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Abstract

We analyze ecosystem management under 'unmeasurable' Knightian uncertainty or ambiguity which, given the uncertainties characterizing ecosystems, might be a more appropriate framework relative to the classic risk case (measurable uncertainty). This approach is used as a formal way of modelling the precautionary principle in the context of least favorable priors and maxmin criteria. We provide biodiversity management rules which incorporate the precautionary principle. These rules take the form of either minimum safety standards or optimal harvesting under precautionary approaches.

Keywords: Knightian uncertainty, uncertainty aversion, ambiguity aversion, risk aversion, precautionary principle, biodiversity, optimal harvesting, robust control.

JEL Classification: D81, Q20

1 Introduction

Biodiversity loss has emerged as a major issue on both academic and policy grounds. As stated in the recent Millennium Ecosystem Assessment report (MEA 2005a, page 4):

> Humans are fundamentally, and to a significant extent irreversibly, changing the diversity of life on earth, and most of these changes represent a loss of biodiversity.

It is estimated, in the same report, that during the past several hundred years, humans have increased the species extinction rate by as much as 1000 times over background rates over the planet's history. In the MEA report (2005b), it is acknowledged that ecosystem management practices that maintain diversity, functional groups, and trophic levels are more likely to decrease the risk of large losses of ecosystem services than practices that ignore these factors.

These statements suggest that the development of management rules that could help to prevent loss of biodiversity is a desirable goal. The attainment of this goal is hindered, however, both by the complexity of ecosystems and by important and interrelated uncertainties, a number of which include sources such as major gaps in global and national monitoring systems; the lack of a complete inventory of species and their actual distributions; limited modelling capacity and lack of theories to anticipate thresholds; emergence of surprises and unexpected consequences. These uncertainties may impede adequate scientific understanding of the underlying ecosystem mechanisms and the impacts of policies applied to ecosystems. For the purposes of our analysis we will refer to the overall uncertainty associated with these sources as scientific uncertainty.

One feature of the uncertainty structure described above is that it might be difficult or even impossible to associate probabilities with uncertain prospects affecting the ecosystem evolution. This is close to the concept of uncertainty as introduced by Frank Knight (1921) to represent a situation where there is ignorance, or not enough information, to assign probabilities to events. Knight argued that uncertainty in this sense of unmeasurable uncertainty is more common in economic decision making. Knightian uncertainty is contrasted to risk (measurable or probabilistic uncertainty) where probabilities can be assigned to events and they are summarized by a subjective probability measure or a *single* Bayesian prior. Thus Knightian uncertainty or 'ambiguity' can be regarded as an appropriate framework for analyzing issues related to scientific uncertainty in biodiversity management.[1] This uncertainty concept has been associated formally with the concept of *multiple priors* (Gilboa and Schmeidler, 1989), as well as with the concept of uncertainty or ambiguity aversion which in general increases with an ignorance parameter (Chen and Epstein, 2002).

In economics, decision making under risk implies expected utility maximization. Under Knightian uncertainty as described above, it was Wald (1950) who suggested that a *maxmin* solution could be a reasonable solution to a decision problem where an *a priori* probability distribution does not exist or is not well known to the researcher. One way to approach the maxmin solution is to use the idea of *least favorable prior* (LFP)[2] decision theory, as developed by Gilboa and Schmeidler (1989), which results in maxmin expected utility theory and represents an axiomatic foundation of Wald's criterion.

Decision theory based on the LFP can be associated with the concepts of *precautionary principle* (PP), safety margin (SM), and *safe minimum standards* (SMS). The precautionary principle is an approach where actions are taken to anticipate and avert serious or irreversible harm, such as for example extinction of species for the case of biodiversity preservation, or prevention of an irreversible climate change, in advance of or without a clear demonstration that such action is necessary. Marchant (2003) states that he PP prescribes how to bring scientific uncertainty into the decision-making processes by explicitly formalizing precaution and bringing it to the forefront of deliberations. On the other hand the ideas of LFP or *worst case scenario* (WCS) and irreversible changes can be intuitively put together, since the emergence of a WCS could lead to an irreversible change. Therefore a direct link can be made between LFP ideas and the PP. Scientific uncertainty or model uncertainty can be manifested in multiple priors. The decision maker cannot choose among them, but one or more of these priors, the LFP, leads to irreversible change. To prevent the irreversible change, which is not clearly demonstrated since the decision maker does not know that the LFP will prevail, a precautionary approach should be taken, which implies that the decision rule should be

[1] In a recent article Shaw and Woodward (2008) very clearly present the high relevance of this analytical framework for environmental and resource economics.

[2] Given a set of prior probability distributions associated with the multiple priors framework, the LFP is the one that corresponds to the least favorable outcomes. It can be associated with the concept of the worst case scenario. Under Knightian uncertainty the researcher cannot choose *one* prior to define expected utility as is done under risk.

based on LFP. Thus, the maxmin expected utility can be used as a conceptual framework for designing management rules which adhere to the PP.

Closely related to the these concepts are the ideas of SM and SMS for the preservation of biodiversity (e.g. Holt and Tisdell, 1993). Safety margins could be defined in terms of feasible variations for land allocations and harvesting values so that, under uncertainty and ambiguity aversion, species biomasses will not deviate more than a prespecified level from a desire steady state. Further, SMS could be defined in terms of minimum viable populations and minimum habitat requirements. Using the LFP and maxmin framework, SM and SMS can be defined so that species extinction is prevented under the least favorable situation associated with the uncertainties obscuring the scientific understanding of the ecosystems' mechanisms. These policies can be regarded as management which embodies some type of PP.

The purpose of this paper is to combine these concepts and provide management rules for preserving biodiversity under scientific uncertainty and ambiguity aversion, which follow a precautionary principle. The precautionary approach is formalized by using multiple priors and LFP ideas, and maxmin decision rules, which lead both to SM or SMS and optimal management rules that embody the PP. Furthermore, by comparing optimal management rules which are obtained by assuming first the traditional risk set up and second ambiguity, it is possible to obtain some quantification of the implications of the PP in terms of decision variables such as harvesting and land allocation rules.

In the rest of the paper we present two approaches to biodiversity management under scientific uncertainty and ambiguity aversion in models of multiple species. In the first approach we apply the k-ignorance approach for specifying the multiple priors model and we derive, in terms of a descriptive non-optimizing model of species interactions, harvesting and land allocation rules for species which are designed to provide safety standards in the sense of either keeping the species populations in some relation to initial values, given an exogenously determined desired steady state for biomasses, or keeping the species biomasses above some minimum safety standard with a given probability. In the second approach we apply robust control methods to derive optimal harvesting rules under model uncertainty. By comparing solutions under risk and under ambiguity we provide a measure of the impact of adopting precautionary approaches in ecosystem management.

2 Modelling Model Uncertainty

Rational expectations models do not permit a self-contained analysis of model uncertainty. Assuming that economic agents have concerns about model misspecification reopens fundamental issues expressed by Knight (1921), Savage (1954) and Elsberg (1961), ideas which, by adopting rational expectations, have been set aside from agents' beliefs.

Knight was the first who made the distinction between risky events for which a true probability distribution can be specified and a worse type of ignorance, where a unique probability measure is not available, that he called uncertainty. Savage believed that as an aspect of rationality, personal probabilities are "correct". On the other hand Fellner (1961) and Elsberg (1961) challenged Savage's theory, on the basis of experimental evidence. Gilboa and Schmeidler (1989), motivated by the Elsberg paradox, formulated, in an atemporal setting, a set of appropriate axioms and incorporated the idea of uncertainty or ambguity aversion into decision making. Dynamic models in which agents are adverse to model ambiguity have been constructed by Epstein and Wang (1994) and Chen and Epstein (2002).

In the recent literature we can distinguish two main, although interrelated, approaches for dealing with ambiguity: the multi priors and the robust control approaches.

2.1 Modelling Uncertainty Using Multiple Priors

Let the set of states of the world be Ω and consider an individual observing some realization $\omega_t \in \Omega$. The basic idea underlying the multiple priors approach is that beliefs about the evolution of the process $\{\omega_t\}$ cannot be represented by a probability measure. Instead, beliefs conditional on ω_t are too vague to be represented by such a single probability measure and are represented by a *set* of probability measures (Epstein and Wang, 1994). Thus for each $\omega \in \Omega$, we consider $\mathcal{P}(\omega)$ as a set of probability measures about the next period's state. Formally $\mathcal{P}$ is a correspondence $\mathcal{P} : \Omega \to \mathcal{M}(\Omega)$ assumed to be continuous, compact-valued and convex-valued and $\mathcal{M}(\Omega)$ is the space of all Borel probability measures.

The individual ranks uncertain prospects or acts which are denoted by α. Let u be a von Neumann-Morgenstern utility function. The utility of any act α in an atemporal model is defined (Gilboa and Schmedler, 1989; Chen and Epstein, 2002) as

$$U(\alpha) = \min_{Q \in \mathcal{P}} \int u(\alpha)\, dQ \tag{1}$$

In a continuous time framework, recursive multiple prior utility, in a finite time setting, is defined as:

$$V_t = \min_{Q \in \mathcal{P}} E_Q \left[\int_t^T e^{-\beta(s-t)} u(\alpha)\, ds \right] \tag{2}$$

where the subjective set of priors $\mathcal{P}$ on a space $(\Omega, \mathcal{F})$ is uniformly absolutely continuous with respect to $Q \epsilon \mathcal{P}$.[3],[4]

These definitions of utility in the context of multiple-priors correspond to an intuitive idea of the 'worst case'. Utility is associated with the utility corresponding to the least favorable prior. With utility defined in this way, decision making by using the maxmin rule follows naturally, since maximizing utility in the multiple-prior case implies maximizing the utility which corresponds to the LFP.

