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THE OPTIMAL EXPLOITATION OF EXHAUSTIBLE RESOURCES, A SURVEY

BY

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1 INTRODUCTION

Although one cannot hope to give a full explanation of the economist's attention for any issue at specific times, in the case of natural resources it is not surprising that the 1970's have produced an abundant literature on the subject. One should however not forget that the subject is not new. In the 19th century land was commonly regarded as the important natural resource (Malthus, Ricardo), whereas W. Jevons predicted the end of the industrial revolution in England as a consequence of the physical limits of coal deposits. In the first decades of the 20th century economic theory was far less concerned with natural resources. Even the long-run economic growth literature of the 1950's and the 1960's has not dealt with them. Dasgupta and Heal (1979) give the following explanation of this phenomenon: "resource constraints were not important for industrialized countries: they either possessed their own resource supplies, which they regarded as adequate, or felt that they could be confident of importing resources in unlimited amounts from developing countries, initially because in many cases they controlled these countries as part of the economic system and subsequently because, although independent, the supplying countries remained politically quiescent, with foreign exchange needs so great that they could be counted on to supply unlimited quantities of their principal (and often only) exports."

Still there is one outstanding economist, Harold Hotelling, who in fact had already founded the contemporary economic theory of exhaustible resources in 1931. His work deserves special attention since it already incorporates almost all the issues which nowadays are considered relevant in

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this field: optimal exploitation, free competition versus monopoly, extraction costs, oligopoly and optimal taxation.

It is tempting to give a survey of all the issues in the literature on exhaustible resources. For reasons of space and clarity we shall however restrict ourselves. Firstly we shall only discuss contributions which refer to pure exhaustible resources. Hence replenishable resources like fish, forests, etc., will be neglected. For a survey of this field we refer to Peterson and Fisher (1977). Secondly, for closed economies we shall only review the literature on socially optimal exploitation, whereas for open economies only the case of given world market prices will be considered. This is a severe restriction: no attention will be paid to the exploitation of a resource owned by a monopolist, nor will the field of interdependence between resource-rich and capital-rich countries be covered. A thorough evaluation of these issues would not only take far more space, but one may also doubt whether the time has arrived to give a full exposition. Indeed, the literature on this subject is still rapidly evolving (see e.g. Kemp and Long (1980)).

Section 2 is devoted to some preliminary remarks about the evaluation of exploitation patterns. In section 3 the basic model, that of Hotelling, will be presented. Section 4 discusses the sensitivity of the optimal program of exploitation for changes in the economy's preference scheme. In section 5 we consider the contributions which fit the theory of exhaustible resources into the theory of optimal economic growth. In section 6 we treat the problem of exploitation of natural resources in an open economy. In section 7 extraction costs are introduced. Section 8 contains some optimistic views concerning natural resources. In section 9 attention will be paid to the issue of uncertainty. Finally section 10 gives some conclusions.

2 WELFARE FUNCTIONALS

In general the theory of the optimal exploitation of exhaustible resources is concerned with the following problem. An economy possesses a finite reserve of a natural resource. Directly or indirectly the exploitation of this resource is important for the economy. In some models the extraction of the resource is the only source of welfare. In other models it serves as an input in a production process having a consumer good as an output. The objective of the economy is to find an exploitation pattern that maximizes welfare. Then the question arises how welfare should be evaluated. Let C_t denote aggregate consumption at time t and let $[C_t]$ be a consumption pattern that fulfils the constraints imposed by the model. Such a pattern is called feasible. Assuming that the set of feasible consumption patterns (F) is not empty, one wishes to rank the elements of this set according to the welfare associated with them. A common feature of the models is that they take the utilitarian point of view. At each t the rate of consumption is evaluated by means of a concave function u, representing the rate of utility, which is possibly discounted at a rate ρ. The best element of F is the pattern that maximizes total discounted utility. Since the problem at hand refers to optimal behaviour over time, one should be concerned with the question of the economy's horizon. In the literature three views can be distinguished. Firstly one can assume that the final time T is given. In this case the welfare functional is:

$$I = \int_0^T e^{-\rho t} u\,(C(t))\,\mathrm{d}t. \tag{1}$$

The feasible consumption pattern that maximizes I maximizes total welfare. The second point of view is that the economy's horizon should be infinite, which gives

$$I = \int_0^\infty e^{-\rho t} u\,(C(t))\,\mathrm{d}t. \tag{2}$$

It might be that no maximum exists since the integral possibly diverges. In some cases this difficulty can be met by invoking the overtaking criterion, defined by Von Weizsäcker (1964). However, we shall not need this criterion.

Finally one may assume that the horizon is not *a priori* finite. This implies that the time to which the planning extends is an endogenous variable as well. A plan $(\hat{T}, \hat{C}(t))$, where $\hat{C}(t)$ is defined for all $t \leqslant \hat{T}$ is optimal if for all T and any $C(t)$, defined for $t \leqslant T$ we have:

$$\int_0^{\hat{T}} e^{-\rho t} u(\hat{C}(t))\,\mathrm{d}t \geqslant \int_0^T e^{-\rho t} u(C(t))\,\mathrm{d}t. \tag{3}$$

If such a final $\hat{T}$ exists then total discounted utility cannot be increased by lengthening the horizon nor by shortening it. If no final $\hat{T}$ exists then the problem reduces to an infinite horizon problem.

There are some basic differences between the three welfare functionals presented above. However, since we do not wish to get involved in the discussions on justice and intergenerational equity, we merely mention the existence of these cases in order to be more concise in the following.

3 THE BASIC MODEL

The simplest model is that of Hotelling (1931) and describes the so-called "cake eating" problem. There exists a nonrenewable resource of known magnitude. The exploitation of this resource is costless and provides a commodity that is available for immediate *consumption*, the sole source of welfare. The economy's horizon is an endogenous variable. There is a strictly positive rate of time preference ρ. Let $R(t)$, $E(t)$ and $C(t)$ denote the size of the resource at time t, the rate of exploitation at time t and the rate of consumption at time t respectively. The economy's objective is to find a final time $\hat{T}$ and a consumption pattern $\hat{C}(t)$ such that

$$I = \int_0^T e^{-\rho t} u(C(t))\, dt$$

is maximized, subject to

$$C(t) = E(t), \tag{4}$$

$$\dot{R}(t) = -E(t), \quad \text{where } \dot{R} = dR/dt, R(0) = R_0, \text{given}, \tag{5}$$

