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Fons Groot; Cees Withagen; Aart de Zeeuw

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NOTE ON THE OPEN-LOOP VON STACKELBERG EQUILIBRIUM IN THE CARTEL VERSUS FRINGE MODEL

Fons Groot, Cees Withagen and Aart de Zeeuw*

In his remarkable article 'Oil prices, cartels and the problem of dynamic inconsistency' Newbery (1981) shows that if the supply side of the oil market can be characterised as one with a dominant cartel and a large number of small producers (called the fringe as a group) the open-loop von Stackelberg equilibrium may give rise to dynamic inconsistency, and should therefore be rejected as an appropriate equilibrium concept. Dynamic inconsistency is also found to occur by Ulph (1982) in a similar model.

The purpose of this note is not to dispute the validity of this result nor its relevance. What we wish to show is that the derivation of the equilibrium is not correct. Apart from the fact that the correct derivation yields qualitatively as well as quantitatively different equilibrium trajectories (although the main conclusion still stands), there is some additional value in the present paper because one of the objectives in Newbery's article was 'to demonstrate a method of analysis which makes the solution of quite complex problems ... accessible to mathematically unsophisticated economists' (p. 619). It is furthermore argued by Newbery that 'This approach also provides scope for intuition so that the mathematically sophisticated can check the plausibility of their solutions, or, given various possible solutions, can choose the correct one' (p. 619). The conclusion of the present note is, however, that in the case at hand intuition is not performing as an entirely reliable guide.

I. THE PROBLEM

Let the world demand schedule of oil be given by

$$x = \bar{p} - p,$$

where x denotes demand, p is the market price and $\bar{p}$ is a choke price. Demand is met by a coherent cartel with constant per unit extraction costs k^c, having an initial endowment of oil amounting to S_0^c, and a large number of identical small suppliers, each with constant per unit extraction costs k^f and aggregate initial resources S_0^f. The common discount rate is denoted by r, a positive constant. In order to have an interesting problem it will be assumed that $\bar{p} > \max(k^c, k^f)$. Each individual fringe member takes the market price as given[1] and maximises

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[1] It would be more elegant to start with a finite number of fringe members and then to see what happens to the equilibrium if that number goes to infinity. It can be shown that indeed price taking behaviour results. We follow Newbery and Ulph who assume price taking behaviour from the outset.

its total discounted profits. The cartel has the same objective and takes the reaction of the fringe into account in determining the optimal extraction path. Let E^f be the equilibrium aggregate extraction trajectory of the fringe and E^c be the extraction trajectory of the cartel.

Then E^f maximises

$$\int_0^{\infty} e^{-rt}(p-k^f)\, E^f\, dt,$$

subject to

$$\dot{S}^f = -E^f, \quad S^f(0) = S_0^f,$$
$$E^f \geqslant 0, \quad S^f \geqslant 0.$$

The time argument is omitted wherever there can be no confusion. It is straightforward to see that we have as necessary conditions

$$h_1 := k^f + \lambda^f e^{rt} - p \geqslant 0.$$
$$E^f h_1 = 0,$$

where $\lambda^f(>0)$ is the constant shadow price of the aggregate in situ oil stocks of the fringe. Since the objective functionals of the fringe members are concave, a trajectory satisfying the necessary conditions is optimal, if $\lambda^f S^f$ approaches zero as t goes to infinity.

The way Newbery and Ulph proceed is then to analyse the optimal strategy of the cartel by ruling out possible sequences of regimes, not using optimal control theory. In contrast, we argue that control theory is a valuable tool in analysing the cartel's problem, be it that no standard theorems can be invoked. We will show that the proper analysis with optimal control theory gives somewhat different results.

