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Focused Power: Experimental Manifestation of the Shapley-Shubik Power Index

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Abstract

Experiments evaluate the fit of the Shapley-Shubik Power Index to a controlled human environment. Subjects with differing votes divide a fixed purse by majority rule in online chat rooms under supervision. Earnings serve as a measure of power. Chat rooms and processes for selecting subjects reduce or eliminate extraneous political forces, leaving logrolling as the primary political force. Initial proposals by subjects for division of the purse allow measurement of effects from focal points and transaction costs. Net results closely fit the Shapley-Shubik Power Index.

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1. Introduction

The Shapley Value and its normalisation the Shapley-Shubik Power Index exist in a game-theoretical, mathematical world of axioms and results. Humans exist in a world of space, time and social relationships. Is their world separated from ours by a chasm or a crack? Is that world close enough to ours to illuminate interesting questions, or is it a beautiful but distant light in the sky? We use relatively unstructured bargaining experiments in an environment tailored to power indices to investigate these questions.

Three reasonable assumptions (efficiency, symmetrical players, and independent games) yield the Shapley Value of participating in a game (Shapley 1953). Normalising the Shapley Value over all players in a game (usually voters with different numbers of votes) gives the Shapley-Shubik Power Index (SSPI) of the capacity for each player to influence election results (Shapley and Shubik 1954). However, the players and votes in these game-theoretical constructs are not the flesh and blood voters casting ballots in the institutions that we experience.

Scholars such as Morriss (1987), Garrett and Tsebelis (1999), Saari (1999), Holler and Napel (2004) and Gelman, Katz and Bafumi (2004) question the usefulness of power indices. Power indices are abstract mathematical representations of the ability of voters to affect group decisions and which focus on votes *per se* in isolation from other political considerations such as resources, traditional alliances, charismatic leadership, or shared agendas. One major concern is that power indices, being general, do not address particulars of human institutions (Garrett and Tsebelis 1999). Morriss (1987) stresses that power is potential, and so is not observable or measurable. Steffen 2002 presents arguments (and counter-arguments) that the abstract mathematics of game theory cannot relate directly to human institutions or human behaviour. Empirical investigations of power indices have shown very weak performance by the SSPI and other power indices, perhaps due to the complexity of the study environments. See Felsenthal and Machover 1995, 1998 for overviews. Gelman, Katz, and Bafumi (2004) empirically determine that power in US and European elections increases with the size of voting blocks, but increases at a decreasing rate. The SSPI, and the Banzhaf Power Index that the authors specifically address, show power tending to increase at an increasing rate with size of voting block.

We accept the sceptics' theoretical position and whole-hearted agree that the Shapley value and the SSPI exist in a world distinct from ours. But we wonder how distinct. We do not accept the empirical results as definitely showing that power indices differ widely from human institutional voting power. Previous empirical studies occur in complex natural institutions which may have caused the consistently weak results.

Whereas many empirical studies indicate a chasm divides the abstract power indices from empirical observations, one recent study found that power as estimated by the SSPI accounted for between 60 and 90 percent of European Union's agricultural spending share per country (Kauppi and Widgrén 2004). This result suggests that the SSPI captures something important about voting power in a human institution. They also refer to earlier non-peer-reviewed publications that found strong links between budget allocations and the SSPI. However due to the complexity of potential political dealing in the EU, Kauppi and Widgrén searched many hypothetical alliances of countries to find vote structures that correspond to the observed subsidies, so it is unclear to what degree their results reflect underlying power in contrast to being a product of the search.

These empirical studies of power do not evaluates relative performance of the index in large versus small blocks of votes. If a power index systematically underestimated the power in large voting blocks but overestimated the power of small blocks, the studies could well show the power index as accurate.

In spite of weak empirical support, power indices have been applied to a variety of political and business environments to predict or account for influence over decisions. Felsenthal and Machover (1998) provide an excellent overview of various applications to the European Union and its predecessors and to the United States. Bilbao *et al* (2000) and Algaba *et al* (2001) apply power indices in the context of the expanding European Union. Other political applications include the United Nations (Penrose 1946), passage of laws in the United States (Shapley and Shubik 1954), United States Presidential elections (Mann and Shapley 1962, Banzhaf 1968, Lambert 1988). The U.S. Supreme Court cases of *Whitcomb v. Chavis*, 403 U.S. 124 (1971), and *New York City Board of Estimate v Morris*, 489 U.S. 688 (1989) are particularly relevant because they reject the use of the Banzhaf index on theoretical and intuitive grounds. One suspects that empirical support for a power index, had it existed at the time, could have affected the court' decisions. (See Felsenthal and Machover 1998 for a listing of other US court cases that use power indices.) Business applications include Birmingham UK airport landing fees (Littlechild and Thompson 1977, with many extensions and replies), costing of joint projects (Tijs and

Branzei 2001), bank ATM interchange fees (Gow and Thomas 1998) and power on corporate boards (Leech 2002, Chen 2003).

This paper uses a novel experimental approach to effectively isolate voting power from other institutional influences while leaving free reign for negotiation among six voters. We experimentally measures the level of power embodied in large blocks of votes by placing sets of six subjects in computer chat rooms, giving them different numbers of votes, issuing them $15, and having them divide the money by majority rule, as will be detailed in Section 3 below. Our measure of power is the percentage of earnings captured by the subject with the most votes. These earnings are averaged over many rounds of play for each of several vote profiles. Effectively, we measure the degree to which power compounds with the size of voting blocks within each profile relative to the degree designated by the SSPI. Results supporting the SSPI would encourage confidence in using it as a control for pure voting power in investigations of other political and socio-economic aspects of group decision making. Results divergent from the SSPI, even in this environment tailored for power indices, would undermine confidence in applying it in more natural institutions.

Section 2 of this paper further explains power indices, specifically defining the SSPI. Section 3 describes the experiment. Section 4 gives our analysis of the experiment: evaluating our pursuit of homogeneity, identifying the effects of focal points and transaction costs. Section 5 presents results and section 6 concludes.

2. Shapley-Shubik power index

The central idea of power indices is estimating the influence of the number of votes held by a player in an environment effectively isolated from other political forces (Felsenthal and Machover 1998). In the world of power indices players face a series of independent decisions; the players share no history; they have no overlap in their interests; they differ in their votes but the players are all identical in other respects.

