Dynamic model of procrastination

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Abstract

Procrastination is the notorious tendency to postpone work for tomorrow. This paper presents a formal model of procrastination based on expectations and prospect theory, which differs signficantly from the prevalent model of O'Donoghue and Rabin. Subject is assumed to work on a task for distant reward which depends on the number of periods actually spent working, where the subject faces varying opportunity costs of working each period before the deadline. In order to assess a hypothesis that procrastination is an evolved and stable habit, the model is rendered dynamic in that past decisions and circumstances affect the present. The model is first explored via qualitative analysis and simulations are performed to further reveal its functionality.

keywords: procrastination, dynamic inconsistency, intertemporal choice, prospect theory, hyperbolic discounting, expectations

JEL Classification: D030, D810, D830, D840, D900, J220

1 Introduction

Postponing work on our tasks and duties is a pervasive phenomenon. Everybody sometimes procrastinates, to some extent or in some circumstances, and later regrets it. The phenomenon is so common that it had not come to theoretic discussion until recently, though it often affects our daily lives considerably. For example, O'Donoghue and Rabin [4] show how even a mild tendency to procrastination may lead to great losses in retirement savings because the subject always plans to take the necessary (and costly) time and choose the best savings plan tomorrow. Many times will we miss an event we would really like to visit (say, an exhibition in a gallery) only because each day we feel that tomorrow's opportunity costs of visiting will be lower than today. People lose large amount of money paying extra fees for lately balanced credit cards, lately paid bills, or lately finished studies. People often rather live in discomfort, say because of the proverbial leaking tap, putting off the fix that would one day turn out to be necessary anyway.

While 'procrastination' literally means putting something off to the next day, the word's connotation is nowadays negative, indicating that is an undesirable behavioural pattern. Most of the definitions provided by psychologists emphasize inefficiency and irrationality of procrastination.[1] From economic perspective, the distinctive feature of procrastination is dynamic inconsistency: my behaviour can be unambigously described as procrastination only if I postpone work which I previously intended to do now.[2] Individual differences in how much or how late people work may be due to different preferences over various goods where the actual outcome is still a result of rational utility maximization. So if somebody decides right from the beginning to do taxes the night before they are due, we may reckon her lazy or reckless; but being ignorant of the reason, we cannot conclude it is a case of procrastination - only if the decision turns out to be time inconsistent.

The prevalent model of procrastination, developed by O'Donoghue and Rabin in a series of articles ([4],[5],[6],[7]) captures its essence by assuming that the subject puts extra salience to present consumption over any future one. The special functional form of subject's intertemporal preferences used in their model is quasi-hyperbolic discounting, defined as:

$$U^t = u(c_t) + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(c_{t+\tau})$$

[1] [1][chpt. 1]

[2] Assuming that the change in one's intentions have not been caused by changes to subject's decision problem, e.g. having more information.

While δ represents discount factor in classic exponential discounting, β represents the special bias for present instantaneous utility. O'Donoghue and Rabin call these preferences present-biased.[3] The per-period discount rate between now and the next period is $(1-\beta\delta)/\beta\delta$ and the per-period discount rate between any two future periods is $(1-\delta)/\delta$. Note that while exponential discounting can explain why people tend to *postpone* work to later periods, it implies that the decision is consistent in time. To see how present-biased preferences can give rise to procrastination, consider the following example from [5, p. 110]. Suppose a subject has to do a task by period 4, receives an immediate reward v and the opportunity cost scheme she faces is $\mathbf{c} = (3, 5, 8, 13)$. Time-inconsistent subject has $\beta = 0.5$ and both subjects have the standard discount factor equal to 1. Time-consistent subject therefore simply maximizes $v - c_t$ and consequently finishes the task at the first period. On the other hand, procrastinator perceives her future costs discounted by the parameter β, so her perception at period 1 can be described as: $E^1(c) = (3, 2.5, 4, 6.5)$. She therefore enjoys the opportunity, thinking that she will do the task tomorrow (in period 2). However, at period 2, she procrastinates again, because $E^2(c) = (-, 5, 4, 6.5)$, and so she does at the third period, leaving the work to period 4 where the opportunity is the highest.

Parameter β in O'Donoghue's and Rabin's model comprises many possible causes of procrastination that psychologists have recognized.[4] While some of the causes are related to psychological concepts that do not invite direct economic interpretation, such as anxiety, and hence are justifiably subsumed under a single parameter, there are other causes of procrastination which can be interpreted in terms of economic theory. Among the latter causes belong underestimation of the time necessary for succesful completion of the task and poor time management skill.[5] This paper presents an expectations-based model of these two causes and explores their effect on procrastination. Furthermore, a question whether procrastination could be a self-reinforcing, stable habit is explored by introducing prospect theory which renders the model dynamic. The intuition behind the question, backed by some psychological findings,[6] is that procrastination can evolve into a habit that is not easily broken.

The paper proceeds in three steps. First, the formal model is specified. Second, working of the model is explored via simulations, because the modeled decision problem is discrete and complex, which precludes use of the

[3] [5]

[4] For a concise overview, see [8].

[5] Cf. [1, chpt. 2]

[6] See [1, p. 13].

standard tools of mathematical analysis. Third, the model is finally assessed and implications are derived.

2 Model specification

This section presents a model of decision making concerning a trade-off between future reward and immediate costs. Unlike most models of intertemporal choice studying such trade-off, the model that follows assumes that a subject decides not only whether to do the task or not but how much effort to invest and when, where the effort is measured by the number of periods spent working. This describes a situation in which procrastination is most likely: working on a long-term project the reward for which depends on the effort invested into it, where the distribution of work is often discontinuous. Writing a dissertation is perhaps the most inviting example.

2.1 Assumptions

Suppose a subject is assigned a task to be done by period T. The subject may put various effort to the task. To finish the task at a passing level, the subject must spend at least m periods working on it. Let $n > m$ denote the maximum efficient effort (in terms of periods) that the subject can invest in the task. Let $r(x)$ be a reward schedule that is a non-decreasing function of number of periods spent working on the task (x). To incorporate subject's decision whether to do the task at all into the model, let $r(x) \leq 0$ for all $x < m$, i.e. the subject receives penalty if she does not complete the task which takes at least m periods to do. Otherwise the subject receives the reward $r(x)$ at period T.

Suppose next that the subject faces alternative occupation each period which therefore represents opportunity costs (OC) of working on the task. OC vary randomly by normal distribution from period to period, unbeknownst to the subject who may not have perfect information about future OC and knows perfectly only OC of the current period. Let $c(t)$ denote the schedule of opportunity costs that the subject is to face during the time she has for completing the task.[7]

Each period the subject can take only either of two actions: work on the task or enjoy the alternative represented by OC. Let $A \subset \mathbb{N} \times \{0; 1\}$ denote schedule of actions taken by the subject each period, where $A(t) = 1$ signifies that the subject works on the task in period t. Assume further that if the

[7]Note that OC may generally represent not only utility of leisure but expected utility of any other occupation, including working on some other task.

subject works at period t or receives a reward at period T, it is the only thing in that period that affects her utility. Thanks to this assumption, we can interpret the opportunity costs and reward directly in terms of time-separable utility and need not assume a specific form for the utility function.

2.2 Decision problem

2.2.1 Time-consistent subject with perfect information

Let's turn now to subject's decision making. Consider first a time-consistent subject with perfect information about all key determinants of the problem - the minimal time m, the maximal efficient time n, reward schedule $r(x)$ and OC schedule $c(t)$.

Suppose the subject is characterized by a discount factor $\delta \in (0;1]$. Let $U^t = (u_t; u_{t+1}; ...u_T)$ represent subject's intertemporal preference (in period t) over instantaneous utilities in future periods. Using standard exponential discounting: $U^t = \sum_{\tau=t}^{T} \delta^{\tau-t} u_\tau$. Let instantaneous utility u_τ be $c(\tau)$ if $A(\tau) = 0$, i.e. the subject enjoys her opportunity costs c in period τ if she does not work on the task; and $u_\tau = 0$ if $A(\tau) = 1$, for simplicity.[8] At period T the subject receives the reward depending on the effort put, hence $u_T = r(x)$. To make things simple, terms of the reward schedule are in the form of instantaneous utility. Thus all the complexities associated with subjective valuation of an objective reward, decreasing marginal utility etc. are delegated to reward schedule which is exogenous to the model.