The problem of the cartel can be formulated as follows.

$$\max_{E^c,\, E^f} \int_0^{\infty} e^{-rt}\, [\bar{p} - E^c - E^f - k^c]\, E^c\, dt,$$

subject to

$$h_1 = k^f + \lambda^f e^{rt} - (\bar{p} - E^c - E^f) \geqslant 0,$$
$$E^f h_1 = 0,$$
$$\dot{S}^f = -E^f, \quad S^f \geqslant 0, \quad E^f \geqslant 0, \quad S^f(0) = S_0^f, \quad \lim_{t\to\infty} S^f(t) = 0,$$
$$\dot{S}^c = -E^c, \quad S^c \geqslant 0, \quad E^c \geqslant 0, \quad S^c(0) = S_0^c,$$
$$\dot{\lambda}^f = 0, \quad \lambda^f \geqslant 0.$$

So the cartel chooses not only its own extraction trajectory but the fringe's as well, subject to the condition that each fringe member is a price taking profit maximiser. The cartel also takes care that the market is always in equilibrium ($x = E^f + E^c$ is therefore inserted into the demand function).

This optimal control problem does not allow for a standard application of the Pontryagin maximum principle because in general the constraint qualification does not hold. However, Neustadt (1976) gives necessary conditions for optimality in this case. We follow Seierstadt and Sydsaeter (1987), who provide a useful formulation. It would go too far to outline all the differences with the standard necessary conditions. In general, the co-state variables need no longer be continuous and the multiplier functions associated with the constraints are not necessarily piece-wise continuous. What remains however, and this is important in the present context, is that the Hamiltonian is maximised with respect to the control variables. The Hamiltonian reads

$$H^c := e^{-rt}[\bar{p} - E^c - E^f - k^c)E^c - \lambda_1^c E^c - \lambda_2^c E^f.$$

Note that, since $\dot{\lambda}^f = 0$, the corresponding co-state variable does not appear in the Hamiltonian. Furthermore, it is evident that the co-state variables λ_1^c and λ_2^c, denoting the value the cartel attaches to a marginal increase in its own initial stock and the fringe's initial stock respectively, will be constants. Hence, although the co-states λ_1^c and λ_2^c need not be continuous in general, they are in the present problem. Along the lines set out by Ulph and Folie (1980) we define the auxiliary prices

$$\begin{aligned} P^1 &= k^f + \lambda^f e^{rt}, \\ P^2 &= k^c + (\lambda_1^c - \lambda_2^c)\, e^{rt}, \\ P^3 &= \tfrac{1}{2}(\bar{p} + k^c) + \tfrac{1}{2}\lambda_1^c e^{rt}. \end{aligned}$$

For reasons that will be clear P^1 is called the competitive price and P^3 is called the monopoly price. P^2 can be interpreted as the marginal costs the cartel incurs when it supplies at the competitive price. P^2 consists of the marginal extraction costs k^c, the opportunity costs $\lambda_1^c e^{rt}$ of extracting its own resource now rather than in the future and the costs $-\lambda_2^c e^{rt}$ the cartel incurs by the fact that producing an additional amount now will cause a higher stock of the fringe (λ_2^c is of course negative). Suppose that it is optimal to have an interval of time with $h_1 = 0$. Then, along this interval of time,

$$H^c := e^{-rt}(P^1 - P^2)\, E^c - \lambda_2^c(\bar{p} - P^1),$$

is maximised subject to

$$0 \leqslant E^c \leqslant \bar{p} - P^1,$$

as can be seen from simple substitution.

If it is optimal to have an interval of time with $h_1 > 0$ then, along this interval of time, the cartel maximises

$$H^c := e^{-rt}[2(\bar{p} - P^3) - E^c]\, E^c$$

subject to $$E^{c'} \geqslant 0.$$

This is so because if $h_1 > 0$ we have $E^f = 0$ and $P^1 > p$. The following is now obvious.

THEOREM

(*a*) *Suppose it is optimal for the cartel to have* $h_1 = 0$. *Then*

$$\text{if } P^1 > P^2 \text{ then } E^c = \max(\bar{p} - P^1, 0), \quad E^f = 0 \tag{a1}$$

$$\text{if } P^2 > P^1 \text{ then } E^c = 0, \quad E^f = \max(\bar{p} - P^1, 0). \tag{a2}$$

(*b*) *Suppose it is optimal for the cartel to have* $h_1 > 0$. *Then* $E^f = 0$, $E^c = \max(\bar{p} - P^3, 0)$.

