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Simone Kohnz:

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Department of Economics
University of Munich

Volkswirtschaftliche Fakultät
Ludwig-Maximilians-Universität München

Ratification quotas in international agreements
An example of emission reduction

Simone Kohnz*

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Abstract

This paper analyses the role of ratification quotas in multilateral agreements over emission reduction. The higher is the quota, the lower is the level of emissions in case the agreement comes into force, but the higher is also the risk of failure. In a setting with incomplete information, two country types and a binary contribution to the provision, I examine the differences between simultaneous and sequential ratification. When the benefits from emission of both types are smaller than the social costs, the outcome in the simultaneous case is essentially identical to the sequential case. The optimal quota is 100% and achieves the first best. With the high type's benefits exceeding the social costs, I find that the optimal quota is as small as possible, if ratification is simultaneous. In the sequential ratification case, I cannot determine the optimal quota. However, I find that the aggregate expected surplus decreases with respect to the simultaneous case.

JEL classification: D71, H41
KEYWORDS: public goods, international bargaining, ratification, emission games

1 Introduction

For the international provision of global public goods, country representatives gather and bargain over individual contribution levels. The resulting agreement not only specifies each country's contribution, typically it also contains a ratification quota, the minimum number of countries that have to ratify the agreement to render it legally binding for the

*Ph.D. student, Munich Graduate School of Economics, Kaulbachstr. 45, 80539 Munich, Germany. Please email to: simone.kohnz@lrz.uni-muenchen.de. I am grateful for the comments of Klaus Schmidt, Martin Hellwig, Florian Herold, Chantelle Bramley and Katri Mikkonen.

ratifying countries. This paper explores the effect of a ratification quota on the provision of a transboundary public good. It focuses on two channels via which the quota might impact the provision of the public good. On the one hand, the higher the quota, the higher is the level of the public good provision whenever the agreement comes into effect. On the other hand, a higher quota may also increase the chance of a contractual breakdown, as an insufficient number of countries ratify the agreement. The present paper captures this trade-off in a three stage model where countries first determine the ratification quota, they then decide whether to ratify and at the last stage provide the public good. The three stage nature is inspired by reality where the national representatives first negotiate the agreement, before typically the legislative body of the country decides on ratification and therewith on the provision of the public good. I examine the differences between the situations where each country decides to ratify simultaneously or where it does so sequentially and identify circumstances where it is optimal to have a high or a low quota.[1]

I consider the special case of emission of a global pollutant, like carbondioxides fostering global warming. I restrict the setting to a binary type space; countries have either high or low benefits from local emission. When deciding on the ratification, countries know their own benefit parameter, but not the parameter of any other country. Their expectations of how many other countries ratify are thus crucial in the analysis. In their ratification decision, countries trade-off the expected gain with the expected costs of ratification. I distinguish between the case where the benefit of the high type is smaller or where it is larger than the social costs. With *simultaneous* ratification, I find that the optimal quota is 100% if individual benefits of the high type do not exceed the social costs of emission. Otherwise, if the high type's benefits are larger than the social costs, the optimal quota is as small as possible such that it still induces the low type to ratify. In this latter case, the optimal quota increases with the benefit parameter as well as with the probability of the low type. Furthermore, the optimal quota increases with the number of countries and the optimal quota relative to the number of countries decreases with the number of countries. With *sequential* ratification, the optimal quota is again 100% if the individual benefits of the high types are smaller than the social costs. However, when the individual benefit of the high type is larger than the social costs, the aggregate expected surplus decreases with respect to the simultaneous case. The sequential structure potentially discloses information inducing some low types to refrain from ratification, whereas in the simultaneous structure all low types ratify.

This paper is closely related to Black, Levi, and de Meza (1993). They simulate the effect of a minimum ratification quota on the provision of a global public good. In a

[1] In reality, ratification quotas differ. The Kyoto Protocol specifies that 55 countries (out of 166 signatories) which have to cover at least 55% of total emissions in 1990 have to ratify. The Convention for the Regulation of Whaling indicates 6 countries (out of 14 signatories) including the Netherlands, Norway, the Union of Soviet Socialist Republics, the UK, and the USA. Finally, for the International Criminal Court to come into force, 60 countries (out of 139 signatories) had to ratify. Mostly, ratification quotas are substantially different from 100% as well as from effectively no quota.

simultaneous ratification setting where the types of countries are continuous, Black, Levi, and de Meza find that the optimal ratification quota relative to the number of countries is relatively robust to variations in the number of countries and in the cost parameter, but that it is not robust with respect to the distribution of benefits. In contrast, for the binary type setting I find that the optimal quota decreases with the number of countries, increases with the cost parameter and also increases with the probability of a low type. These differences stem from the different modelling of types. In a setting with binary types, all low types ratify; whereas with continuous types, there exists a critical value such that only those types exceeding this value ratify. This critical value is influenced by all model parameters.

A strand of coalition theory examines the provision of public goods and in particular emission reduction using a cooperative game theory approach.[2] Part of this literature focuses on the effect of ratification quotas in international environmental treaties, called minimum participation levels in that literature. The paper by Rutz (2001) introduces a minimum participation rule in the context of a two-stage coalition model. It shows that a participation rule can potentially overcome the free-rider problem of transboundary pollution. Carraro, Marchiori, and Oreffice (2003) endogenise the participation rule by extending the model via a preceding stage where countries determine the minimum participation level. They establish that the grand coalition is stable with a 100% quota. Furthermore, they determine conditions under which all players agree to a 100% quota.

The literature on step-level public goods centers on the participation issue in the provision of discrete public goods. Palfrey and Rosenthal (1984) analyse participation to the provision of a binary public good where a fixed number of contributors is needed to provide the public good. In contrast to my paper, they assume complete information of the symmetric players and find that the efficient number of players contribute in equilibrium. They do not consider the role of the fixed number of contributors.

Next, I outline the model and its basic assumptions. For simultaneous ratification, section 3 characterises the optimal quota as well as some comparative statics results. Section 4 presents basic results for the case of sequential ratification. The last section summarises the results and indicates future paths of research.

[2] For example, Carraro and Siniscalco (1993), D'Aspremont, Jacquemin, Gabszewicz, and Weymark (1983), Finus and Rundshagen (2001), Barrett (1994) and Diamantoudi and Sartzetakis (2002) look at the provision of a public good within a two-stage coalition model. On the first stage, a single coalition is formed by simultaneous decisions of all countries. On the second stage, countries contribute in a static or dynamic game. In these models, the stable coalitions are generally small regardless of the number of participating countries. The grand coalition is always efficient, though not stable. For an introduction to the literature on international environmental agreements and coalition theory, see Barrett (2003), in particular chapter 7.

2 Model

N countries participate in an international bargaining process over the emission of a global pollutant. At the first stage, countries bargain over a ratification quota. A ratification quota $Q \in \{1, 2, ..., N\}$ is defined as the absolute number of countries that have to ratify the agreement to render it legally binding. Remember, only those countries that ratify the agreement will enforce the provisions of the agreement. At the second stage, each country decides whether to ratify. At the last stage, if a country ratified, it fulfills its obligations of the agreement, or otherwise emits freely.

Countries benefit from their own, local emission through production and consumption activities. But they suffer from the sum of all emissions, called global emission as these reduce environmental quality globally. The relation between the benefits of local emission and the harm of global emission is expressed by the parameter θ. Countries' utility functions are captured by

$$u_i\left(e_i, \sum_{j=1}^{N} e_j\right) = \theta_i e_i - \sum_{j=1}^{N} e_j$$

where $e_i \in \{0, 1\}$ indicates the emission level of country i. I treat the emission decision as binary.[3] Whenever a country is not member to the agreement, it chooses an emission level independently of all other countries' emissions. If the benefits of emission are small in relation to the costs, i.e. $\theta_i \leq 1$, country i does not emit, while in the reverse case, whenever $\theta_i > 1$, it does emit.

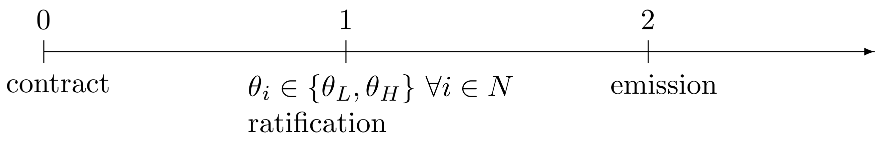
All countries that ratified the agreement satisfy their obligations in case the agreement gets legally binding. I neglect the problem of compliance. With this assumption and the specified utility function, the emission choice on the third stage of the game is uniquely determined.

At the time of deciding on a quota, countries have no information on any of the benefit-damage parameters θ_i. However, they know the distribution function $F(\theta_i)$ which is assumed to be identical and independent for each country i. During the time which elapses between the bargaining over the agreement and its ratification, new information outcrops. Each country i learns its own value of θ_i, but not the realisations of the other countries' parameters. This formulation can be seen as a benchmark analysis to the case where countries do not learn the exact value of their benefit parameter, but get a more precise signal on it. I assume that the parameter can amount two distinct values, a low value θ_L and a high value θ_H, with $\theta_L < \theta_H$. The probability of a low value is denoted by $p = prob\,(\theta = \theta_L)$. After the revelation of information, countries decide whether to

[3] This assumption is particularly helpful as it reduces the dimension of the social maximisation problem. When it comes to the determination of the optimal emission level specified in the agreement, the interesting case is one in which no emission for the ratifying countries is desirable. The agreement therefore essentially determines a level of the ratification quota.

ratify the agreement. I look at two distinct scenarios, the first where countries ratify simultaneously and the second where they do so sequentially, one after the other. In the latter version, when deciding on a quota, countries do not know the order of play. However, they know that it is equally likely to be in any position. Each possible order has the same probability of being drawn after the countries have agreed on the ratification quota. Figure 1 illustrates the time structure of the game.

Figure 1: Timing of the game



In a *first best* world countries internalise the negative externality of their local emission on the rest of the world. As long as the benefit θ_i of local emissions to country i is smaller than the social cost of local emissions N, country i does not emit in a first best world. However, whenever countries are free to emit, they only take into account the damage of their local emission caused on themselves. They neglect the effect of their emission on all other countries. An agreement with a minimum participation level can help to ameliorate this classical free-rider problem. With a minimum participation rule, countries take into account that they may eventually cause a breakdown of the agreement with their decision. An optimal quota is defined to be the number of necessary ratifications that maximises the aggregate expected surplus. A quota that achieves the first best outcome is clearly optimal. Whenever the first best level of pollution is achieved in the Nash equilibrium of the emission game, the design and ratification process of an agreement is of no interest. In these cases, an agreement cannot improve the allocation. The focus of this paper is thus on situations where the Nash equilibrium of the emission game does not achieve the first best outcome. As all players are symmetric at the time of deciding on the quota, the voting process is not essential. All players solve the same maximisation problem and thus decide unanimously on a quota. Therefore, I neglect the design of the voting mechanism.