$$R(t) \geqslant 0,\ E(t) \geqslant 0\,. \tag{6}$$

Mathematically our treatment of this problem will be more rigorous than Hotelling's in order to reveal the assumptions he implicitly makes. Assume that an optimum exists and let ˆ above a variable denote its optimal value. Obviously (4)-(6) have to be fulfilled. Other conditions for an optimum can be found in Takayama (1974). For the present model there must exist a constant p (the shadow price of the resource) such that

$$e^{-\rho t} u'(\hat{E}(t)) = p \ \text{ if } \hat{E}(t) > 0\,, \tag{7}$$

$$e^{-\rho \hat{T}} u(\hat{E}(\hat{T})) = p\hat{E}(\hat{T})\,, \tag{8}$$

$$p\hat{R}(\hat{T}) = 0\,. \tag{9}$$

Henceforth we shall omit the time index t and the hats where there is no danger of confusion. Hotelling assumes that $E(T) = 0$, which implies $u(0) = 0$ from (8). Furthermore this implies $u'(0) < \infty$ in view of (7). The rate of exploitation is always positive since otherwise the optimal path can be improved by postponing the times where $E(t) = 0$ (for $u' > 0$). It follows

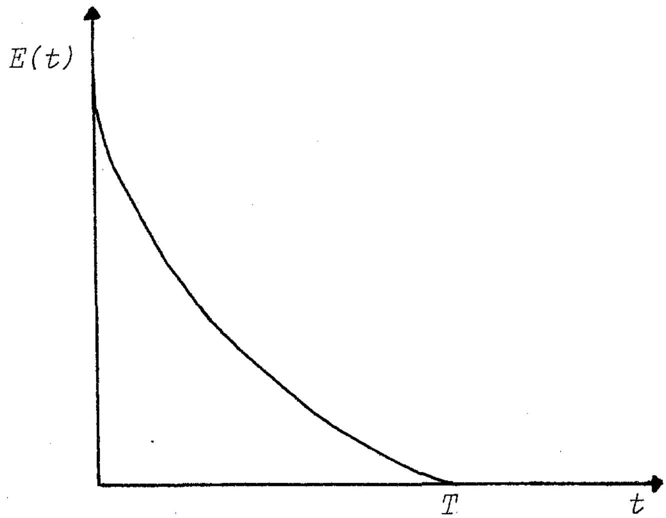

Figure 1

from (7) that the rate of exploitation is decreasing over time. How can the optimal stopping time be determined? Notice that from (7) E can be solved as a function of p and t. Inserting $E(p, t)$ into

$$R_0 = \int_0^T E(p, t)\,\mathrm{d}t \,, \tag{10}$$

gives $T = T(p)$. It can be shown that it is optimal to choose the final time such that $\mathrm{d}T/\mathrm{d}p = 0$. Hence the longest possible exhaustion path must be followed. Due to the concavity of the utility function u the necessary conditions are also sufficient and the finiteness of the resource guarantees the existence of an optimum. An optimum pattern is depicted in Figure 1.

When $u'(0) = \infty$ there will not exist an optimal final time. In this case the resource will never be exhausted. However, as before, the rate of exploitation is always declining. The optimal shadow price of the resource can be determined from (7) and (10), where T is replaced by ∞.

4 ALTERNATIVE SPECIFICATIONS OF THE WELFARE FUNCTIONAL I

The previous section shows that the specification of the welfare functional plays an important role in the characterization of the optimal path. Koopmans (1974) and Vousden (1977) have considered this feature in detail. Koopmans investigates the impact of the *rate of time preference*. In his model a minimum rate of consumption ($\bar{C}$), necessary for the economy to survive, is postulated. This implies that the resource will necessarily be depleted in finite time $(R_0/\bar{C})$. The actual date of depletion is called doomsday.

The function u is strictly concave for all $C > \bar{C}$, $u(\bar{C}) = 0$ and $u'(\bar{C}) = \infty$. Koopmans then proves that if there is no time preference the optimal rate of consumption (which equals exploitation) is a constant, equating marginal and average utility:

$$u(C) = Cu'(C) . \tag{11}$$

From this, doomsday can be derived. When the rate of time preference is positive the optimal rate of consumption and the optimal stopping time are the solution of

$$e^{-\rho t} u'(C(t)) = e^{-\rho T} u'(C(T)) , \text{ where } C(T)u'(C(T)) = u(C(T)) , \tag{12}$$

$$\int_0^T C(t)\,\mathrm{d}t = R_0 . \tag{13}$$

The optimal path shows a decreasing consumption until the level is reached that would be optimal if there were no time preference. The date at which this happens is the moment the resource is exhausted. These results are illustrated in Figure 2.

The conclusion is: "discounting advances doomsday."

Although Vousden (1977) is primarily concerned with something else, as will be shown below, his analysis permits a full treatment of the sensitivity of optimal paths for the specification of the utility function u, the rate of time preference and the economy's horizon. We do not intend to reproduce

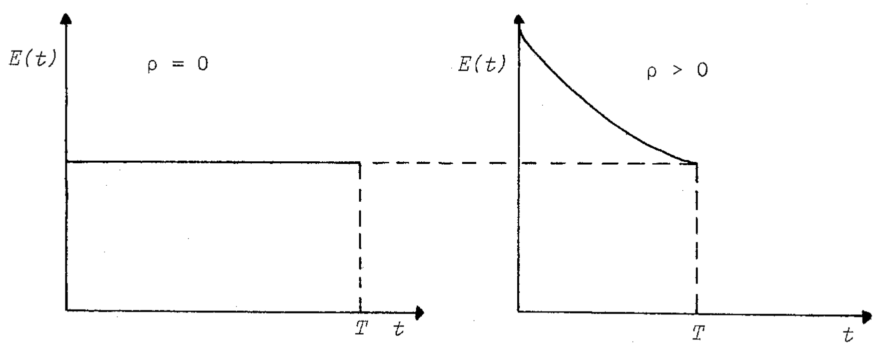

Figure 2

his analysis completely and therefore we shall restrict ourselves to some illustrative examples.

We first deal with the *rate of time preference*. Assume that the horizon is infinite and that $\rho = 0$. In this case no optimal path exists. This can be seen easily from (7). In an optimum the rate of exploitation must be constant as long as it is positive. Moreover, it should always be positive since it cannot be optimal to have jumps in view of the concavity of u. But no positive constant exists which does not exhaust the resource and hence no optimum exists. This result originated with Gale (1967). When, keeping the horizon infinite, there is a positive rate of time preference, an optimum exists and the rate of exploitation is decreasing over time.

