Mixed Duopoly with Price Competition

Roy Chowdhury, Prabal
Indian Statistical Institute, Delhi Center

August 2009

Mixed Duopoly with Price Competition

Prabal Roy Chowdhury
Indian Statistical Institute

Abstract

This paper examines coalition-proof Nash equilibria (CPNE) of a mixed duopoly with price competition where the public firm meets all the demand coming to it. If the private firm is free to supply less than demand, then the unique CPNE involves the competitive price. If however the private firm also has to supply all its demand, then the set of CPNE prices turns out to be an interval, with prices ranging from the socially optimal one, to the price under complete privatization.

JEL Classification: L1, L2, L3.
Keywords: Mixed duopoly; coalition-proof Nash equilibrium; price competition.

‡ Indian Statistical Institute, Delhi Center,
7 S.J.S. Sansanwal Marg, New Delhi 110016, India.
Phone: +91-11-4149 3930.
Fax: +91-11-4149 3981.
E-mail: prabalrc1@gmail.com

1 Introduction

This paper examines a mixed duopoly with price competition where the public firm supplies all the demand coming to it. In the presence of limited cooperation among the firms, we find that the welfare maximizing price is implemented whenever the private firm is free to supply less than demand. If however the private firm has to meet all its demand, then there are multiple equilibria, with the equilibrium price being at least as large as the welfare maximizing one, but can be higher.

Price competition where firms meet all demand dates back to Chamberlin (1933). While the literature explores this formulation in the context of private firms,[1] there is relatively little work in the context of mixed oligopoly. This is surprising given that the very existence of public firms can be traced to governmental concerns with welfare, in particular high prices,[2] and that these same concerns may lead to government regulations that mandate that public firms supply all demand.

Vives (1999) for example mentions that in regulated industries like electricity, or telephone, regulations that firms supply all demand are in force in the USA. Spulber (1989) argues that the common carrier regulation can lead to a scenario where firms supply all demand. This framework is natural whenever costs of turning away customers are high and may arise even in the absence of governmental regulations, e.g. in the presence of reputational costs of turning customers away.[3]

A similar assumption for the private firms may be natural if, for example, the government is concerned with maintaining a level playing field. In the absence of such concerns however, such an assumption may be less compelling. We therefore consider both cases, first where the public firm supplies

[1] See, among others, Vives (1999), Roy Chowdhury and Sengupta (2004), etc.

[2] In fact, Cremer et al. (1989) view the public firm primarily as a tool for regulating an oligopoly market.

[3] Such costs are routinely assumed in the operations research literature (see, Taha (1982)).

all demand but the private firm is free to supply less than demand, and second, where both firms must meet all demand. For ease of reference, we shall call these the semi-Bertrand, and the Bertrand form of price competition respectively.

We allow for the possibility of limited cooperation among the firms, thus focusing on equilibria that are immune to group deviations. Formally, the solution concept is the coalition-proof Nash equilibrium (CPNE), introduced in Bernheim et al. (1987) and Bernheim and Whinston (1987).[4] We however also examine the outcome under Nash equilibrium.

The results depend critically on the form of competition. Under semi-Bertrand competition, interestingly we find that the first best can be implemented in that there is a unique CPNE that involves all firms charging the competitive price. This is surprising given that under quantity competition the mixed oligopoly equilibrium differs from the first best.[5] In fact, as is well known, implementing the first best under mixed oligopoly with quantity competition requires a uniform subsidy.[6] Further, given that a CPNE fails to exist in many games,[7] the existence result is of independent theoretical interest.

Under Bertrand competition however, while the welfare maximizing price can be sustained as a CPNE, so can be the price under complete privatization. We then characterize the set of CPNE prices, demonstrating that it is an interval, with prices ranging from the welfare maximizing one, to the price

[4]Such coordination may be facilitated by the government and its regulatory agencies. In the Indian telecommunications industry for example, private firms, e.g. Bharti-Airtel, etc. compete with government firms like VSNL and MTNL. The industry is regulated by the governmental regulatory authority, TRAI, which coordinates among the various players, keeping an eye on various policies followed by the firms. Further, these firms may regularly interact in various professional bodies, e.g. FICCI, ASSOCHAM, etc., that act on behalf of private firms.

