compact non-empty constraint set, so a solution to the maximization problem, thus an equilibrium, exists.

The comparative static results for the case where $\tilde{\tau} < \delta$ follow from equations (37) and (42). For example, for those with respect to λ, totally differentiate these equations, and use matrix notation:

$$\mathbf{B}\begin{pmatrix} d\tilde{A} \\ d\tilde{\tau} \end{pmatrix} = \mathbf{c}_\lambda d\lambda$$

with

$$\mathbf{B} = \begin{pmatrix} -(\phi-1)\alpha\tilde{A}^{\phi-2} & \begin{array}{c} -\frac{1-\alpha}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}} \times \\ (1-(1-\beta)(1-\lambda)\delta) \end{array} \\ \begin{array}{c} \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}}{(1-\alpha)(1-\beta)} - \frac{\alpha\tilde{A}^{\phi-1}}{(1-\alpha)(1-\beta)} \\ -\frac{(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\delta}{(1-\alpha)} \end{array} & \begin{array}{c} \frac{1}{1-\beta}\frac{1}{1-\alpha}\frac{\alpha-\tilde{\tau}}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\tilde{A} \\ +\frac{1-\lambda}{\alpha}\delta(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\tilde{A} \end{array} \end{pmatrix}$$

and

$$\mathbf{c}_\lambda = \begin{pmatrix} -(1-\beta)\delta(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \\ -\delta\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}}{1-\alpha}\tilde{A} + \frac{\partial \ell(\lambda,\delta)}{\partial \lambda} \end{pmatrix}$$

where $\ell(\lambda,\delta) \equiv \frac{\beta(1-\lambda)^2(\phi-1)(1-\delta)^{\frac{1-\alpha}{\alpha}}\delta A[\delta|\lambda]}{(1-\beta(1-\lambda))(1-\alpha)\phi}$ is the second term in the deviation payoff of the ruler in equation (30), with $\partial\ell(\lambda,\delta)/\partial\lambda < 0$.

By Cramer's rule,

$$\frac{d\tilde{A}}{d\lambda} = \frac{|\mathbf{B}_1|}{|\mathbf{B}|},$$

where $\mathbf{B}_1$ is the matrix obtained by replacing the first column of $\mathbf{B}$ by $\mathbf{c}_\lambda$.

$$\begin{aligned}
|\mathbf{B}| \;=\;& -(1-\beta)^{-1}\alpha^{-1}(1-\tilde{\tau})^{\frac{2-3\alpha}{\alpha}} \times \\
& [1-(1-\lambda)(1-\beta)\delta] \times [(1-\lambda)(1-\beta)\delta - \tilde{\tau}] \\
& -(1-\beta)^{-1}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\tilde{A}^{\phi-1} \times \\
& \left[(\phi-1)\frac{\alpha-\tilde{\tau}}{1-\alpha} + (\phi-1)(1-\lambda)(1-\beta)\delta - \alpha + \alpha(1-\lambda)(1-\beta)\delta\right].
\end{aligned}$$

Now substituting for $\tilde{A}$ from (37), this can be written as:

$$\begin{aligned}
|\mathbf{B}| \;=\;& (1-\beta)^{-1}\alpha^{-1}(1-\tilde{\tau})^{\frac{2-3\alpha}{\alpha}} \times [1-(1-\lambda)(1-\beta)\delta] \times \\
& \left[(\phi-1)\frac{\alpha-\tilde{\tau}}{1-\alpha} + (\phi-1)(1-\lambda)(1-\beta)\delta - \alpha + \alpha(1-\lambda)(1-\beta)\delta + \tilde{\tau} - (1-\lambda)(1-\beta)\delta\right]. \\
=\;& (1-\beta)^{-1}\alpha^{-1}(1-\tilde{\tau})^{\frac{2-3\alpha}{\alpha}} \times [1-(1-\lambda)(1-\beta)\delta] \times \\
& \left[(\alpha-\tilde{\tau})\left(\frac{\phi-1}{1-\alpha}-1\right) + (1-\alpha)(1-\lambda)(1-\beta)\delta\left(\frac{\phi-1}{1-\alpha}-1\right)\right]
\end{aligned}$$

Since $\tilde{\tau} < \delta$, Assumption A4 is sufficient to ensure that $(\phi-1)/(1-\alpha) > 1$, so that $|\mathbf{B}| < 0$. $|\mathbf{B}| < 0$ is also equivalent to the second-order condition for a maximum, so Assumption A4 is also sufficient to ensure that the stationary point we characterized is a maximum.