The individual's set of priors can be further specified for the purposes of the analysis by the so called k-ignorance approach. In this case the individual considers the reference probability measure P and another measure $Q \in \mathcal{M}(\Omega)$. The discrepancy between the two measures is defined by the relative entropy

$$R(Q//P) = \int_0^{+\infty} e^{-\delta t} E_Q[\frac{1}{2}\varepsilon_t^2] dt \tag{3}$$

where ε is a measurable function associated with the distortion of the probability measure P to the probability measure Q. According to the k-ignorance approach, the individual incorporates into her/his decision-making problem the instantaneous relative entropy constraint $Q(\tau) = \{Q : E_Q[\frac{1}{2}\varepsilon_t^2] \leq \tau$, for all $t\}$, which means that probability measures differing from the reference measure P by at least as much as τ should be taken into account. If Q is a probability measure associated with the least favorable outcome, then k-ignorance embodies an LFP or worst case scenario idea.[5]

2.2 Modelling Uncertainty Using Robust Control Methods

Another way to embody decisions makers' concerns about model misspecification is to use robust dynamic control, which is also a minmax approach which

[3] Uniformly absolutely continuous means that for every $\varepsilon > 0$ there is $\delta > 0$ such that $E \in \mathcal{F}$ and $Q(E) < \delta$ implies that $P(E) < \varepsilon\ \forall P \varepsilon \mathcal{P}$.

[4] For further details and behavioral implications of the structure of $\mathcal{P}$ see (Epstein and Wang, 1995, Chen and Epstein 2002).

[5] Another way to specify the set of priors is the so called e-contamination approach (Epstein and wang 1994), where the set of priors is a convex combination of the probability measure P and the measure Q. Thus

$\mathcal{P}_\epsilon = \{(1-\epsilon) P + \epsilon Q : Q \in \mathcal{M}(\Omega)\ ,\ \epsilon \in [0,1]\}$

has been introduced to economics by Hansen and Sargent (see for example Hansen and Sargent (2001)). In this case the decision maker suspects that his/her model is misspecified, in the sense that there is a group of approximate models which are also considered as possibly true given a set of finite data. These approximate models are obtained by disturbing a benchmark model, and the admissible disturbances reflect the set of possible probability measures that the decision maker is willing to consider, or otherwise how ambiguous the decision maker is about the initial estimated model. The objective of this approach is to choose by a minmax criterion, formulated in terms of a differential game where one agent is 'Nature' that 'chooses' the LFP, a rule that will work well under a range of different model specifications. The robust control method which can be regarded as an approach for deriving optimal dynamic policy rules under model uncertainty will be presented in more detail in section 4.

In relation to biodiversity management the approaches described above allow us to model the uncertainties or ambiguities underlying our scientific knowledge about ecosystems in a way that, as will be shown later, leads to well defined policy rules and allows for the quantification of the precautionary principle.

3 Safety Standards and Biodiversity Management: A Non-optimizing Approach

Economists usually try to manage ecosystems and biodiversity in an optimal way despite the fact that the complexity of ecosystems might make optimization exercises difficult, even at a theoretical level. On the other hand, if we are interested in preserving diversity it might be useful to think about managing ecosystems using safety rules, which when applied prevent species or a set of species from becoming extinct.[6] Safety rules in biodiversity preservation could acquire greater importance when the ecosystem manager faces Knightian uncertainty or ambiguity which, as discussed above, is a potentially very relevant case in ecosystem management. In this situation worst case events might cause surprises and extinction of species. Since these irreversible changes have occurred in reality, dealing with worst case scenarios means that ecosystem management and biodiversity preservation are asso-

[6]Safety regulation is a more general issue in economics. For a general discussion of the role of economic analysis in the development of environmental health and safety regulation, see Arrow et al. (1996). For a discussion of safety standards in species protection, see for example Holt and Tisdell (1993).

ciated with a PP, which implies that the management rules are such that species will not become extinct under worst case scenarios.

3.1 Safety Standards in a Deterministic Model

We examine first the determination of safe minimum standards for preventing biodiversity loss, in terms of minimum population levels in the context of a deterministic model. The deterministic model developed here is used as a vehicle for the introduction of uncertainty in analyzing biodiversity management, which is the main target of this paper. In this model population levels are directly controlled by harvesting, and available habitat for each species which is determined by land allocations rules.

We start by considering an ecosystem manager who manages a landscape, normalized to unity, where two species coexist. Let B_{it} for $i = 1, 2$ be the biomasses of the two interacting species at time t, where b_{12}, b_{21} are the interaction coefficients between them. It is assumed that the evolution of the initial biomasses $(B_{10,}B_{20})$ through time can be described by the system of deterministic differential equations:

$$\begin{aligned} \dot{B}_1 &= B_1 f_1(w)[1 - h_1 - B_1 w - b_{12} B_2 w] \\ \dot{B}_2 &= B_2 f_2(1-w)[1 - h_2 - B_2(1-w) - b_{21} B_1 (1-w)] \end{aligned} \tag{4}$$

where $h_i = d_i + \hat{h}_i$, h_i denotes the total removal rate from biomass, $\hat{h}_i$ denotes net harvesting, and d_i are the death rates, for $i = 1, 2$. Since the death rates are assumed known and fixed choosing h_i is equivalent to choosing $\hat{h}_i$, thus we use h_i as our control variable. Furthermore f_i are the intrinsic growth rates with $(w, 1-w)$ being a land allocation rule. It is assumed that the intrinsic growth rate which depends on w or $1 - w$ is increasing and concave in the land allocated to the species with $f_i(0) = 0$.

Using a non-dimensionalization, which is usually applied to models of interacting populations, the model above can be rewritten in a simplified form as:

$$\begin{aligned} \dot{u}_1 &= u_1(1 - u_1 - a_{12}u_2 - h_1) = g_1(u_1, u_2; h_1, h_2) \\ \dot{u}_2 &= u_2 \hat{f}(w)(1 - u_2 - a_{21}u_1 - h_2) = g_2 u_1, u_2; h_1, h_2; \hat{f}(w)) \\ \hat{f}(w) &= \frac{f_2(1-w)}{f_1(w)}, a_{12} = b_{12}\frac{w}{1-w}, a_{21} = b_{21}\frac{1-w}{w}, u_1 = B_1 w, u_2 = B_2(1-w) \end{aligned} \tag{5}$$

In this model carrying capacity is proportional to $1/w$ or $1/(1-w)$ for a space normalized to one.

For $a_{12}a_{21}-1 \neq 0$ the dynamical system (5) has four steady states defined for $\dot{u}_1 = \dot{u}_2 = 0$. In three of them either both or one of the species biomasses are zero. Since we are interested in the preservation of both species we will focus on the fourth steady state where both biomasses are positive and thus both species are preserved in long-run equilibrium. The species biomasses in this steady state are:

$$\begin{aligned} u_1^* &= \frac{1}{a_{12}a_{21}-1}\left(h_1 + a_{12} - h_2 a_{12} - 1\right) \\ u_2^* &= \frac{1}{a_{12}a_{21}-1}\left(h_2 + a_{21} - h_1 a_{21} - 1\right). \end{aligned} \tag{6}$$

It follows that a desired steady state $\mathbf{u}^* = (u_1^*, u_2^*)$ defined through (6) can be attained if there exist non-negative fixed harvesting rules $(h_1^*, h_2^*) = \mathbf{h}^*$ which solve the linear system (6) for the given $\mathbf{u}^* = (u_1^*, u_2^*)$. The stability properties of the desired steady state are characterized in the following proposition.

Proposition 1 *Assume that the non-negative harvesting rule* $(h_1^*, h_2^*) = \mathbf{h}^*$ *attains the desired steady state* $\mathbf{u}^* = (u_1^*, u_2^*)$ *. Then if*

$$(i) \quad \frac{1 - h_1^* - a_{12} + h_2^* a_{12} + \hat{f}(w)(1 - h_2^* - a_{21} + h_1^* a_{21})}{a_{12}a_{21}-1} < 0.$$

$$(ii) \quad (1 - h_1^* - a_{12} + h_2^* a_{12})(1 - h_2^* - a_{21} + h_1^* a_{21})$$
$$-(-h_1^* a_{12} - a_{12}^2 + h_2^* a_{12}^2 + a_{12})(-h_2^* a_{21} - a_{21}^2 + h_1^* a_{12}^2 + a_{21}) > 0.$$

The desired steady state is stable and can be reached from any initial biomass levels in its neighborhood.

