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# Single Transferable Vote with Borda Elimination: A New Vote Counting System

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Working Paper No 2201
May 2002

Published by Deakin University, Geelong, Victoria 3217, Australia

First Published May, 2002

Produced and Distributed by the School of Economics, Deakin University,
Burwood Campus, 221 Burwood Highway,
Burwood, Victoria 3125, Australia



**ISBN: 0 7300 2557 8**
**ISSN: 1325-8435**

**Abstract**

Dummett (1997) notes particular difficulties with single transferable vote (STV) and proposes an alternative vote counting system called "Quota/Borda system" (QBS) to remedy specific difficulties. I propose an alternative system, structurally related to QBS, which accomplishes similar solutions but has some significant differences. This alternative system is identical to STV in all aspects except one. It eliminates candidates in reverse order of their Borda scores rather than by their current ranking of first-place votes. I designate this system STV with Borda elimination (STV-B).

STV-B and QBS share general features. They retain proportional representation from STV. However, they differ from STV is two critical manners. First, both permit some influence on candidate selection to occur between voting blocks. Second, they are much more stable than STV when subjected to small changes in voter preferences.

Outcomes from STV-B differ from QBS outcomes in two ways. Under STV-B, a minority that shares some preferences may elect a candidate even if the minority is not a solid coalition, as is required for minorities under QBS. Further, QBS always selects Borda winners, either for a minority or overall. STV-B may reject a Borda winner through emphasis on each voter's most preferred candidates.

## Section 1: Introduction

This paper presents a new solution to the problem of quasi-chaos in the Single Transferable Vote (STV) system of vote counting. Single Transferable Vote with Borda elimination improves upon STV by basing the elimination of loosing candidates on their Borda scores.[1]Arrow's (1951) famous result showed that all voting systems behave perversely to some extent. However, some behave worse than others do. STV arguably is one of the more problematic voting systems. The popularity of STV is largely due to the proportional representation of minorities which arises under STV. STV-B retains proportional representation and shares features with the Quota\Borda System developed by Dummett (1997).[2]

Single transferable vote (STV) is a method of calculating election results that guarantees proportional representation for solid coalitions – sets of voters who share a set of most preferred candidates -- under reasonable conditions. Several countries and dozens of nongovernmental organizations use STV (Tideman 1995) and it has clear applications for corporate boards. Dummett notes that although STV achieves proportional representation, its selection of particular candidates is problematic. He is particularly concerned with the "quasi-chaotic" nature of STV and the closely related phenomenon that STV considers more of some voters' preferences than it does other voters' preferences (pg 150). Dummett identifies four features of "quasi-chaos." 1) Small changes in voters' preferences can cause large changes in outcomes (pg 142). 2) These changes in outcomes are non-monotonic[3] (pg 148). 3) The changes can affect candidates who were not involved in the changes of preference (pg 146). 4) Voters may not be able to anticipate effects of a change in their preference (pg 142). The latter three features arise from the first.

The first feature provides a link to chaos in the mathematical sense. Weisstein (1999) notes that a characteristic of chaos is "that initially nearby points can evolve quickly into very different states." In chaotic systems series of changes cause further changes so that small variations in initial states can lead to large differences in later states, the "Butterfly Effect". Dummett provides an example, detailed below, in which the role of the metaphorical butterfly is played by a

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I appreciate the suggestions and constructive critiques of Prasanta Pattanaik, Bharat Hazari, and Jamie Mustard. All remaining errors are mine.

1 Detailed descriptions of these voting systems follow preliminary remarks. Since both systems are well over a century old, they may have been combined before. I have found no evidence of it happening, but a more accurate claim should end with "to the author's knowledge".

2 Hereinafter simply "Dummett".

3 Non-monotonicity is also called "nonnegative responsiveness" and means that voters changing preferences by ranking a winning candidate higher can result in that candidate not being elected, or that lowering the ranking of a losing candidate can result in that candidate's election. Nonnegative responsiveness leads to manipulability (in the Gibbard-Satterthwaite sense) at the relevant preference profiles.

fraction of a percent of voters. They flutter wings by switching rankings of one pair of candidates. The resulting tornado is a reversal of fortunes for six of eight candidates, including a counterintuitive election of the candidate who was lowered in the rankings.

