VINTAGE EFFECTS AND THE TIME PATH OF INVESTMENT IN PRODUCTION RELATIONS

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THE ROLE of capital stocks in empirical estimates of production relations is akin to that of a minor vice—we all know there is something wrong with it but persist in the practice for lack of a better substitute. The principal problem is that capital goods are aggregated as if they performed the same functions though they may be drawn from different production processes.

There are two principal ways in which aggregation has been carried out. On the one hand, there are those who combine all capital goods, regardless of their characteristics or the processes in which they are used, into a single conglomerate stock. On the other, there are those who assume that for each vintage of capital there is a separate production function, though, under certain conditions, the separable processes can be described by a single aggregate function. Thus Solow aggregated the activities of separate production units, each associated with a capital of a single vintage, into a single function for the economy as a whole. The difficulties of using a single conglomerate capital stock are well known and require little explanation. Those associated with the second approach are more subtle and need some elaboration.

The trouble with defining production units in a way that limits the scope of each to one vintage of capital is that, in fact, a large proportion of capital goods of differing vintages perform interdependent functions. Consequently, they are inputs in a common production process.

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Hence, conceptual decomposition into independent production units overlooks what we call the interactions among investments. Since these interactions determine the nature of the services and the productivity of new capital goods, any attempt to aggregate investments without reference to them involves the aggregation of units that are nonhomogeneous in function even though of the same vintage.

The primary purpose of this paper is to explain the nature of interactions between various vintages of capital and to develop an analytical framework for taking into account the effects of alternative sequences of investment outlays. Part I presents our general approach to the problem. Part II gives the results of applying the analysis to one industry—namely, the electric utilities.

I

No one who works with economic data remains a purist for long. This is especially apparent when one focuses on the problem of identifying separate production processes. Production units are less observable entities in an objective reality than they are conveniences for organizing information in a useful way. From the standpoint of generating stable parameters, the best level of detail in the choice of production units depends upon two conditions: namely, the homogeneity of the physical process and the degree of interdependence among individual capital goods.

Since there is obviously greater homogeneity both in product structure and production methods at the level of individual plants than at the level of entire industries, this would argue for the measurement of production relations at the plant rather than at the industry level. Should the activities of a plant be further broken down into a set of functional relations for each equipment type? Obviously the technical process described by the production function would be far more homogeneous, but this homogeneity would be purchased at a price. For what the derived relation would miss are the interactions between the equipment types. The elasticity of output with respect to investment in turbines depends upon the boilers with which the turbines are combined, and the relation between output and investment in turbines and boilers depends upon the building in which both are placed. Indeed, production relations at the level of individual plants miss the interactions among the plants of a firm, and those at the level of firms miss the interactions among firms.

An implication of the above is that the choice of observations from the standpoint of the optimal level of detail depends not only on the homogeneity of processes but on the strength of interactions among the inputs of the various units into which the production process can be divided. This implies that the unit of observation which is analytically most relevant will vary among industries. Our intuition, however, is that it will most often be at the plant level. This is not to say that the boundaries that delineate a plant are not often arbitrary. In general, however, physical contiguity is not an accident, and the capital goods that are placed at given plant sites are likely to be functionally interrelated.

Those who have held that technical progress is largely "embodied" in tangible assets have taken one of two approaches to the definition of capital. Given the assumption that capital goods of differing vintages are not homogeneous, one approach (taken by Salter) is to examine production relations only for new plants. This means that all the tangible assets of each plant have approximately the same date of birth. A second approach (taken by Solow) is to aggregate capital across vintages where each vintage of investment is an input in a separate function.

Salter's position is conveniently summarized in his own words: [1]

> The bulk of output is produced with the aid of capital equipment already in existence and is the result of past techniques and investment decisions. . . . This simple but often neglected point is worth emphasizing. As Schumpeter has said "the production function is a planning function in a world of blueprints where every element that is technologically variable can be changed at will." . . . So far as the measurement problem is concerned this implies that capital in the production function must refer to new capital equipment or investment; the part-worn and part-obsolete capital comprising the capital stock cannot be relevant for this is the result of techniques and investment decisions already made.

In short, new investment is based exclusively on the most recent or "best-practice" technology. Technical change then is measured by the changes in investment and labor requirements per unit of output between the best practice investments of successive periods.

The difficulty with this approach is that new investment is made not only for new plants but also for "old" ones. Indeed, for the aggregate of

[1] W. E. G. Salter, "The Production Function and the Durability of Capital," *Economic Record,* April 1959.

manufacturing industries in the United States, over 90 per cent of all capital outlays are made for existing, as distinct from new, plants.[2] Substantially the same is probably true for the public utility industries; and even in the trade and service sectors most investment outlays are probably made in existing establishments rather than for new ones, at least when appropriate allowance is made for capital expenditures customarily charged to current account. The preponderance of investment for existing establishments is likely to be a characteristic of all but very new industries. Therefore, much the same conclusions should apply to any developed industrial economy as to the United States, for in all such economies new industries account for only a minor part of total investment.

Now it is reasonable to assume that decisions to spend on old as distinct from new plants are not arbitrary; they are made because they minimize the requirements of the variable inputs (including new investment) for a given level of output. This must, in turn, mean that the capital goods purchased by these outlays are functionally interrelated with those already in place, for only in this way can the input requirements be reduced below those of best-practice new plants. Consequently, the characteristics of new capital goods will depend upon those of the ones previously purchased; and the choices made today must, in this sense, depend upon yesterday's decisions. Thus, the term "best practice" need not refer only to new plants, and what is best practice for one plant need not be best practice for another. Instead there is a range of "best" alternatives, each contingent upon the endowments of particular plants—that is, upon the level and sequence of their past investments and the kinds of capital goods they already have. The input requirements per unit of output for new plants represent merely upper limits beyond which further investment will not be made on existing plants.

[2] The 1958 *Census of Manufactures* for the United States, I, 5–3, shows that 1958 capital expenditures for plants under construction (presumably unfinished as of the end of the year) were 5.2 per cent of total capital outlays for that year. For 1954, the comparable percentage was 4.7. If we assume an average gestation period of only one year and an even flow of capital outlays over time, expenditures for plants completed during the year will roughly equal those for plants still unfinished as of the end of the year. Under these assumptions, outlays for all plants that were under construction during any part of 1954 or 1958 were equal to about 10 per cent of the total outlays in these years. If, however, the average gestation period is two years, outlays for plants finished during the year would be one-third of those for plants listed as unfinished. On this basis, expenditures for new plants were less than 7 per cent of the total.

The difficulties in Salter's position are reflected in his empirical results. In his work on capital and labor requirements for several hundred Australian factories [3] he examined information for major additions as well as for new plants. The interdependence between old and new investments is perhaps less obvious for major than for minor additions. Nonetheless, the relation between investment and incremental output proved to be highly unstable for existing factories—there was no way of identifying the increments to output that flowed from the new investment alone—and Salter was able to present only his results for new plants.

Our principal objection to Salter's approach is that it leaves us with nothing to say about most capital outlays. However, even for new establishments, there may be no simple relation between investment and the expected or capacity output of a single period. The virtual certainty of subsequent additions is surely taken into account in planning new establishments. Since flexibility for future expansion is not costless, some part of the current investment cost is incurred for the purpose of reducing future capital requirements. In this sense, not all of the investment of a given period is related to the output level of a single interval of time. As an aspect of this, some components of a plant would not be operated at capacity except at an output that substantially exceeds the capacity of the plant as a whole. For reasons we explain later, such reserve capacity is likely to be far greater for new than for older plants and is, therefore, much likelier to bias parameter estimates for new plants.

We do not wish to imply that this problem is necessarily critical, nor to minimize the potential usefulness of estimating, as Salter has done, the capital coefficients of new plants. It is worth remembering, however, that data for new plants offer no simple solutions to estimating investment requirements per unit of output even for the limited segment of total investment to which such data are relevant.

