Staff Economic Study

THE FEDERAL RESERVE-MIT ECONOMETRIC MODEL

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From time to time the Federal Reserve BULLETIN *publishes in full staff studies that are of general interest to the economics profession and others.*

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For the last year and a half, a group of economists at the Massachusetts Institute of Technology and at the Board of Governors of the Federal Reserve System have been working together on a new quarterly econometric model of the United States.[1] This paper is the first report of some of the preliminary results of the model.

Since the number of econometric models of the United States can no longer be counted on the fingers of even two hands, it is natural to wonder why we are adding one more to the list. In this instance the major purpose is to be able to say more than existing models about the effects of monetary policy instruments—both in themselves and in comparison with other policy instruments. No existing model has as its major purpose the quantification of monetary policy and its effect on the economy. As a consequence even those which do contain some treatment of monetary instruments and effects suffer from puzzling results either in their financial sectors or in the response to financial variables in other sectors—results which their proprietors would surely investigate further were the models to be used to say something about monetary developments on a current basis. We have tried to avoid these difficulties by concentrating most of our efforts on the treatment of financial markets and on the links between financial markets and markets for goods and services.

A few comments on some of the differences between the Federal Reserve-MIT and other models may clarify this last point. In the financial sector, the general structure of our equations is similar to some other recent models, but our estimates of the lags are quite different. By experimenting with alternative formulations applied to data

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The MIT group was under the direction of Franco Modigliani and Albert Ando, and included varying proportions of the time of Charles Bischoff, Dwight Jaffe, Morris Norman, Robert Rasche, Harold Shapiro, Gordon Sparks, and Richard Sutch.

The Federal Reserve group currently includes, besides the authors, Enid Miller, Helen Popkin, Alfred Tella, and Peter Tinsley, again with varying proportions of working time. Patric Hendershott was until recently a member of the group.

Views expressed in the paper are those of its authors. All the colleagues listed have helped shape these views but have not edited or corrected this paper.

through 1965 and testing the results against data for 1966 and early 1967, we have tentatively concluded that lags in the demand for money are shorter than many recent estimates, and that the transitory impact effect of open-market operations on interest rates (as contrasted to longer-run effects) is smaller than a number of other models imply. The financial sector also differs from some others by including the market for bank commercial loans as an integral part of the determination of money stock and interest rates, and by including a fairly broad range of interest rates.

In the investment sector, the plant and equipment equations (due to Charles Bischoff) are derived from the neoclassical theory of the business firm, but with allowance for lags in forming expectations, lags between orders and shipments, technological change, and the possibility that substitution between capital goods and other factors of production may be feasible to a much greater degree when new equipment or plant is being ordered than after it has been installed. Interest rates and tax rates enter these equations in the way in which the theory of the firm—after modification for the complications just listed—suggests they should affect returns on investment projects.

The equations for housing distinguish between builders and owners of houses on the one hand, and users of dwelling space on the other. It is in the equation describing decisions by the former group to change the inventory of houses under construction that current and recent interest rates enter with a powerful effect. Nevertheless, our model fails to predict the full extent of the decline in housing starts in 1966, and further work to try to determine whether we are understating the effects of monetary policy on housing is high on our future agenda.

Expenditures and taxes of State and local governments are endogenous in our model, in contrast to any other model of our acquaintance. The equations emphasize the interdependence of spending and taxing decisions, with an important interest rate effect on State and local construction expenditures and a smaller, but still noticeable, effect on the proportion of current expenditures financed by taxes.

Finally, in our consumption equations we have attempted to distinguish the services yielded by stocks of durable goods from expenditures on durable goods which are a part of consumer spending in the national accounts. The sum of the services of durable goods and expenditures on nondurables and services is the consumption variable that we relate to current and past income, whereas the allocation of the sum among its components depends on relative prices, existing stocks, and other variables. One result of this formulation is a small effect of interest rates on the allocation of total consumption (in our sense) and hence on consumer expenditures on durable goods.

These are some of the distinguishing features of our model. The preliminary results suggest that both monetary and fiscal policies have powerful effects on the economy, though monetary policy operates with a longer lag. We also find that the response of money income to both monetary and fiscal policy changes is stronger than that implied by other large-scale econometric models.

At this point in our work, however, we would like to emphasize the tentative nature of any conclusions derived from the model. Not all of the key equations predict well, and the number of observations outside our sample period on which to base an evaluaton is still fairly small. In fitting the model, we have made very extensive use of recent improvements in techniques for estimating lag distributions. But with respect to simultaneous equations difficulties, our efforts so far have been largely confined to using a simultaneous estimation technique or transforming an equation to neutralize the bias only in those cases where we felt the problem was especially likely to be important. Putting the model together in its present form has made us aware of some unexpected system characteristics which need to be examined in more detail. We hope that those who use and read about the model will not simply note and store away its major findings but will suggest alternative specifications that we should consider.

* * *

This first report describes the performance and interaction of three large blocks of equations in the model. The first section deals with the financial block—supply and demand equations for financial claims and their dynamics. The second section deals with the fixed-investment block, covering housing, plant and equipment, and the behavior of State and local governments. The third section deals with the consumption-inventory block, and covers income shares, imports, and Federal personal taxes as well as consumption and inventory investment. Each of these sections includes a general description of the block of equations, results of dynamic prediction tests (that is, predictions which generate their own lagged values as they go along), and results of simulations illustrating the behavior of the block. An appendix lists and briefly describes the individual equations of the blocks. The emphasis on the performance of blocks of equations means that there are only brief references to the theoretical hypotheses and the detailed estimation work underlying individual equations. Papers by those who were directly responsible for individual equations will fill these gaps.

The final section of the paper deals with the three blocks combined. It presents prediction results and simulation experiments. A few miscellaneous tax and income share equations are introduced for the first time in this section. Still to be added is the price (supply equation) and labor market block, on which we are presently at work. The simulations of the final section treat prices and wage rates as exogenous, but since our price-wage sector will show a fair degree of price and wage rigidity in the short run under conditions of moderate slack in labor and product markets, the simulations of the final section do indicate roughly the estimates of the effects of monetary policy which are emerging from our efforts so far.

I. THE FINANCIAL BLOCK

The first block of equations describes the behavior of financial markets, given GNP and its components on the one hand and a number of Federal Reserve policy-determined variables on the other. In this, as in other models of financial behavior, the quantity supplied of an open-market operations variable—for this model, unborrowed bank reserves—is exogenous and the identity relating it to deposits, reserve requirements, and bank free reserves (equation 1 on page 31) is a central equation of the block. Demand equations for the various uses of reserves (equations 2 through 4) depend on interest rates and other variables—most importantly, GNP and its components—with interest rates rising or falling in the short-run to bring quantities demanded into balance with the exogenous supply.

As changes in interest rates affect investment, the short-run interest rate effects of changes in monetary policy variables are reduced and the effects on income increased. Our main goal, partly reached in Section IV of this paper, is to explain these effects on income; but in the financial block we shall take GNP and its components as exogenous and consider solely the equations dealing with supplies and demands in financial markets.

Demand equations for demand deposits, time deposits, and free reserves—the three uses of unborrowed reserves—all include the lagged stock of the dependent variable as one of the explanatory variables. The presence of a lagged stock term makes deposits and free reserves adjust only gradually to changes in their determinants, and therefore implies large temporary jumps in interest rates in order to clear markets in response to a change in unborrowed reserves or reserve requirements. It is difficult to understand why the adjustment in deposits and reserves implied by the introduction of lagged stocks should be as gradual as it usually turns out to be in studies of the demand for money or of bank behavior. In our model adjustments are somewhat faster than in a number of other recent models, and the implied temporary jumps in interest rates are not so large. The simulations described below indicate just how large they are; the estimation and testing procedure which led to this result is outlined in the appendix.

Banks are assumed to accommodate short-run changes in loan demand by their business customers partly by changing their free reserve position. Other bank earning assets are not assumed to have this direct effect on reserve behavior, with the result that the composition of bank credit has a short-run effect on interest rates in the model. The main influence on changes in bank commercial loans is business inventory investment.

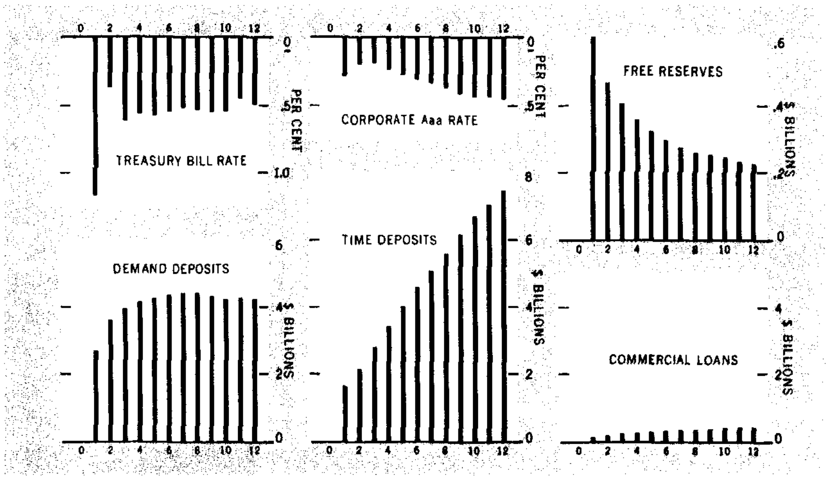
The various interest rates in the financial sector are closely interrelated. A number of equations explain the slower-moving rates largely as complex distributed lags of the more volatile short-term rates or of the corporate bond rate which in turn depends on short-term rates. The dividend yield on common stocks is one of these slower-moving rates; our equation does not explain a high portion of its variance, but does connect it with the corporate rate and thereby relates at least some of the variation in stock prices to developments in other financial markets.

The dividend yield equation also contains terms measuring the past rate of growth of dividends as a proxy for expected capital gains. Apart from these terms, variables reflecting price expectations are absent from the model. It is very difficult to detect such influences in data for this economy during the last two decades, although price expectation effects are clearly present in economies with larger and more variable inflationary spurts.

DYNAMIC PREDICTIONS

Predictions of the financial sector during 1966 and early 1967 are fairly successful. Table 1 shows prediction results based on a dynamic simulation (that is, one generating its own lagged values as it goes along) of the sector starting in the third quarter of 1965. The model successfully predicts the unusually large increases and then declines in interest rates. It does not predict the absolute decline in demand deposit holdings which took place in 1966, but it does predict a very marked slowdown in their rate of growth. Since 1966 and 1967 were outside the sample period used to fit the model, these results are decidedly encouraging. However, the fact that we selected from among several sets tested those equations which performed best in 1966 and early

1 | Effects of a step increase of $1 billion in unborrowed reserves, financial block



Dynamic simulation, initial conditions of 1963 QI.

1967 certainly biases the prediction tests in our favor. It will be some time before we are able to say with confidence how well these equations perform outside the sample period.

As in other sectors of the model, predictions based on one period simulation (that is, using actual values of all lagged variables) are much more accurate than the dynamic predictions in Table 1. Since the model is intended to be of use for evaluating alternative policies over several quarters, however, dynamic predictions are a more relevant test.

TABLE 1

DYNAMIC PREDICTIONS, FINANCIAL BLOCK

Item	1965 QIII	1965 QIV	1966 QI	1966 QII	1966 QIII	1966 QIV	1967 QI
	(In per cent)						
Treasury bill rate:							
Actual	3.86	4.16	4.60	4.58	5.03	5.20	4.52
Predicted	3.40	4.24	4.51	4.98	5.21	5.13	4.48
Corporate Aaa bond rate:							
Actual	4.50	4.61	4.81	5.00	5.32	5.38	5.12
Predicted	4.60	4.82	5.02	5.20	5.35	5.42	5.35
Mortgage rate:							
Actual	5.76	5.78	5.85	6.03	6.17	6.39	6.34
Predicted	5.74	5.82	5.97	6.14	6.30	6.42	6.46
	(In billions of dollars)						
Demand deposits:							
Actual	128.9	131.1	133.3	132.8	132.2	131.6	133.5
Predicted	129.3	131.0	132.1	132.6	133.0	134.1	135.8
Time deposits:							
Actual	142.6	147.5	150.5	155.5	158.1	160.4	167.4
Predicted	141.7	145.8	150.2	153.7	156.9	160.4	164.7
Free reserves:							
Actual	−.15	−.02	−.26	−.36	−.40	−.09	+.21
Predicted	−.17	+.04	−.10	−.27	−.46	−.40	+.02

NOTE.—Financial dollar amounts are all averages of the 2 months surrounding the end of quarter; for example, fourth quarter is average of December and January.