Dummett proposes a vote counting system called the "Quota/Borda system" (QBS) (pg 154) to remedy quasi-chaos. STV-B accomplishes similar solutions but has some significant differences. These differences include potential representation of minorities that are not solid coalitions, emphasis on each voter's most preferred candidates, and a level of stability intermediate between QBS and STV. STV-B is identical to STV in all aspects except one. Candidates are eliminated in reverse order of their Borda scores rather than by their current ranking by first-place votes as in STV.

This paper will proceed through five sections. The introduction continues by explaining the traditional voting systems: Borda and STV. Section 2 explains Dummett's concerns with STV. Section 3 describes Dummett's solutions through QBS. Section 4 presents the novel voting system, STV-B, then compares and contrasts it with STV and QBS. Section 5 concludes.

## Borda

Jean-Charles de Borda in 1770 proposed the vote system now designated by his name. In the Borda system, voters rank candidates[4]through $n$ levels with the most preferred candidate receiving a rank of one and the least preferred receiving a rank of n (Levin and Nalebuff 1995). Borda scores equal:

$$B_c = \sum_{i=1}^{|V|} n - R_{i,c} \ ,$$

where $B_c$ is the Borda score for candidate $c$, $|V|$ is the number of voters, $n$ is the number of candidates to be ranked by each voter, and $R_{i,c}$ is voter $i$'s ranking of candidate $c$.

The candidate with the highest score is the Borda winner. Multiple candidates may be selected by choosing the set of candidates with thehighest scores. Borda voting considers an arbitrarily deep set of preferences for all voters, and that depth can easily be tailored to fit need. An ordinal ranking of candidates from high Borda score to low is called their Borda ranking (Levin and Nalebuff 1995).

[4] I refer to "candidates" however all methods in this paper apply equally to any selection from a specified set of alternatives.

**STV**

Thomas Hare developed STV in 1859. The following description is from Levin and Nalebuff (1995) and Tideman (1995). STV is a vote counting system based upon three principles: listed ordinal voter preferences (as with Borda), a quota of votes required for election, and the transferal of votes between candidates. Voters list candidates from most preferred to least preferred. The quota ($q$) is based upon the number of electoral seats ($e$) to be filled and the number of voters. This quota is the lowest number of votes that could be required for election without the possibility of electing more candidates than the number of seats to be filled:[5]

$$q = \left[ \frac{|V|}{e+1} \right] + 1,$$

where the bracket notation $[x]$ denotes the largest integer less than $x$.

Votes transfer in STV under two conditions. If a candidate receives more than the quota of votes, the excess is distributed to candidates who were ranked lower in voters' preferences. The transfer fraction (excess) is:

$$f_c = \frac{w_c - q}{w_c},$$

where $f_c$ is the fraction of each vote for candidate $c$ that is to be transferred to each voter's next ranked candidate, and $w_c$ is the number of votes for candidate $c$. If no candidate receives the quota of votes, the candidate with the fewest votes is eliminated and votes for that candidate transfer to each supporter's next highest choice. [6]

Vote counting under STV proceeds though the following branched and looped algorithm.

1) Note the number of seats to be filled.
2) Count the number of votes cast.
3) Compute the quota.
4) Count the first place votes for each candidate.

[5] Election of an extra candidate would be prevented by addition of a fraction of a vote rather than a whole vote. In elections with low numbers of voters, significant distortions can result from "rounding up" to the next integer. I will restrict discussion to large sets of voters.

[6] There are many variations in the vote transferral process based on expediency and logical considerations. The method presented here is common for theoretical applications and does not substantially affect any conclusions in this paper.

5) Compare the quota to the number of first place votes for each candidate. If the sum of first place votes is less than the quota, all seats are filled.[7] Stop.
If all candidates have fewer first place votes than the quota, go to step 11.
If at least one candidate has at least the quota of first place votes, proceed with step 6.
6) Declare any candidates with more than the quota of first place votes to be seated.
7) Compute transfer fractions for each seated candidate.
8) Transfer $f_c$ of each vote (or fraction of vote) for each seated candidate *c* to the next ranked candidate for each voter ranking candidate *c* as the most preferred (remaining) candidate.
9) Remove all seated candidates from all voters' rankings.
10) Go to step 4 counting all remaining votes, full and fractional.
11) Identify the candidate *c* with the fewest first place votes.
12) Transfer all votes for candidate *c* to the next highest ranked candidate for each voter for whom *c* is the most preferred (remaining) candidate.
13) Remove the candidate *c* from all voters' rankings.
14) Go to step 4 counting all remaining votes, full and fractional.