From here the analysis can proceed in much the same way as in Newbery and Ulph. Let us concentrate on the continuity of the equilibrium price trajectory. One of the results the authors mentioned above obtain is that in the rather plausible case where the cartel has a cost advantage over the fringe, the advantage however not being extreme ($k^c < k^f < \frac{1}{2}(\bar{p}+k^c)$), and where the cartel has an initial oil stock which is sufficiently large relative to the initial stock of the fringe, there will at some instant of time occur a switch from the fringe supplying at the competitive price P^1 to the cartel producing at the monopoly price P^3. It can be shown along the lines set out by these authors but using our formal control theoretic setting, that this result is correct. It has also been shown by the previous authors, and can be proved to be correct, that in the case at hand the cartel will start supplying at the monopoly price P^3 only after it has supplied at the competitive price P^1.

Since the optimal value of the Hamiltonian should evidently be continuous over time, a discontinuity occurs in the price trajectory at the points in time where a switch takes place from a phase with the fringe supplying to a phase with the cartel supplying at the monopoly price and vice versa. This can be seen as follows.

Along intervals of time where the cartel is the sole supplier at the monopoly price P^3 the value of the Hamiltonian is

$$\hat{H}^c = e^{-rt}(\bar{p} - P^3)^2.$$

Along intervals of time where the fringe is the sole supplier at the competitive price P^1 the value of the Hamiltonian is

$$\hat{H}^c = -\lambda_2^c(\bar{p} - P^1).$$

At the switch point, say s, these values should be equal implying that

$$\lambda_2^c(\bar{p} - P^1) + e^{-rs}(\bar{p} - P^3)^2 = 0.$$

But, if the price trajectory were continuous in s we would have $P^1(s) = P^3(s)$ and hence $P^1(s) = P^2(s) = P^3(s)$. Now assume that at s a switch occurs from the fringe supplying to the cartel supplying at the monopoly price. As mentioned above, there should be an interval of time before s where the cartel is supplying at the competitive price. According to the theorem, $P^1 > P^2$ along that interval, whereas just before s we have $P^2 > P^1$. There must therefore be an instant of time before s where $P^1 = P^2$. But since $k^f \neq k^c$ the curves can intersect only once. So we obtain a contradiction.

For the cost constellation at hand the equilibrium can be depicted as in Figs 1, 2 and 3. Here the symbols C and C^m mean that the cartel is the sole supplier at the competitive price and the monopoly price respectively. F means that the fringe is the sole supplier. The location of the curves P^1, P^2 and P^3 is determined by two equations for the exhaustion of the resources and a third equation for the optimal choice of the co-state variable λ^f. For the technical details of the derivation the reader is referred to Groot *et al.* (1990).

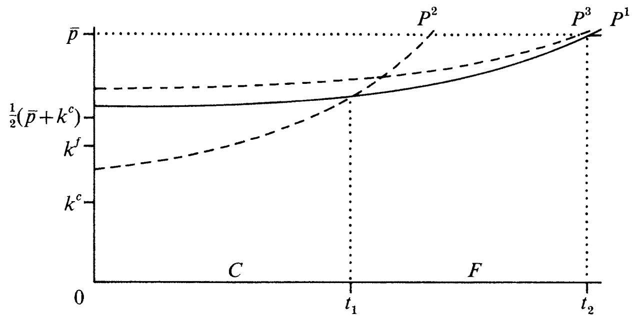

Fig. 1. Equilibrium for S_0^c 'small' relative to S_0^f.

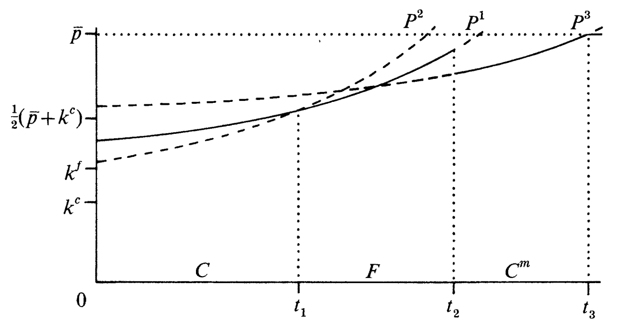

Fig. 2. Equilibrium for 'intermediate' case.