SIMULATION RESULTS

To keep the present paper to manageable size, we present only two simulation results for the financial sector, one tracing out the effects over time of a step increase in unborrowed reserves, and the other tracing out the effects of a step increase in GNP. Simulations of other monetary policy variables in the model—required reserve ratios, the discount rate, and the ceiling rate on bank time deposits—will be the subject of a future presentation.

The unborrowed reserve simulation, illustrated in Chart 1, shows the differences between (a) solution values for the model beginning in 1963 QI with unborrowed reserves $1 billion above actual values and (b) solution values for the model beginning in 1963 QI with actual unborrowed reserves.[2] All other variables exogenous to the financial block are held at actual values for both sets of solution values but in both sets, lagged values of endogenous variables are generated by the model as the solutions progress from quarter to quarter. All the simulation results in the paper follow the same pattern—differences between two sets of dynamic solution values starting in 1963-QI and holding at actual levels all exogenous variables except the one which is the subject of the simulation.

The familiar whiplash effect of open-market operations on interest rates—the large initial impact followed by a smaller permanent effect—is visible in Chart 1, but in much milder form than in a number of other financial models. The impact effect is due to lags in the demand for money and free reserves, and its mildness in Chart 1 is due to the shorter lags in the present model than in some others. For the corporate rate, the initial impact effect is even smaller than the longer-run effect.

Demand deposits soon approach a change about four times the change in unborrowed reserves. This multiple is smaller than the reciprocal of the average reserve requirement against demand deposits for two reasons. First of all, free reserves absorb some of the change in unborrowed reserves—a large fraction initially, and a small fraction even after a lag because of the decline in the bill rate relative to the discount rate, which has not changed. Second, some of the increase in reserves is required to back the increase in time deposits which takes place over a long period because market rates of interest fall relative to the time deposit rate. The fact that not all banks are members of the Federal Reserve System affects the multiple in the opposite direction from the free reserve and time deposit effects.

The second simulation of the financial block deals with a step increase of $10 billion in GNP. It is necessary to make an assumption about how much of the increase goes into inventory investment in order to solve the commercial loan demand equation; the assumption we use is that $4 billion of the $10 billion goes into inventory investment the first quarter, $3 billion of the $10 billion in the second quarter, and so on down to zero in the fifth and succeeding quarters.

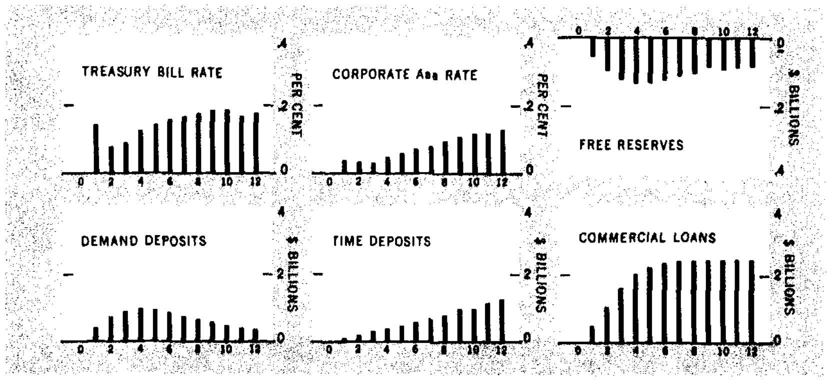
The results, depicted in Chart 2, indicate that, according to our model, income changes have important and fairly prompt effects on financial markets. Interest rates on Treasury bills are driven up sharply, and the corporate bond rate responds after a short lag. The effect on demand deposits builds up to nearly a billion dollars, then falls off as higher interest rates curb the demand for money. The effect on time deposits, as before, develops much more slowly.

II. THE INVESTMENT BLOCK

The investment block of the model consists of components of final demand which are often considered autonomous in simple income-expenditure systems; namely, housing, producers' equipment and structures, and the expenditures and taxes of State and local governments. All of these items are

[2] For the starting point of the simulations, we wanted a fairly recent quarter without abnormal pressures in credit or goods market and preferably with enough slack capacity so that the absence of a price-wage sector would not greatly affect simulation results for the rest of the model. The first quarter of 1963 fills all of these requirements.

2 | Effects of a step increase of $10 billion in GNP, financial block



Dynamic simulation, initial conditions of 1963 QI.

relatively insensitive to the current quarter's income and relatively sensitive to interest rates and relative prices.

The equations for producers' durable equipment and for nonresidential structures (both due to Charles Bischoff) allow interest rates, tax regulations, and relative prices all to affect expenditures through their effect on desired capital-output ratios, in the way suggested by the neoclassical theory of the firm. The empirical fitting of these equations is sufficiently flexible to allow for long lags in adjustment, certain kinds of technological change, and different weights for the corporate bond rate and the dividend-price ratio in measuring the cost of capital.

The fitting also allows for the possibility that capital goods and other factors of production may be much more readily substitutable at the time new capital is being ordered than after it has been installed. For producers' equipment, a "putty-clay" model, in which factor substitution is possible only up until the time of placing orders, turns out to fit the data better than a model in which capital intensity can be altered after—as well as before—installation. One implication of the putty-clay formulation is that a permanent change in interest rates affects investment gradually over the entire time-span needed to replace the existing capital stock rather than in a more concentrated period. For structures, in contrast to producers' equipment, a model allowing for substitution after as well as before installation turns out to fit the data better than a putty-clay model.

Bischoff's equations have other interesting features. With respect to tax laws, they measure the present value of depreciation deductions under various laws, the investment tax credit, and even the effect of the 1964 Long Amendment which changed the tax treatment of equipment eligible for the investment credit. With respect to all cost variables, these equations allow for an elasticity of substitution different from one. For producers' equipment, the central demand variable is not final expenditures but new orders. Orders are translated into expenditures through a variable-weight distributed lag. A technique developed by Peter Tinsley is used to estimate the way the lag lengthens in periods of supply bottlenecks (as measured by a high ratio of unfilled orders to expenditures) and shortens when the bottlenecks disappear.

The housing sector of the model, which follows the work of Gordon Sparks, distinguishes between houses as providing a stream of services for those who live in them and houses as profitable investments for those who own them. The rental price index clears the housing service market. After a lag this rental price rises with real income and population and falls with the supply of houses. In another relationship, the same rental price is the numerator of the investment rate of return on houses, the denominator being the price deflator for houses. This relationship, involving long lags, relates the rate of return on houses to the rate of interest on mortgages. The rate of return does not adjust by the full amount of the change in the mortgage rate, implying that houses and mortgages are not perfect complements.

Builders are assumed to respond to investment demand. If this demand is high, as measured by the investment rate of return on houses relative to mortgage rates, builders carry high levels of inventory under construction, and housing starts and expenditures are high. On the other hand, if mortgage rates are high, housing inventory and starts are low.

The housing sector estimated in this way has interesting dynamic properties. In the long run the mortgage rate and the investment return on houses should be approximately in balance, and housing inventories and expenditures should not be greatly affected by the level of mortgage rates. But in the short- and medium-run, the fact that a change in mortgage rates is only slowly transmitted to a change in the rate of return on houses means that a rise in mortgage rates can have strong depressing effects on housing expenditures.

The housing sector may underestimate the influence of some basic variables on housing expenditures. Income, population, and the stock of houses are all forced to operate through the rental market, and it may be that imperfections in the measured rental price index unduly weaken the effects of these variables. Similarly, it may be that the mortgage rate does not capture all relevant dimensions of the ease or tightness of credit, especially in periods such as 1966 when nonbank financial institutions experienced a marked reduction in deposit inflows. We plan to examine both of these possibilities in more detail and eventually hope to develop a more elaborate treatment of nonbank financial intermediaries and the credit side of the housing market.

The equations explaining the behavior of State and local governments have as their basis the constraint against borrowing on current account faced by these institutions. This constraint introduces strong interdependence of spending and tax decisions for States and localities. Tax revenues are affected by expenditure needs, and expenditures are in turn affected by taxes.

For reasons relating to the simultaneous equations bias, the expenditure equations have been solved directly for their reduced form. Thus expenditures depend on such variables as Federal grants-in-aid, income, interest rates, population, the proportion of the population of school age, and prices. Taxes are affected by the current expenditures that have to be revenue financed together with income, with the proportion of revenue-financing depending on interest rates.

DYNAMIC PREDICTIONS

The investment block was solved dynamically for the six-quarter period from 1965 QIII to 1966 QIV. Most of the equations have been fit through the end of 1965, and since these equations are generally highly dependent on interest rates, 1966 predictions would be of special interest.

The results of the dynamic predictions are given in Table 2. Rather than present actual and calculated values for every variable in the block, Table 2 only shows the important summary variables for each type of spending. Thus the equation for expenditures on housing summarizes the performance of the entire housing market, and so forth.

To review the results briefly, the housing sector predicts actual housing expenditures well until 1966 QIII, when the decline in actual expenditures was much greater than that shown by our model. Our predictions turn down at the right point, but they do not fall nearly enough. Possibly a model allowing for nonprice credit rationing would improve the housing predictions for late 1966.

Actual expenditures for producers' durables did not turn down until early 1967; but here, in contrast to the housing equations, our predictions understate expenditures by turning down two quarters too soon.

TABLE 2

DYNAMIC PREDICTIONS, INVESTMENT BLOCK
(In billions of dollars)

Item	1965		1966			
	QIII	QIV	QI	QII	QIII	QIV
Residential construction:						
Actual	26.4	26.2	26.5	25.3	23.2	20.4
Predicted	26.5	26.4	26.0	25.4	24.7	23.8
Producers' durable equipment:						
Actual	46.8	48.3	50.0	51.2	53.1	55.1
Predicted	46.6	48.8	50.6	51.0	50.8	50.8
Nonresidential construction:						
Actual	25.1	27.3	28.3	27.5	28.2	27.7
Predicted	24.6	25.2	25.8	26.5	27.0	27.3
State and local government expenditures:						
Actual	70.4	72.5	74.3	76.2	78.1	80.2
Predicted	70.5	72.3	74.5	76.3	78.0	80.0
Surplus of State and local governments:						
Actual	1.5	1.1	2.4	2.9	3.3	3.0
Predicted	2.5	2.7	3.2	3.2	3.3	3.2
Total residential and nonresidential construction and producers' durable equipment						
Actual	98.3	101.8	104.8	104.0	104.5	103.2
Predicted	97.7	100.4	102.4	102.9	102.5	101.9

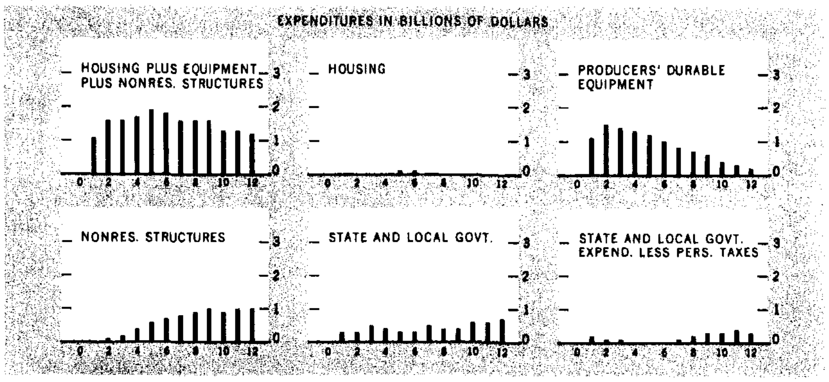
The experience for the other equations is much better. Predicted expenditures for nonresidential structures are low throughout the period, but by the end of the period the model is performing appreciably better than in the beginning. Predicted purchases by State and local governments are extremely accurate throughout the period, as are even the predictions of the entire budget surplus which includes errors for all purchase, tax, and transfer equations.

The last item of the table summarizes the performance of the investment block, by listing the actual and predicted values for expenditures on housing, producers' durables, and structures, which make up gross private domestic fixed investment. Because the errors in housing and producers' durables offset each other, the total gross investment error is relatively small, averaging somewhat less than $1.5 billion.

SIMULATIONS OF BEHAVIOR OF BLOCK

Two basic simulation runs for the investment block trace out the effects of changes in income and the effects of changes in interest rates. As in the financial block, simulation results are differences between a dynamic solution using either higher-than-actual income or higher-than-actual interest rates and a dynamic solution using actual income and interest rates, both solutions starting in 1963 QI. The income simulation inserted a $10 billion step increase in GNP—GNP was put $10 billion above its actual level in each quarter—with appropriate responses in other income variables but no change from actual interest rates; while the interest-rate simulation inserted a percentage point increase in the corporate bond rate with appropriate responses in other interest rates but no departure from actual income.

3 | Effects of a step increase in GNP of $10 billion, investment block



Dynamic simulation, initial conditions of 1963 QI.