Since the subject has time-consistent preferences and perfect information, her optimal solution to the decision problem will appear to be the same in every period (i.e. her action schedule A that maximizes U^t will be identical for all $t \in [1;\ldots;T]$). So the solution of the decision problem by a time-consistent subject with perfect information can be generally described as:

$$A_{TC} = \arg\max_{A_{TC}} U^1(\delta, u_1(A_{TC}, c, r), \ldots, u_T(A_{TC}, c, r)) \tag{1}$$

For the specific functional form with exponential discounting, the solution takes the following specific form:

$$A_{TC} = \arg\max_{A_{TC}} \left[\delta^{T-1} r \left(\sum_{i=1}^{T-1} A_{TC}(i) \right) + \sum_{t=1}^{T-1} \delta^{t-1} c_t [1 - A_{TC}(t)] \right] \tag{2}$$

[8]The model can be easily extended with additional cost schedule signifying possible negative utility associated with working on the task. It should be noted that this negative, task-related utility cannot be completely covered in the reward schedule because of the time gap between reaping the reward and toiling for tomorrow.

The subject looks for such a schedule of actions that maximizes the difference between the corresponding reward (the amount of periods worked, denoted as ones in the action vector A, is thus the sum of all scalars in the vector) and sum of instantaneous utilities prior the deadline weighted by the discount factor and nullified by the action vector in case the subject works in the respective period.

The situation with time-consistent subject is clear: she will work in periods with lowest instantaneous utility (weighted by the discount factor) and only if the reward for the task done exceeds the opportunity costs. The discount factor implies that the subject will, *ceteris paribus*, postpone the work to future periods. Clearly, if the cost schedule $c(t)$ were a constant function, the subject would work on the task in last periods. Solution of the time-consistent subject with perfect information will serve as a benchmark for comparing solutions under different settings where the subject does not have perfect information or/and is time inconsistent.

2.2.2 Time inconsistent subject

If the subject is time inconsistent, the maximizing action schedule may differ from period to period as subject's inconsistent perception of costs and benefits of working may change each period. Since the subject can only decide whether to work in the present period and cannot commit herself to future action, she works in a period t only if it seems to be optimal from t-perspective. Thus the solution can be defined recursively as:[9]

$$\begin{gathered} A(t) \equiv A_t(t), \text{ where} \\ A_t \equiv \arg\max_{A_t} U^t(\beta, \delta, u_t(A_t, F_t(c), E_t(r)), \dots u_T(A_t, F_t(c), E_t(r))) \qquad (3) \\ \text{on condition that for all } \tau < t{:}A_t(\tau) = A(\tau) \end{gathered}$$

The last condition means that the subject knows how much she has progressed with the task and thus her decision takes into account only the remaining part of cost and reward schedule. Parameter β signifies time preference as in the quasi-hyperbolic discounting model used by O'Donoghue and Rabin. Transformations $E_t(r)$, $F_t(c)$, specified below, represent the cognitive distortions recognized as possible causes of procrastination.

Adopting quasi-hyperbolic discounting, the specific form of the general equation 3 takes this form:

[9]The equation describes also the case of a time-consistent agent if $\beta = 1$ and all transformations F_t, E_t are identities.

$$A_t = \arg\max_{A_t} \left[\beta\delta^{T-t} r \left(\sum_{i=1}^{T-1} A_t(i) \right) + u_t + \beta \sum_{\tau=1}^{T-t} \delta^{\tau} u_{\tau} \right],\ u_t = [1 - A_t(t)]c(t) \tag{4}$$

The unique solution (A) can be represented as the diagonal of an action matrix each row of which is generated by the subject's solution of the subproblem whether to work under *current* perception of costs and benefits (A_t). The diagonal of the action matrix is then the schedule of actions actually taken by the subject (A) and corresponds to what [5] coined as "perception-perfect strategy."

Here is a simple example in which the subject procrastinates until the last period and works only one period although the original intention was to work for two periods and right after the first period.

$$\begin{pmatrix} A_1(1) & A_1(2) & A_1(3) & A_1(4) \\ A_2(1) & A_2(2) & A_2(3) & A_2(4) \\ A_3(1) & A_3(2) & A_3(3) & A_3(4) \\ A_4(1) & A_4(2) & A_4(3) & A_4(4) \end{pmatrix} = \begin{pmatrix} \mathbf{0} & 1 & 1 & 0 \\ 0 & \mathbf{0} & 1 & 1 \\ 0 & 0 & \mathbf{0} & 1 \\ 0 & 0 & 0 & \mathbf{1} \end{pmatrix}$$

2.3 Decision problem under cognitive distortions

2.3.1 Imperfect foresight of opportunity costs

Perfect knowledge of opportunity costs in future periods is an ideal state that the subject is unlikely to attain. Poor time management skill can be interpreted as imprecise foresight of future opportunity costs. Suppose therefore that the subject foresees distant opportunity costs with ever decreasing precision and basically predicts mean costs $\overline{c}$. Let subject's expectation (at period t) of opportunity costs in future period τ be:

$$E_t(c(\tau)) = \epsilon^{\tau - t}(c(\tau) - \overline{c}) + \overline{c} \tag{5}$$

$\epsilon \in [0; 1]$ characterizes subject's precision, the higher the better.

Figure 1 illustrates the effects of equation 5 on the perceived cost schedule.

Subject's decision procedure now gets more complicated. The subject calculates, as it were, her intertemporal preference U^t in every period, discounting her *foreseen* opportunity costs. In every period she checks whether from that perspective she is currently in a period with x lowest OC (weighted by the discount factor), where x is the number of periods that seems to be optimal to spend working. If so, the subject works in that period and repeats the same procedure next period.

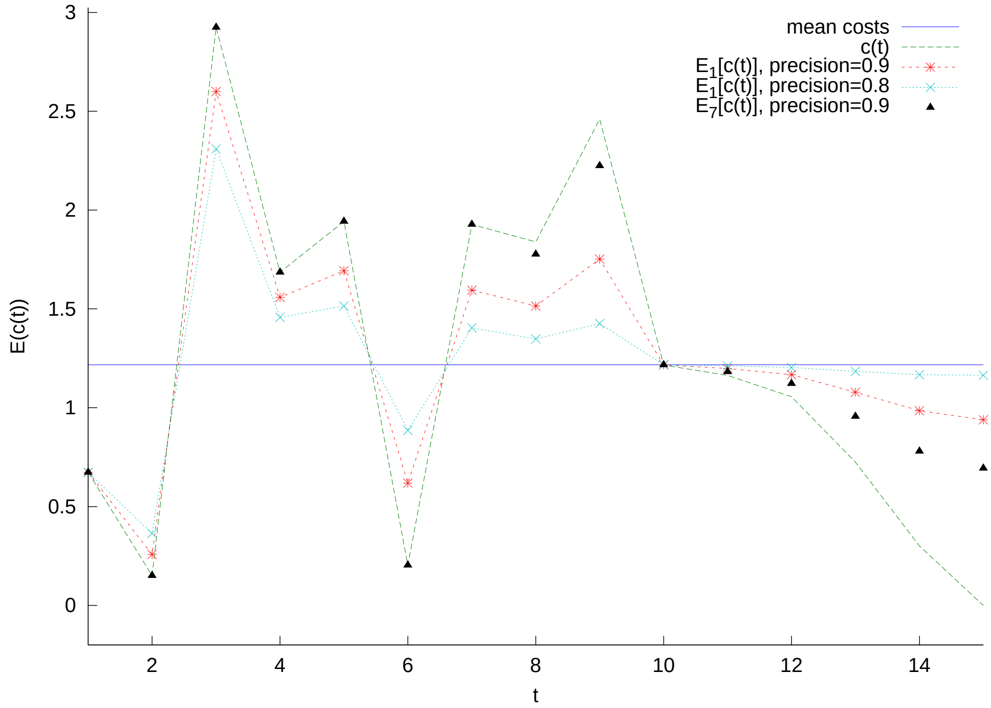

Figure 1: Different transformations of the cost schedule depending on subject's precision and the present period from which expectations are made. The cost schedule is randomly generated using the normal distribution with $\sigma^2 = 1$.

Imperfect foresight may unsurprisingly lead to different and time-incositent decision, depending on subject's precision as well as the cost schedule. Consider a setting in which the opportunity costs in last x periods are quite high while the periods before they are moderate or low. Subject's myopic foresight may make her think that the last periods will present moderate opportunity costs and she may not realize their significance until it is too late. In the extreme case, she could in the end realize it is not worth doing the task anymore because of the high OC she would have to give up ("I will rather fail at exams than miss this concert!"). On the contrary, if costs are very low in last periods and below average at the beginning, a conscientious subject may do the task earlier if she foresees the last periods as moderate.