Each series of one counting of values and seating or exclusion of candidates is called a "round" of vote counting.

## STV example

Consider an example. Table 1 is a set of preferences based upon Dummett page 148 that illustrates the mechanisms and significant characteristics of Borda, STV, QBS, and STV-Borda[8]. Disregard column III_, which will be used in extensions. In our example, we fill four seats. There are 99,995 votes. Thus, the quota is 20,000. Candidate $c_B$ has 21,001 first place votes, exceeding the quota with a residual of 1001 votes. A fraction (20,000/21,001 or 0.952) of each vote is sufficient to seat candidate $c_B$. The remainder of each vote is distributed to candidates $c_A$ and $c_C$, each voter's second preference. Table 2 presents the number of votes held by each candidate through the various rounds of election, elimination, and transfer. A number in italic indicates a candidate's election. An underlined number indicates a candidate's elimination. Dashes indicate that the candidate is no longer under consideration.

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[7] This condition and subsequent parenthetical expressions can only be met after at least one loop through the algorithm.
[8] This table yields results generally consistent with Dummett's. However, I was unable to match his vote tallies exactly.

Table 2 shows the effects of the selection of candidate $c_B$. In the second round, 500 votes transfer to candidate $c_A$ and 501 transfer to candidate $c_C$, in accordance with the preferences listed in Table 1. No candidate meets the quota of votes in round two, so the candidate with the fewest votes, $c_D$, is eliminated. Round three shows $c_D$'s supporters' votes transferred to their next highest choices, 1595 to candidate $c_C$ and 8000 to candidate $c_G$. Again, no candidate has sufficient votes for election. So, the candidate with the fewest current votes, $c_A$, iseliminated. Votes originally assigned to $c_A$ in Table 1, columns I to IV and partial votes transferred to $c_A$ from $c_B$ (column V), transfer to each voter's next highest preference in round four. Some votes, 4611, transfer to $c_C$ and 2722 to $c_H$. Candidate $c_D$would have received 3600 except that $c_D$ has already been eliminated, so the votes transfer to $c_G$, the voters' next highest remaining preference. Candidates $c_C$ and $c_G$ are seated. Residual votes transfer from $c_C$ and $c_G$ to $c_E$ and $c_H$ in round five. Again, no candidate meets the quota and $c_F$ is eliminated. Votes transferred from $c_F$ effect the election of $c_H$, completing the slate of four seats. These results are sensitive to small changes such as from III to III_ in Table 1 as will be addressed in section 2.

**STV and proportional representation**

Although Hare designed STV to reduce the perceived problem of wasted votes, its most popular feature is proportional representation (PR) of solid coalitions of voters (Tideman 1995, Levin and Nalebuff 1995). If at least the quota of voters ranks the same set of candidates before any other candidates that set of voters $V_s$ is a "solid coalition" that "supports" the set of candidates $C_s$. PR arises from the vote transferal process. Proportional representation is inexact due to the discontinuous nature of electoral seats:

$$PR = \left\lfloor \frac{|V_S|}{q} \right\rfloor .$$

Definitions:
$R_{i,c}$ is voter $i$'s ranking of candidate $c$.

A solid coalition is a set of voters $V_s \subset V$ supporting the set of candidates $C_s \subset C$ such that:

1) $q \leq |V_s| < |V|$;
2) $R_{i,s} < R_{i,c} \; \forall i \in V_s, s \in C_s, c \in C \setminus C_s$;
3) There does not exist a $C_s^{'} \subset C_s$ s.t. $C_s^{'}$ satisfies condition 2 and $|C_s^{'}| \geq PR$.

Theorem: Solid coalitions gain proportional representation under STV. A set of candidates $C_e \subseteq C_s$ will be seated; $|C_e| < PR$ iff $|C_s| < PR$.