In the ratification game, players have identical binary action sets $A_i = \{0, 1\}$ with actions labeled {do not ratify, ratify}. A strategy profile $s_i(\theta_i|Q)$ of country i assigns an action $a_i \in A_i$ to every type $\theta_i \in \{\theta_L, \theta_H\}$ given a quota $Q \in \{1, ..., N\}$. There might be multiple equilibria in the ratification game. The aggregate expected surplus therefore depends not only on the ratification quota but also on the specific equilibrium.[4]

Definition 1 *The optimal quota maximises the aggregated expected utility, i.e.* $Q^* = \arg\max_{Q\in\{1,...,N\}} \left\{\arg\max_{s^*(\theta|Q)} N * Eu\left(\left(s^*\left(\theta|Q\right)\right)\right)\right\}$ *where*

[4] For the definition of Bayesian Nash Equilibrium and Perfect Bayesian Equilibrium, see chapter 2.6 and 12.3 in Osborne and Rubinstein (1994).

$s^*(\theta|Q) = (s_1^*(\theta_1|Q), ..., s_N^*(\theta_N|Q))$ *constitutes a symmetric Bayesian Nash Equilibrium (BNE) in pure strategies of the simultaneous ratification game or a symmetric Perfect Bayesian Equilibrium (PBE) in pure strategies of the sequential ratification game.*

The definition of the optimal quota relates to pure strategies. The focus of this paper is to ask what is implementable in pure strategies. The interpretation of mixed strategies is debated among game theorists, see chapter 3 in Osborne and Rubinstein (1994). Considering exclusively pure strategies, avoids the interpretation of mixed strategies in the ratification setting and finds the minimum expected utility that can be implemented in the stronger concept of pure strategies. However, in what follows, I also discuss how the results might change when we allow for mixed strategies.

Next, I solve for the optimal quota in the simultaneous case and establish when it achieves the first best allocation. Furthermore, I conduct some comparative statics on the optimal quota.

3 Simultaneous ratification

In this section, I analyse the case where all countries ratify simultaneously. With simultaneous ratification, countries do not know how many other countries are ratifying. They can merely infer the probability that there is a sufficient number of ratifications such that the agreement gets legally binding. Suppose a country is pivotal, i.e. without this country's ratification the agreement fails. Then, ratification reduces global emissions by the sum of the reduced local emissions of all the ratifying countries. The expected gain of ratification equals the sum of reductions minus the benefit of local emission times the probability of being pivotal. Now, suppose a country is not pivotal. Ratification then implies that this country incurs costs amounting to the foregone benefits minus the damage of local emissions in case the agreement becomes binding. The expected loss of ratification of the country equals these costs times the probability that the quota is satisfied without this country's ratification. A country ratifies if the sum of these net expected gains from ratification exceed naught.

I distinguish between a situation where the agreement aims at inducing cooperation among all types and a situation where it aims at low types only. These two situations differ substantially. In the first case, the optimal quota is as large as possible, while in the second, the reverse is true. The following sections show the reasoning and intuition behind these results.

Case 1: Participation of High Types In this section, I analyse the cases where both types of countries prefer no emissions by any country to maximum emissions by every country. This occurs if the benefits of both types are smaller than the social costs of emission, $\theta_L, \theta_H \leq N$. In the first best allocation, no country emits pollutants and the sum of utilities is zero. However, whenever countries are free to decide on emission, they emit as long as their individual benefits are large enough. That is as long as $\theta > 1$.

An agreement, specifying that a certain amount of countries have to ratify before the agreement comes into force, can ameliorate the free-rider problem inherent in that situation. Suppose the ratification quota is 100%. Then, each country is pivotal for the emergence of the agreement. The potential loss in case of contractual breakdown is large, as emissions might rise from none at all to the maximal level of emissions N. This is an extreme scenario as the agreement induces all countries to take their decision on the background of comparing a situation with no emission to a situation with full emission by all countries.[5] Ratification of all countries and therefore the first best allocation can be achieved.

The above argument relies on the assumption that all countries, regardless of their type, emit if the agreement fails. However, benefits of the low type countries might be smaller than the individual costs, i.e. $\theta_L \leq 1$. Low types might thus abstain from emission regardless of the agreement. In this case, the agreement tries to establish cooperation foremost among the high types. Suppose again that the ratification quota is 100% and every country is pivotal. Whenever the agreement fails, the loss for the high type countries is smaller than in the above scenario as low type countries do not emit for sure. For the agreement to successfully lure high types to participation, the expected gain from ratification must exceed the loss θ_H. Countries that ratify forego the damage of their own emission plus the expected damage of emissions by the remaining $N-1$ countries. Therefore, the expected gain of ratification amounts to $1 + (N-1)(1-p)$. Given that gains exceed losses, ratification of all countries can be achieved by a 100% quota. Proposition 1 summarises this by characterising the conditions under which the symmetric pure strategy to ratify regardless of the benefit type is a BNE and stating the optimal quota.

Proposition 1 *Given $\theta_H \leq N$, a quota $Q = N$ is a necessary condition for the symmetric pure strategy $s(\theta_j|Q) = 1$ for $\theta_j \in \{\theta_L, \theta_H\}$ to be a BNE. If $\theta_L > 1$, this is sufficient. Otherwise, additionally $\theta_H \leq 1 + (N-1)(1-p)$ has to be satisfied. The optimal quota is $Q^* = N$.*

Proof. If $Q < N$, there exists an incentive to deviate from the proposed equilibrium strategy. Given all other countries follow the proposed strategy to ratify regardless of their type, country i's incentive to deviate is

$$\begin{aligned} D(\theta_j, Q < N) &= E(u(0|\theta_j, Q < N)) - E(u(1|\theta_j, Q < N)) \\ &= \theta_j - 1 \end{aligned} \tag{1}$$

[5] Suppose one country abstains from ratification. One might think that it is not credible that there will be no cooperation among the ratifying countries. The literature on coalition formation shows, however, that the maximum number of countries forming a stable coalitions in public good environments is very small, see Carraro and Siniscalco (1993) and D'Aspremont, Jacquemin, Gabszewicz, and Weymark (1983). Therefore the situation is close to the one modelled.

for $j \in \{L, H\}$. At least for the high type θ_H, equation (1) is positive and thus the country has an incentive to deviate from the proposed strategy.

If $Q = N$ and $\theta_L > 1$, there exists no incentive to deviate as each country is pivotal and $D(\theta_j, N) = \theta_j - N \leq 0$ for $j \in \{L, H\}$. If $Q = N$, but $\theta_L \leq 1$ holds, then there is no incentive to deviate for the high types if

$$\begin{aligned} D(\theta_H, N) &= \theta_H - 1 - \sum_{P=0}^{N-1} b(P|N-1, p)(N-1-P) \\ &= \theta_H - N + (N-1)p \leq 0 \end{aligned}$$

where $b(P|N-1, p)$ denotes the binomial distribution
$b(P|N-1, p) = \begin{pmatrix} N-1 \\ P \end{pmatrix} p^P (1-p)^{N-1-P}$. This implies the stated condition.

For $\theta_L > 1$ or $\theta_L \leq 1$ and $\theta_H \leq 1 + (N-1)(1-p)$, a 100% quota implies that there is no global emission. Hence $Q = N$ achieves the first best outcome and is thus optimal. For $\theta_L \leq 1$ and $\theta_H > 1 + (N-1)(1-p)$, all low types do not emit while all high types emit when there is no agreement. No agreement irrespective of the quota can improve on this result. By definition, any quota is therefore optimal. ■

The intuition is straightforward. If each country has a relatively small benefit from its own emission, a 100% quota forces participation of all countries and induces the first best allocation with no emission.[6] The restriction to pure strategies is innocuous in this case. Whenever the pure strategy to ratify regardless of the type can be implemented with a 100% quota, this quota achieves the first best and no other equilibrium can improve on that. Furthermore, if the low type's benefit θ_L is smaller than 1 and the condition that $\theta_H \leq 1 + (N-1)(1-p)$ is not met, then mixed strategies cannot improve the outcome either. Keep in mind that in this situation we cannot induce high types to ratify even if all other countries do ratify. The expected gain from a working agreement is simply not large enough. If we allow high types to mix, then the expected utility of ratification decreases further. Appendix 6.A illustrates that even though the expected utility of no ratification also decreases, it always exceeds the expected utility of ratification. There is thus no mixed strategy equilibrium.

We now turn to the situation where the benefit of the high types exceeds the social costs and high types are thus never going to participate.

Case 2: Participation of Low Types The situation is different when we look at cases where the high types' benefits are above the social costs, $\theta_H \geq N$. It is not possible to induce these countries to ratify an agreement which obliges them to abstain from emission.[7]

[6] In the paper by Black, Levi, and de Meza (1993) this case is not considered. In their simulations, they assume that the benefit parameter is fixed $\theta > 1$ and the costs are drawn from the unit interval $c \in (0, 1)$. Thus, types with low costs prefer to emit than to ratify.

[7] We abstract from the possibility of side payments, see the conclusion for further discussion.

Hence, an agreement can only build up cooperation among the low benefit types. The candidate symmetric pure strategy is to ratify if the country is of low type and to abstain otherwise, $s(\theta_i|Q) = \begin{cases} 1 & \text{if } \theta_i = \theta_L \\ 0 & \text{else} \end{cases}$.

Given a ratification quota Q, there are $N - Q + 1$ possible states at the last stage of the game. There is either no binding agreement or an agreement that is binding for $P \in \{Q, ..., N\}$ countries. The incentive to deviate from the proposed strategy given all other $N - 1$ countries follow the strategy is given by the incentive expression

$$\begin{aligned} D(\theta_L, Q) &= E(u(0|\theta_L, Q)) - E(u(1|\theta_L, Q)) \\ &= (\theta_L - 1) \sum_{P=Q}^{N-1} b(P|N-1, p) - (Q - \theta_L) b(Q-1|N-1, p), \end{aligned} \tag{2}$$

where $(P|N-1, p)$ again denotes the binomial distribution with parameters $N - 1$ and p.[8] The proposed strategy to ratify if of low type and not otherwise is only sustainable in equilibrium if the incentive expression (2) is negative.