The results for the 12 quarters after the change are presented in Charts 3 and 4. The housing sector behaves very much as described above. There is almost no effect of income on housing expenditures, attributable to the fact that the income elasticity of the rent index is very low. Yet there is a very sharp effect of interest rates. This effect reaches its peak of $2.8 billion six quarters after the interest rate change, and then gradually recedes to zero as the mortgage rate and the rate of return on houses come into balance.

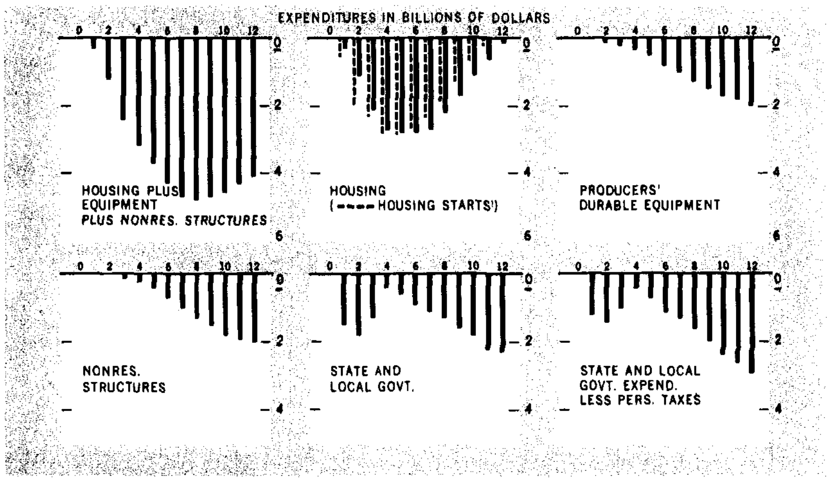
Expenditures for producers' durables also behave as the putty-clay model implies. They respond almost immediately to income and then recede to zero as the desired capital-output ratio is restored. But because of the fixed-factor proportions of installed equipment, the response to interest rates shows a very gradual decline which still has not reached its peak after 3 years.

Lags in the structures equation are very long. The underlying model implies that at some point the response either to income or interest rates will reach a peak, and then fall towards zero. As Charts 3 and 4 indicate, the expenditure response still has not reached its maximum 3 years after the initial change. Yet it is interesting to note that in this case, as opposed to equipment, the lag patterns are similar for income and interest rates.

The purchases made by State and local governments respond fairly rapidly both to income and to interest rates. In the income simulation the budget surplus increases because revenues increase even more than purchases. But for the revenue items that matter—excluding the effect on indirect taxes, which do not feed back to the model to a significant extent—the response is slightly less than the expenditure response, such that States and localities are a slight destabilizing force in the determination of aggregate demand as long as interest rates are held constant.

In the interest rate simulation the initial bulge in the State and local expenditure effect is due to a large postponement effect for wages and salaries, and the long-run effect is due to the delayed response of construction expenditures—which behave in a manner similar to producers' structures. The budget surplus increases more than expenditures decrease in this simulation because high interest rates result in decreased borrowing or in increased tax financing of the expenditures already being made.

4 | Effects of a step increase in the corporate bond rate of 1 per cent, investment block



Dynamic simulation, initial conditions of 1963 QI.

III. THE CONSUMPTION-INVENTORY BLOCK

The third block of equations describes the behavior of consumption, inventory investment, imports, personal income, and taxes and includes the identity adding up the components of GNP. These variables are all tied very closely to the level of, or changes in, current income, and would be considered endogenous in even the simplest income-expenditure system. The multiplier sector would be an appropriate title for this block.

The multiplier implicit in the present model is more complicated than the simple textbook concept. One important reason is that we distinguish between the national accounts version of consumption, which includes expenditures on consumer durables, and our own version, which instead includes the flow of services—as best we can estimate it—from these durables. Following many other students of consumption,[8] we assume our concept has a stable relation to current and past income. This concept of consumption shows smaller variations than the national accounts total since an additional dollar spent on consumer durables raises our consumption by less than a dollar now and by a positive amount in subsequent quarters, rather than by a full dollar now and nothing in subsequent quarters. One implication of this view of consumption is that expenditures on durable goods are quite sensitive to changes in income, because large changes in expenditures are necessary in order to keep our version of consumption in its desired relationship with income.

The allocation of consumption between nondurables and services on the one hand and the services of durable goods on the other depends on relative prices, existing stocks of durables, recent income changes, and to a minor extent interest rates. These forces have all been constrained so that if they increase one component of consumption, they decrease one or more others by an exactly offsetting amount.

[8] For example, Milton Friedman, *A Theory of the Consumption Function* (Princeton University Press, 1957); and Albert Ando, and Franco Modigliani, "The 'Life Cycle' Hypothesis of Saving," *American Economic Review*, March 1963.

A second important reason for a complex multiplier is the inventory investment equation. Our model allows the different components of final demand to affect inventories by different amounts—implying different inventory-sales ratios—and with different lags—implying different periods of production or different ways of forming sales expectations. One prominent example of the differing lags which has received much attention lately is defense spending;[4] in our equation defense spending has an effect on inventories *before* the final expenditures are recorded in the national accounts. Our equation also features a faster speed of adjustment, and therefore a larger accelerator effect, than most other models.

The rest of the block is fairly standard. In the absence of an elaborate treatment of income distribution through the price and wage block, we have a simple equation which relates personal income net of exogenous transfer payments to current and past GNP. Personal income taxes on a liability basis depend on personal income, exemptions, and the average tax rate in equations based on the work of Ando and Brown. As they recommend, the model uses a tax accrual disposable income concept rather than the cash version used in the national accounts.[5]

[4] See Murray Weidenbaum, "The Economic Impact of the Government Spending Process," *The University of Houston Business Review*, Spring 1961.

TABLE 3

DYNAMIC PREDICTIONS, CONSUMPTION-INVENTORY BLOCK

(In billions of dollars)

Item	1965 QIII	1965 QIV	1966 QI	1966 QII	1966 QIII	1966 QIV
GNP level:						
Actual	690.0	708.4	725.9	736.7	748.8	762.1
Predicted	691.8	712.8	729.8	739.5	752.1	763.3
GNP changes:						
Actual	14.6	18.4	17.5	10.8	12.1	13.3
Predicted	16.4	21.0	17.1	9.6	12.7	11.1
Disposable income:						
Actual	479.2	489.1	498.1	505.5	514.9	524.5
Predicted	479.0	489.3	499.7	506.9	516.1	525.1
Consumer expenditures:						
Actual	436.4	447.8	458.2	461.6	470.1	473.8
Predicted	441.6	451.4	461.5	468.1	476.6	483.6
Inventory investment:						
Actual	7.9	8.7	9.6	14.4	12.0	19.0
Predicted	5.4	10.1	10.8	10.8	7.6	9.3
Imports:						
Actual	32.9	34.4	36.0	37.1	39.0	39.7
Predicted	33.6	35.1	36.5	37.3	38.0	38.6

DYNAMIC PREDICTIONS

Dynamic predictions for the consumption-inventory block beginning in 1965 QIII are quite successful. The main exogenous variables on which the predictions depend are fixed investment, exports, and various receipts and expenditures of Federal, State, and local governments. Given actual values of these variables, the model makes only small errors in predicting the course of GNP.

The principal change in the behavior of GNP during the period was the slowdown in quarterly changes starting in the second quarter of 1966. This the model captures at precisely the correct time. A secondary change in the behavior of GNP was the rise in the fourth quarter of 1966 due to extraordinarily high levels of inventory investment. This change the model does not capture, greatly understating inventory investment at the end of 1966.

The understatement of inventory investment in the second and fourth quarters of 1966 is offset in part by an overstatement of consumption expenditures. In part, this offset is a lucky accident. In part, however, it is what we would expect from examining the inventory equation of the model. Consumption expenditures in the current quarter

[5] Albert Ando and E. Cary Brown, "The Effects of the Personal Income Tax Reduction of 1964 on Consumption," to be published.

have an involuntary, negative effect on inventory investment so that an error in predicting current consumption leads to a partially offsetting error in predicting inventory investment in the same quarter.

POLICY SIMULATIONS

Simulations which are most helpful in illustrating the dynamics of this block of equations are those showing how a maintained increase in government or fixed investment spending, or a change in tax rates or transfer payments, multiply into changes in GNP. Before those simulations of the whole block, however, it may be helpful to illustrate the behavior of the consumption equations alone in response to a step increase in disposable income.

Table 4 below traces out the pattern of consumption on the one hand and consumer expenditures on the other in response to a maintained increase of $1 billion in disposable income. Total consumption follows the behavior of equation 11 (page 38), rising by 37.3 per cent of the income change in the first quarter, then rising by smaller increments, and after 3 years reaching 94 per cent of the income change. A portion of this increased consumption goes into the services of durable goods. In order for the services of durable goods to rise, it is necessary for expenditures on durable goods to rise by an accelerated amount at first, then as stocks rise, to fall back gradually toward the new level of consumption. Total consumer expenditures therefore increase initially by 67.5 per cent of the change in disposable income, continue rising until they actually exceed 100 per cent of the income change for a few quarters, and then decline towards 94 per cent.

TABLE 4

EFFECTS OF A $1 BILLION STEP INCREASE IN DISPOSABLE INCOME

(In billions of dollars)

Quarter	Consumption	Consumer expenditures
1	.37	.68
2	.46	.75
3	.54	.82
4	.61	.88
5	.67	.93
6	.73	.97
7	.78	1.01
8	.83	1.04
9	.86	1.05
10	.89	1.06
11	.92	1.07
12	.93	1.07
13	.94	1.06
14	.94	1.04
15	.94	1.02
16	.94	.99

Turning now to the complete consumption-inventory block, we trace out first the effects of a step increase of two percentage points in the aggregate Federal personal income tax rate. As in the simulations of the financial and investment blocks, all variables exogenous to this sector except the tax rate were put at their actual values during 1963-64 while the tax rate was raised by 0.02 above its actual value during each quarter. An increase of two percentage points is roughly a 10 per cent increase, since the actual rate was between 0.20 and 0.23 during the period. It represented a little over $4 billion dollars in tax revenue at actual levels of income during the period. But that dollar amount, like many of the dollar amounts in these simulations, depends on the general size of the economy during the simulation period.

The GNP effects of the policy change are shown in Chart 5 in constant dollars. They begin with two big steps and continue with six much smaller ones. Using $4.15 billion as the initial revenue value of the tax change, we can derive GNP multipliers which begin

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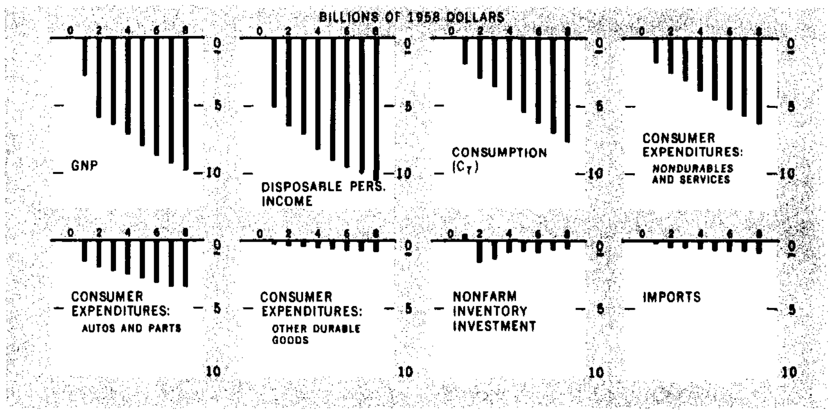
5 | Effects of a step increase in the personal tax rate of .02, consumption-inventory block



Dynamic simulation, initial conditions of 1963 QI.

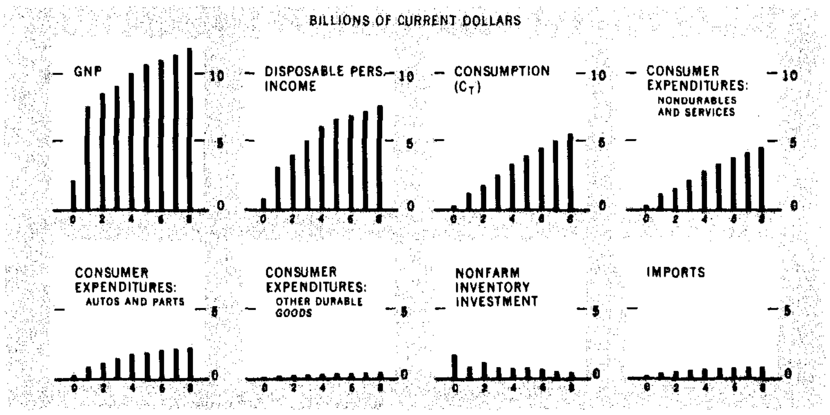
at 0.7 and 1.4 in the first two quarters, and then rise slowly to 2.4 after eight quarters. Disposable income is affected by more than GNP, with the margin between the two declining over time. Most of the change in GNP is due to changes in the components of consumer expenditures; import and inventory effects are quite small. Inventory investment is increased slightly in the first quarter, reflecting unanticipated declines in consumer expenditures; in the second quarter it is decreased by a somewhat greater amount, reflecting the accelerator effects of the decline in consumer spending; and thereafter it is decreased by declining amounts.