Voting power is not a proportionate or simple translation of the size of voting blocks. Suppose there are four players (political parties, shareholders, countries, etc) with 52, 45, 2, and 1 vote each while a majority of at least 51 votes decides the outcome. Compact notation expresses the required majority preceding a semi-colon followed by a list of the votes: {51; 52, 45, 2, 1}. The first player can determine all decisions and so has 100% of the electoral power while the others have none. With the superficially similar profile {51; 46, 45, 6, 3} any two of the first three players can form a majority of votes, and the fourth cannot help any other players form a majority. Thus, the power in this case divides between the four players as follows: 1/3, 1/3, 1/3, and 0. The first three players have equal power even though they are different in their votes, while the latter two with similar numbers of votes are very different in power. The fourth player is a 'dummy' because it cannot turn any losing coalition into a winning coalition. All major power indices – Shapley-Shubik (1954, and Shapley 1953); Penrose (1946), Banzhaf (1965) Coleman (1971), Johnston (1977, 1978), Deegan-Packel (1978), Intervals (Taylor and Zwicker 1997), Holler-Packel (1983), etc – agree with these interpretations. When the number of players increases, results become more complex and different indices can yield different outcomes, even opposite rankings of power (Saari and Sierberg 1999).

In 1953, Shapley developed an abstract measure of the value of playing a game based upon three key assumptions. 1) The game is abstract ‘symmetric’; it is the number of votes controlled by a player that matters, not the player’s personality, name, or any other characteristic. 2) The game is efficient in that all possible gains are captured. 3) If two independent games are merged, then the value for each player in the merged game equals that sum of that player’s value in the two games played separately, ‘additivity’. For example, the value of two lottery tickets from different games, when purchased as a package, equals the sum of the values of the two tickets separately. This assumption does not fit all games; when buying two tickets from the same raffle, the purchase of the second ticket marginally reduces the probability of the first ticket winning.

“It is remarkable that no further conditions are required to determine the value uniquely” (Shapley 1953, 309). However, these are sufficient conditions, not necessary ones. As Shapley 1953 notes, other sets of sufficient conditions exist. By formula, the Shapley value ϕ for a player *i* is:

$$\phi_i[v] = \sum_{S \subseteq N, i \in S} \left(\frac{(s-1)!(n-s)!}{n!} \right) v(S) - \sum_{S \subseteq N, i \notin S} \left(\frac{s!(n-s-1)!}{n!} \right) v(S),$$

where *v* is a game expressed as a set function, *S* is a set of players, *N* is a carrier of *S* i.e. S together with any dummy players, s and n are the sizes of *S* and *N*.

Shapley and Shubik (1954) apply the Shapley value to politics. They normalize the Shapley value so that the sum of all players’ values equals one, by dividing each player’s value by the sum of all players’ values. This normalized form of the Shapley value is the Shapley-Shubik power index.

Power indices estimate the power embodied in blocks of votes that allows players to negotiate using those votes, with no shared interests between players and no other differences between players. Natural environments such as international organizations, legislatures, or board rooms are excellent environments for *application* of power indices. Power indices should distinguish influence deriving from votes in these environments and facilitate identifying influences deriving from other sources. However, these environments may be unsuited for direct *evaluations* of power indices because voters in those environments differ in many dimensions and often share interests, violating symmetry. In contrast, our experiment approximates the conditions that underlie power indices, allowing more effective and direct evaluation of the applicability of the Shapley-Shubik Power Index to human institutional voting power.

3. Experiment

3.1. Experiment Outline

Our experiments consist of six student 'subjects' (actual people) taking the roles of 'players' (game theoretical constructs) and meeting in an online chat room with a proctor and recorder who supervise the play and who assign some number of votes to each subject. (Table 1 shows vote profiles.) The subjects then negotiate using chat room style English and standard abbreviations to divide $15 ('purse') between themselves by simple majority-vote rule. This assignment of votes and distribution of money was repeated twelve or twenty-four times in each session of experiments. Each assignment of votes and distribution of money is called a round.

Consider an example. One subject receives eight votes while the other five receive three votes each, so a majority of at least 12 determines division of the money; {12;8,3,3,3,3,3}.

Identify the subjects with letters consecutively from A (with 8 votes). Subject A may propose to divide the money evenly with B and C. In response, Subjects D, E or F could reply with a proposal to allow B to have six dollars and accept three dollars each themselves. Then A could propose a different division splitting B's and F's money between A, D and E. If each subject has strictly less than half the votes, no matter how the subjects divide the money, there is always a majority (by votes) that can benefit from a different division of the money. That is, all distributions of money in which every member of a majority coalition receives a positive payment form a cycling set of outcomes; the core is empty.

Unlike most economic experiments which evaluate at least some aspects of how closely experimental results approximate individually rational equilibria or maximums, our experiments occur in an environment in which individual rationality does not provide insights about results. All outcomes that divide the full purse among the members of a majority with each receiving a positive payment, and two members receiving more than the minimum possible payment, are in the cycling set and among the individually rational results. The SSPI purports to identify reasonable aggregate outcomes without reference to human behaviour. Our experiments investigate how closely decisions of individual subjects, supposed to be approximately rational, approach an abstract, aggregate and mathematical reasonableness.

3.2. Profiles

We selected 18 vote profiles (Table 1) to meet several criteria. No single player had a majority of votes, so their cores are empty. At least one power index differed widely from the SSPI for each profile so that our results were not driven by some universally agreed concept of power beyond those embodied in the SSPI. The p_i profiles approximate the power distribution in

40 percent of top 400 UK publicly held corporations (Leach 2002). So the power structures relate to natural human institutions.

We used so many profiles for several reasons. Although Binmore (2007) found that fairness focal points did not affect experimental results in a bargaining game, any of many other focal points and transaction costs may have affected earnings outcomes (Schelling 1960, Shubik 2002). Just as power is embodied in votes, focal points are embodied in perceptions of votes. People cannot use votes without perceiving them, nor can any small set of people confidently find all focal points that others may perceive. Sugden (1995) demonstrated the unconscious and un-anticipatable nature of many focal points. We hoped to prevent any focal points from dominating the results by providing opportunities for a variety of focal points to manifest; portfolio diversification in effect. Some profiles have readily observable focal points such as the three sixes that exactly equal the quota in profile u_1. Others have more subtle focal points such as other sums of votes that exactly equal the quota as in w_1. Sometimes apparent focal points favour the large player, and in other profiles they disadvantage the large player. Profiles differed in appearance with the large player differing from others by wide margins in some cases and small margins in others. In some profiles all smaller players had the same number of votes while their votes differed in other profiles. In some profiles, all smaller players had equal SSPI values and in other profiles the values differed.

The 18 profiles fall into seven power-identical sets (Table 1) with each set identified by a letter. Within each of these seven sets, each player by rank has the same power as the same ranked player in each of the other profiles.

3.3. Subjects

Our experimental design is based on approximating the assumptions of power indices in a human institution. The SSPI is based on three assumptions: efficiency, symmetry, and additivity.

We promoted and achieved efficiency through two mechanisms. First, distribution of actual money in the experiments maintained subjects' interest and motivated their active participation. See Smith (1982) for more on the role of sufficient returns for effort and other practical elements of social-scientific laboratory experiments. Pilot study debriefings and observations of play showed $10 per round motivated subjects. However, we used $15 per round to satisfy university human research ethics requirements, resulting in an hourly compensation comparable to that of a research assistant even for relatively low earners.

