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Walsh's Contract and Transparency about Central Bank Preferences for Robust Control

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Abstract: Within a New Keynesian model subject to misspecification, we examine the quadratic contracts in a delegation framework where government and private agents are uncertain about central bank preferences for model robustness. We show that, in the case of complete transparency, the optimal penalty is decreasing in terms of the preference for robustness. In effect, a central bank reacts more aggressively to supply shocks when the model misspecification grows larger. Furthermore, beginning from the equilibrium of perfect transparency and assuming that the average preference for robustness is sufficiently high, the central bank has then an incentive to be less transparent in order to reduce the optimal penalty. Under similar conditions, we also find that greater opacity will increase inflation and output variability.

Keywords: Walsh's contract, robust control, model uncertainty, central bank transparency.

JEL Classification: E42, E52, E58

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1. Introduction

The New-Keynesian approach to macroeconomic modeling is used extensively by the monetary policy literature for almost a decade now. It has produced several important insights in the analysis of monetary policy and is now commonly applied to provide policy prescriptions (Clarida *et al.*, 1999). Most of the studies focus on specific topics and stay unconnected between them. In particular, recent developments on optimal monetary policy in forward-looking models have explored separately various notions of transparency and of model robustness.

Several studies use the New-Keynesian models to discuss monetary policy transparency, while most of them consider that the central bank knows exactly the model structure of the economy. Considering asymmetric losses from output gap, Cukierman (2002) has demonstrated that it may be rational for the central bank to de-emphasize a high flexibility parameter and asymmetric preferences that might raise inflationary expectations. For Jensen (2002), greater transparency about control errors means that policy has a larger impact on future expectations and, via this channel, on current equilibrium inflation. This leads the central bank to be less aggressive in its policy actions. Walsh (2003) examines accountability issues using his inflation contracting approach developed in 1995. Assuming uncertainty about output gap objective of the central bank, he has shown that the fundamental trade-off between accountability and stabilization depends on the degree of transparency, defined as the ability to monitor the central bank's performance. Eijffinger and Tesfaselassie (2005) look at transparency about disclosure of forecasts of future shocks as well as output gap objective of the central bank in a three period New-Keynesian model. Their main result is that advance disclosure of forecasts of future shocks does not improve welfare and in some cases is not desirable.

Nevertheless, as any model, New-Keynesian models rest on a set of assumptions that may or may not be good approximations of true economies. Without the possibility to have a complete description of reality, a policymaker is likely to prefer basing policy on principles that are valid also if the assumptions on which the model is founded differ from reality. In other words, policy prescriptions should be robust to reasonable deviations from the benchmark model. The literature on monetary policy robustness has been developed into two directions[1]. The first one leads to what has been called robustly optimal instrument rules (Svensson and Woodford, 2004; Giannoni and Woodford, 2003a, 2003b). As these instrument rules do not depend on the specification of the generating processes of exogenous disturbances in the model, they are, therefore, robust to misspecification in these processes. The second one, initiated by Hansen and Sargent (2003, 2004), corresponds to robust control approach to the decision problem of agents who face model uncertainty. In the sense of Hansen and Sargent, robust monetary policies are designed to perform well in worst-case scenarios. These policies arise as the equilibrium in a game between the monetary authorities and an evil agent who chooses model misspecification to make the authorities look as bad as possible. While these two approaches to robust policies appear quite distinct, Walsh (2004) has demonstrated that both approaches lead to exactly the same implicit instrument rule for the policy maker in a standard, forward-looking, new Keynesian model of optimal monetary policy.

The role of model robustness has been neglected until now in the literature on transparency. Consider that the central bank knows exactly the model structure of the economy doesn't allow us to study some important strategic interactions between the central bank and private agents. In particular, policy makers may use strategically information they

[1] Another current of research studies about the robustness of a monetary policy rule across different kinds of models (backward-looking, Lucas-type transmission mechanism and forward-looking models). See for example McCallum (1999).

dispose in order to gain benefits in terms of output stabilization. At the other hand, the robustness approaches assume that private agents know exactly the preference parameters and the preferences for robustness of the central bank while the latter does not know exactly the true model structure of the economy. This asymmetry is difficult to justify if the central bank has incentive to not communicate its preferences as well as the degree of model robustness.

In this paper, we make the connection between the literature on transparency and that on robust control of monetary policy by assuming that the central bank doesn't know exactly the true model structure of the economy and is not totally transparent about its model robustness preferences. In a similar framework to that used by Walsh (2003), we examine the interactions between transparency and robustness and their implication for accountability.

In the section 2, we present the basic model. In the section 3, we solve the model to obtain the optimal penalty under perfect transparency about the preference for robustness. The effects of opacity about the central bank preference for robustness are discussed in section 4. We conclude in section 5.

2. The model

Our description of economic environment follows the standard New-Keynesian model based on optimizing private sector behavior and nominal rigidities that has been used extensively in the recent literature on monetary policy (Clarida, Galí, and Gertler, 1999). Instead of formulating monetary policy explicitly in terms of control over the nominal interest rate, we simplify by treating the output gap, i.e. output relative to the flexible-price equilibrium level, as the instrument of monetary policy. It is assumed that the central bank faces model misspecification and its preference for robustness is not perfectly observable by the government and the public.

