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A CASE FOR AFFIRMATIVE ACTION
IN COMPETITION POLICY

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ABSTRACT. We analyze the trade-off faced by competition authorities envisaging a one-shot structural reform in a capitalistic industry. A structure is (1) a sharing of productive capital at some time and (2) a sharing of sites or any other non-reproducible assets. The latter represent opportunities. These two distinct dimensions of policy illustrate the importance of a dynamic theory in which firms may differ in several respects. Though equalization of endowments and rights is theoretically optimal, realistic constraints force competition authorities to adopt second-best solutions. Affirmative action here appears to explain why helping the disadvantaged contributes maximally to social surplus.

KEYWORDS. Competition policy, capacity accumulation, Cournot competition, asymmetric duopoly, regulatory consistency, differential games.

JEL CLASSIFICATION: C73, L13, L40.

1. INTRODUCTION

Restructuring a capital-intensive industry where some incumbent dominates the market is a hard task for competition authorities and regulators. It may not be a reliable option to open the market and let time pass. First, capital may be so long-lived that the incumbent will influence prices for a long time. Second, the incumbent may retain the best sites and know-how, which leaves the incumbent with a permanent superiority vis-à-vis the entrants. These issues are especially pertinent for merger remedies or regulatory reforms in electricity, telecommunication and other spectrum-based industries where investment is durable and capacity expansion is constrained.

Assume that a competition authority has to decide merger remedies such as asset transfer from the newly dominant firm to its competitors. Or assume that a regulator has to ignite competition by transferring assets from an incumbent to an entrant. Theoretically, creating a symmetric oligopoly by splitting the total installed capacity into several identical lots is sensible in both cases. However, such radical moves exceed the routine of competition policy and they are rarely employed.[1] Obvious reasons are that, for efficiency purposes, firms have to keep a degree of geographic or technological integrity.

A complete evaluation of reforms or remedies requires a dynamic theory of the full consequence of reorganizing an industry under certain constraints. In this paper, a firm is characterized by *initial capacity* (i.e. productive forces at divestiture date 0) and *opportunities* (i.e. quality of sites and technologies that the firm inherits). Opportunities, modeled as investment costs, summarize conditions under which the firm will operate and develop.

If firms have the same technologies, investment costs are normally the same for all. In practice, performance depends on location and availability of inputs other than capital, labor or energy. In the case of power generation, regions differ considerably in the comparative and absolute advantages of wind farms, dams or nuclear plants. The difference between nominal and actual power is particularly clear for wind farms: with identical turbines (nominal capacity), the intensity and variability of wind power directly determine production (actual capacity). History and geography may have established asymmetric positioning of firms among available sites. Typically, one firm is a former monopoly and the other a start-up, the structure of the industry has been deeply marked by political interference, or two regions have been interconnected and opened to trade after a long period of isolation. Moreover, in the markets that have been competitive for a long time, the firms' past race to sites may have resulted in clearly differentiated outcomes.

These historical processes, interesting as they may be, are not the focus of this study. Instead, we take their consequences (here different investment costs) as initial conditions for structural reforms. Authorizing investments, reshuffling assets, or splitting dominant firms are

[1]For example, physical divestiture in the electric sector is an appealing solution that has been implemented quite rarely.

especially relevant for mature industries like the energy sector. In this sense, costs and opportunities are controls for the competition authority or the regulator. This paper will discuss the type of constraints under which these controls can be used.

Restructuring an industry poses numerous challenges. Two practical limitations may be encountered. The first comes from technological criteria. Due to economies of scale in technical expertise, production planning and management, it may be proposed to set up technologically uniform firms. For example, restructuring could result in one firm specialized in nuclear plants, a second in gas-fired turbines, a third in dams, and a fourth in windmills. Other groupings are also possible: green versus dirty, thermal versus non-thermal, etc. The second limitations comes from geographic criteria. It may be reasonable to have the plants of a particular firm close to each other. On the contrary, it may be preferred that each firm should be present in each region. In the case of the electricity industry, it is important to decide where the boundaries between regions or sub-regions should be drawn in that interaction between firms via electric grid depends on location. Therefore, when the competition authority or the regulator decides that N firms should be established, the key issue is the attribution of various existing technologies or locations to firms.

The diversity of available policies calls for an abstract version of the trade-off faced by competition authorities or regulators. The literature connected to our question either uses rich static models to address competition policy issues, or sets up complete dynamic descriptive theories. We propose an approach which links these two domains and further explore the dynamic effect of restructuring an industry in competition policy.

In merger control and competition policy, symmetry versus asymmetry is an on-going debate among competition authorities and scholars. Perry and Porter (1985) consider a model of asymmetric competition among sellers with increasing marginal costs that depend on firms' productive capacity. When capacity is tied to physical assets and when there is a fixed amount of such assets available to the sellers in the market, the competitors' ability to increase output in response to rising prices is limited. Asymmetry in the initial distribution of the productive capacity among the sellers can reinforce this effect, making it particularly attractive for two relatively large sellers to merge. Farrell

and Shapiro (1990a,b) also permit firms to differ in their costs. They find that with no synergies the market price will go up post merger, even if one firm is highly inefficient and gets to use the more efficient firm's technology. McAfee and Williams (1992) study mergers between firms with asymmetric but constant average costs. They find that mergers increasing the size of the largest firm will reduce welfare.

Motta *et al.* (2007) raise questions on the recent symmetric settlements of merger remedy in Europe and claim that they increase the potential for tacit collusion and joint dominance. Compte *et al.* (2002) also cast some doubt on standard merger remedies, which favor divesting some capacity of the merged firm and transferring it to other competitors in order to maintain a reasonable amount of symmetry between competitors. The argument is made through a repeated Bertrand game. It confirms that introduction of asymmetric capacities makes tacit collusion more difficult to sustain when the aggregate capacity is limited, which may benefit competition.

In fact, repeated games are not suitable for capital-intensive industries. In these industries, building capacity takes time and thus production flexibility is narrow; moreover, depreciation of capital being slow, firms see their capacities as fixed temporarily, thus limiting their ability to commit to a punishment scheme (which involves increased production).

Besides the static and the repeated games, theories of the dynamics of industries have been well developed. Most of them are descriptive in that they try to retrieve empirical facts like durable asymmetries or to explore the role of preemption. Ingredients may vary as they may concern economies of scale, indivisibilities, idiosyncratic shocks or learning curves.

Tombak (2006) studies asymmetry as a strategic choice by firms and extends Fudenberg's and Tirole's (1984) classic results using the Boeing vs. Airbus case. Besanko and Doraszelski (2004) find, in a model with lumpy investment and idiosyncratic shocks, that asymmetry may tend to persist under certain conditions and symmetry is a rare and temporary coincidence in an ever-moving economy. Koulovatianos and Mirman (2004) find that when large firms have a cost advantage due to their size, asymmetry leads to a decline in the supply of all firms, which suggests that cost advantage is an important aspect to the role of asymmetry in oligopolistic markets. Chen (2008) investigates the price

and welfare effects of mergers through simulations of capacity accumulation. He finds that asymmetric costs may lead to asymmetric sizes post merger even though firms are *ex ante* identical. Ishii and Yan (2007) use an empirical dynamic model to study the "make or buy" decision faced by independent power producers (IPP) in restructured U.S. wholesale electricity markets. Asymmetry in plant characteristics between divested and new assets leads to the difference in expected profit between the two assets. They find that divestiture has encouraged new IPP participation and has not crowded out a large amount of new generation capacity in the long run.