The first term captures the expected costs of ratification whenever the agreement gets legally binding and the country is not pivotal. It is the probability that more than Q countries ratify times the benefit of local emission θ_L minus the damage of local emission of 1. Whereas the second term signifies the gain of ratification if the agreement gets binding and the country is pivotal, i.e. exactly $Q - 1$ other countries ratified. A straightforward insight is that whenever there is no gain from ratification, that is, whenever the quota is smaller than the benefits, the proposed strategy cannot be sustained in equilibrium. This implies a lower bound on the optimal quota $Q > \theta_L$. It is not possible to solve analytically for the quota at which equation (2) equals zero (resp. is just negative). However, Proposition 2 shows that this minimum quota is optimal.

Proposition 2 *With* $1 < \theta_L \leq N < \theta_H$ *and* $p \in (0,1)$, *there exists a unique optimal quota* Q^* *that is the smallest integer for which* $D(\theta_L, Q^*) \leq 0$.

Proof. The aggregate expected surplus is maximised if the individual expected utility is maximised, $pE(u(s(\theta_L|Q))) + (1-p) E(u(s(\theta_H|Q))) \to \max_Q$. Given each country follows the strategy $s(\theta_i|Q) = \begin{cases} 1 & \text{if } \theta_i = \theta_L \\ 0 & \text{else} \end{cases}$, taking first differences of the individual expected utility yields

$$b(Q-1|N-1, p)\, p(\theta_L - N) < 0. \tag{3}$$

The derivation of (3) is found in Appendix 6.C. Thus, the quota that maximises individual and therewith aggregate surplus is as small as possible, still satisfying equation (2).

Next, I turn to characterise the incentive expression (2). For all $Q \in [1, \theta_L]$, the incentive expression is positive as there is no gain from being pivotal, i.e. $D(\theta_L, Q) \geq$

[8] A detailed derivation of the incentive expression can be found in Appendix 6.B.

$(\theta_L - 1)\sum_{i=\lceil\theta_L\rceil}^{N-1} b(i|N-1,p) > 0$. Moreover, whenever the quota is 100%, the incentive expression becomes strictly negative,
$D(\theta_L, N) = b(N-1|N-1,p)(\theta_L - N) < 0$. Taking first differences of the incentive expression using the fact that

$$b(Q|N-1,p) = b(Q-1|N-1,p)\,\frac{p}{1-p}\frac{N-Q}{Q} \tag{4}$$

yields

$$D(\theta_L, Q+1) - D(\theta_L, Q) = -b(Q-1|N-1,p)\left(\frac{pN-Q}{1-p} + \theta_L\right)$$

The incentive expression is therefore decreasing in Q if $Q \leq Q' = p(N-\theta_L) + \theta_L$ and is increasing if $Q \geq Q' = p(N-\theta_L) + \theta_L$. This implies that $D(\theta_L, \cdot)$ jumps the x-axis exactly once. The minimum quota that still satisfies the incentive expression (2) is located at the jump (or just after). ■

Proposition 2 states that it is optimal to set the ratification quota as low as possible to the level where the low type country is just willing to ratify. Intuitively, this is appealing. If the quota is lower than this level, all the low type countries switch to a strategy of no ratification. The agreement does not get legally binding and every country emits. This cannot be optimal. Though, if the quota is higher, then all low type countries ratify. The higher quota implies that more countries have to be of low type. Therefore, the risk of breakdown of the agreement is increased as more low type countries are needed for ratification. This cannot be optimal either.

Next, I discuss some comparative statics results. The impact of an increase in the benefit parameter of the low type is straightforward. The incentive expression (2) increases with an increase in the low type parameter θ_L, as the costs increase in case the country is not pivotal and at the same time the gains decrease in case the country is pivotal. The optimal quota must therefore be increased.

Proposition 3 *With $1 < \theta_L \leq N < \theta_H$ and $p \in (0,1)$, the optimal quota $Q^*(\theta_L, p, N)$ is increasing in θ_L.*

Proof. The incentive expression is increasing with θ_L as

$$\frac{\partial D(\theta_L, p, N)}{\partial \theta_L} = \sum_{i=Q-1}^{N-1} b(i|N-1) > 0$$

Furthermore, we know that the incentive expression decreases with Q for all $Q \leq Q' = p(N-\theta_L)+\theta_L$ and increases for all $Q > Q' = p(N-\theta_L)+\theta_L$. As the incentive expression starts from a positive and ends with a negative value, the cutoff level Q' has to be larger than the optimal quota, $Q^* \leq Q'$. The incentive expression is decreasing around the optimal quota Q^*. This implies that the optimal quota $Q^*(\theta_L, p, N)$ increases with the benefit parameter θ_L. ■

The impact of an increase in the probability of the low type is less straightforward. On the one hand, as the probability of the low type increases, the probability that there are sufficient ratifications from low type countries increases. On the other hand, the effect on the probability of being pivotal is undetermined; the probability could be increasing or decreasing. The following proposition shows that even if the probability of being pivotal increases with an increase of p, the gain associated with this is offset by the costs of the increased probability of sufficient ratifications. The incentive expression increases and the optimal quota increases with the probability of a low type.

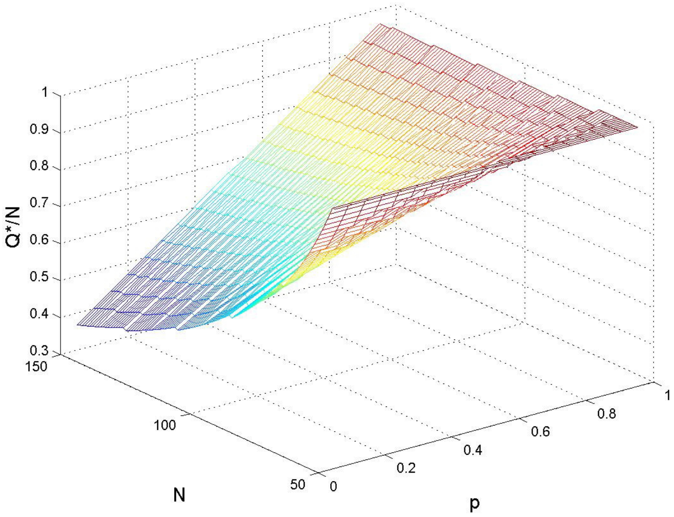
Proposition 4 *With $1 < \theta_L \leq N < \theta_H$ and $p \in (0,1)$, the optimal quota $Q^*(\theta_L, p, N)$ is increasing in $p \in (0,1)$.*

The proof is delegated to Appendix 6.C. At first, the impact of a variation of the number of countries N seems ambiguous. On the one hand, as the number of countries increases, the probability that there are sufficient ratifications from low type countries increases. This is due to the fact that the cumulative distribution function of the binomial distribution with parameter N first order stochastically dominates the binomial distribution with parameter $N-1$. On the other hand, the effect on the probability of being pivotal is undetermined; the probability could be increasing or decreasing with an increase in N. Intuitively, the optimal quota should be increasing in the number of countries N. A constant or even decreasing quota with increasing N does not seem plausible as the probability that sufficient countries out of the $N-1$ other countries are of low type $\sum_{P=Q}^{N-1} b(P|N-1,p)$ is converging to one when N becomes large while the probability of being pivotal $b(Q-1|N-1,p)$ goes to zero. Hence, low type countries have an incentive to abstain from ratification. Therefore, we should observe an increase in the quota with N. Proposition 5 confirms this intuition.

Proposition 5 *With $1 < \theta_L \leq N < \theta_H$ and $p \in (0,1)$, the optimal quota $Q^*(\theta_L, p, N)$ is increasing in N.*

The proof is delegated to Appendix 6.C. One might also be interested in how the relative optimal quota $q^*(\theta_L, p, N) = \frac{Q^*(\theta_L, p, N)}{N}$ evolves with an increasing number of countries N. To see this, I run simulations of the relative optimal quota over a parameter range of $N \in \{3, 4, ..., 150\}$, $p \in \{0.05, 0.1, ..., 0.95\}$ and $\theta_L \in \{2, 4, ..., N\}$. Figure 2 shows the simulated evolution of the relative optimal quota q^* with the number of countries N and the probability p for a given $\theta_L = 50$. The value of θ_L is picked arbitrarily and is in no way particular. The x-axis depicts the number of countries N, while the y-axis shows the values of the probability of a low type p. The graph shows that the relative optimal quota starts with a value close to or equal to one at $N = \theta_L$, regardless of the probability. With an increasing number of countries N the relative optimal quota decreases. This implies that the (absolute) optimal quota Q^* increases underproportionately to the increase in N. Furthermore, the smaller the probability of the low type, the steeper is the decrease in

Figure 2: Relative Optimal Quota Q^*/N for $\theta_L = 50$



the relative optimal quota. The graph also illustrates the result that the optimal quota is increasing in the probability of the low type p.