Second, we selected subjects who were more interested in earning high sums by inviting more subjects than we needed for any particular session, and holding an auction for who we would use. Each potential subject wrote a bid of what they would accept to not participate in the experiments. We paid those with the lowest bids and used the others. Average earnings were $30 for a twelve round game at $15 per round with six subjects. Our payoffs clustered around $25 for twelve round games and $50 for 24 round games. These payoffs, although imprecise and subject to statistical distortion through selectivity, suggest additivity.

Subject homogeneity is the empirical manifestation of symmetry and abstraction. Approximating homogeneity among the subjects is central to the experimental approach of creating a human environment that approaching the assumptions of power indices. We endeavoured to establish this homogeneity through two distinct stages. The first stage was experimental design and selection of subjects prior to the experiment taking place. The second occurred after the experiments took place. We used regression to select a set of empirically homogeneous subjects and identified rounds which involved only these subjects.

In order to approximate the power index assumption of homogeneous players, we did not follow the usual experimental economics approach of regressing results on a set of control variables. Instead we attempted to reduce, eliminate, or negate variation. Significant variation among subjects would seriously compromise the relevance of our results even if we could measure the effects of subject control variables.

A coefficient different than one would not discredit the power index because the test is in a heterogeneous environment. Perhaps the power index functioned perfectly well, but there was an additional effect from heterogeneity. We would not be able to distinguish if the index yielded nonsense or if the index yielded a valid result that was masked by an effect from heterogeneity. Perhaps power compounds differently in a heterogeneous environment than in a homogeneous one. So, in order to be able to argue empirically against application of the SSPI to human institutions, we had to construct an effectively homogenous environment.

Further, we were not investigating the effects of variation among the subjects, an interesting issue in its own right, but distinct from our topic. We performed analyses using control variables to test the effectiveness of our attempts to eliminate effective variation, to estimate potential distortion of our results from residual effective variation. These analyses are detailed in Geller, Mustard and Shahwan (2005) and appendices 3 and 4, and relevant points are summarised here.

Chat rooms permit negotiation based upon logrolling (making deals using one's votes or 'I agree to vote for something you want in return for you agreeing to vote for something I want' Tullock 1976 in Johnston 1977) in an environment with greatly reduced effects from factors other than votes. Side deals and threats are obvious and avoidable in supervised chat rooms. Personality and charisma have much less potential for influence when deals are made using brief typed statements directly relevant to voting. Subjects may be anonymous in chat rooms. In these experiments, subject's identities in the chat rooms consisted of a number shared by all subjects in a particular game and a letter unique to each subject. (For example '528C'; see the screen examples in appendix 1.) For further protection of anonymity, we changed the player-identities of the subjects every six rounds.

We collected a variety of data on the subjects in order to control for and reduce possible effects: psychosocial orientation, risk orientation, nationality and gender. Psychosocial orientation measures individuals' preferences for receiving payments in comparison with payments to others (Van Lange, Otten, De Bruin, and Joireman 1997). This Van Lange *et al* instrument divides subjects into three broad orientations and a residual category. Individualist subjects prefer to receive a higher payment for themselves without regard to payments received by others. Competitive subjects prefer to receive more than others, even to the extent of accepting a lower payment for themselves to cause even lower payments to others. Pro-social subjects will accept a somewhat decreased payment to themselves in order to gain more for others. We excluded subjects with a competitive psychosocial orientation because their willingness to waste money violates the power index assumption of efficiency. Thus, our subjects consisted of individualist and pro-social subjects.

The magnanimity of pro-social subjects was potentially problematic for the study. We countered the effect of social orientation by structuring the experiments such that all subjects would receive approximately equal voting power over the course of the game. Thus, structural equity would replace the perceived need for subjects to pursue equity. Our instructions to subjects stated that the games were fair and that anyone temporarily in a weak position would be in a strong position later, that all had an equal potential to earn money based upon their votes over the course of the game. The instructions also explicitly encouraged subjects to be self-centred or 'greedy'. Binford 2007 notes objections to such instructions. We consider the directions appropriate since we are trying to create an institution that includes self-centeredness, rather than test if a model fits a relatively natural environment.

Pilot-study results showed foreign nationality affected earnings and debriefings suggested English language ability to be the key issue in low foreigner performance. We limited the subjects to those with apparent proficiency in English.

The experimental design mechanisms were largely effective. Gender, psychosocial orientation and national origin did not consistently affect earnings significantly in either the statistical or practical sense. Coefficients on those variables were both small and insignificant, with foreigners, women and pro-social subjects earning insignificantly more than their counterparts (Geller, Mustard and Shahwan 2005 and appendices 3 and 4).

Experience should matter in performance. In the pilot study, we tracked experience in order to document the length of the learning process, if experience continued to matter over time, or if gains from experience were captured within a few trials of the game. Consistent with previous works (for example Kelly and Arrowood 1960; Komorita and Moore 1976), six rounds of play appeared to impart enough experience for proficient play in this relatively transparent game. In our analysis we limited our observations to those in which all subjects had already participated in a practice round and at least six rounds for money.

A simple test (appendix 2) evaluated attitude toward risk: aversion, love, or neutrality. However, risk neutral subjects earned more than others with a confidence level of p=0.05 stratifying the sample by whether the largest subject is in the winning coalition (details in Geller, Mustard and 1995, appendix 4). We developed an additional control for heterogeneity by selecting homogenous subjects by regression analysis as explained below.

3.4. Experiment process

On twelve days, with one or two sessions per day, we conducted two or three concurrent experiments in a classroom-style computer laboratory, with each experiment involving a group of six subjects playing series of twelve or 24 rounds under the supervision of a proctor and a recorder. The subjects sat as widely as possible around the laboratory, maintaining at least one computer between every two subjects and seating subjects in the same experimental group more distantly. Each computer used by a subject had chat rooms for two player-identities, permitting rapid change of identities between every six rounds. The proctor and recorder for each group of six subjects shared a computer, participating in the chats as a single individual. Each proctor's computer had files giving the listings of votes to be used each round and typical messages used during the rounds. Each recorder had a hardcopy sheet giving the votes for each subject and majorities required for each round as backup and verification against electronic records.

We sorted the subjects to create at least one relatively homogeneous group based on social orientation, risk orientation, gender, and nationality. We then assigned the subjects to computers without them knowing each other's subject identities. If new subjects were participating, we distributed and read instructions on the game and played a practice round without money. The instructions (appendix 1) included procedures, rules of the game, suggestions on strategies, and that fifteen dollars would be divided each round. The subjects were students in the university and were familiar with the use of the chat rooms because the platform was used for educational purposes and student communication throughout the university. After we provided instructions, each group of six subjects with a proctor and recorder ran independently of other groups.