[5]See, e.g. De Fraja and Delbono (1990).

[6]See, among others, White (1996), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2004), Tomaru (2006) and Kato and Tomaru (2007).

[7]See, e.g. Bernheim et al. (1987) and Bernheim and Whinston (1987).

under complete privatization.

Given the importance of this market form there is a large and growing literature on mixed oligopoly. The early work of Merrill and Schneider (1966) has been followed by, among others, Cremer et al. (1989, 1991), De Fraja and Delbono (1989) and Anderson et. al. (1997), focusing, among other issues, on the effects of privatization on welfare.[8] We refer the readers to De Fraja and Delbono (1990) for a succinct survey of the early literature. However, while there has been some work on mixed oligopoly with price competition, e.g. Anderson et al. (1995) and Ghosh and Mitra (2008), these are mainly in the context of differentiated products. There is very little work on price competition with homogeneous products where the public firm has to supply all demand. This paper makes a beginning in this respect.

The next section sets up the model and derives some preliminary results. Section 3 solves for the coalition-proof equilibria under both market forms, Bertrand, as well as semi-Bertrand. Section 4 concludes.

2 The Model

There is one public and one private firm, firm 1 and firm 2 respectively, both producing a homogeneous good. The private firm is a profit maximizer, whereas the public firm maximizes social welfare.

The output of the i-th firm is denoted q_i, with both firms producing a homogeneous product with a demand function $D(p)$, and the cost function

[8]While the earlier literature mostly deals with the issue of full socialization versus full privatization, Matsumura (1998), Matsumura and Kanda (2003), Chao and Yu (2006), Fujiwara (2007) and Roy Chowdhury (2009), among others, examine the case of partial privatization. Another branch of the literature deals with the so called irrelevance principle, see, e.g. White (1996), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2004), Tomaru (2006), Kato and Tomaru (2007) and Roy Chowdhury (2009). Further, while many of these papers deal with domestic firms, Fjell and Pal (1996), Pal and White (1998), Fjell and Heywood (2002), Matsumura (2003) and Matsushima and Matsumura (2006), among others, examine mixed oligopoly with foreign firms.

of both firms are denoted by $c(q)$. The following two assumptions on $D(p)$ and $c(q)$ are maintained throughout the analysis.

A1: $D : [0, \infty) \to [0, \infty)$. $D(p)$ is twice differentiable and strictly decreasing for all p such that $D(p) > 0$. There exists a choke-off price K such that $D(p) > 0$ if and only if $p < K$.

A2: $c : [0, \infty) \to [0, \infty)$. $c(q)$ is twice differentiable, strictly increasing and strictly convex for all $q > 0$. Further, $c'(0) < K$.

The firms simultaneously announce their prices. The public firm supplies the whole of the demand coming to it at that price vector. Under Bertrand competition the private firm also supplies all demand, while under semi-Bertrand competition the private firm is free to supply less than the quantity demanded. Let p_i denote the price charged by firm i.

We need some notations and preliminary results.

2.1 Bertrand Competition

Letting $D_i(p_1, p_2)$ denote the residual demand facing firm i under Bertrand competition

$$D_i(p_1, p_2) = \begin{cases} 0, & \text{if } p_i > p_j, \\ \frac{D(p_i)}{2}, & \text{if } p_i = p_j, \\ D(p_i), & \text{if } p_i < p_j. \end{cases}$$

Thus the profit of the i-th firm under Bertrand competition is

$$\pi_i(p_1, p_2) = p_i D_i(p_1, p_2) - c(D_i(p_1, p_2)), \ i = 1, 2, \tag{1}$$

and letting $p = \min\{p_1, p_2\}$, the aggregate welfare under Bertrand competition

$$W(p_1, p_2) = \pi_1(p_1, p_2) + \pi_2(p_1, p_2) + \int_0^{D(p)} D^{-1}(q)dq - pD(p). \tag{2}$$

Thus firm 1 (the public firm) maximizes welfare, i.e. $W(p_1, p_2)$, whereas the private firm maximizes profit, i.e. $\pi_2(p_1, p_2)$. We solve for the pure strategy coalition-proof Nash equilibrium in prices (henceforth CPNE).