2.1 The economy

The economy is characterized by an expectations augmented, forward-looking Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \delta x_t + e_t, \qquad \text{with } 0 < \beta < 1,\ \delta > 0, \tag{1}$$

where π_t is the inflation rate, $E_t \pi_{t+1}$ the private sector's expectation of future inflation, x_t the output gap, e_t an inflation or cost shock, and β the discount rate. The parameter δ is the output gap elasticity of inflation and captures the effects of the gap on real marginal costs and marginal cost on inflation. The cost shock e_t is assumed to be serially uncorrelated.

While the central bank sees the forward-looking Phillips curve described by equation (1) as the most likely specification, it realizes that the true Phillips curve may deviate from the benchmark, although it is unable to specify a probability distribution for deviations. To model such misspecification, we introduce in equation (1) a second type of disturbance, denoted by h_t. In the sense of Hansen and Sargent (2004), the disturbance is controlled by a fictitious "evil agent" representing the policymaker's worst fears concerning specification errors. Thus, the forward-looking Phillips curve with misspecification is given by

$$\pi_t = \beta E_t \pi_{t+1} + \delta x_t + e_t + h_t. \tag{2}$$

2.2 Policy objectives with preference for robustness

In order to study the question of delegation, we distinguish as Walsh (2003) between the objective functions of the principal, referred to as the government (or the public), and the agent, the central bank. The role of the government will be to design the targeting regime under which the central bank conducts policy. The central bank is delegated to attain the target defined by the government. The failure of the central bank to achieve the target is associated with penalty.

The expected social loss function is assumed to take the standard form as following:

$$L_t^S = \frac{1}{2} E \sum_{i=0}^{\infty} \beta^i (\lambda x_{t+i}^2 + \pi_{t+i}^2), \ \lambda > 0 \tag{3}$$

where λ is the relative weight placed on the output objective. Social loss depends on variability of the output gap as well as inflation variability.

In the Walsh's model, the central bank is subject to political pressures for economic expansions, captured by allowing for random fluctuations in the central bank's output target. In the present approach, by eliminating this source of uncertainty about central bank preferences and the relating inflationary bias, we focus on the implications of a new kind of uncertainty concerning the central bank preference for robustness of monetary policy. Consequently, monetary policy is implemented under discretion by a central bank that has the same output target than the government.

The central bank is also charged with an inflation targeting objective, defined by the target and the weight placed on achieving it. As the overly ambitious output target common in the Barro-Gordon framework is absent here, discretionary policy implemented to minimize (3) would not lead to an average inflation bias.

To design the robust monetary policy, the central bank takes into account a certain degree of model misspecification by minimizing its objective function in the worst possible model within a given set of plausible models. Monetary policy is implemented to minimize the conditional expectation of the loss function

$$L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i [\lambda x_{t+i}^2 + \pi_{t+i}^2 + \tau(\pi_{t+i} - \pi_{t+i}^T)^2 - \theta_t h_{t+i}^2], \tag{4}$$

where π_t^T is the period t inflation target, and τ the weight that the central bank places on achieving its inflation target or deviation penalty. θ denotes the preference for robustness which is known only to the central bank, but neither to the government and the private sector.

When taking their decision, the government and the private sector can make guesses about the value of θ_t and believe that θ_t has an average value $\overline{\theta}$ and a variance σ_θ^2. We assume that θ_t is independent of the cost shock e_t so that the covariance $\text{cov}(e_t, \theta_t)$ is zero.

As it is common in the robust control literature (Hansen and Sargent, 2004), we assume that the central bank allocates a budget χ^2 to the evil agent who creates misspecification under the following budget constraint

$$E_t \sum_{j=0}^{\infty} \rho^i h_{t+j}^2 \leq \chi^2 \ . \tag{5}$$

Since parameters τ and π_t^T characterize alternative inflation targeting regimes, thus we can ignore irrelevant constants and terms independent of the central bank's actions. The single period loss function of the central bank's can be rewritten as

$$\frac{1}{2} E^{CB}[\lambda x_t^2 + (1+\tau)(\pi_t - \pi_t^T)^2 + 2\pi_t \pi_t^T - \theta h_t^2]. \tag{6}$$

The loss function (6) can be interpreted as in Walsh (2003) except for two elements. Firstly, there is absence of the random disturbance associated with output target. Secondly, the last term in (6) is specific to the robust control techniques adopted by the central bank. The penalty weight τ in the second term of (6) plays the role of Rogoff's weight conservatism as if the central bank weights more heavily on inflation objective. To insure the consistence of the loss function, we assume that

$$(1+\tau) > 0, \tag{7}$$

i.e. the deviation from inflation target constitutes always a loss.

3. Optimal Walsh's contract under robust control with perfect transparency

We consider that the central bank acts in a discretionary manner when making its policy choices. The government sets up a targeting regime in fixing inflation target and the penalty associated with its realization. Under that regime, the central bank implements the time-consistent discretionary monetary policy which is robust to model misspecification.