Next consider the sensitivity for the *economy's horizon*. Assume that the rate of time preference is zero as well as the utility of zero exploitation ($u(0) = 0$). Given a fixed final time T the optimal rate of exploitation is a constant, equal to R_0/T. If the final time is infinity no optimum exists, as we saw above. Clearly no optimum exists either if the economy's horizon is endogenous. Taking T to be endogenous and assuming a positive rate of time preference the sensitivity for $u(0)$ can be illustrated. No optimal final time exists if $u(0) > 0$ for the economy enjoys utility even after the exhaustion of the resource. If $u(0) = 0$ the results obtained by Hotelling apply. When zero exploitation yields negative utility an optimal final time exists and essentially the solution is equivalent to Koopmans' optimum (12) and (13). As we have seen in the previous section, if $u(0) = 0$ the resource will not be exhausted in finite time when $u'(0) = \infty$, unless of course the final time is *a priori* fixed.

Finally consider the case where the utility function is *convex* for values of C smaller than some $\bar{C} > 0$ and *concave* for values of C larger than $\bar{C}$. Now there is some positive rate of exploitation, denoted by E^*, that maximizes average utility. Vousden shows that it cannot be optimal to have a jump in the rate of exploitation from a value greater than E^* to zero or vice versa. Hence it is not optimal to exploit at a rate for which the utility function is convex and exploitation will not gradually decline to zero.

Our conclusion from this section is that the existence and the form of solutions to the problem heavily depend on the form of the utility function. Linear transformations of the utility function which in standard theory of consumer behaviour do not cause any problem may give completely different results.

5 PRODUCTION

The models presented in the previous section all considered the extraction of the resource to be immediately available for consumption, and the extraction itself an only source of welfare. For most natural resources however the benefits for the economy are not derived from the resource itself but arise from the fact that using the extraction *as a factor of production* gives some other commodity (say a nonresource good) which is the argument of the economy's utility function u. This observation suggests that the theory of the optimal use of exhaustible resources should be fit into the more general theory of optimal economic growth (see Koopmans (1965), Cass (1965)). This has been pointed out by Koopmans (1973) and Anderson (1972) in particular. In this section we shall not survey this area of research historically but the contributions will be ordered according to their complexity.

Restricting ourselves to the well-known one-sector growth model, the problem can be described as follows:

$$\max I = \int_0^T e^{-\rho t} u(C/P)\,\mathrm{d}t\,, \tag{14}$$

subject to

$$\dot{K} = F(K, L, E, t) - \mu K - C,\ K_0 \text{ given}\,, \tag{15}$$

$$\dot{R} = -E\,,\ R_0 \text{ given}\,, \tag{16}$$

$$\dot{L} = \lambda L\,,\ L_0 \text{ given}\,, \tag{17}$$

$$\dot{P} = \pi P\,;\ P_0 \text{ given}\,. \tag{18}$$

Here T is the final time, *a priori* specified or endogenous, P is the size of the population which grows at a rate π, K is the stock of capital, deteriorating at a rate μ, L is the labour force growing at a rate λ. Henceforth we shall always assume that $\lambda = \pi$.

The nonresource commodity is produced according to a production function F, possibly exhibiting technical progress. The factors of production are labour, capital, and the rate of utilization of the resource. Evidently this general formulation does not permit concrete statements about optimal time paths (if any). Indeed most of the results obtained apply to special cases. In the previous section we referred to Vousden (1977) for a closer examination of the welfare function. In doing so however we did not cover his contribution to the field. His point of departure was that the withdrawal from the resource serves as an input to a production process having consumption goods as output. This model can be fitted into the model presented above by putting $\lambda = 0$, $F(K, L, E, t) = F(E)$ and omitting capital. About F it is assumed that $F(0) = 0$ and $F' \geqslant 0$. Then, as in Figure 3, a distinction is made between three types of production functions.

a. F is strictly concave, which is the usual neoclassical case.
b. F is convex for all $E < \bar{E}$ for some $\bar{E}$ and concave for all $E > \bar{E}$. This

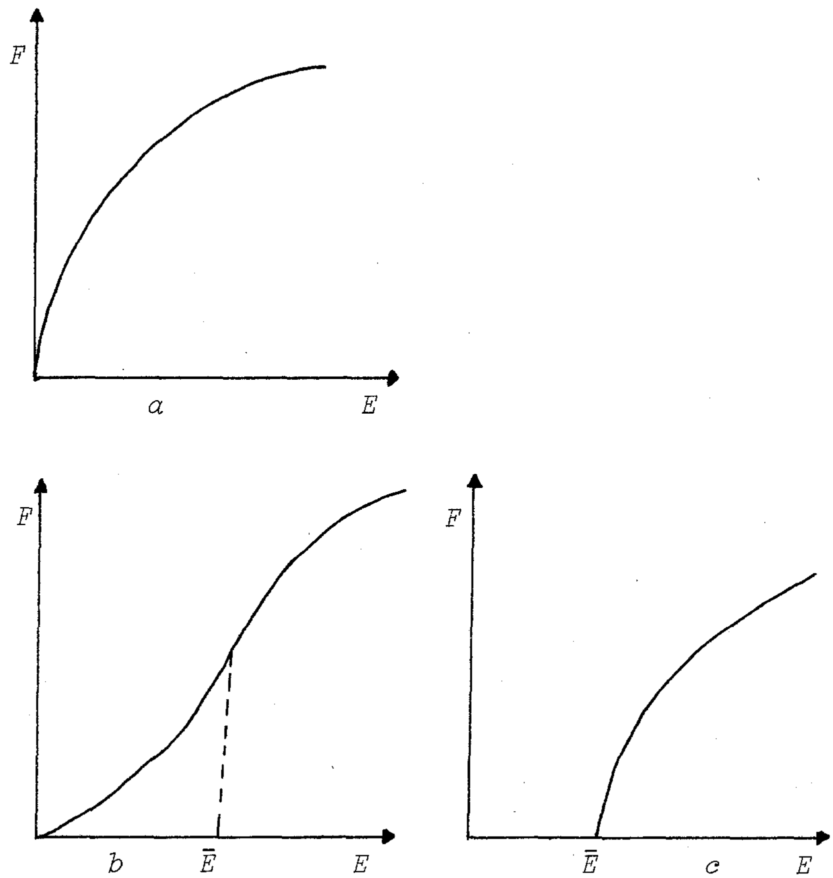

Figure 3

gives the well-known s-shaped production function.

c. $F = 0$ for all $E < \bar{E}$ for some $\bar{E}$ and F is concave for all $E > \bar{E}$. This function allows for set-up costs.

u is assumed to be strictly concave in C. In order to avoid the possibility of having zero consumption, an exogenous stream C' of consumption goods is introduced. It is now easily seen that results similar to those obtained in the previous section can be derived after substitution of $F(E) + C'$ for C.