Subjects within each group had the same information, communicating entirely though chat room windows shared by all group members. At the beginning of each round, the proctor submitted a message to one window, labelled 'Group Chat', on the subjects' monitors saying to wait and do nothing until further notice (appendix 1 has screen examples). Second, the proctor sent a message to another window, labelled 'Vote Vector', on each subject's monitor giving the votes for each subject and majority required for that round. All subjects in the group saw the same message and each knew the votes of all subjects in their group. This was the only message each round sent to the Vote Vector window. Third, the proctor sent a message to the Group Chat requesting that the subjects confirm their votes. Each subject responded with the number of votes assigned to them that round. The proctor and recorder confirmed each number of votes with the data file and hardcopy sheet, correcting any mistakes by the subjects until all subjects reported correct votes.

The proctors submitted a message to each Group Chat to begin the games. Subjects submitted messages proposing, rejecting, revising, or accepting various divisions of the money. Subjects could write plain English statements, use conventional chat room abbreviations, or use brief notation provided during the instructions for the game. Subjects proposing a division of money had to identify that proposal uniquely, using their player-identification letter followed by a number. They were not permitted to use threats, statements that would reveal personal information, or deals for anything other than divisions of money that round. Proctors could end a round without any payment to enforce the rules, but they never had to exercise such a punishment. Recorders and proctors watched the messages for the emergence of a consensus, a difficult task requiring two people. When subjects appeared to have reached a majority decision, the proctors would wait briefly to permit additional proposals and submit a call for votes. Subjects could change their acceptance of a proposal if they wished at any time until they replied to the proctor's call for votes. Each subject could then submit a message supporting one proposal. The proctor and recorder then counted the votes. If there was no winner, the proctor sent a message saying to continue negotiating. When a proposal received enough votes, the proctor sent a message saying that the round was over, which proposal won, and instructing the subjects to send no further messages until the next round started.

At the end of each round, the recorder wrote how much each subject received on the hardcopy sheet and the proctor confirmed the record with the messages in the Group Chat. After each six rounds, the proctors submitted messages instructing subjects to minimise their Group Chat and Vote Vector and open the alternative version of each to proceed for six more rounds with new identities.

After 12 or 24 rounds as time allowed, we ended the game and tallied each subject's winnings. We paid them precisely to five cents, the smallest local denomination coin, and collected a receipt which included each subject's player-identities during the game.

4. Analysis

4.1. Sample

We ran 441 rounds of experiments for money. Many of these rounds included inexperienced subjects or were included as a pre-study for another project. We address 291 observations in this study.

Thirty-one subjects participated in the experiment rounds used in this analysis. Each was the 'largest subject' (in terms of votes) for between one and 26 rounds. We are missing demographic control data for one subject who was the largest subject for three rounds. Twenty subjects were male, 4 were foreign, 16 were individualist, 14 were pro-social, 4 risk averse, 4 risk loving, 15 risk neutral, and 7 gave inconsistent answers in the risk instrument.

4.2. Creating homogeneity

We selected a set of subjects who performed consistently relative to each other and used rounds consisting only of these relatively homogenous-by-performance or 'typical' subjects to establish our results. We regressed (Ordinary Least Squares) earnings of the largest subject per round on the SSPI and a series of dummy variables for each of 30 subjects using 291 observations. Earnings were expressed as a percentage of the total purse. The subject who was the largest subject for the most rounds (26) served as the excluded base for comparison, completing the 31 subjects. For subjects with more than five observations, coefficients varied from a low of -18.9 to a high of 6.8 percent with the extreme low end dominated by subjects with few observations. Regression results are available from the authors but do not warrant presentation since *t* and R^2 statistics are of no interest. There are no issues of generalisation since the regression results apply only to how these subjects performed on these specific experiments.

For subjects with low earnings and few observations, we checked their earnings when they were not the largest subject and found that in all cases except one that they received substantially less than expected in those rounds also. We classified all subjects with coefficients between -8.3 and +6.8 percent as ‘typical subjects’. We also included as ‘typical’ the one subject with few observations, a very low coefficient, but usual earnings when having few votes. For 88 rounds, all six subjects playing in that round were typical. We consider these 88 rounds to have been played by homogeneous subjects, approaching symmetry.

4.3. Measuring distortions

Focal points arise from relative attractiveness, in some sense, of various vote profiles. A large subject could trigger attractiveness or aversion depending on the apparent inequity of votes. The votes themselves may form attractive patterns such as exactly equalling the quota. Focal points may be unconscious, numerous and un-anticipatable (Sugden 1995). Transaction costs likewise make some coalitions more attractive than others. Coalitions consisting of more subjects may be more difficult to form and maintain than coalitions with fewer. Likewise, coalitions of players that appear relatively similar may form easier and be more stable than relatively heterogeneous coalitions. This relative attractiveness may be as apparent in initial proposals by experienced players. Observations allow measurement of and compensation for all possible focal points together with potential transaction costs.

We have chat room records of initial proposals in 274 rounds played by experienced subjects. For each profile we found the number of minimum winning coalitions (winning coalitions without extra players) in the abstract and the number those coalitions that include the largest player. The ratio of the subset over the superset yields the portion of minimum winning coalitions that include the large player.

Differences in attractiveness of the various minimum winning coalitions would cause initial offers to differ consistently from these proportions. We then found the portion of actual, empirical initial proposals that include the large subject for each profile. For each profile, the ratio of the theoretical portion of minimum winning coalitions that includes the large player divided by the portion of the empirical first offers that includes the large subject provides a metric of bias from focal points and transaction costs in favour of the large player, for each profile. The inverse of that ratio, when multiplied by gross earnings of large player, nets out the effects of focal points and transaction costs for each profile.

Initial proposals are largely independent of specific coalitions that win any given round in the experiment. Several to dozens of other proposals occur between initial proposals and final votes. Further, initial proposals and final agreements are different kinds of things. An initial proposal is a statement by a single subject preceding any negotiation over a given vote profile. A final agreement is a consensus of multiple subjects following multiple proposals. Therefore, we feel justified in measuring the effects of focal points and transaction costs using initial proposals, and then using those measures to adjust our gross observations for the effects.

5. Results

Table 2 presents the average earnings for the largest subject in each profile with an adjustment for focal points and transaction costs, adjusted average earnings, and the SSPI. Then it shows the weighted average across all profiles. For ease of comparison across profiles, earnings and the Shapley Value are expressed as percentage of the total purse. Thus the Shapley Value is expressed as the SSPI.

For fourteen of the eighteen profiles (bold in Table 2), including all six cases with more than five observations, the adjustment increases or decreases earnings in the direction to bring it toward the SSPI. The probability that fourteen or more adjustments would be the right direction is only 1.6%, suggesting that the metric captured important aspects of distortions from focal points and transaction costs.