Solow, through an ingenious analytical device, attempts to circumvent the obstacles to aggregating investments of varying vintages. Subject to three assumptions, his model aggregates investments into a capital stock even though capital goods are inputs in differing production processes. These assumptions are: (1) constant returns to scale; (2)

[3] W. E. G. Salter, "Marginal Labor and Investment Coefficients of the Australian Manufacturing Economy," *Economic Record,* June 1962.

substitution between capital and labor such that, at the margin, the productivity of labor is equal regardless of the vintage of capital with which it is combined; and (3) production functions of various vintages differ by a set of time-dependent multiplicative weights. The equation he derives is: [4]

$$Q(t) = Be^{-\delta(1-\alpha)t}L^{\alpha}(t)\left[\int_{-\infty}^{t} I(v)e^{\sigma v}dv\right]^{1-\alpha} \tag{1}$$

where $Q(t)$ = aggregate output

t = time

δ = exponential decay rate of investment

$L(t)$ = aggregate labor

$I(v)$ = gross capital expenditures of period v

$\sigma = \dfrac{1-\alpha}{\lambda} + \delta$, where λ is the time-dependent weight that relates production processes of successive vintages.

The difficulties involved in estimating improvement rates in the context of this model have been discussed by others. What concerns us more are some theoretical implications. Since one unit of capital is interchangeable with any other unit of the same age and vintage, one cannot explain in terms of the model why most investment is made on existing plants. Indeed, if one does not take into account the interdependence between old and new investment, the fact that a large proportion of outlays are made on additions to old plants is exceedingly puzzling since old capital goods are to some extent encumbrances that reduce flexibility in the choice of new capital.

In our model, the time path of investment has a critical role, for it determines the method by which technical change is introduced. Each investment in an existing plant interacts with past investments. The type of capital goods purchased depends, therefore, on the investment history of each plant. Moreover, the input requirements for an incremental unit of output differ not only between new and old plants but between old plants with differing birth dates and investment histories. Stated in another way, the capital expenditures of yesterday enter as

[4] Robert M. Solow, "Investment and Technical Progress," in K. J. Arrow, S. Karlin, and P. Suppes (ed.), *Mathematical Methods of the Social Sciences, 1959,* Stanford, 1960.

factors in today's production function. Today's choice of capital goods therefore depends both upon current knowledge of the most efficient production process and upon the most effective way of adapting to it the capital goods purchased yesterday. This does not mean that each plant is unique and that one cannot infer from the input-output relations of one plant the input requirements of another. The range of variations in birth dates and investment histories over which any given parameter is applicable is an empirical question and has to be solved separately for each industry.

There are three sets of conditions that make it more efficient, most of the time, to spend on extensions and modifications of existing plants rather than on new plants. These conditions define the nature of the interactions between new and old investment. First, new investment may modify the way in which previously purchased capital goods function. This usually entails a smaller cost than the purchase of new assets alone and, in addition, involves a shorter gestation period than the construction of new plants. Second, old investment may raise the productivity of new investment by serving as a stage in the learning and adaptation process necessary for further expansion. Third, some components of old capital will always have spare capacity, with the result that not all of the tangible assets necessary to increase production need to be newly purchased when expanding existing plants.

It is a well-established fact that a large proportion of capital expenditures in the United States constitute outlays on "modernization" [5] —that is, on the adaptation of previously purchased capital goods to new technical processes. A principal variable in the economic life of plants is the adaptability to new production techniques of capital goods purchased by past investments. Expenditures for the replacement of components of plants are a form of interaction between new and old investment and one of the ways by which old processes are adapted to new techniques. Since one rarely replaces capital goods with others of the same specifications, replacement expenditures almost invariably lead to some modifications of earlier processes. As an aspect of this, most "replacement" outlays also affect a plant's capacity.[6] Indeed, because

[5] For example, according to the periodic surveys of business investment conducted by the McGraw-Hill Publishing Company, expenditures on "modernization and replacement" often account for well over half of all capital outlays.

[6] Presumably this is why the McGraw-Hill survey asks for expenditures on "modernization and replacement" rather than for replacement alone.

of this fact, the capital budgets of firms seldom distinguish between outlays for replacement and for expansion and the concept of replacement is, itself, largely nonoperational from the standpoint of decision makers.[7]

Any organizational unit, whether a plant or firm (or even a university) has some maximum efficient growth rate which, if exceeded, leads to sharply rising costs per unit of output of constant quality. These internal diseconomies of high growth rates must be distinguished from diseconomies of scale. They stem from three sources. First, the components of a production unit need to be adapted to each other. The likelihood of partially incompatible systems increases with rapid expansion. Second, it takes time to train managerial and technical personnel. One of the best training grounds is an established plant. In this way past tangible investment is instrumental in the acquisition of trained personnel and consequently exerts an influence on the productivity of new investment. Third, many improvements incorporated in new capital goods, or in the way new capital goods are used, arise from experience in the use of the old ones. This influence is strongest at the level of entire industries rather than of plants because one plant can borrow from the experience of another and, also, because the same equipment producers serve new as well as old plants. But even at the plant level the ability to borrow is important, particularly for the small and undramatic improvements which cumulatively may be quite important. Much of what is sometimes taken as disembodied technical change stems, in fact, from what we have called the interaction between successive streams of capital outlays.

Each plant when operated at its maximum output will have spare capacity in some of its components. This arises partly from indivisibilities in capital goods and partly from the uneven impact of economies of scale on the various parts of an establishment. The incremental cost of additional capacity is quite small for some components. Consequently, imbalances in the system are often the result of deliberate plans and not merely miscalculations. Though most additions to existing plants create some imbalances while reducing others, newly created imbalances are generally larger when entire plants are built. Therefore, additions to plants frequently cost less than building new plants. Indeed, some modernization outlays derive their high returns from the fact that improvements in

[7] Michael Gort, "The Planning of Investment: A Study of Capital Budgeting in the Electric Utilities, I," *Journal of Business,* April 1951.

some components of a plant often lead to a fuller utilization of the other parts.

We are now ready to present the interaction model for a plant with two distinct investment flows—an initial investment and a subsequent expansion or "renewal." In the Cobb-Douglas form, but without the assumption of constant returns to scale, the equation is:

$$(2) \quad \underset{n,n-1}{O}(t) = A(n,n-1)L(t)^{\phi(n,n-1)}M(t)^{\gamma(n,n-1)}I(n)^{\alpha_1(n,n-1)} I(n-1)^{\alpha_2(n,n-1)}$$

where $\underset{n,n-1}{O}(t)$ = expected output in period t for a plant with investment in period n and with one previous investment in period $n-1$

$L(t)$ = expected labor in period t

$M(t)$ = expected materials use in period t

$I(n)$ = actual investment in period n

$I(n-1)$ = actual investment in period $n-1$.

One will note three distinguishing characteristics of this equation. First, *all* the parameters are made to depend upon *both* investment streams. This is because both contribute to determining the production process used. Second, the investment streams are assumed to have a multiplicative rather than an additive effect on output. Third, there is no implicit averaging of the elasticities of output with respect to particular investment streams. Instead, the exponents of the investment streams are allowed to vary.

Labor is assumed in the equation to be homogeneous though, in principle, this assumption is not necessary. Insofar as factor proportions are concerned, we assume that, *ex post,* they are determined by the nature of past investments. That is, once the capital goods have been put in place, factor proportions consistent with long-run equilibrium are fixed in the sense that they can be changed only through new outlays or through a reduction in the output generated by past investments as a result of the obsolescence of these investments. New investment, however, modifies existing plants and consequently can contribute to determining factor proportions on old as well as on new plants. New investment per unit of output, whether on existing or on new plants, can be varied. Moreover, its allocation between new and old plants and among old plants of different vintages should be such as to equate its marginal product in all alternatives (subject, of course, to discontinuities).

Investment in our model is measured in gross rather than net terms with the result that there are no negative values. Also, the equation excludes all zero values (which, in an economic sense, are irrelevant). Consequently, the successive investments do not necessarily relate to consecutive periods.[8] Since the investments are not necessarily consecutive, their position in the sequence of capital flows does not automatically date them. Inasmuch as an industry's technology continues to change independently of whether any given plant has a capital outlay, the expenditures need to be identified by their vintage as well as by their position in the sequence of capital flows.

