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Abstract. This paper shows that a decision maker using the CAPM for valuing firms and making decisions may contradict Modigliani and Miller's Proposition I, if he adopts the widely-accepted disequilibrium NPV. As a consequence, CAPM-minded agents employing this NPV are open to arbitrage losses and miss arbitrage opportunities. As a result, even though the use of the disequilibrium NPV for decision-making is deductively drawn from the CAPM, its use for both valuation and decision should be rejected.

Keywords. Firm value, Free Cash Flow, CAPM, Modigliani and Miller's Proposition I, Net Present Value, disequilibrium, arbitrage, decision making.

JEL code. G31, G32, G11, G12, D46, M21

Introduction

In the corporate finance literature, the classical Capital Asset Pricing Model (henceforth CAPM) is employed for firm valuation as well as capital budgeting decision making (e.g. Rubinstein, 1973; Rao, 1992; Brealey and Myers, 2000; Damodaran, 2001; Fernández, 2002, or any other finance textbook). The value of a firm is given by the discounted free cash flows, where the discount rate is the risk-adjusted cost of capital. The latter is computed by making use of the CAPM pricing relation and equals the risk-free rate plus the product of the beta times the difference between expected market rate of return and risk-free rate of return. The value is the maximum price the buyer is ready to pay for the firm. If the firm's actual price is smaller that the firm value (or, in other terms, if the NPV is positive), then the purchase of the firm is a profitable investment.

It is common in corporate finance to use the CAPM-derived *disequilibrium* NPV to appraise firms and investments (e.g. Bossaerts and Ødegaard, 2001). In the disequilibrium NPV the value depends on the disequilibrium beta, which in turn depends on the project's *cost*. Magni (2007b) shows that the disequilibrium beta is validly deducted from the CAPM for decision making and that the use of the disequilibrium NPV is extensive in corporate finance.

This paper shows that the *disequilibrium* NPV is inconsistent with Modigliani and Miller's (1958) Proposition I (MM-I), which asserts that a firm value is invariant under changes in the debt/equity mix. This implies that the principle of arbitrage, a fundamental principle of economic rationality (Nau and McCardle, 1991; Nau, 1999) is not fulfilled and that the disequilibrium Net Present Value is nonadditive.

The paper is structured as follows. Section 1 makes use of a simple example where two firms generating the same free cash flow are valued; one of them is levered, the other one is unlevered, and it turns out that the disequilibrium CAPM-based NPV provides different values. Section 2 shows that economic agents using this NPV are open to arbitrage losses. In sections 3-4 MM-I is applied to value the two firms. Some remarks on nonadditivity are presented in section 5. The concluding section briefly summarizes the results.

Notational conventions of the paper are collected in Table 1. All the examples refer, for simplicity, to one period, but the same results hold for perpetual cash flows. All numbers are rounded off to the second or third decimal.

1. The example

Consider the security market described in Table 2, where a risky asset and a risk-free asset are traded and two possible states may occur, conventionally labeled 'good' and 'bad', with probability 0.8 and 0.2 respectively.[1] The market is complete, is assumed to be in equilibrium (all marketed assets lie on the SML) and arbitrage is not possible. Suppose now that economic agent B (=buyer) faces the following problem: He is offered the opportunity of purchasing one of two firms, both of which will operate only the next period and then will shut down. One of the firm is equity-financed (firm U), the other one is levered (firm L). Agent U (=unlevered) owns the shares of firm U and is ready to sell the firm at a minimum price of 9000. Firm L's shares are owned by agent E (=equity) which is ready to sell the shares for a minimum of 7000, while agent D (=debt) owns firm L's debt, which is a loan just stipulated for an amount of 2000 with a 7.14% contractual rate. For such a loan agent D is ready to accept not less than the 2000 just lent to the firm. Agent B is willing to evaluate the two firms and decide about possible purchase. To this end, he analyzes the two firms and after thorough investigations, studies and forecasts, he collects the following data (see Table 3):

- The Free Cash Flow of both firms at time 1 will be 13300 in good state and 7800 in bad state[2]
- The Debt Cash Flow of firm L at time 1 will be 7500=7000(1.0714). Given the forecasts on the Free Cash Flow, the debt is not risky.
- The Equity Cash Flow of firm L at time 1 will be consequently 5800 in good state and 300 in bad state.