Proof, following (Tideman 1995).
Case 1: $|C_s| \leq PR$, $C_e = C_s$.
Votes from $V_s$ transfer to other candidates within $C_s$ before transferring to any outside of $C_s$. Therefore, $C_s$ receives at least $q(PR)$ votes and is seated.

Case 2: $|C_s| > PR$ .
Case 2a: Candidates $C_x$ are eliminated from $C_s$. Votes for $C_x$ transfer to other candidates $C_s^{'} = C_s \setminus C_x$ before transferring to any $c \in C \setminus C_s$ .
Case2a1: No $d \in C_s$ receives votes from $V \setminus V_s$, when $|C_s^{'}| = PR$, $C_s^{'}$ receive $q(PR)$ votes and is seated.
Case 2a2: At least one $d$ receives votes from $V \setminus V_S$, $C_s^{'}$ receive more than $q(PR)$ votes and so $C_s^{'}$, $|C_s^{'}| \geq PR$, is seated.
Case 2b: No candidates $C_x$ are eliminated from $C_s$. $|C_s| > PR$ is seated.

**Section 2: A critique of STV**

Consider Table 1 again. The bold font denotes preferences that are eventually considered in the election presented in section one. Notice that STV has considered only the first preference of some voters and considered through the fifth preferences of other voters. Dummett (1997, pg 150) notes that STV is quasi-chaotic because it takes into account only the first choices of some voters, and less preferred choices of others, giving them as much weight as the first choices.

STV election results change dramatically from the small change in preferences from column III to column III' (Table 3 in contrast to Table 2). Dummett shows that under STV this change of preferences of only 100 of 99,995 voters changes outcomes dramatically and non-monotonically. One hundred voters switched from preferring $c_A$ to $c_D$ to preferring $c_D$ to $c_A$. Round one remains the same as in the previous example; however, round two differs in that candidate $c_G$ is now eliminated instead of $c_D$. Notice two consequences. First $c_G$ has been eliminated first whereas previously $c_G$ was seated. Second, the second preferences of $c_G$'s supporters are considered in round three, rather than the second preferences of $c_D$'s supporters as in the case above. Clearly, the process of elimination and election will follow a different path now that different preferences are being considered. The final result of the small change in preferences in column III is rejection of $c_C$, $c_G$ and $c_H$ and election of $c_A$, $c_E$ and $c_F$ in their places.

Notice that lowering $c_A$ in the preference ranking resulted in $c_A$ being seated, a clear example of non-monotonicity. Further, results changed for $c_C$, $c_E$, $c_F$, $c_G$, and $c_H$ although they were not involved in the change of preferences. Dummett stresses that this dramatic change is a result of the order in which candidates are eliminated under STV, notes peculiar results from actual elections, and documents that this instability has been known since at least the Royal Commission on Electoral Reform of 1910.

Tables 1, 2 and 3 illustrate the three step process leading to quasi-chaos in STV. First voters change rankings of candidates. Second, the changed ranks change the order of elimination of candidates directly. The third is the transferal of votes. The third step compounds theeffects of the second by changing the vote rankings of the candidates, thereby potentially changing the order of elimination. These changes propagate through all the rounds of elimination and vote counting in STV. The result is quasi-chaos, small changes trigging large, nonmonotoic, and unanticipatable changes in electoral outcomes.

## Section 3: Quota/Borda System

Dummett proposed the "Quota/Borda system" (QBS) which achieves proportional representation while avoiding quasi-chaos by manually checking for solid coalitions of voters. QBS bases the selection of candidates on Borda rankings while retaining quotas and proportional representation from STV. QBS checks successively for solid coalitions equaling the quota, then for double the quota, through progressively higher multiples. Dummett suggests limiting coalitions to minority coalitions since majorities may not need protection of their proportional representation.