The overall weighted average of adjusted earnings is only four percent greater than the SSPI, which we consider a close fit. The need for weighting the results and the small number of observations per profile preclude establishing a statistical test to potentially reject these empirical results as matching the SSPI. Further, earnings by round for the large subject vary a great deal by the nature of coalition formation. Power indices purport to give average earnings in this environment. In any given round, the large subject would either be out of the winning coalition receiving nothing, or in the coalition and likely receiving more than their SSPI value. This yields a strongly bimodal earnings distribution with high variance. Thus, statistical rejection of matching the SSPI with results differing by only four percent would be unlikely for this sample size. Practically, four percent seems very close and indicates that the SSPI and Shapley value correctly reflect the general nature of voting power.

6. Conclusion

These results support the Shapley Value and SSPI as foundations to investigate power in human institutions. We conclude that these abstract mathematical constructs are closely related to our human world, although usually masked by innumerable complexities. This experiment minimises many of those complexities. The analysis measures effects from focal points and transaction costs, allowing them to be netted from gross experimental results. Our aggregated net results show that the subjects with the most votes in a variety of vote profiles earned only four percent more than their SSPI values. In a human institution tailored to power indices, the Shapley Value normalised here to the SSPI performed remarkably well.

We conclude that a small gap separates our human world from the abstract world of power indices. The SSPI provides an empirically suitable foundation for interpreting actions in human institutions in that it allows researchers to control for the power of votes per se. With raw voting power in controlled, observation researchers may focus more precisely on social, economic and other political influences.

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Table 1: Vote profiles with largest player's power by index

Profile	S.S.P.I.
p_1={31:11,10,10,10,10,10}	33.3
p_2={36:14,12,12,11,11,11}	33.3
p_3={22:8,7,7,7,7,7}	33.3
p_4={16:11,4,4,4,4,4}	33.3
p_5={20:14,5,5,5,5,5}	33.3
p_6={12:8,3,3,3,3,3}	33.3
r_1={13:8,4,3,3,3,3}	36.7
r_2={18:10,7,5,4,4,4}	36.7
r_3={9:5,3,2,2,2,2}	36.7
s_1={10:5,3,3,3,2,2}	31.7
s_2={25:10,8,8,8,7,7}	31.7
s_3={13:6,4,4,4,3,3}	31.7
t_1={24:10,8,8,7,7,6}	30.0
t_2={18:8,6,6,5,5,4}	30.0
t_3={36:14,12,12,11,11,10}	30.0
u_1={18:7,6,6,6,5,5}	30.0
v_1={22:8,7,7,7,7,6}	26.7
w_1={13:8,5,3,3,3,3}	40.0

S.S.P.I = Power of largest player according to the Shapley-Shubik Index

Table 2 Earnings Adjusted for Focal Points

Profile	Average Earnings Percent	Adjustment Factor	Adj Ave Earnings	N	Adj Ave Earnings Percent	SSPI	Empirical / SSPI
p_1	**42.2**	**0.94**	5.98	6	39.9	33.3	1.20
p_2	**53.3**	**0.81**	6.48	3	43.2	33.3	1.30
p_3	**42.2**	**0.82**	5.20	6	34.6	33.3	1.04
p_4	**11.7**	**1.08**	1.89	4	12.6	33.3	0.38
p_5	**29.2**	**1.41**	6.16	8	41.0	33.3	1.23
p_6	42.7	1.19	7.59	5	50.6	33.3	1.52
r_1	28.3	1.03	4.38	4	29.2	36.7	0.80
r_2	**49.5**	**0.83**	6.19	7	41.3	36.7	1.13
r_3	**66.7**	**0.93**	9.29	3	61.9	36.7	1.69
s_1	20.0	0.88	2.65	3	17.7	31.7	0.56
s_2	50.0	1.00	7.50	4	50.0	31.7	1.58
s_3	**36.7**	**0.88**	4.86	4	32.4	31.7	1.02
t_1	**46.7**	**0.67**	4.70	3	31.3	30.0	1.04
t_2	**52.7**	**0.75**	5.94	5	39.6	30.0	1.32
t_3	**40.0**	**0.62**	3.69	4	24.6	30.0	0.82
u_1	**4.8**	**1.88**	1.34	7	8.9	30.0	0.30
v_1	**39.2**	**0.72**	4.21	8	28.0	26.7	1.05
w_1	**55.0**	**0.64**	5.25	4	35.0	40.0	0.88
Weighted overall average				88			**1.040**

Bold: 14 of 18 profiles, multiplier is suitable magnitude relate to 1.

Appendix 1: Subject instructions

We provided these written instructions to subjects when they were seated in the computer laboratory. Before experiments with subjects who had not previously played, we reviewed the instructions verbally, emphasising: the procedure of the game; use of the chat room. eg. do not close the chat rooms in between rounds; that players should try to make as much money as they could; ethical considerations such as consensual participation, equal potential for all participants to earn money, the possibility of earning nothing; that there should be no communication outside of the chat room or between rounds; and that all proposals could be changed to the advantage of a new coalition of players, and that the purse is really $15 per round in contrast to the written directions.

Player Instructions for the Power Index Game

You're about to become a 'player' in an experimental game that investigates the influence that voting power of electoral groups has on democratic decisions. Confusing? A player could represent a political party and all its voters or a corporate board member where votes could be the number of your shares. The procedures are really quite simple and you get to make some big money!

The Game is as Follows:

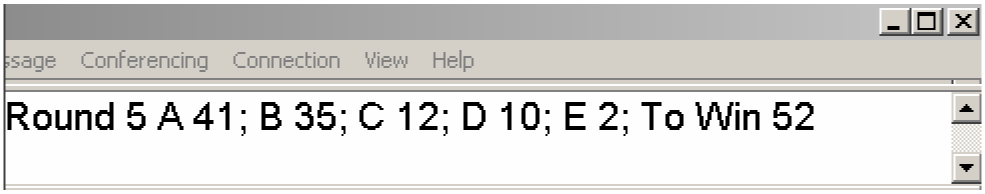
- You will be divided into groups, but you won't know whom you're playing against.
- You will communicate only through open First Class chat rooms (not private).
- You will be given a username such as player **A,** or **B**.
- A game consists of twelve rounds.
- In each round you are allocated a different number of votes, e.g. 12 or 2 votes.
- In each round your group has $15 to divide democratically.
- Division of money is done by offering a proposal, and accepting the proposal.
- Each round goes for a maximum of 5 minutes.
- At the end of all the rounds you get the sum of what you earned in CASH.

Your job is to be GREEDY and get as much money as possible!

Game Notation:

It is difficult to grasp the relationship between votes and power and so is getting used to the games notation.

You will see a 'vote vector' like this:



This tells us that:
- We are in round 5
- Player A has 41 votes
- Player B has 35 votes etc…
- To win this game the players who agree on a proposal must have at least 52 votes combined to win.