[1] The market is a very simple one just for the sake of convenience (for a market with three securities and some results dealing with capital budgeting decisions see Magni, 2007a, 2007b).
[2] The free cash flow of firm U is obviously an equity cash flow, given that the firm is unlevered.

Agent B applies the disequilibrium NPV. To value the firms, he needs the beta of firm U as well as the betas of both equity and debt of firm L. But the betas are functions of the rates of return and the latter are in turn functions of the outlay required for receiving the cash flows. In general, if P_U, P_e, P_D are the costs for acquiring firm U's equity, firm L's equity, and firm L's debt, respectively, we have:

$$
\begin{aligned}
\tilde{r}_U &= \frac{\mathrm{FCF}^*}{P_U} - 1, \\
\tilde{r}_e &= \frac{\mathrm{ECF}^*}{P_e} - 1 \\
\tilde{r}_D &= \frac{\mathrm{CFD}^*}{P_D} - 1.
\end{aligned}
\tag{1}
$$

The betas are then

$$
\begin{aligned}
\beta_U &= \frac{\mathrm{cov}(\tilde{r}_U, \tilde{r}_m)}{\sigma_m^2} = \frac{\mathrm{cov}\left(\frac{\mathrm{FCF}^*}{P_U} - 1, \tilde{r}_m\right)}{\sigma_m^2} = \frac{1}{\sigma_m^2 P_U}\mathrm{cov}\left(\mathrm{FCF}^*, \tilde{r}_m\right) \\
\beta_e &= \frac{\mathrm{cov}(\tilde{r}_e, \tilde{r}_m)}{\sigma_m^2} = \frac{\mathrm{cov}\left(\frac{\mathrm{ECF}^*}{P_e} - 1, \tilde{r}_m\right)}{\sigma_m^2} = \frac{1}{\sigma_m^2 P_e}\mathrm{cov}\left(\mathrm{ECF}^*, \tilde{r}_m\right) \\
\beta_D &= \frac{\mathrm{cov}(\tilde{r}_D, \tilde{r}_m)}{\sigma_m^2} = \frac{\mathrm{cov}\left(\frac{\mathrm{CFD}^*}{P_D} - 1, \tilde{r}_m\right)}{\sigma_m^2} = \frac{1}{\sigma_m^2 P_D}\mathrm{cov}\left(\mathrm{CFD}^*, \tilde{r}_m\right)
\end{aligned}
\tag{2}
$$

The further step is to compute the required rates of return using the SML (Security Market Line) equation:

$$
\begin{aligned}
k_U &= r_f + \beta_U (r_m - r_f) \\
k_e &= r_f + \beta_e (r_m - r_f) \\
k_D &= r_f + \beta_D (r_m - r_f).
\end{aligned}
\tag{3}
$$

Substituing (2) in (3) we have

$$\begin{aligned}
k_U &= r_f + \frac{(r_m - r_f)}{\sigma_m^2 P_U}\operatorname{cov}\left(\text{FCF}^*, \tilde{r}_m\right) \\
k_e &= r_f + \frac{(r_m - r_f)}{\sigma_m^2 P_e}\operatorname{cov}\left(\text{ECF}^*, \tilde{r}_m\right) \\
k_D &= r_f + \frac{(r_m - r_f)}{\sigma_m^2 P_D}\operatorname{cov}\left(\text{CFD}^*, \tilde{r}_m\right)
\end{aligned} \tag{4}$$

so that the values are

$$\begin{aligned}
V_U &= \frac{\text{FCF}}{1+k_U} = \frac{\text{FCF}}{1 + r_f + \frac{(r_m - r_f)}{\sigma_m^2 P_U}\operatorname{cov}\left(\text{FCF}^*, \tilde{r}_m\right)} \\
E &= \frac{\text{FCF}}{1+k_e} = \frac{\text{ECF}}{1 + r_f + \frac{(r_m - r_f)}{\sigma_m^2 P_e}\operatorname{cov}\left(\text{ECF}^*, \tilde{r}_m\right)} \\
D &= \frac{\text{CFD}}{1+k_D} = \frac{\text{CFD}}{1 + r_f + \frac{(r_m - r_f)}{\sigma_m^2 P_D}\operatorname{cov}\left(\text{CFD}^*, \tilde{r}_m\right)}.
\end{aligned} \tag{5}$$