Definitions

The set of QBS winners $Q$ consists of candidates $Q_s$ selected by solid coalitions $V_s$ and candidates $Q_o$ selected by voters at large.

$$\forall V_s, \exists Q_s \subseteq C_s \text{ s.t. } B_j > B_k \forall j \in Q_s, k \in C_s \setminus Q_s, |Q_s| = \min(PR, |C_s|)$$

Let $Q_o$ be the set defined by

$$Q_o \subseteq C \setminus \left( \bigcup_s Q_s \right)$$

where

$$\forall j \in Q_o, k \in C \setminus \left( \bigcup_s Q_s \cup Q_o \right) B_j > B_k,$$

and

$|Q_o| = e - \left|\bigcup_s Q_s\right|$.

The QBS winners are $Q = \left(\bigcup_s Q_s\right) \cup Q_o$.

Dummett reasonably makes no provision for ties as they are very unlikely given the large Borda scores involved in real elections.[9]

Dummett built QBS on the Borda system, and so it includes information from voters' complete ordinal rankings of candidates. It is weakly monotonic in changes of preferences. If voters change their preferences ranking so that some candidate X is higher in their preferences, candidate X is no less likely to be seated thanwithout the change in preferences because candidate X's Borda score must increase while others must decrease. Selection of candidates only changes under QBS when Borda rank or solid coalitions change. Therefore, QBS is not quasi-chaotic.

All voters contribute to choosing the solid coalitions' candidates. The solid coalition chooses a supported set of candidates. If the supported set is greater than *PR*, the voters as a whole decide which candidates are selected from the supported set. Dummett points out that this arrangement is arguably more democratic than STV in that the preferences of more people are considered in the selection of candidates. He argues further that this democratic element does not negate coalition preferences as the coalition can, by limiting their pool of candidates to the number of seats assigned to them, choose candidates without influence from other voters.

## Section 4: Single Transferable Vote with Borda Elimination

Single Transferable Vote with elimination based on Borda scores (STV-B) is the same as STV except in the rule for eliminating candidates. Under STV, candidates are eliminated in any particular round in the order oftheir vote tally in that round. In STV-B candidates are eliminated based on their Borda scores as computed using the voters'complete (initial) rankings. Make two changes to the algorithm for STV to get the algorithm for STV-B. Add a line 3' "Compute Borda scores." Replace line 11 with "Identify candidate *c* with the lowest Borda score of the (remaining) candidates."[10]

[9] Pairwise comparison overall then within solid coalitions could resolve ties.
[10] Ties may be resolved with pairwise comparison as suggested for QBS.

Consider STV-B in practice with the preferences from Table 1 resulting in the Borda scores in Table 4. The preferences ignored by STV in the selection above were ordered so that STV-B would yield identical results to STV prior to the change of preferences in column III. With STV-B the reason that each candidate is eliminated has changed. Candidate $c_D$ is eliminated first, $c_A$ second and $c_C$ third because they have the lowest remaining Borda scores at the time. All other processes remain the same as in the STV initial example. With the sole exception that $c_A$ receives 100 fewer votes and $c_D$ receives 100 more in rounds 1 and 2 than they do in Table 2. Revising column III to III' does not alter the rankings of the candidates Borda scores, and so does not affect the election.

STV-B shares some features with STV and with QBS, but it does not result in the same outcomes as either. STV-B achieves proportional representation without quasi-chaos but without the restriction of solid coalitions of QBS. Both STV-B and QBS consider complete preference rankings by using Borda scores. Also, both permit arguably more democratic outcomes than STV in that all voters potentially contribute to selection of candidates from sets supported by coalitions. STV-B is nonmonotonic like STV, and unlike QBS. In addition, like STV and QBS[11], under STV-B switching rankings of two candidates can affect other candidates. However, in the non-quasi-chaotic environment of STV-B, these latter two traits emerge as linked to a kind of minority representation. Nonmonotonicity and impacts on other candidates arises from vote transferal, which also promotes election of candidates from minorities that share preferences but are not solid coalitions.

**STV-B yields proportional representation of solid coalitions.**

The proof for proportional representation for STV applies equally to STV-B. Since STV-B transfers votes from higher ranked to next lower ranked candidates, it shares PR with STV. Votes cast by a solid coalition transfer to supported candidates before any of the votes transfer to lower ranked candidates.