Lets say you are playing for $10 in this round. To offer a proposal in the open chat room you could use the following notation:

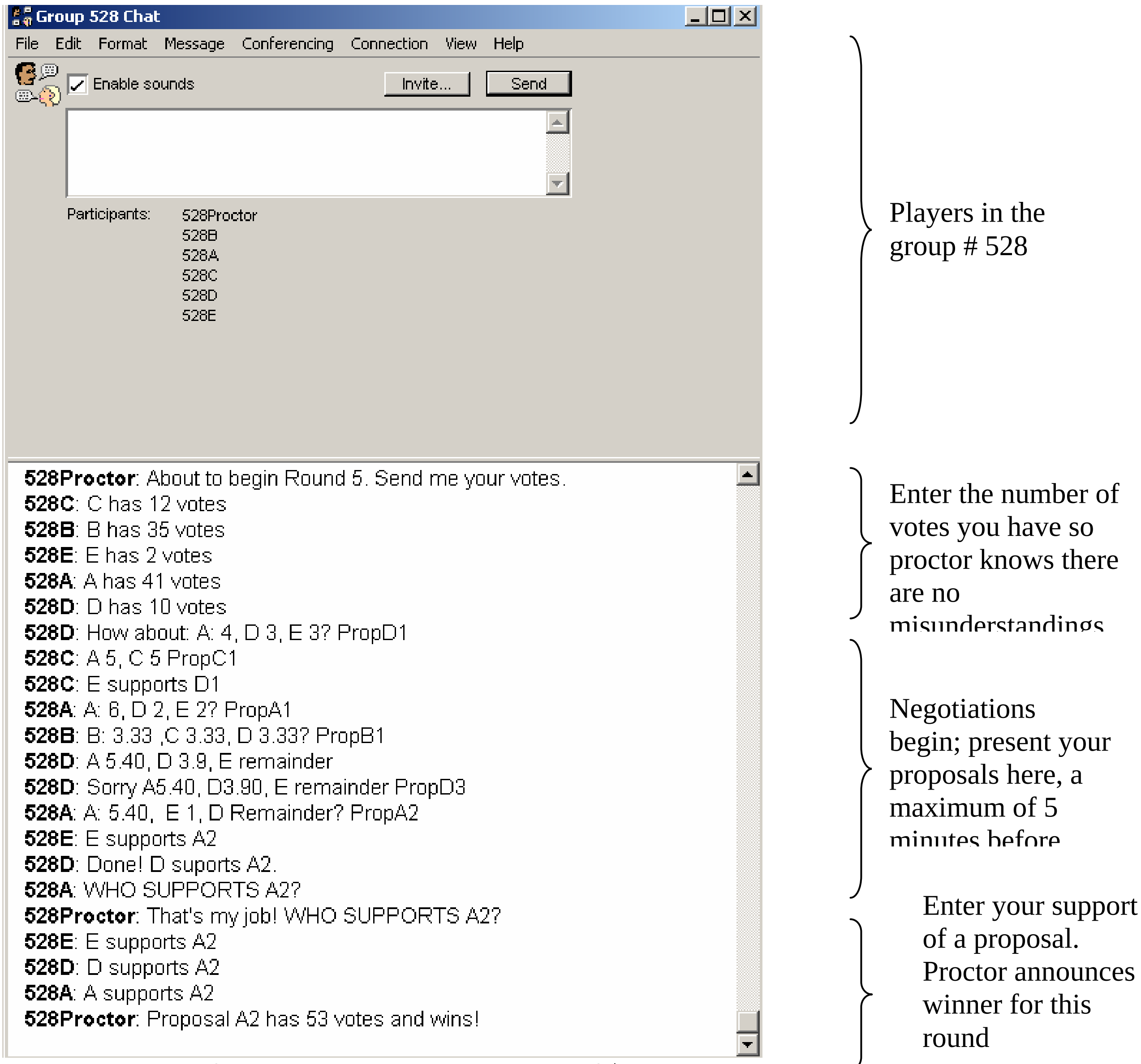
Prop B1: A5 B5

This tells us that:
- Player B is making his/her first proposal (prop B1)
- Player B offers A $5 and B $5
- The combined votes (if A accepts) is 76, this proposal has majority votes and is hence accepted.

If Player A doesn't accept and player B wishes to make a second proposal it should be titled **Prop B2,** or just **B2.**

To accept a proposal Player A could write:
Supports B1

An example of a game for the above vote vector is as follows:



Note: Example is from the pre-study with a purse of $10.

Here you can see that the 'proctor' announces which proposal wins. Proposal A2 has since players A, D, and E all support A2 giving a total of exactly 53 votes. Whereby, **A** receives $5.40, **E** $1 and **D** $3.60. i.e. A 5.40, E 1, D remainder Prop A2

It is also interesting to note that just because you're allocated are large amount of votes doesn't necessarily mean you will get the biggest earnings! Player B with 35 votes completely missed out! On one hand, Player B should have (once he realised that he may not get anything) offered a proposal like; Prop B2: A 8, B 2. He could then get $2 instead of **0.** In this case player A could be greedier and accept this proposal, forgetting any proposals that included players C, D, or E!

Playing the Game

At the start of the game, the proctor will send a message:
“About to begin round 1. Send me your votes.”

Reply stating your letter and number of votes:
“A has 41” or **“B has 35”** etc.

Then the proctor will announce the beginning of the round:
“Start round 1”

And the end of the round or when sufficient support is shown before a proposal:
“End of round Prop ## wins. Stop voting” At this point no more proposals can be offered or accepted

If a proposal appears to have enough votes to win, the proctor will call a vote. (If the proctor has not noticed support for a proposal, you may send a message pointing out that a proposal has received enough support.) The proctor will say, for example, ‘Call for votes.’ At that time you cast a vote for one proposal: ‘I vote for PropE89’. There is no need to vote against a proposal because the Proctor will count no reply as a vote against. So, if you want a proposal to pass, vote for it even if it is your proposal. Proctors will declare if a proposal has passed or not.

- It is also not imperative that you stick to the notation examples, plain English is fine, however we have found that short hand is easily understood.
- A good message could be ‘Prop F43 is nearly winning. Hey F, C&D and I will give you more. B 3.25 C 2.00 D 2.00 F 2.75: PropB46.’
- You can argue about a proposal, try to persuade other players to support your proposal…whatever gets you the most money!

You will want to find other players whose votes can be added to yours to equal or exceed the minimum votes required to divide the money. You get them to cooperate with you by offering them some of the money. Of course, you want to keep as much as you can for yourself. Other players will propose deals that give you nothing. Perhaps you can get some of them to share with you by offering them more than their current deal.

Once the game is over, wait for the proctor to give you instructions to begin a new game.

Game Rules:

- No Private Chat rooms are allowed!

