

Optimal use of information in synchronization of a supply chain dedicated to customized mass production

ABSTRACT: This article deals with the optimal use of available information by the customer and the supplier in order to guarantee the supply for a given work station of the customer's assembly line. Because the daily delivery is defined to avoid the stockout, the respect of these requests by the supplier is a constraint for his organization (effectiveness criterion). The intelligent use of all the provided information enables him to produce in an efficient and effective way. In the study of the customer's requirements, it is necessary to take into account the batch constraints related to transport, which compels the customer to hold safety stocks even if the organization's design guarantees that orders will be duly honored. The determinants of these stocks will be given.

KEY-WORDS: mass production of customized products, alternate components, optimal use of customer's information, decoupling of plants, supply chain synchronization, safety stocks, stochastic demand, risk analysis, rank change, lot-sizing.

1. Introduction

Four points define the framework retained for this analysis: the actors of the studied supply chain, the consequences of stockout, the coordination principles used in the supply chain and the choice of a replenishment policy.

For Christopher (1998), a supply chain is a network of firms linked by exchanges of products, services and information, to meet a final demand. Even if some papers consider a supply chain with a significant number of links (Chen 1998), the majority of literature on supply chain management focuses on a supply chain composed of two consecutive links. There is more analyses about downstream supply chains (i.e. about the units being downstream from the last unit contributing to a product transformation step), than for upstream supply chain. In the downstream supply chain, the case of a distribution centre delivering N retailers is the most frequently examined; some studies deal with the mechanisms of competition between retailers, restricted to 2 retailers (Zhang 2002). The upstream supply chains are generally composed of one supplier who delivers one industrial customer (B2B), which constitutes our study field. Relations between a supplier and N customers, or N suppliers for one customer, are less common (Zou *et al.* 2004 position their study in this context).

An industrial customer order to his supplier must be delivered at a date set by contract. If this cannot be done, the demand can be differed or lost. Traditional replenishment policies have the purpose of defining the order system by a minimization of a cost function that integrates an average carrying cost, an average stock out cost (whose definition changes according to whether the unsatisfied demand is differed or lost) and, if order periodicity is not predetermined, an average order cost. This article is interested by the case, little studied, of the "customer-supplier" relations implying a negligible stockout probability and a predetermined periodicity of decision-making. This approach returns to the preceding one if the stockout cost is relatively very high compared to carrying cost. The problem of the periodicity predetermination is linked to the overall logistic organization.

For Kouvelis *et al.* (2006) "actions or approaches which lead supply chain partners to act in ways that are best for the chain as a whole are known as supply chain coordination." The two main modes of supply chain coordination are contracts and information sharing.

- Contracts determine the exchange conditions: quantities, prices, dates, return conditions (which depend on the speed of obsolescence). Our study pertains essentially to contracts concerning the downstream supply chain (between retailers and the producer). In a Vendor-Managed-Inventory contract (Disney & Towill 2003), the supplier chooses how many units to deliver to his customer,

but he is paid only for the sold quantities. In profit sharing contracts (Cachon & Lariviere 2005), a share of the retailer's earnings returns to the supplier. VMI and profit sharing contracts can be coupled. The wholesale-price-based contracts take place at two times: first the supplier chooses the price and then the retailer chooses the quantity (Gerchak & Wang 2004). To coordinate a supply chain, it is necessary to associate buybacks, which are unsold quantities returned to the supplier who buys them back at a predetermined lower price (Emmons & Gilbert 1998). Supply chain coordination by contract supposes that the supplier has the capacity to produce the quantities required by his customer. No problems will occur if the supplier works systematically in sub-capacity, but they will if the supplier's production capacity is barely sufficient to cover the customer's demand and if this demand concerns different products.

We will retain as a principle of supply chain coordination the use and transmission of information to the upstream part of a supply chain (i.e. starting from the last production process).

- In information sharing models, various degrees of sharing exist. The spectrum goes from the successive supply chain customers' order history (Axsäter 1993) to the real-time transmission of inventories positions and final demands (Cachon & Fisher 1997), passing by the final sales forecasts (Forslund & Jonsson 2007) or the customer's order policy parameters and the final demand distribution (Gavirneni *et al.* 1999). The purpose of information sharing is to reduce the bullwhip effect (Viswanathan *et al.* 2007) and some authors consider that the supply chain approach was initially imagined to protect oneself from this effect (Medan & Gratacap 2008). The base stock strategies implemented within this framework are thus based upon two main ideas: the repercussion along the supply chain of all information available and the determination of the orders based on inventory position instead of actual inventory level in every network node. The shared information allows for a decrease in carrying costs and stock out costs in the supply chain (Hariharan & Zipkin 1995), because it aims at improving the supplier's decisions on order quantities (Lee & Tang 2000) and the product allowance between retailers (Chen & Samroengraja 2000). The problem of earnings sharing has been widely developed in literature, for example by Lee & Tang (2000), but is ignored here.

In all the articles the final demand is uncertain or stochastic; sometimes the distribution is known and the total demand is stable (Cachon & Fisher 2000). Ordered (or produced) quantity must cover the demand occurring between two deliveries, with a given probability of stock shortage. Several reasons justify the use of the periodic replenishment policy. Firstly, it facilitates the organization of the orders making and of deliveries reception when many parts must be supplied from many suppliers. Secondly, it facilitates the flows' synchronization in the supply chain by a transactions regular clocking which looks like the "Drum - Buffer - Rope" technique in the theory of constraints (Goldratt & Cox, 1984). We will study the information' use within the framework of periodic replenishment policies.

Customer orders are firm information for the supplier; on the one hand they can't be based on a known final demand because of insufficient anticipation if the supplier is distant, on the other hand, the supplier can improve his efficiency by exploiting all his customer's known information, beyond those of the firm orders (demand pattern); the proper use of this information will be approached here. To our knowledge, only Bourland *et al.* (1996) consider the case where customers communicate their orders to their supplier every week for the following two or three weeks, as well as their forecasts for the five or six weeks beyond. They explain why the plants need stocks to counter the effects of uncertain orders and deliveries, but they only consider the case of a single good. The transmission of firm information is only approached under the Production To Order and the OPP (Order Penetration Point) analysis (Giard & Mendy 2008). We will show that the upstream flow of information along the supply chain entails the creation of a plurality of OPPs that are locally defined.

The impact of batch constraints on the supply chain control doesn't seem to be approached in literature. The lot-sizing problem is taken into account through scheduling as well as its impact upon production capacity due to set-up times (White & Wilson 1977). Optimal batch sizes are the result of a minimized cost function which integrates both carrying and set-up costs. The ex-ante determination of the batch size can be related to packaging constraints or to storage constraints near a work station, preventing from having the variety required by the consumed components. Packaging is also approached in transportation problems through its influence upon transportation capacities, but it is

not taken into account in the determination of orders to be sent to the supplier. Here we will examine the impact of batch constraints on supply chain flow controls, respective to both the transformation of the orders to be made and the necessity of holding a safety stock.