Dasgupta and Heal (1974) study the case where not only the resource but also the stock of *capital and labour* are factors of production. The labour force and the population are kept constant; there is no depreciation and no technical progress. The horizon is infinite. The production function is assumed to exhibit constant returns to scale. The utility function is strictly concave,

$$u'(0) = \infty \text{ and } \infty > \lim_{C \to 0} (-Cu''(C)/u'(C)) = \eta > 0 .$$

Whether exploitation will continue forever or not depends on the importance of the resource as a factor of production. The resource is important if its marginal product approaches infinity as the resource input approaches zero. Intuitively it is clear that the resource will not be exhausted in finite time. If the resource is not essential to production in the sense that production can take place without it and if moreover its marginal product is bounded, exploitation will come to an end within finite time.

Then Dasgupta and Heal derive more explicit statements about the existence and profile of an optimum. To this end specific assumptions are made about the utility function and the production function. In particular it is assumed that the elasticity of marginal utility $(-\eta)$ is constant. This implies that $u(C) = C^{1-\eta}/(1 - \eta)$ for $\eta \neq 1$, $u(C) = \ln C$ for $\eta = 1$. This function is frequently used in the growth literature (see (e.g. Tinbergen (1960), and Mirrlees (1967)). Moreover only CES production functions are considered. The elasticity of substitution between capital and the resource is denoted by σ. Define ϕ as the marginal product of capital when the capital-resource ratio approaches infinity. Now the following proposition holds. If the rate of time preference is sufficiently large $(\rho > \phi(1 - \eta))$ and $0 < \sigma < \infty$, an optimum exists and the growth rates of the rate of consumption $(\dot{C}/C)$ and the stock of capital $(\dot{K}/K)$ approach $(\phi - \rho)/\eta$. Furthermore $\dot{E}/E$ tends to $-\sigma\phi + (\phi - \rho)/\eta$. Let us try to interpret these results. The condition on the rate of time preference will look familiar to those who have studied the economic growth literature. It insures convergence of the welfare functional. The growth rates of the rate of consumption and the stock of capital are positive if marginal product of capital is large enough, as one would expect. Finally it is obvious that exploitation will decline faster the larger substitution possibilities are.

The case of a linear production function ($\sigma = \infty$) gives some mathematical difficulties. But let us assume that σ is very large. Then it is optimal to exhaust the resource almost completely at the beginning of the planning period to build up the stock of capital, which from $t = 0$ on is "eaten up." There is some relation between the elasticity of substitution σ and the limit of marginal productivity of capital (ϕ). If $0 \leqslant \sigma < 1$ the substitution possibilities are small and marginal product tends to zero. In this case any feasible consumption pattern declines towards zero. When substitution possibilities are large ($\sigma > 1$) the marginal product of capital tends to a nonzero constant. In this case the resource is not essential to production and the rate of nonresource consumption is not necessarily decreasing.

It seems interesting to consider the Cobb-Douglas case ($\sigma = 1$) in more detail. On an optimal path the rate of consumption is falling, but it has been shown by Solow (1974) that a constant level of consumption can be sustained if the elasticity of production with respect to capital is larger than the elasticity of production with respect to the resource. Hence the Cobb-Douglas case is the limiting case between a technology which is not able to sustain a constant rate of consumption and a technology in which essentially there is no resource problem. A modest defense of a Cobb-Douglas production function for the case of essential inputs can be found in Kemp and Long (1980). Using a Cobb-Douglas technology Stiglitz (1974) has pointed out that "there are at least three *economic forces offsetting the limitations imposed by natural resources*: technical change, the substitution of man-made factors of production (capital) for natural resources and returns to scale."

This proposition is illustrated by means of a model which resembles the Dasgupta/Heal model but takes into account a growing labour force, Hicks neutral technical progress at a rate γ and possibly returns to scale. The technology is described by:

$$F(K, L, E, t) = e^{\gamma t} K^{\alpha_1} L^{\alpha_2} E^{\alpha_3}, \tag{19}$$

where α_i ($i = 1, 2, 3$) is the elasticity of production with respect to capital, labour and the natural resource respectively (henceforth we shall call them shares). The labour force is a constant proportion of the population and grows at a rate λ. As in the previous model the utility function is iso-elastic. The argument of the function is per capita consumption.

Before describing optimal solutions, some results concerning feasibility are summarized. Assume that production is intertemporarily efficient, which means that the return on capital equals the rate of change of the marginal product of the natural resource ($F_K = \mathrm{d}\, ln\, F_R/\mathrm{d}t$). This condition is one of the necessary conditions for an optimum. If there is no technical progress nor population growth a necessary and sufficient condition for sustaining a constant rate of per capita consumption is that the share of capital is larger than the share of the natural resource. If there is technical progress and population growth a necessary and sufficient condition for sustaining a constant level of per capita consumption is that the ratio of technical change to the rate of population growth is larger than the share of the natural resource. If in the latter case there are returns to scale ($\Sigma\alpha_i > 1$), this condition becomes $\gamma/\lambda > (1-\alpha_1-\alpha_3)$. Next the optimal paths are calculated. The horizon is taken to be infinite. A necessary and sufficient condition for the existence of an optimum is that $(1-\alpha_1)(\rho+(1-\eta)\lambda) > (\gamma+\alpha_2\lambda)(1-\eta)$. In an optimum the capital-output ratio and the savings rate converge to constants (which will not be given here). The rate of consumption eventually grows if and only if $\alpha_3(\rho+(1-\eta)\lambda) < \gamma + \alpha_2\lambda$. If this inequality holds, the stock of capital decreases. Finally, an appealing result is that if $\eta = 1$ ($u = ln\, C/P$) a constant percentage ρ of the existing pool of the natural resource should be exploited at any instant of time when the stationary savings rate is reached.