Applying (5) to the particular case at hand, agent B finds (see Table 3)

$$V_U = 9150,\ E = 2380,\ D = 6521.$$

This valuation contradicts MM-I, since

$$V_U = 9150 \neq 2380 + 6521 = 8901 = V_L.$$

Financially, agent B commits a nonsense: He faces two equivalent assets generating the cash-flow stream ($-P_U$, FCF). Yet, agent B values them in different ways.[3]

2. Arbitrage losses

Not only is agent B irrational in that he computes different values for financially equivalent alternatives, but he is also susceptible to arbitrage losses. Let us see. Suppose, for the sake of

[3] It is worth noting that this striking result does not depend on the fact that the debt rate is different from the risk-free rate. Even if we had $r_D = r_f$ the valuation would not be consistent with MM-I. Assuming $r_D = 15\%$ we have $P_D = 6521 = D$ and using eq. (5) we still have $V_U = 9150 \neq 8901 = V_L$.

convenience, that a single agent DEU owns shares and debt of both firms U and L.[4] Agent DEU offers agent B the following course of action:

"We borrow 9149 from you and will repay the amount FCF* at time 1".

Agent B accepts, since

$$-9149+\frac{\text{FCF}}{1+k_U}=-9149+V_U=-9149+9150=1>0$$

Agent DEU then offers agent B another course of action:

"We lend you 2479 and you will repay us the amount ECF* at time 1".

Agent B accepts again, since

$$2479-\frac{\text{ECF}}{1+k_e}=2479-\frac{0.8(5800)+0.2(300)}{1+0.975}=2479-2380=99>0$$

Finally, agent DEU offers agent B the following course of action:

"We lend you 6620 and you will repay us the amount CFD at time 1".

Again, agent B accepts, since

$$6620-\frac{\text{CFD}}{1+k_D}=6620-\frac{7500}{1+0.15}=6620-6521=99>0.$$

But so doing, agent B is trapped in an arbitrage loss (while agent DEU realizes an arbitrage profit): He spends 50 today and receives nothing at time 1 (the cash flows for agent B are collected in Table 4. Agent DEU's cash flow are the same reversed in sign).

3. Firm value according to MM-I

Let us calculate the firm value using MM-I, therefore making use of the principle of arbitrage. As for firm U, consider a portfolio of 55 shares of the risky security and 36.739 units of the risk-free asset. The value of such a portfolio today is 9173.9=55(100)+36.739(100). At time 1, the owner of such a portfolio will receive 13300=55(165)+36.739(115) in the good state and 7800=55(65)+36.739(115) in the bad state. This portfolio replicates firm U's free cash flow.

[4] This is not restrictive at all. We could have keep on dealing with agents D, U and E, but a single representative agent DEU makes presentation simpler and shorter.

Therefore, the one-price law leads us to $V_U = 9173.9$. Also, consider a portfolio consisting of a long position on the risky asset (55 shares) and a short position on the risk-free security (28.478 units). Its value is $2652.2 = 55(100) - 28.478(100)$. Such a portfolio replicates firm L's equity cash flow: 5800=55(165)−28.478(115) in the good state, and 300=55(65)−28.478(115) in the bad state. Accordingly, the one-price law tells us that the value of firm L's equity $E = 2652.2$.

Finally, consider a portfolio consisting of 65.217 units of the risk-free asset. Its value is $6521.7 = 65.217(100)$. Such a portfolio replicates the debt cash flow of firm L: 7500=65.217(115) in both states, so that the debt value is $D = 6521.7$.

Consequently, we have

$$V_U = 9173.9 = 2652.2 + 6521.7 = E + D = V_L.$$

To sum up, the CAPM-based values of firm U and firm L do not coincide each other and both are inconsistent with the (unique) value found via arbitrage pricing (i.e. via MM-I), which guarantees that the one-price law holds.

It is also noteworthy that agent B is missing an arbitrage opportunity. He actually rejects to purchase firm L (equity+debt). But he could sell short 36.739 units of the risk-free security and 55 shares of the risky security, while buying firm L for the total amount of 9000. At time 0, he would have a net gain of 55(100)+36.739(100)−9000=173.9 whereby at time 1 he could use the free cash flow of firm L to close off the position in the security market, with no net expenditure. Therefore, users of CAPM-based (disequilibrium) NPV are not only subject to arbitrage losses, but they may even miss some arbitrage opportunities.