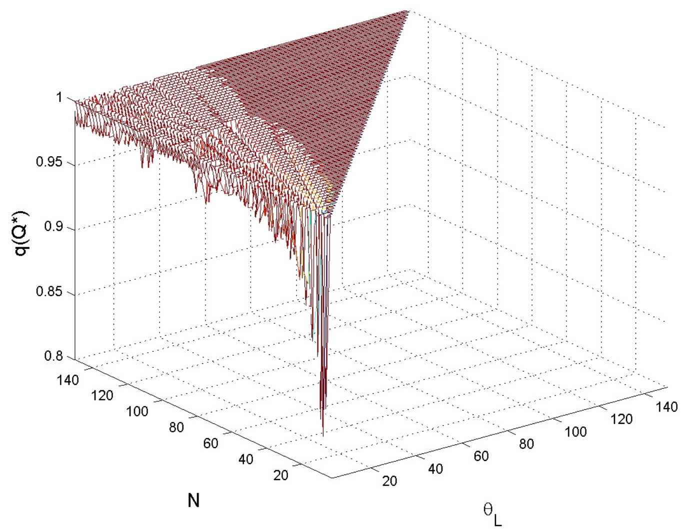
So far, we answered the question what can be optimally achieved when countries either ratify or not. Unlike case 1, the restriction to pure strategies might be binding. If low type countries chose to ratify with probability $q < 1$, it might be possible to increase aggregate expected utility by lowering the quota. Suppose all low types follow the symmetric mixed strategy to ratify with probability q. Given all other countries employ the mixed strategy, country i has no incentive to deviate if the expected utility from ratification equals the expected utility from no ratification. The incentive expression (2), where the probability p is substituted by the probability $\widetilde{p} = pq$, has to hold with equality,

$$(\theta_L - 1) \sum_{P=Q}^{N-1} b\left(P|N-1, \widetilde{p}\right) - \left(Q - \theta_L\right) b\left(Q-1|N-1, \widetilde{p}\right) = 0. \tag{2'}$$

Whether there exists a $q \neq 0$ that solves this equation, depends on the parameter constellations.[9] In the following example, we find that the modified incentive expression has got an inner solution with $q \in (0,1)$ and that the aggregate expected utility increases in comparison to the pure strategy equilibrium. Suppose the number of countries is $N = 7$, the low benefit parameter is $\theta_L = 4$ and the probability of a low type is $p = \frac{1}{2}$. With pure strategies, the incentive expression (2) is positive for a quota of $Q = 5$ at $D\left(4,5\right) = \frac{6}{64}$ and becomes negative for $Q = 6$ with $D\left(4,6\right) = -\frac{9}{128}$. Thus, the optimal quota is $Q = 6$ resulting in an expected aggregate utility of $-2.8359 + \frac{1}{2}\theta_H$. Contrary, when we allow

[9] As shown in Appendix 6.C, the incentive expression is increasing in $\widetilde{p}$ for all $\widetilde{p} \geq \frac{Q-\theta_L}{N-\theta_L}$. Thus, it starts in the origin, decreases with q until $q = \frac{Q-\theta_L}{(N-\theta_L)p}$ and increases thereafter. If the reversal point $\frac{Q-\theta_L}{(N-\theta_L)p}$ is large, it is likely that there exists no solution $q \neq 0$. The expression is thus negative for all $q \neq 0$, and we are in a corner solution where each low type country choses to ratify with certainty, $q = 1$.

Figure 3: Mixing Probability $q\left(Q^{*}\right)$ given $p = 0.5$



for mixed strategies, the incentive expression (2') equals zero at the mixing probability $q = \frac{8-\sqrt{14}}{5}$ for a quota of $Q = 5$. The attached aggregate expected utility amounts to $-2.7252 + \frac{1}{2}\theta_H$, which exceeds the expected utility with pure strategies.

Simulations can give an indication how results change when we allow for mixed strategies. I simulate the scenario with mixed strategies for a parameter range of $N \in \{3, 4, ..., 150\}$, $\theta_L \in \{2, 4, ..., N\}$ and $p \in \{0.05, 0.1, ..., 0.95\}$. First, I compute the probability of ratification $q\left(Q\right)$ given a quota Q. Next, I calculate the quota Q^{*} that maximises the aggregate expected utility given the probability $q\left(Q^{*}\right)$. For a low type probability of $p = 0.5$, Figure 3 shows the simulated mixing probability $q\left(Q^{*}\right)$ given the optimal quota Q^{*}. The x-axis depicts the number of countries N, while the y-axis shows the benefit parameter of the low type θ_L. I find that the corner solution, where the probability is $q\left(Q^{*}\right) = 1$, is the rule rather than the exception. There is only an inner solution when the benefit parameter θ_L is relatively small. Moreover, the smallest value for the mixing probability is $q\left(Q^{*}\right) = 0.83$. It occurs when the number of countries is $N = 7$, the benefit parameter equals $\theta_L = 2$ and the optimal quota is $Q^{*} = 3$. The simulation results for $p = 0.5$ are in no way different to those with other values for p. We include further simulation results in Appendix 6.D. The simulation results illustrate that the limitation to pure strategies is not very restrictive and that it loses importance with an increasing benefit parameter θ_L.

Summarising the case of simultaneous ratifications, we have seen that, if the benefit of the high type is smaller than the social costs, the optimal quota is 100% and the first best can be achieved. Otherwise, if the benefit of the high type exceeds the social costs, the optimal quota is as low as possible, taking into account that low type countries must still have an incentive to ratify. Thus, the results in theses two cases are diametrically opposed.

4 Sequential Ratification

In this section, I investigate the implications of sequential rather than simultaneous play. Instead of deciding all at once, countries ratify one after the other, observing the decisions made by all previous countries. One crucial difference between these two scenarios is that with sequential ratification, the process of ratifying stops as soon as a sufficient number of ratifications occurred. No country has an incentive to keep on ratifying after the sufficient number of ratifications is reached as long as its benefit type exceeds one, i.e. $\theta_i > 1$. Therefore, with sequential ratification, the equilibrium outcome is either no binding agreement or an agreement with exactly Q ratifying parties.

Sequential ratification is a game of incomplete information. The country only faces uncertainty regarding the type of the countries moving after it. The solution concept is Perfect Bayesian Equilibrium (PBE). In contrast to the case of simultaneous ratification, the PBE is essentially unique.[10] The history of the game can be summarised by the number of countries that have ratified so far. The restriction to pure strategies is innocuous when countries ratify sequentially. There exists no mixed strategy equilibrium. Consider the ratification decision of the last country when this country is pivotal. If the benefit parameter θ_i is smaller than Q, the last country's optimal strategy is to ratify with certainty. Further, if the second last country is pivotal, it is also going to ratify for sure. However, if it is not pivotal and the quota is not yet satisfied, then the country compares the expected utility from ratifying with the expected utility from not ratifying. Depending on the model parameters, one of the two expected utilities exceeds the other. It is thus optimal to play the pure strategy that leads to the higher expected utility. Only in the case, where the model parameters are such that both expected utilities are identical, does any mixed strategy belong to the set of optimal strategies. The same reasoning applies to all previous countries.

In analysing sequential ratification, I distinguish again between the two previous cases; the case where cooperation aims at the high types and where it aims at the low types. The outcome of the first case is largely the same as under simultaneous ratification, the optimal quota is as high as possible. Although I cannot determine the optimal quota in the second case, I find that the expected aggregate surplus is always higher under simultaneous than under sequential ratification.

[10] It is unique up to a variation of the strategy after histories where it is not possible to achieve a sufficient number of ratifications. Suppose it is the turn of country $(N - i)$, P countries have ratified before it and Q have to ratify in total. A sufficient number of ratifications cannot be achieved, if too few countries have ratified so far, that is, if $P < Q - i - 1$.

Case 1: Participation of High Types This is the case where the benefit of local emission to both types of countries is smaller than the social damage of local emission, $\theta_H < N$. Again the optimal quota is 100% and it achieves the first best outcome, i.e. no emission by any country. Under a 100% quota each country is pivotal regardless of the order of play. The strategy to ratify regardless of the type and the history of the game is implementable if either $\theta_L > 1$ or if $\theta_L \leq 1$ and the condition of Proposition 1 that $\theta_H \leq 1 + (N-1)(1-p)$ is satisfied. In contrast to simultaneous ratifications, where there are multiple equilibria, this is the unique PBE. In case the low benefit types emit whenever the agreement fails, that is if $\theta_L > 1$, the intuition for an optimal quota of 100% is the same as in the simultaneous case. Countries, when ratifying, chose between a situation of no emission and a situation with full emission by all countries. Every country thus has an incentive to ratify. Contrary, if the low benefit types do not emit in any case, it has to hold that the benefits from emission do not outweigh the expected gains from ratification. That is, the condition of Proposition 1 that $\theta_H \leq 1 + (N-1)(1-p)$ has to be met. This condition does not depend on the position in the order of play as the decisions of the previous countries do not reveal information. Effectively, the simultaneous and sequential specifications yield the same outcome.

Whenever the high type is large, i.e. $\theta_H > 1 + (N-1)(1-p)$, then an agreement cannot help ameliorate the free-rider problem, just as in the simultaneous case. A 100% quota cannot induce cooperation among the high types. Lowering the quota reduces the expected gain of an agreement as fewer countries participate. Thus a lower quota is even less capable of inducing participation of the high types.

The result of the sequential game is therefore identical to the result in the simultaneous case. We either achieve the first best by implementing a 100% quota, or we cannot improve upon the situation at all.[11]

Case 2: Participation of Low Types In this case, high types never participate voluntarily in any agreement, as their individual benefits outweigh the social damage, $\theta_H \geq N$. The aggregate expected utility of the emission game is given by $U(0) = \sum_{i=0}^{N} \begin{pmatrix} N \\ i \end{pmatrix} p^i (1-p)^{N-i} \left(i\theta_L + (N-i)\theta_H - N^2\right)$ if no contracting stage is preceding it.

A 100% ratification quota is clearly better than no agreement at all as it can achieve cooperation of all countries in the case where all countries are of low type. Each country is pivotal and the optimal strategy of a low type country is to ratify regardless of the history. The aggregate expected surplus is

[11] The analysis of sequential ratification is closely related to sequential voting mechanisms. In particular, Dekel and Piccione (2000) show that in unanimity games, essentially the whole set of equilibria is the same in all sequential structures. There, sequential structures range from the one-period voting game, which would be the purely simultaneous case, over combined simultaneous and sequential structures to a purely sequential structure where each voter decides in a distinct period.

$U(N) = \sum_{i=0}^{N-1} \begin{pmatrix} N \\ i \end{pmatrix} p^i (1-p)^{N-i} \left(i\theta_L + (N-i)\theta_H - N^2\right)$. This is clearly better than no agreement as $U(N) - U(0) = -p^N N (\theta_L - N) \geq 0$.

Reducing the quota to $Q = N-1$ implies that, on the one hand, one country free-rides surely, but on the other hand, the risk of a breakdown of the agreement is reduced. The optimal strategy for low types depends on the history of the game and on the position in the order of play. Whenever a country is pivotal,[12] the optimal strategy is to ratify. If it is not pivotal and the quota is not yet fulfilled, the country trades-off the gains from free-riding with the probability that the agreement fails due to its decision, and the associated loss. The probability that the agreement fails due to its decision depends on how many previous countries have ratified and how many more countries are to follow. If the benefits from emission are sufficiently large, the country takes the risk and tries to free-ride. Finally, if the quota is already satisfied, the optimal strategy is to abstain from ratification and free-ride. For a quota $Q = N-1$, the optimal strategy for the $(N-i)^{\text{th}}$ country is given by

$$s^{N-i}(P|\theta_L, N-1) = \begin{cases} 1 & \text{if } P \leq Q-i-1 \text{ and } \theta_L \leq Q \text{ or} \\ & \text{if } P = Q-i,\ i \neq 0 \text{ and } \theta_L \leq (1-p)Q \\ 0 & \text{else} \end{cases}$$

for all $i \in \{0, ..., N-1\}$. P signifies the number of countries that have already signed the agreement.[13] The proof of this strategy being the equilibrium strategy can be found in Appendix 6.E.