The organization of this article is the following one. After giving an analysis framework in a second section, we will determine, in section 3, the periodic replenishment policies of customer and the periodic supplier production. In section 4, we will propose a method for determining the safety stock which makes it possible to counter the effects of the lot-sizing used in transport or production, in the case of supply of alternate components.

2. Specifications of the studied context

We are interested in an elementary supply chain consisting in a production unit (indicated in this article as the customer) configured as an assembly line allowing mass production of diversified products (an automotive production line, for example) and another unit (indicated as being the supplier) producing alternate components (e.g. car engines) or optional components (e.g. sunroof) assembled on a work station of this line and contributing to the required diversity (Anderson & Pine 1997). From the point of view of the analyzed problem, the case of the optional components is a particular case of the alternate components. Figure 1 describes the problem parameters.

After a few weeks, the daily production of the customer ($= n$) is predetermined by the opening duration of the line (stable on this horizon) and the line's cycle time. The total daily demand of the alternate components to be ordered from the supplier is thus known. From the final demand (vehicle orders), the industrial customer determines his production schedule on a horizon of K days, which in turn determines, on this horizon, the ordered list of the alternate components to assemble each day on the assembly line work station where they are mounted. Beyond this horizon K , the customer only has information about the average structure of the demand. At the beginning of every day t , the schedule of days t to $t+K-1$ is kept and the new orders of day $t+K$ are sequenced according to the logic of revolving planning. The schedule's update is immediately transmitted to the supplier.

The delivery request, transmitted by the customer for the beginning of day t , is determined by the assembly plan of the alternate components. If the delivery time λ does not exceed the horizon K , then the demand is transmitted no later than the beginning of day $t-\lambda$. Otherwise, the delivery request is a periodic replenishment policy which will be presented in this article. Whatever the determination made by the customer (firm demand and/or structure of forecast demand), the requisitions that are transmitted are firm demands for the supplier. *If the organization relating to the supplier's production and transportation guarantees that deliveries are in conformity with requisitions, there is decoupling between the customer and the supplier in the supply chain*

Customer orders, which are firm information for the supplier, determine an OPP in his process. This supplier in turn sends requisitions to his own supplier that are firm orders to the supplier's supplier, creating a new OPP. All along the supply chain, upstream information transmission creates local OPPs disconnected from final demand.

In the case where the articles are bulky, trucking capacity can lead to split the delivery into several successive shipments throughout the day. Conversely, this delivery can cover the needs of day t and $\theta-1$ following days, if the interval between two deliveries is of θ days; in that case, the demand concerns a production schedule transmitted no later than at the beginning of day $t-\lambda+1-\theta$.

The alternate components are most often manufactured with common production means which can require specific equipment. For efficiency reasons (amount of the set-up costs), the supplier may have an interest in successively producing the alternate references on a cycle of H days (H can be equal to 1). The periodic production of the references must take into account the fact that daily deliveries made by the supplier integrate all references. The delay separating the date t' of the beginning of the production of reference i from its delivery is noted L_i .

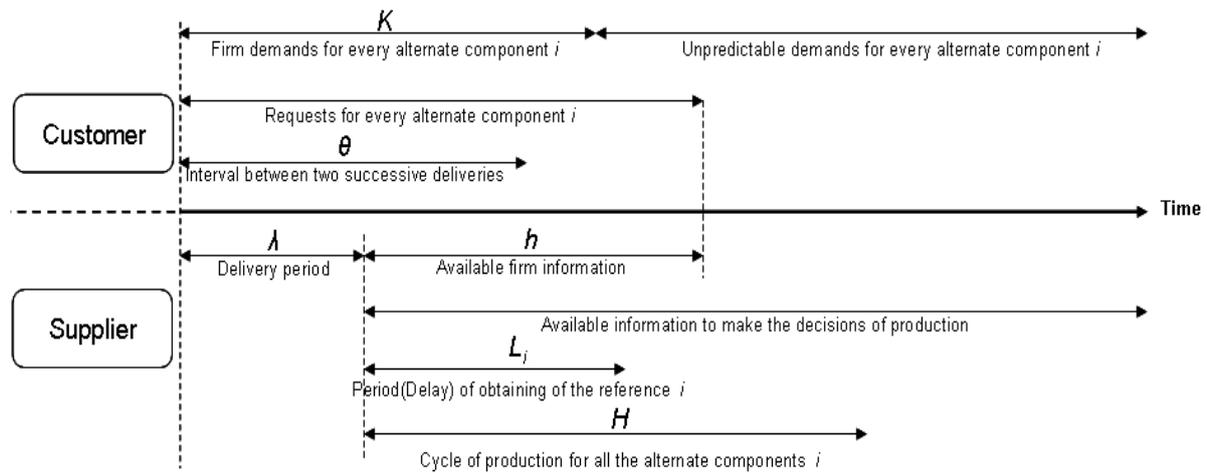


Figure 1. Problem parameters.

3. Determination of the periodic replenishment policies of customer demand and supplier production

We rely on the example of motor assembly line in which a work station takes up the engine desired by the final customer; this one has the choice between six engines (alternate components). The daily production of this assembly line is 962 vehicles.

Table 1. Distribution of engines' demand.

Engine i	1	2	3	4	5	6
p_i	54.46 %	13.29 %	3.58 %	21.51 %	5.13 %	2.03 %

The observed structure on any day differs necessarily from this average structure, because we are in presence of a realization of the Multinomial distribution with parameters $\{n = 962; p_i\}$ and $\sum_{i=1}^6 X_i = 962$, where X_i represents the daily demand of the requested engine i . For the determination of a confidence interval of X_i , we use binomial law $\mathcal{B}(n, p_i)$ because the analysis focuses on this reference against the set of all other references.

Determination of the order-up-to level of a periodic replenishment policy

The demand X_{iD} of reference i on D consecutive days follows the law $\mathcal{B}(nD, p_i)$ that we can approximate by the Normal distribution $\mathcal{N}(nDp_i, \sqrt{nDp_i(1-p_i)})$, due to the high value of nD .