As there is no average inflation bias in the present model, the inflation target is assumed to be zero ($\pi_t^T = 0$). The central bank's problem is to solve under the economic constraint with model misspecification (equation (2)) the following program:

$$\min_{x_t} \max_{h_t} L_t^{CB} = \frac{1}{2} E_t^{CB} [\lambda x_t^2 + (1+\tau)\pi_t{}^2 - \theta h_t^2]. \tag{8}$$

The only state variable in this model is the exogenous cost shock e_t. We assume it is serially uncorrelated, the central bank treats expectations of future inflation as given ($\mathrm{E}_t \pi_{t+1} = 0$) in setting x_t and h_t. The first-order conditions for the central bank's decision problem are:

$$\frac{dL_t^{CB}}{dx_t} = 0 \Rightarrow E_t^{CB} [\lambda x_t + (1+\tau)\pi_t \delta] = 0, \tag{9}$$

$$\frac{dL_t^{CB}}{dh_t} = 0 \Rightarrow E_t^{CB} [(1+\tau)(\delta x_t + e_t) + (1+\tau-\theta)h_t] = 0. \tag{10}$$

The second-order condition for the maximization program of evil agent is derived from (10) as follows:

$$\frac{d^2 L_t^{CB}}{dh_t^2} < 0 \Rightarrow E_t^{CB} [(1+\tau-\theta)] < 0. \tag{11}$$

The condition (11) yields

$$\theta > 1+\tau. \tag{11'}$$

Equations (2), (9) and (10) can be solved jointly for the equilibrium output gap and inflation rate under discretion with a non-state contingent target of zero inflation. It yields

$$\pi_t = \frac{\theta\lambda}{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)} e_t , \tag{12}$$

$$x_t = -\frac{\theta(1+\tau)\delta}{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)} e_t , \tag{13}$$

$$h_t = \frac{\lambda(1+\tau)}{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)} e_t . \tag{14}$$

where the expression $\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau) > 0$ according to condition (11'). So the current inflation and the model misspecification react positively to the inflationary shock, while the output negatively. When $\theta \to \infty$, the central bank is certain about its economic model. In this case, the solution becomes:

$$\pi_t = \frac{\lambda}{\lambda + (1+\tau)\delta^2} e_t , \tag{12'}$$

$$x_t = -\frac{(1+\tau)\delta}{\lambda + (1+\tau)\delta^2} e_t , \tag{13'}$$

$$h_t = 0 . \tag{14'}$$

The central bank is a risk aversion agent who wants to avoid particularly bad outcomes, and therefore needs policy to be robust against specification errors that could have particularly severe consequences. Unambiguously, the preference for robustness will affect the outcomes of macroeconomic variables.

The equilibrium solution of inflation π_t, output x_t and misspecification errors h_t react to the stochastic and unverifiable realization of e_t. The accountability of the central bank modifies the effects of e_t shocks on these variables as follows:

$$\frac{d^2\pi_t}{de_t d\tau} = \frac{\theta\lambda(\lambda - \theta\delta^2)}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} < 0 , \quad \text{if } \lambda - \theta\delta^2 < 0 , \tag{15}$$

$$\frac{\partial^2 x_t}{\partial e_t \partial\tau} = -\frac{\theta^2\delta\lambda}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} < 0 , \tag{16}$$

$$\frac{\partial^2 h_t}{\partial e_t \partial \tau} = \frac{\lambda^2 \theta}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} > 0 . \tag{17}$$

The effect of e_t on inflation is decreasing in τ if the parameter representing preference for robustness, θ, is larger enough to ensure that $\lambda - \theta\delta^2 < 0$. The effects of e_t on output and model specification errors are decreasing and increasing respectively in τ. The amplitude of these effects depends also on θ as shown in (15)-(17). That can be shown by the following derivatives:

$$\frac{d^2 \pi_t}{de_t d\theta} = \frac{-\lambda^2(1+\tau)}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} < 0 , \tag{18}$$

$$\frac{\partial^2 x_t}{\partial e_t \partial \theta} = \frac{(1+\tau)^2 \delta\lambda}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} > 0, \tag{19}$$

$$\frac{\partial^2 h_t}{\partial e_t \partial \theta} = \frac{-\lambda(1+\tau)[\lambda + (1+\tau)\delta^2]}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} < 0 . \tag{20}$$

The effect of e_t on inflation is decreasing in θ. This means that if the central bank has a lower preference for robustness (higher θ), it is less aggressive in response to inflationary shocks[2]. The effects of e_t on output and model specification errors are increasing and decreasing respectively in θ.

The government, when deciding the optimal targeting weight (penalty), faces the trade-off between the need for accountability and the need for stabilization in taking account of the fact the central bank uses the robust control approach in implementing the targeting regime.

The optimal target weight is obtained by minimizing (3) with respect to τ in taking account of the solution of π_t and x_t in equations (12)-(13). The first order condition for the optimal τ leads to

[2] See also Leitemo, Kai & Ulf Söderström (2007).

$$\frac{dL_t^S}{d\tau} = 0 \Rightarrow E_t(\lambda x_t \frac{dx_t}{d\tau} + \pi_t \frac{d\pi_t}{d\tau}) = 0\,, \tag{21}$$

where

$$\frac{dx_t}{d\tau} = \frac{-\lambda\theta^2\delta e_t}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2}\,, \tag{22}$$

$$\frac{\partial\pi_t}{\partial\tau} = \frac{\theta\lambda e_t(\lambda - \theta\delta^2)}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2}\,. \tag{23}$$

Manipulating (21) and using equations (12), (13), (22) and (23), the first-order condition becomes

$$E_t\left[\frac{(\tau\theta\delta^2 + \lambda)\lambda^2\theta^2(e_t)^2}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^3}\right] = 0\,. \tag{24}$$

When θ is certain, solving (24) gives the solution of τ as follows

$$\tau = \frac{-\lambda}{\theta\delta^2} < 0\,. \tag{25}$$

The optimal solution of penalty implies that, the condition (7), i.e. $1+\tau > 0$, is equivalent to the condition $\lambda - \theta\delta^2 < 0$ which is used to obtain the result reported in (15). In the case where $\theta \to \infty$, i.e. absence of robust control, we have[3] $\lim_{\theta\to\infty} \tau = 0$. It follows from (25) that $\frac{d\tau}{d\theta} = \frac{\lambda}{\theta^2\delta^2} > 0$. These results can be summarized in the following proposition.