**Coalitions affect other coalitions' selections in STV-B and QBS.**

Suppose that a solid coalition supports more candidates than the number of seats they may fill by PR. At least one of those candidates will be eliminated. Since all voters contribute to candidates' Borda scores and candidates are eliminated by Borda ranking, all voters potentially contribute to the selection of candidates from sets supported by solid coalitions. Likewise, minority coalitions contribute to the

[11] Suppose Borda scores for three candidates are close. If some voters reverse the rankings of the candidates with the highest and lowest scores, the Borda rank of the intermediate candidate can change.

selection of candidates supported by the majority. It may be reasonable to expect voters to list moderate candidates supported by other voting blocks (coalitions or majority) above more extreme candidates supported by those other voters. If so, STV-B promotes election of moderate candidates,as does QBS for the same reason.

Theorem

If minorities support more than *PR*, then the majority can force the elimination of any candidate supported by the minority.

Proof

Let $B_{x,c}$, the "subset specific Borda score," be the Borda score for candidate *c*, considering only the votes cast by voters in set $V_x \subset V$ . $B_{N,c}$ denotes the Borda score of some candidate *c* considering only the votes cast by the voters in a minority solid coalition $V_N$, where "minority" indicates that $|V_N| < 0.5|V|$. Order the candidates such that $B_{N,1} > B_{N,2} > ... > B_{N,|C|}$.

The maximum difference in the minority specific Borda scores between candidates $C_N$ supported by a minority solid coalition $V_N$:

$$B_{N,1} - B_{N,|c_N|} = |V_N|(|C_N| - 1)$$

Suppose that each voter in $V \backslash V_N$ ranks the candidates $C_N$ in the reverse order of their minority coalition specific Borda scores. The difference in majority specific Borda scores for the minority candidates is:

$$B_{J,1} - B_{J,|c_N|} = |V_J|(|C_N| - 1)$$

where $V_J = V \backslash V_N$.

The candidate most preferred by a minority solid coalition would be eliminated first among that coalition's supported candidate, if the minority's most preferred candidate had the lowest overall Borda score of all the supported candidates. In order for the majority to reverse the Borda rankings of candidates supported by a minority solid coalition the following inequality must hold:

$$B_{J,1} - B_{J,|c_N|} > B_{N,1} - B_{N,|c_N|} .$$

This simplifies to:

$$|V_J| > |V_N|,$$

which is true by definition of minority

Majorities can likewise force the elimination of a minority's most preferred candidate under QBS by the same proof.

**STV-B is not quasi-chaotic.**

Changes in preferences can alter STV outcomes in three ways. 1) A candidate can be raised above or lowered below the quota. 2) The change in preferences can directly change the order of elimination of candidates. 3) The firsttwo ways can change the transferal of votes, which in turn can alter the order of elimination. The first way applies to any voting system and applies equally to STV and STV-B. The other two show substantial differences.

The second way functions more precisely in STV-B than in STV. Borda scores contain more information than do tabulations of current first place votes in STV. Since they contain more information, Borda scores allow for more robust differentiation between candidates. In STV, depending upon which elements of preferences happen to be counted at each round of elimination, large changes in preferences may not alter the order of elimination, but small changes may. In STV-B the relationship between changes in preferences and changes in order of elimination is precise. For the Borda rankings of any two candidates $c$ and $d$ to change, excluding the possibility of ties, individual voter rankings must change to the following degree:

$$B_c - B_d < \sum_{i=1}^{V} R_{i,c} - R_{i,d},$$

$$c,d \in C.$$

The third way drives quasi-chaos (Dummett, pp 142, 149). STV'squasi-chaos is a result of vote transferal interacting with the rule of candidate elimination. In STV-B the initial Borda scores fix the order of elimination. Small changes in preferences can change outcomes between nearly tied candidates, but effects do not compound through changing the order of elimination. Thus, STV-B is not quasi-chaotic.

**Minorities that are not solid coalitions may gain representation under STV-B.**

Theorem

STV-B can elect candidates popular among non-solid coalitions even without those candidates receiving votes from outside the non-solid coalition.

Proof

Let a non-solid coalition be a set of voters that support a set of candidates according to the definition of solid-coalition except that some of the voters

$V_X \subseteq V_S$ in the non-solid coalition rank one unsupported candidate $x$ before at least one supported candidate $d$:

$\forall x \in V_X \exists! g \in C \setminus C_s$, s.t. $R_{x,g} < R_{x,d}, d \in C_s$.