In the general case of a random variable X according to a Normal distribution of parameters \bar{x} and σ , the value R of X such as $P(X > R) = \alpha$ is $R = \bar{x} + t_\alpha \sigma$, where α is the accepted risk and t_α , the value of the variable T according to the law $\mathcal{N}(0, 1)$ such as $P(T > t_\alpha) = \alpha$. It follows that the value R_{iD} such as $P(X_{iD} > R_{iD}) = \alpha$ is given by the relation 1.

$$R_{iD} = nDp_i + t_\alpha \sqrt{nDp_i(1-p_i)} = nDp_i(1 + t_\alpha \sqrt{\frac{1-p_i}{nDp_i}}) \tag{Relation 1}$$

The periodic replenishment policy of the supply of a product i is characterized by making an order q_{it} for the component i at the beginning of the period t , calculated as the difference between its order-up-to level R_i and its inventory position P_{it} at the beginning of the period t ; the interval between two successive decisions being θ . The determination of the optimal value of R_i is economically based on a trade-off between a carrying cost and a stockout cost, leading to the determination of an optimal value of risk α . As we look at the supply of alternate components to be taken up on an assembly line, the cost of a component shortage triggering a line stoppage is much greater than the carrying cost. It is

then acceptable to use a very low risk α , for instance 0.01%. In the relation 1, R_{iD} is analyzed as an order-up-to level and $t_{\alpha} \sqrt{nDp_i(1-p_i)}$, as a safety stock. Contrary to what some practitioners recommend, this safety stock cannot be defined as a constant percentage of the average demand, it depends clearly on 4 parameters: α , n , p and D .

If the delivery delay λ is not null (the assumption made here), we have to consider the random demand expressed between t and $t+\theta+\lambda$, in order to determine the order-up-to level (noted $R_{i,\theta+\lambda}$). If the unsatisfied demands are delayed and/or if the probability of stock shortage is negligible (assumptions made here), then demands over two different periods are independent. The demand over the period $\theta+\lambda$ then follows the law $\mathcal{N}(n(\theta+\lambda)p_i, \sqrt{n(\theta+\lambda)p_i(1-p_i)})$, the order-up-to $R_{i,\theta+\lambda}$ is defined for the risk α as follow:

$$R_{i,\theta+\lambda} = n(\theta+\lambda)p_i + t_{\alpha} \sqrt{n(\theta+\lambda)p_i(1-p_i)} \quad \text{Relation 2}$$

Under the conditions selected, when an order is placed, the stock position P_{it} is the sum of the observed stock, during this time, and of the expected deliveries ($k = \lfloor \lambda / \theta \rfloor$, where $\lfloor A \rfloor$ represents the lower roundness of A). Then, we obtain the following relation:

$$P_{it} = S_{it} + \sum_{j=1}^k q_{t-j\theta} \quad \text{Relation 3}$$

Periodic demands of the customer to his supplier

Periodically (interval θ), the customer transmits to his supplier the specifications of the next delivery. We suppose that this requisition is daily ($\theta=1$), without loss of generality (θ becoming the unit of time). This customer orders to his supplier, at the beginning of the day t , q_{it} units of the component i to be delivered at the beginning of the day $t+\lambda$.

If $\lambda \leq K$, we obtain relation 4; the supplier can mobilize the techniques of the synchronous production (Giard & Mendy 2008) under certain conditions ($H \leq 2h$):

$$q_{it} = x_{i,t+\lambda} \quad (\sum_i q_{it} = \sum_i x_{i,t+\lambda} = n) \quad \text{Relation 4}$$

Otherwise ($\lambda > K$; $h=0$), this demand q_{it} is determined by relation 5 which is an adaptation of (the) relations 2 and 3.

$$q_{it} = [n(1+\lambda)p_i + t_{\alpha} \sqrt{n(1+\lambda)p_i(1-p_i)}] - [S_{it} + \sum_{j=1}^{\lambda} q_{i,t-j}] \quad \text{Relation 5}$$

The stockout probability being negligible, the average residual stock before delivering (SM_{it}) is equal to the safety stock. Then, we obtain relation 6:

$$SM_{it} = t_{\alpha} \sqrt{n(1+\lambda)p_i(1-p_i)} \quad \text{Relation 6}$$

With $R_{i,1+\lambda} = q_{it} + [S_{it} + \sum_{j=1}^{\lambda} q_{i,t-j}] = q_{i,t-1} + [S_{i,t-1} + \sum_{j=1}^{\lambda} q_{i,t-1-j}]$ and $S_{it} = S_{i,t-1} - x_{i,t-1} + q_{i,t-1-\lambda}$, we obtain the valid relation 7 in steady state under the stated conditions:

$$q_{it} = x_{i,t-1} \quad (\sum_i q_{it} = \sum_i x_{i,t-1} = n) \quad \text{Relation 7}$$

However, at the initialization and every time we take into account a change of characteristics of the steady state, relation 5 must be used.

In summary, if the delivery period is lower than the customer's programming horizon, the demand pertains to the firm final demand ahead (relation 4); otherwise, it corresponds to the previous period's consumption in order to reduce the stock position to the order-up-to level (relation 7). As it involves alternate components taken up on the same work station of the assembly line and all provided within the same delivery period at the same supplier, the level of the total daily order of these alternate components is constant ($=n$) because we are in the presence of a Multinomial distribution.

Table 2 illustrates the order of engine 2 on the assumption of a distant supply ($\lambda > K$) from day 100. The demands were generated randomly and the starting inventory at the beginning of day 100 was arbitrarily fixed (using relation 5 for day 100). For the following days, the use of relations 5 and 7 leads to the same results.

Table 2. Example of periodic replenishment policy.

Day	96	97	98	99	100	101	102	103	104	105	106	107	108	Daily demand: $\mathcal{D}(962; 13,29\%)$ $\lambda = 4$ Demand on 5 days: $\mathcal{N}(639,25; 23,54)$ $\alpha = 0,01\% \rightarrow R = 727$
Initial stock					125	101	95	90	74	77	80	100	115	
Order	111	126	132	118	115	135	132	137	134	112	132	112	122	
Delivery					111	126	132	118	115	135	132	137	134	
Demand					135	132	137	134	112	132	112	122	146	
Final stock					101	95	90	74	77	80	100	115	103	

Periodic programming of the supplier

The supplier must make sure that the daily deliveries required by his customer are honored in a nearly certain way. He organizes the production of alternate components on a cycle of H days during which each component is successively produced. If H is lower or equal to $2h$ (see Figure 1), then the supplier can produce during a cycle the exact quantities which will be sent during the following cycle and can work in synchronous production. Otherwise, which is our interest here, he must make the best use of the firm information that he received ($x_{i,t+\lambda+1}$ to $x_{i,t+\lambda+h}$) and of the structural information (probability p_i and level n of the customer's daily production).

This periodic replenishment policy takes into account an average achievement delay L_i for component i , with production being considered available only at the end of this period. Without loss of generality, we will assume that L_i is an integer number of days and that during one day several references are successively produced, a common lead time being then shared by those references. In a cycle beginning at t , and if the other components were produced earlier during this cycle, the earliest date of production of component i is $t' > t$ (the previous launch in production of this component had begun at $t'-H$).