Proof. The proof follows directly for the Jacobian of the dynamical system (5) evaluated at the desired steady state. The Jacobian is

$$J\left(u_1^*, u_2^*\right) = \begin{bmatrix} \frac{1-h_1^*-a_{12}+h_2^*a_{12}}{a_{12}a_{21}-1} & \frac{-h_1^*a_{12}-a_{12}^2+h_2^*a_{12}^2+a_{12}}{a_{12}a_{21}-1} \\ \hat{f}(w)\frac{-h_2^*a_{21}-a_{21}^2+h_1^*a_{12}^2+a_{21}}{a_{12}a_{21}-1} & \hat{f}(w)\frac{1-h_2^*-a_{21}+h_1^*a_{21}}{a_{12}a_{21}-1} \end{bmatrix}$$

Condition (i) implies negative trace, while condition (ii) implies positive determinant. Therefore the Jacobian matrix has eigenvalues with negative real parts and the desired steady state is stable. ■

At this stage the desired steady state is rather loosely determined, without any reference to optimality criteria. It can be assumed, however, that this steady state is determined through some political process, which is a situation very often encountered in reality, where competing conservation and harvesting objectives determine some equilibrium desired steady state. The process of arriving at this steady state is not modelled here.

3.2 Safety Standards in Stochastic Environments

3.2.1 Safety Standards under Risk Aversion

Having defined the desired deterministic state as a benchmark we consider, in this section, the more realistic case where the evolution of biomasses is stochastic. We assume at this stage that the manager of the ecosystem has a single subjective prior distribution. A single prior is the main characteristic of the vast majority of continuous time dynamic models which assume probabilistic sophistication, implying that we analyze the problem under risk (measurable uncertainty). We follow this approach because it is an intuitive way to proceed to the case of Knightian (unmeasurable) uncertainty, but also because it allows us, by comparing solutions under risk and solutions under uncertainty, to obtain a quantification of the precautionary principle, since as discussed in the introduction, PP can be associated with the Knightian uncertainty framework.

Keeping that same structure with the deterministic model, we assume that the evolution of the initial biomasses $B_{10,}B_{20}$ through time is given by a system of stochastic differential equations, which in the nondimensionalized form can be written as:

$$\begin{aligned} du_1 &= u_1\left(1-u_1-a_{12}u_2-h_1\right)dt+\sigma_1(h,u_1)dz_1 \\ du_2 &= \hat{f}\left(w\right)u_2\left(1-u_2-a_{21}u_1-h_2\right)dt+\sigma_2(h,u_2)dz_2 \end{aligned} \tag{7}$$

where dz_1, dz_2 denote two correlated Brownian motions, with ρ being the correlation coefficient between them.[7]

To obtain a better understanding of the problem, we analyze a first order linear approximation (see Flemming, 1971) of the stochastic differential equations (7) in the neighborhood of the desired deterministic steady state (u_1^*, u_2^*), (h_1^*, h_2^*).

[7] For an appropriate specification of the dynamics of the uncontrolled system, the two biomasses should not take negative values, which means that $\sigma_i(\mathbf{h}, 0) = 0$ for $i = 1, 2$. Alternatively if the variances are independent of $\mathbf{u}$, it can be assumed that the u_i represents logarithms of the nondimensionalized biomasses so that they follow a lognormal distribution.

Let $(u_i - u_i^*) = x_i, du_i = dx_i$, $(1 - 2u_1^* - a_{12}u_2^* - h_1^*) = \delta_{11}, -a_{12}u_1^* = \delta_{12}, -\hat{f}(w)\, a_{21}u_2^* = \delta_{21}, \hat{f}(w)(1 - 2u_2^* - a_{21}u_1^* - h_2^*) = \delta_{22}$, where $\delta_{ij}, i, j = 1, 2$ depend on the harvesting and the land allocation parameters at the desired deterministic steady state. Using matrix notation, the first-order linear approximation of the stochastic differential equations (7) in the neighborhood of the desired deterministic steady state can be written as:

$$
\begin{aligned}
& dx = Axdt + \Sigma dz \quad \text{where} && (8)\\
& dx = \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}, \; A = \begin{bmatrix} \delta_{11} \, \delta_{12} \\ \delta_{21} \, \delta_{22} \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}, dz = \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix}.
\end{aligned}
$$

The following proposition can be stated:

Proposition 2 *Given a land allocation rule and a harvesting rule* (w, h_1, h_2) *and initial values* (x_{10}, x_{20}), *the expected values of the two biomasses are given by*

$$
\begin{aligned}
\mathcal{E}x_{1t} &= A_{1t}x_{10} + A_{2t}x_{20} && (9)\\
\mathcal{E}x_{2t} &= A_{3t}x_{10} + A_{4t}x_{20}.
\end{aligned}
$$

Proof. *System (8), multiplied from the left by a suitable matrix, becomes (see Oksendal (2000)):* [8]

$$
\begin{aligned}
& d(e^{-At}x_t) = e^{-At}dx - e^{-At}Axdt = e^{-At}\Sigma dz && (10)\\
& \textit{where } e^F = \sum_{n=1}^{\infty} \frac{1}{n!}F^n = F + \frac{1}{2!}F^2 + \frac{1}{3!}F^3 + ... && (11)\\
& \textit{where } F = -At. && (12)
\end{aligned}
$$

Equivalently:

$$
\begin{aligned}
e^{-At}x_t - x_0 &= \int_0^t e^{-As}\Sigma dz_s && (13)\\
x_t = e^{At}x_0 &+ \int_0^t e^{A(t-s)}\Sigma dz_s \; , x_0 = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} && (14)
\end{aligned}
$$

[8] In our case F is the matrix $-At$. The elements of this matrix converge to a real number. This holds because each element of this matrix is upper bounded by the sum $a_q = \sum_{q=1}^{\infty} \frac{2^{q-1}}{q!}(-tx)^q$, with x being the maximum of the four elements of matrix A in equation (8). For the above general term a known convergence criterion holds: $\limsup |\frac{a_{q+1}}{a_q}| < 1$ and therefore the series converge.

where

$$e^{At} = \begin{bmatrix} A_1 \, A_2 \\ A_3 \, A_4 \end{bmatrix} \tag{15}$$

with A_i for $i = 1, ..., 4$ depending on the values of the interaction coefficients a_{ij} and on f_i, h_i. The A_i can be calculated using relationship (11). Using relationships (13) and (15), we can derive:

$$x_{1t} = A_{1t}x_{10} + A_{2t}x_{20} + \int_0^t G_1 dz_1 + \int_0^t G_2 dz_2 \tag{16}$$

$$x_{2t} = A_{3t}x_{10} + A_{4t}x_{20} + \int_0^t G_3 dz_1 + \int_0^t G_4 dz_2 \tag{17}$$

with G_i being functions of f_i , h_i, and σ_i, with the property that they belong to the class $V = V(0, T)$.[9] *The four integrals in equations (16) and (17) are stochastic integrals with the property that for all the possible combinations of i, j,*

$$\mathcal{E} \int_0^t G_i dz_j = 0. \tag{18}$$

Therefore taking expected values in (16) and (17), condition is (9) obtained. ■

In expressions (9), A_{it} is defined as $A_{it} = A_i(w, h_1, h_2, t)$ and thus the associated expected deviations from the desired steady state depend on the land allocation weights $(w, 1-w)$ and on the harvesting rules (h_1, h_2). Since $\mathcal{E}x_{it} = \mathcal{E}\left(u_{it} - u_i^*\right) = \mathcal{E}u_{it} - u_i^*$, (9) can be written as:

$$\begin{aligned} \mathcal{E}u_{1t} &= A_1(w, h_1, h_2, t)\left(u_{10} - u_1^*\right) + A_2(w, h_1, h_2, t)\left(u_{20} - u_2^*\right) + u_1^* \\ \mathcal{E}u_{2t} &= A_3(w, h_1, h_2, t)\left(u_{10} - u_1^*\right) + A_4(w, h_1, h_2, t)\left(u_{20} - u_2^*\right) + u_2^* \end{aligned} \tag{19}$$

Suppose that for a given (u_1^*, u_2^*) the manager wants the expected biomasses at time t to satisfy a certain exogenous safety standard by being a certain proportion of initial biomasses, $\mathcal{E}u_{it} = \gamma_i u_{i0}$. The land allocation and harvesting rule (w_t, h_{1t}, h_{2t}) that satisfies (19) for $\mathcal{E}u_{it} = \gamma_i u_{i0}$, provided that it exists,[10]

[9] V is the set of measurable and adapted functions f with the property $\mathcal{E}\int_0^T f(t,\omega)^2 dt < \infty$. Then for the corresponding stochastic integral it holds that $\mathcal{E}\int_0^T f(t,\omega) dz_t = 0$.

[10] System (19) is a nonlinear system with two equations and three unknowns (w, h_1, h_2). Solution means that by fixing one of the unknowns, say w, the other two will be determined as functions $h_i = h_i(w)$. Thus for a given land allocation w, $h_i(w)$ is the harvesting rule which satisfies the safety margin. A solution will exist if the Jacobian determinant of (19) with respect to (h_1, h_2) does not vanish in an appropriate neighborhood which contains the solution.

can be regarded as a *safety rule*, which under conditions of risk prevents expected species biomasses from moving below the safety standard at time t. If the safety standard is determined by a rule $\mathcal{E}u_{it} = \gamma_i u_{it-1}$, $t = 1, 2, ...$ then the sequence $(w_{t+1}, h_{1t+1}, h_{2t+1})$, $t = 0, 1, 2, ...$ will determine an adaptive safety rule which will not let expected biomasses go below an exogenous safety standard.