Definition. A price vector(p_1, p_2) constitutes a Nash equilibrium if and only if $W(p_1, p_2) \geq W(p_1', p_2)$ $\forall p_1'$, and $\pi_2(p_1, p_2) \geq \pi_2(p_1, p_2')$ $\forall p_2'$. Let Γ be the set of Nash equilibrium price vectors.

Definition. A price vector $(p_1, p_2) \in \Gamma$ constitutes a coalition-proof Nash equilibrium (CPNE) if and only if there exists no other $(p_1', p_2') \in \Gamma$ such that $W(p_1', p_2') > W(p_1, p_2)$ and $\pi_2(p_1', p_2') > \pi_2(p_1, p_2)$.

Next let $\overline{W}(p)$ denote the welfare when both firms charge the same price p, whereas $\underline{W}(p)$ denotes the social welfare when one of the firms charge a price p that is strictly lower than that charged by the other firm. Formally

$$\begin{aligned} \overline{W}(p) &= W(p_1, p_2)|_{p_1=p_2=p}, \\ \underline{W}(p) &= W(p_1, p_2)|_{p_i=p<p_j}, \end{aligned} \tag{3}$$

From the convexity of $c(q)$ it is easy to see that $\overline{W}(p) > \underline{W}(p)$, $\forall p < K$.

Similarly, let $\overline{\pi}(p)$ denote the profit when of both the firms when they both charge the same price p, whereas $\underline{\pi}(p)$ denotes the the profit of a firm that charges a price p that is strictly lower than that charged by the other firm. Formally

$$\begin{aligned} \overline{\pi}(p) &= \pi_i(p_1, p_2)|_{p_1=p_2=p}, \\ \underline{\pi}(p) &= \pi_i(p_1, p_2)|_{p_i=p<p_j}. \end{aligned} \tag{4}$$

We need another assumption.

A3: $\overline{\pi}(p)$, $\underline{\pi}(p)$, $\overline{W}(p)$ and $\underline{W}(p)$ are all strictly quasi-concave in p.

Remark 1 *Note that A3 is satisfied for linear demand and quadratic cost functions. Further, A3 implies that for* $\overline{\pi}(p)$, $\underline{\pi}(p)$, $\overline{W}(p)$ *and* $\underline{W}(p)$ *the* *maximizing price is unique whenever it exists. Further, these functions are all monotonic to the left and to the right of the respective maximizing prices.*

Let p^C denote the competitive price that maximizes $\overline{W}(p)$. Clearly, p^C satisfies

$$p = c'(\frac{D(p)}{2}). \tag{5}$$

Moreover, let $\hat{p} = \arg max\, \overline{\pi}(p)$, so that $\hat{p}$ solves

$$p = c'(\frac{D(p)}{2}) - \frac{D(p)}{D'(p)}. \tag{6}$$

In Lemmas 1 and 2 below we develop some preliminary results that we shall need for our subsequent analysis.

Lemma 1 Let A1-A3 hold. (i) $p^C < \hat{p}$.
(ii) $\overline{\pi}(p^C) \geq 0$.
(iii) $\overline{\pi}(p^c) \geq \underline{\pi}(p)$, $\forall p \leq p^C$.
(iv) $\overline{\pi}(0) > \underline{\pi}(0)$.