The second policy change we trace through this block is a maintained increase of $5 billion in defense spending—that is, a level of defense spending $5 billion above actual levels starting in the first quarter of 1963, with other variables exogenous to this sector held at actual levels. Chart 6 sets out the results. Since the $5 billion is in current prices, results for this simulation are also presented in current dollars, in contrast to the results of the tax rate change simulation.

The GNP effects of the rise in defense spending begin before the rise is actually recorded as a final expenditure, since inventory investment depends in part on next quarter's defense expenditures. This initial effect is a small one, however, amounting to less than half of the rise in defense spending. Large effects begin in quarter 1 and increase by generally declining amounts thereafter. GNP multipliers are 1.5 in quarter 1, 1.7 in quarter 2, and small increases thereafter up to 2.4 in quarter 8. Effects on consumption are less important in this simulation than they were in the tax change, for the initial shock to the system only gradually spreads to disposable income in this simulation; whereas it has its full initial impact on disposable income in the tax change case. Import and inventory effects again are small.

Comparison of the tax and the defense expenditure multipliers brings out some interesting characteristics of this block. In the long run, the expenditure multiplier is

6 | Effects of a step increase in defense expenditures of $5 billion, consumption-inventory block



Dynamic simulation, initial conditions of 1963 QI.

slightly larger than the multiplier of a lump-sum tax change because of the leakage into personal saving. In the eight-quarter simulations depicted in Charts 5 and 6, however, the expenditure multiplier is larger than the tax multiplier only in the first few quarters, and slightly smaller thereafter. One reason for this similarity is that in the medium run the response of consumer spending to changes in income is quite large, even exceeding one in some quarters. A second reason is that the tax simulation is a simulation of a *rate* change, not a lump-sum change, and therefore has effects which grow as the economy grows. A final reason is that the inventory effects of a change in exogenous spending are bunched around the time of the expenditure, whereas the inventory effects of a tax change are spread over a long period when consumer expenditures are changing.

IV. THE THREE BLOCKS COMBINED

We now combine the three blocks already described—the financial block, the investment block, and the consumption-inventory block—into a single group of simultaneous equations. The principal exogenous variables which ultimately drive the system are: population and other demographic variables; Federal Government expenditures and tax rates; monetary policy variables; exports; and wages and prices (except for rents and the price of houses, which are explained in the investment block). A fourth block containing price (supply) equations and labor market equations, now under development, will remove prices and wages from the exogenous list.

The simulations in this section illustrate how the first three blocks interact. We have run some simulations (not shown below) including preliminary price and labor market equations,[6] which suggest that except in conditions of high resource utilization, the major results for the three blocks will continue to hold for the entire model.

[6] Some of these equations are described in Alfred Tella, and Peter Tinsley, "The Labor Market and Potential Output," *Proceedings* of the American Statistical Association, December 1967 meetings, to be published.

TABLE 5

DYNAMIC PREDICTIONS, THREE BLOCKS COMBINED

Item	1965 QIII	1965 QIV	1966 QI	1966 QII	1966 QIII	1966 QIV
			(In billions of dollars)			
GNP level:						
Actual	690.0	708.4	725.9	736.7	748.8	762.1
Calculated	690.7	709.4	725.4	736.1	745.6	753.1
GNP changes:						
Actual	14.6	18.4	17.5	10.8	12.1	13.3
Calculated	15.3	18.7	16.0	10.7	9.5	7.4
Consumer expenditures:						
Actual	436.4	447.8	458.2	461.6	470.1	473.8
Calculated	441.1	450.3	460.1	466.7	473.8	479.7
Residential construction:						
Actual	26.4	26.2	26.5	25.3	23.2	20.4
Calculated	26.6	26.3	25.7	25.1	24.2	23.3
Producers' equipment and nonresidential structures:						
Actual	71.9	75.8	78.3	78.7	81.3	82.8
Calculated	71.2	74.2	76.3	77.1	76.9	76.4
Inventory investment:						
Actual	7.9	8.7	9.6	14.4	12.0	19.0
Calculated	5.4	9.8	10.4	10.6	7.0	6.6
Demand deposits:						
Actual	128.9	131.1	133.3	132.8	132.2	131.6
Calculated	129.1	130.8	132.2	132.3	132.5	132.8
			(In per cent)			
Corporate bond yield:						
Actual	4.50	4.61	4.81	5.00	5.32	5.38
Calculated	4.65	4.84	4.95	5.29	5.39	5.51
Treasury bill rate:						
Actual	3.86	4.16	4.60	4.58	5.03	5.20
Calculated	3.60	4.26	4.22	5.42	5.16	5.47

DYNAMIC PREDICTIONS

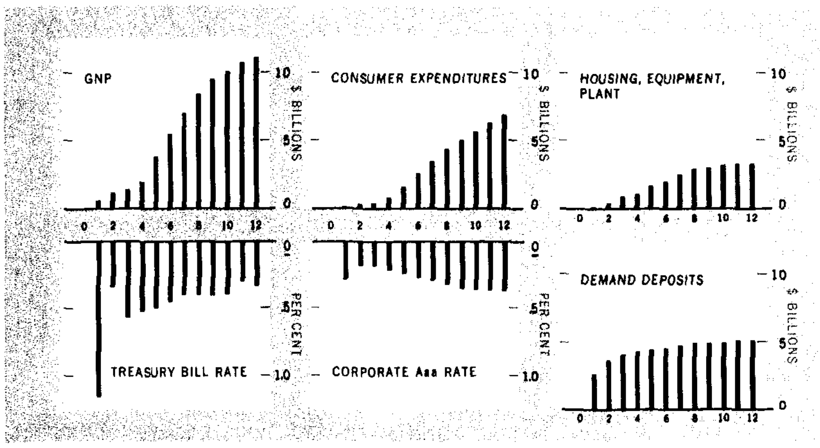
Dynamic predictions starting in 1965 QII for the three blocks combined are not as good as predictions for individual blocks, but they are nevertheless decidedly encouraging. For GNP, as Table 5 shows, the model predicts the marked slowdown in growth which begins in 1966 QII. It fails to predict the slight pickup in growth in the fourth quarter, but that pickup was short-lived, and it is likely that the model would be on track again in 1967 QI.

A rise in interest rates of something like the right magnitude is predicted, though there is a large error in the bill rate prediction for 1966 QII. Whether the three blocks together predict the decline in interest rates in 1967 QI, as the financial block alone did, we will not know until we finish collecting 1967 data for all three blocks. The demand deposit predictions fail to catch the absolute decline in deposits during 1966, but they do show a marked slowdown in their rate of growth.

SIMULATION RESULTS

The simulation experiments of this section trace out the effects of three policy changes already investigated for individual blocks—namely, a $1 billion step increase in unborrowed reserves, a $5 billion step increase in defense spending, and a 0.02 (10 per cent) increase in the personal tax rate. In the near future we plan to simulate the effects of a much wider range of government policy variables. As before, the results represent differences between a dynamic solution including the policy change and one excluding the policy change, with both solutions starting in 1963 QI.

7 | Effects of a step increase of $1 billion in unborrowed reserves, three blocks combined



Dynamic simulation, initial conditions of 1963 QI.

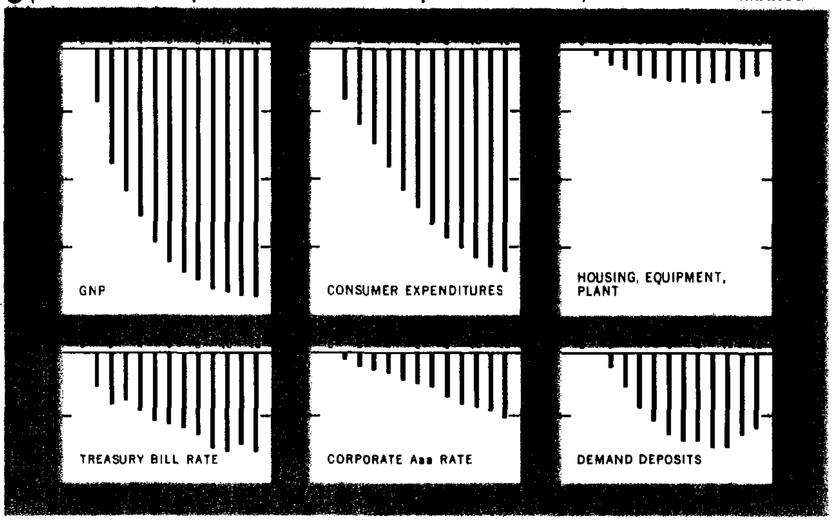
For the unborrowed reserve simulation (Chart 7), effects on fixed investment (partly due to lower interest rates and partly due to higher income) build up gradually to a little more than $3 billion. Effects on GNP are small in the first few quarters; they accelerate as the increase in fixed investment has its multiplier influence, and then decelerate as fixed investment reaches a peak. At the end of the 3 years GNP has increased by more than $11 billion, which implies a somewhat higher multiplier for unborrowed reserves than is shown by most other models (see Table 6). This simulation says, then, that monetary policy is ultimately quite powerful but that the lags are long. To that extent, these tentative results suggest that monetary policy is difficult to use as a stabilization device. The powerful impact of a policy change will not come into play until one year hence, when it is inevitably more difficult to predict the needs of stabilization policy.

Both fiscal policy simulations tell different stories. As in the consumption-inventory block, changes in defense spending operate faster than income tax changes, which depend on the delayed response of consumption. Also it remains true that the medium-term (2- to 3-year) multiplier for income taxes is higher than that for defense spending. This result follows from the fact that the medium-term consumer expenditure propensity is greater than one (see Section III), and from the fact that the income tax multiplier has not yet entered the reversal range that occurs when actual stocks of capital and consumer durables approach their target levels.

It is interesting to observe the behavior of fixed investment in these simulations. In both cases the income change induces more investment than the stabilizing interest rate change shuts off, and fixed investment reinforces the multiplier action. This property contradicts the argument that the induced rise in interest rates will restrict fixed investment enough to offset part of the initial expenditure change.

The simulations indicate that fiscal policy suffers less from the lag problems that plague monetary policy. Both multipliers, especially that for purchases, approach their maximum levels rapidly, and are responsible for strong

8 | Effects of a step increase of .02 in the personal tax rate, three blocks combined



Dynamic simulation, initial conditions of 1963 QI.

effects on GNP less than half a year after the policy change. The lag problems that may interfere with the effectiveness of fiscal measures are lags between recognition of the need for action and actual changes in tax rates or expenditures, not lags in the economy's response to the policy changes.

TABLE 6

COMPARISON OF 3-YEAR MULTIPLIERS OF DIFFERENT MODELS

(Ratios)

Model	Un-borrowed reserves	Defense spending	Personal tax cut
Federal Reserve—MIT model	11.2	3.2	4.2
Brookings model [1]	8.2	2.7	1.2
Wharton School model [2]	2.9	2.9	2.4
Michigan model [3]	n.a.	2.5	1.7

[1] Gary Fromm and Paul Taubman, *Policy Simulations with an Econometric Model,* Chapter II, forthcoming.

[2] Michael K. Evans and Lawrence R. Klein, *The Wharton Econometric Forecasting Model,* University of Pennsylvania Study in Quantitative Economics No. 2, 1967.

[3] Unpublished results of the University of Michigan model supplied by Daniel B. Suits.