2.3.2 Optimism about future workload

Among the cognitive distortions that procrastinators exhibit most often is the belief that the task will take less time to complete than it actually does.

This optimistic view about the necessary workload can be modeled as a transformation of the actual reward schedule $E[r(t)]$ that satisfies the following conditions:

1. $\max_t E(r(t)) = \max_t r(t)$, $\min_t E(r(t)) = \min_t r(t)$ The subject knows the penalty for not finishing the task and also the maximal achievable reward. She is only mistaken about the time it takes to do the task at a target quality.

2. $E(r(t)) \geq r(t)$ The equality holds only for t such that $E(r(t)) = \min_t r(t)$ or $\max_t r(t)$. For a pessimistic subject, the inequality would be reversed.

A simplified[10] functional form of such transformation is:

$$E(r(t)) = r(t/\omega) \tag{6}$$

where $\omega \in (0;1]$ defines subject's optimism - the lower value, the more optimistic the subject is. For $\omega = 1$ the subject's perception of the reward schedule is not distorted. The ω parameter causes the active part of the reward schedule (i.e. for all $t < n$) to "shrink" in its length. For illustration, see figure 2.

The figure shows the actual reward schedule and its various transformations, including a "pessimistic" one which results from setting $\omega > 1$. Pessimistic view corresponds to subject who thinks in advance that the task is much more difficult (demanding) to complete than it actually is. Pessimism about the difficulty is also listed among irrational beliefs that lead to procrastination.[11]

The figure also shows a case in which the subject has already worked for some periods and estimates the rest of the reward schedule. It is assumed that the subject correctly recognizes the progress she has made and thus her optimism affects expectations of future workload only, therefore the more work the subject has done, the closer her estimate is to the actual reward schedule, *ceteris paribus*. Although it would be rational to alter one's optimism after noticing that the progress is not so fast as expected, optimism

[10] Since r is a vector, the index t/ω must be a natural number. Therefore, strictly speaking, only the integer part of t/ω should appear at the right-hand side of the equation. In the simulation, the integer part is taken and the corresponding value is adjusted for what was left out by adding the corresponding part of the mean of two neighbouring values, i.e. $(t/\omega - [t/\omega])(r([t/\omega]+1) - r([t/\omega]))/2)$, where $[x]$ is a function returning the integer part of x.

[11] See discussion on self-efficacy in[8].

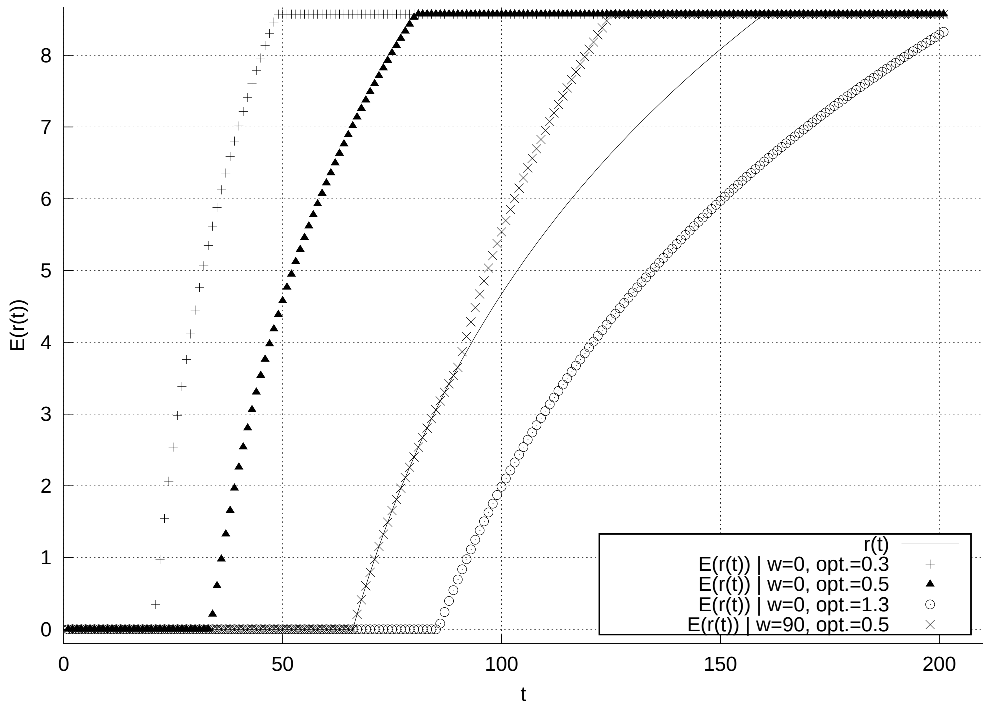

Figure 2: Different transformations of the reward schedule, including a pessimistic one. 'opt.' is the optimism parameter; 'w' denotes the number of periods already spent working on the task.

as a cause procrastination is an enduring cognitive distortion, hence the ω parameter is assumed fixed for all periods.

Optimism about the future workload can cause dynamic inconsistency simply because subject's perception of the reward schedule changes hand in hand with her progress. Optimistic subject will *ceteris paribus* start working on the task later than TC because she thinks less time is needed to achieve the same reward.

2.4 Decision problem under relative valuation

Let us divert from the standard economic assumption that subjects derive their instantaneous utility from absolute levels of consumption or states of wealth. Assume instead that terms of the utility function are gains and losses relative to a reference point. This idea originates from the prospect theory put forward by [2].

Suppose therefore that intertemporal preferences are defined over outcomes of the value function $v(c - R)$, where c is the level of consumption in absolute terms and R is the reference point. This conforms to the familiar intuition that people's preferences depend on what they are used to (or more precisely, what they reckon their level, to include such phenomena as peer pressure or status quo bias among the evidence for relative valuation). As an example (used later in simulation) of a value function that meets conditions formulated by [3], I propose the following functional form. For $x = c - R$:

$$v(x) = \begin{cases} \log_a(1+kx) & \text{if} \quad x \geq 0 \\ \ln(1-kx) & \text{if} \quad x < 0 \end{cases}, a > e = 2.718\ldots \tag{7}$$

Figure 3 shows the function graphically. The base of logarithm a determines the intensity of loss aversion (the higher a/e, the more loss averse the subject) and k determines an interval of x-values in absolute terms where the value function has a reasonable linear approximation, which is important for calibration of the model later in simulation.

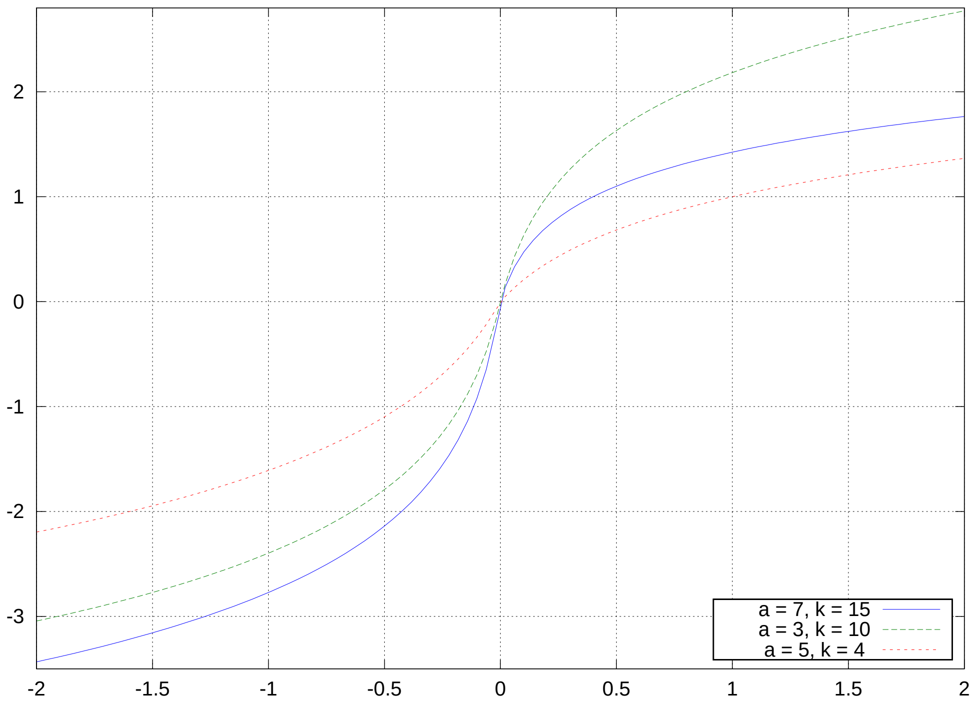

Figure 3: Examples of diferrent courses of the value function depending on paramater setting.