Proof. (i) Follows from Eq. (5) and (6).
(ii) $\overline{\pi}(p^C) = \frac{D(p^C)}{2}[p^C - \frac{c(D(p^C)/2)}{D(p^C)/2}] > \frac{D(p^C)}{2}[p^C - c'(D(p^C)/2)] = 0$, where the first inequality follows since $c(q)$ is strictly convex and $D(p^C) > 0$.
(iii) Consider $p < p^C$. Then $D(p) > D(p)/2 > c'^{-1}(p)$. Since $pq - c'(q)$ is strictly convex in q, and is maximized at $c'^{-1}(p)$, it follows that $\overline{\pi}(p) > \underline{\pi}(p)$. Consequently,

$$\overline{\pi}(p^C) \geq \overline{\pi}(p) > \underline{\pi}(p),$$

where the first inequality follows from Lemma 1(i) and the fact that $\overline{\pi}(p)$ is increasing for $p \leq \hat{p}$.
(iv) Follows since $\overline{\pi}(0) = -c(\frac{D(0)}{2}) > -c(D(0)) = \underline{\pi}(0)$. ■

We then have

Lemma 2 *Let A1-A3 hold. At any p solving $\overline{\pi}(p) = \underline{\pi}(p)$, both $\underline{\pi}(p)$ and $\underline{W}(p)$ are increasing in p.*

Proof. Note that any such p solves

$$p = \frac{c(D(p)) - c(D(p)/2)}{D(p)/2},$$

so that from strict convexity of $c(q)$ we have that $p < c'(D(p))$.

Consequently, note that at such a p,

$$\underline{\pi}'(p) = D(p) + D'(p)[p - c'(D(p))] > 0,$$

and

$$\underline{W}'(p) = D'(p)[p - c'(D(p))] > 0,$$

from the previous paragraph.■

Next define $\overline{p}$ to be the maximal $p < K$, such that $\overline{\pi}(p) = \underline{\pi}(p)$.

A4: $\overline{\pi}(p)$ and $\underline{\pi}(p)$ are strictly concave in p.

We need one more lemma.

Lemma 3 *Given A1-A4, there is a unique price $\overline{p}$ such that $\overline{\pi}(p) = \underline{\pi}(p)$.*

Proof. Given Lemma 2, $\underline{\pi}(p)$ is positively sloped at $p = \overline{p}$. Hence if there are multiple such intersections, then concavity of $\overline{\pi}(p)$ must be violated.■

2.2 Semi-Bertrand Competition

Let the residual demand facing firm i under semi-Bertrand competition be denoted by $D_i'(p_1, p_2)$. Given that the public firm meets all demand, the residual demand facing the private firm takes the same form as that under Bertrand competition, so that $D_2'(p_1, p_2) = D_2(p_1, p_2)$. The residual demand for the public firm however will change form given that the private firm will supply $\min\{D_2'(p_1, p_2), c'^{-1}(p_2)\}$. Thus assuming that the residual demand facing firm 1 is parallel in nature

$$D_1'(p_1, p_2) = \begin{cases} D(p_1), & \text{if } p_1 < p_2, \\ D(p_1) - \min\{\frac{D(p_1)}{2}, c'^{-1}(p_1)\}, & \text{if } p_1 = p_2, \\ \max\{D(p_1) - \min\{D(p_2), c'^{-1}(p_2)\}, 0\}, & \text{if } p_1 > p_2. \end{cases}$$

Thus the profit function under semi-Bertrand competition is

$$\pi_i'(p_1, p_2) = p_i D_i'(p_1, p_2) - c(D_i'(p_1, p_2)),\ i = 1, 2. \tag{7}$$

Next let $CS(p_1, p_2)$ denote the consumers' surplus.[9] Then the aggregate welfare

$$W'(p_1, p_2) = \pi_1'(p_1, p_2) + \pi_2'(p_1, p_2) + CS(p_1, p_2). \tag{8}$$

Thus firm 1 maximizes $W'(p_1, p_2)$, whereas the private firm maximizes $\pi_2'(p_1, p_2)$. The CPNE for this case can be defined, with appropriate modifications, in a manner analogous to that for the Bertrand case.

3 The Analysis

We then solve for the set of CPNE under both semi-Bertrand and Bertrand competition.