4. Generalizing

The examples above shown are just particular cases of a more general result. Let $V(D)$ be the firm value seen as a function of the debt.[5] Formally, MM-I may be rephrased saying that

[5] With this notation, we have the value of the unlevered firm is $V_U = V(0)$.

$$V(D_1)=V(D_2)\ \text{ for any }\ D_1\neq D_2\,. \tag{6}$$

We now show that if firm valuation is realized via disequilibrium values, eq. (6) above is not satisfied. Bearing in mind that ECF=FCF−CFD and assuming that the cash flow to debt is riskless, we have

$$\begin{aligned}V(D)&=\frac{\mathrm{FCF}-\mathrm{CFD}}{1+r_f+\dfrac{r_m-r_f}{\sigma_m^2P_e}\operatorname{cov}(\mathrm{FCF}^*-\mathrm{CFD}^*,\tilde{r}_m)}+\frac{\mathrm{CFD}}{1+r_f}\\&=\frac{\mathrm{FCF}-(1+r_f)\dfrac{\mathrm{CFD}}{1+r_f}}{r_f+\dfrac{r_m-r_f}{\sigma_m^2P_e}\operatorname{cov}(\mathrm{FCF}^*,\tilde{r}_m)}+\frac{\mathrm{CFD}}{1+r_f}\\&=\frac{\mathrm{FCF}-(1+r_f)D}{r_f+\dfrac{r_m-r_f}{\sigma_m^2P_e}\operatorname{cov}(\mathrm{FCF}^*,\tilde{r}_m)}+D.\end{aligned} \tag{6}$$

Taking the derivative with respect to *D*, we have

$$\frac{\mathrm{d}V(D)}{D}=1-\frac{(1+r_f)D}{r_f+\dfrac{r_m-r_f}{\sigma_m^2P_e}\operatorname{cov}(\mathrm{FCF}^*,\tilde{r}_m)}.$$

In general, we have $\dfrac{\mathrm{d}V(D)}{D}\neq 0$,[6] which means that $V(D)$ is not constant. This boils down to saying that $V(D)$ is not invariant under changes in *D*, i.e. eq. (6) is not fulfilled. In particular, we have $V(0)\neq V(D)$ whenever $D\neq 0$. The example in section 1 above is just a particular case of this general result where we have picked $D=6521$, so that $V_U=V(0)\neq V(6521)=V_L$.

5. Nonadditivity

The results above shown may be rephrased in terms of additivity. To see a project or a firm as an aggregate quantity generating free cash flow (firm U) or a disaggregate quantity generating equity cash flow and debt cash flow (firm L) is only a matter of convention, and the property of

---

[6] We have $\dfrac{\mathrm{d}V(D)}{D}=0$ only for $D=\dfrac{r_f+\dfrac{r_m-r_f}{\sigma_m^2P_e}\operatorname{cov}(\mathrm{FCF}^*,\tilde{r}_m)}{(1+r_f)}$.

additivity should be fulfilled by any rational methodology of asset valuation. In other terms, we should have

$$\text{NPV}(E)+\text{NPV}(D)=\text{NPV}(E+D).$$

But the previous sections just imply that the NPV is nonadditive, since

$$\begin{aligned}\text{NPV}(E)+\text{NPV}(D) &= (-P_e+E)+(-P_D+D) = -P_U+E+D \\ &= -P_U+V_L \neq -P_U+V_U = \text{NPV}(E+D)\end{aligned}$$

(see also Magni, 2007a, 2007b, for issues of nonadditivity).