In the previous contributions the exploitation rate of the natural resource enters into the economy's production function as just another factor of production. One popular view, however, recently expressed again by Sassin (1980), is that for example energy and national product are *complementary*. This case has been treated by Anderson (1972). The economy's production possibilities are described by

$$Y = F(K, L)\,, \tag{20}$$

$$E = e^{-\gamma t}\, Y\,, \tag{21}$$

where Y is the nonresource output, F is a production function for which the usual neoclassical properties hold. Equation (21) says that the resource requirement is a fixed proportion of the economy's nonresource output. This burden is possibly less severe if there is technical progress (γ). In Anderson's model labour grows at a rate λ. As before the rate of depreciation of the stock of capital is denoted by μ. The utility function u is linear: $u = C/P$. The horizon is fixed and the final time is T; the final stocks of the resource and capital are bound to be larger than some given values R_T and K_T. The problem presented here has been studied extensively by Shell (1967) for the case where the resource constraint is not binding, which means that R_0 is infinite or that $R_0 - R_T$ exceeds the resource use on any feasible path. We shall not give all of his results but restrict ourselves to the cases which are dealt with by Anderson. Defining the modified golden rule path of the economy as the path where the per capita consumption is derived from a constant capital-labour ratio equating marginal product and the sum of the depreciation rate, the rate of population growth and the rate of time preference, a typical optimal path can be characterized as follows (assuming $k_T > k_0$):

> for $0 \leqslant t \leqslant t_1$ for some t_1 and for $t_2 < t \leqslant T$ for some t_2 the savings rate equals unity and for $t_1 \leqslant t \leqslant t_2$ the savings rate is set such that the modified golden rule path prevails.

This concept is illustrated in Figure 4. It is assumed that T is large enough to have $0 < t_1 < t_2 < T$. $\hat{k}$ is the modified golden rule capital-labour ratio. Anderson shows that in the case where technical progress in resource requirements just offsets population growth and time preference ($\gamma = \lambda + \rho$) and if

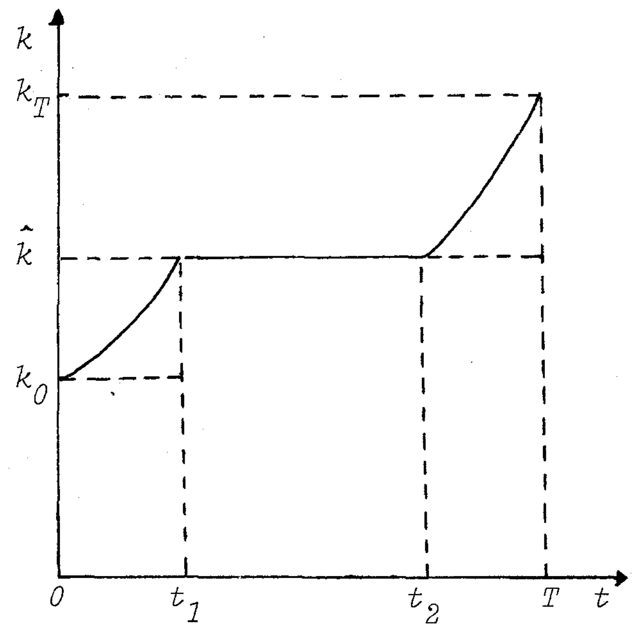

Figure 4

in the unconstrained case the optimal path is as shown above, a constrained optimum can be depicted as in Figure 5. Here k^* is the solution of

$$F'(K/L, 1) = (\mu + \lambda + \rho)/(1 - q), \tag{22}$$

where q is the constant shadow price of the resource. q is positive and smaller than unity and it is determined such that the final pool of the resource equals the minimal prescribed reserve. Compared with the unconstrained case the accumulation is postponed. If $\gamma > \rho + \lambda$, there is also postponement of accumulation but if T increases the optimal path monotonically approaches the unconstrained optimal path. In this case the resource is essentially not scarce. If $\gamma < \rho + \lambda$ the existence of an optimum for all final T is not guaranteed. But if an optimum exists a typical trajectory will look like the path depicted in Figure 6. The intervals during which all output is saved are even longer now and in the intermediate stage ($t_1'' < t < t_2''$) the capital-labour ratio is decreasing. What happens if T is infinite? If $\gamma > \rho + \lambda$ the optimal

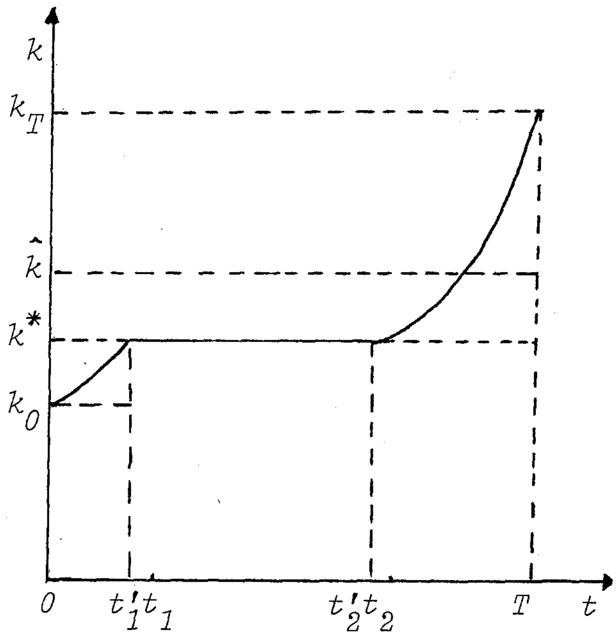

Figure 5

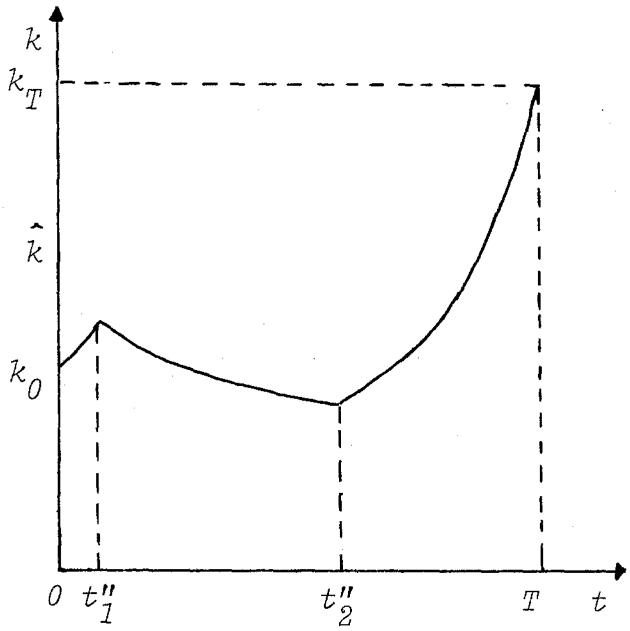

Figure 6

path will approach the modified golden rule path. If $\gamma < \rho + \lambda$ the capital-labour ratio necessarily declines to zero.

Finally we mention a contribution by Withagen (1980) who characterizes the optimal solution for the case where capital and the resource are complements.