$|\mathbf{B}_1|$ is more straightforward:

$$\begin{aligned}
|\mathbf{B}_1| \;=\;& -\delta(1-\beta)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \\
& \times\left[\frac{\alpha-\tilde{\tau}}{\alpha(1-\alpha)}\frac{(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}}{1-\beta}\tilde{A} + \frac{1-\lambda}{\alpha}\delta(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\tilde{A}\right] \\
& -\left[\frac{1}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\right] \times [1-(1-\lambda)(1-\beta)\delta] \\
& \times\left[\delta(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{A} - (1-\alpha)\frac{\partial\ell(\lambda,\delta)}{\partial\lambda}\right].
\end{aligned}$$

Using the fact that $\partial\ell(\lambda,\delta)/\partial\lambda < 0$, this immediately implies $|\mathbf{B}_1| < 0$, so that

$$\frac{d\tilde{A}}{d\lambda} = \frac{|\mathbf{B}_1|}{|\mathbf{B}|} > 0.$$

A similar argument shows that when $\tilde{\tau} < \delta$,

$$\frac{d\tilde{A}}{d\delta} < 0.$$

To see this, it suffices to note that in this case

$$\mathbf{B}\begin{pmatrix} d\tilde{A} \\ d\tilde{\tau} \end{pmatrix} = \mathbf{c}_\delta d\delta,$$

and

$$\mathbf{c}_\delta = \begin{pmatrix} (1-\beta)(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} \\ \\ \frac{(1-\lambda)(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}}{1-\alpha}\tilde{A} + \frac{\partial \ell(\lambda,\delta)}{\partial \delta} \end{pmatrix},$$

and $\partial \ell(\lambda,\delta)/\partial\delta > 0$. The same steps immediately imply that $|\mathbf{B}_1| > 0$, so $d\tilde{A}/d\delta < 0$.

Finally, we have to check that the citizens' incentive compatibility constraint, (34), holds. Rewriting this as:

$$\begin{aligned} \frac{1}{1-\beta}\frac{\alpha}{1-\alpha}(1-\tilde{\tau})^{1/\alpha}\tilde{A} \quad \geq \quad & \frac{1}{1-\alpha}(1-\tilde{\tau})^{(1-\alpha)/\alpha}\left(\alpha(1-\tilde{\tau})+\eta\tilde{\tau}\right)\tilde{A} + \\ & \frac{\beta}{1-\alpha}(1-\delta)^{(1-\alpha)/\alpha}\left(\alpha(1-\delta)+\lambda\eta\delta\right)\tilde{A} \\ & +\frac{\beta^2}{1-\beta}\frac{1}{1-\alpha}(1-\delta)^{(1-\alpha)/\alpha}\left(\alpha(1-\delta)+\lambda\eta\delta\right)A[\delta \mid \lambda]. \end{aligned}$$

Dividing by $\tilde{A}$ and multiplying by $(1-\alpha)(1-\beta)$, we have

$$\begin{aligned} \beta\alpha(1-\tilde{\tau})^{1/\alpha} \quad \geq \quad & (1-\beta)(1-\tilde{\tau})^{(1-\alpha)/\alpha}\eta\tilde{\tau} + \beta(1-\beta)(1-\delta)^{(1-\alpha)/\alpha}\left(\alpha(1-\delta)+\lambda\eta\delta\right) \\ & +\beta^2(1-\delta)^{(1-\alpha)/\alpha}\left(\alpha(1-\delta)+\lambda\eta\delta\right)\frac{A[\delta \mid \lambda]}{\tilde{A}}. \end{aligned}$$

Now using the facts that $\tilde{\tau} < \delta$, that $(1-\tilde{\tau})^{(1-\alpha)/\alpha}\tilde{\tau}$ is increasing in $\tilde{\tau}$ for $\tilde{\tau} < \alpha$, and $A[\delta \mid \lambda] \leq \tilde{A}$, and rearranging, we obtain the following sufficient condition for (34) to hold:

$$\beta^2\lambda\eta\delta \geq (1-\beta)\eta\delta.$$

This condition is satisfied and (34) holds for sure, as long as

$$\beta \geq \beta^* \equiv \frac{\sqrt{1+4\lambda}-1}{2\lambda} < 1.$$

This establishes that $\beta \geq \beta^*$ is sufficient for this form of the equilibrium to exist and for the comparative static results. **QED**

Proof that (36) is upward sloping in the $\left(\tilde{\tau}, \tilde{A}\right)$ space: Note that the left-hand side of (36) is always increasing in $\tilde{\tau}$ and a strictly concave function of $\tilde{A}$. Moreover, it is maximized at $\hat{A}[\tilde{\tau}] = \left(\alpha^{-1}(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}[\tilde{\tau}-(1-\lambda)(1-\beta)\delta]\right)^{1/(\phi-1)}$. (37) implies that when $\tilde{\tau} < \delta$, $\tilde{A} = \left((1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}[1-(1-\lambda)(1-\beta)\delta]\right)^{1/(\phi-1)}$. Since $\tilde{\tau} < \alpha$ and $(1-\lambda)(1-\beta)\delta > \alpha(1-\lambda)(1-\beta)\delta$, this implies $[1-(1-\lambda)(1-\beta)\delta] > \alpha^{-1}[\tilde{\tau}-(1-\lambda)(1-\beta)\delta]$, and therefore $\tilde{A} > \hat{A}[\tilde{\tau}]$. Since (36) is strictly concave with a unique maximum at $\hat{A}[\tilde{\tau}]$ and $\tilde{A} > \hat{A}[\tilde{\tau}]$, it is decreasing in $\tilde{A}$ (in the neighborhood of the values for $\tilde{A}$ consistent with (37)), and consequently, (36) traces an upward sloping curve in the $\left(\tilde{\tau}, \tilde{A}\right)$ space.**QED**

