Socially Optimal Criminal Justice System
Waiting Times: A More General Theoretical Analysis

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Abstract

Criminal justice system delay, which is defined as the time elapsing between the defendant being charged and the case being listed at court, has up to some point, socially valuable flexibility value. This insight borrowed from the options literature in finance, makes it potentially possible to formally model and estimate socially optimal criminal justice system waiting times. This exercise is attempted in this paper by randomising the defendant’s probability of conviction, which is a critical argument in the defendant’s expected cost of going to trial and the monetary value of a conviction to society. The probability of conviction is assumed to conform to a uniform probability distribution with increasing variability until the trial date. What drives this variability is a continuous stream of information impacting on the defendant’s probability of conviction, which is continuously evaluated by the defendant and prosecutor as they formulate their different strategies given their respective conflicting objectives. Endogenising the probability of conviction in this way enables the respective payoffs of the defendant and prosecutor to be expressed as dynamic net present values per unit of time. The economic value of this flexibility to wait is measured as the rate of change in the sum of these dynamic NPVs per unit of time. Socially optimal delay is defined as the trial wait, which maximises the dollar value of the sum of the economic value of flexibility to the prosecutor (society) and the defendant simultaneously, and is the optimal time to list the case at the court. The corresponding dollar value is the total marginal social value of delay. Corresponding to the socially optimal wait and the total marginal social value of delay will be the optimal sentence discount for the prosecutor (society) to offer the defendant and the defendant’s optimal plea. Society makes a trade off between two conflicting objectives, the need to ensure justice to the defendant as legally and morally defined with the need to resolve the issue as quickly as possible.
1. Introduction

Gravelle (1990) has analysed the demand for civil trials by bringing delay explicitly into the analysis with the objective of showing that rationing by waiting (being placed on a court waiting list rather than standing in a queue) can increase social welfare. The author models the impact of court delay, which he defines as the time elapsing from the listing of the case until its determination, on both the parties’ pre and post dispute decisions. For example, prior to the accident resulting in a civil suit, the defendant would have made a decision about how much care to take in potentially risky situations so as to minimise the expected cost of any mishap. While strict rationality compels the defendant to consider the marginal damage and precaution costs in the care decision, aspects of the former such as likely compensation payable to an injured plaintiff and possible trial costs if litigation ensues, may well be underestimated if not ignored completely. Any inefficient decision by the potential defendant about the level of care to take will not be corrected by charging a higher price for a court trial because this decision is very much anterior to any court proceedings, and therefore cannot provide any incentive to consider the care decision more carefully. Court delay however can impact on the value of any likely defendant payout or court costs because for example, the probability of winning or losing changes with time and the time value of money declines. It follows that delay can be a benefit or a cost to society depending on its impact on the accident rate through care levels and the discounted cost of a trial.

In this paper the focus is on criminal justice system delay, which is defined as the time elapsing between the defendant being charged and the listing of the case at the court. Up to some point this delay has a flexibility value. The charged defendant wants the lowest possible penalty (acquittal or a low sentence following a guilty plea), while the prosecutor wants the highest possible punishment consistent with the offence, and with justice as understood by the community being accorded to the victim and accused. The level of penalty depends on the defendant’s probability of conviction if a trial is sought or on the sentence discount if offered for a relatively early guilty plea. Before optimal decisions can be made by the prosecutor and defendant they need the maximum amount of information pertaining to the defendant’s guilt or innocence. However, relevant evidence comes in randomly and requires constant evaluation. Similarly irrelevant information has to be discarded. Delay then is essential as it gives both parties the option and flexibility of acting on better subsequent information.

Criminal court delays (initiation of the case to final disposition) are considerable in Australia, although there is considerable variation between the states and disposition modes. This can be seen from the following information extracted from the ABS series on Higher Criminal Courts (2000-01).1

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1 Later data cannot be provided because subsequent issues of this publication have discontinued providing information in this format.
Criminal Court Delay.

Initiation to Verdict (weeks), 2000-01.