For n successive investments, the function is:

$$(3) \quad O_i(t) = A(i)L_i(t)^{\phi(i)}M_i(t)^{\gamma(i)}I(v_n)^{\alpha_1(i)}I(v_{n-1})^{\alpha_2(i)}\ldots I(v_1)^{\alpha_n(i)}$$

The letter v refers to the vintage of the investment. The letter i denotes the technology of the plant and depends upon the birth date of the plant and its pattern of subsequent investment streams, that is, the sequence and magnitudes of the investments. Thus $O_i(t)$ is expected output for the period t for a plant using process i, and i is a function of all investments from vintage 1 to n. A strict interpretation of the model is that i is unique to a specific investment history. As a practical matter, however, plants can be grouped into technology classes, with each class encompassing some variation in birth dates and subsequent investment flows. The choice of grouping criteria is an empirical question and is discussed in that context later. At this juncture we need only note that all the parameters depend upon i.

While the gross investment streams could be depreciated for physical decay, assuming such information were available, this is not necessary. The constant term and the exponents of the investments will reflect, among other things, both the rate of physical decay and the differential impact of technical change on the efficiencies of investments of different vintages. Consequently, information on the economic life of assets is not essential. This permits us to circumvent a somewhat tenuous estimat-

[8] If, however, maintenance expenditures were included in capital outlays, the record would probably show a positive investment stream each year. Maintenance expenditures that are expected to yield returns over more than one accounting period are conceptually equivalent to other capital outlays.

ing procedure that is a crucial element in the application of all models in which the stock of capital is a variable. It can be shown that capital stock estimates are highly sensitive to assumptions about the economic life of assets and that all measures of economic life are subject to a largely unknown, and potentially serious, error. Moderate differences in measures of economic life can lead to significant differences in estimated growth rates of capital. Thus errors that stem from mismeasurement of the variables themselves are likely to be reduced if one can dispense with a capital stock variable.

In equation (3), technical change is reflected in the multiplicative term of the equation or, alternatively, in the partial elasticities, or in both. To the extent it affects the multiplicative term, it can be visualized as a force that affects the efficiencies of all the inputs of a plant uniformly. It is important to note, however, that the constant is not simply a trend term—that is, it depends not on time but on the technology of each plant, which, in turn, is a function of the plant's investment history. The parameter may be significantly different for a plant which has had no investment of vintage n from one for which $I(v_n)$ is a significant positive value. Technical change, in this context, can be envisaged as resulting from the introduction of new processes through tangible investment.

The second way in which both technical change and interactions are revealed is through the partial elasticities of output with respect to investments of different vintages and ages. There are four questions one can ask. First, what is the relation between the exponents for the same vintage of investment made on new and on old plants? Second, what is the relation between the exponents of investments of successive vintages within a given (single) production function and at one point in time? Third, how does the exponent of a given vintage of investment change over time (that is, as the investment ages). Fourth, given two identically ordered sequences of investments but with one sequence n years later than the other, is there a predictable relation between the exponents of the two sequences?

In an interaction model, the relation between the successive partial elasticities for investments of differing vintages is complex. What is crucial is not the separate flow of productive services associated with any investment but the effect of new capital expenditures on the efficiency of the entire production process and thus on the incremental output of the plant as a whole. Because of interactions, it is usually cheaper to spend on existing rather than on new plants even when the partial elasticity of output with respect to the last investment on an existing plant is lower than the elasticity of investment in a new plant.

To clarify our analysis, let

$$(4) \qquad O_R(t) = A(R)\, I(v_n)^{\alpha_1(R)} I(v_{n-1})^{\alpha_2(R)} \ldots I(v_1)^{\alpha_n(R)} L_R(t)^{\gamma(R)}$$

where $O_R(t)$ is the output at time t of a plant with an initial investment and $n-1$ subsequent "renewals" or additions. Now let

$$(5) \qquad D = A(R)\, I(v_{n-1})^{\alpha_2(R)} \ldots I(v_1)^{\alpha_n(R)} L_R(t)^{\gamma(R)}$$

so that, $O_R(t) = DI(v_n)^{\alpha_1}$. Similarly for $O_{v_n}(t)$ the output of a newly created plant, with investment of vintage v_n, we have the equation

$$(6) \qquad O_{v_n}(t) = BI(v_n)^{\beta(v_n)} L_{v_n}(t)^{\gamma(v_n)}$$

If we define G such that

$$(7) \qquad G = BL_{v_n}(t)^{\gamma(v_n)}$$

then $O_{v_n}(t) = GI(v_n)^{\beta(v_n)}$. It is our conclusion that, usually, D is greater than G, α_1 is less than β (with both exponents greater than zero and less than 1), and $\alpha_1 D$ is greater than βG.

D will usually exceed G because modifications of an existing plant can greatly increase the efficiency of the plant as a whole, because of the presence of spare capacity in some components of the plant, etc. However, the constraints under which investment is carried out on existing plants are greater than those associated with plants under construction. For the latter, the plans are not encumbered by the limitations of old capital goods with the result that the number of feasible alternative designs and combinations of components is greater. Hence there is more flexibility to investment in new plants and this, in turn, leads to a greater elasticity for capital expenditures in new than in old plants. Indeed, as a plant ages, its technology becomes increasingly less adaptable to new techniques with the result that, beyond some point, no further investments are made on old plants. In most instances, however, the difference between D and G more than offsets the effects of a higher β than α_1, with the result that most investment is, in fact, carried out on existing plants.

In general, with each successive investment in a given plant, the flexibility in the planning of further renewals or additions should decline. However, this may be offset by developments in techniques which make new investment more adaptable to the old than it previously had been. Consequently, the exponents for successive renewals in a single production function can either rise or fall.

As a given vintage of investment becomes obsolete, its uses are degraded. To the extent that obsolete capital goods are held for special uses such as reserve capacity that goes on stream for only short intervals, small amounts of such capital goods are likely to find more uses, per dollar of past investment, than larger amounts. The volume of investment that was optimal for its original functions is more than would have been purchased for its new and less valued services. Thus, as capital goods age, the usefulness of increments to the total amount of such equipment placed in a given plant diminishes. This should have the effect of reducing elasticity. At the least, it is clear that as capital goods age, the exponent for an investment of a specified vintage is most unlikely to rise.

For two identically ordered sequences of investment, the entire process for the later one should be more productive. It is difficult, however, to say, a priori, what the effect will be on the exponents of the investments. The greater efficiency may be reflected in an increase in the elasticities of all the I's, but it may also generate some movements of the exponents in opposite directions.

An alternative way of representing technical change might be to weight the capital expenditures themselves, that is, to make the technical change capital augmenting. For example,

$$I(v_n) = e^{\lambda n} I(v_0) \tag{8}$$

where λ is the rate of technical improvement. This would imply that there is a component to technical change that is dependent on time alone as distinct from the production process and investment history of a particular plant. Within the Cobb-Douglas production function such a redefinition of capital expenditures has the same effect as a time-dependent multiplicative trend.

Most of the characteristics of our model, as contrasted with the familiar trend relation in which K is a homogeneous capital stock, are apparent from the discussion above. One of these, however, warrants further elaboration. The simple trend model with constant returns to scale implies that each increment to the capital stock generates a proportionate increase in output provided the ratio K/L remains unchanged. In contrast, the interaction model with constant returns to scale indicates that, for a given set of past investment streams, increases in current investment accompanied by proportionate increases in all factors other than investment are associated with less than proportionate increases in output. This has several critical implications from the standpoint of economic planning. One implication is that as the growth rate of output rises, investment requirements per unit of additional capacity will also rise. Hence, for this reason alone, increases in the growth rate of output for the economy as a whole should be associated at least temporarily with successively higher values of $I/\Delta GNP$. Moreover, because of the same set of forces, the investment requirements for new capacity in a specified industry are greater in an economy in which the industry in question is itself new. Consequently, unless offset by differences in factor proportions, the investment cost per unit of new capacity should generally be higher in underdeveloped economies than in those with a mature industrial base.