COMPARISON OF MULTIPLIERS OF ALTERNATIVE MODELS

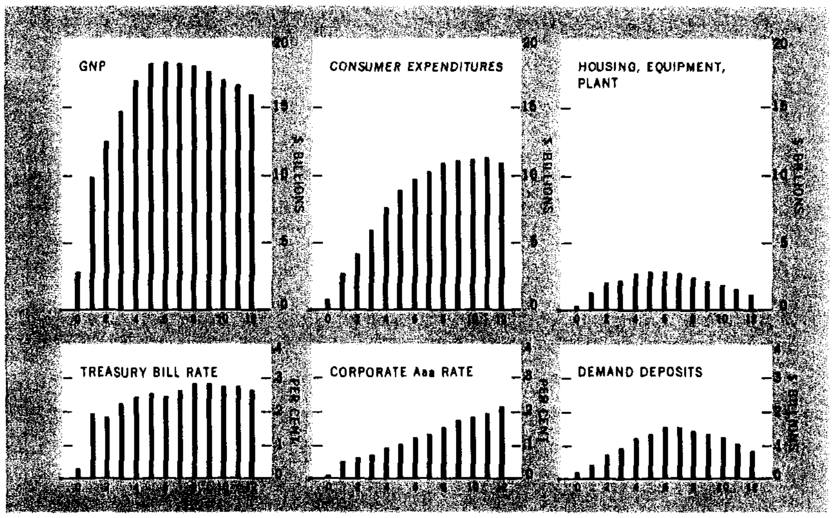
A brief comparison of our 3-year multipliers with those estimated by a few other models are given in Table 6.

The table indicates that all of our multipliers are higher than those for other models. The difference is due primarily to the fact that our version of consumption gives rise to a much higher medium-run consumer propensity to spend than is shown by other models. The same factor accounts for our higher personal tax multiplier.[7]

[7] Our multiple tax multiplier seems to be unusually high relative to our expenditure multiplier in Table 5. Although this result is partly explained by the greater-than-one medium-run expenditure propensity mentioned above, the major share of the explanation lies in the time period chosen as the basis of the multiplier calculation. Had we presented 2-year results, the expenditure and tax multipliers would have been quite similar. The same is true of longer-run 5- to 7-year multipliers. It is only in the 3- and 4-year range, when the expenditure multiplier has begun to decline from its maximum value while the tax multiplier has not, that the unusual result of Table 5 obtains.

We should like to emphasize here also that these findings are preliminary. It may be that further experimentation with consumption will lead to different functional forms, statistical estimates, and multiplier calculations.

9 **Effects of a step increase in defense expenditures of $5 billion, three blocks combined**



Dynamic simulation, initial conditions of 1963 QI.

V. CONCLUSION

It is apparent from the limited number of policy simulations conducted that our model finds monetary policy to be quite powerful —much more so than is found in other econometric models. Future refinements of our model, of which an examination of the financial intermediary-credit rationing process in the mortgage market is a basic one, could increase the relative power that we attribute to monetary policy and might shorten the lags.

These findings follow strictly from our best specification of the way in which monetary policy affects the economy. They are not caused by simple expedients such as throwing in the money supply whenever nothing else works. Although we emphasize that our conclusions are tentative and caution against using them as a basis for generalizations about stabilization policy, we think it significant that a more intensive examination of monetary policy than is usual in econometric models finds monetary factors to be more important than they are usually found to be.

APPENDIX

Financial Block

The central supply identity of the financial block (equation 1) relates unborrowed reserves, taken as exogenous, to three endogenous uses of reserves—reserves against private demand deposits, reserves against time deposits, and free reserves —and one exogenous use—Federal Government demand deposits. The open-market variable which the Federal Reserve System controls to the last dollar is not unborrowed reserves but its own portfolio of government securities. Yet changes in the two are closely related, and there is no doubt that the System can, and often does, consciously offset movements in the other elements in its balance sheet so as to influence unborrowed reserves.

The next three equations (equations 2 to 4) explain demands for the three endogenous components of unborrowed reserves. Bank holdings of free reserves—excess reserves minus borrowing from the Federal Reserve System—are thought of as an inventory held because of uncertainty about changes in deposits and in loans to regular customers which affect banks' reserve positions. The target amount of free reserves depends on the Treasury bill rate, representing earnings foregone on other assets, and the discount rate, representing the cost of raising reserves. Actual levels of free reserves depend not only on target amounts but also on changes in the balance-sheet items against which free reserves are a buffer stock; that is, on movements in deposits or—for the banking system as a whole—unborrowed reserves and on movements in commercial loans. Demands for demand deposits and time deposits (equations 3 and 4) depend on GNP and interest rates, variables which are suggested by either an inventory or a portfolio theory approach to the public's financial behavior.

Several varieties of statistical difficulty complicate the estimation of equations 2 to 4. There are simultaneous equations problems, bias problems in the coefficient of the lagged dependent variable, and, in the case of the time deposit yield, measurement error problems. In view of these difficulties, the three equations were each estimated in two ways, once with quantities as the dependent variable and once with interest rates as dependent.[1] There are reasons for believing that coefficients estimated in these alternative ways should bracket "true" coefficients with respect to these difficulties. A number of alternative combinations of quantity and interest rate versions of the equations were used to predict the extremely large movements in financial variables during 1966 and early 1967 —the equations were fit through 1965. The results were not a clear-cut victory for any one set of estimates; the best combination on many grounds, and the one used in this presentation, is the quantity version of the free reserves and time deposit equations and the interest-rate version of the demand deposit equation (equation 3: it has been solved for the quantity for listing here).

Equation 5 explains business borrowing in the form of commercial loans. The public's demand for currency is not an integral part of the model as it now stands; interest rates and consumer expenditures affect currency holdings, but there is no feedback from currency. There would be a feedback if currency plus unborrowed reserves were the open-market variable of the model; and a currency equation is included in the model (equation 6) in order to make possible simulation experiments with this alternative policy variable.

Interest rates appear in all of the equations 2 through 6, and it is increases or declines in these rates which, in the short run, make commercial banks and the public willing to hold exactly those quantities of deposits and free reserves which use up the unborrowed reserves supplied by the Federal Reserve.

The remaining equations of the model—equations 7 through 12—describe interrelations among various interest rates. Equation 7 is a term-structure relationship based on the work of Modigliani and Sutch, and derived ultimately from the expectations hypothesis combined with the hypothesis that expectations about interest rates are a combination of extrapolative elements (with re-

[1] In the interest rate versions, dependent variables were: an average of R_{CP} and R_{TD} in the demand deposit equation, R_{TD} in the time deposit equation, and the differential R_{TB}-R_{DIS} in the free reserve equation.

spect to recent changes) and regressive elements (with respect to longer-term changes). Equation 8 is a supply equation for time deposits, relating the yield at which banks are willing to accept time deposits to other rates, portfolio composition, and the exogenous ceiling rates set by the regulatory authorities. Equations 9, 10, and 11 are simply empirical relationships among interest rates. Experiments with a full-scale mortgage market in place of equation 11 were not far enough along to use in the present version of the model, but we are continuing work on an expanded mortgage-housing treatment.

The final equation of the financial sector deals with the yield on common stocks, which is related to corporate bond rates and past rates of growth of dividends, along lines suggested by Modigliani and Miller. Stock market yields are thus endogenous to our model, responding to other long-term rates of interest with coefficients which sum to a little less than one. The standard error of this equation is quite large, indicating that much of the quarter-to-quarter change in stock market yields is unexplained by the variables in our equation. However, the equation does indicate a strong link to other financial markets.

EQUATIONS

1. Reserve identity

$$RU = (RF + .074\,S_1 - .005\,S_2 + .051\,S_3 - .121\,S_4) + R_{EQD} \cdot k_D \cdot DD + R_{EQD} \cdot DD_{GM} + R_{EQT} \cdot k_T \cdot DT + e_{RF}$$

2. Demand for free reserves

$$\Delta RF = -.232 + .081\,S_2 + .025\,S_3 + .197\,S_4 + .463\,(\Delta RU + RREL) + .180\,R_{DIS} - .127\,R_{TB} - .078\,\Delta CL - .289\,RF_{-1}$$

3. Demand for demand deposits ($ln\,R_{CP} + ln\,R_{TD}$ dependent variable)

$$ln\,DD = -.3453 - .0718\,ln\,R_{CP} - .0718\,ln\,R_{TD} + .3333\,ln\,Y + .6667\,ln\,DD_{-1} + .840\,\hat{u}_{-1}$$

4. Demand for time deposits

$$DT/Y = .01386 + .00941\,R_{TD} - .00158\,R_{TB} - .00348\,R_C + .8953\,(DT/Y)_{-1} - .1477\,(\Delta Y/Y_{-1})$$

5. Demand for commercial loans

$$\Delta CL = .1167\,E_I - .450\,\Delta R_{CL} + .544\,\Delta CL_{-1}$$

6. Demand for currency

$$ln\,CURR = -.3382 - .035\,ln\,R_{TD} + .157\,ln\,C_P + .843\,ln\,CURR_{-1} + .758\,\hat{u}_{-1}$$

7. Term structure

$$R_C = 1.1225 + .337\,R_{CP} + \sum_{i=1}^{18} w_i\,R_{CP_{-i}}$$

$w_1 = -.024$	$w_7 = .064$	$w_{13} = .023$	
$w_2 = .015$	$w_8 = .059$	$w_{14} = .018$	
$w_3 = .041$	$w_9 = .052$	$w_{15} = .015$	$\Sigma w_i = .587$
$w_4 = .057$	$w_{10} = .044$	$w_{16} = .012$	
$w_5 = .065$	$w_{11} = .036$	$w_{17} = .009$	
$w_6 = .067$	$w_{12} = .029$	$w_{18} = .005$	

8. Supply of time deposits

$$R_{TD} = -.362 + .390\left(\frac{CL}{DD + DT}\right) + .800\left(\frac{CL}{DD + DT}\right)_{-1} + .390\left(\frac{CL}{DD + DT}\right)_{-2} + .008\,(R_C + R_{C_{-1}}) + .463\,R_{MAX} - .360\,R_{MAX_{-1}} + .869\,R_{TD_{-1}}$$

9. Commercial paper rate

$$R_{CP} = .5775 + .7234\,R_{TB} + .3178\,R_{TB_{-1}} - .2469\,D_{UCD}$$

10. Commercial loan rate

$$R_{CL} = .448\,R_C + .160\,R_{CP} + .341\,R_{CL_{-1}} + .762$$

11. Mortgage rate

$$\Delta R_M = .078\,\Delta R_{CP} + .362\,\Delta R_{C_{-1}} + .417\,\Delta R_{M_{-1}}$$

12. Stock market yield

$$R_D = \sum_{i=0}^{4} w_i R_{C_{-i}} - \sum_{i=4}^{14} w'_i \left[\frac{\Delta Y_{CD_{-i}}}{Y_{CD_{-i-1}}}\right] + .9452\,\hat{u}_{-1}$$

$w_0 = .0294$	$w'_4 = .49$	$w'_{10} = .89$
$w_1 = .1608$	$w'_5 = .82$	$w'_{11} = .79$
$w_2 = .2235$	$w'_6 = 1.03$	$w'_{12} = .62$
$w_3 = .2176$	$w'_7 = 1.12$	$w'_{13} = .32$
$w_4 = .1431$	$w'_8 = 1.11$	$w'_{14} = .14$
$\Sigma w_i = .7744$	$w'_9 = 1.03$	$\Sigma w'_i = 8.26$

GLOSSARY

* indicates exogenous to financial block.

Interest rates, in per cent

R_{CL} = bank business loans

R_C = Aaa seasoned corporate bonds (Moody's)

R_{CP} = 4-6 month commercial paper

R_D = dividend yield on common stock (Moody's)

* R_{DIS} = discount rate, N. Y. Federal Reserve Bank

R_M = new conventional mortgages, FHLBB

R_{TB} = 3-month Treasury bills

R_{TD} = bank time and savings deposits (incl. CD's)

* R_{MAX} = ceiling rate on bank time deposits

Amounts, in billions of dollars (seasonally adjusted, except *RF* and *RREL*)

CL = bank commercial loans

* C_P = consumer expenditures (annual rates)

$CURR$ = currency

DD = demand deposits

* DD_{GM} = govt. demand deposits at member banks

* e_{RF} = error in reserve identity (statistical discrepancy)

* E_I = inventory investment (annual rates)

RF = free reserves, not seasonally adjusted

* $RREL$ = reserves released through reserve requirement changes

* RU = unborrowed reserves

DT = time deposits

* Y_{CD} = dividend payments (annual rates)

* Y = GNP (annual rates)

* D_{UCD} = CD dummy (1 after 1962)

Ratios (between zero and one)

* k_D = proportion of demand deposits at member banks; seasonally adjusted

* k_T = proportion of time deposits at member banks; seasonally adjusted

* R_{EQD} = weighted average reserve requirement ratio against member bank demand deposits

* R_{EQT} = reserve requirement ratio against member bank time deposits

NOTES—$\hat{u}_{t-1}$ in this and the following blocks indicates that the variables were transformed into the semi-first-difference form

$$x_t - bx_{t-1}$$

before estimation, where b is the coefficient of $\hat{u}_{t-1}$

S_1 through S_4 are seasonal dummy variables.