6 OPEN ECONOMIES

A natural extension of the models just described seems to consider economies which face *world markets for the nonresource good and for the natural resource* they own. The justification of incorporating this feature into the theory of exhaustible resources is clearly evident in view, for example, of the case of energy.

Most of the models consider the problem of a small country in possession of a natural resource for which there is a given constant relative world market price p. Hence we may assume throughout that the resource extracted is entirely sold on the world market. If there is no domestic production of consumption commodities the problem reduces to:

$$\text{maximize } I = \int_0^T e^{-\rho t} u(C(t))\, \mathrm{d}t\,,$$

subject to

$$C = pE \tag{23}$$

$$\dot{R} = -E\,. \tag{24}$$

Equation (23) says that (the current account of) the balance of payments has to be in equilibrium. Substitution of pE for C in the welfare functional shows that all the results from section 2 apply. When domestic production is introduced the picture becomes only slightly different. Equation (23) has to be replaced by

$$C = F(E_f) + C_m - C_x\,, \tag{25}$$

$$p(E - E_f) = C_m - C_x\,, \tag{26}$$

where it is assumed that domestic production (F) requires the resource good. C_m and C_x are the imports and the exports of the consumption commodity. If the production function allows for an equality of the world market price p and marginal product, then it is optimal to import the resource at a constant rate such that $F'(E_f) = p$ and again the problem of the optimal exploitation can be solved along the lines exposed in section 2. The problem becomes more interesting if more inputs are introduced. Suppose that the domestic production possibilities are described by $F(K, L, E_f, t)$ where t refers to technical progress. Let K_d be the domestically owned stock of capital and assume r to be the given world market rate of interest. Then equilibrium on the current account requires

$$pE + rK_d + F(K, L, E_f, t) = C + rK + pE_f + \dot{K}_d\,. \tag{27}$$

This equation says that the revenue from the exploitation plus the revenue from capital and nonresource output equals consumption plus expenditure on capital and resource imports plus net investments. An interesting conclusion is that the rate of exploitation cannot be positive for an interval of time. This can be seen immediately from the necessary conditions for an optimum. Hence the resource should be depleted at once in the beginning of the planning period and the returns be invested in capital. Intuitively the result is evident: the resource is not more nor less than a lump of gold which can be traded on the world market. One should however not forget that this seemingly striking result is obtained under rather specific assumptions. It will for example not occur when exploitation costs are introduced, as will be shown below. The assumptions concerning world market conditions constitute another, major problem. In the real world, markets are far less perfect than we have postulated. The oil market provides an example. Furthermore world market prices are not constant. Finally the existence of a perfect capital market can be doubted. Since it is intended to limit the scope of this article, we shall only give one example of how these problems can be tackled.

Dasgupta and Heal (1979) introduce a small resource-rich country, behaving monopolistically, trading with a capital-rich world. It is then shown that the optimal decisions concerning the inputs of capital and the resource good are independent of the decisions concerning the rate of consumption and the rate of accumulation of capital. In fact the Fisher separation theorem applies: utility maximization requires income maximization. This result is important since in order to find the optimal rate of exploitation it is not necessary to know the parameters of the welfare functional. It is also clear that the argument can be extended to the case of many nonresource goods. Still there remains the problem of the intermediate case where there are a few competing resource-rich countries.

7 EXTRACTION COSTS

In general, extraction costs occur if the exploitation of a resource requires labour and/or capital and hence reduces the output of the consumption commodity. For a closed economy this can be modelled in the following way:

$$C = F(K_1, L_1, E, t) - \dot{K}, \tag{28}$$

$$E = G(K_2, L_2, R), \tag{29}$$

$$L_1 + L_2 = L, \tag{30}$$

$$K_1 + K_2 = K, \tag{31}$$

where L_2 (K_2) is the amount of labour (capital) devoted to exploitation activities. In the exploitation function G the stock of the natural resource may appear as a variable to indicate that extraction might be easier or more laborious as more of the resource has already been extracted. Unfortunately for the model presented here no general conclusions can be found. It seems, however, (see Dasgupta and Heal (1974)) that introduction of extraction costs in closed economies will in general not alter the results drastically. Another concept of extraction costs is introduced by Solow and Wan (1976). In their model there are several deposits of the natural resource, each requiring a different but constant amount of capital per unit of extraction. Using a Cobb-Douglas technology for the nonresource output they prove that it is optimal to exhaust the cheapest deposits first.

Let us now turn to the case of an open economy and consider a model presented by Vousden (1974). The sole input in the production of consumption commodities is labour (L_1). For the extraction of the resource *labour* is also necessary (L_2). Hence the problem is to find an optimal allocation of the (constant) labour force over both activities. In Vousden's contribution the resource good appears in the welfare functional as well. In order to keep the exposition simple, however, we shall neglect this feature. The length of the horizon is fixed. The model is

$$\text{maximize } I = \int_0^T e^{-\rho t} u(C(t))\, \mathrm{d}t\,, \tag{32}$$

subject to

$$C = F(L_1) + C_m - C_x\,, \tag{33}$$

$$\dot{R} = -E\,, \tag{34}$$

$$E = G(L_2)\,, \tag{35}$$

$$pE = C_m - C_x\,, \tag{36}$$

$$L_1 + L_2 = L\,, \text{given.} \tag{37}$$

Here F and G are assumed to be neoclassical. Let E^* denote the rate of exploitation that maximizes national product, $F + pE$, subject to the labour constraint. It can be shown that the optimal rate of exploitation is E^* until the resource is exhausted. Obviously this occurs in finite time. After exhaustion the economy specializes in producing consumption commodities. Remark that the optimal rate of exploitation is independent of the economy's preferences.

Kemp and Suzuki (1975) consider a similar model, but in their contribution the resource good is a factor of production of the consumption good. Moreover extraction costs are also affected by the amount of the *resource already extracted*. Hence (33) and (35) are replaced by:

$$C = F(L_1, E_f) + C_m - C_x, \tag{38}$$

$$E = G(L_2, R). \tag{39}$$

It is assumed that $G(L_2, R) = L_2 g(R)$, where $g(0) = 0, g(\infty) = \infty, g'(0) = \infty$, $g'(\infty) = 0$. Hence it gets more difficult to extract the resource. Their conclusion is that depending on the parameters of the model the economy either specializes in the C-industry, importing the resource good or it specializes in the resource industry until some finite T and afterwards all labour is allocated to the C-industry. In the latter case the resource will not be exhausted. Hence in either case the economy eventually specializes in the C-industry.