Still another aspect of interactions between successive investment streams is that relatively small amounts of new investment can have large impacts on output. This may resolve a puzzle as to how "embodied" technical change can explain a significant proportion of technical advance in our economy, notwithstanding the fact that new investment is always small relative to the accumulated stock of capital. If new investment changes the efficiency of the entire process in which all past as well as current investments are inputs, the effect on technical change of even modest new capital expenditures can be very large. Moreover, if (as indicated earlier) tangible investment is instrumental in the learning process, capital expenditures will affect the rate of advance in our knowledge of techniques as well as the rate at which new techniques are applied to tangible assets.

The interaction model presented earlier needs to be modified to make it more generally applicable. First, not all new outlays interact with all past outlays. Second, while it is difficult to accelerate the learning process,[9] the implications of our equation with respect to continuously

[9] Partly for this reason, the isoquant for combinations of new and old investment is convex to the origin.

diminishing returns to new investment are too rigid. Apart from limitations in the supply of labor and materials, it is plausible that returns to successive increments of new investment will, beyond some point, approximate those associated with newly built plants in which interactions with past investments are irrelevant.

Consequently, a more general model is one with additive as well as interaction terms. Thus, leaving out labor and materials, the equation can be written as follows:

$$(9)\quad O_i(t) = B(v_n)I(v_n)^{\beta(v_n)} + B(v_{n-1})I(v_{n-1})^{\beta(v_{n-1})} \ldots + B(v_1)I(v_1)^{\beta(v_1)} + A(i)I(v_n)^{\alpha_1(i)}I(v_{n-1})^{\alpha_2(i)} \ldots I(v_1)^{\alpha_n(i)}$$

The terms are defined as before. The β's are shown to depend on vintage. The α's depend upon vintage and upon the position of the investments in the capital expenditure sequence. That is, they depend upon i where, as before, i is a function of all past investments. The above equation has only one interaction term. As a practical matter, this may be sufficient. In principle, however, there can be a separate interaction term for all combinations of n investments. We hope, however, that this degree of complexity is unnecessary.

The general interaction equation with additive terms encompasses as special cases both the "pure" interaction and the additive models. If the interactions are strong, the exclusion of additive terms may not seriously impair the usefulness of the equation for the purpose of estimating investment requirements.

II

In the statistical analysis that follows, we try to test an interaction model in which output depends upon the level and sequence of investments of successive vintages. One objective is to see if the individual investments are associated with significant differences in exponents. Another is to establish if the differences in exponents between investments of various vintages, both within a single production unit and among production units, are consistent with the analysis presented above. Thus our primary purpose is to examine the nature of interactions among investments rather than to "explain" variations in output.

With this objective in mind, we tried to select observations that are least affected by short-run deviations from equilibrium relations among the variables. The tests were carried out with cross-section data for individual plants in the electric power industry. Because the industry is characterized by steady growth with relatively few other-than-seasonal downturns in the outputs of individual plants, the problem of selecting points that correspond to "capacity" production is substantially reduced. Moreover, for most of the analysis, the measure of output we employed was peak demand—that is, the maximum production achieved for a single hour in the course of the year. This further reduced the likelihood that the observations related to nonequilibrium outputs.

As a simplification, additive terms were excluded from the equations tested. The relations tested were log-linear functions in which output or peak demand was expressed as a function of the individual investments only. Thus the parameters were estimated indirectly by means of a restricted reduced form equation where all the investments enter as predetermined variables. In this way we attempt to avoid some of the statistical biases and specification problems associated with a number of alternative approaches. In a reduced form equation of this type, labor and materials inputs have no role as independent variables.

For the electric utilities, given our purpose, there are compelling reasons for excluding labor and materials as variables on either side of an equation. First, there is no measure of a labor input that is relevant to peak demand. Annual or monthly data on labor (or materials) inputs are clearly inappropriate. Second, variations in the number of employees in electric power plants are largely attributable to construction work and, in this sense, are more germane to the measure of capital inputs than of labor engaged in current production. Moreover, for the electric power industry, statistical relations in which output is expressed as a function of capital (or investment), labor, and materials are subject to high collinearity among the factor inputs. For this reason, Komiya [10] found a single equation of the Cobb-Douglas type unsatisfactory and, as a result, sought to explain technical progress through equations each of which expressed a single input as a function of output.

One alternative to estimating production functions in which output is expressed as a function of all the factor inputs is to estimate a complete

[10] R. Komiya, "Technological Progress and the Production Function in the United States Steam Power Industry," *Review of Economics and Statistics,* May 1962.

demand equation for the factors. Nerlove[11] and Barzel[12] have used equations in which factor demand is expressed as a function of output, relative prices, and other indexes. This approach, however, suffers to some extent from the absence of reliable indexes of relative prices. Indeed, the cost of capital has thus far eluded an acceptable measure and the investigator is faced with the option of using an unsatisfactory measure or none.

If the proportions between investment and both labor and materials are fixed by a given technology, exclusion of the latter two variables should not bias the elasticities of output with respect to the individual investments. If, on the other hand, there is significant substitution among the factor inputs, the exponents will be biased upward. However, from the standpoint of our problem, the relevant question is whether the relative values of the exponents are changed materially. The equation with output as a function of the investments only can be interpreted as a restricted version of a reduced form equation in which the dependent variable is a function of the exogenous and predetermined variables. Thus, if our structural equations are all log-linear, we have the equation:

$$O = GI(v_n)^{\gamma_1} I(v_{n-1})^{\gamma_2} \dots I(v_1)^{\gamma_n} Z_1^{\phi_1} Z_2^{\phi_2} \dots Z_m^{\phi_m}, \tag{10}$$

where the Z's are the relevant exogenous variables (other than investment). If the vectors of exponents of the investments in all structural equations other than the production function are scalar multiples of the vector of investment exponents in the production function, then $\gamma_j = k\alpha_j$ where γ_j is the exponent of the jth investment in the reduced form equation, α_j is the exponent of that investment in the original production function, and k is a constant. The problem is to ascertain the impact on the relative values of the exponents of the I's if one or more of the exogenous variables are excluded. Assume that there is only one relevant exogenous variable, z. Then,

$$E(\hat{\gamma}_R) = \gamma + [(X'X)^{-1}X' \log z]\,\phi, \tag{11}$$

where $E(\hat{\gamma}_R)$ is the expected value of the vector of exponents in the restricted equation, and $(X'X)^{-1}X' \log z$ is the vector of least squares

[11] Marc Nerlove, "Returns to Scale in Electricity Supply," in C. Christ *et al.* (ed.), *Measurement in Economics, Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld,* Stanford, 1963.

[12] Y. Barzel, "The Production Function and Technical Change in the Steam Power Industry," *Journal of Political Economy,* April 1964.

estimates of the log of z on the logs of the capital expenditures. Consequently, if the regression coefficients for the regression of the log of z on the logs of I's do not differ markedly among the I's, the absolute differences between the exponents of the I's should not be materially affected by the exclusion of one or more Z's. That is,

$$E(\gamma_{1R}/\gamma_{2R}) \simeq (\gamma_1 + C/\gamma_2 + C).$$

There appears to be little reason to infer that the regression of any exogenous variable on the various I's is associated with coefficients that differ markedly among the I's. In fact, there is reason to doubt that the value of C is so large relative to the γ's as to alter significantly even the ratio between any two exponents. In this connection, our estimates of elasticity for plants with but a single investment do not differ much from those derived in the several other studies of the electric power industry (mentioned above) in which the estimating techniques differed from ours. Plants with but one investment do not involve problems of decomposition of capital; so our results for these plants can be readily compared with the earlier work of others.

While the lags used in the various equations we tested differed, in no case was the most recent investment allowed to lead output by less than one year. The long gestation period associated with investment projects in the electric utilities assures that the errors in decisions about investments are determined independently of the random component of current output or peak demand. This permits us to assume that the I's are predetermined variables. For most of our results, peak demand (the dependent variable) was measured at its maximum point over the period between the last investment and the next successive capital expenditure for the plant.