The quantity launched in production has to guarantee that between two deliveries there is almost no chance of being out of stock. The order-up-to level would cover the unknown demand.

Two rules to determine production $q_{it'}$ are possible, the first one uses all the available information at t'_i (relations 8)¹:

$$\begin{aligned}
 \text{Rule 1} \quad & \left\{ \begin{aligned}
 R_{i,H-h+L_i} &= n(H-h+L_i)p_i + t_\alpha \sqrt{n(H-h+L_i)p_i(1-p_i)} \\
 q_{it'_i} &= \sum_{j=1}^h x_{i,t'_i-1+j} + R_{i,H-h+L_i} - S_{it'_i} = \sum_{j=-(H-h)}^{h-1} x_{i,t'_i+j}
 \end{aligned} \right. \quad \text{Relations 8}
 \end{aligned}$$

This relation establishes that during the steady state, the quantity to be launched is the sum of h next deliveries and $H-h$ last ones. These have unknown values during the last launch decision taken at the beginning of day $t'_i - H$. The safety stock that the supplier establishes for the same risk α for each of the references is then:

$$SS_{i,H-h+L_i} = t_\alpha \sqrt{n \sum_i \sqrt{(H-h+L_i)p_i(1-p_i)}} \quad \text{Relation 9}$$

This rule has the effect of leading to a variable total quantity from one cycle to another, which complicates the organization of production, while the total quantities shipped each day are constant. The quantities produced correspond to the sum of quantities shipped on H consecutive days. However, this set of consecutive days is not the same for all the references. In order to stabilize production, this set of consecutive days should be the same for all the references, that which is

¹ The second formulation of $q_{it'_i}$ is obtained by taking into account the fact that:

$$R_{i,H-h+L_i} = q_{it'_i} + [S_{it'_i} - \sum_{j=1}^h x_{i,t'_i-1+j}] = q_{i,t'_i-H} + [S_{i,t'_i-H} - \sum_{j=1}^h x_{i,t'_i-1-H+j}] \quad \text{and} \quad S_{it'_i} = S_{i,t'_i-H} - \sum_{j=1}^H x_{i,t'_i-1-H+j} + q_{i,t'_i-H}$$

obtained with rule 2 which determines production at the beginning of period t regardless of the information available between t and t' . Considering J_i to be the period for obtaining the produced quantity of component i from t , we obtain relation 10 by modifying both relations 8 and 9. The counterpart of having constant production is an increase of the safety stock.

$$\text{Rule 2} \quad \left\{ \begin{array}{l} R_{i,H-h+J_i} = n(H-h+J_i)p_i + t_\alpha \sqrt{n(H-h+J_i)p_i(1-p_i)} \\ q_{it'} = \sum_{j=1}^h x_{i,t-1+j} + R_{i,H-h+J_i} - S_{it} = \sum_{j=-(H-h)}^{h-1} x_{i,t+j} \\ SS_{i,H-h+J_i} = t_\alpha \sqrt{n} \sum_i \sqrt{(H-h+J_i)p_i(1-p_i)} \end{array} \right. \quad \text{Relation(s) 10}$$

With rule 2, the sum of stocks at t is constant because on the one hand the daily total demand is constant (n), on the other hand the previous relation allows one to write:

$$\sum_i S_{it} = \sum_i \sum_{j=1}^h x_{i,t-1+j} + \sum_i R_{i,H-h+J_i} - \sum_i \sum_{j=-(H-h)}^{h-1} x_{i,t+j} = hn + \sum_i R_{i,H-h+J_i} - Hn = \sum_i R_{i,H-h+J_i} - (H-h)n$$

As long as no delivery is made, the sum of all stocks remains constant and continues at the beginning of the subsequent periods.

Table 3 illustrates the application of rules 1 and 2, assuming that $\lambda=2$, $K=5$ or 6 ($h=3$ or 4), $n=962$, $H=5$. In rule 1, safety stocks vary in the opposite direction of h . In the example, the fact that we increase the firm request visibility for one day allows the supplier to win 16 % on the level of safety stocks. This provides elements for the evaluation of the value of the information transmitted to the supplier. The change from rule 1 to rule 2 leads to an increase of 26 % for the safety stock: the increase of the carrying cost can be compensated with the savings brought by the passage to constant daily production.

A simulation of rules 1 and 2 is proposed in tables 4 and 5, for $h = 3$.

Table 3. Parameters of the tested replenishment policies, order-up-to levels, and safety stocks.

Engine i	p_i	L_i	Delivery day J_i	Rule 1 • $h=3$			Rule 1 • $h=4$			Rule 2 • $h=3$		
				$H+L-h$	$R_{i,H-h+L}$	$SS_{i,H-h+L}$	$H+L-h$	$R_{i,H-h+L}$	$SS_{i,H-h+L}$	$H+J-h$	$R_{i,H-h+J}$	$SS_{i,H-h+J}$
1	54,46%	2	2	4	2211	115,38	3	1672	100,28	4	2211	115,38
2	13,29%	1	4	3	452	68,45	2	312	56,30	6	864	96,90
3	3,58%	1	4	3	141	37,68	2	100	31,12	6	260	53,36
4	21,51%	1	3	3	703	82,22	2	481	67,15	5	1141	106,37
5	5,13%	1	4	3	193	44,95	2	135	36,30	6	359	62,90
6	2,03%	1	4	3	87	28,41	2	63	23,94	6	158	40,83
				Σ	377,09		Σ	315,09		Σ	475,74	

Table 4. Simulation of the use of rule 1, with $h = 3$.