Proof of Proposition 5: The analysis follows the one for the case where $F_\lambda = F^*_\lambda$. In particular, the incentive compatibility constraint for the citizens can be formulated by only checking their payoff from deviating when $\theta = 0$. This is given by

$$\begin{aligned} U^d\left(\tilde{\tau}, \tilde{A} \mid \tilde{A}\right) &= \frac{1}{1-\alpha}(1-\tilde{\tau})^{(1-\alpha)/\alpha}\left(\alpha\left(1-\tilde{\tau}\right)+\eta\tilde{\tau}\right)\tilde{A} + \qquad (43) \\ &\frac{\beta}{1-\alpha}(1-\bar{\tau}(\tilde{A}))^{(1-\alpha)/\alpha}\left(\alpha\left(1-\bar{\tau}(\tilde{A})\right)+\lambda F\left(\mathcal{T}\left(\bar{\tau}(\tilde{A})\right)\right)\bar{\tau}(\tilde{A})\eta\right)\tilde{A} - \lambda\int_0^{\mathcal{T}\left(\bar{\tau}(\tilde{A})\right)}\theta\tilde{A}dF(\theta) \\ &+\frac{\beta^2}{1-\beta}\frac{1}{1-\alpha}(1-\bar{\tau})^{\frac{1-\alpha}{\alpha}}\left(\alpha(1-\bar{\tau})+\lambda F\left(\mathcal{T}\left(\bar{\tau}\right)\right)\bar{\tau}\eta\right)A[\bar{\tau}] - \lambda\int_0^{\mathcal{T}(\bar{\tau})}\theta A[\bar{\tau}]dF(\theta), \end{aligned}$$

where $\bar{\tau}(A)$ denotes the best response of a ruler after a deviation by the citizens when public goods are equal to A, and the integral terms incorporate the expected cost of replacing the ruler.

Let us first ignore this incentive compatibility constraint. Then the first-order conditions for the new maximization problem are similar to (40), (41) and (42). Since $\hat{\tau}(\tilde{A})$ is chosen optimally for the ruler from (39), we can use the envelope theorem to simplify the effect of $\tilde{A}$ on the incentive compatibility constraint of the ruler. In particular, we have

$$\frac{\partial\mathcal{L}}{\partial\tilde{A}} = \frac{1}{1-\beta}\frac{\alpha}{1-\alpha}(1-\tilde{\tau})^{\frac{1}{\alpha}}+\mu^V\left[\frac{1}{1-\beta}\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}}{1-\alpha} - \frac{1}{1-\beta}\frac{\alpha\tilde{A}^{\phi-1}}{1-\alpha} - \frac{(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\delta}{1-\alpha}\right] = 0, \tag{44}$$

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial\tilde{\tau}} &= -\frac{1}{1-\beta}\frac{1}{1-\alpha}(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{A} + \qquad (45) \\ &\mu^V\left[\frac{1}{1-\beta}\frac{1}{1-\alpha}((1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}} - \frac{1-\alpha}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\tilde{\tau})\tilde{A} + \frac{1-\lambda F\left(\hat{\tau}(\tilde{A})\right)}{\alpha}(1-\tilde{\tau})^{\frac{1-2\alpha}{\alpha}}\hat{\tau}(\tilde{A})\tilde{A}\right] \geq 0 \end{aligned}$$

and $\tilde{\tau} \leq \delta$, with complementary slackness. Finally,

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial\mu^V} &= \frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}\tilde{A}}{(1-\alpha)(1-\beta)} - \frac{1}{1-\beta}\frac{\alpha\tilde{A}^\phi}{(1-\alpha)\phi} \\ &- \frac{(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\hat{\tau}(\tilde{A})\tilde{A}}{1-\alpha} \\ &- \frac{\beta(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))(1-\lambda F(\bar{\tau}))(\phi-1)(1-\bar{\tau})^{\frac{1-\alpha}{\alpha}}\bar{\tau}A[\bar{\tau}]}{(1-\beta(1-\lambda F(\bar{\tau})))\phi(1-\alpha)} = 0. \end{aligned}$$

With an analysis similar to before, we obtain the following two equations characterizing an interior solution:

$$(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\left(1-(1-\beta)\left(1-\lambda F\left(\hat{\tau}(\tilde{A})\right)\right)\hat{\tau}(\tilde{A})\right) - \tilde{A}^{\phi-1} = 0,$$

$$\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}\tilde{A}}{1-\alpha}-\frac{1}{1-\beta}\frac{\alpha\tilde{A}^{\phi}}{(1-\alpha)\phi}-$$
$$\frac{(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\hat{\tau}(\tilde{A})\tilde{A}}{1-\alpha}-\frac{\beta(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))(1-\lambda F\left(\bar{\tau}\right))(\phi-1)(1-\bar{\tau})^{\frac{1-\alpha}{\alpha}}\bar{\tau}A[\bar{\tau}]}{(1-\beta(1-\lambda F\left(\bar{\tau}\right)))\phi(1-\alpha)}=0.$$