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These reported waiting times are the outcome of many decisions made by numerous participants in the criminal justice system, police, prosecutors, defendants, defendants’ lawyers, court administrators and judges. A relatively large literature reviewed in Torre (2008) emphasises the potential rationing function of these waiting times. In that paper the author shifted the analysis from rationing to benefits and costs of delay. Simple static NPV analysis in non continuous time was utilised to define the socially optimal amount of pre-trial delay for defendants not granted bail and an attempt was made to estimate the net social benefits yielded by these waits. It was argued that public perceptions including those of policy makers, should move away from always exclusively focussing on the costs to offenders of being held on remand, while ignoring any possible net gains to society from these arrangements. This paper is a further extension of this line of argument in which the option value of delay is analysed in a dynamic rather than a static framework in continuous time. Again this richer analysis leads to the conclusion that the focus on court waiting times (listing of the case until final adjudication) may be misplaced.

11. Analytics

(a) Defendant

(i) Not on Bail:

When a defendant is charged with an offence, the court may or may not grant bail. It is necessary to consider each of these cases separately since the value of the expected sentence cost following a trial differs. In both instances, the defendant is legally aided and risk neutral, which enables the expected cost payoff to be written in its simplest form. A lower bound estimate of the cost of imprisonment to the defendant is the foregone income during incarceration. The expected cost of a trial is given by

\[ E(C) = (Y, P, D, r, T) \]

Where, \( P \) is the probability of being convicted following a trial, and \( T \) is the time elapsing from the time of remand until the conclusion of the trial.

The static net present value of \( (ii) \) is :

\[
E(C) = \int_0^T Y e^{-rx} \, dx + P \left[ \int_0^{T+D} Y e^{-rx} \, dx - \int_0^T Y e^{-rx} \, dx \right] - (1-P)(0)
\]

\[ = \frac{Y}{r} \left( 1 - P e^{-r(T+D)} \right) - \left( 1 - P \right) e^{-rT} \]

Expression (2) is the static NPV because any decision made according to this algorithm is a once and for all decision. It does not capture the learning and responding and possibly strategic feature of the passing of time in the context of the criminal justice process because any future flexibility is ruled out. In order to convert (2) into a dynamic expression (dynamic NPV) \( P \) needs to be endogenised by

\[ ^2 \text{All derivations are shown in an appendix to this paper.} \]
writing P as a function of T, P(T). This is done by assuming that P tracks a uniform probability distribution with randomness increasing over time i.e.

$$\left[ \frac{\alpha}{1+T}, \frac{\alpha+T}{T+1} \right]$$

(3)

In (3) $\alpha$ is the defendant’s probability of conviction when the defendant is charged. As T increases, the range over which randomness can fluctuate also increases. For example, if $\alpha = 0.4$ initially and $T = 0$, the lower value of the distribution is 0.4 and the upper value is also 0.4. If $T$ increases to 1, the lower value falls to 0.2 and the higher value increases to 0.7.

The mean value $E(P)$ of (3) is given by:

$$E(P) = \frac{2\alpha + T}{2 + 2T}$$

(4)

Substituting $E(P)$ in (4) into (2) yields:

$$= \frac{Y}{r} \left( 1 - \frac{2\alpha + T}{2 + 2T} e^{-r(T+D)} - \left(1 - \frac{2\alpha + T}{2 + 2T} e^{-rT}\right) \right)$$

(5)

Expression (5) measures dynamic not static net present value because unlike (2), as T changes so does P(T) due to information randomly arriving and being discarded, which enables the defendant and prosecutor, to continuously evaluate their respective options and strategies. The economic value of this flexibility can be measured as the change in dynamic NPV per unit of time. This is found by differentiating (5) with respect to T, which yields the following expression:

$$\frac{\partial E(C)}{\partial T} = \frac{Y}{r T^2} \left[ 1 + \frac{(e^{-D} - 1)}{2 + 2T} - \left( \frac{2\alpha + T - 2 - 4\alpha}{r(2 + 2T)} \right) \right]$$

(6)

The sign of (6) depends on the value of $\alpha$ the defendant’s probability of conviction at a trial. If $\alpha = 0.5$ then (6) is always an increasing positive number for $T \geq 0$; for values of $\alpha \leq 0.5$ it is always a declining positive number and for values of $\alpha \geq 0.5$ it is initially a decreasing negative and then an increasing positive number.