In a recent contribution Aarestad (1978) has generalized the previous models by introducing capital into the model. However the resource does not enter as a means of production. In addition to the usual equations (32), (34) and (36) the model reads:

$$C = F(K, L) + pE - G(E) - \mu K - \dot{K} + \bar{C}, \tag{40}$$

$$0 \leqslant E \leqslant \bar{E}, \tag{41}$$

$$\dot{L} = \lambda L. \tag{42}$$

$\bar{C}$ denotes an exogenous stream of consumption goods. The rate of exploitation is bounded from above by $\bar{E}$. $G(E)$ gives the costs of exploitation. The analysis of the problem becomes rather complicated. Perhaps the most interesting result is that the resource is exhausted in finite time.

Summarizing this section we may say that for closed economies some results are obtained which do not seem to alter the outcomes drastically compared with the case where no extraction costs prevail. For open competitive economies the results point to the end of exploitation in finite time.

8 NONCONVENTIONAL EXPLOITATION AND RESEARCH AND DEVELOPMENT

Thus far it has been assumed that the size of the resource is finite. Recently the idea of absolute finiteness of resources has come under discussion. Smith (1974) argues, for example, that society will find *substitutes for resources*, such as solar energy for oil and natural gas and furthermore he points to the possibility of recycling. Heal (1976) puts forward that "there is evidence that many of the resources commonly regarded as exhaustible are in fact available in effectively unlimited quantities but under a range of different supply conditions" (see also Nordhaus (1973)). Apart from these considerations it is argued in the literature (by e.g. Chiarella (1980) and Takayama (1980)) that research and development are important to the case of nonrenewable resources.

We first consider the Smith model. Let S be the output from the *backstop technology* (recycling, solar energy). Assume that the economy's labour force L is given and has to be allocated to conventional exploitation (L_1) and to the nonconventional production (L_2). The technology is described as follows:

$$S = \gamma L_1, \gamma \text{ constant}, \tag{43}$$

$$E = G(L_2, R), \ \partial G/\partial L_2 < \infty . \tag{44}$$

The economy's objective is to maximize discounted total resource output:

$$I = \int_0^{\infty} e^{-\rho t} (E + S) \, dt . \tag{45}$$

If the initial size of the resource is "large," the economy specializes initially in conventional exploitation. The rate of exploitation declines. At some finite time T labour is transferred out of "mining" and then follows a stage in which both activities are carried out, more and more labour being transferred into nonconventional exploitation. However the resource will not be exhausted in finite time. If the initial size of the resource is small then the stage of specialization is omitted. For the case of a Cobb-Douglas technology in conventional exploitation it is shown that the higher the production costs in the nonconventional technology ($1/\gamma$) or the greater the initial stock of the resource or the smaller the supply of labour, the longer is the period of specialization. Finally if the economy were interested not in maximizing discounted output but in the utility of output, then qualitatively the results are not affected. Obviously Smith's results strongly depend on his rather specific assumptions about the technology. For a more rigorous treatment of transitions from one resource to another, incorporating more factors of production, we refer to Hanson (1978). In general the cheapest resource will be exhausted first, the cheapness depending on substitution possibilities in the different sectors. Heal approaches the problem rather unconventionally. It is assumed that the resource has an *infinite size*. The costs of the exploitation depend on the amount already extracted (X). Defining

$$X_t = \int_0^t E(\tau)\, \mathrm{d}\tau\,, \tag{46}$$

the model is

$$\dot{K} = F(K, E) - C - K_1, \tag{47}$$

$$K_1 = G(X_t) \cdot E, \tag{48}$$

where for $0 \leqslant X_t \leqslant X$ for some X, $G'(X_t) > 0$ and for $X_t > X$, $G(X_t) = \beta$ ($\geqslant G(X_t)$ if $X_t \leqslant X$). K_1 denotes the investments in exploitation. It then follows that under the usual assumptions about F the backstop technology ($G(X_t) = \beta$) is going to prevail within finite time.

Finally we would like to mention two contributions incorporating *research and development*. In Takayama (1980) and Chiarella (1980) the nonresource output is produced according to a Cobb-Douglas production function having capital, the rate of utilization of the resource and labour as inputs. In Chiarella the labour force is constant. Output is devoted to consumption, net investments in the nonresource industry and to research and development, which benefits the nonresource output. Assuming that $u(C) = \ln C$ it is shown that under certain conditions there exists a unique and stable equilibrium where consumption and the capital stock are growing whilst the resource declines exponentially at the rate of time preference ($R = R_0 e^{-\rho t}$). In Takayama technical knowledge is acquired by means of labour, which can be allocated to research and development and to the nonresource sector. We shall not repeat all of his conclusions here. However again the resource declines exponentially at the rate of time preference.

Admittedly it is useful to study the case of only relative scarcity of natural resources, but it cannot be denied that uncertainty about future developments plays an important role and should be taken into account.

9 UNCERTAINTY

The issue of uncertainty is clearly important to the analysis of natural resources. Uncertainty may occur in different ways. For many natural resources the reserves are not exactly known. As already indicated another field where uncertainty plays a role is research and development: it might be that the time of a technological breakthrough is uncertain or that the cost associated with a new technology is not known. In this section we briefly review the contributions made in these areas.

In Gilbert (1979) and Loury (1978) the initial size of the *reserve is a random variable* with a given probability distribution F. Associated with every plan $[E(t)]$ there is a terminal date of consumption T which is also stochastic, depending on the initial stock R_0 and the rate of consumption. The problem is then to maximize

$$I = V \int_0^T e^{-\rho t} u(C_t)\, \mathrm{d}t \tag{49}$$

subject to

$$\int_0^T E(t)\, \mathrm{d}t \leqslant R_0, \tag{50}$$

$$E(t) = C(t) . \tag{51}$$

Here V denotes expected value. Denote by $\hat{E}(0)$ the optimal initial rate of exploitation if the size of the reserve is known with certainty to be the expected value of R_0. Gilbert proves that in the case of uncertainty the optimal $E(0)$ is smaller than $\hat{E}(0)$. One would expect that the riskier the distribution F the more conservative the exploitation policy would be. Loury shows, however, that this does not hold in general. Finally it is shown by both authors that if the distribution is exponential the optimal rate of exploitation is a constant, not depending on the utility function u nor on the rate of time preference.

Robson (1979) deals with the problem of *several deposits* of a natural resource which are perfect substitutes. Suppose that there are two deposits, the first of which has known size $R_1(0)$. Assume that the size $R_2(0)$ of the second reserve is unknown and can be described by a probability distribution. In this case it is optimal to exhaust the uncertain reserve before exploiting the known one. If the size of the first deposit is also unknown but its distribution F dominates that of the second then the optimal strategy will exhaust the second deposit before exploiting the first.