Since the reward is assumed to be consumed in a single period, whereas opportunities have to be given up many times to fulfill the task, two distinct reference points must be assumed - one for costs, another for reward.[12] Moreover, introduction of different reference points conforms to the intuition that procrastination may be domain specific - someone could be very conscientious in work and at the same time buy presents the day before Christmas Eve.

Let us briefly discuss comparative statics of the model with relative valuation. The most important aspect of the value function is loss aversion: the "disutility" from a loss of x is greater than "utility" from a gain of x. Consider now isolated effect of a change in reference point for reward (R_r). Suppose the subject is indifferent between working y periods for reward A and $y+1$ for reward $B > A$, her reference point being R_r, the value of working in any period is $-X$ and the value of opportunity forlorn when working more is Z. The indifference implies

$$-Xy + Z + v(A - R_r) = -X(y+1) + v(B - R_r)$$
$$Z + v(A - R_r) = -X + v(B - R_r)$$

Now, if $v'(A - R_r) > v'(B - R_r)$, then the subject will strictly prefer working more if the reference point increases and working less if the reference point decreases. Furthermore, if $A - R_r > 0$, the value function operates on the positive part where the first derivative is decreasing with x and thus $v'(A-R_r)$ will always exceed $v'(B-R_r)$. The opposite is true if $B-R_r < 0$.[13] Thus whether increasing the reference point will lead to working more or less depends on whether the reference point is below or above the considered reward (and *mutatis mutandis* for decreasing the reference point). This is an important result because it affects the dynamics of the changing reference point.

Let's turn finally to model specification. The value function v can be simply interpreted as a special transformation of cost and reward schedule introduced in the previous model. To compare with the previous model, note that were the value function a linear transformation of $c(t)$, $r(x)$, the "super-rational" TC (with $\delta = 1$) would take the same sequence of actions no matter what the reference point is, provided it is common to both costs and reward, and it would be identical to her action schedule over unaltered $c(t)$, $r(x)$.[14]

[12] If a single reference point were assumed then only a little change to the cost schedule would have considerable effect on the decision, relative to the change in reward schedule, for the sum of the value of missed opportunities is compared with the value of reward.

[13] If $A - R_r < 0 < B - R_r$ then the actual derivatives would have to be calculated.

[14] Assuming, of course, that the penalty for working at a period changes with the reference point, i.e. from 0 to $-R$.

Also note that for fixed reference points and no discounting, the choice of the subject with relative valuation is time-consistent (since nothing changes subject's valuation of past and future opportunities over time). However, introducing exponential discounting into the model with value function no longer preserves time-consistency, because the effect of discounting is then that all values approximate to the reference point - the same effect, actually, as that of imperfect foresight described in 2.3.1. Exponential discounting therefore generates dynamic inconsistency if the terms of subject's valuation are gains or losses *and* if the decision problem involves comparing long-term utilities.

Equation 3 also covers the solution of the decision problem by a subject with relative valuation (GL subject, henceforth), where transformation $F[c(t)]$ can be defined as $v[c(t)-R_c]$ and transformation $E[r(t)]$ as $v[r(t)-R_r]$. Solution to the decision problem by a GL subject can thus be defined as:

$$A(t) \equiv A_t(t), \text{ where}$$

$$A_t \equiv \arg\max_{A_t} \left[\delta^{T-t} v[r(x) - R_r] + \sum_{\tau=t}^{T-1} \delta^{\tau-1} v[c_\tau(1 - A_t(\tau)) - R_c] \right] \tag{8}$$

$$\text{on condition that for all } \tau < t\text{:} A_t(\tau) = A(\tau) \text{ , and } x = \sum_{i=1}^{T-1} A_t(i)$$

All it takes now to make the model dynamic is supplying it with equations for reference points. The intuition behind those equations is that one's domain-specific reference point is what one is used to in that particular domain. To be sure, there are countless other psychologically plausible determinants of the reference point level (ambition, for example), but this one depends straightforwardly on past actions and circumstances and thus the reference point can be modeled as an endogenous variable. To avoid further complexity, I have decided that reference points adjust only after one particular decision problem has been solved, that is, after period T. Let R_c denote the current (old) reference point for costs and R_c^+ the new adjusted reference point.

$$R_c^+ \equiv \alpha R_c + (1 - \alpha) \sum_{\tau=1}^{T-1} c_\tau [1 - A_{GL}(\tau)] \tag{9}$$

Similarly for R_r:

$$R_r^+ \equiv \alpha R_r + (1 - \alpha) r \left(\sum_{\tau=1}^{T-1} A_{GL}(\tau) \right) \tag{10}$$

Parameter $\alpha \in [0, 1]$ specifies the inertia of past reference points or, in other words, the strength of the habit - larger α implies slower adjustment to changes in the environment. Basically, the new reference point for costs is a weighted sum of the current reference point and average utility enjoyed throughout the time available for completing the task, including the periods spent working.

Let's now discuss the dynamics of the model. First, consider changes in reference point for reward. It has been derived earlier that increasing R_r when $r(x) - R_r > 0$ will induce more work, *ceteris paribus*. More work results, *ceteris paribus*, in higher achieved reward, which again raises the reference point according to equation 10. Thus R_r will keep increasing until $r(x) - R_r < 0$ where further increase would actually induce the subject to work less and consequently lower the reference point again. This reasoning shows, and was confirmed by simulations, that the system is stable - for a fixed cost schedule, reward schedule and value function, there is R_r^* such that it is the attractor to which the reference point R_r converges. Naturally, cost schedule and reward schedule differ from case to case, but stability is a theoretically important characteristic of the system.

Dynamics of the reference point for costs is more subtle because it comprises effects of many periods on the overall decision problem. Furthermore, as has been derived above, the effect of change of the reference point depends on relative distances to 0 and c_t respectively. Simulations indicate, however, that the reference point always neatly converges to a stable point.

If reference points for both costs and reward are allowed to change simultaneously, it is not obvious whether they will reach a stable point. While simulation shows that they do reach stable points, it is possible that there be more than one pair of attractors for reference points - depending on initial states.[15] Note that this question is directly related to the issue whether procrastination is a self-reinforcing, stable habit. For if there were two and more pairs of stable points, one with low reference point for reward, it would mean that procrastination is one stable pattern among many.

[15] What has been actually sometimes observed in simulations is that the attractor may consist of a stable set of reference points and corresponding action schedules. This anomaly, however, is caused due to relatively low resolution of the timeline into periods (usually 20 periods). Because of the discrete nature of the decision problem, working x periods may be too much and working $x - 1$ periods too little, for example. Thus the subject oscillates between these options and so does the reference point. Increasing the resolution to more periods resulted in lower frequency of these anomalous attractors, which supports the reasoning above.

3 Simulations and results

Since subject's behaviour is described as a result of sequence of several discrete decision problems, it cannot be specified by first-order conditions and simulations have been used to explore functionality and dynamics of the model. Each simulation generates hundreds of 'cases', i.e. pairs of cost and reward schedule, for every combination of control variables, so that the resulting means reflect only the isolated effect of control variables on subject's behaviour. Details and settings of every simulation are specified in tables at page 27.

The cost schedule is randomly generated using normal distribution with $\sigma^2 = 1$, $\mu = 0$. Unless specified otherwise, the cost schedule is increased by 1.5 of the minimal generated value so that it ranges only over positive numbers. Number of periods in a 'case' may vary depending on simulation needs, but the minimal length used is 20 periods. Figure 1 at page 8 shows one such generated cost schedule.

Reward schedule depends deterministically on the generated cost schedule and has the following properties (unless specified differently):

1. The minimal amount of work needed to fulfill the task at a passing level (i.e. m in the notation introduced in section 2) is set to 1/5 of T.
2. The maximal efficient effort n is set to 1/2 of T.
3. For $t \in (n, m)$, $r(t)$ exhibits diminishing marginal reward from effort.