Under the assumption that $\theta_L \leq (1-p)Q$, the expected aggregate surplus increases with a smaller quota, if $p \leq \frac{N(N-1)}{N(N-1)+1}$. Otherwise, *under the assumption that* $\theta_L > (1-p)Q$, the expected aggregate surplus increases with a smaller quota, if $p \leq \frac{N-1}{N}$.[14] Both conditions are relatively mild, in particular if the number of countries N is large. Thus, it is profitable to reduce the quota from $Q = N$ to $Q = N-1$ in most cases. This finding is intuitive as the gain from the reduction in the risk of breakdown of the agreement is high if the quota is large. Yet, the loss remains small, as a quota of $Q = N-1$ allows only one country to free-ride.

A further reduction in the quota complicates the equilibrium analysis. Yet, countries still face the basic trade-off between risk of failure and free-riding. A complete characterisation of the PBE and therewith the optimal quota is cumbersome since it relies on many case distinctions, as the simple example of a $(N-1)$-quota illustrates.[15] In what follows,

[12] Suppose P countries have already ratified. Then, there need to be $Q-P$ more ratifications. A country $(N-i)$ is pivotal if the number of countries i following that country equals the number of countries still needed to satisfy the ratification quota minus 1, i.e. $i = Q-P-1$.

[13] The number of countries P that have ratified before the $(N-i)^{\text{th}}$ country equals at most the number of countries preceding it, i.e. $P \leq N-i-1 = Q-i$.

[14] For the derivation, see Appendix 6.E.

[15] It would be interesting to simulate the sequential equilibrium depending on the model parameters θ_L, p, and N, and compare the resulting optimal quota with the simultaneous case.

I characterise two special cases. In the first case, benefits from emission for low types are very high such that every country's incentive to free-ride is large. Countries therefore only ratify if they are pivotal. In the other extreme, benefits from emission for low types are small and each country rather prefers to ratify than to risk the failure of the agreement. Proposition 6 characterises these two cases.

Proposition 6 *Given a quota Q, the optimal strategy for a low type country at position $(N-i)$ in the order of ratification*

Case 1) if $Q \geq \theta_L \geq \overline{\theta}(p,Q) = (1-p)\,Q$, is

$$s^{N-i}(P|\theta_L,Q) = \begin{cases} 1 & \text{if } P \leq Q-i-1 \\ 0 & \text{else} \end{cases},$$

Case 2) if $\theta_L \leq \underline{\theta}(p,Q) = (1-p)^{N-Q}\,Q$, is

$$s^{N-i}(P|\theta_L,Q) = \begin{cases} 1 & \text{if } P \leq Q-1 \\ 0 & \text{else.} \end{cases}$$

Proof. ad 1) Suppose $Q \geq \theta_L \geq (1-p)\,Q$ and suppose every country follows the proposed strategy, a country $(N-i)$ has no incentive to deviate: After histories where so few countries have ratified, that it is not possible to fulfill the quota with the remaining countries anyway, i.e.$P < Q-i-1$, to ratify is among the optimal actions. Whenever the number of participating countries is such that exactly $i+1$ ratifications are still needed, $P = Q-i-1$, the country is pivotal and it prefers to ratify, as long as $\theta_L \leq Q$. Suppose the country is not pivotal and $P = Q-i+k$ with $k \in \{0,...,i\}$, that is, $i-k$ ratifications are needed for satisfying the quota. If country $(N-i)$ ratifies, the following $k+1$ countries do not ratify, regardless of their type. The countries thereafter are pivotal and ratify if of low type. The expected utility is $E(u(1)) = p^{i-k-1}Q + \left(1-p^{i-k-1}\right)\theta_L - N$. If, however, the country $(N-i)$ does not ratify, then only the k following countries do not ratify, regardless of their type. The expected utility of no ratification is $E(u(0)) = \theta_L - N + p^{i-k}Q$. The $(N-i)^{th}$ country has an incentive to deviate from the proposed strategy if $E(u(1)) - E(u(0)) > 0$. That is equivalent to $(1-p)\,Q > \theta_L$. As by assumption $\theta_L \geq (1-p)\,Q$, no country has an incentive to deviate from the proposed strategy. Finally, if the quota is already satisfied, $P \geq Q$, the country has no incentive to ratify.

ad 2) Suppose $\theta_L \leq (1-p)^{N-Q}\,Q$ and suppose every country follows the proposed strategy to ratify as long as the quota is not satisfied. A country $(N-i)$ has no incentive to deviate: Suppose $P = Q-1-k$ with $k \in \begin{cases} \{0,1,...,Q-1\} & \text{if } i \leq N-Q \\ \{Q-N+i,...,Q-1\} & \text{if } i > N-Q \end{cases}$, that is $k+1$ ratifications are needed for satisfying the quota. If $k > i$, then it is not possible to fulfill the quota with the remaining countries, i.e. $P < Q-i-1$, one optimal action is to ratify. If instead $k \leq i$, the agreement is feasible. Regardless of country $(N-i)$, all following low type countries ratify until the ratification quota is met. The utility of ratification is thus $E(u(1)) = \sum_{j=k}^{i} b(j|i,p)\,Q + \left(1 - \sum_{j=k}^{i} b(j|i,p)\right)\theta_L - N$, whereas the expected utility of no ratification is $E(u(0)) = \sum_{j=k+1}^{i} b(j|i,p)\,Q + \theta_L - N$. There is a positive incentive to deviate if $E(u(0)) - E(u(1)) > 0$, which is equivalent to

$$\theta_L > \frac{b(k|i,p)}{\sum_{j=k}^{i} b(j|i,p)} Q = \widetilde{\theta}(k,i)\,. \tag{5}$$

In Appendix 6.F, we show that the following ordering holds

$$\widetilde{\theta}(k-1,i-1) \leq \widetilde{\theta}(k,i) \leq \widetilde{\theta}(k,i-1)\,.$$

- The incentive to deviate for countries following the Q^{th} country, that is, for countries with $i \leq N-Q$, is never positive. Equation (5) can never be satisfied as by assumption $\theta_L \leq b(0|N-Q)\,Q = \widetilde{\theta}(0,N-Q)$ and $\widetilde{\theta}(0,N-Q) \leq \widetilde{\theta}(k,i)$ for all $i \leq N-Q$ and $k \geq 0$.
- The incentive to deviate for countries before the Q^{th} country, that is for countries with $i > N-Q$, is also never positive as $\theta_L \leq \widetilde{\theta}(0,N-Q) \leq \widetilde{\theta}(1,N-Q+1) \leq \widetilde{\theta}(2,N-Q+2) \leq ... \leq \widetilde{\theta}(Q-1,N-1)$.

If $P \geq Q$, the contract comes into force for sure and each country prefers to free-ride.

■

Clearly, the threshold levels $\underline{\theta}(p,Q)$ and $\overline{\theta}(p,Q)$ for the benefit parameter depend both on the probability of the low type as well as on the quota. The higher the quota, the less likely is case 1 and the more likely is case 2. The reverse holds for the probability of the low type. The higher p, the more likely we are in case 1 and the less likely we are in case 2.

In particular, the second case is interesting where, regardless of the history, each low type country ratifies as long as the quota is not yet satisfied. In this case, the simultaneous as well as sequential representation lead to exactly the same probability of success of the agreement with a given quota $Q \geq Q^*$ larger than the optimal quota of the simultaneous case. However, under the sequential representation, exactly Q countries ratify, whereas with simultaneous ratification, most probably more than Q countries ratify. Thus, the expected aggregate surplus is larger under simultaneous than under sequential ratification for any quota. Furthermore, for any given quota, the probability of success of the agreement is largest whenever we are in the equilibrium of case 2. Therefore, the expected aggregate surplus is larger with simultaneous rather than sequential ratification. The following proposition summarises this result.

Proposition 7 *For a given quota $Q \geq Q^*$, the expected aggregate surplus is higher under simultaneous than under sequential ratification.*

The proof is straightforward and therefore omitted. In the simultaneous ratification game, every low type country ratifies, given the quota is larger than the optimal quota

$Q \geq Q^*$. Turning from simultaneous to sequential play, some low type countries might abstain from ratification with the same quota. That lowers, on the one hand, the number of participating countries and, on the other, the probability of a success of the agreement. It remains to be noted that ex post some countries are better off in the sequential rather than the simultaneous game.[16]

5 Concluding remarks

The present model, proposes a three stage international bargaining game where countries first determine a ratification quota. Then, they decide whether to ratify and finally they decide over emission of a global pollutant. In a setting with incomplete information, two country types and a binary contribution to the provision, I examine the differences between simultaneous and sequential ratification. When the benefits from emission of both types are smaller than the social costs, the outcome in the simultaneous case is essentially identical to the sequential case. The optimal quota is 100% and achieves the first best. With the high type's benefits exceeding the social costs, I find that the optimal quota is as small as possible, if ratification is simultaneous. In the sequential ratification case, I cannot determine the optimal quota. However, I find that the aggregate expected surplus decreases with respect to the simultaneous case.

The crucial assumption driving the results of the model is the informational structure. The risk of failure of the agreement is introduced through the (costless) acquisition of new information concerning the benefits of a country. In reality, ratification processes differ from country to country.[17] They often involve the legislative body of a country to decide on the acceptance of the agreement. These processes take a substantial amount of time. The outcrop of new information can realistically occur during that period of time. In the case of the Kyoto Protocol, the research group of the Intergovernmental Panel on Climate Change has published several special reports as well as a new Assessment Report since the Kyoto Conference in 1997, which constituted the start of the ratification period. Moreover, in the time period between the signature and the ratification of an agreement, the internal political situation of a country as well as the political and economic relationships to other countries can change. All this can influence the benefits accruing from emission. I do not model the national political processes leading to the ratification decision explicitly. These are definitely very important, but beyond the scope of the present paper.