An alternative to peak demand as a measure of output is annual production of kilowatt-hours. One may expect important differences in the exponents of successive investments when output is measured by peak demand rather than by annual production. During peaks, older capital goods are used much more intensively than in off-peak intervals and the proportions in which new and old capital goods are used are relatively fixed. That is, all resources tend to be stretched as far as possible. In contrast, in off-peak periods there is substitution of the newer for the older capital goods in some of the functions performed by the old assets during peaks in production. As a result, the exponents for the later investments should rise markedly when output is measured by annual production and $\alpha_1(KWH)/\alpha_2(KWH)$, the ratio of the elasticities of KWH production with respect to $I(v_n)$ and $I(v_{n-1})$ should be greater than $\alpha_1(pd)/\alpha_2(pd)$, the ratio of the elasticities of peak demand with respect to the same two investments. Also, the correlation coefficients with output measured by KWH production should be lower than when output is measured by peak demand, since variables other than investment have a larger role in annual than in peak output.

Data for peak demand and kilowatt-hours were drawn from the annual issue of the Federal Power Commission's *Steam-Electric Plant Construction Cost and Annual Production Expenses.* Gross capital expenditures were derived from data on the gross book value of structures and of equipment (separately) as shown in the same source. Since retirements have been small and infrequent for most electric power plants,[13] the net change between consecutive years in the gross book value of structures and equipment is usually a valid measure of capital expenditures. In the few instances where it was appropriate, however, estimated retirements were added to the net change in book value.[14] When the derived capital expenditures were aggregated for the plants of individual companies, they were generally consistent with reported investment outlays of the companies.[15] However, the lag between expenditures and the time they were recorded differed between the two sources because of a difference in accounting practice. Capital expenditures for structures were deflated by a construction cost index, and those for equipment were deflated by an index derived from two wholesale price indexes, namely, for electrical machinery and for engines and turbines, with equal weights for the two.[16]

The specific subsamples used for each equation and the periods covered by the analysis are shown in the section below together with our results. These subsamples were drawn from a total sample of 198

[13] In the entire period 1948–63, retired capacity, measured in nameplate megawatts, was only 10 per cent of the nameplate capacity that existed in 1947 (Federal Power Commission, *Steam-Electric Plant Construction Cost and Annual Production Expenses,* 1962–63, Table 2, p. xix).

[14] Data for the value of retirements for individual plants were derived mainly from Federal Power Commission, *Statistics of Electric Utilities in the United States,* 1948–63. While the data in this source are classified by company, the information, when used in conjunction with available plant data, was sufficient to permit the allocation of retirements by plants.

[15] Company capital expenditures are reported annually since 1948 in *ibid.*

[16] Unpublished data of Office of Business Economics, Department of Commerce.

TABLE 1

Number of Plants in Each Year with Capital Expenditures, 1948-63[a]

(number of plants)

Year	Plants with First Investment Expenditures	Existing Plants with Investment Expenditures	
		Plants Built Since 1948	Plants Built Before 1948
1948	6		12
1949	10	2	25
1950	10	7	16
1951	16	6	19
1952	9	11	18
1953	11	13	16
1954	14	15	14
1955	8	14	12
1956	5	9	7
1957	6	12	8
1958	7	10	17
1959	5	17	8
1960	6	13	6
1961	7	13	6
1962	4	11	5
1963	1	14	1

Source: See accompanying text.

[a]Restricted to sample of 198 plants.

steam-electric plants, of which 74 existed before 1948 and 124 were built since 1948. The sample was selected with a view to covering a wide spectrum of plants of different ages. The plants selected, however, are somewhat newer and larger than the average for the industry. Table 1 shows the number of newly built plants in our sample in each year since 1948. For each year, it also shows the number of existing plants with capital expenditures. An interesting fact is that only a few of the plants built before 1948 had investment outlays after 1958. Apparently the adaptability to new capital goods of plants built before 1948 was largely exhausted by 1959. However, most of the expenditures after 1958 were made for *existing* plants born after 1948 rather than for completely new plants. Despite a growth rate in demand and production far above that for most industries, less than 30 per cent of the kilowatt capacity added by the steam-electric power industry in 1962–63 was in new plants completed in those two years.[17]

The statistical analysis was carried out on the basis of three classes of observations. First, we examined the investment-output relations for new plants built in 1948 or later. Second, we examined a subset of plants born in 1948 or later, namely those that had at least one "renewal" or expansion outlay. Third, we examined plants born in the pre-1948 period that had at least one outlay in 1948 or later. The composition of plants in the cross sections differed among the equations tested. The main selection criterion was the degree of homogeneity in investment history, but this criterion was constrained by the need for a sufficient number of observations.

We first present our results for newly built plants which, by definition, have but a single investment. For cross sections of plants built in each of three five-year periods, 1948–52, 1953–57, and 1958–62, the following equation was estimated:

$$PD(v_1 + 1) = AI(v_1)^{\pi} \qquad (12)$$

where v_1 denotes the birth year of the investment,[18] PD is peak demand measured in megawatts, and I is investment in thousands of dollars (deflated to 1954 prices). The results appear in Table 2 (the expressions in parentheses are the t ratios).

In general π appears not to differ much from the results obtained in other studies. For example, Barzel[19] reports a coefficient of .815 for capital as a function of kilowatt capacity, when the other explanatory variables are the load factor, the price of fuel, and the price of labor. The reciprocal, that is the measure of returns to scale for investment, is

[17] Federal Power Commission, *Steam-Electric Plant Construction Cost. . . .*

[18] New plants for which there appeared to be initial investment outlays extending over more than one year were excluded in these estimates.

[19] *Op. cit.*

TABLE 2

Results for Equation (12)

Period	A	π	R^2	n
1948-52	.0042	1.05 (13.6)	.840	37
1953-57	.0009	1.22 (17.9)	.917	31
1958-62	.0043	1.09 (10.7)	.838	25

then 1.23–a value that does not differ very much from our range of 1.05 to 1.22 for π.

For plants with at least one renewal or expansion outlay in the period beginning in 1948, the cross sections were also divided by period. The need for an adequate number of observations permitted only two periods defined by the year of the first renewal or expansion outlay after 1948.[20] The two periods were 1949–57 and 1958–62. With the variables measured in the same units as before, the following two equations were estimated for the plants born in 1948 or later:

$$PD(v_2 + 1) = AI(v_2)^{\pi_1} I(v_1)^{\pi_2} \tag{13}$$

with $v_2 - v_1$ more than one and less than five years, and

$$PD(v_2 + 1) = A[I(v_2) + I(v_1)]^{\beta} \tag{14}$$

with v_2 and v_1 the same as in equation (13). In equation (14) the impacts of the investments on output are additive, and $I(v_2) + I(v_1)$ is a form of capital stock. The results for the two equations are shown in Table 3.

For the post-1948 plants, Table 3 shows that π_2, the exponent for the initial investment, is greater than π_1 for both cross sections. As expected, the flexibility associated with the initial investment was apparently greater than that for the second investment, with the consequence that the exponent was higher. However, $(\pi_2 - \pi_1)$ is much smaller when the second investment comes later, reflecting possibly an improvement in

[20] To eliminate the possibly unstable effects of very small expenditures only outlays of $10.5 million or more were deemed to be renewals of expansions.

TABLE 3

Results for Equations (13) and (14)

Equation	Period[a] of v_2	A	π_1	π_2	β	R^2	n
(13)	1949-57	.360	.27 (3.1)	.63 (4.7)		.628	19
(14)	1949-57	.254			0.87 (5.2)	.617	19
(13)	1958-62	.031	.48 (8.2)	.69 (5.5)		.943	22
(14)	1958-62	.020			1.13 (17.4)	.938	22

[a]Period in which renewal or expansion outlays were made.

the techniques of adapting new to older capital goods. $(\pi_1 + \pi_2)$ differ only slightly from β, but because of the large difference between π_1 and π_2, the implications of the two equations for the planning of investment outlays are quite different. As an explanation of the variance in output, equation (14) does about as well as equation (13).