The multi-species case Our model can be extended to the multi-species case. In this case, the evolution of the biomass of the k^{th} species is given by:

$$du_k = u_k \hat{f}_k(\mathbf{w})[1 - h_k - u_k w_k - \sum_{j \neq k} b_{kj} u_j w_k]dt + \sigma_k dz_k \ , \ \ j, k = 1, ..., n \quad (20)$$

with $\mathbf{w} = (w_1, ..., w_n)$, $\sum_k w_k = 1$ being the land allocation rule and $\hat{f}_k(w_k) = \frac{f_k(\mathbf{w})}{f_1(\mathbf{w})}$. Following the same procedure as above, the expected values of species biomasses are defined as:

$$\mathcal{E}x_{kt} = \sum_{i=1}^{n} A_{tk_i} x_{i0} \ , \ k = 1, ..n. \quad (21)$$

3.2.2 Safety Margins under Knightian Uncertainty and Precaution

Suppose now that the ecosystem manager operates under conditions of ambiguity or Knightian uncertainty, which could be a realistic approximation of the actual ecosystem conditions. Along the lines of our previous discussion, this type of uncertainty can be modelled in terms of the multiple priors approach. In particular, we assume that the manager has multiple priors regarding the evolution of the species biomasses. We further specify the set of priors by following the k-ignorance approach.

For the two species case the ecosystem's dynamics can now be written as:

$$dx = Axdt + \Sigma R d\hat{z} \quad \text{where} \quad (22)$$

$$dx = \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}, \ A = \begin{bmatrix} \delta_{11} \ \delta_{12} \\ \delta_{21} \ \delta_{22} \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1 - \rho^2} \end{bmatrix}, \ \ d\hat{z} = \begin{bmatrix} d\hat{z}_1 \\ d\hat{z}_2 \end{bmatrix}$$

with all the variables defined as in (8), ρ being the correlation coefficient between the two Brownian motions in the initial system (8), and $d\hat{z}_1, d\hat{z}_2$ being two independent Brownian motions.

In the k-ignorance approach, the landscape manager has reference priors about the biomasses' evolution, which are expressed by dz_i. Because of ambiguity the manager considers a decision-making problem with multiple priors. In this problem the prior, which according to the manager's beliefs is further away from the reference prior, does not differ from the reference prior, in terms of relative entropy, more than a positive number. This means that the manager is characterized by a subjective 'maximum' level of ignorance, and believes that all sources of uncertainty that make him/her ambiguous about the reference model (or reference prior) cannot lead to a model that differs from the reference model by more that a certain level. To obtain the set of priors which reflect ambiguity, using as the benchmark model the model of the reference priors (8), we consider measurable drift distortions to the reference priors. More specifically the initial Brownian motions, dz_i, $i = 1, 2$, of the stochastic system (8) are replaced by

$$z_i(t) = \widehat{z}_i(t) + \int_0^t \varepsilon_i(s)ds \quad , \; i = 1, 2 \tag{23}$$

where $\widehat{z}_i$ are Brownian motions and ε_i are measurable functions. By doing this, system (22) takes the form:

$$dx = Axdt + \Sigma REdt + \Sigma Rd\widehat{z} \text{ where } E = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}. \tag{24}$$

Following the same approach as in the proof of proposition 1, the evolution of species biomass under ambiguity (unmeasurable uncertainty) is given by:

$$x_t = e^{At}x_0 + \int_0^t e^{A(t-s)}\Sigma REds + \int_0^t e^{A(t-s)}\Sigma Rd\widehat{z}_s. \tag{25}$$

If we compare equation (25) with (13), it can be seen that the extra term, $\int_0^t e^{A(t-s)}\Sigma REds$, acts as a measure of precaution and reflects the impact of Knightian uncertainty or ambiguity, relative to the case of risk. This has as a result the introduction of two extra terms in equations (16) and (17). Therefore the expected values change depending on the structure of the problem's parameters.

In particular, when considering distortions in the benchmark model, the initial measure P is replaced by another probability measure Q. The discrepancy between the two measures is measured by the relative entropy, $R(Q//P) = \int_0^{+\infty} e^{-\delta t} E_Q[\frac{1}{2}\varepsilon_t^2]dt$. According to the k-ignorance framework, we consider the instantaneous relative entropy constraint[11] $Q(\tau) = \{Q : E_Q[\frac{1}{2}\varepsilon_t^2] \leq \tau$, for all $t\}$, which restricts the set of models the decision maker considers at each instant of time. This constraint means that the deviation between the reference prior and the distorted priors cannot be more than τ_i.[12] Then, the worst case perturbation is:

$$\varepsilon_{it}^* = -\sqrt{2\tau_i}. \tag{26}$$

It should be clear that (26) reflects the idea of the LFP with the multiple priors model. By adopting this approach the distortions are now constant negative numbers and therefore we can calculate the adjusted values. In particular, examining one of the possible cases of the signs of the matrix $e^{A(t-s)}\Sigma RE$ in the integral $\int_0^t e^{A(t-s)}\Sigma RE ds$ which reflects the impact of Knightian uncertainty on decision making, the following proposition can be stated:

Proposition 3 *Given a land allocation rule and a harvesting rule* (w, h_1, h_2), *the differences in expected values of the biomasses of species* $i = 1, 2$, *under ambiguity formalized in terms of k-ignorance, relative to the risk (measurable uncertainty) case, are given by* $\int_0^t e^{A(t-s)}\Sigma RE ds$.

Proof. Regarding the sign choice of the matrix $e^{A(t-s)}\Sigma RE$ we have that: [13]

$$if \quad e^{A(t-s)}\Sigma = \begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix}, E^* = \begin{bmatrix} -\sqrt{2\tau_1} \\ -\sqrt{2\tau_2} \end{bmatrix} \quad RE^* = \begin{bmatrix} -\sqrt{2\tau_1} \\ -\sqrt{2\tau_1}\rho - \sqrt{2\tau_2}\sqrt{1-\rho^2} \end{bmatrix}$$

$$e^{A(t-s)}\Sigma RE^* = \begin{bmatrix} \Theta^- - (\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})\Theta^- \ , \Theta^- < 0 \\ \Theta^+ - (\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})\Theta^+ \ , \Theta^+ > 0 \end{bmatrix}. \tag{27}$$

Therefore from equation (27) we obtain that if the term $(\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})$ is less than or equal to zero, then the second element of the matrix is positive

[11] This is in contrast to the robust control approach where we consider a lifetime constraint.

[12] This τ_i reflects the manager's beliefs about ambiguity and model uncertainty. If $\tau = 0$ then the manager is risk averse in the traditional sense and believes that the reference prior is an adequate representation of uncertainty.

[13] Depending on the values of the elements of matrix A, other possible cases can be examined. For the specific case of choices of signs in Proposition 4, which turns out to be the more interesting one, we provide numerical results.

and the first is negative. When $\tau_1 = k\tau_2$ the above condition is satisfied if $\sqrt{k}\rho + \sqrt{1-\rho^2} \leqslant 0$. Particularly if $k = 1$, the previous condition is satisfied if $\rho \leqslant -\frac{\sqrt{2}}{2}$. ∎

Condition (27) implies that under ambiguity aversion and given land allocation and harvesting rules, the path of the expected deviations from the desired steady state changes relative to the risk aversion case. In particular, when $\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2} \leqslant 0$, the expected value that corresponds to the first species can be lower, and the expected values corresponding to the second species can be higher as compared to the values obtained under risk aversion. In general, however, the final outcome depends on the type of correlation between the two biomasses and the regulator's beliefs about the worst case scenario. This could be of interest since it implies that our approach of dealing with ambiguity does not lead simply to wider values and uniformly more conservative behavior regarding both species, but takes into account the structure of the ecosystem and the relationship among species.

Further insights can be obtained by considering land allocation and harvesting rules which correspond to the *same* expected deviations for the biomasses under both risk aversion and ambiguity aversion. Consider a rule (w^r, h_1^r, h_2^r) under risk aversion (Proposition 2) and a rule (w^a, h_1^a, h_2^a) under ambiguity aversion (Proposition 4) which both attain the same safety standard for the same initial biomass values. Comparison of the rules could provide some quantitative measure associated with precaution. In particular deviations $|w^r - w^a|$, $|h_i^r - h_i^a|$ $i = 1, 2$, would quantify the impact of being precautious in terms of harvesting and land allocation. This is because they represent the necessary changes in the harvesting and land allocation that will preserve under a least favorable prior, the same amount of biomasses relative to the risk aversion case, which is the case where the manager is confident about the reference model.

3.3 Numerical Approximations

To obtain a better understanding of the structure of the solution of the above problem some numerical results are provided.[14] We assume that a suitable fixed land allocation rule $\hat{\mathbf{w}} = (\hat{w}, 1 - \hat{w})$ has been chosen such that $\hat{f}(w) = 1$

[14] The linearization which we adopt for our numerical simulations produces clearly interpretable results, but it should be noticed that these results hold in a bounded time interval, otherwise linearity might lead to unbounded solutions. Nevertheless even with this limitation the linear approximation provides clear insights into the effects of precaution. A full scale empirical implementation with appropriate curvature assumptions is beyond the scope of the present paper and represents an area for further research.

and $a_{21} = a_{12} = 0.7$.[15] For the harvesting rule at the deterministic case we assume $h_1^* = h_2^* = 0.5$. This rule implies that the desired deterministic steady state (DDSS) defined by (6) is $u_1^* = u_2^* = 0.2941$. This steady state is stable since the trace of the associated Jacobian matrix is negative, while the determinant is positive.