A major restriction of the model is the abstraction from compliance problems, in particular, as compliance could depend on the number of countries that have ratified the

[16] This is related to a result in an early version of the paper by Börgers (2004) that explores the effect of sequential voting. Under certain circumstances, Börgers shows that sequential voting weakly Pareto-dominates simultaneous voting. With sequential voting fewer agents incur the costs of voting. Hence, the public good is provided at lower total costs. The crucial difference to the ratification game is that in the voting setting, the "amount" of the public good does not change with the number of voters.

[17] For an overview of the different processes within Europe, see Stoiber and Thurner (2000).

agreement. The more countries join the agreement, the larger is potentially the pressure from these countries on non-complying members. Furthermore, I do not allow for transfer payments. Transfer payments from the low benefit to the high benefit countries could potentially induce all countries to participate in an agreement. However, low benefit countries can have an incentive to pretend to be of high type.[18] The assumption of no side-payments allows me to concentrate on the participation decision of each country.[19]

When introducing sequential ratification, further issues arise such as renegotiation and the order of ratification. The exogenously given order of ratification allocates bargaining power in favour of countries that are positioned later in the order. These countries might be able to exploit the ratification of previous countries. During the ratification process of the Kyoto Protocol, the case of Denmark suggests that countries do renegotiate with others that have already committed to ratification. The European Union's target was that all member states ratify until the World Summit of Sustainable Development in Johannesburg 2002. Denmark threatened not to do so, if its share of the entire union's reduction burden would not be lowered. An extension of the model would therefore consider renegotiation. Endogenising the order of ratifications, by allowing each country to chose its ratification time, represents another interesting possibility to extend the model.[20]

6 Appendix

6.A Simultaneous ratification, case 1: mixed strategies

In case 1 where $\theta_L < 1$ and $\theta_H > 1 + (N-1)(1-p) = \underline{\theta}_H(N,p)$ and countries ratify simultaneously, the restriction to pure strategies is innocuous. To see this, suppose the quota is $Q = N - s$ for $s \in \{0, 1, ..., N-1\}$ and all high type countries ratify with a probability $q \in (0,1)$. For this to be an equilibrium, the expected utility of *no ratification* has to equal the expected utility of *ratification*. However, in the following, I illustrate that the expected utility of no ratification exceeds the expected utility of ratification for all $q \in (0,1)$. The expected utility of no ratification consists of a) the probability that the agreement succeeds times the benefit θ_H minus the sum of all emissions and b) the probability of a failure times the benefit θ_H minus the associated aggregate emissions, i.e. $E(u(0|\theta_H, N-s)) = \sum_{h=1}^{s}\left(\sum_{P=0}^{N-h} b(P|N-1,p)\, b(N-P-h|N-1-P,q)(\theta_H - h)\right)$

[18] Generally, incentive problems are tackled by the mechanism design literature. For a good survey article, see Moore (1992). Mechanism design focuses on whether there exists a mechanism that implements the efficient level of a public good. In contrast to this, I postulate a given institution and analyse the provision of the public good within that institution.

[19] For a recent paper that analyses the role of transfer schemes in international environmental agreements within the framework of coalition theory, see Carraro, Eyckmans, and Finus (2005).

[20] There is some literature on dynamic games of voluntary contributions to a public project, for example Marx and Matthews (2000) and the literature cited there. In their paper, Marx and Matthews assume that players have perfect information concerning the utility functions of every player. Furthermore, they neglect the impact of the minimum participation rule.

$+\sum_{P=0}^{N-s} b\left(P|N-1,p\right)\sum_{j=0}^{N-s-P-1} b\left(j|N-1-P,q\right)\left(\theta_H+P-N\right)$. The expected utility of ratification is defined analogously and equals
$E\left(u\left(1|\theta_H,N-s\right)\right)=-\sum_{h=1}^{s+1}\sum_{P=0}^{N-h} b\left(P|N-1,p\right)b\left(N-1-h-P|N-1-P,q\right)\left(h-1\right)$
$+\sum_{P=0}^{N-s-1} b\left(P|N-1,p\right)\sum_{j=0}^{N-s-P-2} b\left(j|N-1-P,q\right)\left(\theta_H+P-N\right)$. The difference in expected utilities is given by

$$\begin{aligned} D\left(s\right) &= E\left(u\left(0|\theta_H,N-s\right)\right)-E\left(u\left(1|\theta_H,N-s\right)\right) \\ &= \sum_{h=1}^{s}\left(\sum_{P=0}^{N-h} b\left(P|N-1,p\right)b\left(N-P-h|N-1-P,q\right)\left(\theta_H-1\right)\right) \\ &\quad+\sum_{P=0}^{N-s-1} b\left(P|N-1,p\right)b\left(N-s-P-1|N-1-P,q\right)\left(\theta_H+P-\left(N-s\right)\right). \end{aligned}$$

From the initial condition that

$$\begin{aligned} \theta_H &> \underline{\theta}_H\left(N,p\right) \\ &\leftrightarrow \sum_{P=0}^{N-1} b\left(P|N-1,p\right)\left(\theta_H+P-N\right)>0, \end{aligned}$$

we know that the difference is positive for the starting value $s=0$,
$D\left(0\right)=\sum_{P=0}^{N-1} b\left(P|N-1,p\right)b\left(N-P-1|N-1-P,q\right)\left(\theta_H+P-N\right)>0$. Furthermore, I checked numerically that the first differences $D\left(s\right)-D\left(s-1\right)$ are positive. To do this, I simulated

$$\begin{aligned} & D\left(s\right)-D\left(s-1\right) \\ = & \sum_{P=0}^{N-s-1} b\left(P|N-1,p\right)b\left(N-s-P-1|N-1-P,q\right)\left(\theta_H+P-N+\frac{s}{1-q}\right) \end{aligned}$$

over a parameter range of $N\in\{3,4,...,50\}$, $p,q\in\{0.05,0.1,...,0.95\}$ and
$\theta_H\in\{\underline{\theta}_H\left(N,p\right),...,N\}$. The simulations show that the difference $D\left(s\right)-D\left(s-1\right)$ is positive, implying that $D\left(\cdot\right)$ increases in s. As $D\left(\cdot\right)$ starts with a positive value at $s=0$, it is positive for all s. There is thus no probability $q\in\left(0,1\right)$ that equates the expected utility of no ratification and the expected utility of ratification.

6.B Derivation of the incentive function

In section 3, case 2, the proposed symmetric pure strategy is to ratify if of low type and to abstain from ratification otherwise. The incentive function (2) gives the incentive to deviate from this strategy for a low type country given all other countries follow it. On the one hand, the expected utility of *no ratification* is given by a) the benefit of emission θ_L, plus b) the damage of global emission if the agreement gets binding, that is $P\geq Q$, times the probability that this happens, plus c) the damage if the agreement fails times the probability, i.e. $E\left(u\left(0|\theta_L,Q\right)\right)=\theta_L-\sum_{P=Q}^{N-1} prob\left(P|N-1\right)\left(N-P\right)-\sum_{P=0}^{Q-1} prob\left(P|N-1\right)N$. On the other hand, the expected utility of *ratification* is given by a) the damage of global emission if the agreement gets binding, that is $P \geq Q-1$, times the attached probability, plus b) the damage if the agreement fails minus the benefit of local emission times the probability, i.e. $E(u(1|\theta_L, Q)) = -\sum_{P=Q-1}^{N-1} prob(P|N-1)(N-1-P) - \sum_{P=0}^{Q-2} prob(P|N-1)(N-\theta_L)$. The probability that P countries ratify out of the $N-1$ remaining countries is given by the binomial distribution

$$b(P|N-1,p) = \begin{pmatrix} N-1 \\ P \end{pmatrix} p^P (1-p)^{N-1-P}.$$

The incentive function is thus

$$\begin{aligned} D(\theta_L, Q) &= E(u(0|\theta_L, Q)) - E(u(1|\theta_L, Q)) \\ &= (\theta_L - 1)\sum_{P=Q}^{N-1} b(P|N-1,p) - (Q-\theta_L)\, b(Q-1|N-1,p). \end{aligned}$$

6.C Proofs

Details to the proof of Proposition 2 Proposition 2 establishes that the optimal quota is the smallest integer that renders the incentive expression equal to naught. To proof this, we first show that, given the proposed strategy $s(\theta_i) = \begin{cases} 1 & \text{if } \theta_i = \theta_L \\ 0 & \text{else} \end{cases}$, the individual expected utility is decreasing in the quota Q. The inidividual expected utility is

$$\begin{aligned} U(Q) &= pE(u(\theta_L|Q)) + (1-p)E(u(\theta_H|Q)) \\ &= p\left(-\sum_{P=Q-1}^{N-1} b(P|N-1,p)(N-1-P) + (\theta_L - N)\sum_{P=0}^{Q-2} b(P|N-1,p)\right) \\ &\quad + (1-p)\left(\theta_H - \sum_{P=Q}^{N-1} b(P|N-1,p)(N-P) - N\sum_{P=0}^{Q-1} b(P|N-1,p)\right). \end{aligned}$$

Taking first differences yields

$$\begin{aligned} & U(Q) - U(Q-1) \\ = \;& p\,(b(Q-1|N-1,p)(N-Q) + (\theta_L - N)\, b(Q-1|N-1,p)) \\ & + (1-p)\,(b(Q|N-1,p)(N-Q) - Nb(Q|N-1,p)) \\ = \;& b(Q-1|N-1,p)\left(p(-Q+\theta_L) - (1-p)\frac{p}{1-p}\frac{N-Q}{Q}Q\right) \text{ with the use of (4)} \\ = \;& b(Q-1|N-1,p)\, p\,(\theta_L - N) < 0. \end{aligned}$$

This shows that the individual expected utility is decreasing in the quota Q.

Proof of Proposition 4 Proposition 4 states that the optimal quota is increasing the the probability of a low type p. To proof this, I first show that the probability of sufficient ratifications is increasing in the probability of the low tpye in the following way $\frac{\partial \sum_{P=Q}^{N-1} b(P|N-1,p)}{\partial p} = \begin{pmatrix} N-1 \\ Q \end{pmatrix} Qp^{Q-1}(1-p)^{N-1-Q} > 0$. This is shown by induction.