The above regressions excluded observations for plants with an initial or a second investment program that extended over more than one year. Hence, there were no observations for projects with a long gestation period. In addition, the assumed one-year lag in peak demand may be shorter than warranted. Accordingly, for the next set of estimates, we took sequences of investment expenditures cumulated over periods defined by the number of consecutive years with capital outlays. Peak demand was the highest peak attained from the last year of the second sequence to the next expenditure sequence. Where there was no third investment sequence, the maximum peak was taken at its highest point from the end of the second sequence to 1963. Actually, it usually occurred within the first several years following the last outlays. The following two equations were tested with results shown in Table 4.

(15) $$\text{Max}\, PD(s_2, s_1) = AI(s_2)^{\pi_1} I(s_1)^{\pi_2}$$

and the capital stock version of this equation:

TABLE 4

Results for Equations (15) and (16)

Equation	Period[a]	A	π_1	π_2	β	R^2	n
(15)	1950-57	11.3	.29 (4.0)	.81 (9.2)		.819	29
(16)	1950-57	5.6			1.09 (12.7)	.858	29
(15)	1958-62	20.2	.45 (4.1)	.50 (4.9)		.813	32
(16)	1958-62	8.4			0.92 (11.5)	.816	32

[a]Period which encompasses all $I(s_2)$ expenditures.

(16) $$\text{Max}\, PD(s_2, s_1) = A[I(s_2) + I(s_1)]^{\beta}$$

where $I(s_1)$ = gross capital expenditures (in millions of dollars in 1954 prices) from the birth of the plant to the end of the first set of consecutive annual outlays.

$I(s_2)$ = second set of consecutive gross capital expenditures (in millions of dollars in 1954 prices).

$\text{Max}\, PD(s_2, s_1)$ = the maximum of peak demands (in megawatts) from the end of the second investment sequence to the beginning of the next one or to 1963.

Table 4 shows that for the 1950–57 period π_2 was much greater than π_1. The average age from birth to second expenditure sequence was higher for the plants with a second expenditure sequence in 1958–62 than for those with a second sequence in 1950–57. The aging of the initial investment appears to be reflected in a decline in π_2. On the other hand, the rise in π_1 may perhaps be attributed to increased flexibility of later investments as a consequence of technical advance. An awkward result from the standpoint of the interaction model is the somewhat higher R^2 for the equation with a capital stock variable, when estimated for the sample with renewals or expansions for 1950–57. This aspect of the problem is discussed later.

Estimates for equations with sequences of investments as variables were also made for the plants born before 1948. More than nine-tenths of these plants were in existence before 1938, the first year of Federal

TABLE 5

Results for Equations (17) and (18)

Equation	A	π_1	π_2	π_3	β	R^2	n
(17)	29.0	.258 (5.6)	.223 (4.8)	.360 (5.9)		.911	20
(18)	9.5				.87 (10.1)	.849	20

Power Commission data. As a result, our measure of what we call $I(s_1)$ is, in fact, a form of capital stock. It was derived by deflating the gross book value [21] of 1938 plant and equipment and adding to it gross capital expenditures (also deflated) for the period 1939–47. Actually, capital outlays in the 1939–47 period were not large, with the result that the stock of 1938 represented most of $I(s_1)$. After 1947, investment expenditures for these plants were very heavy until 1958, and thereafter fell abruptly. Only a few of the pre-1948 plants in our sample had capital outlays after 1958. Consequently, our analysis for these plants could not go beyond 1958, and for convenience, was carried out for plants with a third investment sequence that terminated in 1957 or earlier.

For the plants born before 1948, we estimated equations (17) and (18). The results are shown in Table 5.

$$\text{(17)} \qquad \text{Max } PD(s_3, s_2, s_1) = AI(s_3)^{\pi_1} I(s_2)^{\pi_2} I(s_1)^{\pi_3},$$

and the capital stock version of this equation is

$$\text{(18)} \qquad \text{Max } PD(s_3, s_2, s_1) = A[I(s_3) + I(s_2) + I(s_1)]^{\beta}$$

where Max $PD(s_3, s_2, s_1)$ = maximum peak demand (in megawatts) after the last year of $I(s_3)$ for a plant with $I(s_3)$, $I(s_2)$, and $I(s_1)$. The technique for determining the maximum peak was the same as described earlier.

$I(s_1)$ = gross capital in 1947 (millions of dollars in 1954 prices).

$I(s_2)$ = first investment sequence in 1948–57 period (millions of dollars in 1954 prices).

$I(s_3)$ = second investment sequence in 1948–57 period (millions of dollars in 1954 prices).

[21] Price indexes were taken from unpublished data of the Office of Business Economics, Department of Commerce.

Equation (17), the interaction model, does significantly better in explaining the variance in peak demand than equation (18). The exponent of the capital stock, π_3, is fairly low as compared with the exponent of the initial investment sequence, π_2, in equation (15). It is also less than the sum of π_2 and π_3 in equation (17). This reflects, in all likelihood, the adverse effects of age on the adaptability of old capital goods to the changing technology of existing plants.

Still another grouping criterion for plants of pre-1948 birth consisted of dividing the 1948–56 period into two arbitrarily chosen intervals, 1948–53 and 1954–56. The following two equations were then estimated with the dependent variable alternatively peak demand and kilowatt-hours. Table 6 indicates the results.

$$Y(1957) = A\left(\sum_{'54}^{'56} I\right)^{\pi_1}\left(\sum_{'48}^{'53} I\right)^{\pi_2}\left(\sum_{v_1}^{'47} I\right)^{\pi_3} \tag{19}$$

$$Y(1957) = A\left(\sum_{v_1}^{'56} I\right)^{\beta} \tag{20}$$

All the investments are measured in millions of dollars in 1954 prices. $\sum_{v_1}^{'47}$ and $\sum_{v_1}^{'56}$ are, respectively, the capital stocks of 1947 and 1956 (v_1 signifies the year of birth of the plant). $Y(1957)$ denotes alternatively megawatts of peak demand in 1957 and millions of kilowatt-hours produced in 1957.

TABLE 6

Results for Equations (19) and (20)

Equation	Dependent Variable	A	π_1	π_2	π_3	β	R^2	n
(19)	KWH	.0075	.25 (5.1)	.49 (6.5)	.54 (5.3)		.696	40
(20)	KWH	.0041				1.16 (7.4)	.590	40
(19)	PD	.0290	.15 (6.1)	.27 (7.3)	.54 (10.7)		.837	40
(20)	PD	.0126				0.92 (13.7)	.831	40

For reasons developed earlier, the partial elasticities for the later investments are substantially higher when the dependent variable is kilowatt-hours rather than peak demand. The R^2 is, however, higher for peak demand as the dependent variable, since variables other than the investments have a much larger role in determining kilowatt-hours than peak demand. With kilowatt-hours as the dependent variable, the interaction model, equation (19) explains more of the variance, than equation (20).

There remains still another methodological question. Suppose that $PD(s_2, s_1) = AI(s_2)^{\pi_1}I(s_1)^{\pi_2}$ [where both $I(s_2)$ and $I(s_1)$ are greater than zero] is a correct specification of the relation of peak demand to the investments. Under what conditions will

$$PD(s_2, s_1) = [BI(s_2) + I(s_1)]^{\beta}$$

yield a stable and meaningful β? The sufficient condition is that $I(s_1) = CI(s_2)$ for the investments of every plant. In that event

$$\beta = \pi_1 + \pi_2$$

that is, the exponent of the capital stock will equal the sum of the exponents of the individual investments in the "true" model. Statistically, correlations between $I(s_2)$ and $I(s_1)$ weaken the precision of the estimates of π_1 and π_2. The standard errors for the estimates of the exponents in the interaction equation will, under these conditions, be higher than the error for the exponent of the capital stock.

To sum up our results thus far, for plants born before 1948 the interaction equation is somewhat better as an explanation of the variance in output than an equation with a capital stock variable. For the newer plants, the two equations yield about the same R^2 in three out of four tests, with the capital stock version somewhat better in the fourth.[22] The level of R^2, however, is not the crucial test of the relative usefulness of the two alternative models. The specification of the interaction equation tested was somewhat arbitrary, especially insofar as it excluded all additive terms. The inclusion of additive terms along with interactive terms is probably a better description of the relevant process. Our primary purpose in this paper is not to defend a particular specification of

[22] The ratio of the highest to the lowest individual investment in each cross section was very high, and many times greater than, the ratio of largest to the smallest capital stock. This probably reduced the goodness of fit for the interaction equation.

the model but to explain certain relations among successive investments which need to be taken into account.