Consider two special cases.

1) If each g has either been seated or eliminated at the time of transferal of votes to each g, then all votes transfer back to supported candidates and the non-solid coalition gains identical representation as if it were a solid coalition.
2) Suppose that $|V_X| = q$, all voters not in $V_N$ rank the supported candidates last, and that at least one g is seated with votes equaling the quota including votes from $V_X$. No votes transfer back from g to the supported candidates. Then, the non-solid coalition will not have enough votes to effect the election of any supported candidate.

Thus, non-solid coalitions may gain proportional representation under STV-B through the vote transferal process, but will not necessarily gain representation.

**STV-B is nonmonotonic.**

STV-B is nonmonotonic as is demonstrated in Table 5. This table shows a simple election with seven voters filling two seats. The quota is three. Columns I to VII present initial rankings. The STV-B selection process begins with eliminating $c_Y$ in the first round as $c_Y$ has the lowest Borda score. In round two, $c_W$ and $c_Z$ have three votes each and take their seats. If we replace the rankings in columns VI and VII with VI' and VII' candidate $c_Z$ increases in Borda score. With the new rankings, round one sees $c_Y$ eliminated. In round two, $c_X$'s vote transfers to $c_Y$ and $c_Y$ is seated. In round three no candidate meets the quota, so the remaining candidate with the lowest Borda score, $c_Z$, is eliminated. Increasing $c_Z$'s Borda score has resulted in $c_Z$ loosing a seat. STV-B is nonmonotonic.

**Under STV-B, changes in rankings of two candidates can alter outcomes for other candidates.**

Table 5 also demonstrates that under STV-B, switching the rankings of two candidates can alter outcomes for other candidates. Rankings changed for $c_X$ and$c_Z$ whereas outcomes changed for $c_Y$. If a change in preferences causes the election of one candidate rather than another, different votes transfer to remaining candidates.

**<u>STV-B emphasizes higher ranked preferences.</u>**

STV-B retains STV's emphasis on each voter's higher ranked candidates. When selecting for election, both are structured to consider a more preferred candidate before less preferred candidates. Herein lies the central difference between STV-B and QBS. QBS selects according to Borda rank, first for solid coalitions, and then for the entire body of voters. Effectively, QBS removes the lowest Borda-ranked candidates, except those supported by solid coalitions. STV-B removes the lowest Borda-ranked candidates, while checking for solid coalitions or any other set of at least $q$ voters whose highest remaining preferences support a candidate.

## Section 5: Conclusions

Dummett's QBS and the currently proposed STV-B both offer significant improvements over STV. QBS achieves proportional representation for solid coalitions of minority voters and monotonicity while avoiding quasi-chaos and reducing effects on additional candidates when voters switch rankings of some candidates. STV-B and QBS both permit voting blocks (coalitions or majority) to contribute to the selection of candidates from other voting blocks. STV-B also achieves PR without quasi-chaos and reduces effects on additional candidates, but is nonmonotonic, emphasizes each voter's higher ranked candidates, and is compatible with representation of minorities that do not constitute solid coalitions.

Electoral systems that use STV should consider switching to STV-B or QBS in order to avoid quasi-chaos while retaining proportional representation. Electoral systems should also consider that changing from STV to STV-B or QBS also results in voting blocks potentially contributing to the selection of candidates from other voting blocks. The selection between STV-B and QBS pivots on the trade-off between representation of nonsolid minorities and counter-intuitive results such as Nonmonotonicity and effects on third candidate outcomes when voters change the rankings of other candidates.

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Table 1: Preference ranking of 99995 voters.