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[3] Following Walsh (2003), we can introduce in the loss function of the central bank, a parameter u_t, i.e. the mean zero period t realization of the net pressures for economic expansion. The realization of u_t is known by the central bank, although it is assumed to be unverifiable private information. Therefore, the optimal penalty will be $\tau = [(\lambda\theta\delta^2 - 2\lambda\delta^2 + \theta\delta^4)\sigma_u^2 - \lambda\sigma_e^2]/(\lambda\delta^2\sigma_u^2 + \theta\delta^2\sigma_e^2)$ with $\partial\tau/\partial\theta > 0$ and $\partial\tau/\partial\sigma_u^2 > 0$. The result is reduced to that of Walsh if $\theta \to \infty$ (i.e., in the absence of model misspecifications): $\lim_{\theta\to\infty} \tau = (\lambda + \delta^2)\sigma_u^2/\sigma_e^2$.

Proposition 1. *When monetary policy is perfectly transparent concerning the preference for robustness* ($\sigma_\theta^2 = 0$)*, the optimal target weight* τ *is negative and decreasing in terms of preference for robustness.*

We remark that in the absence of transparency issue ($\sigma_\theta^2 = 0$) and robust control (i.e. $\theta \to \infty$), it is not necessary to impose any penalty since $\lim_{\theta \to \infty} \tau = 0$. Departing from this situation, when the central bank practices the robustness control (θ decreases), the inflation rate increases and this implies higher social loss that the government seeks to minimise. In the classical Walsh's contract, this will incite the government to increase the penalty (τ) in order to diminish the inflationary pressures resulted from an inflationary bias. In our framework, the central bank has not this kind of inflationary bias as its certain output target is the same as the potential output. Once we assume model uncertainty, an increase in the penalty will not necessarily attain the same goal, since it has a double effect on inflation rate. Higher penalty incites the central bank to reduce inflation rate (direct effect) in favor of increased model misspecifications leading to higher inflation (indirect effect). Furthermore, a higher preference for robustness control has also a positive effect on model misspecifications leading to higher inflation. In our analysis, the negative direct effect of a higher penalty on inflation is dominated by its second indirect effect and the effect of higher preference for robust control on model misspecifications. Consequently, the government will counterbalance the inflationary effect of higher preference for robustness (lower θ) by decreasing the penalty.

4. The effect of opacity about the robustness preference of the central bank

The results summarized in Proposition 1 are obtained under the assumption that the information about the preferences for robustness of the central bank is transmitted to the government and the private agents. In the following, we relax this assumption in admitting that this information is private to the monetary policymaker. That leads us to investigate whether the lack of transparency about the preferences for robustness (θ) can be used strategically by the central bank in order to avoid penalties inflicted by the government in the case of deviation from the targeting regime.

Substituting the solution of π_t and x_t in equations (12) and (13) into expected social loss function (3) leads to

$$
\begin{aligned}
L_t^S &= \frac{1}{2}E_t\left\{\lambda\left[\frac{-\theta(1+\tau)\delta e_t}{\theta[\lambda+(1+\tau)\delta^2]-(1+\tau)\lambda}\right]^2+\left[\frac{\theta\lambda e_t}{\theta[\lambda+(1+\tau)\delta^2]-(1+\tau)\lambda}\right]^2\right\} \\
&= \frac{1}{2}E_t\left\{\frac{(\lambda\theta^2\delta^2T^2+\theta^2\lambda^2)(e_t)^2}{\left\{\theta[\lambda+(1+\tau)\delta^2]-(1+\tau)\lambda\right\}^2}\right\}
\end{aligned}
\quad . \qquad (26)
$$

The application of second-order Taylor approximation to the expected social loss function (26) yields

$$
L^S = \frac{1}{2}\frac{[\lambda\bar{\theta}^2\delta^2(1+\tau)^2+\bar{\theta}^2\lambda^2]\sigma_e^2}{\left\{\theta[\lambda+(1+\tau)\delta^2]-(1+\tau)\lambda\right\}^2}+\frac{\Omega}{2}\sigma_\theta^2 . \qquad (27)
$$

with $\Omega = -\dfrac{-\left\{[\delta^2(1+\tau)^2+\lambda](1+\tau)^2\lambda^3+2[\delta^2(1+\tau)^2+\lambda](1+\tau)\bar{\theta}\lambda^2[(1+\tau)\delta^2+\lambda]\right\}\sigma_e^2}{\left\{\bar{\theta}[\lambda+(1+\tau)\delta^2]-(1+\tau)\lambda\right\}^4}$.

Proposition 2. *Starting from the initial equilibrium characterized by perfect transparency* ($\sigma_\theta^2 = 0$)*, an increase in the opacity about the preferences for robustness in monetary policymaking will incite the government to decrease the optimal target weight (i.e.* $d\tau / d\sigma_\theta^2 < 0$*) if the average preference for robustness is sufficiently low, i.e.* $\bar{\theta} > \max\{1/\delta^2, \lambda/\delta^2\}$.

Proof : See Appendix.