The decisive question is whether the elasticities of output with respect to the successive investments differ significantly from the elasticity of output with respect to a capital stock. Our results in general show marked differences among the exponents of successive investments and therefore between most of these exponents and that for a capital stock. If interactions are present, equations with a single capital stock variable can be seriously misleading from the standpoint of predicting the effects of new investment.

COMMENT

ANNE P. CARTER, Harvard Economic Research Project

When I told my ten-year-old son that I was going to New York to speak at a meeting, he was puzzled. He finally found a rationalization: "It must be one of those programs where they represent all walks of life!" You will have to forgive me if my remarks are not quite detached—if I tend to stress a few general articles of faith in place of detailed textual criticism. This is what you risk when you admit "all walks of life" to a conference on so serious a subject as production relations.

The paper is indeed serious, thoughtful—even provocative. It contains new and interesting ideas, not to be treated lightly. As the authors suggest, a lot of clever things have been said in this area, but there are important unsolved problems. If we do not learn to cope with them, empirically as well as theoretically, we shall be handicapped in dealing with the broader problems of economic growth and development, at home and abroad.

At the level of individual plant cross sections (I am not talking about broad aggregates. That's a "different game."), research in this area has not been very rewarding thus far. There are at least two major sources of difficulty: First, as Professors Gort and Boddy stress so carefully, capital goods of recent vintage may have very different effects on the operation of a plant or an industry, depending on the nature of the process and of the new investment. After all, there are many different kinds of processes to start with, many kinds of change in them, and as many kinds of new investment goods as you care to distinguish. Who could reasonably expect the installation of a computer to have the same effect on the production function as a new parking facility of the same cost? (It is hard to be factual and realistic without getting very specific.)

The old-fashioned "layering" approach—explaining changes in the average production function by weighting in strata of best-practice technique associated with new capacity and dropping out old strata associated with retired capacity is, admittedly, simple-minded. Nevertheless, even this approach works rather well in a few industries where the ties of process to capital are strong—notably in textiles (or it *used* to) and in electric power generation. (I shall return to this later.) These are especially rigid—simple-minded—industries. (It takes all kinds!) When I tested my faith in this approach in tin cans, and in ball and roller bearings, I emerged, as Kuh so aptly pointed out, an agnostic. No one really knows, as yet, just how well the layering approach or Gort-Boddy interaction, or any alternative approach, scores in the universe of roughly 500-odd four-digit SIC industries that make up the American economy.

Certainly there must be many industries, many innovations, many types of capital goods, many initial situations for which the layering approach is quite inadequate. Professors Gort and Boddy make an important contribution in emphasizing this. The dichotomy between "embodied" and "disembodied" change is too crude. Within the category of embodied change we have a spectrum of possibilities: New capital goods may remain aloof—may go about their business and produce as if old capital goods were not there, or, much more interesting, the old and new may interact. To allow for interaction of new and old capital goods, Gort and Boddy make the parameters of a plant's production function depend on its entire investment history. Good! The broader conception forces us to consider the many different ways in which new capital actually affects the productive process—and these ways vary from the inert to the catalytic. A new generating unit is inert: It just adds a layer of new capacity. Breaking a bottleneck, that is, correcting a state of initial imbalance in the capacities of different types of capital goods, activates idle capital, and thus yields a relatively rich return per dollar of new investment. Some types of new investment will accelerate an entire process, will multiply all old capacity by a predictable factor: Oxygen injection into blast furnaces and in open hearths boosts the productivity of all existing equipment; automated control of assembly processes may do the same. These last two seem to be the prototypes for the Gort and Boddy model—and well they might be—they are much the most exciting of the three types, to business, and to economists who are impatient with a pedestrian conception of progress. To implement what they call an "interaction model" in a given industry, however, one must know which kind of interaction predominates.

This brings me to the second major source of difficulty in this area of research: At the plant level very little data are readily available for studying these problems. And even though our profession puts a premium on ideas, on broadened conceptions, the bottleneck in this field is still information. I have had plans, for about five years, and even a small grant stashed away, for studying the impact of capital expenditures on input patterns of individual plants, using the Census individual-plant continuous time series records. Those records have been a long time in preparation. They seem to be just about ready now. When asking me to comment on this paper, Murray Brown said that it would be an investigation of vintage effects using Census data, and I frankly had high hopes of finding that Gort and Boddy had plunged into the work that Census (and I) had been so slow about. The fact that the empirical section deals with electric power generation tells us that the really interesting implementation of the Gort-Boddy model had to be postponed. The reasons are easy to guess: The Census tapes probably were not available in time. In any case, the individual-plant time series sample will be fraught with complicated problems of product mix, small samples in each sector, cumbersome arrangements for using the Census tapes. Materials inputs would have to be tabulated by hand.

Even with the best intentions, we are lost without our old standby: electric power generation. The man from Mars who scans the economic literature (strange idea?) may get the impression that electric power generation is our only real industry. For the empirical worker who requires adequate homogeneous samples for regressions, it is!

The situation is deplorable on general grounds, but particularly unfortunate for the Gort-Boddy interaction theory. Of all industries in the economy, electric power generation seems to be the one which is best described by the naïve layering approach, the one where the multiplicative interactions between new and old capital are least important. In 1959 Komiya and I, just as data-starved as Gort and Boddy, did a study of this industry, in which we projected fuel requirements from 1938 to 1956 using the layering thesis—assuming that all new capacity could be characterized by fuel requirements of a sample of new units. The results were generally within 1 per cent of actual fuel requirements! (The study was, irresponsibly, never published, although it was documented in a Harvard Economic Research Project Progress report. The reasons, I am afraid, were purely personal: Candidly, the theory was too simple-minded to enhance our reputations. I had been pushing this naïve hypothesis for years, and we both understood, a priori and a posteriori, that this industry is very special. Anyway, I had a new baby at home.)

If layering works so well in the power generation industry, how can the interaction model be appropriate? I am not sure it can. But, since "everything correlates" in electric power generation, I felt it necessary to do some further homework on the problem this week. This kind of homework, incidentally, is often very useful at the outset of an industry study: I spent a scant hour with a reputable industrial consultant in the field of electric power generation, a man who designs power networks, and posed my questions about interaction of old and new capital directly. In his opinion these interactions are negligible in this industry. His explanation was as follows: Existing steam capacity, with very minor exceptions, has been installed, in fully "optimized," that is, balanced, units (Engineers use jargon, too!) with boilers and turbogenerators paired—one to one—in carefully matched capacity. That fact is, incidentally, documented in a 1964 report of the Federal Power Commission. Above the $10.5 million cutoff point, which Gort and Boddy use, new capital expenditure means, mainly, more pairs of matched, balanced units, operated independently of existing capacity, except for some shared yard facilities of minor importance. (I think the problem of which units are used for peaking is not really relevant to the main point here.)

Why then, I asked, are new units added to old plants? Why is not all capital invested in new plants? The same factors which make the original site favorable for initial capacity may make it favorable for additional capacity: existing load distribution patterns and transmisson capacity, availability of space in crowded areas, adequate cooling water.

Capital charges contribute roughly 50 per cent of generating cost. The bulk of the remainder is fuel cost. In view of the problems of peak versus average production, of associating an appropriate output with the capital input, it would probably be preferable to concentrate on the fuel-output rather than the capital-output relation, if a single-input production function is to be used in this industry.

I do not understand the results of the Gort-Boddy regressions sufficiently to attempt to reconcile them with the frankly contradictory evidence which I have just mentioned. In all fairness to the central idea, one must remember that the authors did not really test the interaction theory in their empirical section. They fitted an exponential function distinguishing capital expenditures for different time periods. But the exponents are not made to depend explicitly on investment history. The regressions themselves are not production functions. I am not sure what they tell us.