Period t	Engine 1			Engine 2			Engine 3			Engine 4			Engine 5			Engine 6			$\sum_i x_{it}$	$\sum_i S_{it}$	$\sum_i q_{it}$
	S_{1t}	q_{1t}	x_{1t}	S_{2t}	q_{2t}	x_{2t}	S_{3t}	q_{3t}	x_{3t}	S_{4t}	q_{4t}	x_{4t}	S_{5t}	q_{5t}	x_{5t}	S_{6t}	q_{6t}	x_{6t}			
1	1200	2555	520	600	0	137	200	0	32	700	0	204	350	0	46	90	0	23	962	3140	
2	680	0	508	463	0	121	168	0	35	496	0	233	304	0	49	67	0	16	962	2178	
3	2727	0	516	342	0	146	133	0	38	263	1062	192	255	0	51	51	0	19	962	3771	4649
4	2211	0	531	196	628	137	95	150	30	1133	0	205	204	146	46	32	108	13	962	3871	
5	1680	0	511	687	0	109	215	0	36	928	0	225	304	0	64	127	0	17	962	3941	
6	1169	2594	518	578	0	126	179	0	38	703	0	210	240	0	47	110	0	23	962	2979	
7	651	0	534	452	0	133	141	0	36	493	0	203	193	0	44	87	0	12	962	2017	
8	2711	0	500	319	0	137	105	0	35	290	1058	224	149	0	52	75	0	14	962	3649	4779
9	2211	0	514	182	628	107	70	174	38	1124	0	228	97	239	50	61	86	25	962	3745	
10	1697	0	550	703	0	119	206	0	36	896	0	193	286	0	50	122	0	14	962	3910	
11	1147	2653	524	584	0	132	170	0	29	703	0	213	236	0	43	108	0	21	962	2948	
12	623	0	555	452	0	120	141	0	19	490	0	203	193	0	49	87	0	16	962	1986	
13	2721	0	510	332	0	138	122	0	33	287	1021	205	144	0	56	71	0	20	962	3677	4819
14	2211	0	526	194	646	132	89	164	31	1103	0	211	88	246	43	51	89	19	962	3736	
15	1685	0	544	708	0	117	222	0	41	892	0	189	291	0	52	121	0	19	962	3919	
16	1141	2625	509	591	0	139	181	0	40	703	0	213	239	0	46	102	0	15	962	2957	
17	632	0	526	452	0	134	141	0	33	490	0	206	193	0	42	87	0	21	962	1995	
18	2731	0	520	318	0	126	108	0	35	284	1037	212	151	0	48	66	0	21	962	3658	4796
19	2211	0	549	192	641	134	73	178	23	1109	0	201	103	213	39	45	102	16	962	3733	
20	1662	0	527	699	0	129	228	0	47	908	0	205	277	0	37	131	0	17	962	3905	
21	1135	2656	510	570	0	118	181	0	40	703	0	220	240	0	47	114	0	27	962	2943	
22	625	0	521	452	0	119	141	0	28	483	0	222	193	0	53	87	0	19	962	1981	

Table 5. Simulation of the use of rule 2, with $h = 3$.

Period t	Engine 1			Engine 2			Engine 3			Engine 4			Engine 5			Engine 6			$\sum_i x_{it}$	$\sum_i S_{it}$	$\sum_i q_{it}$
	S_{1t}	q_{1t}	x_{1t}	S_{2t}	q_{2t}	x_{2t}	S_{3t}	q_{3t}	x_{3t}	S_{4t}	q_{4t}	x_{4t}	S_{5t}	q_{5t}	x_{5t}	S_{6t}	q_{6t}	x_{6t}			
1	1200	2555	520	600	668	137	200	165	32	700	1070	204	350	155	46	90	126	23	962	3140	4739
2	680	0	508	463	0	121	168	0	35	496	0	233	304	0	49	67	0	16	962	2178	
3	2727	0	516	342	0	146	133	0	38	263	0	192	255	0	51	51	0	19	962	3771	
4	2211	0	531	196	0	137	95	0	30	1141	0	205	204	0	46	32	0	13	962	3879	
5	1680	0	511	727	0	109	230	0	36	936	0	225	313	0	64	145	0	17	962	4031	
6	1169	2594	518	618	642	126	194	175	38	711	1067	210	249	253	47	128	79	23	962	3069	4810
7	651	0	534	492	0	133	156	0	36	501	0	203	202	0	44	105	0	12	962	2107	
8	2711	0	500	359	0	137	120	0	35	298	0	224	158	0	52	93	0	14	962	3739	
9	2211	0	514	222	0	107	85	0	38	1141	0	228	106	0	50	79	0	25	962	3844	
10	1697	0	550	757	0	119	222	0	36	913	0	193	309	0	50	133	0	14	962	4031	
11	1147	2653	524	638	616	132	186	155	29	720	1042	213	259	248	43	119	96	21	962	3069	4810
12	623	0	555	506	0	120	157	0	19	507	0	203	216	0	49	98	0	16	962	2107	
13	2721	0	510	386	0	138	138	0	33	304	0	205	167	0	56	82	0	20	962	3798	
14	2211	0	526	248	0	132	105	0	31	1141	0	211	111	0	43	62	0	19	962	3878	
15	1685	0	544	732	0	117	229	0	41	930	0	189	316	0	52	139	0	19	962	4031	
16	1141	2625	509	615	648	139	188	180	40	741	1031	213	264	231	46	120	95	15	962	3069	4810
17	632	0	526	476	0	134	148	0	33	528	0	206	218	0	42	105	0	21	962	2107	
18	2731	0	520	342	0	126	115	0	35	322	0	212	176	0	48	84	0	21	962	3770	
19	2211	0	549	216	0	134	80	0	23	1141	0	201	128	0	39	63	0	16	962	3839	
20	1662	0	527	730	0	129	237	0	47	940	0	205	320	0	37	142	0	17	962	4031	
21	1135	2656	510	601	604	118	190	178	40	735	1049	220	283	219	47	125	104	27	962	3069	4810
22	625	0	521	483	0	119	150	0	28	515	0	222	236	0	53	98	0	19	962	2107	

4. Determination of safety stocks allowing to counter rank-change induced disturbances

Impact of lot-sizing on demand and respect for demand

The customer’s demand for alternate components is expressed in the form of a continuously updated ordered list of components. The order of M components placed earlier by the customer (gap θ) is delivered periodically (periodicity λ). Packaging constraints can lead to mandatory grouping of the references delivered by batches of m identical components, which will lead to a delivery of $y = M/m$ batches.

Lot-sizing has two consequences. At the arrival of each delivery one must perform a reconstruction of the sequence of M alternate components respecting the order of assembly. The batch of M delivered components has practically no chance to coincide with the batch which will be assembled; the missing number of components corresponds to the number of surplus components. As a result, it will be necessary to have a safety stock available for each component. To fully understand the results, it is useful to clarify the rank-change probability distribution of a component, as the studied variable corresponds to the number of ranks gained or lost at delivery for any alternate component, with regard to the demand of assembly (a similar phenomenon has been studied by Giard *et al* (2001a, 2001b) in the off-line treatment of quality problems in an automotive industry production line).

Before starting that study, it should be noted that an identical mechanism can be noticed at the supplier’s. His demand to be fulfilled is γ batches. Some technical constraints can lead to a programming of the supplier’s production in a succession of sequences of ν batches, not necessarily based on the same alternate component but all sharing an identical characteristic such as including an identical elementary component (for example an identical crankcase used by several engines). This form of lot-sizing is a little more complex, since it does not imply homogeneity but, as previously, it involves rank-changes and thus the necessity for the supplier to this time get a safety stock allowing him to meet the demand.

In both cases, batch rules are taken into account when a deterministic demand is to satisfy leads to rank-changes (Figure 2) and it is necessary to set up safety stocks to avoid stockout.