In this paper we produce a simple and workable model in which policies can be evaluated analytically. The industry dynamic models that inspire us are in Reynolds (1987) and Hanig (1986). Differential games are practical to analyze the accumulation of productive capacity in an imperfectly competitive market. As capacity accumulation takes time, initial capacities and investment cost are the crucial factors of differentiation between firms. These are the channels that the competitive authorities or the regulators use to improve market performance.

We introduce affirmative action which captures the best response to various constraints faced by competition authorities and regulators: if for some reasons, full-symmetrization of initial capacities and opportunities is impossible, compensation has to be implemented. The firm that receives lesser opportunities should receive more initial capacity and the other way around. These two dimensions of policy, and the way they can compensate each other, illustrate the importance of having and using a dynamic theory in which firms may differ in several respects. Affirmative action appears to explain why helping the disadvantaged firms maximizes social surplus.

Our paper is organized as follows. In Section 2, we set up the model and present the key assumptions on the constraints faced by the policy-maker. In Section 3, we solve the model and establish the general features of the investment trajectories. Section 4 describes the trajectories, in particular their dependency on the choice of the policy-maker. Section 5 provides the comparative statics of the steady state. Section 6 shows the importance of affirmative action. Section 7 discusses the robustness of the results with respect to synergies. Section 8 gives a series of extensions. A conclusion follows.

2. THE MODEL

2.1. **Capacity and market for final product.** The game is played
in continuous time with an unbounded horizon; in the following game,
t is a date in $\mathbb{R}^+$. There are two firms serving the market, i denotes
an arbitrary firm while j denotes the other. Capacity is the maximum
a firm can produce at a given date t. Capacity accumulation by firm i
follows

$$
(1) \qquad \overset{\bullet}{k_i} \equiv \frac{dk_i(t)}{dt} = I_i(t) - \delta_i k_i(t),
$$

where $I_i(t)$ and $k_i(t)$ are, respectively, firm i's investment and capacity
at date t, and δ_i is a constant depreciation rate.

The instantaneous cost of investment is quadratic

$$
(2) \qquad C_i(I_i) = \gamma_i^0 + \gamma_i I_i + \frac{I_i^2}{2\theta_i},
$$

with γ_i^0, γ_i and θ_i non-negative reals.[2] A fundamental difference be-
tween our model and Reynolds' (1987) or Cellini's and Lambertini's
(2003) is that, in our case, the two firms may have different technolo-
gies, namely, different investment costs and depreciation rates. The
product is homogeneous and firm i's marginal production cost is con-
stant and equal to c_i. The global constraint of the industry is an
important modeling choice: we expose in Subsection 2.2 how we model
the joint condition on $C_i(\cdot)$ and $C_j(\cdot)$.

The inverse demand function is linear. Thus denoting $q_i(t)$ the quan-
tity sold by firm i at time t, the price is

$$
(3) \qquad P(t) = A - q_i(t) - q_j(t).
$$

Firm i's production $q_i(t)$ is a proportion $\alpha_i(t)$ of its capacity $k_i(t)$, with
$\alpha_i(t) \in [0,1]$; thus $q_i(t) = \alpha_i(t)k_i(t)$.

As firm i's instantaneous profit is

$$
(4) \qquad \pi_i(t) = (P(t) - c_i)q_i(t) - C_i(I_i(t)),
$$

firm i's objective is to maximize the present value of its profit flow

$$
(5) \qquad \int_0^\infty \pi_i(t)e^{-\rho_i t}dt,
$$

---

[2]Hanig (1986) defines a quadratic adjustment cost which depends on the *net*
investment $I_i(t) - \delta_i k_i(t)$; the advantage and the limitation are that investment
cost is null in the stationary state. Fudenberg and Tirole (1983), in a model based
on Spence (1979), assume no depreciation.

where ρ_i is firm i's discount rate. The control variables are the instantaneous investment rate $I_i(\cdot)$ and the rate of capacity utilization $\alpha_i(\cdot)$; accumulation equation (1) and firm j's strategy are the constraints.

2.2. **Sites and investment costs.** We propose a simple theory of Ω, the feasibility constraint faced by the competition authority or the regulator in the allocation of costs. Let's focus on the trade-off between $C_i(\cdot)$ and $C_j(\cdot)$. Clearly, sensible comparative statics has to be made along certain efficiency frontier, i.e. where $C_i(\cdot)$ cannot be decreased without increasing $C_j(\cdot)$. Our ideas are close in inspiration to those expressed in Perry and Porter (1985) or Farrell and Shapiro (1990a,b) for the analysis of mergers. However, the substantial difference is that, in our model, restructuring of an industry impacts investment rather than production costs. Our focus on investment costs only is motivated by its direct relevance for the industry dynamics.

Assume that there is a continuum of sites parameterized by $\theta \in [\underline{\theta}, \bar{\theta}]$. The investment cost attached to site θ is $\gamma(\theta)z + \frac{z^2}{2\theta}$ where z is the site specific rate of investment and $\gamma(\cdot)$ is a positive function. Firm i can be described by the sites it owns. Ownership is summarized by $\omega_i(\theta)$, the mass of θ-sites that firm i owns out of an exogenous total $h(\theta)$, with $\omega_i(\theta) + \omega_j(\theta) = h(\theta)$. When it invests a total I_i, firm i optimally spreads its capacity augmentation across the sites it owns, which yields the aggregate $C_i(I_i)$. We find the expression of costs already given above (see equation 2):

Given that γ_i is the average $\gamma(\theta)$ (weighted by $w_i(\theta) \cdot \theta$) and that θ_i is the weighted (by $w_i(\theta)$) sum of θ, we have the overall constraints:

$$\text{(6)} \qquad \theta_i + \theta_j = \Theta,$$

$$\text{(7)} \qquad \theta_i \gamma_i + \theta_j \gamma_j = \Gamma,$$

where Θ and Γ are economy-wide (i.e. independent of site sharing between firms) investment costs. All calculations and exact expressions are in Appendix A.1.