- You are not allowed to make references to the world outside of the game. You may not use your names or any other real world identifier. You may not make side deals such as offers to do homework for or threats of violence to other players in the game. Players who violate these rules will forfeit their earnings and be removed from the game

- Be ***Greedy***! Try to get as much money as possible. In previous experiments players have earned $60 after a couple of rounds! Don't feel sorry for players with a small amount of votes. You never know in the next round they could have the majority! Everyone has the same potential to earn money in the game.

- If you identify two proposals with the same number and one of them passes, we will give you the lower amount of money.

Summary:

Let us review. In each round of this game you will divide $15 between yourself and other players. You will get some votes which may change between every round. You will use a Group chat room to discuss how the money should be divided. You will vote on who gets the money in the Group chat. Whatever you earn in this game is yours to keep.

Consider some possible strategies. You could:

- Look at every player's votes to see what combinations can win.

- Who can help you? Who can you help?

- Are any players dependent on your votes in particular?

- Make offers quickly hoping to make big earnings before others catch on.

- If you are left out of a proposal, think of a way to divide the money so that some of the players in the proposal get more by voting with you and your votes are enough with theirs to create a majority. That is, break up coalitions that leave you out.

Please keep several points in mind:

- You are playing for real money.

- You may earn nothing in this experiment.

- You don't know who is in your group and you cannot speak during rounds

- Check the Votes Vector area because votes may change between rounds.

- Be careful not to close the group chat room or the vote vector.

- It is a majority of votes, not of players that decides how the money gets divided.

- You may not offer any deal that involves anything from outside of the game. That is, you cannot offer to do someone's homework or threaten them to make them cooperate with a deal.

- Every majority coalition of players can be broken.

- This game is divided into rounds. In each round, your group of players will divide $15 among yourselves.

- Each round will last up to five minutes, but may be shorter if players reach an agreement sooner. Any later and no one gets any money

- Keep an eye out for bogus proposals. Player B for example names a proposal 'D2'. This is technically possible, against the rules, probably pointless and silly, but it could cause confusion.

- Make sure that your proposals contain players that will give you the minimum amount of votes 'To win'. For example if B has 35 and C has 2 votes, To win is 52, then Prop B1 B5, C5 cannot work since the summation of the votes is only 37

- Make sure your proposals add up! If you're dividing $15, make sure that you don't offer A3 B9, C5. This adds up to $17…not $15

Appendix 2: Risk aversion instrument

In this section the alternatives are different chances that you will receive different amounts of money, given that we select your reply. We will select twenty replies for this section and each of these twenty will have the following chances to win the following rewards.

Consider this example:

	A	B	C
(10) Chance:	100%	50%	10%
Winnings	$5.00	$10.00	$50.00

Suppose that this reply was one of the twenty we chose for payment. If you selected alternative A, we would give you five dollars. IF you chose B we would flip a coin, and if it came up 'heads' we would give you ten dollars. If you chose C we would roll a ten sided dice, and if it came up 10, we would give you fifty dollars.

Choose and circle A, B or C for each of the following alternatives.

	A	B	C
(10) Chance:	100%	50%	10%
Winnings	$5.00	$10.00	$50.00

	A	B	C
(11) Chance:	100%	50%	10%
Winnings	$4.00	$12.00	$40.00

	A	B	C
(12) Chance:	100%	50%	10%
Winnings	$4.50	$10.00	$45.00

	A	B	C
(13) Chance:	100%	50%	10%
Winnings	$4.00	$10.00	$40.00

Note: This is not part of questionnaire. Risk loving answers are: C,C,C,C; C,B,C,B; and C,B,C,C. Risk neutral answers are: B,B,B,B; A,B,B,B; and C,B,B,B. Risk averse answers are: A,A,A,A; A,B,A,B; and A,B,A,A. Other combinations are intransitive.

Appendix 3: Data description: Characteristics of the subject with the most votes per round and of profiles

	Number	Mean %	St Dev %	Min %	Median %	Max %
Earnings	291	35.4	22.8	0	40.0	73.3
Shapley-Shubik	291	32.8	3.0	26.7	33.3	40.0
Male	291	72.5	44.7			
Foreign	291	7.6	26.5			
Individualist	291	50.5	50.1			
Risk Defined	288	86.5	34.3			
Risk Averse	288	16.7	37.3			
Risk Neutral	288	56.9	49.6			
Risk Loving	288	12.8	33.5			
In Winning Coalition	291	74.6	43.6			
Equal	291	38.5	48.7			
Unequal	291	43.0	49.6			
Homogeneous	291	30.2	46.0			

	Earn%	SS%	In WC	Male	Foreign	Individ.	Risk Averse	Risk Neutral	Risk Loving	Risk Defined	Equal	Unequal	Hom.
Earnings	1.00	0.17	0.91	-0.02	0.01	-0.06	-0.10	0.09	-0.02	0.00	-0.06	-0.07	0.08
Shapley Shubik	0.17	1.00	0.05	0.02	-0.06	-0.04	-0.12	0.07	0.00	-0.03	-0.43	0.51	-0.04
In Winning Coalition	0.91	0.05	1.00	-0.02	0.02	0.01	-0.06	0.05	-0.01	0.00	0.01	-0.11	0.04
Male	-0.02	0.02	-0.02	1.00	-0.46	0.13	-0.31	0.07	0.24	0.00	0.03	-0.03	-0.01
Foreign	0.01	-0.06	0.02	-0.46	1.00	0.21	0.50	-0.30	-0.11	0.00	-0.09	-0.04	-0.08
Individualist	-0.06	-0.04	0.01	0.13	0.21	1.00	0.45	-0.04	-0.36	0.07	-0.02	0.04	-0.29
Risk Averse	-0.10	-0.12	-0.06	-0.31	0.50	0.45	1.00	-0.51	-0.17	0.18	-0.01	-0.06	-0.16
Risk Neutral	0.09	0.07	0.05	0.07	-0.30	-0.04	-0.51	1.00	-0.44	0.46	0.00	0.11	-0.11
Risk Loving	-0.02	0.00	-0.01	0.24	-0.11	-0.36	-0.17	-0.44	1.00	0.15	-0.05	0.01	0.13
Risk Defined	0.00	-0.03	0.00	0.00	0.00	0.07	0.18	0.46	0.15	1.00	-0.06	0.09	-0.20
Equal	-0.06	-0.43	0.01	0.03	-0.09	-0.02	-0.01	0.00	-0.05	-0.06	1.00	-0.69	0.00
Unequal	-0.07	0.51	-0.11	-0.03	-0.04	0.04	-0.06	0.11	0.01	0.09	-0.69	1.00	-0.10
Homogeneous	0.08	-0.04	0.04	-0.01	-0.08	-0.29	-0.16	-0.11	0.13	-0.20	0.00	-0.10	1.00