Low average preference for robustness (higher values of $\overline{\theta}$) implies a decrease in inflation rate. In the case of perfect transparency, this leads to a higher penalty or smaller reward (see Proposition 1). The opacity of the central bank concerning its preferences for robustness introduces uncertainty for the government and increases the social loss. The optimal solution for the government is to reduce the penalty in order to counterbalance the negative effect of opacity on social welfare. In the contrary, if the central bank has weak anti-inflation credentials in its robust control policy (lower values of $\overline{\theta}$, i.e. $\lambda/\delta^2 < \overline{\theta} < 1/\delta^2$), low degree of transparency about its preference could eventually incite the government to increase the penalty or to decrease award.

The effects of opacity about the model robustness on the macroeconomic performance can be summarized in the following proposition.

Proposition 3. *Starting from the initial equilibrium characterized by perfect transparency ($\sigma_\theta^2 = 0$), an increase in the opacity about the preferences for robustness in monetary policymaking will increase output and inflation variability if the average preference for robustness is sufficiently low, i.e.* $\overline{\theta} > \max\{1/\delta^2, \lambda/\delta^2\}$.

Proof : Deriving equations (12)-(13) relative to σ_θ^2 leads to

$$\frac{\partial^2 \pi_t}{\partial e_t \partial \sigma_\theta^2} = \frac{\theta\lambda(\lambda - \theta\delta^2)}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} \frac{\partial \tau}{\partial \sigma_\theta^2} > 0, \quad \text{since } \lambda - \theta\delta^2 < 0, \qquad (28)$$

$$\frac{\partial^2 x_t}{\partial e_t \partial \sigma_\theta^2} = \frac{-\theta^2\delta\lambda}{\{\theta[\lambda + (1+\tau)\delta^2] - \lambda(1+\tau)\}^2} \frac{\partial \tau}{\partial \sigma_\theta^2} > 0. \qquad (29)$$

The effects of opacity about robustness on macroeconomic performance depend on the sign of its impacts on the penalty as summarized in Proposition 2. Under the condition, *i.e.* $\overline{\theta} > \max\{1/\delta^2, \lambda/\delta^2\}$, guaranteeing $d\tau / d\sigma_\theta^2 < 0$, the effect of opacity on output and inflation variability is positive. As we have discussed before, under the optimal solution of τ (25), the condition (7), i.e. $1+\tau > 0$, is equivalent to assume that $\lambda - \theta\delta^2 < 0$ or $\theta > \lambda/\delta^2$. The results summarized in the above proposition are closely linked to the negative effect of opacity on the penalty assigned by the government. Given the average preference for robust control, a decreased penalty leads to an amplified reaction of inflation rate to inflationary shocks as its direct effect on inflation rate dominates its indirect effect on inflation rate via model misspecifications. However, if the central bank has weak anti-inflation credentials in its robust control policy (lower values of $\overline{\theta}$, i.e. $\lambda/\delta^2 < \overline{\theta} < 1/\delta^2$), $d\tau / d\sigma_\theta^2$ could eventually be positive for some values of model parameters (see Appendix) as we have mentioned above. Then the effect of opacity on output and inflation variability could eventually be negative. The opacity of the central bank concerning its preferences for robustness, introducing uncertainty for the government, permits to reduce this penalty.

5. Conclusion

In this paper, using a New-Keynesian framework, we examine the interaction between monetary policy uncertainty (lack of transparency) and model uncertainty (robust control). In particular, we link them by assuming that the central bank doesn't know exactly the true model structure of the economy and is not totally transparent about its preferences for model robustness. We have found several implications of these two kinds of uncertainty for the accountability of the central bank vis-à-vis the government.

Firstly, in the absence of transparency issue and robust control, it is not necessary to impose any penalty since we do not introduce inflationary bias in the loss function of the central bank. If the central bank introduces robustness policy with total transparency, any increase in the preferences for robustness leads to a lower penalty or higher award.

Secondly, given high average preference for robustness, low degree of transparency of the central bank about its preferences for robustness reduces the penalty relative to the deviation of inflation rate from its target. In the contrary, if the government considers that the central bank has weak anti-inflation credentials in its robust control policy, low degree of transparency about its preference could eventually incite the government to increase the penalty.

Finally, the effect of monetary policy opacity on inflation and output variability is positive if penalty decreases with the degree of opacity. In the contrary, if the central bank has average low preference for robustness, low degree of transparency about its preference could eventually lead to decreased inflation and output variability.

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Appendix: Demonstration of Proposition 2

The expected social loss function (26) can be linearized using second-order Taylor development as:

$$L^S = \frac{1}{2}\frac{(\lambda\bar{\theta}^2\delta^2T^2+\bar{\theta}^2\lambda^2)E_t(e_t)^2}{[\theta(\lambda+T\delta^2)-T\lambda]^2}-\frac{1}{2}\frac{\lfloor(\delta^2T^2+\lambda)T^2\lambda^3+2(\delta^2T^2+\lambda)T\theta\lambda^2(T\delta^2+\lambda)\rfloor E(e_t)^2}{[\theta(\lambda+T\delta^2)-T\lambda]^4}E_t(\theta-\bar{\theta})^2,$$

$$\text{with } T=(1+\tau). \tag{A.1}$$

The first-order condition of government minimization problem is:

$$\begin{aligned}\frac{\partial L^S}{\partial\tau} &= \frac{1}{2}\frac{2\lambda\bar{\theta}^2\delta^2TE_t(e_t)^2[\bar{\theta}(\lambda+T\delta^2)-T\lambda]-(\lambda\bar{\theta}^2\delta^2T^2+\bar{\theta}^2\lambda^2)E_t(e_t)^2(-\lambda+\bar{\theta}\delta^2)}{[\bar{\theta}(\lambda+T\delta^2)-T\lambda]^3}\\ &+\frac{1}{2}\frac{\begin{pmatrix}-2\delta^2T^3\lambda^3-2(\delta^2T^2+\lambda)T\lambda^3\\ -4\delta^2T^2\bar{\theta}\lambda^2(T\delta^2+\lambda)\\ -2(\delta^2T^2+\lambda)\bar{\theta}\lambda^2(T\delta^2+\lambda)\\ -2(\delta^2T^2+\lambda)T\bar{\theta}\lambda^2\delta^2\end{pmatrix}[\bar{\theta}(\lambda+T\delta^2)-T\lambda]-\begin{pmatrix}-(\delta^2T^2+\lambda)T^2\lambda^3\\ -2(\delta^2T^2+\lambda)T\bar{\theta}\lambda^2(T\delta^2+\lambda)\end{pmatrix}(-\lambda+\bar{\theta}\delta^2)}{[\bar{\theta}(\lambda+T\delta^2)-T\lambda]^5}E(e_t)^2E_t(\theta-\bar{\theta})^2=0.\end{aligned}$$

$$\tag{A.2}$$

According to equation (11'), we have $\theta > 1+\tau = T$. That implies $-T\lambda+\theta(\lambda+T\delta^2)>0$. Since $-T\lambda+\theta(\lambda+T\delta^2)\neq 0$ and $E(e_t)^2>0$, $E_t(\theta-\bar{\theta})^2=\sigma_\theta^2$, we can rewrite equivalently condition (A.2) as:

$$\begin{aligned}\Gamma &= \begin{Bmatrix}2\lambda\bar{\theta}^2\delta^2T[\bar{\theta}(\lambda+T\delta^2)-T\lambda]\\ -(\lambda\bar{\theta}^2\delta^2T^2+\bar{\theta}^2\lambda^2)(-\lambda+\bar{\theta}\delta^2)\end{Bmatrix}[\bar{\theta}(\lambda+T\delta^2)-T\lambda]^2\\ &+\left\{\begin{pmatrix}-2\delta^2T^3\lambda^3\\ -2(\delta^2T^2+\lambda)T\lambda^3\\ -4\delta^2T^2\bar{\theta}\lambda^2(T\delta^2+\lambda)\\ -2(\delta^2T^2+\lambda)\bar{\theta}\lambda^2(T\delta^2+\lambda)\\ -2(\delta^2T^2+\lambda)T\bar{\theta}\lambda^2\delta^2\end{pmatrix}[\bar{\theta}(\lambda+T\delta^2)-T\lambda]-\begin{pmatrix}-(\delta^2T^2+\lambda)T^2\lambda^3\\ -2(\delta^2T^2+\lambda)T\bar{\theta}\lambda^2(T\delta^2+\lambda)\end{pmatrix}(-\lambda+\bar{\theta}\delta^2)\right\}\sigma_\theta^2=0.\end{aligned}$$

$$\tag{A.3}$$

According to the Theorem of implicit function, totally differentiating (A.3) leads to

$$\frac{\partial\Gamma}{\partial\tau}d\tau+\frac{\partial\Gamma}{\partial E_t(\theta-\bar{\theta})^2}d\sigma_\theta^2=0. \tag{A.4}$$

where

$$\frac{\partial\Gamma}{\partial\sigma_\theta^2}=\begin{bmatrix}-2\delta^2T^3\lambda^3\\-2(\delta^2T^2+\lambda)T\lambda^3\\-4\delta^2T^2\overline{\theta}\lambda^2(T\delta^2+\lambda)\\-2(\delta^2T^2+\lambda)\overline{\theta}\lambda^2(T\delta^2+\lambda)\\-2(\delta^2T^2+\lambda)T\overline{\theta}\lambda^2\delta^2\end{bmatrix}[\overline{\theta}(\lambda+T\delta^2)-T\lambda]-\begin{bmatrix}-(\delta^2T^2+\lambda)T^2\lambda^3\\-2(\delta^2T^2+\lambda)T\overline{\theta}\lambda^2(T\delta^2+\lambda)\end{bmatrix}(-\lambda+\overline{\theta}\delta^2)$$

$$=\lambda^5\overline{\theta}^2+\begin{pmatrix}4\lambda^5\overline{\theta}\\-\lambda^4\overline{\theta}^2\delta^2\end{pmatrix}T+\begin{pmatrix}3\lambda^4\overline{\theta}^2\delta^2\\+\lambda^4\overline{\theta}\delta^2\\+\lambda^5\\-2\lambda^3\overline{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\overline{\theta}^2\delta^4\\+3\lambda^4\overline{\theta}\delta^2\end{pmatrix}T^3. \tag{A.5}$$

$$\frac{\partial\Gamma}{\partial\tau}=\lambda^2\overline{\theta}^3\delta^2[\overline{\theta}(\lambda+T\delta^2)-T\lambda]^2+2[T\lambda^2\overline{\theta}^3\delta^2-(-\lambda+\overline{\theta}\delta^2)\lambda^2\overline{\theta}^2][\overline{\theta}(\lambda+T\delta^2)-T\lambda](-\lambda+\overline{\theta}\delta^2)$$
$$+\left[\begin{pmatrix}4\lambda^5\overline{\theta}\\-\lambda^4\overline{\theta}^2\delta^2\end{pmatrix}+\begin{pmatrix}6\lambda^4\overline{\theta}^2\delta^2\\+2\lambda^4\overline{\theta}\delta^2\\+2\lambda^5\\-4\lambda^3\overline{\theta}^2\delta^4\end{pmatrix}T+\begin{pmatrix}9\lambda^3\overline{\theta}^2\delta^4\\+9\lambda^4\overline{\theta}\delta^2\end{pmatrix}T^2\right]\sigma_\theta^2. \tag{A.6}$$