(ii) On Bail:

In the case of an on bail defendant there is no sentence backdating for time spent on remand if conviction and imprisonment ensues following a trial. Convicted on bail defendants are less likely to be imprisoned than their not on bail counterparts, therefore another probability is inserted into the expected cost of a trial expression, $\lambda$ which is the probability of imprisonment. The static NPV of a trial is given by (7):
Substituting $P(T)$ into (7) yields the more useful dynamic version:

$$\text{NPV} = \frac{2 \alpha + T}{2 + 2T} \frac{Y}{r} e^{-\alpha T} \left[ 1 - e^{-rD} \right]$$

(8)

Differentiating (8) with respect to $T$ gives the rate of change in dynamic NPV or the economic value of flexibility:

$$\frac{\partial E(C)}{\partial T} = \frac{\lambda Y e^{-\alpha T}}{r} \left( \frac{2 - 4\alpha}{2 + 2T} \right) \left( 1 - e^{-rD} \right) - \lambda Y e^{-\alpha T} \frac{2\alpha + T}{2 + 2T} \left( 1 - e^{-rD} \right)$$

(9)

$$= \frac{\lambda Y e^{-\alpha T}}{r (2 + 2T)^2} \left[ 2 - 4\alpha - r (2\alpha + T) (2 + 2T) \right]$$

The sign of (9) depends on the value of $\alpha$; if $\alpha \geq 0.5$ then it is negative with the negative number becoming smaller for $T \geq 0$ and if $\alpha < 0.5$ it is positive with the positive number increasing for $T \geq 0$.

(b) Society

An important function of punishment is to compensate society for the often substantial social cost of the crime. Social compensation proportional to the social loss from the offence will only be forthcoming if the defendant is convicted following a guilty plea or a trial and depends on the sentence length. It is assumed that the expected marginal social value of a conviction diminishes per unit of time. A functional form, which satisfies these properties, is given by:

$$E(V) = P(T) L e^{(\sqrt{D} - rT)}$$

(10)

Substituting $P(T)$ into (10)

$$E(V) = \frac{2\alpha + T}{2 + 2T} L e^{(\sqrt{D} - rT)}$$

(11)

Where $L$ is the social loss or cost of the offence and $r$ is the social discount rate. Since $P(T)$ this is a measure of dynamic $E(V)$. Differentiating (11) with respect to $T$: 
Expression (12) gives the economic value of flexibility or court delay to society.

For $\alpha \geq 0.5$ (12) is initially negative becomes positive reaches a maximum and then declines, for $\alpha < 0.5$ the expression is initially negative becomes positive reaches a maximum and then declines for $T \geq 0$.

Socially optimal delay is defined as the trial wait, which maximises the value of the sum of (6) and (12) if the defendant is not on bail, and the sum of (9) and (12) if the defendant is on bail. Since each of the functions is a marginal relationship, each of these summations is called the total marginal social value of delay. The curve showing the relationship between trial delay and the total marginal social value of delay is typically shaped like an inverted parabola, initially increasing in $T$ reaching a maximum value, and then decreasing in $T$. The one exception to this appears to be the not on bail case when $\alpha = 0.5$. Figure 1 shows these outcomes diagrammatically.
111. Social Optimality

(a) Simultaneous Equilibrium

In figures 1 and 2 the socially optimal wait is denoted by $T^*$ and the corresponding total marginal social value of delay by $S^*$. These need to be interpreted carefully. In addition to justice, society also wants a speedy resolution of the case, and is prepared to offer sentence discounts for early guilty pleas. The optimal wait $T^*$ then is the time when the prosecutor should offer a sentence discount and when the defendant should decide whether to accept it or reject it and go to trial. It also corresponds to the time when the case should be listed in the guilty plea or trial lists at the court, and is the social value of criminal justice delay. It is assumed not unrealistically that the defendant will choose between the two options, by comparing the cost of a guilty plea with the sentence discount with the expected cost of the trial.