The rest of the analysis is similar. In particular, we can again write:

$$\mathbf{B}\begin{pmatrix} d\tilde{A} \\ d\tilde{\tau} \end{pmatrix}=\mathbf{c}_{\lambda}d\lambda,$$

and using the envelope theorem,

$$\mathbf{B}=\begin{pmatrix} -(\phi-1)\,\alpha\tilde{A}^{\phi-2} & \begin{array}{c}-\frac{1-\alpha}{\alpha}\left(1-\tilde{\tau}\right)^{\frac{1-2\alpha}{\alpha}}\times \\ \left(1-\left(1-\beta\right)\left(1-\lambda F\left(\hat{\tau}(\tilde{A})\right)\right)\hat{\tau}(\tilde{A})\right)\end{array} \\ \begin{array}{c}\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}\tilde{\tau}}{(1-\alpha)(1-\beta)}-\frac{\alpha\tilde{A}^{\phi-1}}{(1-\alpha)(1-\beta)} \\ -\frac{(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))\hat{\tau}(\tilde{A})(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}}{(1-\alpha)}\end{array} & \begin{array}{c}\frac{1}{1-\beta}\frac{1}{1-\alpha}\frac{\alpha-\tilde{\tau}}{\alpha}\left(1-\tilde{\tau}\right)^{\frac{1-2\alpha}{\alpha}}\tilde{A} \\ +\frac{(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))\hat{\tau}(\tilde{A})}{\alpha}\left(1-\tilde{\tau}\right)^{\frac{1-2\alpha}{\alpha}}\tilde{A}\end{array} \end{pmatrix}$$

and

$$\mathbf{c}_{\lambda}=\begin{pmatrix} -\left(1-\beta\right)\lambda\hat{\tau}(\tilde{A})\hat{\tau}(\tilde{A})\left(1-\tilde{\tau}\right)^{\frac{1-\alpha}{\alpha}} \\ -\frac{(1-\tilde{\tau})^{\frac{1-\alpha}{\alpha}}F\left(\hat{\tau}(\tilde{A})\right)\hat{\tau}(\tilde{A})}{1-\alpha}\tilde{A}+\frac{\partial\bar{\ell}(\lambda,\delta)}{\partial\lambda} \end{pmatrix}$$

where now $\bar{\ell}\left(\lambda,\delta\right)\equiv\frac{\beta(1-\lambda F\left(\hat{\tau}(\tilde{A})\right))(1-\lambda F(\bar{\tau}))(\phi-1)(1-\bar{\tau})^{\frac{1-\alpha}{\alpha}}\bar{\tau}A[\bar{\tau}]}{(1-\beta(1-\lambda F(\bar{\tau})))\phi(1-\alpha)}$, so $\partial\bar{\ell}\left(\lambda,\delta\right)/\partial\lambda<0$. Applying Cramer's rule again gives identical expressions and establishes $d\tilde{A}/d\lambda>0$. The analysis leading to $d\tilde{A}/d\delta<0$ is identical.

Finally, we also still have $\lim_{\beta\to1}U^{c}\left(\tilde{\tau},\tilde{A}\mid\tilde{A}\right)>\lim_{\beta\to1}U^{d}\left(\tilde{\tau},\tilde{A}\mid\tilde{A}\right)$, thus $\beta^{**}<1$ exist such that (34) is slack at the solution.**QED**

7 Appendix B: Endogenous Size of Government

The analysis in the text related the output and efficiency to the distribution of rents between the state and the citizens (the economic strength of the state) is captured by the parameter δ. But what does δ correspond to in reality? Here, I give a brief analysis of the situation where δ is related to the size of government, and the involvement of the government in the economy.

Suppose that there are $[0,1]$ sectors, and individuals randomly choose which sector to produce in. The output of all sectors are perfect substitutes, and the production function is still given by (2). The government can only tax the sectors that it inspects or controls. Intuitively, if there is no government inspection or government involvement in a particular sector, citizens can hide their output effectively.

The government chooses which sectors to inspect, with a restriction that one employee can inspect one sector. So the size of the government will determine δ. In particular, in equilibrium agents will distribute themselves evenly across sectors and the government will randomize over which sectors to inspect, so

$$\delta = n_t,$$

where n_t is the number of agents hired by the government at time t.[33]

Since all agents are risk neutral, Propositions 1 and 2 from Section 2 continue to apply. To simplify the analysis, assume that the size of the government is chosen at the beginning of every period independently from past sizes of government.

Then the maximization problem of the government can be written as:

$$V\left(A_t\right) = \max_{n_t, A_{t+1}} \left\{ T\left(A_t, n_t\right) - w\left(A_{t,} n_t\right) n_t - \frac{\alpha}{\phi(1-\alpha)} A_{t+1}^{\phi} + \beta V\left(A_{t+1}\right) \right\}$$

where $w\left(A_{t,} n_t\right)$ is the wage that the government has to pay to agents to convince them to work in the public sector.

Let us first assume that, in this maximization problem, the government treats the wage $w\left(A_{t,} n_t\right)$ as a constant at ω (or alternatively, the wage is exogenously given at ω). I endogenize the wage later and compare the optimal value of n_t with the result for the constant case.