3.1 Semi-Bertrand Competition

We first consider the case where the private firm is free to supply less than the quantity demanded, but the public firm supplies all demand. We find that there is a unique CPNE that involves both firms charging the competitive price p^C. Thus under semi-Bertrand mixed oligopoly, the outcome involves the first best. Further, implementing the first best does not involve any subsidy.

Proposition 1 *Let A1-A3 hold. Under semi-Bertrand competition, the unique CPNE involves both firms charging the welfare maximizing price* p^C*.*

Proof. We begin by showing that both firms charging p^C constitutes a CPNE. Note that in this case both firms supply till marginal cost. We first

[9] Thus, for $p_1 \leq p_2$, $CS(p_1, p_2) = \int_0^{D(p_1)} D^{-1}(q)dq - p_1 D(p_1)$. For $p_1 > p_2$, $CS(p_1, p_2) = \int_0^{\max\{D(p_1)-\min\{D(p_2), c'^{-1}(p_2)\}, 0\}+\min\{D(p_2), c'^{-1}(p_2)\}} D^{-1}(q)dq - p_2 \min\{D(p_2), c'^{-1}(p_2)\} - p_1 \max\{D(p_1) - \min\{D(p_2), c'^{-1}(p_2)\}, 0\}$.

argue that neither firm has an incentive to deviate from p^C. Consider the public firm. Since p^C maximizes welfare, no deviation can be profitable.

Next consider the private firm, i.e. firm 2. If it charges a higher price, then its profit drops to zero. Given Lemma 1(ii), this is not profitable. Undercutting is not profitable either since it supplies till marginal cost under both situations, but the price is lower in case it undercuts. Next note that no profitable joint deviation from p^C exists since p^C maximizes welfare.

We next argue that no other CPNE exists. If both firms charge the same price p', where $p' > p^C$, then firm 2 has an incentive to undercut. This follows since firm 2 is demand contstrained at this p', i.e. $D(p')/2 < c'^{-1}(p')$. Hence if it undercuts to $p' - \epsilon$ and supplies till $\min\{D(p' - \epsilon), c'^{-1}(p' - \epsilon)\}$, then its profit goes up for ϵ sufficiently small. In case both firms charge p'' where $p'' < p^C$, then a joint deviation to p^C increases welfare. Further, the profit of firm 2 increases as it supplies till marginal cost under both situations but $p^C > p''$.

Finally, we cannot have an equilibrium where the lower priced firm charges a price p''' different from p^C. If firm 2 is charging p''', then firm 1 can match this price when welfare goes up. Whereas if firm 1 is charging p''', then firm 2 can match this price and supply till its marginal cost, when it gains.■

Remark 2 *Note that the proof of Proposition 1 does not depend on the fact that the residual demand facing firm 1 is parallel, and thus goes through in case the residual demand function is proportional as well.*

Remark 3 *It is easy to check that p^C can be sustained as a Nash equilibrium, and further that no price larger than p^C can be sustained as a Nash equilibrium.*

3.2 Bertrand Competition

We next consider the case where both firms must meet all demand. This is of interest if, for example, the government insists on maintaining a level playing field for all firms.

In the next three propositions we characterize the set of prices that can be sustained as CPNE under Bertrand competition. To begin with we can mimic the argument in Proposition 1 to show that the competitive price can be sustained as a CPNE.[10]

Proposition 2 *Let A1-A3 be satisfied. Under Bertrand competition, the welfare maximizing price p^C can be sustained as a CPNE with both firms charging p^C.*

Roy Chowdhury and Sengupta (2004) demonstrate that the unique CPNE outcome under complete privatization (where both firms maximize profits) involves both firms charging a price of $\min\{\hat{p}, \overline{p}\}$. Note that $\min\{\hat{p}, \overline{p}\} > p^C$. Somewhat surprisingly, it turns out that this price can be sustained as a CPNE of the Bertrand mixed duopoly game.

Proposition 3 *Let A1-A3 be satisfied. Under Bertrand competition,* $\min\{\hat{p}, \overline{p}\}$ *(the price under complete privatization) can be sustained as a CPNE with both firms charging* $\min\{\hat{p}, \overline{p}\}$.