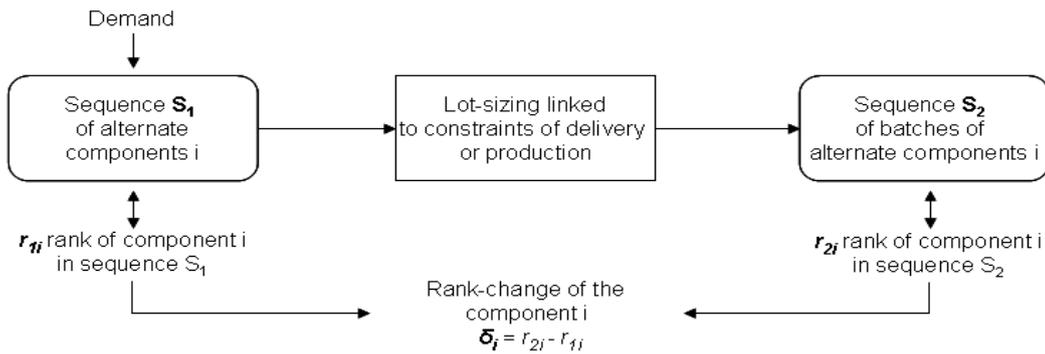


Figure 2. Origin of operations of rank-change

It is important to state that a rank-change does not systematically entail a stockout of stock and then the constitution of a safety stock: the supplier’s deliveries to the customer are performed in rounds (deliveries of γ batches at interval θ), which implies that being out of stock is not determined by sequence S_2 but by the constitution of the group formed by the γ batches produced and in stock, and permutations within this are possible without any consequences.

For any component i , when using the known demand in set S_1 , the last batch of m components may be incomplete (K_i elements $< m$). One can decide to only use the firm demand and wait for the demand of following days (ω components) to get the missing item $m-K$. The probability of filling up the batch is given by the Negative Binomial distribution $\mathcal{NB}(m - K_i, \omega, p_i)$. The risk of being unable to complete a batch may be significant, for instance, in the case of engine 4 for which $p_4=0,9\%$, with $\omega = 1,000$ and $K_i = 1$, the risk increases by 11,3 %). The other solution consists in deciding to launch a full batch as soon as a component is required in S_1 . This second solution, adopted henceforth, is equivalent to considering a set S_1 of infinite size.

Analysis of rank-change operations inferred by lot-sizing

The simulations of this second industrial example are performed with Simul8 software and concern six million engines distributed according to table 1. The 12,000 first components are excluded from the simulation since they represent the transient state before the steady state that interests us. The rule used to create S_2 is the following one, and deals with the treatment of vehicle y in sequence S_1 .

- If the required engine for vehicle y is in stock, that engine is taken away from the stock, whose level falls by 1.
- Otherwise, a batch of six engines for that required engine is launched; that engine is taken from the stock, whose level drops to 5.
- Let’s move on to vehicle $y+1$ in S_1

Table 6 summarizes the simulation results for each engine E_i having the probability of use p_i , their average $\bar{\delta}_i$ of rank-changes, the standard deviations σ_{δ_i} of the rank-changes and the safety stocks SS_i , required to avoid stockout:

Table 6. Rank change(s) expectations – standard deviations of rank-changes– and safety stocks per engine.

Engine E_i	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%	0.4%	0.9%	3.0%	6.5%	6.1	3.5	6.1%	11.3%	2.6%
$\bar{\delta}_i$	-98.73	6.99	-65.20	-230.57	-230.57	39.40	29.44	-145.72	-230.45	-576.22	-586.58	-229.12	-36.57	8.95	7.52	-24.64	6.40	25.20	-48.29
σ_{δ_i}	134.62	36.88	103.10	257.68	258.46	9.13	16.75	178.10	259.09	585.75	588.51	256.36	76.30	34.62	36.12	65.44	36.80	20.18	87.62
SS_i	5	10	6	4	4	33	17	5	4	3	3	5	8	11	11	8	10	16	8

With the notations of Figure 2, a negative value of δ_i means that the engine which has just arrived is early with regard to the vehicle on which it will be assembled; a positive value means that the engine came late with regard to the vehicle on which it has to be assembled, which implies a withdrawal from the safety stock. In a deterministic universe with deliveries in accordance with requirements and lot-sizing, it is thus necessary to anticipate rank-changes with safety stocks. A safety stock is made

up beforehand for each engine to avoid any line stoppage; this stock is fed by engines coming too early (which reduces the required safety stock) and by engines coming too late (which restores the safety stock). The weighted sum of rank-change $\overline{\delta}_i$ by probability p_i is null². The engine rank-change curves are not identical: the mathematical expectation of the rank-change of engines varies in the same way as their use probabilities, whereas the standard deviations of engine rank-change vary in inverse order of their use probability (Figure 3). Therefore, those engines in low demand are delivered somewhat ahead of schedule, whereas the delivery of engines in high demand is somewhat delayed. It ensues that engines without great demand are delivered somewhat ahead of schedule, but the extent of their dispersion leads to the creation of safety stocks; engines in high demand are delivered somewhat late, but their small dispersion limits the need for safety stocks.

The rank-change function of an engine i depends both on the number I of engines ($i=1,..,I$) and on the distribution of the demand p_i . Figure 4 illustrates the case of an engine in low demand and the most demanded engine; the scales used not being identical.

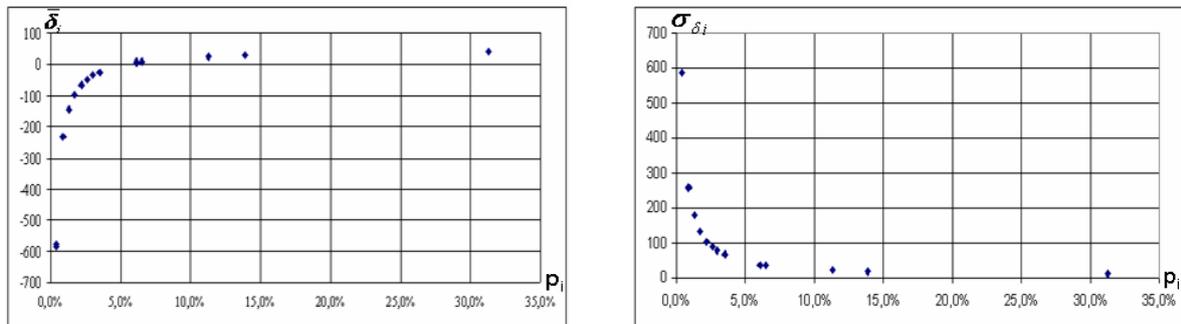


Figure 3. Evolution of the mathematical expectation and the standard variation of rank-change of engines according to their probabilities p_i of assembly