Since the choice of n_t does not affect the choice of A_{t+1}, the first-order condition for the case of constant wage is:

$$\frac{(1-n_t^c)^{(1-\alpha)/\alpha}}{1-\alpha} A_t \frac{(\alpha - n_t^c)}{\alpha(1-n_t^c)} = \omega. \tag{46}$$

[33] Notice that there is a very large number of citizens (a continuum of them), so any mixed strategy will lead to a deterministic allocation of agents across sectors. An allocation in which agents distribute themselves unevenly across the sectors cannot be an equilibrium, since it would be a best response for the government to inspect the sectors where there are more agents with a greater probability, making it suboptimal for them to enter these sectors. This leaves the allocation considered in the text. In principle, if we consider correlated equilibria, there can be other equilibrium allocations as well, but these would be equivalent to the one considered in the text, because they would have exactly the same expected tax revenue and the ruler's utility is linear in revenue.

Now to make the comparison, suppose that $\omega = w\left(A_{t,}n_t^c\right)$, where

$$w\left(A_{t,}n_t\right) = \frac{\alpha}{1-\alpha}\left(1-n_t\right)^{1/\alpha} A_t \tag{47}$$

is the wage that will make individuals indifferent between entering public employment and working as producers.

Combining (46) and (47), we obtain n_t^c as:

$$\frac{(1-n_t^c)^{(1-\alpha)/\alpha}}{1-\alpha}\frac{(\alpha-n_t^c)}{\alpha(1-n_t^c)} = \frac{\alpha}{1-\alpha}\left(1-n_t^c\right)^{1/\alpha}. \tag{48}$$

Let us now contrast this to the case where the wage is endogenous, given by (47), and the dependence of the wage on the level of public employment is recognized by the ruler in his maximization decision. This then gives the first-order condition:

$$\frac{(1-n_t^*)^{(1-\alpha)/\alpha}}{1-\alpha}\frac{(\alpha-n_t^*)}{\alpha(1-n_t^*)} = \frac{\alpha}{1-\alpha}\left(1-n_t^*\right)^{1/\alpha} - \frac{(1-n_t^*)^{(1-\alpha)/\alpha}}{1-\alpha}n_t^*. \tag{49}$$

Comparing this expression with (48) immediately implies that $n_t^* > n_t^c$: when the endogeneity of the wage rate is taken into account, there is an additional negative term on the right-hand side, indicating that the effective cost to the ruler of hiring one more worker into the public sector is lower, and thus his demand for labor is higher with an endogenous wage. This is because of a "reverse monopsony" problem. By hiring one more worker, the ruler is increasing taxes, and this depresses earnings in the private sector. Recognizing this, the effective cost of hiring one more worker is lower, since it reduces the public wage bill. Of course, this reverse monopsony problem is inefficient: as the ruler hires more employees, and taxes increase, investment in the private sector decreases.

We saw in the text section how the level of δ, the "organization of society", most preferred by the ruler, δ^r, is greater than that preferred by a net output maximizing social planner, δ^{wm}, thus loosely speaking inefficiently high. The analysis in this Appendix identifies another potential inefficiency that will arise when a self-interested ruler controls the organization of society, not by directly choosing δ, but by deciding the size of the public sector, which influences δ. Because of the reverse monopsony problem, whereby the opportunity cost of hiring more workers into public employment declines in the size of public employment, the ruler will choose an even larger level of δ and public employment.

8 References

Acemoglu, Daron (2003) "Why Not a Political Coase Theorem?" *Journal of Comparative Economics*, 31, pp 620-652.

Acemoglu, Daron (2005a) "The Rise of the Strong State" in progress.

Acemoglu, Daron and James A. Robinson (2002) "Economic Backwardness In Political Perspective" NBER Working Paper 8831.

Acemoglu, Daron, Simon Johnson and James A. Robinson (2001) "Colonial Origins of Comparative Development: An Empirical Investigation," *American Economic Review*, 91, pp. 1369-1401.

Acemoglu, Daron, Simon Johnson, and James A. Robinson (2002) "Reversal of Fortune: Geography and Institutions in the Making of the Modern World Income Distribution," *Quarterly Journal of Economics* vol 117, November, 1231-1294.

Acemoglu, Daron, James A. Robinson and Thierry Verdier (2004) "Kleptocracy and Divide-and-Rule: A Model of Personal Rule, The Alfred Marshall Lecture", *Journal of the European Economic Association Papers and Proceedings,* April-May 2004, volume 2, pp 162-192.

Aghion, Philippe, Alberto Alesina and Francesco Trebbi (2004) "Endogenous Political Institutions" *Quarterly Journal of Economics*, volume 119, 527-565.

Alesina, Alberto and Dani Rodrik (1994) "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 109, 465-490.

Aldrich, John H. (1983) "A Downsian Spatial Model with Party Activism." *American Political Science Review* 77: 974-90.

Atkinson, Anthony B. and Joseph E. Stiglitz (1980) *Lectures on Public Economics*, McGraw-Hill Book Co., New York.

Austen-Smith, David, and Jeffrey S. Banks (1999) *Positive Political Theory I: Collective Preference*. Ann Arbor MI: University of Michigan Press.