Figure 2 shows one such generated cost schedule, which is nevertheless adjusted for the purpose of illustration of transformations by setting $m = T/3$ and $n = 2T/3$. The marginal rewards are generated by decreasing ratio of median costs. This ensures that subjects across all cases face similar reward scheme but different distributions (in the ordinary sense of the word) of costs.

3.1 Welfare comparisons

Welfare comparisons are always problematic in case of time-inconsistent preferances. Despite the theoretical intricacies, I have decided to follow O'Donoghue and Rabin who make welfare comparisons by means of long-run utility.[16] The long-run utility of agent X, adapted to our model, is a simple sum of the reward achieved and opportunities taken:

[16] See [5][pp. 112-116]

$$\overline{U}_X(A_X, c, r) \equiv r\left(\sum_{i=1}^{T-1} A_X(i)\right) + \sum_{i=1}^{T-1} c(i)(1 - A_X(i)) \tag{11}$$

A measure of efficiency used in the subsequent simulations is a ratio of an agent's long-run utility to the long-run utility of a time-consistent agent with perfect information and same discount factor.

$$\eta_{TC}^{X} \equiv \overline{U}_X / \overline{U}_{TC} \tag{12}$$

Note that for a discount factor lower than 1 even the time-consistent agent may not achieve the maximum possible long-run utility because her (rational) impatiance can cause her miss the unweighted minimal costs.

3.2 Model simulation

3.2.1 Comparing TCs and $\beta\delta$ procrastinators

Let us first examine working of O'Donoghue's and Rabin's model with present-biased preferences under the new setting with variable workload assumed in this paper.

First simulation compared the long run utility of a $\beta\delta$ subject to the long run utility of a TC having equal discount factor. As figure 4 shows, the simulation confirms the intuition that efficiency is an increasing function of β - the less present-biased the subject is, the closer her long-run utility is to that of a TC subject with equal discount factor. Also the lower the discount factor, the higher the marginal efficiency of β (β being equal). For better idea about how significant the relative differences are, note that mean costs compared to the long-run utility of a TC are approximately 0.04. This means that the efficiency loss in terms of missed opportunities for $\beta = 0.55$,$\delta = 0.9$, for instance, is about 1.5 average opportunity costs.

To see the direct effect on procrastination, see figure 5. Working defintion of "the rate of procrastination" is "the number of cases in which the subject either (1) works less periods than the TC subject, or (2) works the same number of periods but later (at least for one working period), all divided by the total number of cases generated for a given n-tuple of independent variables." Although working less might not be regarded as procrastination, it needs to be incorporated in the definition, because it is due to the possibility of putting various effort into the task that the same mechanism, which unambigously causes procrastination if the task is done in a single period, gives rise to two different behavioural patterns.

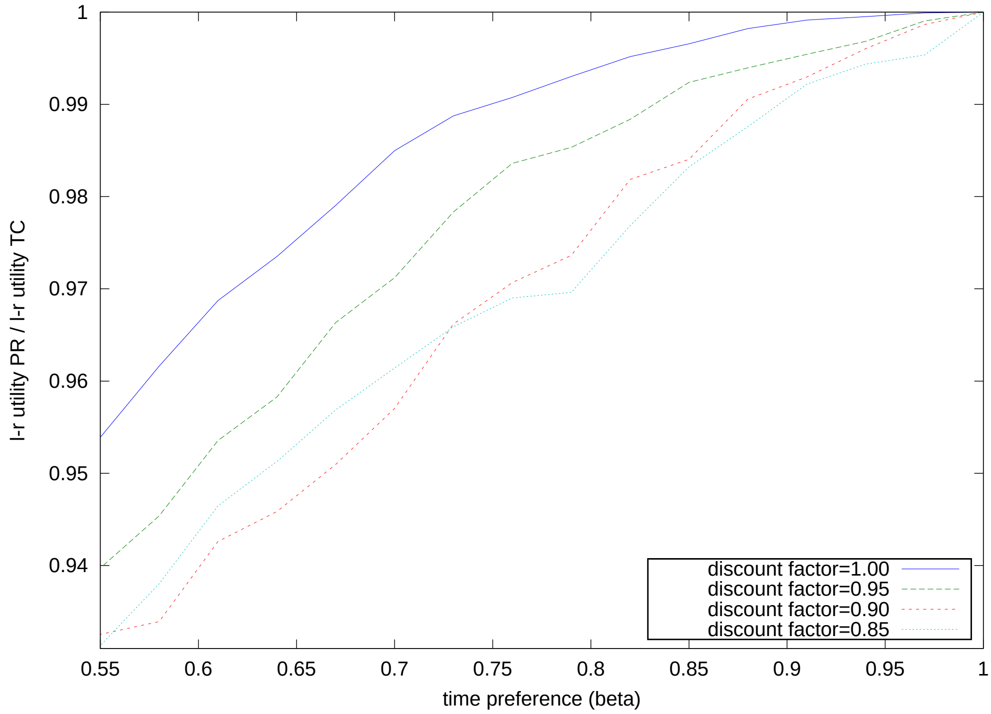

Figure 4: The influence of β the efficiency defined as the ratio of long-term utilities; for three different discount factors. (mean costs/$\overline{U}_{TC} \simeq 0.04$)

3.2.2 Comparing TCs and subjects with imperfect foresight

Imperfect foresight of future costs, specified by equation 5, can induce both procrastination and "preproperation".[17] In case the lower opportunity costs are distributed at the beginning and the higher costs at the end of the timeline, the subject with imperfect foresight mispredicts the future costs to be average and may decide to postpone her work because of exponential discounting. When she realizes that the later costs are high, the periods with lower costs have already passed and the subject either works less than TC or at greater costs. On the other hand, if the lower costs are near the end of the timeline and the high costs at the beginning, the subject decides to wait for later anyway, and even more so if the discount factor is low. Thus procrastination appears to be more likely than preproperation, *ceteris paribus*. However, since cost are randomly distributed and the length of the timeline is 20 periods, it is very unlikely that the generated cost schedule turns out to be sorted, with lower costs crammed at one side of the timeline.

[17] A term coined by O'Donoghue and Rabin in their 1999a article which means taking action before it is optimal.

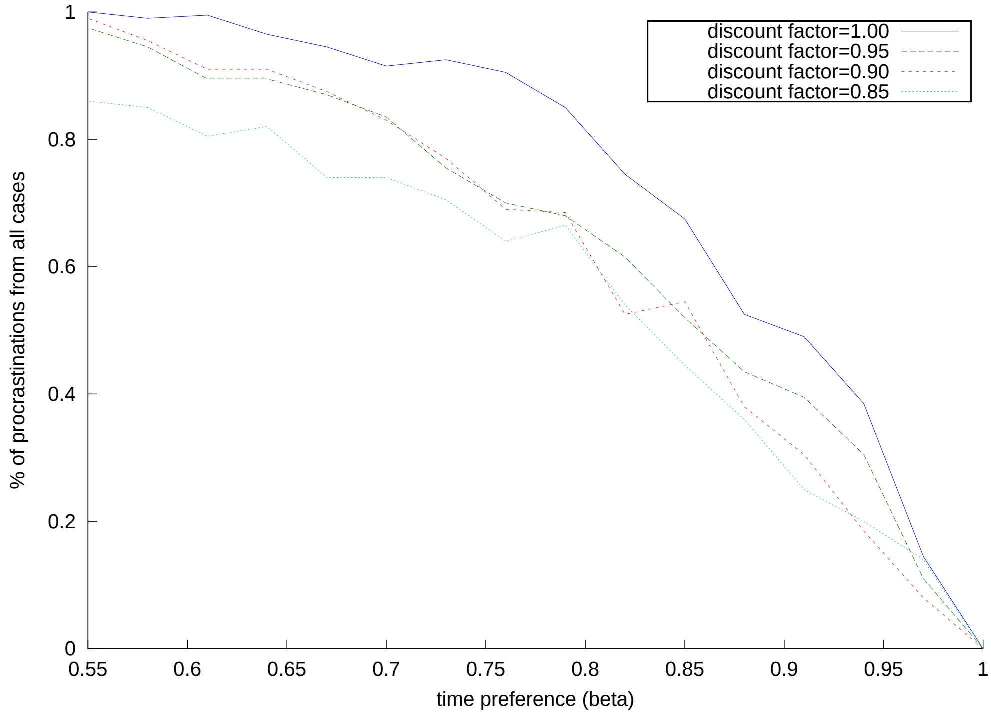

Figure 5: The influence of β on the ratio of cases in which the $\beta\delta$ subject spends less time working than TC.