Appendix 4: Largest subject earnings by control variables

		p_i	profiles	not p_i	profiles	all	profiles
Overall average	Mean	1.010		1.131		1.078	
	St Dev	0.671		0.698		0.688	
	N	128		163		291	
	Mean in WC	1.390		1.486		1.445	
	St Dev in WC	0.296		0.330		0.319	
	in WC	93		124		217	
	Conf Int H	1.054		1.176		1.110	
	Conf Int L	0.965		1.086		1.045	
		1	0	1	0	1	0
Male = 1	Mean	1.015	0.999	1.104	1.212	1.067	1.106
Female = 0	St Dev	1.000	0.683	0.706	0.677	0.691	0.684
	N	88	40	123	40	211	80
	Mean in WC	1.395	1.378	1.476	1.515	1.443	1.450
	St Dev in WC	0.283	0.330	0.335	0.321	0.316	0.330
	in WC	64	29	92	32	156	61
	Conf Int H	1.066	1.088	1.157	1.303	1.104	1.170
	Conf Int L	0.963	0.910	1.052	1.121	1.030	1.042
Foreign = 1	Mean	0.755	1.031	1.433	1.107	1.125	1.074
Australian = 0	St Dev	0.723	0.665	0.516	0.706	0.695	0.689
	N	10	118	12	151	22	269
	Mean in WC	1.258	1.399	1.564	1.479	1.456	1.444
	St Dev in WC	0.425	0.287	0.263	0.336	0.350	0.317
	in WC	6	87	11	113	17	200
	Conf Int H	0.963	1.077	1.579	1.154	1.256	1.107
	Conf Int L	0.547	0.986	1.288	1.059	0.994	1.041
Individualist = 1	Mean	0.919	1.106	1.136	1.125	1.039	1.117
Pro-Social = 0	St Dev	0.683	0.650	0.641	0.755	0.667	0.709
	N	66	62	81	82	147	144
	Mean in WC	1.333	1.429	1.416	1.564	1.388	1.503
	St Dev in WC	0.312	0.278	0.333	0.313	0.325	0.304
	in WC	45	48	65	59	110	107
	Conf Int H	0.982	1.169	1.203	1.184	1.085	1.161
	Conf Int L	0.855	1.044	1.070	1.067	0.992	1.073

Earnings are expressed as a portion of the Shapley-Shubik Power Index value, for ease of comparison.
Mean in WC and St Dev in WC give the mean and standard deviation of earnings of the largest subject when the largest subject is in the winning coalition.
The mean and standard error for the largest subject's earnings when not part of the winning coalition were, of course, zero.
in WC gives the number of observations with the largest subject in the winning coalition.
Conf Int H Conf Int L give the upper and lower bound of the earnings of the earnings of the largest subject whether or not the largest subject was in the winning coalition.
Conf Int H, Conf Int L = Mean $\pm$ 2(in WC/n)[St Dev in WC/(in WC)$^{1/2}$]

Appendix 4: Largest subject earnings by control variables, continued

		p_i profiles		not p_i profiles		all profiles	
		1	0	1	0	1	0
Risk defined =1	Mean	0.997	1.063	1.139	1.081	1.077	1.073
	St Dev	0.679	0.680	0.697	0.718	0.692	0.694
	n	109	16	140	23	249	39
	Mean in WC	1.393	1.417	1.490	1.462	1.449	1.443
	St Dev in WC	0.297	0.289	0.330	0.341	0.320	0.316
	in WC	78	12	107	17	185	29
	Conf Int H	1.045	1.188	1.188	1.203	1.112	1.160
	Conf Int L	0.949	0.938	1.090	0.958	1.042	0.986
Risk Averse = 1	Mean	0.844	1.031	1.020	1.157	0.958	1.100
	St Dev	0.695	0.674	0.718	0.694	0.707	0.686
	n	17	108	31	132	48	240
	Mean in WC	1.305	1.409	1.437	1.497	1.393	1.459
	St Dev in WC	0.335	0.289	0.328	0.332	0.331	0.316
	in WC	11	79	22	102	33	181
	Conf Int H	0.975	1.078	1.119	1.208	1.037	1.135
	Conf Int L	0.713	0.983	0.921	1.106	0.879	1.065
Risk Neutral =1	Mean	1.055	0.928	1.170	1.084	1.117	1.022
	St Dev	0.683	0.667	0.683	0.718	0.983	0.700
	n	76	49	88	75	164	124
	Mean in WC	1.432	1.337	1.493	1.478	1.466	1.424
	St Dev in WC	0.294	0.290	0.330	0.335	0.314	0.324
	in WC	56	34	69	55	125	89
	Conf Int H	1.113	0.997	1.233	1.150	1.160	1.071
	Conf Int L	0.997	0.859	1.108	1.018	1.074	0.973
Risk Loving = 1	Mean	0.881	1.023	1.183	1.123	1.052	1.080
	St Dev	0.646	0.682	0.741	0.694	0.708	0.690
	n	16	109	21	142	37	251
	Mean in WC	1.282	1.412	1.552	1.477	1.442	1.449
	St Dev in WC	0.248	0.299	0.345	0.329	0.333	0.317
	in WC	11	79	16	108	27	187
	Conf Int H	0.984	1.072	1.314	1.171	1.146	1.115
	Conf Int L	0.778	0.975	1.051	1.075	0.958	1.045
Homogeneous=1	Mean	1.088	0.984	1.214	1.087	1.168	1.038
	St Dev	0.682	0.640	0.729	0.681	0.697	0.682
	n	32	96	56	107	88	203
	Mean in WC	1.392	1.389	1.581	1.436	1.511	1.415
	St Dev in WC	0.297	0.298	0.320	0.327	0.323	0.314
	in WC	25	68	43	81	68	149
	Conf Int H	1.180	1.035	1.289	1.142	1.229	1.076
	Conf Int L	0.995	0.933	1.139	1.032	1.107	1.000

Appendix 4: Largest subject earnings by control variables, continued

		p_i	profiles	not p_i	profiles	all	profiles
		1	0	1	0	1	0
Relatively equal vote distribution=1	Mean	1.162	0.895	1.002	1.200	1.081	1.076
	St Dev	0.573	0.719	0.765	0.653	0.679	0.695
	n	55	73	57	106	112	179
	Mean in WC	1.389	1.390	1.503	1.479	1.441	1.448
	St Dev in WC	0.268	0.325	0.335	0.330	0.303	0.330
	in WC	46	47	38	86	84	133
	Conf Int H	1.228	0.956	1.074	1.258	1.131	1.119
	Conf Int L	1.096	0.834	0.930	1.142	1.031	1.033
Relatively unequal vote distribution=1	Mean			1.054	1.167	0.961	1.165
	St Dev			0.685	0.705	0.707	0.662
	n			52	111	125	166
	Mean in WC			1.503	1.479	1.397	1.476
	St Dev in WC			0.335	0.330	0.334	0.306
	in WC			38	86	86	131
	Conf Int H			1.133	1.222	1.011	1.207
	Conf Int L			0.975	1.111	0.911	1.123