2.3. **The competition authority's problem.** The competition authority has to evaluate policies by taking into account their full consequences, summarized by capital trajectories $(k_i(t), k_j(t))_{t \geq 0}$. We will first determine these trajectories by solving the game and then calculate the present profits, the present consumer surplus and the present social surplus.
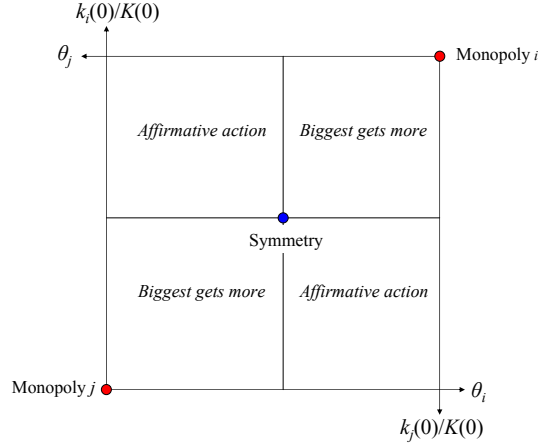
FIGURE 1. A kind of Edgeworth box.

The constraint on capacity at reform date 0 is

$$(8) \qquad\qquad k_i(0) + k_j(0) = K(0),$$

where $K(0)$ is total initial capacity. To simplify policy analysis, we shall assume at the evaluation stage that sites differ only with respect to the quadratic part ($\gamma(\cdot)$ is then a constant function). The two constraints on costs (6) and (7) boil down to (6):

A policy is summarized by $(k_i(0), k_j(0), \theta_i, \theta_j)$, a choice of initial capacities and perspectives for the two firms under the two constraints (8) and (6). Some of our results will be illustrated with the Edgeworth boxes in Figure 1, where each point represents a policy. Two regions represent the policies that compensate an advantaged firm in one dimension by a disadvantage in the other (affirmative action), and vice versa. The other two regions are giving advantages to one firm in both dimensions. We shall see that affirmative action is generally preferable.

3. The Cournot-Nash equilibrium

3.1. **The equilibrium.** The existing literature on differential games focusses on two types of strategies: open-loop and closed-loop strategies.[3] Both types can form a Nash equilibrium. In the open-loop equilibrium, strategies are just functions of time; they are time consistent inasmuch as the opponent doesn't deviate. In the closed-loop

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[3]See Dockner *et al.* (2000) for a complete survey.

equilibrium, one player defines actions that depend on what the other has done and the equilibrium is sub-game perfect.

We explore the open-loop solution in the study for three reasons. First, the solution is unique and analytically tractable. This is extremely convenient for pursuing the comparative statics that the competition authorities need to decide on policy. In contrast, due to the multiple solutions of the nonlinear characteristic equations, the calculation of the closed-loop equilibrium requires a selection that seems resistant to algorithmic treatment.[4] Second, far from being an inferior concept, the open-loop equilibrium represents specific assumptions on players' information (Dockner *et al.*, 2000, chapter 4). For example, the other's position may be imperfectly observed, e.g. with delay or noise; also, if investment has to be programmed in advance, reaction to the opponent's decisions may not be immediate and sharp. Since delayed state variables are hard to handle, the open-loop equilibrium may be a reasonable approximation.[5] Third, open-loop strategies capture well the ability to commit on the part of investors. Playing strong is known to be an individually beneficial strategy; it can become a mutually beneficial one if it limits temptation to play a preemption war (a dynamic version of the prisoner's dilemma).[6] Whether a firm plays strong is a modeling choice that can be discussed rather than a logical necessity.

3.2. **General solution.** The law of motion followed by firm i's investment is

$$(9) \quad (\rho_i + \delta_i)I_i - \overset{\bullet}{I}_i + (2\alpha_i k_i + \alpha_j k_j)\alpha_i \theta_i = ((A - c_i)\alpha_i - (\rho_i + \delta_i)\gamma_i)\theta_i.$$

See proof in Appendix A.2.

In the Cournot game of the two-period setting of Kreps and Scheinkman (1983) or the differential game in Dockner (1992), full utilization of capacity is assumed. In contrast, firms in our model can choose to leave a fraction of their capacities idle. For example, if a firm inherits huge

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[4]Reynolds' (1987) ingenious resolution of the fully symmetric case is evocative of the difficulty one faces with the closed-loop equilibrium in our more general case.

[5]It may be felt that the open-loop equilibrium is inadequate for representing reaction to unexpected shocks. However, the closed-loop equilibrium doesn't address this problem (it is designed as a theory of reaction to *voluntary* deviations by the competitor, which is a totally different issue).

[6]Reynolds (1987) shows that the closed-loop equilibrium is more competitive—less profitable—than the open-loop equilibrium for that reason.

capacity in a small market (e.g. due to permanent reduction of demand caused by the introduction of a substitute), withholding capacity makes sense. The method we follow is simple. We characterize in detail trajectories along which firms fully utilize their capacities. We check equilibrium conditions *ex post* and show the clear practical relevance of the full utilization scenario (see also Section 4).

Using accumulation equation (1), we eliminate investments to simplify (9) as

$$(10) \quad \overset{\bullet\bullet}{k}_i + \delta_i \overset{\bullet}{k}_i - (2\theta_i + (\rho_i + \delta_i)\delta_i)\, k_i - \theta_i k_j + (A - c_i - (\rho_i + \delta_i)\gamma_i)\theta_i = 0.$$

We show in Appendix A.3 that capacities, as functions of time, have the form

$$(11) \qquad k_i(t) = k_i^* + k_i^{(1)} e^{\lambda_1 t} + k_i^{(2)} e^{\lambda_2 t},$$

where λ_1 and λ_2 are two strictly negative reals.[7] We also provide restrictions on the other parameters. The practical consequence is that once initial capacities $k_i(0)$ and $k_j(0)$ and investment costs θ_i and θ_j are chosen or known, trajectories are in fact fully determined.

3.3. **Simplification.** The capacity trajectories are entirely solvable with asymmetric costs. Calculation of the surpluses, which is on the basis of any policy evaluation, require a few simplifications:

(1) Perfect capital markets: Firms have the same interest rate ρ.
(2) Homogenous capital: Firms have the same depreciation rate δ.
(3) $\gamma_i(\cdot) = $ Constant. The only constraint on costs is $\theta_i + \theta_j = \Theta$.[8]
(4) Identical production cost ($c_i = c_j = c$).

Moreover, we will denote

$$(12) \qquad \overline{A} \equiv A - c - (\rho + \delta)\gamma.$$

With these simplifications, λ_1 and λ_2 can be calculated explicitly:

$$(13) \qquad \lambda_1 = -\tfrac{\delta}{2} - \tfrac{1}{2}\sqrt{5\delta^2 + 4\left(\delta\rho + \theta_i + \theta_j + \sqrt{\theta_i^2 + \theta_j^2 - \theta_i\theta_j}\right)},$$

$$(14) \qquad \lambda_2 = -\tfrac{\delta}{2} - \tfrac{1}{2}\sqrt{5\delta^2 + 4\left(\delta\rho + \theta_i + \theta_j - \sqrt{\theta_i^2 + \theta_j^2 - \theta_i\theta_j}\right)}.$$

[7]They are the negative eigenvalues of the matrix characterizing the dynamics of the system.
[8]See Subsection 2.3.

Note that $\lambda_1 < \lambda_2 < 0$. These explicit expressions simplify the characterization of the trajectories. Convergence speeds only depends on opportunities, not on the sharing of capacities.