1. Suppose the above holds for Q, than for $Q-1$:
$\frac{\partial \sum_{P=Q-1}^{N-1} b(P|N-1,p)}{\partial p} = \frac{\partial \sum_{P=Q}^{N-1} b(P|N-1,p)}{\partial p} + \frac{\partial b(Q-1|N-1,p)}{\partial p}$
$= \begin{pmatrix} N-1 \\ Q \end{pmatrix} Qp^{Q-1}(1-p)^{N-1-Q} +$
$\begin{pmatrix} N-1 \\ Q-1 \end{pmatrix} \left((Q-1)p^{Q-2}(1-p)^{N-Q} - (N-Q)p^{Q-1}(1-p)^{N-Q-1}\right)$. After a couple of transformations and using (4) this yields the required result
$\frac{\partial \sum_{P=Q-1}^{N-1} b(P|N-1,p)}{\partial p} = \begin{pmatrix} N-1 \\ Q-1 \end{pmatrix} (Q-1)p^{Q-2}(1-p)^{N-Q}$.

2. The statement holds for the starting value of $Q = N-1$,
$\frac{\partial \sum_{P=N-1}^{N-1} b(P|N-1,p)}{\partial p} = \frac{\partial p^{N-1}}{\partial p} = (N-1)p^{N-2}$.

Next, I determine the sign of the derivative of the incentive expression with respect to p:
$\frac{\partial D(\theta_L,p,Q)}{\partial p} = (\theta_L - 1)\frac{\partial \sum_{P=Q}^{N-1} b(P|N-1,p)}{\partial p} - (Q-\theta_L)\frac{\partial b(Q-1|N-1,p)}{\partial p}$
$= (\theta_L - 1)\begin{pmatrix} N-1 \\ Q \end{pmatrix} Qp^{Q-1}(1-p)^{N-1-Q}$
$-(Q-\theta_L)\begin{pmatrix} N-1 \\ Q-1 \end{pmatrix}\left(p^{Q-2}(1-p)^{N-Q-1}\right)((Q-1)(1-p)-(N-Q)p)$
After a couple of transformations and the use of $\begin{pmatrix} N-1 \\ Q \end{pmatrix} Q = \begin{pmatrix} N-1 \\ Q-1 \end{pmatrix}(N-Q)$, this is equivalent to

$$\begin{aligned} &\frac{\partial D(\theta_L, p, N)}{\partial p} \\ = \; &\frac{b(Q-1|N-1,p)}{(1-p)p}(Q-1)((N-\theta_L)p-(Q-\theta_L))\begin{cases} \geq 0 & \forall p \geq \frac{Q-\theta_L}{N-\theta_L} \\ < 0 & \forall p < \frac{Q-\theta_L}{N-\theta_L} \end{cases}, \end{aligned}$$

for all $Q \geq \theta_L$.

Furthermore, we know from the proof of Proposition 3 that $Q^* \leq p(N-\theta_L)+\theta_L = Q'$. This implies that $\frac{Q^*-\theta_L}{N-\theta_L} \leq \frac{p(N-\theta_L)+\theta_L-\theta_L}{N-\theta_L} = p$. Hence, around the optimal quota the incentive expression is increasing with p which translates into the optimal quota itself being weakly increasing with p. ■

Proof of Proposition 5 Proposition 5 says that the optimal quota is increasing in the number of countries N. To see this, I first show that the binomial distribution function with parameter $N+1$ first oder stochastically dominates the distribution with parameter

N in the following way $\sum_{P=0}^{x}\left(b\left(P|N,p\right)-b\left(P|N+1,p\right)\right)=b\left(x|N,p\right)p>0$ for all $x\in\{0,1,...,N\}$. This is shown by induction.

1. Suppose the above holds for x, then for $x+1$, $\sum_{P=0}^{x+1}\left(b\left(P|N,p\right)-b\left(P|N+1,p\right)\right)$
   $=b\left(x|N,p\right)p+b\left(x+1|N,p\right)-b\left(x+1|N+1,p\right)$
   $=\begin{pmatrix} N \\ x \end{pmatrix}p^{x+1}\left(1-p\right)^{N-x}+\begin{pmatrix} N \\ x+1 \end{pmatrix}p^{x+1}\left(1-p\right)^{N-x-1}-\begin{pmatrix} N+1 \\ x+1 \end{pmatrix}p^{x+1}\left(1-p\right)^{N-x}$
   $=p^{x+1}\left(1-p\right)^{N-x-1}\left(\left(\left(\begin{pmatrix} N \\ x \end{pmatrix}-\begin{pmatrix} N+1 \\ x+1 \end{pmatrix}\right)\right)\left(1-p\right)+\begin{pmatrix} N \\ x+1 \end{pmatrix}\right)\right)$
   $=\begin{pmatrix} N \\ x+1 \end{pmatrix}p^{x+2}\left(1-p\right)^{N-x-1}=b\left(x+1|N,p\right)p$ where we use (6).
   The two binomial coefficients $\begin{pmatrix} N \\ x \end{pmatrix}$, $\begin{pmatrix} N+1 \\ x+1 \end{pmatrix}$ can be combined such that
   $$\begin{pmatrix} N \\ x \end{pmatrix}-\begin{pmatrix} N+1 \\ x+1 \end{pmatrix}=-\begin{pmatrix} N \\ x+1 \end{pmatrix} \tag{6}$$
2. For the starting value $x=0$, we know that $\sum_{P=0}^{0}\left(b\left(P|N,p\right)-b\left(P|N+1,p\right)\right)=\left(1-p\right)^{N}p=b\left(0|N,p\right)p$.

Next, I establish that the incentive function increases with N around the optimal quota,

$$\begin{aligned} D\left(N+1\right)-D\left(N\right) &= \left(\theta_L-1\right)\left(\sum_{P=Q}^{N}b\left(P|N,p\right)-\sum_{P=Q}^{N-1}b\left(P|N-1,p\right)\right) \\ &\quad -\left(Q-\theta_L\right)\left(b\left(Q-1|N,p\right)-b\left(Q-1|N-1,p\right)\right). \end{aligned} \tag{7}$$

Using the fact that the binomial distribution with N first order stochastically dominates the binomial with $N-1$, we get that

$$\begin{aligned} &\sum_{P=Q}^{N}b\left(P|N,p\right)-\sum_{P=Q}^{N-1}b\left(P|N-1,p\right) \\ =&\sum_{P=0}^{Q-1}b\left(P|N-1,p\right)-\sum_{P=0}^{Q-1}b\left(P|N,p\right)=b\left(Q-1|N-1,p\right)p. \end{aligned}$$

Substituting this into equation (7), we get

$$\begin{aligned} D\left(N+1\right)-D\left(N\right) &= \left(\theta_L-1\right)b\left(Q-1|N-1,p\right)p \\ &\quad -\left(Q-\theta_L\right)\left(b\left(Q-1|N,p\right)-b\left(Q-1|N-1,p\right)\right) \\ &= -b\left(Q-1|N-1,p\right)p-Q\left(b\left(Q-1|N,p\right)-b\left(Q-1|N-1,p\right)\right) \\ &\quad +\theta_L\left(b\left(Q-1|N,p\right)-\left(1-p\right)b\left(Q-1|N-1,p\right)\right) \\ &= \frac{b\left(Q-1|N,p\right)}{N\left(1-p\right)}\left(Q-1\right)\left(p\left(N+1-\theta_L\right)+\theta_L-Q\right). \end{aligned}$$

We therefore know that the incentive function increases for all small Q and decreases thereafter, i.e.

$$D(N+1) - D(N) \begin{cases} > 0 & p(N+1-\theta_L) + \theta_L > Q \\ \leq 0 & \text{else} \end{cases} .$$

As we know, the optimal quota Q^* is smaller than $Q' = p(N - \theta_L) + \theta_L$. Therefore, the incentive expression increases around the optimal quota and the optimal quota increases in the number of countries N. ■

6.D Simulation results for mixed strategies

The following two figures report simultation results on the optimal mixing strategy $q(Q^*)$ for different values of the low type probability p. The x-axis depicts the number of countries N, while the y-axis shows the benefit parameter of the low type θ_L. Panels 1-4 of Figure 4 show the results for small probabilities p and panels 1-4 of Figure 5 for large probabilities p. Furthermore, low type countries optimally ratify with a relatively large probability $q(Q^*)$. The smallest probability over the entire parameter range is $q(Q^*) = 0.76$ with $Q^* = 5$. It occurs at $N = 21$, $p = 0.1$ and $\theta_L = 4$. Again, I find that the corner solution is the rule rather than the exception. There is only an inner solution when the benefit parameter θ_L is relatively small.

Figure 4: Mixing probability, $p \in \{0.1, ..., 0.4\}$

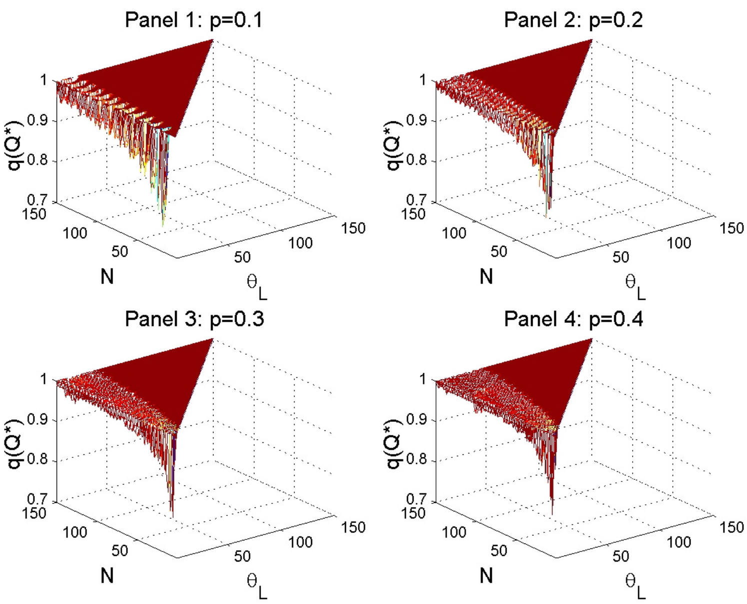
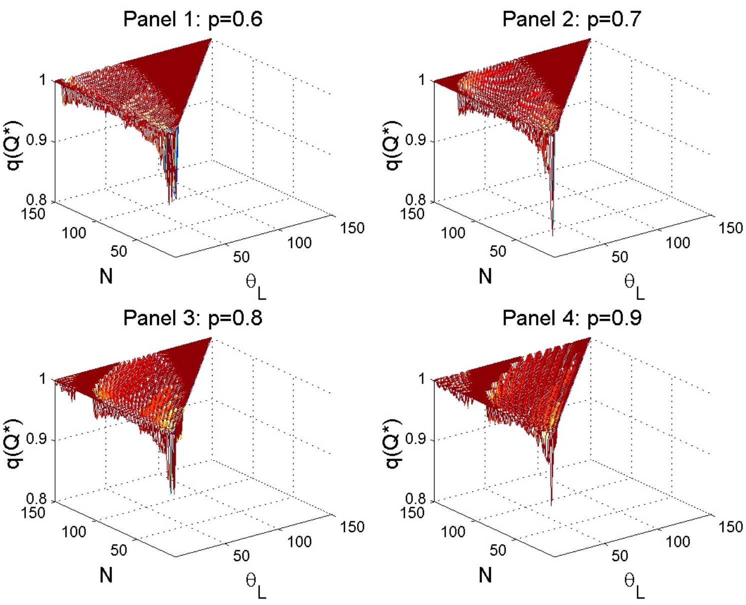