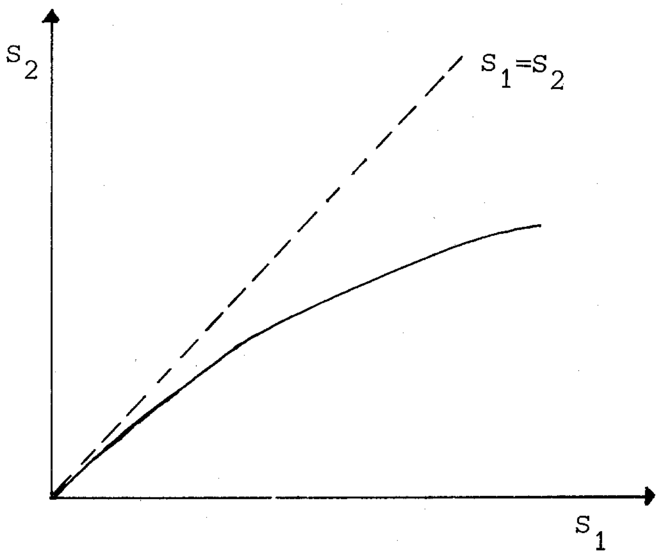

Figure 7

The concept of dominance is illustrated in Figure 7. S_1 denotes the size of the first deposit and S_2 is the size of the second deposit. Hence if the stock of the first deposit turns out to be zero, the second deposit will be empty as well. The larger the first deposit the larger is the second, but the first one will always contain more of the resource than the second. If the reserves are independently distributed then of course the sequence of exploitation is immaterial if these distributions are identical. However if the distributions differ and if they are both exponential it is optimal to exhaust the reserve with the higher expected value (and variance) first.

Hoel (1978) considers the case where the size of the reserve is known. But at some specific time T a *substitute* becomes available which has a unit cost of production q. This unit cost is unknown until it is revealed at T. Let $\hat{q}$ be the expected value of q. It is optimal to exhaust the resource in the interval $[0, T]$ if and only if this is also optimal when $q = \hat{q}$ with certainty. If it is optimal to leave some of the resource at T then less will be left if uncertainty about q is increased (preserving mean cost). In the case of a more general welfare functional I it can be shown that increased risk aversion of society is an incentive to leave more of the resource at T.

In Davison (1978) the economy is capable of developing a *resource independent technology* by increasing "knowledge" via capital input. It is, however, uncertain how much knowledge has to be accumulated before the new technology becomes available. Hence the time at which the breakthrough takes place is, contrary to Hoel's assumption, a random variable. The nonresource output is produced according to a Cobb-Douglas production function having capital (K_1) and the resource good (E) as inputs. Output is divided over consumption and net investments:

$$\dot{K} = K_1{}^{\alpha} E^{1-\alpha} - C, \ 0<\alpha<1 . \tag{52}$$

Capital is also allocated to research activities:

$$K = K_1 + K_2 . \tag{53}$$

Denoting knowledge by N and assuming a linear research function we have:

$$\dot{N} = \beta K_2, \ \beta>0 . \tag{54}$$

The utility function is assumed to be

$$u(C) = \frac{1}{1-\eta} C^{1-\eta} + u_0, \ \eta>1 . \tag{55}$$

When $N = \bar{N}$, the required knowledge, the economy will be governed by a different set of equations. In particular it is assumed that after the discovery of the new technology $u = u_0$, which means that a situation of bliss is achieved. Hence capital and the resource have become useless. $\bar{N}$ is a random variable with known probability distribution F (taken to be an exponential or gamma distribution). The most striking result is that there exists some time t_f such that no more capital will be devoted to research activities after t_f. Knowledge will remain constant also. It follows that if the new technology has not been discovered before time t_f a technological breakthrough will never occur. t_f varies inversely with the time discount rate. In the case of a gamma distribution research is not even started if initial knowledge is below a certain level. These results stand in sharp contrast with those obtained by Kamien and Schwartz (1978) and Dasgupta, Heal and Majumdar (1977). In their approach increasing knowledge comes about if net investments are allocated to the *research and development* sector. In Davison research and development activities have a stock as an input, whereas in Dasgupta *et al.* the input is a flow. Furthermore they have an alternative specification of the knowledge production function. In brief, (52) and (54) are replaced by:

$$\dot{K} = K_1^{\alpha} E^{1-\alpha} - C - K_2, \tag{56}$$

$$\dot{N} = K_2^{\beta} , \beta < 1 . \tag{57}$$

Their conclusion is that expenditure on research and development is always positive but approaches zero. This decrease is faster the higher the discount rate and the less risk-averse society is. Kamien and Schwartz obtain similar results.

In our opinion the work presented above constitutes a major contribution to the theory of exhaustible resources. However, it contains an important conceptual weakness. It is assumed that the output of research and development is more or less predictable. Citing Dasgupta *et al*: "But modelling the rational allocations of resources to the development of ideas and techniques that we cannot even conceptualize at present and of whose potential existence we are completely unaware, is obviously a far more challenging problem."

10 CONCLUSIONS

What conclusions can be drawn from the preceding analyses? Beyond any doubt it has enriched economic theory by incorporating natural resources in optimal growth models. Still many problems remain. These are summarized in the following list of recommendations and objections:

- the existence and characterization of optimal solutions strongly depend on the welfare functional, whose determinants are the length of the horizon, the time discount and the utility function. This dependence is even more striking than in traditional growth models. Therefore Koopmans' (1965) argument applies *a fortiori*: "the . . . aim is to argue against the complete separation of the ethical or political choice of an objective function from the investigation of the set of technologically feasible paths . . . Ignoring realities in adopting 'principles' may lead one to search for a nonexistent optimum or to adopt an 'optimum' which is open to unanticipated objections."
- the case of open economies does not seem to be solved satisfactorily. The results obtained strongly depend on the assumption of given world market prices which permits the application of the Fisher separation theorem. However, for many natural resources, free competition does not prevail.
- the models in which research and development towards a backstop technology are present, especially those involving uncertainty, need further elaboration in view of the efforts, currently made in the real world, concerning this issue.

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Summary

THE OPTIMAL EXPLOITATION OF EXHAUSTIBLE RESOURCES, A SURVEY

In this article a survey is given of the economic theory on the optimal exploitation of exhaustible resources. Attention is paid to Hotelling's pioneering work, the specification of welfare objectives, open economies, extraction costs, uncertainty, research and development and the relation between optimal economic growth and optimal extraction.