Table 1 below depicts the deviations of the expected values of the two biomasses from the DDSS $(0.2941, 0.2941)$ during three time periods, if harvesting of the species during these periods is kept at the level of $(h_1, h_2) = (0.5, 0.5)$.

	$t = 1$	$t = 2$	$t = 3$
$x_1 = u_1 - u_1^*$	0.4281	0.2597	0.1575
$x_2 = u_2 - u_2^*$	0.4281	0.2597	0.1575

Table 1: *Deviations of expected values for the two biomasses as a function of* t, where $h_1 = h_2 = 0.5$, and $DDSS = (u_1^*, u_2^*) = (0.2941, 0.2941), a_{21} = a_{12} = 0.7, \hat{f}_1(w) = 1$.

Table 1 indicates that the expected values of the two biomasses in the first three periods are $0.4281 + 0.2941 = 0.7222$, $0.2597 + 0.2941 = 0.5538$, and $0.1575 + 0.2941 = 0.4516$, respectively. Expected biomasses decline as they converge towards the DDSS according to the stability properties of this steady state.

We turn now to the case of Knightian uncertainty and precaution. Calculating the terms that correspond to the integral $\int_0^t e^{A(t-s)} \Sigma R E dt$, we can quantify the impact of precaution at the expected values of the two biomasses. We adopt the same parameter values as in table 1, that is $\hat{f}(w) = 1$, $a_{12} = a_{21} = 0.7$. Furthermore, we assume that the standard deviation is the same for the two biomasses, that is $\sigma_1 = \sigma_2 = 0.1$, that the correlation coefficient is $\rho = \sqrt{2}/2$, and that the parameter τ_i which reflects manager's ambiguity and 'maximum' ignorance is equal to 0.15 for each one of the two biomasses and each instant of time. Table 2 presents the changes in the deviations of the expected values of the two biomasses under ambiguity, relative to the traditional risk aversion case.

	$t = 1$	$t = 2$	$t = 3$
change in x_1	-0.0412	-0.0628	-0.0728
change in x_2	-0.0629	-0.1044	-0.1326

[15] Matlab has been used for numerical calculations.

Table 2: *Changes in the expected values for the two biomasses due to precaution as a function of* t where, $h_1 = h_2 = 0.5$, $\rho = \sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$.

Model uncertainty and ambiguity aversion induces a reduction in the expected values of the two biomasses relative to the risk aversion case. The results of table 2 can be interpreted in the following way. If the manager is ambiguity averse with ignorance parametrized by $\tau_i = 0.15$ and follows the harvesting rule $h_1 = h_2 = 0.5$, then in the first period expected biomasses will be 0.0412 and 0.0629 less than expected biomasses under risk for species 1 and 2 respectively. Thus taking into account that a worst case senario (or a least favorable prior), which is parametrized by the value of τ_i, may emerge because the manager is ambigous about his/her reference model, then the expected values of biomasses are less relative to the case where the manager is confident about the reference model and the worst case scenario is ruled out. The biomass evolution for the first three periods are shown in table 3.

	$t = 1$	$t = 2$	$t = 3$
$\mathcal{E}u_1$	0.681	0.491	0.379
$\mathcal{E}u_2$	0.6593	0.4494	0.319

Table 3: *Expected values for the two biomasses under ambiguity aversion,* where $h_1 = h_2 = 0.5$, $\rho = \sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$.

The convergence to the DDSS is faster relative to the risk aversion case, but eventually the expected biomasses will fall below the DDSS due to ambiguity. Thus under ambiguity aversion expected values tend to be less relative to risk aversion for the same level of harvesting.

A plausible question emerging from this result is: 'how much should the harvesting rule under ambiguity aversion change, relative to the risk aversion rule, so that expected biomasses under risk and ambiguity aversion will be the same?' The difference in harvesting between risk and ambiguity can be regarded as a measure of precaution in the following sense. If, because of ambiguity about the reference model, the manager is to take into account the possible emergence of a worst case scenario, then in order to keep expected biomasses at the the same level as if the reference model was known, harvesting should be changed by a certain amount relative to the harvesting when the reference scientific model is known. The necessary changes in harvesting in order to keep the same expected biomasses under different uncertainty structures, while land allocation is kept constant, are shown in table 4.

t	1	2	3
change in h_1	-0.0679	-0.1324	-0.2637
change in h_2	-0.0727	-0.1372	-0.2747
u_1^*	0.3275	0.3654	0.4341
u_2^*	0.3435	0.3814	0.4708

Table 4: *The impact of precaution in terms of harvesting and on steady state, where* $\rho = \sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$, $h_1 = h_2 = 0.5$, $n = 2$

Harvesting should be reduced in the first period by -0.0679 and -0.0727 for species 1 and 2 respectively, when the manager operates under model uncertainty and ambiguity aversion, so that expected values are the same as in the case where the manager operates without model uncertainty and he/she is just risk averse. In a sense these reductions can be regarded as reflecting the cost of been precautious, in terms of reduced harvesting, under model uncertainty. Since however harvesting changes, the DDSS, which is also the expected desired steady state (EDSS) for the biomasses which is implied by the new harvesting rules, changes. The sequence of new expected DDSS is shown in the two last rows of table 4. The results suggest that ambiguity aversion implies an increase in expected steady state biomass values relative to risk aversion. This can be regarded as the effect of precaution. Since the manager is ambiguous about the reference model, in order to take into account the emergence of a worst case scenario, harvesting should be reduced and expected biomass values should be increased relative to the case where scientific uncertainty does not exist. The specific structure of anbiguity implied by the k-ignorance approach allows the quantification of the precaution effect in terms of harvesting.

By keeping all the parameter values as above except for the value of the correlation coefficient which we set at $\rho = -\sqrt{2}/2$, we repeat the calculations leading to table 2. We present the new results in table 5 below.

	$t = 1$	$t = 2$	$t = 3$
change in x_1	-0.0478	-0.0848	-0.1147
change in x_2	0.0047	0.0156	0.0296

Table 5: *Changes at the expected values for the two biomasses due to precaution as a function of* t, where $h_1 = h_2 = 0.5, \rho = -\sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$

It can be seen that in this case precaution induces a reduction in the expected values only for the first biomass. For the second species there is an increase in the expected values. The impact of precaution, as defined in table 4, can be quantified in terms of harvesting units as shown in table 6 below.

t	1	2	3
change in h_1	-0.0326	-0.0580	-0.0944
change in h_2	-0.0237	-0.0444	-0.0732
u_1^*	0.3255	0.3469	0.3787
u_2^*	0.2958	0.3016	0.3081

Table 6: *The impact of precaution in terms of harvesting, where* $\rho = -\sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$, $h_1 = h_2 = 0.5$

In table 6 both harvesting and the DDSS move towards the same direction as in table 3, with the only difference being the magnitude of the change.

The analysis above suggests that ambiguity aversion induces a different EDSS than the one corresponding to risk aversion, which is in general higher. Another way of approaching the problem is to keep the EDSS fixed at the original level of $(u_1^*, u_2^*) = (0.2941, 0.2941)$, and calculate the changes in harvesting rates which under ambiguity aversion will provide the *same* expected values as in the case of risk aversion.

Using the results of tables 1 and 2 we obtain table 7.

	$t = 1$	$t = 2$	$t = 3$
change in h_1	-0.1259	-0.1889	-0.3443
change in h_2	-0.1337	-0.2001	-0.3630

Table 7: *The impact of precaution in terms of harvesting, where* $\rho = \sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$, $h_1 = h_2 = 0.5$, $n = 2$.

Using the results of tables 1 and 5 we obtain table 8.

	$t = 1$	$t = 2$	$t = 3$
change in h_1	-0.061	-0.0844	-0.1248
change in h_2	-0.043	-0.0607	-0.0928

Table 8: *The impact of precaution in terms of harvesting,* $\rho = -\sqrt{2}/2$, $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$, $h_1 = h_2 = 0.5$.

In order to explore the impact of the correlation coefficient ρ we derive the change in harvesting in order to keep constant the expected biomass values under different uncertainty structures. The results are shown in table 9, for $\rho = -0.9, -0.95, -0.99$ and $t = 1$. Table 9 depicts, for one time period and for various values of the corelation coefficient the impact on harvesting rates for keeping the *same* EDSS under ambiguity aversion as in the case of risk aversion.

$\rho \backslash t = 1$	Changes in (h_1, h_2)
-0.9	$(-0.0410, -0.0150)$
-0.95	$(-0.0336, -0.0047)$
-0.99	$(-0.0248, +0.0076)$

Table 9: *The impact of precaution in terms of harvesting where,* $\tau_i = 0.15$, $\sigma_i = 0.1$, $i = 1, 2$, $h_1 = h_2 = 0.5$.

It can be noticed from the last row of table 9 ($\rho = -0.99$) that if the manager wants to follow a precautionary principle and to keep the expected requirement within the same values as in the risk aversion case under certain values of our parameter space, harvesting should be reduced for the first species but increased for the second. Furthermore the associated steady state could have lower biomass values than the values proposed under risk Thus depending on the correlation among the two species biomasses, precaution could imply conservative behavior towards one species and aggressive behavior towards the other relative to risk aversion. It should be emphasized that this conservative/aggressive behavior keeps the expected values within the same levels as in the risk aversion case.