4. DYNAMICS

The case illustrated in Figure 2 will deliver the essential message. If $\theta_i = \theta_j$, the picture is symmetric with respect to the 45° line. E is the steady state. Trajectories combine two movements:

(1) the fastest, parallel growth, associated with λ_1. Trajectories follow the 45° line: AE arrives from below, CE arrives from above.

(2) the slowest, difference reduction, associated with λ_2. Trajectories follow FE and $F'E$, along which total capacity is constant.

At each instant, capital is predetermined but production decisions are controlled by utilization rates and are thus very flexible. An understanding of the instantaneous Cournot game (in which only demand and the linear production costs are considered) suffices to calculate α_i and α_j. More precisely in Figure 2, CD supports player i's Cournot reaction function $k_i = \frac{1}{2}(A - c - k_j)$, and BC supports player j's Cournot reaction function $k_j = \frac{1}{2}(A - c - k_i)$. Thus the quadrilateral $ABCD$ is the region in the plane (k_i, k_j) in which players are constrained by their actual capacities; necessarily, $\alpha_i = \alpha_j = 1$.

The steady state E is in the interior of $ABCD$, meaning that capacity is fully employed when the economy converges.[9] Finding a large and realistic set of initial conditions such that capacity is fully utilized all along the accumulation path is now straightforward. It suffices to look at the trajectories that converge to the steady states, follow them backwards, and eliminate their portions outside the full-utilization region $ABCD$. We propose a simple (though not maximal) "safety zone" in which assuming full-utilization all along the trajectories is consistent: $AGFCF'G'$ in Figure 2.[10]

In view of this, we can now state:

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[9] In the steady state, it is immediate to check that the left hand side of (36) is strictly positive, thus we conclude that $\mu_i > 0$, i.e. $\alpha_i = 1$.

[10] If a trajectory starts somewhere in the shaded area $AGFCF'G'$, then it stays therein. This safety zone is bounded by the axes, the *Cournot* best responses (BC and CD) and edges GF and $F'G'$ that are parallel to the 45° line.
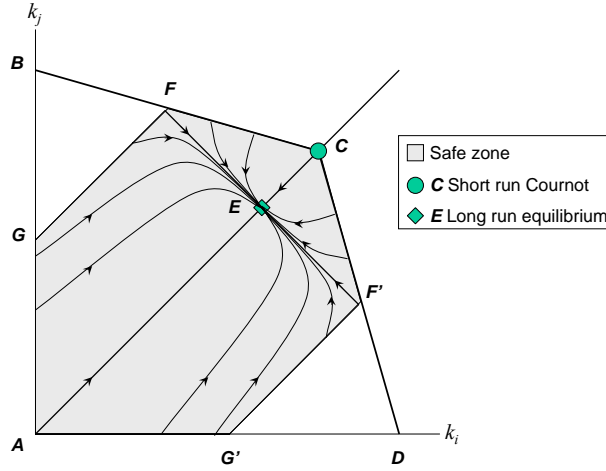
FIGURE 2. Trajectories.

Proposition 1 (Relevance of full-utilization scenarios)**.** *In any equilibrium, capacities become fully-utilized and remain so as the long-run is sufficiently approached.*

Along all trajectories except the 45° line, one capacity is monotonic whereas the other peaks and then decreases. This is a consequence of the different speeds of the two pure movements described above. In this sense, we have an overshooting effect for at least one of the firms.[11,12]

Overshooting here can be interpreted as a form of transitory preemption. The intuition is that building capacity takes time, thus the firm starting with large capacity is able to take advantage of its advance and to play a temporary monopoly strategy; indeed, having more capacity is akin to moving first, i.e. preempting the market. This temptation vanishes as the small firm catches up: in the long run, maintaining the advance is too costly since it would suppose sustaining a higher rate of investment (higher marginal cost) for the same marginal revenue.

As effects are continuous with respect to cost allocation, generalization to $\theta_i \neq \theta_j$ is direct. One movement could be labeled *approximate* parallel growth, and the other *approximate* difference reduction, where one capacity-unit less for one firm corresponds to about one capacity-unit more to the other firm. Overshooting for one of the firms is also typical. Overshooting of *total* capacity now becomes prevailing on one

---

[11]Strictly speaking, if a firm starts big and close to the steady state, we may observe the contraction of capacity only for this firm.

[12]This confirms Hanig's (1986) simulations.

side of the approximate parallel growth trajectory; on the other side, total capacity simply grows.

5. THE LONG RUN

In the steady state $k_i^* = I_i^*/\delta$. Thus we find

$$
(15) \qquad k_i^* = \frac{\left(1 + \frac{(\rho+\delta)\delta}{\theta_j}\right)}{\left(2 + \frac{(\rho+\delta)\delta}{\theta_i}\right)\left(2 + \frac{(\rho+\delta)\delta}{\theta_j}\right) - 1} \cdot \overline{A}.
$$

A higher θ_i decreases the cost of sustaining any level of capital. Consistently, the equilibrium capacity of firm i is increasing with respect to θ_i and increasing with respect to θ_j. Note that when θ_i is close to an extreme point (0 or Θ), the economy behaves as if the market were monopolized.

Proposition 2. *The steady state profit π_i^* is such that*

$$
(16) \qquad \frac{\partial \pi_i^*}{\partial \theta_i} > 0 \qquad and \qquad \frac{\partial \pi_i^*}{\partial \theta_j} < 0.
$$

Proof. See Appendix A.4. □

Each firm's steady state profit decreases in its own instantaneous investment cost, but increases in its rival's instantaneous investment cost. When we take the constraint on the allocation of sites into account, these two effects draw in the same direction, thus

$$
(17) \qquad \left.\frac{d\pi_i^*}{d\theta_i}\right|_{\theta_i+\theta_j=\Theta} > 0 \qquad and \qquad \left.\frac{d\pi_i^*}{d\theta_j}\right|_{\theta_i+\theta_j=\Theta} < 0.
$$

Proposition 3. *Equalizing investment costs*
   (1) *maximizes the long run total capacity, and thus the consumer surplus, and*
   (2) *maximizes the total surplus.*

Proof. See Appendix A.5. □

The second point is not a consequence of the first, since the proposition says nothing of profits. In fact, profits are maximized when one firm collects all sites and monopolizes development and sales. So when a regulator has to assign sites to firms, full symmetry of capacities and opportunities gives the most propitious conditions for competition *in the long run*.

The intuition for these results is very similar to that of static Cournot models.

6. Affirmative action

How should competition authorities or regulators approach the long run optimum? Firms could be given equal numbers of plants of various types of technologies; in theory, there is considerable flexibility in the way sites can be reallocated to yield similarly performing firms if the only constraint were $\theta_i + \theta_j = \Theta$ and $k_i(0) + k_j(0) = K(0)$. In practice, however, the flexibility in the grouping of plants or technologies may be limited: the competition authority may follow a geographical or technological logic when it comes to redefining the two firms; more importantly, sites cannot be reshuffled without reshuffling assets.