From a decisional point of view, the nonadditivity of the valuation has serious consequences for decision making: Agent B has the opportunity of purchasing firm U's shares, or, alternatively, buying both equity and debt of firm L. The two alternatives are just the same from a financial point of view. Yet, as Table 3 shows, agent B considers it profitable to buy U's equity (NPV=150), whereas he considers it not worth purchasing equity and debt of firm L, (NPV= −99). He then takes two different decisions for the same course of action. This absurd behavior is just due to the nonadditivity of the NPV. Nonadditivity means that valuation and/or decision changes if the problem at hand is differently framed, although the descriptions of the problem are logically equivalent. Financially, the cash flow generated by a firm should be valued by decision makers univocally, irrespective of whether it is considered an aggregate quantity (FCF) or a disaggregate quantity (ECF+CFD). Therefore, agent B incurs what behavioral scholars call a "framing effect" (Kahneman and Tversky, 1979; Tversky and Kahneman, 1981; Kahneman and Tversky, 1984; Qualls and Puto, 1989; Roszkowski and Snelbecker, 1990).

Conclusions

This paper shows that:

- The use of CAPM-based disequilibrium NPV for valuing firms is not consistent with arbitrage pricing
- The CAPM-based disequilibrium NPV changes under changes in the debt/equity mix, so infringing Modigliani and Miller's Proposition I (and the principle of arbitrage)

- Agents using disequilibrium NPVs are open to arbitrage losses and may miss arbitrage opportunities
- The disequilibrium Net Present Value is nonadditive
- Agents using disequilibrium NPVs are subject to framing effects

Although the NPV as a decision rule is deductively drawn from the CAPM, its use for valuation and for decision-making is a mis-use, leading to biases, arbitrage losses, misses of arbitrage profits, framing effects.[7]

[7] It is worth noting the use of the *equilibrium* NPV does not guarantee correct valuations (see Dybvig and Ingersoll, 1982, Magni, 2007b).

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Table 1. Notations	
FCF*	Free Cash Flow (random) of firms U and L
ECF*	Equity Cash Flow (random) of firm L
CFD*	Debt Cash Flow (random) of firm L
FCF	Free Cash Flow (expected) of firms U and L
ECF	Equity Cash Flow (expected) of firm L
CFD	Debt Cash Flow (expected) of firm L
E	Equity value
D	Debt value
V_U	Value of firm U
V_L	Value of firm L
$V(D)$	Firm value as a function of debt
P_U	Selling price of firm U's equity
P_e	Selling price of firm L's equity
P_D	Selling price of firm L's debt
$\tilde{r}_u$	Rate of return of firm U's equity
$\tilde{r}_e$	Rate of return of firm L's equity
r_D	Rate of return of firm L's debt
β_u	Beta of firm U's equity
β_e	Beta of firm L's equity
β_D	Beta of firm L's debt
$\tilde{r}_m$	Market rate of return (random)
r_m	Market rate of return (expected)
r_f	Risk-free rate
cov	Covariance
k_u	Cost of equity of firm U
k_e	Cost of equity of firm L
k_D	Cost of debt of firm L
NPV	Net present value
MM-I	Modigliani and Miller's Proposition I

Table 2. The security market

	Security				
	Risky	Risk-free	Market	State	Probability
Outstanding shares	10	10	10		
Cash Flow	165	115	1650	Good	0.8
	65	115	650	Bad	0.2
Rate of return (%)	65	15	65	Good	0.8
	-35	15	-35	Bad	0.2
Expected rate of return (%)	45	15	45		
Covariance with the market rate of return	0.16	0	0.16		
Beta	1	0	1		
Value	100	100	2000		

Table 3. Firm valuation			
Firm U		Firm L	
		ECF*	$\begin{cases}5800\\300\end{cases}$
		CFD*=CFD	7500
FCF*	$\begin{cases}13300\\7800\end{cases}$	FCF*	$\begin{cases}13300\\7800\end{cases}$
		P_e	2000
P_U	9000	P_D	7000
$\tilde{r}_u$ (%)	$\begin{cases}47.77\\-13.33\end{cases}$	$\tilde{r}_e$ (%)	$\begin{cases}190\\-85\end{cases}$
		r_D (%)	7.14
β_u	1.222	β_e	5.5
		β_D	0
k_u (%)	33.33	k_e (%)	97.5
		k_D (%)	15
		E	2380
		D	6521
V_U	9150	V_L	8901
NPV	150	NPV	−99

Table 4. Arbitrage loss		
	Cash flow at time 0	**Cash flow at time 1**
1st course of action (agent B lends)	−9149	FCF*
2nd course of action (agents B borrows)	2479	−ECF*
3rd course of action (agent B borrows)	6620	−CFD
Overall	−50	0