This result about conservative/aggressive behavior might be regarded as counterintuitive, since one expects precaution to induce uniformly conservative behavior, as has been detected in the areas of monetary policy (Onatski and Williams, 2003) and portfolio selection (Vardas and Xepapadeas, 2007). In the case examined in the present paper it is the very strong negative correlation coefficient that allows behavior to be aggressive regarding one species, since the emergence of a least favorable prior will move species biomasses in different directions.

3.4 Probabilistic Safety Minimum Standards under Risk and Uncertainty

Another way of approaching biodiversity management in terms of exogeneous safety minimum standards is to ask the question: ‘Under what harvesting and land allocation rules will species biomasses exceed a minimum level set exogeneously with a given probability'? This rule is formulated in the following proposition.

Proposition 4 *Given land allocation and harvesting rules* $(w_1, ..., w_n; h_1, ..., h_n)$, *upper and lower bounds can be determined for the probabilities that the biomasses of species* $i = 1, 2, ..., n$ *are higher than* $\frac{1}{\gamma}$ *of the initial biomasses values. The safety rules and the corresponding bounds are characterized by*

$$(\frac{A_{k1}}{l_{1k}} + A_{kk} + \frac{A_{kn}}{l_{nk}}) - \frac{1}{\gamma} \leq \Pr(x_{kt} > \frac{1}{\gamma} x_{k0}) \leq \gamma(\frac{A_{k1}}{l_{1k}} + A_{kk} + \frac{A_{kn}}{l_{nk}})$$
$$\text{with } l_{jk} = \frac{x_{k0}}{x_{j0}} \quad k = 1, ...n \quad j \neq k. \tag{28}$$

For n=2 species the above relationship takes the form

$$(A_1 + \frac{A_2}{l}) - \frac{1}{\gamma} \leq \Pr(x_{1t} > \frac{1}{\gamma} x_{10}) \leq \frac{\gamma}{x_{10}}(A_1 x_{10} + A_2 x_{20}) = \gamma(A_1 + \frac{A_2}{l}) \tag{29}$$

$$(lA_3 + A_4) - \frac{1}{\gamma} \leq \Pr(x_{2t} > \frac{1}{\gamma} x_{20}) \leq \frac{\gamma}{x_{20}}(A_3 x_{10} + A_4 x_{20}) = \gamma(lA_3 + A_4) \tag{30}$$

$$\text{where } l = \frac{x_{10}}{x_{20}}.$$

Proof. *Proofs follow directly from (8), using standard operations from probability theory.* ■

In expressions (29) and (30), A_i is defined as $A_i = A_i(w, h_1, h_2, t)$ and thus the associated probability bounds depend on the land allocation weights $(w, 1 - w)$ and on the harvesting rules (h_1, h_2). The land allocation and harvesting rule (w, h_1, h_2) that satisfies proposition 4 therefore provides a *probabilistic safety rule*, since it bounds the probability of having the biomasses at any point in time above the level $\frac{1}{\gamma} x_{i0}$, $i = 1, 2$. By choosing this level, that is by choosing $1/\gamma$, relations (29) and (30) can be used to determine a land allocation and a harvesting rule (w, h_1, h_2) for desired probability bounds. For example, a rule $(w, h_1, h_2)|_p^x$ could be specified such that the biomasses during the planning period exceed by $x\%$ the initial biomasses, with a probability that is between p and $p + \Delta p$. Thus $x\%$ can be regarded as a probabilistic SMS for keeping species from extinction with probability p. Then the rule $(w, h_1, h_2)|_p^x$ can be regarded as a probabilistic safety rule which may prevent the loss of biodiversity or the irreversible extinction of a species with a given probability.

A proposition similar to 4 can also be derived under model uncertainty and ambiguity aversion. The basic result is that to keep the same probabilitic SMS between risk and ambiguity aversion, harvesting in general should be reduced when model uncertainty exists. As indicated however by numerical results, there exist parameter constellations such that, when a strong negative correlation among species exists, the combined conservative/aggressive behavior noted in the previous section also emerges.

4 Optimal Harvesting Rules under Uncertainty: Risk vs Ambiguity

4.1 Optimal Harvesting under Risk

In the previous section we analyzed harvesting rules which would seek to secure SF and SMS for biodiversity preservation under alternative assumptions regarding the structure of uncertainty. In this section we turn to the derivation of optimal harvesting rules under alternative uncertainty structures. In particular we study the impact of model uncertainty and uncertainty, or ambiguity, aversion on optimal harvesting rules and we try to quantify precaution, measured as the deviation between optimal harvesting rules under uncertainty aversion relative to traditional risk aversion.

In the two species model, we consider the problem of choosing harvesting paths for a fixed land allocation $\mathbf{w} = (w, 1-w)$ which will maximize expected discounted benefits defined as:

$$\max_{\{h_1(t),h_2(t)\}} \mathcal{E}_0 \int_0^T e^{-\delta t} \left[\sum_{i=1,2} \left(\alpha_i h_i - \frac{1}{2}\beta_i h_i^2 \right) + \sum_{i=1,2} \left(\zeta_i u_i - \frac{1}{2}\xi_i u_i^2 \right) \right] dt \tag{31}$$

$$\begin{gathered} subject \;\; \text{to} \\ du_1 = u_1 \left(1 - u_1 - a_{12} u_2 - h_1\right) dt + \sigma_1(h) dz_1 \\ du_2 = \hat{f}\left(w\right) u_2 \left(1 - u_2 - a_{21} u_1 - h_2\right) dt + \sigma_2(h) dz_2. \end{gathered}$$

It should be noted that the objective function includes both consumptive benefits assosiated with harvesting and non-consumptive benefits, like existence values, assosiated with the levels of existing biomasses. Thus problem (31) can be regarded as the regulator's or the biodiversity manager's problem. For $\zeta_i = \xi_i = 0$ the problem can be associated with a private agent who does not attach any welfare weights to existing biomass but cares only about consumptive benefits.

We start by analyzing first the deterministic solution, which will be used as a benchmark. In the deterministic case where $\sigma_i = 0$ for $i = 1, 2$, the current value Hamiltonian function is defined as:

$$\begin{aligned} G &= J + p_1 F^1 + p_2 F^2 \\ &\quad \textit{with} \\ J &= \sum_{i=1,2} \left(\alpha_i h_i - \frac{1}{2} \beta_i h_i^2 \right) + \sum_{i=1,2} \left(\zeta_i u_i - \frac{1}{2} \xi_i u_i^2 \right) \\ F^1 &= u_1 \left(1 - u_1 - a_{12} u_2 - h_1 \right) \\ F^2 &= \hat{f}\left(w \right) u_2 \left(1 - u_2 - a_{21} u_1 - h_2 \right). \end{aligned} \tag{32}$$

Pontryagin's maximum principle implies the following set of optimality conditions:

$$a_i - \beta_i h_i - p_i f_i u_i = 0 \ , i = 1, 2 \ , f_1 = 1, f_2 = \hat{f}\left(w \right) \tag{33}$$

$$\zeta_1 - \xi_1 u_1 + \dot{p_1} + \delta p_1 + p_1 f_1 (1 - u_1 - a_{12} u_2 - h_1) - p_1 f_1 u_1 - p_2 f_2 u_2 a_{21} = 0 \tag{34}$$

$$\zeta_2 - \xi_2 u_2 + \dot{p_2} + \delta p_2 + p_2 f_2 (1 - u_2 - a_{21} u_1 - h_2) - p_2 f_2 u_2 - p_1 f_1 u_1 a_{12} = 0 \tag{35}$$

$$f_1 u_1 (1 - u_1 - a_{12} u_2 - h_1) - u_1 = 0 \tag{36}$$

$$f_2 u_2 (1 - u_2 - a_{21} u_1 - h_2) - u_2 = 0. \tag{37}$$

The optimal deterministic short-run harvesting rules are obtained by solving (33), for h_i , $i = 1, 2$ as:

$$h_i = \frac{a_i - p_i f_i u_i}{\beta_i}. \tag{38}$$

Substituting (38) into (34)-(37) we obtain the modified Hamiltonian dynamic system. The steady state of this system determines the optimal long-run equilibrium for biomasses. An optimal steady state with preservation of both species is characterized by $\mathbf{u}^* = (u_1^*, u_2^*) > (0, 0)$. Assume that such a steady state exists. Using (36)-(37), we can solve for

$$u_i^* = \frac{(1-\frac{1}{f_1}-\frac{a_1}{\beta_1})(1-\frac{p_2 f_2}{\beta_2}) - a_{ij}(1-\frac{1}{f_2}-\frac{a_2}{\beta_2})}{\prod_{i=1,2}(1-\frac{p_i f_i}{a_i}) - a_{12}a_{21}} \quad i = 1, 2. \tag{39}$$

Substituting (39) into (34)-(35), we can solve for the steady state costate vector $\mathbf{p}^* = (p_1^*, p_2^*)$ and in the sequence using (39),(38) we can obtain the optimal steady state harvesting $\mathbf{h}^* = (h_1^*, h_2^*)$.

Assume that the modified Hamiltonian dynamic system (34)-(37) has a steady state solution $(u_1^*, u_2^*, p_1^*, p_2^*)$ which is a local saddle point, and let $(\boldsymbol{\chi}, \boldsymbol{\gamma}, \boldsymbol{\eta}) = (\mathbf{u} - \mathbf{u}^*, \mathbf{h} - \mathbf{h}^*, \mathbf{p}^* - \mathbf{p})$, with $\mathbf{u} = (u_1, u_2), \mathbf{h} = (h_1, h_2), \mathbf{p} = (p_1, p_2)$ denote deviations from the steady state.