The former argument says that, presumably, there are constraints on the policies that can be chosen in effect in the Edgeworth box (see Subsection 2.3). Whether the fully symmetric allocation of plants is feasible is a matter of circumstances. We examine now the consequences of these restrictions for policy. We start with descriptive comparative statics and we continue with normative assessments.

The first approach to the dynamics is to draw "half-lives", which is the time needed to cover half the distance between the current state and the steady state. As there are two combined processes, we have half-lives T_1 and T_2 with

$$(18) \qquad T_1 = -\log[2]/\lambda_1 < T_2 = -\log[2]/\lambda_2.$$

Proposition 4 (Redistributing opportunities)**.** *More asymmetric opportunities foster capacity growth (T_1 decreases) and cause longer durability of initial differences in capacities (T_2 increases).*

Proof. See Appendix A.6.   □

We fix parameters at plausible values.

Table 1: Parameters		
Demand	$A = 1$	
Production cost	$c = 0$	
Investment cost	$\Theta = 1/10$	with $\theta_i + \theta_j = \Theta$
Rates	$\delta = .05$	$\rho = .08$
Capacity	$K(0) = 1/2$	with $k_i(0) + k_j(0) = K(0)$

Figure 3 shows how T_1 and T_2 vary with θ_i. We see that T_1 (growth) is relatively insensitive to the sharing of opportunities. On the contrary, T_2 (difference reduction) becomes relatively high with unequal sharings:

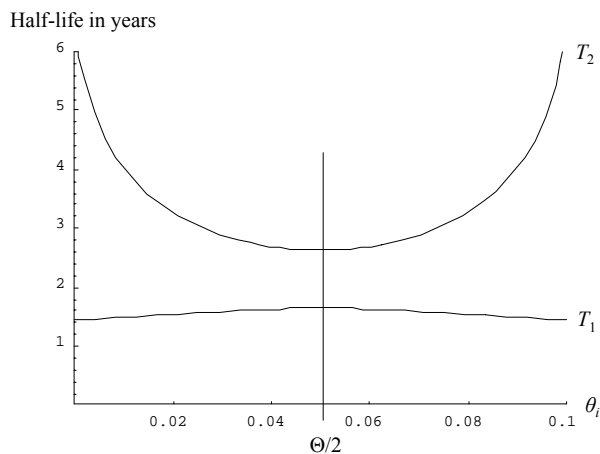half-life towards the steady state takes as much as 6 years when a firm is given essentially all sites.

Half-life in years

FIGURE 3. Half-lives.

Note also that a faster depreciation (higher δ) accelerates convergence along the two dimensions. Intuitively, since investment is less durable, the current state is less durably affected by the past, which means that long-term values can be attained quickly. In addition, the steady state capacities decrease as δ increases; thus an economy starting with no capacity will always approach its long run equilibrium faster if δ is higher (less distance to be covered and higher speed).

These descriptive results depict the behavior of the economy in a useful way. However, competition authorities need firmer normative guidance. To look more precisely at the impact of asymmetry on consumer welfare, a natural (and simple) angle is to look at total capacity over time, which represents total consumption. We start with the impact of the initial conditions.

Proposition 5 (Redistributing initial capacity)**.** *Fix θ_i and θ_j. Without loss of generality, assume that $\theta_i \geq \theta_j$ (Firm i has the least costs).*

Denote total capacity as $K(t) \equiv k_i(t) + k_j(t)$. Fix total initial capacity $K(0)$. $k_i(0)$ is the portion allocated to Firm i, while $K(0) - k_j(0)$ goes to Firm j.

(1) *Total capacity at date 0 and in the long run are independent of initial sharing.*

(2) *Total capacity $K(t)$ increases more slowly at date 0 for higher $k_i(0)$.*

(3) *Total capacity $K(t)$ at any date $t > 0$ is smaller for higher $k_i(0)$.*

*Proof.* See Appendix A.7.   □

The proposition provides a strong result: initial conditions determine uniform ranking of capacity over time. We can directly conclude the impact on consumer's welfare without further calculations: consumers prefer that less efficient firms be compensated by better opportunities. The result is even stronger: consumers would prefer maximal compensation, i.e. giving all the capacities to the less efficient firm. See Figure 4(a). A corollary is that if costs are symmetric ($\theta_i = \theta_j$), total capacity as a function of time is independent of the initial allocation of the existing capacity.

The intuition is that longer survival of the inefficient firm constrains the efficient firm for a longer time. The efficient firm has to accommodate its non-trivial rival in production while preparing, with smooth investment, its future dominance. Though the initial conditions on capital vanishes in the long run, the transition is so important for consumers that they want such extreme remedies if exact symmetry is not possible.

The appreciation by firms is of course very different, as we illustrate in Figure 4(b). Firms collectively prefer monopolies. Low contours have not been traced for legibility.
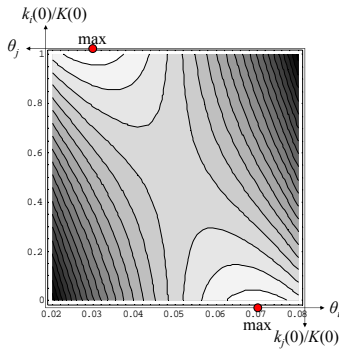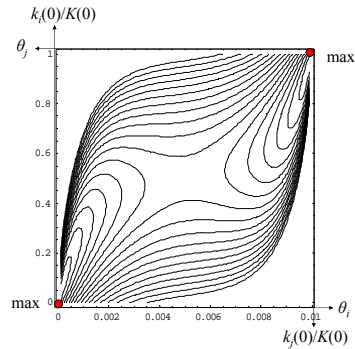
(a) Present consumer surplus.   (b) Present total profit.

FIGURE 4. Synthesis.

The symmetric initial allocation $\theta_i = \theta_j = \Theta/2$ and $k_i(0) = k_j(0) = K(0)/2$ is a singular point. Indeed, all firms are symmetric in cost parameters and initial capacities; thus in the Edgeworth box, social

welfare (or consumers' surplus or total profit) exhibits a central symmetry. It is less clear why it should be a maximum, a minimum or a saddle-point. In fact, all possibilities are open.

Figure 5 shows the contours of the present value of total surplus in the Edgeworth box.[13] We retrieve numerically the maximum for the fully symmetric situation. The stretched shape illustrates that constraints on $k_i(0)$ (respectively θ_i) have to be compensated by distortion on θ_i (respectively on $k_i(0)$).[14] Indeed, the firm with lesser investment opportunities has to be compensated with more initial capacity if the competition authority or the regulator seeks maximum efficiency.
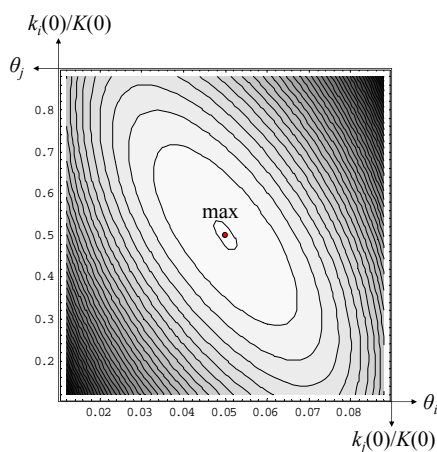

FIGURE 5. Present total surplus.