Let the sentence discount $D$ equal imprisonment in years upon conviction following a trial $M$ multiplied by the defendant’s foregone income while incarcerated $Y$ minus the sentence following a guilty plea $S$ and $\beta =$ the maximum percentage sentence discount, so that $S = \beta MY$, in which case

$$D = MY - S \quad (13)$$

Since at $T = \bar{T}$ (trial date allocated by the court), the sentence discount will be equal to zero, (13) can be rewritten as follows:

$$D = MY - \left[ \beta + (1 - \beta) \left( 1 - \frac{\bar{T} - T}{T} \right) \right] MY$$

$$D = MY \left[ 1 - \beta - (1 - \beta) \left( 1 - \frac{\bar{T} - T}{T} \right) \right]$$

$$D = MY \left[ 1 - \beta - (1 - \beta) + (1 + \beta) \frac{(\bar{T} - T)}{T} \right]$$

$$D = (1 - \beta) \frac{(\bar{T} - T)}{T}$$

$$D = MY (1 - \beta) \left( 1 - \frac{T}{\bar{T}} \right) \quad (14)$$

Expression (14) is a linear function in the trial wait $T$. 
The information in figures 1, 2 and 3 and expressions (5) and (8) can now be brought together into one framework showing the simultaneous determination of the optimal wait, the social value of delay, the optimal sentence discount and the defendant’s optimal plea. For illustrative purposes let the value of $\alpha = 0.5$ for both the not on bail and bail cases. Beginning with the not on bail case the following parameter values are used: $Y = $10,000; $r$ (defendant and social) = 0.05; $L = $10,000; $\alpha = 0.5$; $D = 1$ year; $M = 1$ year; $\beta = 0.2$ and $\bar{T} = 2$ years. From the summation of expressions (6) and (12) the estimated optimal wait $T^*$ is 1 year. Using expression (14) the optimal sentence to offer the defendant at $T^*$ would result in a cost of $4,000 to the defendant. Finally, using expression (5) at $T = 0$ the dynamic NPV of a trial is $4877 > $4,000, which means that for all $T$ until $\bar{T}$ the offered sentence discount is the cheaper option, therefore a guilty plea is the optimal choice. The optimal time to plead guilty and list the case at the court is one year after the defendant has been charged. This solution is shown diagrammatically below in figure 4.

Figure 3: Sentence Discount
Figure 4
In the bail case the assumed parameter values are: \( y = 10,000 \); \( r = 0.05 \) (defendant and social); \( \alpha = 0.5 \); \( \lambda = 0.3 \); \( D = 1 \) year; \( L = 10,000 \); \( M = 1 \) year; \( \beta = 0.2 \) and \( \bar{T} = 2 \) years. From the summation of expressions (9) and (12) the estimated optimal wait \( T^* \) is 1 year.. Using expression (14) the optimal sentence to offer the defendant at \( T^* \) would result in a cost of $4,000 to the defendant. Finally, using expression (8) at \( T = 0 \) the dynamic NPV of a trial is $1463 < $4,000, which means that for all \( T \) until \( \bar{T} \), the trial is the cheaper option, therefore a not guilty plea is the optimal choice .The optimal time to plead not guilty and list the case for trial is 1.4 years after the defendant has been charged. This solution is shown diagrammatically below in figure 5.

(b) Comparative Statics

From a policy perspective, it is useful to assess the impact of changes in each of the values of the parameters in the model on the optimal \( T^* \). Preliminary work using the parameter values used to
construct figures 4 and 5 reveal that for both the not on bail and on bail cases, the optimal value of $T^*$ is most sensitive to changes in the value of $\alpha$ the probability of conviction. In the simulation exercise beginning with the initial values outlined above and then increasing $\alpha$ from 0.5 to 0.8, and holding everything else constant while only increasing $T$, increased $T^*$ from 1 year to 2.2 years and 1.5 years respectively for the no bail and on bail cases. Increases in income from $10,000 to $20,000 reduced $T^*$ from 1 to 0.9 years in both cases, while increases in the defendant and social discount rates from 0.05 to 0.10 reduced $T^*$ from 1 to 0.9 years and 0.8 years respectively for the on and no bail cases. Increases in the sentence after a trial from 1 to 2 years lowered $T^*$ from 1 to 0.9 years for both categories of defendant, and similarly, increases in the social loss of the offence from $10,000 to $20,000 lowered $T^*$ from 1 to 0.9 years in both instances. Finally an increase in the probability of imprisonment following conviction after a trial for an on bail defendant from 0.3 to 0.5 lowers $T^*$ from 1 to 0.9 years.