Proof. We first argue that no profitable joint deviation exists that is proof to individual deviations. If $\hat{p} \leq \overline{p}$, then there is nothing to prove. So let $\overline{p} < \hat{p}$. Then for any decrease in price $\overline{\pi}(p)$ falls, whereas for an increase in price $\overline{W}(p)$ decreases. We then examine if there can be profitable individual deviations. Consider the public firm. By Lemma 2, $\underline{W}(p)$ is increasing to the left of $\overline{p}$, and moreover $\overline{W}(p) > \underline{W}(p)$, so undercutting is welfare reducing. Next consider the private firm. Since $p \leq \min\{\overline{p}, \hat{p}\}$, $\overline{\pi}(p) \geq \underline{\pi}(p)$. Thus by Lemma 2, undercutting is not profitable for firm 2.■

We next argue that under an additional assumption A4, all prices $p \in (p^C, \min\{\overline{p}, \hat{p}\})$ can be sustained as a CPNE. Further, no other CPNE exists.

[10] The argument that the private firm has no incentive to undercut follows from Lemma 1(iii) for this case.

Proposition 4 *Let A1-A4 hold. Any price* $p \in [p^C, \min\{\overline{p}, \hat{p}\}]$ *can be sustained as a CPNE. Further, no price less than* p^C*, or greater than* $\min\{\overline{p}, \hat{p}\}$ *can be sustained.*

Proof. Note that for any $p \in (p^C, \min\{\overline{p}, \hat{p}\})$ no profitable joint deviation exists. By Lemma 2, the public firm does not have a profitable individual deviation. Thus for any such p to be sustained as a CPNE, it is sufficient to rule out undercutting by the private firm. This follows since from Lemma 3, $\overline{\pi}(p)$ and $\underline{\pi}(p)$ has a unique intersection at $\overline{p}$.

We then argue that no other price can be sustained as a CPNE. Consider an outcome where the active firm charges a price less than p^C. Then a joint deviation to p^C clearly increases welfare. Further, firm 2's profit increases as $\hat{p} > p^C$ and $\overline{\pi}(p)$ is increasing for $p < p^C$. Further, this joint deviation to p^C is proof to individual deviations.

Next consider an outcome where the active firm charges $p > \overline{p}$. If both firms are active, then firm 2 can undercut and gain. So let only one firm be active. If firm 1 is the active firm, it can reduce price, when welfare increases. Whereas if firm 2 is the active firm, then firm 1 can match this price, when welfare increases. So let $\hat{p} < p \leq \overline{p}$. Then a joint deviation to $\hat{p}$ benefits both firms (for firm 1 welfare increases since $p^C < \hat{p}$) and is proof to individual deviations. ■

Remark 4 *At this point it may be of interest to examine the set of Nash equilibrium prices under mixed Bertrand duopoly. Let* $\tilde{p}$ *denote the minimal* p *satisfying* $\overline{\pi}(p) = 0$*. It is straightforward to check that under A1-A4, the set of Nash equilibrium prices is* $[\tilde{p}, \overline{p}]$*.*[11] *Note that* $\tilde{p} < p^C$*, so that under Nash equilibrium prices lower than the competitive price may be sustainable.*

It is often argued that public firms curb the private firms' incentive to charge high prices. As our analysis shows, while this intuition goes through under semi-Bertrand Competition (with the welfare maximizing price being implemented), under Bertrand competition this intuition turns out to be only

[11] A proof is available on request.

half-true given there are multiple equilibria. While the welfare maximizing price turns out to be a CPNE, there are other equilibria that involve higher prices. In particular the price under complete privatization turns out to be an equilibrium.

4 Conclusion

This paper examines a mixed duopoly with price competition, focusing on two aspects that are new in the literature, first that the public firm supplies all demand, and second, allowing for limited cooperation among the firms. In contrast to the literature on quantity competition, under semi-Bertrand competition we find that the outcome involves the first best. Further, implementing the first best does not involve any subsidy. If however the private firm also has to supply all demand, then while the welfare maximizing price turns out to be a CPNE, there are other equilibria that involve higher prices. Finally, turning to robustness issues, while we restrict attention to the duopoly case, preliminary analysis suggests that the results go through qualitatively if there is one public firm facing $n \geq 1$ private firms.