I think the important ideas in this paper deserve a better "break." To give them a fair chance, however, it may be necessary to do more spadework along two closely related lines before submitting the program to the computer:

1. Industries, their technologies and types of capital, must first be screened qualitatively to locate bona fide, nontrivial interactions. Such interactions are common, but not universally important.

2. The interactions themselves must be surveyed, qualitatively, to identify the relevant input and output variables and to specify the forms of interactions. Are we talking about automation, with potential speedup of all existing capacity, or about adding balanced, independent lines of more conventional process equipment? How important are balancing and bottlenecks in a given industrial picture? Can they be related primarily to building expenditures alone?

I realize that there is grave danger in my recommendation of ignoring the forest for the trees. What I am really suggesting is that we follow a policy of informed, selective cutting. Otherwise, we will soon run out of wood, and we risk turning it all into pulp.

Our supply of individual plant input and investment data is expanding, but it is still very inadequate. Because we do not have ideal statistical materials, we must plan their use very carefully. Under the circumstances, perhaps under any circumstances, we must supplement the regression techniques we know and trust with outside technical information. The industrial specialist, or the literature, can help us to specify appropriate production functions for each industry. The specialist can help us to identify the important capital, input, and output variables, to anticipate specific kinds of interactions. Production functions, and changes in them, may be very dissimilar in different sectors. We cannot ignore the differences at a detailed level.

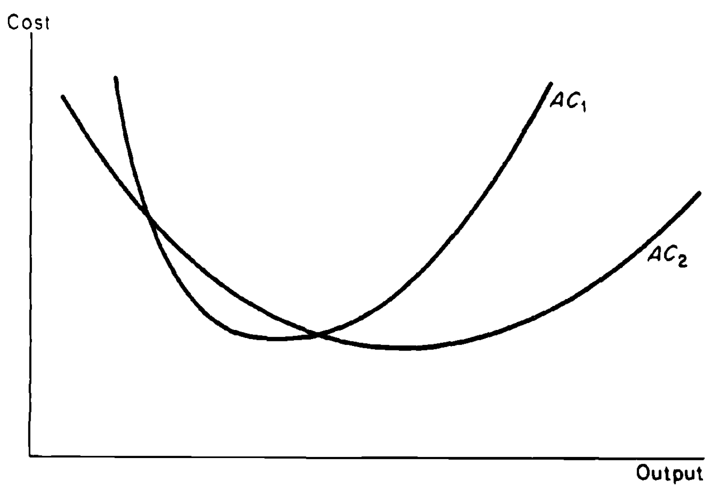
The engineer cannot, should not, do the whole job. The second, essential stage is to quantify the relationships which we have formulated, using economic statistics. But the more we know, concretely, before we run our regression programs, the better chance we have of getting meaningful answers to meaningful questions.

PETER A. DIAMOND

The authors raise a number of very interesting and extremely thorny questions in both production and investment theories on the way to developing the model they have estimated. I want to restate their general framework to see its relation to their specific model, and to point up certain alternative paths that might have been followed.

They are concerned with the interaction of present and future investment, an interaction which raises three fundamental questions: the effects of future additional investment on the current choice of technique and the future productivity of current investment and the effect of current investment on the productivity of future investment. Clearly the questions are interrelated, for the answers to the latter two contain essential information for the choice of technique.

Stated in terms of cost rather than production functions, the first question parallels the choice of technique faced by a firm with variable demand (or uncertainty about its level). For example the firm might have the two alternatives whose average cost curves are given in the diagram below.



The choice between these techniques depends on the range of possible outputs. If future investment rather than, as the story is conventionally told, labor is viewed as the variable factor, we have a representation of the problem at hand. As the authors point out, the capital cost per unit output can differ when the economy is functioning normally from what might be achieved if, in a crash program, say, preparation for future possibilities are omitted. An empirical discussion of the choice of technique would be a difficult matter indeed, but one can, following the authors, assume a single available technique and still ask the remaining two questions.

When progressing from a general to a specific model, the view of the general model colors the choice of the specific model; and equivalent general models will, when viewed differently, lead to different specific models. The authors have viewed the interaction of new and old capital as analogous to the interaction of any two inputs into the production process. Alternatively, they could have concentrated on the modernization or replacement aspect of the interaction. An example may show the type of model to which this view might lead.

Output might be determined by the inputs of labor and two types of capital goods and by the vintages of the two capital goods. For example

$$O = F(L, K_1, K_2; V_1, V_1)$$

assuming both capital goods of vintage V_1. A modernization or replacement expenditure could convert one of the capital goods to a later vintage and thus alter the output level. A variety of capital goods would be necessary to permit sensitivity of output to small expenditures on key items.

In a more aggregated framework we could pursue this line and assume fixed coefficients. A new capital good might have particular output-capital and labor-capital ratios (perhaps with choice ex ante). Over time these coefficients could be changed with a cost of modernization. The cost of changing coefficients would decline over time with advancing technical knowledge but would probably rise with the increasing age of the original capital good (or perhaps with the age of the original good adjusted somewhat for modernization). Technical progress could then be viewed as embodiable; for, unlike embodied change, it affects old capital but unlike disembodied change only at a cost. Since with these assumptions new capital has greater improvement potential than improved capital and so probably a greater future quasi-rent stream, we would expect the marginal product of investment in new plants to be lower than in old plants if investment is carried on to equate the present values of the rental streams.

Now let us turn to the additional input approach to examine the setting of the authors' specific model. With future investment treated as an additional input, present and future output produced in the same plant are expressible by the same function of labor, present investment, and future investment.

$$O_1 = F(L, I_1, O)$$
$$O_2 = F(L, I_1, I_2).$$

By contrast, with homogeneous capital, present and future output would be written

$$O_1 = G(L, I_1)$$
$$O_2 = G(L, I_1 + I_2).$$

(With embodied technical change, the capital input in the future would be a weighted sum of the two investment streams.) In their general forms, the homogeneous capital approach is a special case of the additional input approach. However, this ceases to be true in general once the two functions are specified, as Cobb-Douglas functions for example.

Having selected the additional input approach, the authors chose to approximate the production function by two separate functions rather than a single function. For each plant there are two observations, at the points (I_1, O) and I_1, I_2). All the observations presumably lie along the same production function. The authors felt that the two types of observations differed sufficiently to warrant the use of two separate Cobb-Douglas functions, rather than a single function.

This discussion is my interpretation of the authors' view of the relation of their specific and general models. I hope that I have not done serious violence to their views.

REPLY by Gort and Boddy

It is true that the interactions among successive investments are weaker for the electric power industry than they are for most other industries. Our choice of industry was constrained by considerations that

Mrs. Carter explains all too well. However, she goes too far in saying that interactions for power-generating plants are negligible and that additions are normally made in combinations of perfectly balanced components.

A possible reason for Mrs. Carter's impression is the tendency to think of investment in terms of the purchase of the principal equipment units of a given production process—in the case of electric power, boilers and turbogenerators. In fact, however, investment expenditures are made on a large variety of items, many of them relatively small individually but large in the aggregate. For example, in electric power plants, there are outlays on transformers, switchgear, modifications of turbogenerators, the conversion of boilers from one fuel to another, improvements in structures, and many other items. Moreover, one form of interaction is the addition of new boilers and turbogenerators to an existing building.

The pattern of post-1947 outlays on plants built before 1948 is too distinctive to attribute to chance influences. If, for example, the principal reason why an outlay is made on an existing rather than on a new plant is the presence of unused yard space, the number of successive outlays made on a plant after its birth will depend on chance factors such as how much real estate was available at the time of a plant's construction. Accordingly, the number of successive after-birth outlays should vary widely among plants. Similarly, if successive outlays are made in the same location until the geographic pattern of demand shifts, variations in experience among plants should once again be great. As a matter of fact, there was a distinct modal category of two investment "programs" (sequences of consecutive years with outlays) after 1947 for plants built in or before 1947. Hardly any of these plants had investment outlays after 1958. Is it not more plausible that this pattern is explained by the character of interactions and the limits to interactions imposed by technical change rather than by chance factors such as the availability of yard space?