7. SYNERGIES

The constraint on site allocation $\theta_i + \theta_j = \Theta$ is without synergies. We could assume instead that grouping sites is efficiency enhancing or degrading. A simple formulation is

$$(19) \qquad \theta_i + \theta_j + \frac{a\theta_i\theta_j}{\Theta} = \Theta \text{ with } a \in (-\infty, 1).$$

Positive a means that concentration degrades the economy's investment potential, thus reinforcing the interest of promoting two equal firms. Negative a means that there are synergies: $\theta_i + \theta_j$ is maximal for a monopoly. This may lead the competition authority to prefer

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[13]Note that the function can be calculated exactly with *Mathematica*, but its full expression takes several pages.

[14]The sign of the off-diagonal terms of the Hessian matrix informs on how contours are stretched.

to asymmetric structure. We study this case now using parameters in Table 2.

Table 2: Parameters		
Demand	$A = 1$	
Production cost	$c = 0$	
Investment cost	$\Theta = 1/100$	with $\theta_i + \theta_j + \frac{a\theta_i\theta_j}{\Theta} = \Theta$
Synergy parameter	$a < 0$	
Rates	$\delta = .05$	$\rho = .08$
Capacity	$K(0) = .17$	with $k_i(0) + k_j(0) = K(0)$

The contours of the long-run total present surplus in Figure 6(a) have the control variable $\theta_i - \theta_j$ on the horizontal axis and the exogenous synergy parameter a on the vertical axis. For a below -3.5, the optimal structure in the long run is the monopoly. Above -3.5, symmetric duopoly is optimal.

To show in this context the importance of transition, we took $a = -.5$. Though in this case symmetry is preferred in the long-run, the value of a implies that splitting sites into two equal lots decreases $\theta_i + \theta_j$ by about 10% compared to the maximum. In Figure 6(b), we traced the contours of the total present surplus with $K(0) = .17$ as in Table 2. Low contours have not been traced for legibility. We see two maxima involving both an asymmetric structure of the industry (they are equivalent by permutation). They represent the optimal social trade-off between concentrating sites for investment efficiency (due to $a < 0$) and balancing market power by avoiding the establishment of a monopoly. This is the affirmative effect again: a disadvantaged firm in one dimension is compensated in the other. This is entirely due to the dynamics, since the optimum policy here differs from the long-run optimal choice.

8. Extensions

Extension 1. Some new sites may be discovered; others may be exhausted or degraded. The impact on the parameters could be a change of Θ. Assume that we expand Θ keeping $\theta_i = \theta_j = \Theta/2$; we take $\gamma = c = 0$ to shorten expressions. The steady state flow of profit is then

$$(20) \qquad \pi^*_{\text{sym}}(\Theta) = \frac{\Theta(\Theta + (2\rho + \delta)\delta)}{(3\Theta + 2(\rho + \delta)\delta)^2} \cdot A^2.$$
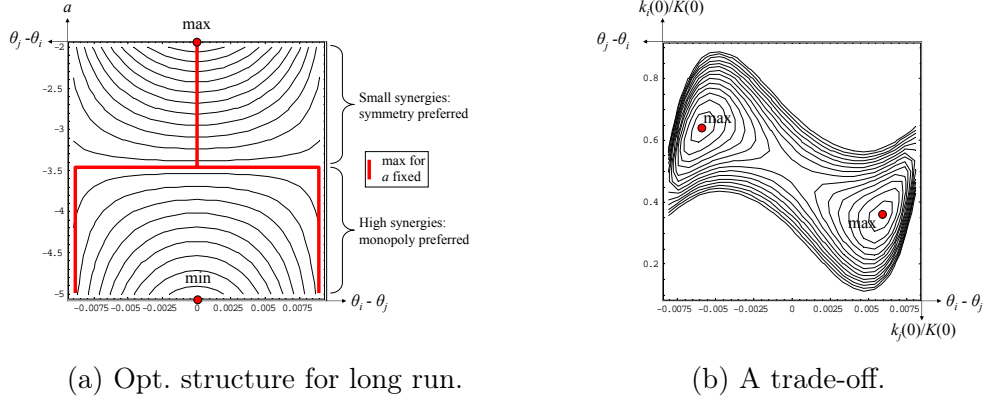
(a) Opt. structure for long run.   (b) A trade-off.

FIGURE 6. Total surplus with synergies.

We find

$$(21) \qquad \frac{\partial \pi^*_{\text{sym}}(\Theta)}{\partial \Theta} = \frac{\delta\left(2\delta^3 + 6\delta^2\rho + \delta\left(\Theta + 4\rho^2\right) - 2\Theta\rho\right)}{(3\theta + 2(\rho+\delta)\delta)^3} \cdot A^2 \qquad \lesseqgtr 0,$$

$$(22) \qquad \frac{\partial \pi^*_{\text{sym}}(\Theta)}{\partial \delta} = -\frac{2\Theta\left(2\delta^3 + 6\delta^2\rho + \delta\left(\Theta + 2\rho^2\right) - \Theta\rho\right)}{(3\Theta + 2(\rho+\delta)\delta)^3} \cdot A^2 \qquad \lesseqgtr 0.$$

To simplify the discussion, let's assume that Θ is "large".

The sign of $\frac{\partial \pi^*_{\text{sym}}(\Theta)}{\partial \Theta}$ is given approximately by $\delta - 2\rho$. Very patient (impatient) players gain (lose) if their investment costs decrease: they see (they don't see) the durable impact of investment on profits and thus tend (not) to restrict themselves.

The sign of $\frac{\partial \pi^*_{\text{sym}}(\Theta)}{\partial \delta}$ is given approximately $\rho - \delta$. Very patient players suffer more from higher depreciation rate, which is the intuitive result (investment cost raises and commitment through investment regresses). For very impatient players, a higher depreciation rate is beneficial as it diminishes equilibrium capacities, a discipline they are not able to impose on themselves. They benefit from a higher depreciation rate mostly because they are discouraged from investing (direct effect).

This comparative statics shows that patient players react less aggressively to cost reductions: they prefer to benefit from cheaper capital which reduces their bill, rather than increase capacity by too much, which would undermine profitability. In contrast, impatient players can't resist the temptation to grow. In both comparisons above, they may benefit from higher costs.

Extension 2. A has been treated up to now as a constant. In fact, it can be replaced by $A(t)$, an arbitrary function of time without changing the algebra. More precisely, in the expressions in which A appears, it can be revised by replacing A with $A(t)$. The conditions for full utilization

of capacity may become complex, but the case $A(t) = A \exp(gt)$, with $g > 0$ a growth rate, could be used to avoid this problem.

9. CONCLUSION

The dynamic oligopoly model presented in this paper makes a case for affirmative action in competition policy. The results have to be examined with care, however, since misunderstanding the effects would lead to suboptimal decisions.