Financial dollar amounts are all averages of the 2 months surrounding end of quarter; for example, fourth quarter is average of December and January.

Investment Block

Equations 3 and 4 give expressions for the rate of return at which future earnings should be discounted, both for producers' durables and for structures. These rates of return are functions of the dividend-price ratio, the industrial bond yield, corporate tax rates, and the desired long-run debt-equity ratio. The parameters are estimated in nonlinear fashion together with the investment functions.

Equations 5 and 6 use these rates of return to derive expressions for the imputed rent which must be earned by a new machine to make its purchase worthwhile. Along with rates of return, the implicit rental also depends on the price of capital equipment, the rate of depreciation, and various features of the tax law—the investment credit, the present value of depreciation deductions under various laws, and tax rates.

The desired capital-output ratios for equipment and structures are given in equations 7 and 8. Their ratios depend on the price of output, the imputed rent, and a trend factor reflecting technological change. The exponents of the relative price terms are the elasticities of substitution between factors, which are again estimated in a nonlinear fashion.

The important equations of the business fixed investment sector are 9 and 14, which explain respectively the new orders of producers' durable equipment and the actual purchases of structures. Both of these equations represent desired capital in real terms as a distributed lag function of the product of real output and the desired capital-output ratio. For structures, the dynamics of this lag are complicated by the presence of the lagged capital stock (equation 15). For producers' durables, the dynamics are even further complicated by a separate distributed lag between orders and expenditures (equation 10). This lag depends on the orders-shipments ratio, with supply bottlenecks, as represented by a high ratio, postponing the lag. The orders-shipments relationships are described in equations 11 to 13.

For the housing sector, equation 19 explains the rental price. This price then determines the price of houses by equation 20. Equation 20 imposes the requirement that in the long run the net return on investment in houses is brought into equilibrium with mortgage rates. There should be a variable approximating capital gains in the net rate of return equation, but we have not yet been able to find one which works well.

Equation 17 explains housing inventory under construction. This equation assumes that builders are influenced only by rates of return on houses and mortgage rates. As mentioned in the text, it may not give enough importance to other variables.

The remaining equations close up the housing sector. Equation 16 defines the variable we use to approximate the inventory of (unsold) houses under construction. The weights in the equation are based on construction and sales statistics, along with an assumption about the effect on housing starts of a sold but unstarted house. The equation is used to solve for real housing starts in value terms. Equation 18 then gives expenditures as a distributed lag on starts, with a constant and time trend to pick up the coverage difference (additions and alterations). Equation 21 uses expenditures to calculate the real stock of housing and feedback into determination of the rent index.

Equations 24 to 27 explain the purchases and transfer payments of State and local governments. These values, along with equation 31 and identity 23, are the total net expenditures which go into tax equations 28 to 30 and 32. Of this total, all but construction needs to be revenue-financed, and the construction ratios in the tax equations are a way of adjusting for this difference. It would have been difficult to proceed otherwise because grants-in-aid would have had to be allocated between construction and all other. The income variable used in the State and local sector, defined in equation 22, is net of Federal taxes and plus Federal transfers.

EQUATIONS

A. INVESTMENT IN PLANT AND EQUIPMENT

Gross business product identity

1. $Y_B = Y - C_N - E_{FW} - E_{SW} - Y_F$

Cost of capital relationships

2. $R_{CBI} = -.0133 + .9325\, R_C + .0635\, R_D + .0045 \text{ time} + .8512\, \hat{u}_{-1}$

3. $R_E = (1 - \alpha_2 t_c)\,(-1.30 + .6290\, R_{CBI} + .2160\, R_D)$

4. $R_S = (1 - \alpha_2 t_c)\,(-1.833 + .0264\, R_{CBI} + .7258\, R_D)$

Identities defining current dollar rent per unit of new investment

5a. $P_{QE} = \dfrac{P_{PD}\,(.01\, R_E + \alpha_3)\,(1 - t_c z_e - z_k + z_k t_c z_e)}{1 - t_c}$ for 1948–63

5b. $P_{QE} = \dfrac{P_{PD}\,(.01\, R_E + \alpha_3)\,(1 - t_c z_e - z_k)}{1 - t_c}$ for 1964–66

6. $P_{QS} = \dfrac{P_{PS}\,(.01\, R_S + \alpha_4)\,(1 - t_c z_s)}{1 - t_c}$

Identities defining equilibrium capital-output ratio

7. $V_E = \left(\dfrac{P_B}{P_{QE}}\right)^{.877} e^{.00081\,(\text{time} - 42.5)}$

8. $V_S = \left(\dfrac{P_B}{P_{QS}}\right)^{.450} e^{-.00295\,(\text{time} - 42.5)}$

Demands for orders and expenditures, producers' durable equipment

9. $\dfrac{O_{PD}}{P_{PD}} = \sum_{i=0}^{16} \dfrac{w_i\, V_{E_{-i-1}}\, Y_{B_{-i}}}{P_{B_{-i}}} + \sum_{i=1}^{17} \dfrac{w'_i\, V_{E_{-i}}\, Y_{B_{-i}}}{P_{B_{-i}}}$

w_0 =	.0502	w'_1 =	−.0495
w_1 =	.0422	w'_2 =	−.0401
w_2 =	.0339	w'_3 =	−.0310
w_3 =	.0256	w'_4 =	−.0223
w_4 =	.0174	w'_5 =	−.0141
w_5 =	.0095	w'_6 =	−.0064
w_6 =	.0022	w'_7 =	.0006
w_7 =	−.0045	w'_8 =	.0068
w_8 =	−.0105	w'_9 =	.0120
w_9 =	−.0154	w'_{10} =	.0163
w_{10} =	−.0192	w'_{11} =	.0194
w_{11} =	−.0217	w'_{12} =	.0212
w_{12} =	−.0228	w'_{13} =	.0217
w_{13} =	−.0222	w'_{14} =	.0208
w_{14} =	−.0198	w'_{15} =	.0182
w_{15} =	−.0154	w'_{16} =	.0140
w_{16} =	−.0088	w'_{17} =	.0079
Σw_i =	.0207	$\Sigma w'_i$ =	−.0045

10. $\dfrac{E_{PD}}{P_{PD}} = \sum_{i=0}^{5} w_i \left(\dfrac{O_{PD}}{P_{PD}}\right)_{-i} + \sum_{i=0}^{5} w'_i \left(\dfrac{O_{PD}}{P_{PD}}\right)_{-i} \left(\dfrac{O_{UME}}{E_{ME}}\right)_{-i-1}$

w_0 =	.6475	w'_0 =	−.3575
w_1 =	.2555	w'_1 =	−.0724
w_2 =	.0598	w'_2 =	.1061
w_3 =	−.0018	w'_3 =	.1781
w_4 =	.0090	w'_4 =	.1431
w_5 =	.0302	w'_5 =	.0022
Σw_i =	1.0002	$\Sigma w'_i$ =	−.0004

11. $O_{UME} = O_{UME-1} + .25\,(O_{ME} - E_{ME})$

12. $O_{ME} = 6.0640 + .9665\, O_{PD} - .0289 \text{ time}$

13. $E_{ME} = 8.2212 + .8554\, E_{PD} + .0041 \text{ time}$

Demand for nonresidential structures

14. $\dfrac{E_{PS}}{P_{PS}} = \sum_{i=1}^{16} \dfrac{w_i\, V_{S_{-i}}\, Y_{B_{-i}}}{P_{B_{-i}}} - .2710\, K_{SR}$

w_1 =	.0038	w_{11} =	.0021
w_2 =	.0051	w_{12} =	.0019
w_3 =	.0056	w_{13} =	.0017
w_4 =	.0057	w_{14} =	.0015
w_5 =	.0053	w_{15} =	.0013
w_6 =	.0048	w_{16} =	.0008
w_7 =	.0042	Σw_i =	.0528
w_8 =	.0035		
w_9 =	.0030		
w_{10} =	.0025		

Identity defining stock of nonresidential structures

15. $K_{SR} = \left(1-\frac{\alpha_i}{4}\right) K_{SR_{-1}} + .25\left(\frac{E_{PS}}{P_{PS}}\right)_{-1}$

B. HOUSING SECTOR

Housing inventory, starts, and expenditures

16. $K_{HIR} = \sum_{i=0}^{3} w_i \left(\frac{H_S}{P_H}\right)_{-i}$

$w_0 = .55$	$w_3 = .03$
$w_1 = .47$	$\Sigma w_i = 1.20$
$w_2 = .15$	

17. $\Delta K_{HIR} = \sum_{i=0}^{7} w_i (\Delta R_M)_{-i} + \sum_{i=0}^{7} w_i \left(\Delta \frac{P_R}{P_H}\right)_{-i} + .5170\,\hat{u}_{-1}$

$w_0 = -.3584$	$w'_0 = 1.76$
$w_1 = -.2771$	$w'_1 = 3.06$
$w_2 = -.2062$	$w'_2 = 3.92$
$w_3 = -.1458$	$w'_3 = 4.35$
$w_4 = -.0958$	$w'_4 = 4.35$
$w_5 = -.0562$	$w'_5 = 3.91$
$w_6 = -.0270$	$w'_6 = 3.04$
$w_7 = -.0083$	$w'_7 = 1.74$
$\Sigma w_i = -1.1748$	$\Sigma w'_i = 26.13$

18. $E_H = 2.9658 + \sum_{i=0}^{2} w_i H_{S-i} + .0408\ \text{time} + .7109\,\hat{u}_{-1}$

$w_0 = 1.8541$	$w_2 = .5079$
$w_1 = 1.8271$	$\Sigma w_i = 4.1891$

Rent and house prices

19. $\Delta \ln P_R = .5972 - .0957 \ln \frac{K_{HR}}{N} + .0207 \ln \frac{Y_D}{P_c N} - .0782 \ln \frac{P_{R-1}}{P_c} - .2031\,\hat{u}_{-1}$

20. $\Delta\left(\frac{P_R}{P_H}\right) = -.0037 + \sum_{i=1}^{12} w_i (\Delta R_M)_{-i} + .2545\,\hat{u}_{-1}$

$w_1 = .0102$	$w_8 = .0060$
$w_2 = .0099$	$w_9 = .0050$
$w_3 = .0095$	$w_{10} = .0039$
$w_4 = .0090$	$w_{11} = .0027$
$w_5 = .0084$	$w_{12} = .0014$
$w_6 = .0077$	$\Sigma w_i = .0806$
$w_7 = .0069$	

Identity defining housing stock

21. $K_{HR} = .994\, K_{HR_{-1}} + .25 \frac{E_H}{P_H}$

C. STATE AND LOCAL GOVERNMENTS

Expenditures

22. $Y_S = Y - T_{FP} - T_{FC} - T_{FI} - T_{FS} - T_{FE} - e_T + G_{FP} + G_{FU} + G_{FS} + INT_F$

23. $E_{ST} = E_{SW} + E_{SC} + E_{SO} + G_{SP} + INT_S - T_{SS} - G_{FG}$

24. $\frac{E_{SC}}{NP_G} = .302 + \sum_{i=0}^{11} w_i \left(\frac{Y_S}{NP_G}\right)_{-i} + \sum_{i=0}^{11} w'_i \left(\frac{Y_S}{NP_G}\right)_{-i} (R_C)_{-i} + \sum_{i=0}^{1} w''_i \left(\frac{G_{FG}}{NP_G}\right)_{-i} + \sum_{i=0}^{1} w'''_i \left(\frac{P_S}{P_G}\right)_{-i} \left(\frac{Y_S}{NP_G}\right)_{-i} + .1744 \left(\frac{N_{20}}{N}\right)\left(\frac{Y_S}{NP_G}\right)$

$w_0 = -.0428$	$w'_3 = -.0003$
$w_1 = -.0273$	$w'_4 = -.0003$
$w_2 = -.0150$	$w'_5 = -.0004$
$w_3 = -.0053$	$w'_6 = -.0004$
$w_4 = .0016$	$w'_7 = -.0005$
$w_5 = .0062$	$w'_8 = -.0005$
$w_6 = .0089$	$w'_9 = -.0004$
$w_7 = .0100$	$w'_{10} = -.0003$
$w_8 = .0096$	$w'_{11} = -.0002$
$w_9 = .0081$	$\Sigma w'_i = -.0050$
$w_{10} = .0057$	$w''_0 = .2924$
$w_{11} = .0030$	$w''_1 = .0986$
$\Sigma w_i = -.0372$	$\Sigma w''_i = .3910$
$w'_0 = -.0006$	$w'''_0 = -.0198$
$w'_1 = -.0006$	$w'''_1 = .0340$
$w'_2 = -.0005$	$\Sigma w'''_i = .0142$