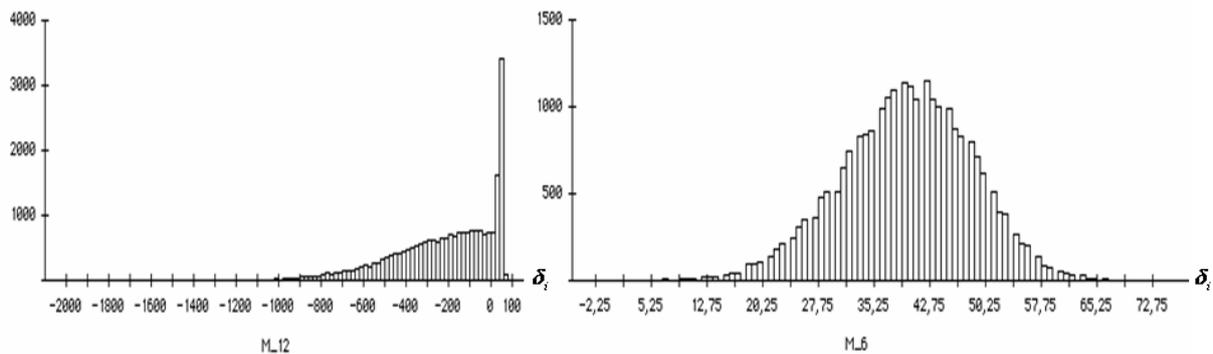


Figure 4. (Curves of r)Rank(s)-change(s) curves of the engines ($p_{12} = 0,9\%$; $\overline{\delta}_{12} = -229$) and E_6 ($p_6 = 31,3\%$; $\overline{\delta}_6 = 39,4$)

Determination of safety stocks intended to counter the effects of lot-sizing

In deterministic universe, safety stocks depend on the rank-changes and on the lead time θ . The rank-changes themselves depend on the range I of engines and on their assembly probabilities p_i . An evaluation of their appropriate level can be performed only through an approach using simulation of

² In steady state, there is inevitably compensation between the won ranks and the lost ranks. On a sample, the average observed (here $\overline{\delta}_i$) coincides only exceptionally with the mathematical hope (0).

the steady state. At the beginning of the simulation, the available quantities of alternate components at the customer's are fixed to a value W . The first delivery is made immediately before the first component of sequence S_1 is taken. This first delivery corresponds to the θ first components of sequence S_2 and the first withdrawn component corresponds to the first component of sequence S_1 . Each simulation concerned the assembly of 6 million components so as to empirically obtain safety stocks SS_i having an insignificant probability to lead to a stockout. The safety stock level is calculated as the difference between W and the lowest inventory level during the simulation, since W must be big enough to avoid an empty stock. The safety stock of components varies in the same way as their use probabilities (Figure 5), which was not obvious a priori. In that industrial example, an approximately linear relation can be noticed ($\hat{\rho}^2=0.983$; $SS_i=0.95889 p_i+3.008$).

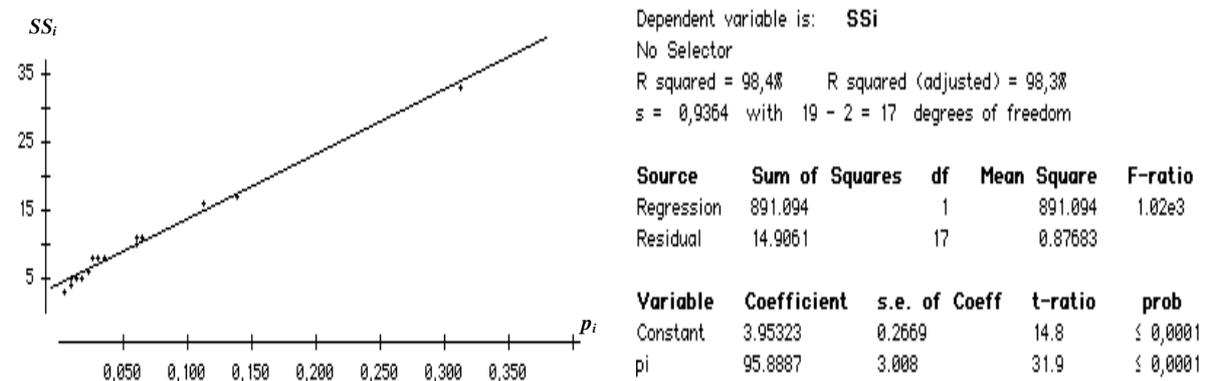


Figure 5. Evolution of the safety stock SS_i according to the probabilities p_i of the assembly of engines

We would think that the dispersion of probabilities influences the level of safety stocks. Thus, new simulations have been performed by replacing the probabilities of Table 6 by distributions of type $p_i = p_1 \times k^{i-1} + b$ with $b = 0.005$ (asymptote), k being the coefficient of decreasing under control (the closer it is to 1 k , the more equal probabilities of component demands there is). An approximately linear relation can be noticed ($\hat{\rho}^2=0.991$; $\overline{SS} = 0.0438 k + 0.1579$).

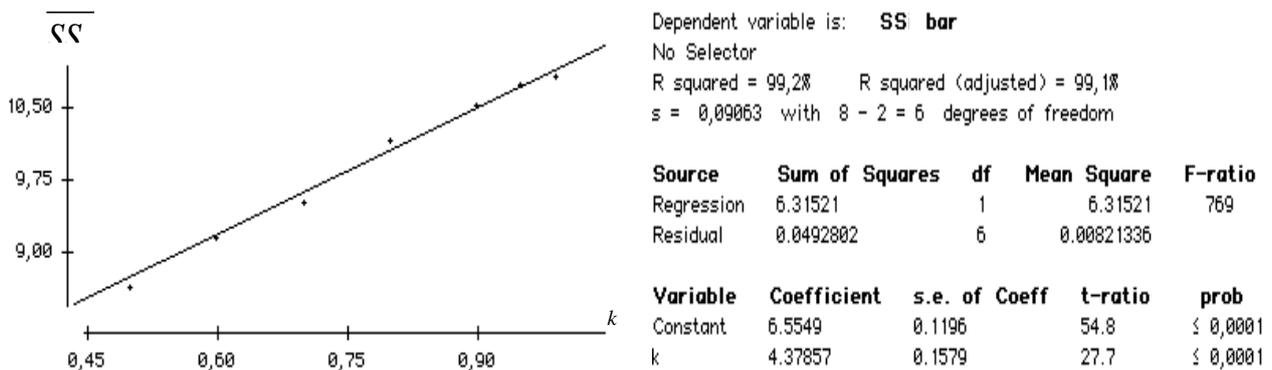


Figure 6. Evolution of the average safety stock \overline{SS} , according to k

Then, the impact of the periodicity of supplying was studied by performing successively the same simulation with various values of θ . This analysis was lead through three structures of demand with 5, 10 and 19 alternate components and equal probabilities of component demands. We finally made a more detailed analysis of the impact of the range I on the average safety stock, for equal probabilities of component distributions.