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | XIII | XIV | XV | XVI | XVII | XVIII | III' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4111 | 3500 | 100 | 2722 | 10490 | 10511 | 13936 | 1595 | 8000 | 7639 | 4000 | 12051 | 4095 | 3153 | 2422 | 1020 | 2150 | 8500 | 100 |
| **A** | **A** | **A** | **A** | **B** | **B** | **C** | **D** | **D** | **E** | **E** | **F** | **G** | **G** | **G** | **H** | **H** | **H** | D |
| **C** | **D** | **D** | **H** | **A** | **C** | **H** | **C** | **G** | F | H | **B** | **A** | **A** | **A** | C | E | F | A |
| **H** | **G** | **G** | C | **C** | **H** | D | **H** | **E** | D | B | **H** | **C** | **D** | **H** | D | G | E | G |
| E | **C** | **C** | G | **H** | F | F | F | **H** | H | C | G | **H** | **H** | E | E | C | G | C |
| G | **H** | **H** | F | E | G | E | B | B | B | G | E | B | **C** | F | G | F | A | H |
| D | E | B | D | G | A | A | E | F | G | D | C | D | E | B | B | B | C | B |
| F | B | F | E | F | E | B | G | C | A | A | A | F | B | D | A | A | D | F |
| B | F | E | B | D | D | G | A | A | C | F | D | E | F | C | F | D | B | E |

First row gives column identification. Second row is number of voters with each preference. Third row gives first preference of each set of voters. Lower preferences are given in descending order through the tenth row. Bold indicates preferences considered by STV.

Table 2: STV results using initial preferences

| Round | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | 10433 | ***21001*** | 13936 | 9595 | 11639 | 12051 | 9670 | 11670 |
| 2 | 10933 | ----- | 14437 | <u>9595</u> | 11639 | 12051 | 9670 | 11670 |
| 3 | <u>10933</u> | ----- | 16032 | ----- | 11639 | 12051 | 17670 | 11670 |
| 4 | ----- | ----- | ***20643*** | ----- | 11639 | 12051 | ***21270*** | 14392 |
| 5 | ----- | ----- | ----- | ----- | 12116 | <u>12051</u> | ----- | 15828 |
| 6 | ----- | ----- | ----- | ----- | 12116 | ----- | ----- | ***27879*** |
| 7 | ----- | ----- | ----- | ----- | 19995 | ----- | ----- | ----- |

Bold indicates candidate selected. Underline indicates candidate eliminated.

Table 3: STV results using changed preferences

| Round | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | 10333 | ***21001*** | 13936 | 9695 | 11639 | 12051 | 9670 | 11670 |
| 2 | 10833 | ----- | 14437 | 9695 | 11639 | 12051 | <u>9670</u> | 11670 |
| 3 | ***20503*** | ----- | 14437 | 9695 | 11639 | 12051 | ----- | 11670 |
| 4 | ----- | ----- | 14650 | <u>9859</u> | 11639 | 12051 | ----- | 11796 |
| 5 | ----- | ----- | 16431 | ----- | 19639 | 12051 | ----- | <u>11874</u> |
| 6 | ----- | ----- | 17596 | ----- | ***21848*** | ***20551*** | ----- | ----- |
| 7 | ----- | ----- | 19995 | ----- | ----- | ----- | ----- | ----- |

Bold indicates candidate selected. Underline indicates candidate eliminated.

Table 4: Borda scores of the candidates in Table 1.

| | III | III' | Ranking |
|---|---|---|---|
| A | 295245 | 295**1**45 | 7 |
| B | 335273 | 335273 | 5 |
| C | 385070 | 385070 | 2 |
| D | 258283 | 258**3**83 | 8 |
| E | 346245 | 346245 | 3 |
| F | 342037 | 342037 | 4 |
| G | 334311 | 334311 | 6 |
| H | 503396 | 503396 | 1 |

Bold indicates changes induced from change in preferences from column III to III'.

Table 5: Example of nonmontonicity and third candidate effects in STV-B

| I | II | III | IV | V | VI | VII | VI' | VII' |
|---|---|---|---|---|---|---|---|---|
| Z | Z | Y | W | X | W | Y | W | Y |
| W | W | Z | Z | Y | X | W | **Z** | W |
| X | X | W | X | W | Z | X | **X** | **Z** |
| Y | Y | X | Y | Z | Y | Z | Y | **X** |

| | Borda Scores | Borda' Scores |
|---|---|---|
| Z | 11 | 13 |
| Y | 8 | 8 |
| X | 9 | 7 |
| W | 14 | 14 |

Election with seven voters selecting between four candidates to fill two seats. Quota is three.