Figure 5: Mixing probability, $p \in \{0.6, ..., 0.9\}$



6.E Equilibrium under sequential ratification and $Q = N - 1$

For a quota $Q = N - 1$, the proposed equilibrium strategy for the $(N-i)^{\text{th}}$ country is

$$s^{N-i}(P|\theta_L, N-1) = \begin{cases} 1 & \begin{array}{l} \text{if } P \leq Q - i - 1 \text{ and } \theta_L \leq Q \text{ or} \\ \text{if } P = Q - i,\ i \neq 0 \text{ and } \theta_L \leq (1-p)\,Q \end{array} \\ 0 & \text{else} \end{cases}$$

for all $i \in \{0, ..., N-1\}$. To see that this is indeed the optimal strategy, we distinguish between histories after which the country is pivotal, not pivotal or the quota is satisfied.

1) Suppose $P < Q - i - 1$. There are only i countries to follow the $(N-i)^{\text{th}}$ country. Even if all $i+1$ countries ratify, the agreement does not get binding. It lacks at least one ratification. Therefore the strategy after this history is irrelevant. The proposed strategy belongs therefore to the optimal ones.

2) Suppose $P = Q - i - 1$. Then the $(N-i)^{\text{th}}$ country's decision is pivotal. As long as $\theta_L \leq N - 1 = Q$, it ratifies.

3) Suppose $P = Q - i$. For the last country $i = 0$, this implies that the quota is satisfied. The last country will therefore not ratify. For all other countries, the trade-off between free-riding and the increase in risk of contractual breakdown becomes relevant. Suppose all countries follow the above strategy. If the $(N-i)^{\text{th}}$ country does not ratify, another i ratifications are needed. All remaining i countries ratify if they are of low type, as they are pivotal. The agreement gets binding with probability p^i. The expected utility of no ratification for the $(N-i)^{\text{th}}$ country is given by

$$E(u) = \theta_L - N + p^i Q. \tag{8}$$

If instead the $(N-i)^{\text{th}}$ country does ratify and only $i-1$ further ratifications are needed, then the optimal strategy of the remaining i countries depends on the benefit parameter.

3.1) If $\theta_L \leq (1-p)\,Q$, then all countries, except the last, ratify whenever they are of low type. The probability of sufficient ratifications is $p^i + ip^{i-1}(1-p)$. The expected utility of ratification is given by

$$E(u) = \theta_L - N + \left(p^i + ip^{i-1}(1-p)\right)(Q - \theta_L)\,. \tag{9}$$

The incentive to deviate from the proposed strategy for the $(N-i)^{\text{th}}$ country is given by the difference in expected utility (8)>(9)

$$\leftrightarrow \theta_L > \frac{i(1-p)}{(i-(i-1)\,p)}Q.$$

This condition contradicts the assumption that $\theta_L \leq (1-p)\,Q$. For all $i \geq 1$, it holds that $\frac{i(1-p)}{i-(i-1)p} \geq (1-p)$. Therefore, the $(N-i)^{\text{th}}$ country has no incentive to deviate from the proposed strategy to ratify.

3.2) If $\theta_L > (1-p)\,Q$, then the first country following the $(N-i)^{\text{th}}$ country does not ratify. All the remaining $i-1$ countries ratify, if they are of low type. As $i-1$ ratifications are still needed, the probability of sufficient ratifications is given by p^{i-1}. The expected utility of ratification is given by

$$E(u) = \left(1 - p^{i-1}\right)\theta_L + p^{i-1}Q - N. \tag{10}$$

The incentive to deviate for the $(N-i)^{\text{th}}$ country is given by the difference in expected utility (10)>(8)

$$(1-p)\,Q > \theta_L.$$

This condition contradicts the assumption that $\theta_L > (1-p)\,Q$. Therefore the $(N-i)^{\text{th}}$ country has no incentive to deviate from the proposed strategy which is not to ratify.

The aggreate expected surplus *under the assumption that* $\theta_L \leq (1-p)\,Q$ is given by

$$\begin{aligned} U(N-1) &= p^N(\theta_L - N) + Np^{N-1}(1-p)(\theta_H - N) \\ &\quad + \sum_{P=0}^{N-2}\begin{pmatrix} N \\ P \end{pmatrix} p^P (1-p)^{N-P}\left(P\theta_L + (N-P)\,\theta_H - N^2\right). \end{aligned}$$

It increases with a smaller quota if

$$\begin{aligned} & U(N-1) - U(N) \\ = \; & p^N(\theta_L - N) + Np^{N-1}(1-p)(\theta_H - N) \\ & + \sum_{P=0}^{N-2}\begin{pmatrix} N \\ P \end{pmatrix} p^P (1-p)^{N-P}\left(P\theta_L + (N-P)\,\theta_H - N^2\right) \\ & - \left(\sum_{P=0}^{N-1}\begin{pmatrix} N \\ P \end{pmatrix} p^P (1-p)^{N-P}\left(P\theta_L + (N-P)\,\theta_H - N^2\right)\right) \\ = \; & (N - \theta_L)\,p^{N-1}\left(N(N-1)(1-p) - p\right) \geq 0, \end{aligned}$$

which is equivalent to $p \leq \frac{N(N-1)}{N(N-1)+1}$. For a large number of countries N, this is a relatively mild condition. The aggregate expected surplus *under the assumption that* $\theta_L > (1-p)Q$ is given by

$$\begin{aligned} U(N-1) &= p^N(\theta_L - N) + p^{N-1}(1-p)(\theta_H - N) \\ &\quad + (N-1)p^{N-1}(1-p)\left((N-1)\theta_L + \theta_H - N^2\right) \\ &\quad + \sum_{P=0}^{N-2}\begin{pmatrix} N \\ P \end{pmatrix} p^P(1-p)^{N-P}\left(P\theta_L + (N-P)\theta_H - N^2\right). \end{aligned}$$

Thus the expected aggregate surplus increases with a smaller quota, if

$$\begin{aligned} & U(N-1) - U(N) \\ = \; & p^N(\theta_L - N) + p^{N-1}(1-p)(\theta_H - N) \\ & + (N-1)p^{N-1}(1-p)\left((N-1)\theta_L + \theta_H - N^2\right) \\ & - \begin{pmatrix} N \\ N-1 \end{pmatrix} p^{N-1}(1-p)\left((N-1)\theta_L + \theta_H - N^2\right) \\ = \; & p^{N-1}(N - \theta_L)((1-p)(N-1) - p) \geq 0, \end{aligned}$$

which is equivalent to $p \leq \frac{N-1}{N}$. Again, for a large number of countries N, this is a mild condition.

6.F Comparative statics on $\widetilde{\theta}(k,i)$

In the proof to Proposition 6, I derive a threshold level on the benefit parameter $\widetilde{\theta}(k,i) = \frac{b(k|i,p)}{\sum_{j=k}^{i} b(j|i,p)}$. Here, I show the ordering of the threshold levels to be

$$\widetilde{\theta}(k-1,i-1) \leq \widetilde{\theta}(k,i) \leq \widetilde{\theta}(k,i-1).$$

1. Show that $\widetilde{\theta}(k,i) \geq \widetilde{\theta}(k-1,i-1)$:

$$\begin{aligned} & \frac{\begin{pmatrix} i \\ k \end{pmatrix} p^k(1-p)^{i-k}}{\sum_{j=k}^{i}\begin{pmatrix} i \\ j \end{pmatrix} p^j(1-p)^{i-j}} Q \geq \frac{\begin{pmatrix} i-1 \\ k-1 \end{pmatrix} p^{k-1}(1-p)^{i-k}}{\sum_{j=k-1}^{i-1}\begin{pmatrix} i-1 \\ j \end{pmatrix} p^j(1-p)^{i-1-j}} Q \\ \leftrightarrow\quad & \frac{i}{k}\sum_{j=k}^{i}\begin{pmatrix} i-1 \\ j-1 \end{pmatrix} p^j(1-p)^{i-j} \geq \sum_{j=k}^{i}\begin{pmatrix} i \\ j \end{pmatrix} p^j(1-p)^{i-j} \\ \leftrightarrow\quad & \sum_{j=k}^{i} p^j(1-p)^{i-j}\begin{pmatrix} i-1 \\ j-1 \end{pmatrix}\frac{i(j-k)}{kj} \geq 0 \end{aligned}$$

2. Show that $\widetilde{\theta}(k,i) \leq \widetilde{\theta}(k,i-1)$:

$$\frac{\begin{pmatrix} i \\ k \end{pmatrix} p^k (1-p)^{i-k}}{\sum_{j=k}^{i} \begin{pmatrix} i \\ j \end{pmatrix} p^j (1-p)^{i-j}} Q \leq \frac{\begin{pmatrix} i-1 \\ k \end{pmatrix} p^k (1-p)^{i-1-k}}{\sum_{j=k}^{i-1} \begin{pmatrix} i-1 \\ j \end{pmatrix} p^j (1-p)^{i-1-j}} Q$$

$$\leftrightarrow \quad p^i \geq \sum_{j=k}^{i-1} p^j (1-p)^{i-j} \left(\frac{i}{i-k} \begin{pmatrix} i-1 \\ j \end{pmatrix} - \begin{pmatrix} i \\ j \end{pmatrix} \right)$$

$$\leftrightarrow \quad p^i \geq \sum_{j=k}^{i-1} \begin{pmatrix} i-1 \\ j \end{pmatrix} p^j (1-p)^{i-j} \left(\frac{i(k-j)}{(i-k)(i-j)} \right)$$

3. Show that $\widetilde{\theta}_L(k,i) \geq \widetilde{\theta}_L(k-1,i)$: Follows from 1 and 2.

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