IV. Conclusions and Future Directions

Further analysis of criminal court waiting times suggests again that a distinction should be made between delay elapsing from the laying of charges until the listing of the case at the court and the listing of the case until final disposition by a guilty plea or a trial. The latter time period is probably more relevant in considering the internal operation of the courts as an economic organisation. Only the former time period is considered here and by explicitly modelling the probability of conviction as a stochastic variable, and utilising dynamic rather than static net present value as an analytical tool, it has been possible to theoretically redefine the concept of a socially optimal wait for the problem at hand. The framework is applicable to all defendants not on bail and those on bail. In this instance the theoretical specification of the relevant wait is not immediately obvious a priori.

As well as optimum waits, the study’s methodology is useful for placing a dollar value on waiting time and illustrates the circumstances under which it is likely to be a benefit or a cost. This information is potentially an important input into cost benefit studies of the criminal justice system. The methodology can also be carried over to civil disputes. Empirical studies are now required to accompany this work, which is the next step in this research programme. The analytical technique adopted here may well allow the utilisation of an innovative approach to the empirics, namely option pricing models rather than standard econometric or simulation methods. The empirics pose some challenges, in particular valuing the social losses associated with different offences and the issue of defendant and social discount rates in the criminal justice system context.
References

ABS (2000-01) Higher Criminal Courts. Catalogue Number 4513.0


Appendix

Static NPV of the expected cost of a trial to the defendant: (Not on Bail)

\[
E(C) = \int_{0}^{T} Y e^{-rT} \, dx + P \left[ \int_{0}^{T+D} Y e^{-rT} \, dx - \int_{0}^{T} Y e^{-rT} \, dx \right] - (1 - P)(0)
\]

\[
\int_{0}^{T} Y e^{-rT} \, dx = \left[ -\frac{Y}{r} e^{-rt} - \left( -\frac{Y}{r} e^{-r(0)} \right) \right] = \frac{Y}{r} (1 - e^{-rt})
\]

\[
P \int_{0}^{T+D} Y e^{-rT} \, dx = \left[ -\frac{Y}{r} e^{-r(T+D)} - \left( -\frac{Y}{r} e^{-r(0)} \right) \right] = P\left(\frac{Y}{r} (1 - e^{-r(T+D)}) \right)
\]

\[
= \frac{Y}{r} (1 - e^{-rt}) + P \left[ \frac{Y}{r} (1 - e^{-r(T+D)}) - \frac{Y}{r} (1 - e^{-rt}) \right]
\]

\[
= \frac{Y}{r} \left[ (1 - e^{-rT})(1 - P) + P(1 - e^{-r(T+D)}) \right]
\]

\[
= \frac{Y}{r} \left[ 1 - e^{-rT} - P + P e^{-rT} + P - P e^{-r(T+D)} \right]
\]

\[
= [1 - (1 - P) e^{-rT} - P e^{-r(T+D)}]
\]

\[
= \frac{Y}{r} (1 - P e^{-r(T+D)} - (1 - P) e^{-rT})
\]
P(T)

Lower and upper bounds of the uniform probability distribution with increasing randomness:

\[
[ \frac{\alpha}{1+T}, \frac{\alpha+T}{T+1} ]
\]

\[
\frac{1}{2} \left( \frac{\alpha}{1+T} + \frac{\alpha+T}{1+T} \right)
= \frac{\alpha}{2+2T} + \frac{\alpha+T}{2+2T}
= \frac{\alpha+\alpha+T}{2+2T}
\]

\[
E(P) = \frac{2\alpha+T}{2+2T}
\]

Dynamic NPV of the expected cost of a trial to the defendant: (Not on Bail)

\[
= \frac{Y}{r} \left( 1 - \frac{2\alpha+T}{2+2T} e^{-r(T+D)} - (1- \frac{2\alpha+T}{2+2T} e^{-rT}) \right)
\]

\[
= \frac{Y}{r} \left( 1 - \frac{2\alpha+T}{2+2T} e^{-D} e^{-rT} - (1- \frac{2\alpha+T}{2+2T} e^{-rT}) \right)
\]

\[
= \frac{Y}{r} \left( 1 - e^{-rT} \left( \frac{2\alpha+T}{2+2T} e^{-D} + 1- \frac{2\alpha+T}{2+2T} \right) \right)
\]