Simulations produced some unexpected results. The most notable thing is that, safe for $\delta = 1$, the precision of foresight does not significantly affect relative efficiency (see fig.6), even though the foreseeing subject is not time consistent.[18] When the efficiency is measured in comparison with the superrational TC, all inefficiency seems to be mostly due to discount factor. Comparing η^F_{TC} with $\eta^F_{TC^*}$, it appears that the efficiency loss due to precision is negligible - only about 1/8 of average OC (safe for $\delta = 1$). Interestingly, the correlation between precision and η^F_{TC} is ambiguous. For $\delta = 0.95$, imperfect foresight actually slightly increases subject's long-run utility because it combats the negative effect of discounting by taking early action. However, the efficiency gain is not significant either. The lesson to be learnt from this finding is this: uncertainty about future opportunity costs may, under some circumstances, prove (long-run) utility increasing by motivating the subject to work whenever the opportunity is sufficiently low, for it is not worth risking that the unforeseen opportunities will be too high.

[18]The ratio of time-inconsistent decisions quickly reaches 100% as soon as the precision falls below 0.9.

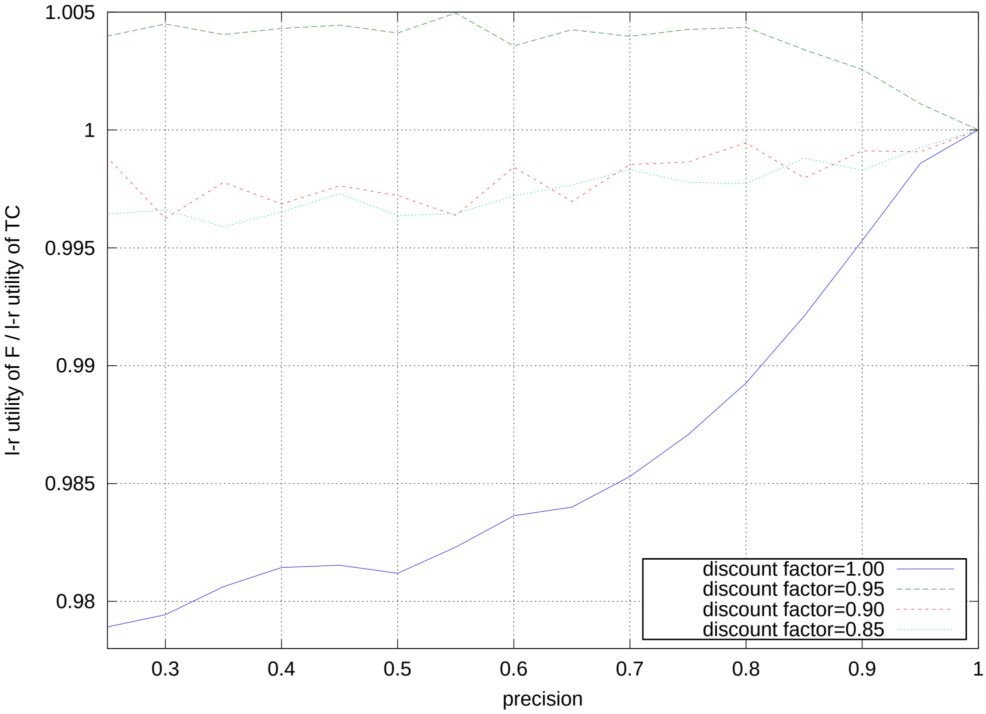

Figure 6: The influence of precision (ϵ) on the efficiency compared to TC subject (η^F_{TC}). (the average of mean costs/$\overline{U}_{TC^*}$ is 0.04)

Another startling result is that the rate of preproperations is much higher than the rate of procrastinations for higher discount factors. A general observation is that both procrastinations and preproperations occur more often with decreasing precision of foresight. The reason for the high rate of preproperations at $\delta = 1$ seems to be that without discounting, the subject tends to work at any period with below-average costs as these appear to be exceptionally rare due to leveling of future costs to the mean (see figure 1). When the subject later happens to face unexpectedly low opportunity costs, she may decide to work more on the task because the marginal reward seems still higher. Obviously, the positive correlation between efficiency and precision of foresight at discount factor equal to 1 is due to growing number of preproperations for decreasing precision. Since the relation between precision and the rate of preproperations (or procrastinations) is less significant for $\delta < 1$, precision has in these cases little effect on efficiency. It is so because low discount factors render distant periods' costs so low that the subject decides to postpone her work to the final periods anyway. In other words, the relative effect of precision is much lower than the effect of discounting and is therefore pronounced only if discounting does not occur.

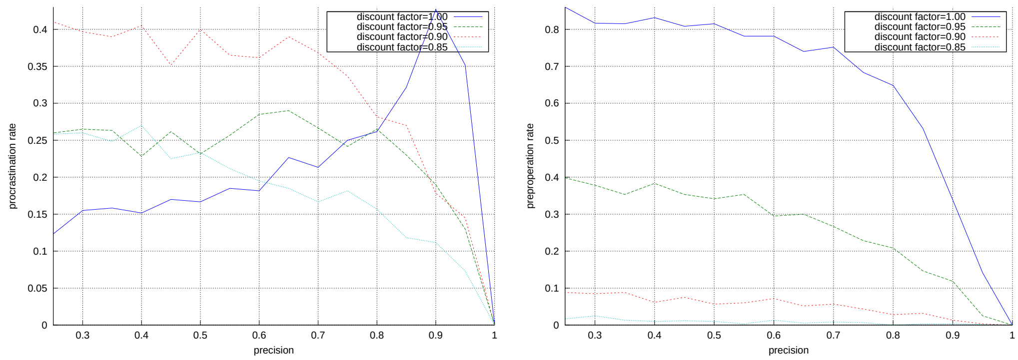

Figure 7: Rates of procrastination and preproperation with changing precision of foresight

3.2.3 Comparing TCs and optimistic/pessimistic subjects

Biased but improving expectations of the reward schedule can also cause dynamic inconsistency and hence procrastination. Simulations unambigously show that efficiency decreases with increasing bias of reward schedule expectations towards optimism. Also the lower the discount factor, the higher the marginal efficiency (optimism being equal). Pessimism ($\omega > 1$) does not seem to influence efficiency until a threshold is passed, when it becomes utterly inhibitive - with growing pessimism, the subject is more likely to conclude *ex ante* that the task is not worth doing at all. The rate of procrastination increases with both growing optimism and pessimism, while preproperations occur from time to time without clear pattern. The efficiency results are plotted in figure 8.

The results therefore corroborate the psychological finding that optimism about the workload is positively correlated with procrastination. If we compare procrastination induced by present-biased preferences with procrastination induced by optimism (in terms of efficiency losses), procrastination is more sensitive to changes of the optimism parameter. For the same discount factor equal to 0.9, the efficiency loss caused by $\omega = 0.8$ (relatively small bias) is matched by the efficiency loss caused by $\beta = 0.55$.

3.2.4 Simulation of the dynamic model with relative valuation

The dynamic model contains many more variables and parameters than the static model discussed previously. Consequently it is difficult to control for each variable and paramater. Although some paramaters have only local influence that can be assessed directly (e.g. the persistence of a habit defined