Baron, David P., and John Ferejohn (1989) "Bargaining in Legislatures." *American Political Science Review* 83: 1181-206.

Barro, Robert (1990) "Government Spending in a Simple Model of Endogenous Growth" *Journal of Political Economy*, 98, S103-125.

Barro, Robert (1999) *Determinants of Economic Growth: A Cross-country Empirical Study,* MIT University Press, Cambridge MA.

Barro, Robert and Xavier Sala-i-Martin (1992) "Public Finance Models of Economic Growth" *Review of Economic Studies*, 59, 645-661.

Bates, Robert H. (1981) *Markets and States in Tropical Africa*, University of California Press, Berkeley CA.

Bates, Robert H. (2001) *Prosperity and Violence,* Cambridge; Harvard University Press.

Besley, Timothy, and Stephen Coate (1998) "Sources of Inefficiency in a Representative Democracy: a Dynamic Analysis" *American Economic Review,* 88 139-156.

Benhabib, Jess, Aldo Rustichini, and Andres Velasco (2001) "Public Spending and Optimal Taxes without Commitment" *Review of Economic Design*, 6,371-96.

Buchanan, James M., and Gordon Tullock (1962) *The Calculus of Consent.* Ann Arbor MI: University of Michigan Press.

Bueno De Mesquita, Bruce, Alastair Smith, Randolph Siverson, and James D. Morrow (2003) *The Logic of Political Survival*, MIT University Press, Cambridge.

Burdett, Kenneth (1979) "A useful restriction on the offer distribution in job search models," in Gunnar Eliasson, Bertil Holmlund, and Frank P. Stafford (eds.) *Studies in labor market behavior: Sweden and the United States.* Stockholm: IUI Conference Report

Dixit, Avinash (2004) *Lawlessness and Economics: Alternative Modes of Governance*, Princeton University Press, Princeton.

Dixit, Avinash and John Londregan (1995) "Redistributive Politics and Economic Efficiency," *American Political Science Review*, 89, 856-866.

Evans, Peter, (1995) *Embedded Autonomy: States and Industrial Transformation*, Princeton University Press.

Ferejohn, John (1986) "Incumbent Performance and Electoral Control," *Public Choice* 50: 5-25.

Grossman, Herschel I. (1991) "A General Equilibrium Model of Insurrections," *American Economic Review*, 81, pp. 912-921.

Grossman, Herschel I. and S.J. Noh (1994) "Proprietary Public Finance and Economic Welfare," *Journal of Public Economics*, 53, pp. 187-204.

Grossman, Sandford and Oliver Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy,* 94, 691-719.

Hall, Robert E. and Charles I. Jones (1999) "Why Do Some Countries Produce so Much More Output per Worker than Others?," *Quarterly Journal of Economics*, CXIV, 83-116.

Hart, Oliver (1995) *Firms, Contracts, and Financial Structure,* Oxford University Press.

Hart, Oliver and John Moore (1990) "Property Rights and the Nature of the Firm" *Journal of Political Economy* Vol 98, pp 1119-1158.

Herbst, Jeffery I. (2000) *States and Power in Africa: Comparative Lessons in Authority and Control,* Princeton University Press, Princeton NJ.

Huer, Jon (1989) *Marching Orders: The Role of the Military in South Korea's Economic Miracle, 1961-1971*, Greenwood Press, New York.

Huntington, Samuel (1968) *Political Order in Changing Societies*, Yale University Press, New Haven.

Jones, Eric L. (1981) *The European Miracle: Environments, Economies, and Geopolitics in the History of Europe and Asia*, Cambridge University Press, New York.

Kang, David (2002) *Crony Capitalism: Corruption and Development and South Korea in the Philippines,* Cambridge, Cambridge University Press.

Knack, Steven and Philip Keefer (1995) "Institutions and Economic Performance: Cross-Country Tests Using Alternative Measures," *Economics and Politics, 7, 207-227.*

Krusell, Per and José-Victor Ríos-Rull (1996) "Vested Interests in a Theory of Growth and Stagnation," *Review of Economic Studies*, 63, 301-329.

La Ferrara, Eliana and Robert H. Bates (2001) "Political Competition in Weak States" *Economics and Politics*, 13,159-84.

Lizzeri, Alessandro and Nichola Persico (2001) "The Provision of Public Goods under Alternative Electoral Incentives" *American Economic Review,* 91,221-35.

McGuire, Martin C. and Mancur C. Olson (1996) "The Economics of Autocracy and Majority Rule," *Journal of Economic Literature*, 34, pp. 72-96.

Meltzer, Allan H. and Scott Richard (1981) "A Rational Theory of the Size of Government" *Journal of Political Economy* volume 89, #5, 914-927.

Migdal, Joel (1988) *Strong Societies and Weak States: State-Society Relations and State Capabilities in the Third World*, Princeton University Press, Princeton.

Myerson, Roger (1995) "Analysis of Democratic Institutions: Structure, Conduct and Performance" *Journal of Economic Perspectives* 9: 77-89.

North, Douglass C. (1981) *Structure and Change in Economic History*, New York: W.W. Norton & Co.