The first (obvious) result is that firms do better if competition is minimized, namely, if one firm gathers all opportunities and capacity at the starting point, and quickly builds and sustains a monopoly position in the long run.

The second intuitive results is that, if investment costs cannot be set equal, as far as consumer surplus is concerned, symmetric allocation of initial capacity is no longer optimal: firms with less investment opportunities (higher investment cost) should be compensated (or allocated) with more capacities at the initial stage. That is what we called affirmative action.

The third result shows that these two kinds of logic compensate each other when the social surplus is considered. If competition authorities or regulators are free to set investment cost and initial capacity, symmetry maximizes social surplus. By keeping firms in equal positions, the competition authority or the regulator avoids creating quasi-monopoly or quasi-Stackelberg situations, which would be detrimental to consumers.

The analysis above also directly uncovers the important issue of consistency of competition policy. Assume the competition authority can intervene frequently but is limited, every time it acts, in its ability to reshuffle assets. Once the short run benefits of asymmetry are reaped, it will wish to restructure the industry again to get another crop of short run benefits. This may be feasible because the constraint that prevented full equalization in the first place is likely to relax over time. However, the rational anticipation of such discretionary interventions by firms would perturb and invalidate the notion of equilibrium we have studied. Whether it is preferable to commit not to restructure again is an open question.

Future research along these lines might consider other aspects of competition policy. One example is planned capacity transfer from one

firm to the other at dates posterior to the first reform. They might be associated with monetary transfer (ceding conditions). For example in 2001, the European Commission urged the French energy giant, Electricité de France (EDF), to sell part of its capacities through auction as an EDF/EnBW merger remedy.[15] These capacities are called virtual power plants, which are a form of financial instead of physical divestiture as analyzed in this paper. It is worthwhile to investigate whether the welfare effect of affirmative action still hold when the timing of the game is changed and when financial transactions are involved.

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[15]http://ec.europa.eu/comm/competition/mergers/cases/decisions/m1853_en.pdf

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Appendix A. Proofs.

A.1. **Constraint on investment costs.** Investment is represented by the function $z_i(\theta)$; the corresponding cost for site θ is $\gamma(\theta)z_i(\theta) + \frac{z_i(\theta)^2}{2\theta}$.

Let's define

$$(23) \qquad \langle f(\theta), g(\theta) \rangle \equiv \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)g(\theta)d\theta, \qquad \forall g, f.$$

Therefore, $C_i(I)$ as defined in Section 2 solves the following program

$$(24) \qquad C_i(I) = \min_{z_i} \left\langle \omega_i(\theta), \gamma(\theta)z_i(\theta) + \frac{z_i(\theta)^2}{2\theta} \right\rangle,$$

$$\text{s.t. } I = \langle \omega_i(\theta), z_i(\theta) \rangle.$$

The first order condition gives (λ is the Lagrange multiplier)

$$(25) \qquad z_i(\theta) = (\lambda - \gamma(\theta))\theta, \qquad \forall \theta.$$

The relationship between λ and I can now be calculated:

$$(26) \qquad \lambda = \frac{I + \langle \omega_i(\theta), \gamma(\theta)\theta \rangle}{\langle \omega_i(\theta), \theta \rangle}.$$

We can now express firm i's investment cost

$$(27) \quad C_i(I) = \frac{\langle \omega_i(\theta), \gamma(\theta)\theta \rangle^2 - \langle \omega_i(\theta), \theta \rangle \langle \omega_i(\theta), \gamma(\theta)^2\theta \rangle}{2\langle \omega_i(\theta), \theta \rangle} + \frac{\langle \omega_i(\theta), \gamma(\theta)\theta \rangle}{\langle \omega_i(\theta), \theta \rangle}I + \frac{1}{2\langle \omega_i(\theta), \theta \rangle}I^2.$$

The (constant) first term γ_i^0 equals 0 if $\gamma(\cdot)$ is constant, as we assume in the text for policy analysis. We can identify directly γ_i and θ_i:

$$(28) \qquad \gamma_i = \frac{\langle \omega_i(\theta), \gamma(\theta)\theta \rangle}{\langle \omega_i(\theta), \theta \rangle},$$

$$(29) \qquad \theta_i = \langle \omega_i(\theta), \theta \rangle.$$

Given that $\omega_i(\theta) + \omega_j(\theta) = h(\theta)$ (all sites are allocated), we have $\langle \omega_i(\theta), \cdot \rangle + \langle \omega_j(\theta), \cdot \rangle = \langle h(\theta), \cdot \rangle$, therefore

$$(30) \qquad \theta_i + \theta_j = \Theta = \text{Constant},$$

$$(31) \qquad \theta_i\gamma_i + \theta_j\gamma_j = \Gamma = \text{Constant},$$

with $\Theta = \langle h(\theta), \theta \rangle$ and $\Gamma = \langle h(\theta), \gamma(\theta)\theta \rangle$.

A.2. **Fundamental dynamic equations.** Using the inverse demand function, we can write the Hamiltonian function of firm i as

$$(32)$$

$$H_i(I, \alpha, k) = (A - \alpha_i k_i - \alpha_j k_j - c_i)\alpha_i k_i - \gamma_i I_i - \frac{I_i^2}{2\theta_i} + \lambda_i(I_i - \delta_i k_i) - \mu_i\alpha_i.$$

where λ_i is the co-state variable associated with k_i and μ_i is the Lagrange multiplier of the constraint forcing capacity utilization not to exceed 1.

The first order conditions are

(33) $$\frac{\partial H_i}{\partial \alpha_i} = 0,$$

(34) $$\frac{\partial H_i}{\partial I_i} = 0,$$

and the adjoint equation is

(35) $$-\frac{\partial H_i}{\partial k_i} = \overset{\bullet}{\lambda}_i - \rho_i \lambda_i.$$

Consequently,

(36) $$k_i \left(A - 2\alpha_i k_i - \alpha_j k_j - c_i \right) = \mu_i,$$

(37) $$-\gamma_i - I_i/\theta_i + \lambda_i = 0,$$

(38) $$\delta_i \lambda_i - \alpha_i \left(A - 2\alpha_i k_i - \alpha_j k_j - c_i \right) = \overset{\bullet}{\lambda}_i - \rho_i \lambda_i.$$

From (37), we can derive

(39) $$\overset{\bullet}{\lambda}_i = \overset{\bullet}{I}_i/\theta_i.$$

Plugging these results into (38), we get equation (9) in the text.

A.3. **Trajectories.** Let's define two functions of time $h_i \equiv \overset{\bullet}{k}_i$ and $h_j \equiv \overset{\bullet}{k}_j$. Let's denote $A - c_i - (\rho_i + \delta_i)\gamma_i$ by A_i.