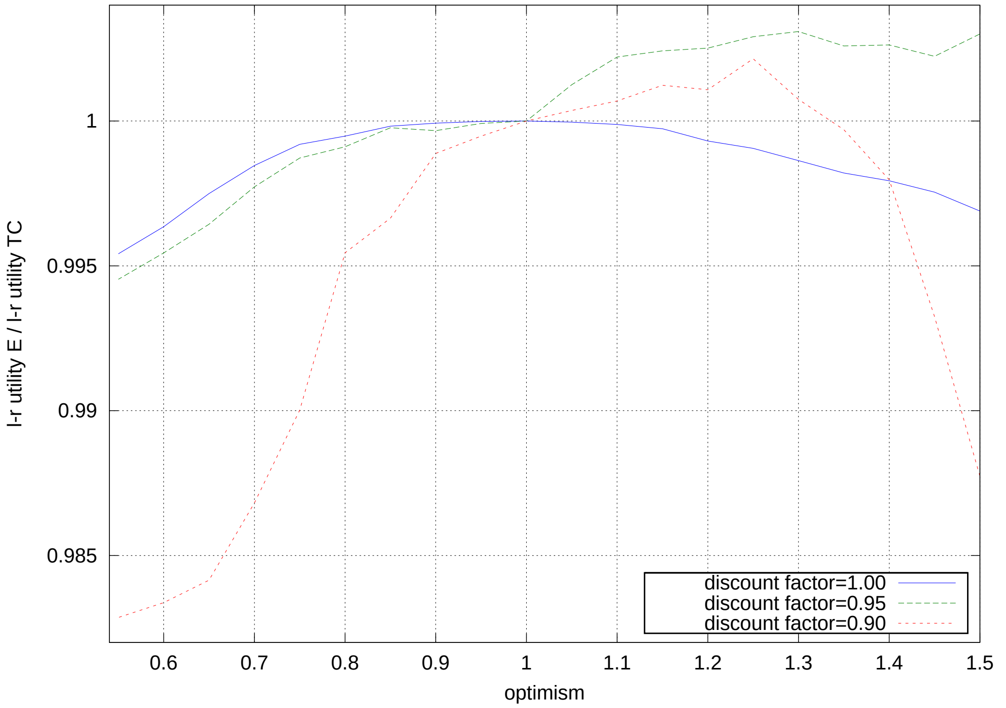

Figure 8: The influence of optimism/pessimism ($\omega < 1$ / $\omega > 1$) on the efficiency compared to a TC with equal discount rate. (η^E_{TC}). (the average of mean costs/$\overline{U}_{TC}$ is 0.038)

by α in equation 7), other variables, while having a rather unambiguous effect under the *ceteris paribus* condition (e.g. controling for the isolated effect of shifts in reference point for reward), may turn out to have ambiguous (if not opposite) effect if different variables shift.

The main purpose of the dynamic model is to explore the influence of changing reference points on procrastination / preproperation, therefore simulations concerning efficiency, which served mainly to calibrate the value function to a realistic setting, have been excluded.

The first simulation focuses on the isolated effect of changing the reference point for cost, holding other variables constant, except for cases. The simulation (fig. 9) unambiguously shows that the rate of procrastination is a decreasing function of the reference point for costs. Controling for discount factor showed that increasing δ does not change the slope but shifts it downward (the subject procrastinates less often). On the other hand, increasing discount factor increased the difference in the time spent working - thus the subject procrastinated less often but more severely. Controling for the k-modifier of the value function showed that increasing k shifts R_c upwards and flattens its slope (which remains decreasing). While procrastination rate decreases with R_c, preproperation increases. The figure clearly shows these two are negatively correlated (the actual correlational coefficient for the data plotted is -0.96).

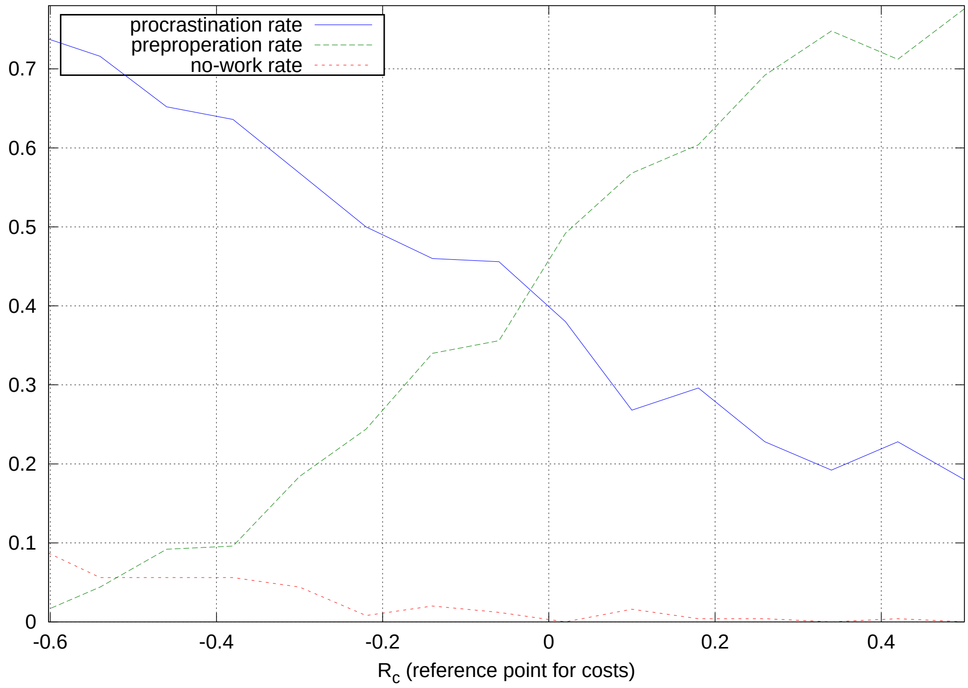

Figure 9: Changing rate of procrastination and preproperation with changing R_c. Parameter setting: $\delta = 0.9$,$R_r = 0$, $a = 3$, $k = 15$.

Similar results were obtained for the reference point for reward. The rate of procrastination decreases with R_r and the rate of preproperation increases. As in the previous simulation, the only significant result is that the slope is negative for procrastination and positive for preproperation, because the actual rates depend on parameter setting. Checking for various combinations of paramaters, I have not found any that violated this general finding.

Finally, let's turn to the question whether the habit (determined by the reference points) that *ceteris paribus* induces procrastination is stable or not. Previous results have showed that procrastination is more likely for low reference points (both for cost and reward). It was established in section 2.4 that for fixed cost schedule and reward schedule the respective reference points converge to stable points according to equations 9 and 10. Next simulation therefore allows both reference points to change in response to past actions and also generates new case each time (thus the cost schedule and reward schedule are no longer fixed). Bearing in mind limits of the model, this is as realistic setting as can be.

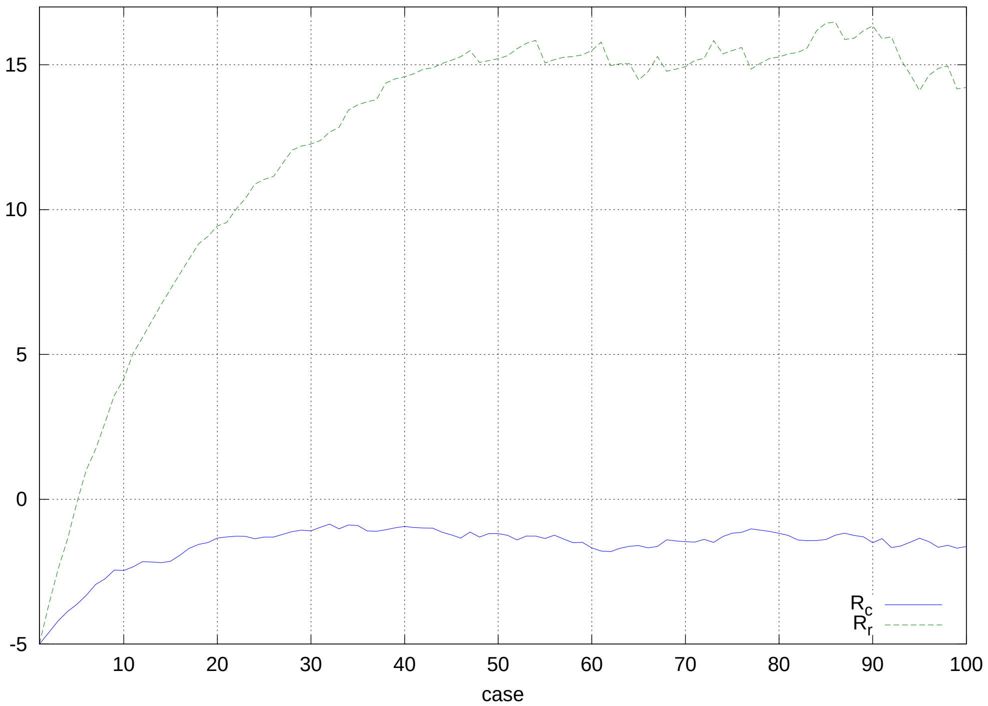

Figure 10: Evolution of reference points for costs (R_c) and reward (R_r) in response to taken actions and faced circumstances. Parameter setting: $\delta = 1$,$\alpha = 0.9$, $a = 3$, $k = 8$.