The condition (A.4) and equations (A.5) and (A.6) allow us to find the relationship between penalty parameter (τ) and degree of opacity (σ_θ^2).

$$\frac{\partial\tau}{\partial\sigma_\theta^2}=-\frac{\frac{\partial\Gamma}{\partial\sigma_\theta^2}}{\frac{\partial\Gamma}{\partial\tau}}=-\frac{\lambda^5\overline{\theta}^2+\begin{pmatrix}4\lambda^5\overline{\theta}\\-\lambda^4\overline{\theta}^2\delta^2\end{pmatrix}T+\begin{pmatrix}3\lambda^4\overline{\theta}^2\delta^2+\lambda^4\overline{\theta}\delta^2\\+\lambda^5-2\lambda^3\overline{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\overline{\theta}^2\delta^4\\+3\lambda^4\overline{\theta}\delta^2\end{pmatrix}T^3}{\begin{array}{c}\lambda^2\overline{\theta}^3\delta^2[\overline{\theta}(\lambda+T\delta^2)-T\lambda]^2+2[T\lambda^2\overline{\theta}^3\delta^2-(-\lambda+\overline{\theta}\delta^2)\lambda^2\overline{\theta}^2][\overline{\theta}(\lambda+T\delta^2)-T\lambda](-\lambda+\overline{\theta}\delta^2)\\+\left[\begin{pmatrix}4\lambda^5\overline{\theta}\\-\lambda^4\overline{\theta}^2\delta^2\end{pmatrix}+\begin{pmatrix}6\lambda^4\overline{\theta}^2\delta^2+2\lambda^4\overline{\theta}\delta^2\\+2\lambda^5-4\lambda^3\overline{\theta}^2\delta^4\end{pmatrix}T+\begin{pmatrix}9\lambda^3\overline{\theta}^2\delta^4\\+9\lambda^4\overline{\theta}\delta^2\end{pmatrix}T^2\right]\sigma_\theta^2\end{array}} \tag{A.7}$$

The following solution is obtained at the equilibrium where $\sigma_\theta^2=0$. Since the optimal solution for the penalty is $\tau=-\frac{\lambda}{\overline{\theta}\delta^2}$ at equilibrium, so that $T=1+\tau=1-\frac{\lambda}{\overline{\theta}\delta^2}=\frac{\overline{\theta}\delta^2-\lambda}{\overline{\theta}\delta^2}$

and consequently $(-\lambda+\bar{\theta}\delta^2)=\bar{\theta}\delta^2 T$. Using the condition $\sigma_\theta^2=0$, (A.7) can be simplified to

$$\frac{\partial\tau}{\partial\sigma_\theta^2}=-\frac{\lambda^5\bar{\theta}^2+\begin{pmatrix}4\lambda^5\bar{\theta}\\-\lambda^4\bar{\theta}^2\delta^2\end{pmatrix}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^3}{\lambda^2\bar{\theta}^3\delta^2[\bar{\theta}(\lambda+T\delta^2)-T\lambda]^2+2[T\lambda^2\bar{\theta}^3\delta^2-(-\lambda+\bar{\theta}\delta^2)\lambda^2\bar{\theta}^2][\bar{\theta}(\lambda+T\delta^2)-T\lambda](-\lambda+\bar{\theta}\delta^2)}. \quad \text{(A.8)}$$

Substituting $(-\lambda+\bar{\theta}\delta^2)$ by $\bar{\theta}\delta^2 T$ into the denominator of (A.8) leads to,

$$\frac{\partial\tau}{\partial\sigma_\theta^2}=-\frac{\lambda^5\bar{\theta}^2+\begin{pmatrix}4\lambda^5\bar{\theta}\\-\lambda^4\bar{\theta}^2\delta^2\end{pmatrix}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^3}{\lambda^2\bar{\theta}^3\delta^2[\bar{\theta}(\lambda+T\delta^2)-T\lambda]^2}. \quad \text{(A.9)}$$

Since the denominator is positive, the sign of $\frac{\partial\tau}{\partial\sigma_\theta^2}$ will be the opposite sign of the numerator.