We rewrite the linear second-order system of equations as a four-dimensional first-order system:

(40) $$\overset{\bullet}{H} = MH - N,$$

where $H = (k_i, k_j, h_i, h_j)^T$, $N = (0, 0, A_i \theta_i, A_j \theta_j)^T$ and

(41) $$M = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2\theta_i + (\rho_i + \delta_i)\delta_i & \theta_i & -\delta_i & 0 \\ \theta_j & 2\theta_j + (\rho_j + \delta_j)\delta_j & 0 & -\delta_j \end{pmatrix}.$$

We can now state:

Proposition 6. *In the regime where capacities are fully utilized, capacities, as functions of time, have the form*

(42) $$k_i(t) = k_i^* + k_i^{(1)} e^{\lambda_1 t} + k_i^{(2)} e^{\lambda_2 t}$$

where λ_1 and λ_2 are the two strictly negative eigenvalues of matrix M.

These six parameters $k_i^*, k_i^{(1)}$ and $k_i^{(2)}$ (there are two firms) are such that

(1) *k_i^* and k_j^* are uniquely defined as the particular solution $M^{-1}N$.*

(2) $(k_i^{(m)}, k_j^{(m)})$ with $m = 1, 2$ are the 1st and 2nd coordinate of an eigenvector of M associated with eigenvalue λ_m. This fixes the ratio between $k_i^{(m)}$ and $k_j^{(m)}$.

(3) If $k_i(0)$ is firm i's initial capacity, $k_i(0) = k_i^* + k_i^{(1)} + k_i^{(2)}$.

Proof. The eigenvalues of M are denoted by λ_s with $s = 1, 2, 3, 4$. At least one of them is negative since $\text{Tr}[M] = -(\delta_i + \delta_j) < 0$. In fact,

$$(43) \qquad \text{Det}[M] = (2\theta_i + \delta_i(\delta_i + \rho_i))(2\theta_j + \delta_j(\delta_j + \rho_j)) - \theta_i\theta_j > 0,$$

meaning that there is an even number (namely 2 or 4) of negative eigenvalues. Moreover, the coefficient of the 2nd order in the characteristic polynomial $\text{Det}[M - \lambda I]$ is

$$(44)$$
$$\frac{(-1)^2}{2!} \cdot \sum_{\substack{s,s'=\{1,2,3,4\} \\ s \neq s'}} \lambda_s \lambda_{s'} = -2(\theta_i + \theta_j) - (\delta_i^2 + \delta_j^2 - \delta_i\delta_j + \rho_i\delta_i + \rho_j\delta_j) < 0,$$

meaning that eigenvalues can't be all negative. We conclude that there are two negative eigenvalues (noted λ_1 and λ_2) and two positive ones (noted λ_3 and λ_4). The weights given to diverging exponentials must be null (otherwise capacity diverges to $\pm\infty$ as $t \to +\infty$). $\qquad\square$

A.4. **Proof of Proposition 2.** We have

$$(45)$$
$$\pi_i^* = \left[\frac{\left(1 + \frac{(\rho+\delta)\delta}{\theta_j}\right)\left(2 + \frac{(2\rho+\delta)\delta}{\theta_i}\right)}{2\left(2 + \frac{(\rho+\delta)\delta}{\theta_i}\right)\left(2 + \frac{(\rho+\delta)\delta}{\theta_j}\right) - 1} \cdot \overline{A} \right]^2 + \gamma\rho \frac{\left(1 + \frac{(\rho+\delta)\delta}{\theta_j}\right)}{\left(2 + \frac{(\rho+\delta)\delta}{\theta_i}\right)\left(2 + \frac{(\rho+\delta)\delta}{\theta_j}\right) - 1} \cdot \overline{A}.$$

As $\overline{A}$ doesn't depend on θ_i nor θ_j, straightforward calculations show that

$$(46) \qquad\qquad \frac{\partial \pi_i^*}{\partial \theta_i} > 0 \text{ and } \frac{\partial \pi_i^*}{\partial \theta_j} < 0.$$

A.5. **Proof of Proposition 3.** Total long-run capacity is

$$(47) \qquad\qquad k_i^* + k_j^* = \frac{\left(2 + \frac{(\rho+\delta)\delta}{\theta_i} + \frac{(\rho+\delta)\delta}{\theta_j}\right)}{\left(2 + \frac{(\rho+\delta)\delta}{\theta_i}\right)\left(2 + \frac{(\rho+\delta)\delta}{\theta_j}\right) - 1} \cdot \overline{A}.$$

Variations with respect to θ_i can be analyzed directly. The derivative changes sign only once from positive to negative at $\theta_i = \Theta/2$.

The second point is proved with the help of the formal calculator *Mathematica*. The derivative of the total long run surplus has three roots. One is $\Theta/2$; the other two are symmetric with respect to $\Theta/2$ and one of them is negative (the expression takes several lines). These conditions guarantee that the surplus has a unique extremum (at $\Theta/2$)

in $[0, \Theta]$ when θ_i varies. The second-order condition at $\Theta/2$ is easy to verify as symmetry simplifies the expression.

A.6. **Proof of Proposition 4.** The variations of the eigenvalues only depend on the variations of $\theta_i^2 + \theta_j^2 - \theta_i\theta_j$. Along the efficiency frontier, $\theta_i + \theta_j = \Theta$, thus we have to analyse $\theta_i^2 + \theta_j^2 - \theta_i\theta_j = \Theta^2 - 3\theta_i(\Theta - \theta_i)$.

We find that $|\lambda_1|$ increases and $|\lambda_2|$ decreases as the situation becomes more symmetric. The consequence is that, for more symmetric distributions of sites, growth of total capacity (related to λ_2) is slower whereas the difference reduction (related to λ_1) is faster.

A.7. **Proof of Proposition 5.** It is obvious from the analysis of the steady state (i.e. the particular solution to the differential system) that the total capacities at date 0 and in the long run are independent of the initial allocation rule $(k_i(0), k_j(0))$. At date 0, the slope of the total capacity is denoted by ξ with

$$(48) \qquad \xi = \left.\frac{\partial K(t)}{\partial t}\right|_{t=0} = \lambda_1(k_i^{(1)} + k_j^{(1)}) + \lambda_2(k_i^{(2)} + k_j^{(2)}).$$

Calculations of the eigenvectors and eigenvalues plus utilization of Proposition 6 (point 2) gives us the effect of $k_i(0)$ on the slope ξ

$$(49) \qquad \frac{\partial \xi}{\partial k_i(0)} = -\frac{(\theta_i - \theta_j)(\lambda_1 - \lambda_2)}{2\sqrt{\theta_i^2 - \theta_i\theta_j + \theta_j^2}} < 0.$$

The total capacity increases more at date 0 when more of the initial capacity is given to the efficient firm.

If we look at the total capacity $K(t)$ at any date $t > 0$, the derivative of slope $\xi(t)$ with respect to $k_i(0)$ will be

$$(50) \qquad \frac{\partial \xi(t)}{\partial k_i(0)} = -\frac{(\theta_i - \theta_j)(e^{\lambda_1 t} - e^{\lambda_2 t})}{2\sqrt{\theta_i^2 - \theta_i\theta_j + \theta_j^2}} < 0,$$

which proves the proposition.

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