5 References

Anderson, S. P., de Palma, A., Thisse J., 1997. Privatization and efficiency in a differentiated industry. European Economic Review 42, 1635-54.

Bernheim, B.D., Peleg, B., Whinston, M.D., 1987. Coalition-proof Nash equilibria I: Concepts. Journal of Economic Theory 42, 1-12.

Bernheim, B.D., Whinston, M.D., 1987. Coalition-proof Nash equilibria II: Applications. Journal of Economic Theory 42, 13-22.

Chamberlin, E.H., 1933. The theory of monopolistic competition. Cambridge: Harvard University Press.

Chao, C., Yu, E. S. H., 2006. Partial privatization, foreign competition and optimum tariff. Review of International Economics 14, 87-92.

Cremer, H., Merchand, M., Thisse, J.F., 1989. The public firm as an instrument for regulating an oligopolistic market. Oxford Economic Papers 41, 283-301.

Cremer, H., Merchand, M., Thisse, J.F., 1991. Mixed oligopoly with differentiated products. International Journal of Industrial Organization 9, 43-53.

De Fraja, G., Delbono, F., 1989. Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41, 302-311.

De Fraja, G., Delbono, F., 1990. Game theoretic models of mixed oligopoly. Journal of Economic Surveys 4, 1-17.

Fjell, K., Heywood, J. S., 2002. Public Stackelberg leadership in a mixed oligopoly with foreign firms. Australian Economic Papers 41, 267-81.

Fjell, K., Pal, D., 1996. A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29, 737-743.

Fujiwara, K., 2007. Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92, 51-65.

Ghosh, A., Mitra, M., 2008. Comparing Bertrand and Cournot outcomes in the presence of public firms. UNSW, School of Economics DP 2008/18.

Kato, K., Tomaru, T., 2007. Mixed oligopoly, privatization, subsidization, and the order of firms' moves: several types of objectives. Economics

Letters 96, 287-292.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70, 473-483.

Matsumura, T., 2003. Stackelberg mixed duopoly with a foreign competitor. Bulletin of Economic Research 55, 275-287.

Matsumura, T., Kanda, O., 2005. Mixed oligopoly at free entry markets. Journal of Economics 84, 27-48.

Matsushima, N., Matsumura, T., 2006. Mixed oligopoly, foreign firms and location choice. Regional Science and Urban Economics 36, 753-72.

Merrill, W., Schneider, N. 1966. Government firms in oligopoly industries. Quarterly Journal of Economics 80, 400-412.

Myles, G., 2002. Mixed oligopoly, subsidization and the order of firm's moves: an irrelevance result for the general case. Economics Bulletin 12, 1-6.

Pal, D., White, M. D., 1998. Mixed oligopoly, privatization and strategic trade policy. Southern Economic Journal 65, 264-81.

Poyago-Theotoky, J., 2001. Mixed oligopoly, subsidization and the order of firm's moves: an irrelevance result. Economics Bulletin 12, 1-5.

Roy Chowdhury, P., Sengupta, K., 2004. Coalition-proof Bertrand equilibria. Economic Theory 24, 307-324.

Roy Chowdhury, P., 2009. Mixed oligopoly with distortions: First best with budget-balance and the irrelevance principle, Economics Bulletin 29, 1885-1900.

Spulber, D.F., 1989. Regulation and Markets. Cambridge, MA: The MIT Press.

Tomaru, Y., 2006. Mixed oligopoly, partial privatization and subsidization. Economics Bulletin 12, 1-6.

White, M., 1996. Mixed oligopoly, privatization and subsidization. Economics Letters 53, 189-195.

Vives, X., 1999. Oligopoly Pricing: Old Ideas, New Tools. Cambridge, MA London: The MIT Press.