North, Douglass C. and Barry R. Weingast (1989) "Constitutions and Commitment: The Evolution of Institutions Governing Public Choice in Seventeenth-Century England" *Journal of Economic History*, 49, pp. 803-832.

Olson, Mancur C. (1965) *The Logic of Collective Action; Public Goods and the Theory of Groups*, Cambridge: Harvard University Press.

Olson, Mancur (1982) *The Rise and Decline of Nations: Economic Growth, Stagflation, and Economic Rigidities,* Yale University Press, New Haven and London.

Olson, Mancur C. (2000) *Power and Prosperity*, New York: Basic Books.

Person, Torsten, Gerard Roland and Guido Tabellini (1997) "Separation of Powers and Political Accountability" *Quarterly Journal of Economics*, volume 112, 1163-1202.

Persson, Torsten and Guido Tabellini (1994) "Is Inequality Harmful for Growth? Theory and Evidence," *American Economic Review*, 84, 600-621.

Persson, Torsten and Guido Tabellini (2000) *Political Economics: Explaining Economic Policy*, Cambridge: MIT Press.

Reno, William (1999) *Warlord Politics and African States*, Lynn Weiner Publishers, Boulder London.

Stokey, Nancy, Robert E. Lucas and Edward Prescott (1989) *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge.

Tilly, Charles (1990) *Coercion, Capital and European States, AD 990-1990*, Blackwell, Cambridge MA.

Tullock, Gordon (1983) *The Economics of Income Redistribution*, Kluwer-Nijhoff, Boston MA.

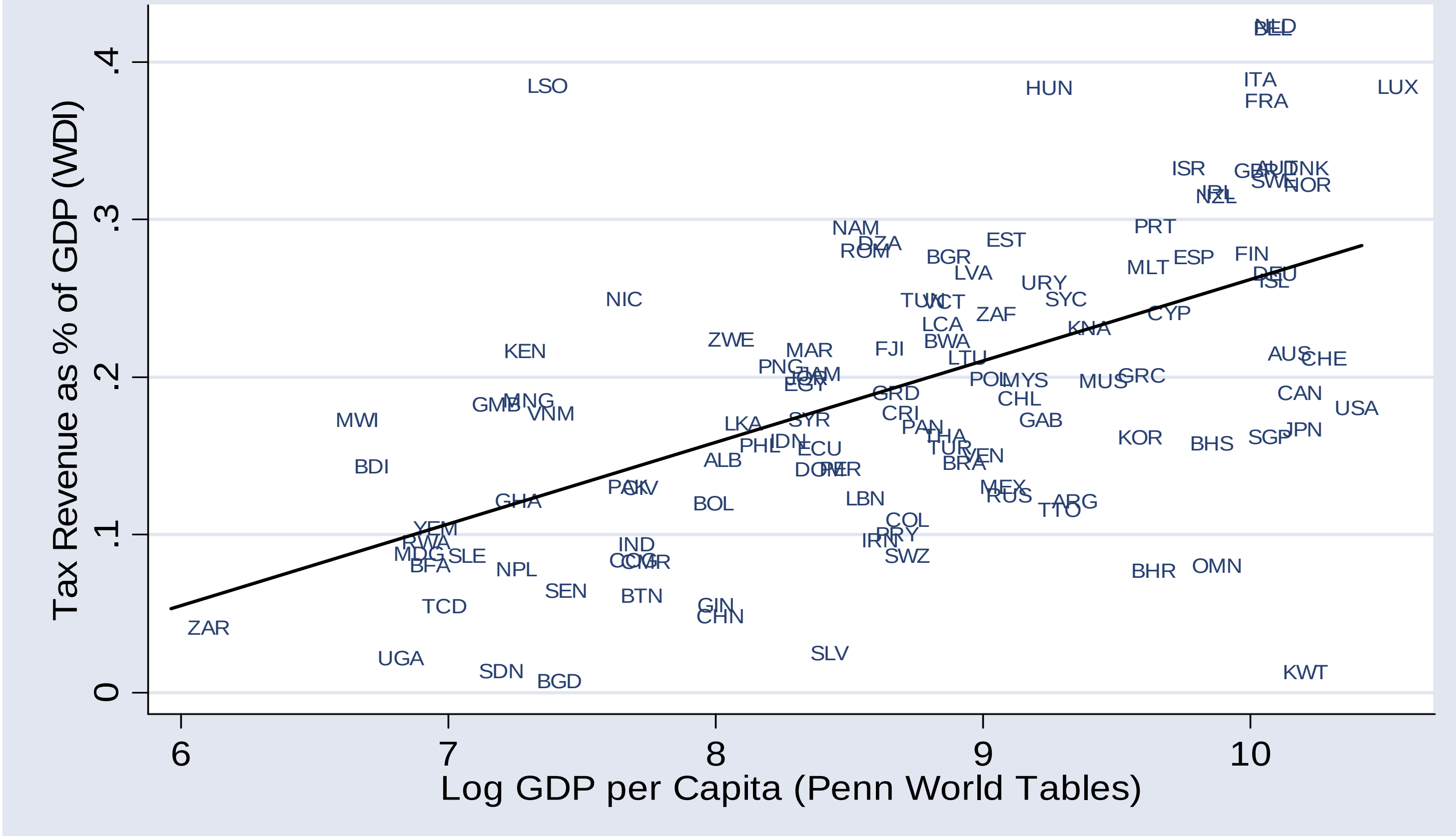
Wade, Robert (1990) *Governing the Market: Economic Theory and the Role of Government in East Asian Industrialization*, Princeton, Princeton University Press.