25. $\frac{E_{SO}}{NP_G} = 4.394 - .0272 \frac{Y_S}{NP_G} - .0104 \left(\frac{P_S}{P_G}\right)\left(\frac{Y_S}{NP_G}\right) + .1334 \left(\frac{N_{20}}{N}\right)\left(\frac{Y_S}{NP_G}\right) - .0006\, R_C \left(\frac{Y_S}{NP_G}\right) + \sum_{i=0}^{1} w_i \left(\frac{G_{FG}}{NP_G}\right)_{-i}$

$w_0 = .1970$	$w_1 = .1679$	$\Sigma w_i = .3649$

26. $\frac{E_{SW}}{NP_G} = 32.722 + .3086 \frac{G_{FG}}{NP_G} + .0835 \left(\frac{P_S}{P_G}\right)\left(\frac{Y_S}{NP_G}\right) + .1298 \left(\frac{N_{20}}{N}\right)\left(\frac{Y_S}{NP_G}\right) + \sum_{i=0}^{1} w_i \left(\frac{Y_S}{NP_G}\right)_{-i} + \sum_{i=0}^{3} w_i (R_C)_{-i} \left(\frac{Y_S}{NP_G}\right)_{-i}$

$w_0 = -.0762$	$w'_0 = -.0017$
$w_1 = -.0277$	$w'_1 = -.0002$
$\Sigma w_i = -.1039$	$w'_2 = .0016$
	$w'_3 = .0021$
	$\Sigma w'_i = .0018$

27. $$\frac{G_{SP}}{NP_G} = 8.041 + .1074\frac{G_{FG}}{NP_G} - .0104\left(\frac{L_E + L_A}{N}\right)\left(\frac{Y_S}{NP_G}\right) + \sum_{i=0}^{1} w_i \left(\frac{Y_S}{NP_G}\right)_{-i} + \sum_{i=0}^{1} w'_i (R_C)_{-i}\left(\frac{Y_S}{NP_G}\right)_{-i}$$

$w_0 = .0219$, $w_1 = -.0126$, $\Sigma w_i = .0093$; $w'_0 = -.0004$, $w'_1 = .0007$, $\Sigma w'_i = .0003$

Taxes and profits of government enterprises

28. $$\frac{T_{SP}}{N} = -37.671 + .0258\frac{Y_H}{N} + .2470\frac{E_{ST}}{N} - .0053(R_C)\left(\frac{E_{ST}}{N}\right) - .5358\left(\frac{E_{SC}}{E_{ST} + G_{FG}}\right)\left(\frac{E_{ST}}{N}\right)$$

29. $$\frac{T_{SI}}{N} = -14.944 + \sum_{i=0}^{1} w_i\left(\frac{Y_S}{N}\right)_{-i} + \sum_{i=0}^{1} w'_i\left(\frac{E_{ST}}{N}\right)_{-i} - .0053(R_C)\left(\frac{E_{ST}}{N}\right) - 1.2280\left(\frac{E_{SC}}{E_{ST} + G_{FG}}\right)\left(\frac{E_{ST}}{N}\right)$$

$w_0 = -.0014$, $w_1 = .0277$, $\Sigma w_i = .0263$; $w'_0 = .7684$, $w'_1 = .1149$, $\Sigma w'_i = .8833$

30. $$T_{SC} = -.418 + .0177\,Y_C + .0365\,E_{ST} - .0514\left(\frac{E_{SC}}{E_{ST} + G_{FG}}\right)E_{ST} - .0010\,R_C\,E_{ST}$$

31. $$T_{SS} = .0900 + .1110\,E_{SW}$$

32. $$G_{SS} = -.301 + \sum_{i=0}^{1} w_i (Y_S)_{-i} + .0859\,E_{ST} - .1520\left(\frac{E_{SC}}{E_{ST} + G_{FG}}\right)(E_{ST}) - .0033(R_C)(E_{ST})$$

$w_0 = -.0038$ $w_1 = .0064$ $\Sigma w_i = .0026$

GLOSSARY

* indicates exogenous to investment block.

Investment in plant and equipment

Y_B = gross business products, current dollars

*Y = GNP, current dollars

*C_{NW} = output originating in households, current dollars

*E_{FW} = Federal compensation of employees, current dollars

E_{SW} = State and local compensation of employees, current dollars

*Y_F = output originating abroad, current dollars

*R_{CBI} = Moody's industrial bond yield, per cent

*R_D = Moody's industrial dividend-price ratio for common stocks, per cent

R_C = Moody's Aaa corporate bond rate, per cent

R_E = cost of capital, equipment, per cent

R_S = cost of capital, structures, per cent

P_{QE} = current dollar rent, equipment, decimal

P_{QS} = current dollar rent, structures, decimal

*P_{PD} = price deflator, producers' durable equipment, decimal

*P_{PS} = price deflator, producers' structures, decimal

P_B = price deflator, gross business product, decimal

V_E = equilibrium capital-output ratio, equipment

V_S = equilibrium capital-output ratio, structures

O_{PD} = orders for producers' durable equipment, current dollars

E_{PD} = expenditures on producers' durable equipment, current dollars

O_{ME} = orders for machinery and equipment, current dollars

E_{ME} = shipments of machinery and equipment, current dollars

O_{UME} = stock of unfilled orders for machinery and equipment, current dollars

E_{PS} = expenditures on producers' structures, current dollars

K_{SR} = capital stock of producers' structures, 1958 dollars

*time = 1 in 1948 QI, increments by one every quarter

*t_c = corporate tax rate, decimal

*z_k = rate of tax credit for investment in producers' durable equipment, decimal

*z_s = present value of depreciation deduction per dollar of new producers' durable equipment, decimal

*z_s = present value of depreciation deduction per dollar of new producers' durable structures, decimal

*α_2 = desired proportion of debt in corporate capital structures, = .2, 1948-66

*α_3 = annual rate of depreciation of producers' durable equipment, = .16, 1948-66

*α_4 = annual rate of depreciation of producers' structures, = .06, 1948-66

e = base of natural logarithm

$\hat{u}_{-1}$ = previous period error term

Housing

E_H = expenditures for residential construction, current dollars

H_S = housing starts times average value per start, current dollars quarterly rates

K_{HR} = stock of houses, 1958 dollars

K_{HIR} = housing inventory under construction, 1958 dollars

P_R = rental price component of consumer price index, decimal

P_H = housing price deflator in the national income accounts, decimal

*P_C = consumer expenditure price deflator, national income accounts, decimal

*N = total population, billions

*Y_D = disposable personal income using personal tax liabilities, current dollars

*time = 1 in 1948 QI, increments by one every quarter

$\hat{u}_{-1}$ = previous period residual

State and local governments

(All flow variables measured in current dollars, seasonally adjusted annual rates)

Y_S = net income of citizens of States and localities

E_{ST} = total net expenditures of States and localities

*Y = GNP

*T_{FP} = Federal personal taxes, liability basis

*T_{FC} = Federal corporate profits tax accruals

*T_{FI} = Federal indirect taxes

*T_{FS} = Federal social insurance contributions

*T_{FE} = Federal estate and gift taxes

*e_T = Federal personal taxes on national income accounts basis less taxes on a liability basis

*G_{FP} = Federal transfer payments to persons less unemployment insurance benefits

*G_{FU} = unemployment insurance benefits

*G_{FS} = Federal subsidies less current surplus of government enterprises

*INT_F = Federal net interest payments

*G_{FG} = Federal grants-in-aid to State and local governments

E_{SW} = State and local compensation of employees

E_{SC} = State and local construction expenditures

E_{SO} = State and local other purchases

G_{SP} = State and local transfer payments

*INT_S = State and local net interest payments

G_{SS} = State and local surplus of government enterprises

T_{SP} = State and local personal taxes

T_{SC} = State and local corporate taxes

T_{SI} = State and local indirect taxes

T_{SS} = State and local social insurance contributions

*Y_H = personal income

*Y_C = corporate profits before tax (does not include IVA)

*R_C = Moody's Aaa corporate bond rate, per cent

*P_G = GNP deflator, decimal

*P_S = price deflator for State and local purchases, decimal

*N = population

*N_{20} = population under age 20, billions

*L_E = employed labor force, billions

*L_A = armed forces, billions

Consumption-Inventory Block

Equations 1 and 2 of this block are identities adding up the components of GNP and consumer expenditures, respectively.

The next set of equations gets us from GNP to disposable income in four steps. Personal income less certain exogenous transfer payments depends on current and lagged GNP in such a way as to change less abruptly than GNP (equation 3). After we add the price-wage block to the model, we plan to replace equation 3 with one in which the relation of personal income and GNP depends on price, wage, and manhour changes. Taxable income depends on personal income and exemptions (equation 4), income tax accruals are equal to taxable income multipled by an exogenous tax rate (equation 5), and disposable income depends, through an identity, on personal income, income tax accruals, and two categories of tax payments exogenous to this block.

Equations 7 through 10 get us from disposable income to total consumption (in our sense of including the services of durable goods) and its allocation. Equation 7 is the basic equation in the consumption sector. It relates our version of consumption to current and lagged disposable income with a long distributed lag. To guard against simultaneous equations bias, the relationship was actually estimated in ratio form with everything divided by current income.[2] We have experimented with wealth effects on consumption but were unable to get usable results due to collinearity between wealth and income. To some extent, the lag in income can serve as a proxy for wealth effects.

Equations 8 to 10 explain the distribution of total consumption among its three components—consumption of nondurables and services and the imputed services of autos and of other durables. These equations have been estimated with the sum of the coefficients for total consumption constrained to equal one, and the sum of all other sets of coefficients constrained to equal zero. This means that variables such as relative prices, interest rates, population, and the stocks of durables influence the distribution of consumption among its components, but not the over-all total. Equation 11 is the definition of the real income variable appearing in equations 8 to 10.

The remaining consumption equations are identities. Equations 12 and 13 show the relation between consumption and consumer spending for autos and for other durables. Since stocks (K_{AR} and K_{DR}) are not measured at annual rates, whereas the other variables are, these equations imply that on a quarterly basis consumption in our sense equals between 6 and 7 per cent of the initial stock plus a fraction of current purchases. For durables other than autos, the fraction of current purchases is also between 6 and 7 per cent, but for autos it is nearly 12 per cent. All of these coefficients represent estimated depreciation rates plus an interest imputation. For other durables, depreciation rates come from a standard declining-balance formula based on data on lengths of life of durables in Goldsmith.[3] For autos depreciation rates come from a declining-balance formula with roughly double depreciation in the first quarter, based on a regression analysis of data compiled by Charles Friedman.[4] Autos are thus assumed to depreciate in value (and hence to yield services) at a much faster rate in the first quarter of their existence than in subsequent quarters.

Equations 14 and 15 define total consumer expenditures in real terms and total consumption in real terms. Equation 15 is listed simply to make clear the relation between the two consumption concepts; it is redundant in the complete consumption-inventory block since it is the sum of equations 8, 9, and 10.

Equations 16 and 17 explain stocks of autos and other durables by using declining-balance formulas. The depreciation rates are the ones already discussed in connection with equations 12 and 13.

Equation 18 explains inventory investment. As described in the text, the equation estimates different inventory-sales ratios and different lags on the various components of final demand. Thus goods-in-process inventories connected with defense spending show up before expenditures, whereas consumer expenditures have a small negative unanticipated effect in the current quarter.[5]

Equation 19 explains imports. This equation allows the average propensity to import to rise as GNP rises (as if imports were a luxury good). Relative price effects were tried in this equation, but they proved to be unimportant, perhaps because of errors of measurement in the import price index.

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[2] See Ando and Modigliani, *op.cit.*, pages 69-70.

[3] Raymond Goldsmith, *The National Wealth of the United States in the Postwar Period* (Princeton University Press, 1962).

[4] Charles Friedman, "The Stock of Automobiles in the United States," *Survey of Current Business*, October 1965.

[5] In estimating this effect we made use of an instrumental variable, or two-stage technique, with new orders, lagged potential bank deposits, defense spending, and a number of other predetermined variables entering the first stage.