To obtain tractable and interpretable results for the stochastic case, we use Magill's (1977) method for replacing a nonlinear stochastic optimal control problem by its linear-quadratic approximation around the deterministic steady state.

Taking a first order linear approximation of the stochastic differential equations given by (7) around the optimal deterministic steady state $(\mathbf{u}^*, \mathbf{h}^*, \mathbf{p}^*)$ and following Magill (1977) problem (31) is replaced by:

$$\max_{\gamma} \mathcal{E}_{\chi(0)} \int_0^{+\infty} e^{-\delta t} L^0(\chi, \gamma) dt \tag{40}$$

$$\textit{subject to}$$

$$d\chi_1 = (\Lambda_*^{11}\chi_1 + \Lambda_*^{12}\chi_2 + M_*^1\gamma_1)dt + \sigma_1 dz_1$$

$$d\chi_2 = (\Lambda_*^{22}\chi_1 + \Lambda_*^{21}\chi_2 + M_*^2\gamma_2)dt + \sigma_2 dz_2, or$$

$$d\boldsymbol{\chi} = (\Lambda\boldsymbol{\chi} + M\boldsymbol{\gamma})dt + \Sigma dz, \tag{41}$$

$$\boldsymbol{\chi} = \begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix}, \Lambda = \begin{bmatrix}\Lambda_*^{11} & \Lambda_*^{12}\\ \Lambda_*^{21} & \Lambda_*^{22}\end{bmatrix}, \Sigma = \begin{bmatrix}\sigma_1 & 0\\ 0 & \sigma_2\end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix}$$

$$M_*^1 = f_1 u_1^*, M_*^2 = -f_2 u_2^*, M = \begin{bmatrix}M_*^1 & 0\\ 0 & M_*^2\end{bmatrix}, dz = \begin{bmatrix}dz_1\\ dz_2\end{bmatrix}.$$

$$with\ \Lambda_*^{11} = f_1\left(1 - 2u_1^* - a_{12}u_2^* - h_1^*\right), \Lambda_*^{12} = -f_1 a_{12} u_1^*, \sigma_1 = \sigma_1(h^*)$$

$$\Lambda_*^{22} = f_2\left(1 - 2u_2^* - a_{21}u_1^* - h_2^*\right), \Lambda_*^{21} = -f_2 a_{21} u_2^*, \sigma_2 = \sigma_2(h^*)$$

where

$$L^0(\boldsymbol{\chi}, \boldsymbol{\gamma}) = \frac{1}{2}\begin{bmatrix}\boldsymbol{\chi}\\ \boldsymbol{\gamma}\end{bmatrix}' \begin{bmatrix}A & N\\ N' & B\end{bmatrix} \begin{bmatrix}\boldsymbol{\chi}\\ \boldsymbol{\gamma}\end{bmatrix} \tag{42}$$

with

$$
\begin{bmatrix} A & N \\ N' & B \end{bmatrix} = \{ \begin{bmatrix} J_{\mathrm{uu}}(u^*, h^*) & J_{uh}(u^*, h^*) \\ J_{hu}(u^*, h^*) & J_{hh}(u^*, h^*) \end{bmatrix} + \tag{43}
$$
$$
\sum_{i=1,2} p_i^* \begin{bmatrix} F^i_{\mathrm{uu}}(u^*, h^*) & F^i_{uh}(u^*, h^*) \\ F^i_{hu}(u^*, h^*) & F^i_{hh}(u^*, h^*) \end{bmatrix} \}.
$$

In this case the Hamilton-Jacobi-Belman (HJB) equation implies, for the value function V, that:[16]

$$
\delta V = \max_{\boldsymbol{\gamma}} \left\{ L^0(\boldsymbol{\chi}, \boldsymbol{\gamma}) + V_{\boldsymbol{\chi}}(\Lambda \boldsymbol{\chi} + M\boldsymbol{\gamma}) + \frac{1}{2} trace(\Sigma^T \partial^2 V \Sigma) \right\} \tag{44}
$$
$$
where, V\boldsymbol{\chi} = \begin{bmatrix} V_{\chi_1} & V_{\chi_2} \end{bmatrix}, \partial^2 V = \begin{bmatrix} V_{\chi_1\chi_1} & V_{\chi_1\chi_2} \\ V_{\chi_2\chi_1} & V_{\chi_2\chi_2} \end{bmatrix}.
$$

Since problem (40) is a linear quadratic problem, the value function should be linear quadratic as well, of the form:

$$
V(\boldsymbol{\chi}, t) = \frac{1}{2} \boldsymbol{\chi}' Q \boldsymbol{\chi} + r.
$$

Then the maximizer γ satisfies the folowing relationship:

$$
\boldsymbol{\gamma}^* = -B^{-1}(M'Q' + N')\boldsymbol{\chi}. \tag{45}
$$

Substituting this value in (44) we obtain after manipulations that:

$$
r = \frac{1}{2\delta} trace(\Sigma' Q' \Sigma).
$$

Matrix Q can be detrmined by the following matrix equation

$$
\frac{1}{2}(NB^{-1}N' + A) = -\frac{1}{2} N(B^{-1} - (B^{-1})')M'Q' + \tag{46}
$$
$$
Q(\Lambda - \delta \frac{I}{2} - MB^{-1}N') + QM(\frac{1}{2}(B^{-1})' - B^{-1})M'Q.
$$

After the determination of matrix Q, the optimal harvesting rule $\boldsymbol{\gamma}^*$ can be obtained from (45).

[16] We use either the notation T *or* $\prime$, to denote the transpose of a matrix.

4.2 Optimal Harvesting under Uncertainty: A Robust Control Approach

Following Hansen and Sargent (2001), problem (40) can be regarded as a benchmark model. If the decision maker was sure about the benchmark model, then there would be no concerns regarding scientific uncertainty and model misspecification. In such a case the solutions derived in the previous section would have been adequate for characterizing the optimal harvesting rule. If however there are concerns about model uncertainty, the decision making framework needs to account for uncertainty or ambiguity aversion. Model uncertainty in this case is modelled by a family of stochastic perturbations, so that:

$$z_i(t) = \hat{z}_i(t) + \int_0^t \omega_i(s)ds \ \ , \ i = 1, 2 \tag{47}$$

where $\{\hat{z}_i(t) : t \geq 0\}$ are Brownian motions and $\{\omega_i(t) : t \geq 0\}$ are measurable drift distortions.

Consider again the first-order linear approximation around the optimal deterministic stationary state $(\mathbf{u}^*, \mathbf{h}^*, \mathbf{p}^*)$. Then the dynamics of our system take the form:[17]

$$\begin{aligned} d\chi_1 &= [f_1 (1 - 2u_1^* - a_{12}u_2^* - h_1^*) \chi_1 - f_1 a_{12} u_{11}^* \chi_2 - f_1 u_1^* \gamma_1 + \omega_1 \sigma_1] \, dt + \\ &\quad \sigma_1 d\widehat{z}_1 \\ d\chi_2 &= [f_2 (1 - 2u_2^* - a_{21}u_1^* - h_2^*) \chi_2 - f_2 a_{21} u_2^* \chi_1 - f_2 u_2^* \gamma_2 \\ &\quad + \rho\omega_1\sigma_2 + \omega_2 \sqrt{1-\rho^2}\sigma_2 \Big] \, dt + \sigma_2 \rho d\widehat{z}_1 + \sqrt{1-\rho^2}\sigma_2 d\widehat{z}_2. \end{aligned} \tag{48}$$

Under model misspecification, a multiplier robust control problem (Hansen et al., 2002) can be associated with the problem of maximizing discounted benefits under model uncertainty. This problem can be written in the linear quadratic approximation form as:

$$\begin{aligned} V(\theta_1, \theta_2, u_1, u_2) &= \max_{h_i} \min_{\omega_i} \mathcal{E}_0 \int_0^\infty e^{-\delta t} J_{robust} dt \ , \ J_{robust} = \left[L^0(\boldsymbol{\chi}, \boldsymbol{\gamma}) + \sum_{i=1,2} \theta_i \frac{\omega_i^2}{2} \right] \\ &\text{subject to (48).} \end{aligned} \tag{49}$$

In this optimization problem 'Nature' acts as a 'mean' agent seeking to

[17] Using the correlation coefficient matrix R as in equation (22), we obtain independent distorted Brownian motions.