Williamson, Oliver (1975) *Markets and Hierarchies: Analysis and Antitrust Implications,* Free Press, New-York.

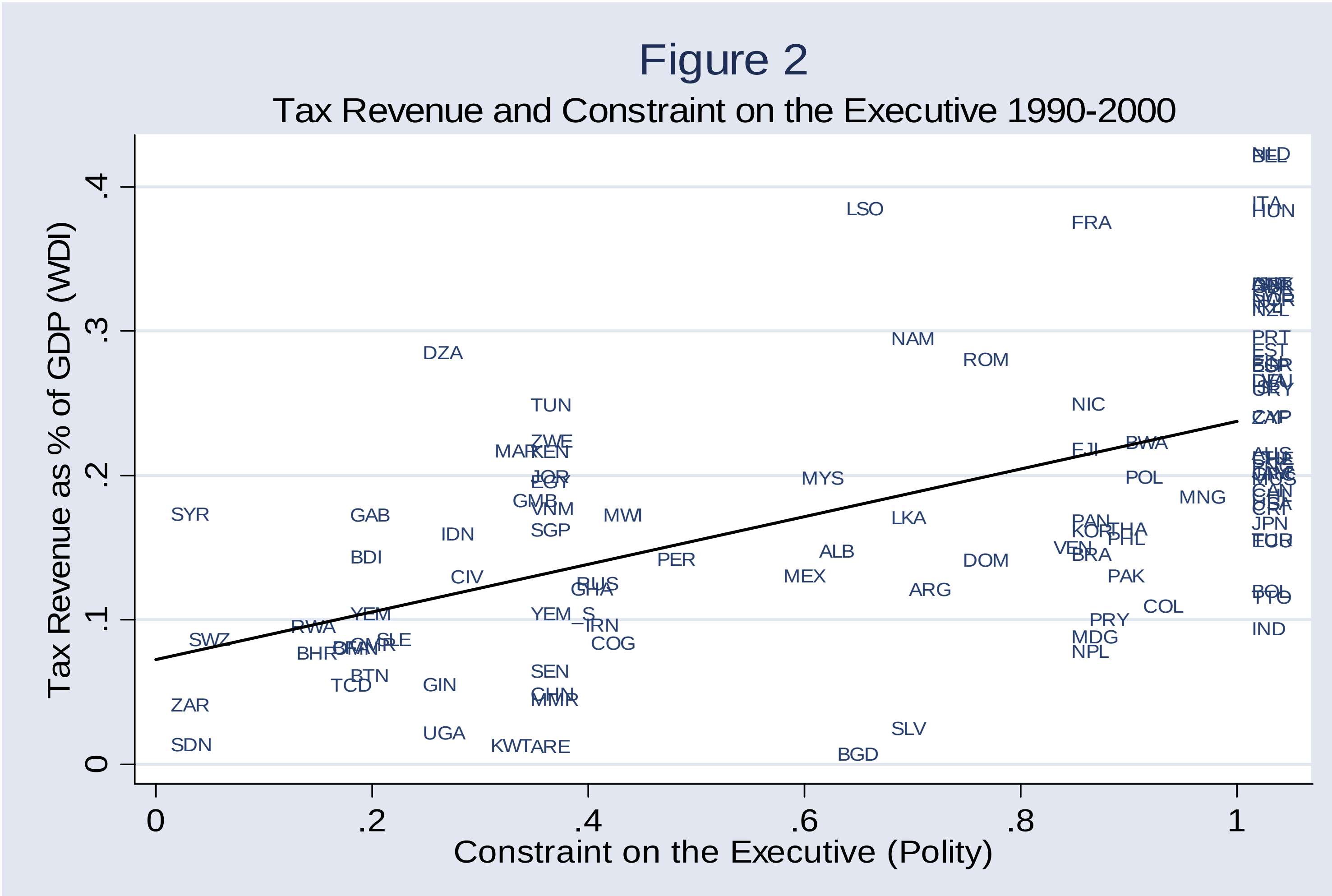
Wintrobe, Ronald (1998) *The Political Economy of Dictatorship*, New York: Cambridge University Press.

Figure 1

Tax Revenue and Income 1990-2000



Log GDP per Capita is the average log GDP per capita between 1990 and 1999 in 1996 dollars from Penn World Tables 6.1 in Summers, Heston and Atten (2002). Tax Revenue as percent of GDP is the average between 1990 and 1999 and is from the World Bank's World Development Indicators (2003).



Constraint on the executive is the average constraint on the executive index normalized from 0 to 1 between 1990 and 1999 from Polity IV explained in Marshall and Jaggers (2004). Tax Revenue as percent of GDP is the average between 1990 and 1999 and is from the World Bank's World Development Indicators (2003).

Figure 3