\[
= \frac{Y}{r} \left( 1 - e^{-rT} (1+ \frac{2\alpha+T}{2+2T} (e^{-D} -1)) \right)
\]
Economic value of flexibility or option value to the defendant (Not on Bail):

\[
\frac{\partial E(C)}{\partial T} = \frac{Y}{r} \left( 1 - e^{-rT} \left( 1 + \frac{2\alpha + T}{2 + 2T} (e^{-D} - 1) \right) \right)
\]

\[
= \frac{Y}{r} e^{-rT} \left( 1 + \frac{2\alpha + T}{2 + 2T} (e^{-D} - 1) \right) - \frac{Y}{r} e^{-rT} (e^{-D} - 1) \frac{2 + 2T - 2(2\alpha + T)}{(2 + 2T)^2}
\]

\[
= Y e^{-rT} \left( 1 + \frac{2\alpha + T}{2 + 2T} (e^{-D} - 1) \right) - \frac{Y}{r} e^{-rT} (e^{-D} - 1) \frac{2 - 4\alpha}{(2 + 2T)^2}
\]

\[
= Y e^{-rT} \left[ 1 + \left( \frac{e^{-D} - 1}{2 + 2T} \right) \left( 2\alpha + T - \frac{2 - 4\alpha}{r(2 + 2T)} \right) \right]
\]

Static NPV of the expected cost of a trial to the defendant: (On bail)

\[
E(C) = \lambda P \int_0^{T+D} Y e^{-rx} \, dx - (1 - P)(0) = \lambda P \frac{Y}{r} [e^{-rT} - e^{-r(T+D)}]
\]

Dynamic NPV of the expected cost of a trial to the defendant: (On bail)

\[
\text{NPV} = \lambda \frac{2\alpha + T}{2 + 2T} \frac{Y}{r} e^{-rT} [1 - e^{-rD}]
\]

Economic value of flexibility: (On bail)

\[
\frac{\partial E(C)}{\partial T} = \lambda \frac{Y}{r} e^{-rT} \left( \frac{2 - 4\alpha}{(2 + 2T)^2} (1 - e^{-rD}) - \frac{2\alpha + T}{2 + 2T} (1 - e^{-rD}) \right)
\]

\[
= \lambda Y e^{-rT} \left( 1 - e^{-rD} \right) \frac{(2 - 4\alpha - r(2\alpha + T)(2 + 2T))}{r(2 + 2T)^3}
\]
Expected value of a conviction to society:

\[ E(V) = P(T) \cdot L \cdot e^{\sqrt{\alpha} - rT} \]

\[ E(V) = \frac{2\alpha + T}{2 + 2T} \cdot L \cdot e^{\sqrt{\alpha} - rT} \]

Economic value of flexibility or an option to society:

\[ \frac{\partial E(V)}{\partial T} = L \left[ \left( \frac{2-4\alpha}{(2+2T)^2} \right) e^{\sqrt{\alpha} - rT} + \frac{2\alpha + T}{2+2T} e^{\sqrt{\alpha} - rT} (-r) \right] \]

\[ = L \left[ \left( \frac{2-4\alpha}{(2+2T)^2} \right) e^{\sqrt{\alpha} - rT} - r \left( \frac{2\alpha + T}{2+2T} \right) e^{\sqrt{\alpha} - rT} \right] \]

\[ = L \cdot e^{\sqrt{\alpha} - rT} \left[ \left( \frac{2-4\alpha}{(2+2T)^2} \right) - r \left( \frac{2\alpha + T}{2+2T} \right) \right] \]

\[ = L \cdot e^{\sqrt{\alpha} - rT} \left[ \left( \frac{(2-4\alpha)(2+2T)-r(2\alpha + T)}{(2+2T)^2} \right) \right] \]

\[ = L \cdot e^{\sqrt{\alpha} - rT} \left[ \left( \frac{4+4T-8\alpha - 8\alpha T - 2\alpha r - rT}{(2+2T)^2} \right) \right] \]

\[ = L \cdot e^{\sqrt{\alpha} - rT} \left[ \left( \frac{4(1+T) - 8\alpha (1+T) - r(2\alpha + T)}{(2+2T)^2} \right) \right] \]