'choose' the worst possible distortion. Thus the manager's obgective is maximin: 'Choose the harvesting rule which maximizes discounted net benefits by taking into account the fact that the benchmark model could be misspecified and biomass growth might be far less than the one suggested by the benchmark model.' Using matrix notation the problem can be written as:

$$\max_{\boldsymbol{\gamma}} \min_{\boldsymbol{\varpi}} \mathcal{E} \int_0^{+\infty} e^{-\delta t} J_{robust} dt \tag{50}$$

subject to

$$d\boldsymbol{\chi} = (\Lambda \boldsymbol{\chi} + M\boldsymbol{\gamma} + \Omega_\rho \boldsymbol{\varpi})dt + \Omega_\rho d\hat{z}, with\ d\hat{z} = \begin{bmatrix} d\hat{z}_1 \\ d\hat{z}_2 \end{bmatrix}, \Omega_\rho = \begin{bmatrix} \sigma_1 & 0 \\ \sigma_2 \rho & \sqrt{1-\rho^2}\sigma_2 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \Lambda_*^{11} & \Lambda_*^{12} \\ \Lambda_*^{21} & \Lambda_*^{22} \end{bmatrix}, M_*^1 = -f_1 u_1^*, M_*^2 = -f_2 u_2^*, M = \begin{bmatrix} M_*^1 & 0 \\ 0 & M_*^2 \end{bmatrix}, \boldsymbol{\varpi} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

$$\Lambda_*^{11} = f_1 \left(1 - 2u_1^* - a_{12} u_2^* - h_1^*\right), \Lambda_*^{12} = -f_1 a_{12} u_1^*, \sigma_1 = \sigma_1(h^*)$$

$$\Lambda_*^{22} = f_2 \left(1 - 2u_2^* - a_{21} u_1^* - h_2^*\right), \Lambda_*^{21} = -f_2 a_{21} u_2^*, \sigma_2 = \sigma_2(h^*).$$

In the above problem θ_i for $i = 1, 2$ denotes the robustness parameters which reflects the intensity of concerns about model misspecification for the biomasses dynamics. A value of $\theta_i = \infty$ indicates the manager is confident about the benchmark model and he/she is not concerned about possible model misspecification, with no preference for robustness. This case can be regarded as the traditional risk aversion case. A value of $\theta_i = 0$ indicates that there is no knowledge about the initial model.

Using the results of Fleming and Souganidis (1989) regarding the existence of a recursive solution to the multiplier problem, Hansen et al. (2002) show that problem (50) can be transformed into a stochastic infinite horizon two-player game between the biodiversity manager and Nature. Nature plays the role of a "mean agent" and chooses a reduction $\boldsymbol{\varpi}$ in the mean return of biomasses to reduce the agent's revenue function.

The Bellman-Isaacs condition for this game implies that the value function V satisfies the following equation:

$$\delta V = \max_{\boldsymbol{\gamma}} \min_{\boldsymbol{\varpi}} \left\{ J_{robust} + \partial V(\Lambda \boldsymbol{\chi} + M\boldsymbol{\gamma} + \Omega_\rho \boldsymbol{\varpi}) + \frac{1}{2} trace(\Omega_\rho^T \partial^2 V \Omega_\rho) \right\} \tag{51}$$

$$\partial V = \begin{bmatrix} V\chi_1 & V\chi_2 \end{bmatrix}, \partial^2 V = \begin{bmatrix} V_{\chi_1\chi_1} & V_{\chi_1\chi_2} \\ V_{\chi_2\chi_1} & V_{\chi_2\chi_2} \end{bmatrix}.$$

As in the risk aversion case examined in the previous section, the value function should be quadratic of the form:

$$V(\boldsymbol{\chi}) = \frac{1}{2}\boldsymbol{\chi}' Q \boldsymbol{\chi} + r.$$

Solving initially for $\boldsymbol{\varpi}$, the distortion chosen by 'Nature', and taking into account the expression of J_{robust}, we obtain:

$$-\Theta^{-1}\Omega'_{\rho} Q' \boldsymbol{\chi} = \boldsymbol{\varpi}, with\ \Theta^{-1} = \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix}.^{18} \tag{52}$$

Optimal harvesting is determined by the the maximizer of (51), which is:

$$\boldsymbol{\gamma}_U^* = -B^{-1}(M'Q' + N')\boldsymbol{\chi}. \tag{53}$$

Substituting the values of (53),(52) into (51), we can initially determine r, Q through the relationships

$$\frac{1}{2}(NB^{-1}N' + A) = -\frac{1}{2}N(B^{-1} - (B^{-1})')M'Q' + \tag{54}$$

$$Q(\Lambda - \delta\frac{I}{2} - MB^{-1}N') + QM(\frac{1}{2}(B^{-1})' - B^{-1})M'Q' - \frac{1}{2}Q(\Omega_{\rho}\Theta^{-1}\Omega'_{\rho})Q,$$

$$r = \frac{1}{2\delta} trace(\Omega'_{\rho} Q' \Omega_{\rho}). \tag{55}$$

Then using (53), we can solve for optimal harvesting $\boldsymbol{\gamma}_U^*$.

4.3 Quantifying the Precautionary Principle

Relationships (45), (46) and (53), (54) characterize optimal harvesting under risk aversion and ambiguity aversion respectively. The impact of the change in the structure of uncertainty, from a single prior to multi priors, is embodied in matrix Q determined by (46) for the risk aversion case and (54) for the ambiguity aversion case. Comparing relationships (46) and (54), it can be seen that in (54), relative to (46), there is only one extra term, $-\frac{1}{2}Q(\Omega_{\rho}\Theta^{-1}\Omega'_{\rho})Q'$.This term quantifies the concerns about model uncertainty and indicates a different harvesting rule relative to the one suggested under the risk aversion case. It can therefore be regarded as reflecting precaution. This extra term can be

[18] $\frac{1}{2}\boldsymbol{\varpi}'\ \Theta\ \varpi = \sum_{i=1,2}\theta_i\frac{\omega_i^2}{2}$.

written as:

$$Z = -\frac{1}{2}Q(\Omega_{\rho}\Theta^{-1}\Omega'_{\rho})Q' = -\frac{1}{2}Q\begin{bmatrix} \frac{\sigma_1^2}{\theta_1} & \frac{\sigma_1\sigma_2\rho}{\theta_1} \\ \frac{\sigma_1\sigma_2\rho}{\theta_1} & \frac{(1-\rho^2)\sigma_2^2}{\theta_2} \end{bmatrix} Q'. \quad (56)$$

Depending on $\Omega_{\rho}\Theta^{-1}\Omega'_{\rho}$, that is, on the magnitude of the parameters σ_i, ρ, θ_i, through (56) the elements of the matrix Q in the robust control (ambiguity aversion) case will have different values, and will indicate a different harvesting rule than the rule emerging from standard risk aversion case. It is clear that if $\theta_i \to \infty, i = 1, 2$, that is there is no concern for model uncertainty, $Z = 0$, and the optimal harvesting rules under risk aversion and ambiguity aversion coincide. In this case precaution vanishes and only adjustments for traditional risk (measurable uncertainty) affect the decision rule. Thus for finite θ_i we have

$$PR = \|\boldsymbol{\gamma}^* - \boldsymbol{\gamma}_U^*\| \neq 0.$$

This deviation can be regarded as the quantification of precaution, since it measures the deviations between optimal harvesting rules under risk aversion and ambiguity aversion.[19]

5 Concluding Remarks

We introduce the conceptual frameworks of multiple priors in order to analyze unmeasurable Knightian uncertainty (or ambiguity) which, given the multiple types of uncertainty characterizing ecosystems, might be regarded as a more appropriate framework relative to the classic risk case (measurable uncertainty). We believe that this approach can be regarded as a formal way of modelling the precautionary principle and providing policy rules for biodiversity management under model uncertainty and precaution. We specify the multiple priors framework using the k-ignorance and the robust control approaches, which are associated with decision making under uncertainty or ambiguity aversion, in the context of least favorable priors and maxmin criteria.

First, we apply the k-ignorance approach to a descriptive non-optimizing dynamic model of interacting species and we provide safety standards through land allocation and harvesting rules which could guarantee that species will

[19] A similar result can be obtained if we choose optimal harvesting by using the multiple prior structure implied by k-ignorance, with the worst case perturbation defined as $\varepsilon_{it}^* = -\sqrt{2\tau_i}u_i$.

not become extinct under scientific uncertainty and ambiguity. We solve the problem both under risk aversion and under uncertainty or ambiguity aversion and, by comparing solutions, we provide a measure of the impact from adopting a precautionary approach. By considering a simplified linearized version of the general model, we obtain numerical results which confirm and quantify our theoretical findings and we show that the cost of being precautious can be quantified in terms of reduced harvesting. Rules could indicate, depending on the type of species interactions, conservative behavior towards one group of species and aggressive towards another. Furthermore, we provide land allocation and harvesting rules for keeping biomasses above some minimum safety level with a given probability.

Second we consider an optimizing framework where robust control methods are used to specify multiple priors approaches and maxmin optimal harvesting rules. We compare solutions under risk and under uncertainty aversion and show how a measure of precaution can be formulated.

It should also be noted that the impact of ambiguity depends on the subjective parameters, τ for k-ignorance and θ for *robust control*, which represent the manager's beliefs regarding possible deviations from the reference model and the structure of least favorable priors. Although these parameters are subjective, their effects can be traced by considering a set of solutions for different values of these parameters, since land allocation or harvesting rules are a function of either τ or θ, depending on the case. For $\tau = 0$ or $\theta = \infty$, rules under ambiguity are the same as rules under tradional risk aversion and incentives for precautionary behavior vanish.

Our conceptual framework can be extended along two possible lines. The conceptual framework of Knightian uncertainty or ambiguity can be extended to formal prey-predator or mechanistic resource-based models of species competition, along with numerical simulation to obtain a sense of the quantitative results. Finally, it might be worth exploiting the possibility of combined presence of measurable (risk) and unmeasurable (ambiguity) uncertainty in models described by two qualitatively different but interrelated dynamic systems. These could be, for example, coevolutionary models where population dynamics which evolve in a fast time scale are characterized by measurable uncertainty and a single prior, while trait dynamics which evolve in slow time are characterized by unmeasurable uncertainly and multiple priors.

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