Figure 10 shows that both reference points ultimately reach stable interval in which they oscillate. Reference for costs is positively correlated with mean costs in previous 'case' - this is just intuitive if we realize that the reference point in case i is to great extent determined by mean costs in the earlier case $i - 1$. More importantly, the reference point for reward is relatively high which means that the subject is not likely to procrastinate - the rate of procrastination for the reached stable pair of reference points is only about 12% while preproperations occur in a quarter of cases. Cases in which both the GL and the benchmark subject decided not to work at all have been excluded from the analysis because these mean that the task had not been *ex ante* evaluated as worth doing. It is implausible that in such case the reference point shifts because the subject interprets the result of not doing the task as an externally imposed penalty she would never agree to in a contract. It is questionable whether cases in which the benchmark subject does work and the GL subject does not should be excluded as well. On the one hand, it means that the GL subject does not find the task worth doing *ex ante*. On the other hand, since the rational benchmark subject does work, it could be interpreted as an expression of laziness rather than procrastination.

4 Discussion and conclusion

Specification of the decision problem in terms of a task that takes many periods to complete is a fruitful one, for it allows to study procrastination (and intertemporal choice in general) along two dimensions: how *late* and how *much* the subject works, comparing to some other subject.

The optimism / pessimism extension of the model shows that deviations from the unbiased perception of the reward schedule lead to efficiency loss (comparing with a rational, time-consistent agent). Two important conclusions can be derived from the model of optimism: 1) if marginal reward in the active part of the reward schedule (periods m to n) is low, the optimistic subject is more likely to procrastinate because she thinks she can start working late and still complete the task at an acceptable level; 2) if the marginal reward is high, the optimistic subject may eventually work more because she expects the marginal reward to be higher than it is, no matter how much time she has so far spent working. Under both conditions, the subject works inefficiently comparing to the rational, time-consistent agent. Thus whether procrastination or excessive work will be the result of optimism depends mainly on the shape of the reward function. Since optimism about future workload is listed as a cognitive distortion particularly relevant to procrastination, one could infer that those procrastinators have a step-like reward function[19] - zero until $m\omega$, then increasing sharply at some "satisficing" level and then flat again. It was derived that imperfect foresight will cause procrastination mainly in case high opportunity costs are near the deadline. The most important result of simulations is that, for unbiased estimate of mean opportunity costs, the imperfect foresight does not, on average, affect efficiency. Therefore, although the imperfectly foreseeing subject is frequently time-inconsistent, all efficiency loss is due to "rational" discounting. Thus imperfect foresight seems to severely affect long-run utility only if the distribution of costs is particularly unfavourable.

Simulation of the relative valuation model indicates that procrastination as a result of low reference points, which reflect the results of past actions, is not a stable habit. Two qualifications must be made. First, it does *not* follow that procrastination cannot be a stable habit - I have examined only

[19] Note that the reward function reflects the perceived *utility* from some objective reward, not the objective reward itself.

one plausible reference point adjustment among many. Second, the result is admittedly not robust enough to claim with certainty that the examined reference point adjustment will under *any* circumstances lead to a stable point where procrastination is not likely - not every possible parameter combination has been controlled for. The general finding that the rate of procrastination is a decreasing function of the reference point for reward is strong. It intuitively follows from the shape of the value function: the more accustomed the subject is to high reward, the less likely she is to procrastinate, because the threat of not meeting her standard is more motivating due to loss aversion. Generally, loss aversion implies that prospective losses motivate people more than gains, *ceteris paribus*.

Let us proceed to policy implications of the model. The model of imperfect foresight suggests that if one could achieve better foresight with respect to high cost periods, say by having better information, one could avoid cases of very costly procrastination. As simple a precaution as keeping a calendar could often enhance the precision of foresight significantly, because although one is generally aware that some high opportunities are to come, one may not realize they are to at that very time.

The optimism model naturally implies that one ought to curb her optimistic view of the future workload. In reality, this optimism can take various forms. Apart from really expecting the project to take less time, one could, for example, get too optimistic after making the first easy steps. Having an unbiased estimate of the workload is arguably a matter of experience with that particular kind of work. Nevertheless, our memory is selective and we are likely to forget the discrepancy between our past estimate and the actual time the project took us, so we may eventually be optimistic again next time. That's why psychologists, in order to eradicate this cognitive distortion, ask procrastinators to note their estimate at the beginning of the project and keep track of the time spent working throughout.[20] Having the discrepancy before one's eyes significantly helps to correctly adjust the estimate.

The relative valuation model implies that penalties, or anything that the agent will regard as a loss compared to status quo, motivate more than rewards. However, negative motivation alone will not reduce procrastination, since the subject must first get used to some positive result, so that there be a motivating prospective negative result. Unless the principal has the authority to impose penalties on the agent, the former cannot rely on negative motivation only, because the latter would never accept a contract whose expected value is negative. The agent also achieves high reference point for

[20] Cf. [1][chpt. 9]

reward (which inhibits procrastination) only if the marginal reward of effort is high enough to motivate working above the passing level.

It goes without saying that the most reliable mean to limit one's own procrastination tendencies is precommitment, but since the model presented here does not allow the subject to limit her future options, discussion of precommitment was omitted.

In conclusion, the paper shows how the dynamic inconsistency typical for procrastination can be modeled by different means than hyperbolic discounting in which many different potential causes of procrastination are subsumed under a single paramater β. Although the presented model is more complex and thus more difficult to analyze than the original O'Donoghue and Rabin model, it provides a finer analysis of the behavioural patterns involved in procrastination.

-	simulation with $\beta\delta$ prefs.	simulation with foresight	simulation with optimism	simulation with rel. value function
total of cases generated	12, 800	38, 400	24, 000	12, 000 / 10, 000
cases per comb. of control variables	200	600	400	200 / 200
control variables	δ,β	δ,ϵ	δ, ω	δ, k / k, a
range / increment of c. variables	$\delta \in [0.85, 1]$ / 0.05 $\beta \in [0.55, 1]$ / 0.03	$\delta \in [0.85, 1]$ / 0.05 $\epsilon \in [0.25, 1]$ / 0.05	$\delta \in [0.9, 1]$ / 0.05 $\omega \in [0.55, 1.50]/0.05$	$\delta \in [0.85, 1]$ / 0.05; $k \in [2, 30]$ / 2 $k \in [2, 10]$ / 2; $a \in [2.8, 7.3]/0.5$
dependent variables	A_{TC},$A_{\beta\delta}$, $\eta_{TC}^{\beta\delta}$, $\Pi(\beta,\delta)$	A_{TC},A_F, η_{TC}^F, $\Pi(\delta,\epsilon)$	A_{TC},A_E, η_{TC}^E, $\Pi(\delta,\omega)$	A_{TC},A_{GL}, η_{TC}^{GL}, $\Pi(k,a,\ \delta)$
other independent variables	$c(t)$,$r(x)$	$c(t)$,$r(x)$	$c(t)$,$r(x)$	$c(t)$,$r(x)$, $a = 3$ / $\delta = 0.9$
nr. of periods (T) in a case	20	20	20	20 / 20
minimal workload (m)	4	4	4	4/4
maximal workload (n)	10	10	10	10 / 10

Table 1: Simulation setting for the four alternative models ($\Pi(x)$ means procrastination rate depending on x)

-	R_c convergence	R_r convergence	R_r & R_c convergence	pr. rate for R_c	pr. rate for R_r	free play
total of cases generated	1	1	1	3, 750	3, 300	100
cases per comb. of control variables	-	-	-	250	300	-
control variables	-	-	-	$R_c \in [-0.7; 0.5]$ / 0.08	$R_r \in [-5; 5]$ / 1	-
dependent variables	R_c, A_{GL}	R_r,A_{GL}	R_c, R_r, A_{GL}	A_{GL}, $Pi(R_c)$	A_{GL}, $Pi(R_r)$	A_{GL}, R_r, R_c
δ	1	1	0.9	0.9	0.9	1
a	3	3	3	3	3	3
k	15	7	15	15	7	8
other independent variables	$c(t)$,$r(x)$, $\alpha = 0.8$	$c(t)$,$r(x)$, $R_r = 0$, $\alpha = 0.8$	$c(t)$,$r(x)$, $R_c = 0$, $\alpha = 0.8$	$c(t)$,$r(x)$, $R_r = 0$	$c(t)$,$r(x)$, $R_c = 0$	$c(t)$,$r(x)$, $\alpha = 0.9$
nr. of periods (T) in a case	25	20	20	20	20	30
minimal workload (m)	5	4	4	4	4	6
maximal workload (n)	13	10	10	10	10	15

Table 2: Simulation setting for the dynamic model

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