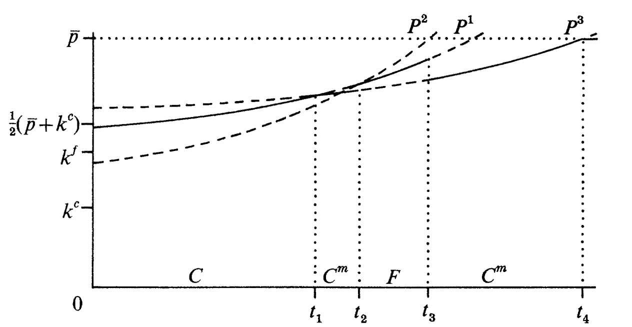

Fig. 3. Equilibrium for S_0^c 'large' relative to S_0^f.

From these figures it can be seen that for more or less the same parameter values as in Newbery and Ulph the conclusion that dynamic inconsistency arises remains valid: with the parameter values we departed from there is always an initial interval of time where the cartel supplies at the competitive price. Note also that, contrary to earlier findings, there are parameter values for which there is a transition from the cartel supplying at the monopoly price to the fringe supplying, accompanied by an upward discontinuity. A numerical example can show the differences in the equilibrium trajectories obtained by us and Ulph and Newbery.

Suppose $\bar{p} = 40$, $k^f = 20$, $k^c = 15$, $r = 0{\cdot}1$ and $S_0^f = 15$. For $S_0^c = 50$ the equilibrium will be as in Fig. 2, with $t_1 = 0{\cdot}47$, $t_2 = 8{\cdot}1$ and $t_3 = 11{\cdot}8$. Cartel's profits are 596·8, whereas Newbery would obtain 594·2. If, *ceteris paribus*, $S_0^c = 150$, then the equilibrium is as in Fig. 3, with $t_1 = 6{\cdot}2$, $t_2 = 7{\cdot}2$, $t_3 = 9{\cdot}3$ and $t_4 = 21{\cdot}4$. In this case the cartel's profits are 1,096·4 where Newbery would obtain only 1,089·4.

An interesting question is why the intuition of the previous authors has failed to reveal the correct solution. The main reason is that it should be taken into account that the stock of the fringe has a negative shadow value to the cartel, i.e. λ_2^c is strictly negative. For the situation where the cartel is bound to supply at the competitive price the Hamiltonian reads

$$e^{-rt} P^1(t)\, E^c(t) - e^{-rt}(k^c + \lambda_1^c e^{rt} - \lambda_2^c e^{rt})\, E^c(t) - \lambda_2^c[\bar{p} - P^1(t)].$$

The first term just gives the revenues. The first part of the second term are the direct production costs, the second part constitutes the opportunity costs of supplying now instead of in the future and the third part gives the price the cartel is willing to pay for a marginal increase of the stock of the fringe. If the cartel would produce one unit more, the fringe would produce one unit less and therefore the remaining stock of the fringe increases by one unit, which is to be considered as a cost for the cartel. The final term is a fixed cost factor: if the cartel does not produce at all, the fringe supplies $\bar{p} - P^1$ and hence the stock of the fringe decreases by that amount.

One might not like outcomes with discontinuous price trajectories, because they can be deemed to open arbitrage opportunities in reality. However, if one wants to avoid this phenomenon the forces behind the absence of arbitrage should be modelled explicitly. For the present model price discontinuities are not excluded and can be beneficial for the cartel.

II. CONCLUSION

It has been shown that the derivation of the open-loop van Stackelberg equilibrium in the cartel versus fringe model of the oil market by Newbery and Ulph has not been entirely correct. In several cases the true price trajectories will display one or two discontinuities. The main conclusion that the open-loop von Stackelberg equilibrium can be dynamically inconsistent is not affected and its consequences regarding the appropriateness of this equilibrium concept are not under dispute. However, our claim is that in deriving the equilibrium one should not only rely on intuition and let formal mathematical analysis do its work in checking for the merits of intuition.

Free University Amsterdam

Eindhoven University of Technology

Tilburg University

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