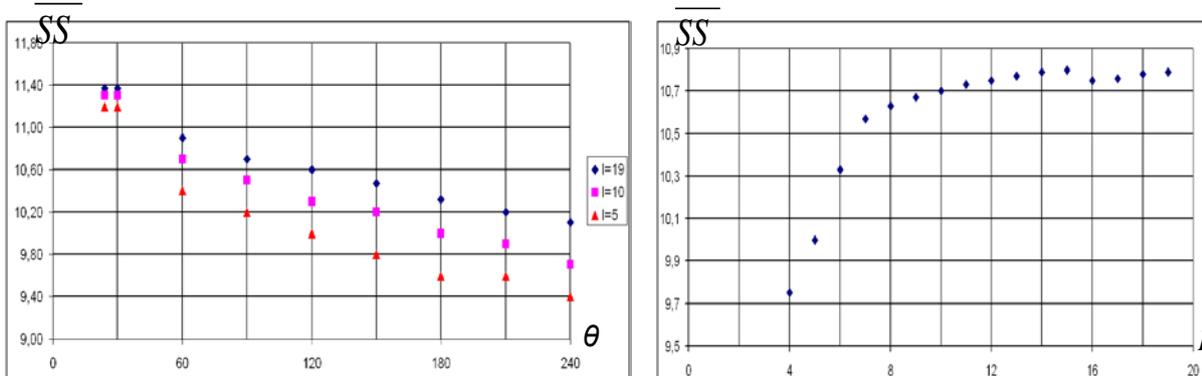


Figure 7. Evolution of the average safety stock \overline{SS} according to θ for various values of I

Safety stocks also depend on other risks such as quality problems in a production context, variation of transportation time or modification of the definite sequence of final demand. A combination of disruptions leads to risk pooling; thus the safety stock needed to face several disruptions is lower than the sum of necessary safety stocks if one disruption is considered regardless of the other.

5. Conclusion

We have studied synchronization and decoupling of the control of the last two links of a supply chain dedicated to customized mass production. Knowledge of the production sequence set in response to a known final demand as well as of the demand structure modifies the traditional policies of piloting the flow, making improvements in effectiveness and efficiency possible. Taking into account lot-sizing entails the creation of safety stocks; the explanatory factors of their importance were analyzed. All of these elements take on significance in the context of the increasing geographical dispersion of the links of large worldwide logistic chains.

6. Bibliography

- Axsäter S. (1993), Exact and approximate evaluation of batch-ordering policies for two-level inventory systems, *Operations research*, vol. 41, issue 4, p. 777-785.
- Anderson D. & Pine J. (1997), *Agile Product Development for Mass Customization: How to Develop and Deliver Products for Mass Customization, Niche Markets, JIT, Build-to-Order and Flexible Manufacturing*, McGraw-Hill.
- Bourland K., Powell S. & Pyke D. (1996), Exploiting timely demand information to reduce inventories, *European journal of operational research*, vol.92, p. 239-253.
- Cachon G. & Fisher M. (1997), Campbell Soup's continuous product replenishment program : evaluation and enhanced decision rules, *Production and operations management*, vol.6, issue 3, p. 266-276.
- Cachon G. & Fisher M. (2000), Supply chain inventory management and the value of shared information, *Management science*, vol. 46, issue 8, p. 1032-1048.
- Cachon G. & Lariviere M. (2005), Supply chain coordination with revenue sharing: strength and limitations, *Management science*, vol. 51, issue 1, p. 30-44.
- Chen F. (1998), Echelon reorder points, installation reorder points, and the value of centralized demand information, *Management science*, vol.44, issue 12, p. S221-S234.
- Chen F. & Samroengraja R. (2000), A staggered ordering policy for one-warehouse, multiretailer systems, *Operations research*, vol.48, issue 2, p. 281-293.
- Christopher M. (1998), *Logistics and Supply Chain Management*, 2d edition, London, Prentice Hall.
- Disney S. & Towill D. (2003), The effect of vendor managed inventory (VMI) dynamics on the Bullwhip Effect in supply chains, *International journal of production economics*, vol. 85, issue 2, p. 199-215.
- Emmons H. & Gilbert S. (1998), The role of returns policies in pricing and inventory decisions for catalogue goods, *Management science*, vol. 44, issue 6, p. 276-283.
- Forslund H. & Jonsson P. (2007), The impact of forecast information quality on supply chain performance, *International journal of operations & production management*, vol. 27, issue 1, p. 90-107.

- Gavirneni S., Kapuscinski R. & Tayur S. (1999), Value of information in capacitated supply chains, *Management science*, vol.45, issue 1, p16-24.
- Gerchak Y. & Wang Y. (2004), Revenue-sharing vs. wholesale price contracts in assembly systems with random demand, *Production and operations management*, vol. 13 issue 1, p. 23-33.
- Giard V., Danjou F. & Boctor F. (2001a), Rank changes in production/assembly lines: impact and analysis, *3^e congrès international de Génie Industriel*.
- Giard V., Danjou F. & Boctor F. (2001b), Analyse théorique des décyclages sur lignes de production, *Journal européen des systèmes automatisés*, vol. 35, issue 5, p. 623- 45.
- Giard V. & Mendy F. (2008), Exploitation of the knowledge of the final demand in the piloting of a logistics chain, *Production planning and control*. (to appear)
- Goldratt E. & Cox J. (1984), *The goal*, North River Press.
- Hariharan R. & Zipkin P. (1995), Customer-order information, lead-times and inventories, *Management science*, vol. 41, issue 10, p. 1599-1607.
- Kouvelis P., Chambers C. & Wang H. (2006), Supply chain management research and production and operations management: review, trends, and opportunities, *Production and operations management*, vol. 15, issue 3, p. 449-469.
- Lee H. & Tang K. (2000), The value of information sharing in a two-level supply chain, *Management science*, vol. 46, issue 5, p. 626-643.
- Medan M. & Gratacap A. (2008), *Logistique et supply chain management*, Dunod.
- Viswanathan S., Widiarta H. & Piplani R. (2007), Value of information exchange and synchronization in a multi-tier supply chain, *International journal of production research*, vol. 45, issue 21, p5057-5074.
- White C. & Wilson R. (1977), Sequence dependent set-up times and job sequencing, *International journal of production research*, vol. 15, issue 2, p. 191-202.
- Zhang H. (2002), Vertical information exchange in a supply chain with duopoly retailers, *Production and operations management*, vol. 11, issue 4, p531-546.
- Zou X., Pokharel S. & Pipliani R. (2004), Channel coordination in an assembly system facing uncertain demand with synchronized processing time and delivery quantity, *International journal of production research*, vol.42, issue 22, p4673-4689.