EQUATIONS

A. GNP AND CONSUMPTION IDENTITIES

1. $Y = C_P + E_I - E_M + Z$ (identity)

2. $C_P = C_N + (E_{CAR} + E_{CDR})\, P_{CAD}$ (identity)

B. INCOME SHARES AND TAXES

3. $$\frac{Y_H - G_{FP}}{Y} = .789 - .0005\ \text{time} - .271\left(\frac{\Delta Y}{Y_{-1}}\right) - .218\left(\frac{\Delta Y_{-1}}{Y_{-2}}\right) - .105\left(\frac{\Delta Y_{-2}}{Y_{-3}}\right)$$

4. $$\ln\left(1 - \frac{Y_T}{Y_H}\right) = .123 - .3274\ \ln\left(\frac{Y_H}{N}\right) + .2808\ \ln EX$$

5. $T_{FP} = t_h\,(Y_T)$

6. $Y_D = Y_H - T_{FP} - T_{FE} - T_{SP}$ (identity)

C. TOTAL CONSUMPTION AND ITS COMPONENTS

Total consumption

7. $$C_{TR}\,(.774\,P_{CN} + .226\,P_{CAD}) = .3734\,Y_D + \sum_{i=1}^{12} w_i\,Y_{D_{-i}} + .90\,\hat{u}_{-1}$$

$w_1 = .0849$ $w_5 = .0584$ $w_9 = .0300$
$w_2 = .0785$ $w_6 = .0514$ $w_{10} = .0227$ $\Sigma w_i = .5673$
$w_3 = .0718$ $w_7 = .0442$ $w_{11} = .0153$
$w_4 = .0651$ $w_8 = .0373$ $w_{12} = .0077$

Components

8. $$C_N/P_{CN} = .9518\,C_{TR} - .0191\,Y_{DR} + \left(\frac{1}{10}\sum_{i=2}^{7} w_i\,R_{C_{-i}} - .0919\,\frac{P_{CN}}{P_{CAD}}\right)K_{AR-1} - .1625\,K_{AR-1} - .1497\,K_{DR-1} + 32.640\,N$$

9. $$C_{AR} = .0369\,C_{TR} + .0205\,Y_{DR} + \left(\frac{1}{10}\sum_{i=2}^{7} w'_i\,R_{C_{-i}} + .0781\,\frac{P_{CN}}{P_{CAD}}\right)K_{AR-1} + .1778\,K_{AR-1} - .1324\,K_{DR-1} - 21.666\,N$$

10. $$C_{DR} = .0112\,C_{TR} - .0014\,Y_{DR} + \left(\frac{1}{10}\sum_{i=2}^{7} w''_i\,R_{C_{-i}} + .0138\,\frac{P_{CN}}{P_{CAD}}\right)K_{AR_{-1}} - .0154\,K_{AR_{-1}} + .2821\,K_{DR_{-1}} - 10.965\,N$$

$w_2 = -.0154$	$w'_2 = .0165$	$w''_2 = -.0011$
$w_3 = .0038$	$w'_3 = .0004$	$w''_3 = -.0041$
$w_4 = .0163$	$w'_4 = -.0103$	$w''_4 = -.0060$
$w_5 = .0222$	$w'_5 = -.0157$	$w''_5 = -.0065$
$w_6 = .0214$	$w'_6 = -.0158$	$w''_6 = -.0056$
$w_7 = .0140$	$w'_7 = -.0106$	$w''_7 = -.0035$
$\Sigma w_i = .0623$	$\Sigma w'_i = -.0355$	$\Sigma w''_i = -.0268$

$w_2 + w'_2 + w''_2 = 0$
etc.

11. $Y_{DR} = Y_D/(.774\,P_{CN} + .226\,P_{CAD})$ (identity)

D. RELATION OF REAL CONSUMER EXPENDITURES TO REAL CONSUMPTION

12. $C_{AR} = .2551\,K_{AR_{-1}} + .11625\,E_{CAR}$ (identity)

13. $C_{DR} = .2675\,K_{DR_{-1}} + .06251\,E_{CDR}$ (identity)

14. $C_{PR} = C_N/P_{CN} + E_{CAR} + E_{CDR}$ (identity)

15. $C_{TR} = C_N/P_{CN} + C_{AR} + C_{DR}$ (identity)

E. STOCKS OF CONSUMER DURABLES

16. $K_{AR} = .9457\,K_{AR_{-1}} + .8884\,(E_{CAR}/4)$ (identity)

17. $K_{DR} = .9426\,(K_{DR_{-1}} + [E_{CDR}/4])$ (identity)

F. INVENTORY INVESTMENT

18. $$E_{IR} = .424\,E_{IR_{-1}} - .138\,\Delta C_{PR} + .573\,(\Delta C_{PR})_{-1} + .387\,\Delta\left(\frac{E_{FD}}{P_F}\right)_{+1} + \sum_{i=0}^{4} w_i\,\Delta\left(\frac{O_{PD}}{P_{PD}}\right)_{-i} - .276\,\Delta S_{TR}$$

$w_0 = -.020$ $w_3 = .335$
$w_1 = .214$ $w_4 = .224$
$w_2 = .331$ $\Sigma w_i = 1.084$

19. $E_I = E_{IR}\,(1.202\,P_{NB} - .095\,W_{NB}) + .009$

G. IMPORTS

20. $$E_M/Y = .665\,(E_M/Y)_{-1} + \frac{1}{100}\left(.0009\,\frac{Y}{P_G} + .0140\left(\frac{1}{1.05 - U_M}\right) + 1.0095\right) + .0032\,D_{UDS} + .0027\,D_{USS}$$

H. CAPACITY UTILIZATION, MATERIALS INDUSTRIES

21. $$\Delta U_M = \frac{1}{(C_N/P_{CN})}\left(1.832\,\Delta E_{IR} + 1.905\,\Delta\,(E_{CAR} + E_{CDR}) + .821\,\Delta\,\frac{C_N}{P_{CN}}\right) - .0156$$

GLOSSARY

(Dollar amounts, in billions, seasonally adjusted at annual rates except where noted. * indicates exogenous to consumption-inventory block)

C_{AR} = consumption of the services of autos and parts, 1958 dollars

C_{DR} = consumption of the services of durables except autos and parts, 1958 dollars

C_N = expenditures on nondurables and services, current dollars

C_P = total consumer expenditures, current dollars

C_{PR} = total consumer expenditures, 1958 dollars

C_{TR} = total "consumption" (see text), 1958 dollars

* D_{UDS} = dummy variable for 1965 dock strike

* D_{USS} = dummy variable for 1959 steel strike

E_{CAR} = consumer expenditures on autos and parts, 1958 dollars

E_{CDR} = consumer expenditures on durables except autos and parts, 1958 dollars

* E_{FD} = Federal expenditures on defense goods, current dollars

E_I = nonfarm inventory investment, current dollars

E_{IR} = nonfarm inventory investment, constant dollars

E_M = imports, current dollars

* EX = per capita exemptions under Federal personal income tax, in dollars

* G_{FP} = Federal transfer payments to persons, except unemployment benefits.

K_{AR} = stock of consumer autos and parts end of quarter, not at annual rates, 1958 dollars

K_{DR} = stock of consumer durables except autos and parts, end of quarter, not at annual rates, 1958 dollars

* N = total U.S. population, billions

* O_{PD} = new orders for producers' durable equipment, current dollars

* P_{CAD} = implicit deflator for consumer durables (including autos), 1958 = 1.00

* P_{CN} = implicit deflator for consumer nondurables and services, 1958 = 1.00

* P_F = implicit deflator for Federal purchases, 1958 = 1.00

* P_{NB} = implicit deflator for nonfarm business GNP, 1958 = 1.00

* P_{PD} = implicit deflator for producers' durable equipment, 1958 = 1.00

* P_G = implicit deflator for GNP, 1958 = 1.00

* R_C = yield on seasoned Aaa corporate bonds, in per cent

* S_{TR} = man-days idle in excess of 10,000, in thousands

* $time$ = time; 1948 I = 1, 1948 II = 2, etc.

* T_{FE} = Federal estate and gift tax payments, current dollars

T_{FP} = Federal personal income tax liabilities, current dollars

* t_h = average tax rate under Federal personal income tax; 20 per cent = .2, etc.

* T_{SP} = State and local personal taxes, current dollars

U_M = utilization rate for materials industries; 90 per cent = .9, etc.

* W_{NB} = average wage rate for nonfarm business, dollars per hour

Y = GNP, current dollars

Y_D = disposable personal income with taxes measured on a liability basis, current dollars

Y_{DR} = disposable personal income divided by a weighted average of deflators for consumer expenditures

Y_H = personal income, current dollars

Y_T = taxable personal income, current dollars

* Z = autonomous spending; the sum of exports, government expenditures, fixed investment, and farm inventory investment

The Three Blocks Combined

The important equation of this block is number 5, which explains corporate dividend payments. Dividends have an effect on stock prices and will also be included in personal income when we finish the labor market side of the model. They depend on corporate cash flows with a distributed lag.

Corporate cash flows are described by the identity in equation 3 and Federal corporate taxes by equation 4. The equation for State and local corporate taxes is part of the investment block.

Equation 1 explains corporate profits. Except for statistical discrepancy and inventory valuation adjustment, this equation would be an identity, and equation 2 is an informal way of dealing with the obscure residual items. Equations 6 through 10 then fill in the missing links in equation 1. This treatment gives a very minor importance to indirect taxes, but our conclusions on this matter will change when we add the price and labor market blocks to the model. Both taxes would then have a direct effect on personal income, and indirect taxes will also influence output prices.

EQUATIONS

Corporate profits, cash flows, and dividends

1. $Y_C = Y - DEP - T_{FI} - T_{SI} + G_{FS} - G_{SS} - T_{FS} - T_{SS} + Y_{CD} + INT + G_{FP} + G_{SP} - Y_H + ERR$

2. $\frac{ERR}{Y_{-1}} = -.0238 + .0386\, U_M - .0776 \left(\frac{Y}{Y_{-1}} - 1\right) + .6585\hat{u}_{-1}$

3. $Y_{CF} = Y_C + DEP_C - T_{FC} - T_{SC}$

4. $\ln (T_{FC} + \alpha_1 z_k E_{PD}) = -.4161 + .7262 \ln t_c + 1.0177 \ln (Y_C - T_{SC}) + .8591\hat{u}_{-1}$

5. $Y_{CD} = .9906 + \sum_{i=0}^{7} w_i Y_{CF-i} + .5000\hat{u}_{-1}$

$w_0 = .0600$	$w_3 = .0309$	$w_6 = .0119$
$w_1 = .0485$	$w_4 = .0239$	$w_7 = .0061$
$w_2 = .0390$	$w_5 = .0177$	$\Sigma w_i = .2380$

Federal indirect and social insurance taxes

6. $T_{FI} = T_{FX} + T_{FCD}$

7. $\ln T_{FX} = 1.0883 + .6315 \ln C_P + 1.1027 \ln t_x$

8. $T_{FS} = T_{FO} + T_{FU} + T_{FSO}$

9. $\ln T_{FO} = \ln t_o + .9473 \ln Y_H - .4384$

10. $\ln T_{FU} = \ln t_u + .4480 \ln Y_H + 1.2887 \ln t_{uic} + 2.9812$

GLOSSARY

(All flow variables from national income accounts, seasonally adjusted annual rates, billions of current dollars)

$*Y$ = GNP

$*DEP$ = total depreciation allowances

$*DEP_c$ = corporate depreciation allowances

Y_c = corporate profits before tax (does not include IVA)

$*Y_H$ = personal income

Y_{CD} = corporate dividend payments

Y_{CF} = corporate cash flows

$*E_{PD}$ = expenditures on producers' durables equipment

$*C_P$ = personal consumption expenditures

$*INT$ = net interest paid by government and by consumers

ERR = Federal unemployment benefits less statistical discrepancy less net wage accruals less inventory valuation adjustment

T_{FI} = Federal indirect taxes

T_{FS} = Federal social insurance contributions

$*G_{FS}$ = Federal subsides less current surplus of government enterprises

$*G_{FP}$ = Federal transfer payments to persons less unemployment insurance benefits

T_{FC} = Federal corporate profits tax accruals

T_{FX} = Federal excise taxes

$*T_{FCD}$ = Federal customs duties

T_{FO} = Federal social insurance contributions for old-age, survivors, disability insurance

T_{FU} = Federal social insurance contributions, unemployment insurance

$*T_{FSO}$ = Federal social insurance contributions, other

$*T_{SI}$ = State and local indirect taxes

$*G_{SS}$ = State and local current surplus of government enterprises

$*T_{SS}$ = State and local social insurance contributions

$*G_{SP}$ = State and local transfer payments

$*T_{SC}$ = State and local corporate taxes

$*U_M$ = FRB capacity utilization rate for materials, decimal

$*z_k$ = rate of tax credit for producers' durable equipment, decimal

$*t_c$ = Federal corporate tax rate, decimal

$*t_x$ = Federal excise tax rate, decimal

$*t_o$ = Federal OASDI tax rate, decimal

$*t_u$ = Federal unemployment insurance tax rate, decimal

$*t_{uic}$ = labor force covered by unemployment insurance over total labor force, decimal

$*\alpha_1$ = proportion of producers durable equipment eligible for tax credit, = .4139 from 1962-66

$\hat{u}_{-1}$ = previous error term in appropriate equation