Using $(-\lambda+\bar{\theta}\delta^2)=\bar{\theta}\delta^2 T$, the numerator can be developed successively as follows:

$$\lambda^5\bar{\theta}^2+\begin{pmatrix}4\lambda^5\bar{\theta}\\-\lambda^4\bar{\theta}^2\delta^2\end{pmatrix}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^3$$

$$=\lambda^5\bar{\theta}^2+\begin{pmatrix}\lambda^5\bar{\theta}+3\lambda^5\bar{\theta}\\-\lambda^4\bar{\theta}^2\delta^2\end{pmatrix}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^3$$

$$=\lambda^5\bar{\theta}^2+\begin{pmatrix}+3\lambda^5\bar{\theta}\\(\lambda-\bar{\theta}\delta^2)\bar{\theta}\lambda^4\end{pmatrix}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^3$$

$$=\lambda^5\bar{\theta}^2+3\lambda^5\bar{\theta}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4-\bar{\theta}\lambda^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^3$$

$$=\lambda^5\bar{\theta}^2+3\lambda^5\bar{\theta}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4-\bar{\theta}\lambda^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}^2\delta^4\\+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}\frac{(-\lambda+\bar{\theta}\delta^2)}{\bar{\theta}\delta^2}T^2$$

$$=\lambda^5\bar{\theta}^2+3\lambda^5\bar{\theta}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4-\bar{\theta}\lambda^4\end{pmatrix}T^2+\begin{pmatrix}3\lambda^3\bar{\theta}\delta^2(-\lambda+\bar{\theta}\delta^2)\\+3\lambda^4(-\lambda+\bar{\theta}\delta^2)\end{pmatrix}T^2$$

$$=\lambda^5\bar{\theta}^2+3\lambda^5\bar{\theta}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2\\+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4-\bar{\theta}\lambda^4\\3\lambda^3\bar{\theta}\delta^2(-\lambda+\bar{\theta}\delta^2)\\+3\lambda^4(-\lambda+\bar{\theta}\delta^2)\end{pmatrix}T^2$$

$$=\lambda^5\bar{\theta}^2+3\lambda^5\bar{\theta}T+\begin{pmatrix}3\lambda^4\bar{\theta}^2\delta^2+\lambda^4\bar{\theta}\delta^2+\lambda^5-2\lambda^3\bar{\theta}^2\delta^4-\bar{\theta}\lambda^4\\-3\lambda^4\bar{\theta}\delta^2+3\lambda^3\bar{\theta}^2\delta^4-3\lambda^5+3\lambda^4\bar{\theta}\delta^2\end{pmatrix}T^2$$

$$= \lambda^5\bar{\theta}^2 + 3\lambda^5\bar{\theta}T + \begin{pmatrix} 3\lambda^4\bar{\theta}^2\delta^2 + \lambda^4\bar{\theta}\delta^2 \\ -\bar{\theta}\lambda^4 + \lambda^3\bar{\theta}^2\delta^4 - 2\lambda^5 \end{pmatrix} T^2$$

$$= \lambda^5\bar{\theta}^2 + 3\lambda^5\bar{\theta}T + \begin{pmatrix} 3\lambda^4\bar{\theta}^2\delta^2 - \lambda^5 - \bar{\theta}\lambda^4 \\ +(\lambda^4\bar{\theta}\delta^2 - \lambda^5) + \lambda^3\bar{\theta}^2\delta^4 \end{pmatrix} T^2$$

$$= \lambda^5(\bar{\theta}^2 - T^2) + 3\lambda^5\bar{\theta}T + \begin{pmatrix} 3\lambda^4\bar{\theta}^2\delta^2 - \bar{\theta}\lambda^4 \\ +(\lambda^4\bar{\theta}\delta^2 - \lambda^5) + \lambda^3\bar{\theta}^2\delta^4 \end{pmatrix} T^2$$

$$= \lambda^5(\bar{\theta}^2 - T^2) + 3\lambda^5\bar{\theta}T + \begin{pmatrix} 2\lambda^4\bar{\theta}^2\delta^2 + \lambda^4\bar{\theta}(\bar{\theta}\delta^2 - 1) - \bar{\theta}\lambda^4 + \lambda^4 T \\ +\lambda^3\bar{\theta}^2\delta^4 \end{pmatrix} T^2 > 0 \text{ if } \bar{\theta} > \frac{1}{\delta^2}. \qquad \text{(A.10)}$$

The term $(\bar{\theta}^2 - T^2)$ is always positive since $\bar{\theta} > (1+\tau) = T$ according to condition (11'). The condition $\bar{\theta} > \frac{1}{\delta^2}$ is a sufficient condition to ensure that the numerator is positive. Since the condition $(1+\tau) = T > 0$ implies $\lambda - \theta\delta^2 < 0$ in the case of transparency. So, beginning form an equilibrium without opacity, we have in average $\lambda - \bar{\theta}\delta^2 < 0$ or equivalently $\bar{\theta} > \frac{\lambda}{\delta^2}$. So the general condition guaranteeing the numerator to be positive is then,

$$\bar{\theta} > \max\left\{\frac{1}{\delta^2}, \frac{\lambda}{\delta^2}\right\}. \qquad \text{(A.11)}$$

Consequently, under the condition (A.11):

$$\frac{\partial\tau}{\partial\sigma_\theta^2} < 0. \qquad \text{(A.12)}$$

The expression $\lambda^5\bar{\theta}^2 + \begin{pmatrix} 4\lambda^5\bar{\theta} \\ -\lambda^4\bar{\theta}^2\delta^2 \end{pmatrix} T + \begin{pmatrix} 3\lambda^4\bar{\theta}^2\delta^2 + \lambda^4\bar{\theta}\delta^2 \\ +\lambda^5 - 2\lambda^3\bar{\theta}^2\delta^4 \end{pmatrix} T^2 + \begin{pmatrix} 3\lambda^3\bar{\theta}^2\delta^4 \\ +3\lambda^4\bar{\theta}\delta^2 \end{pmatrix} T^3$ could be negative for small values of δ and λ. Notably, when $\delta \to 0$ and $\lambda \to 0$, the above expression is reduced to $(-2\bar{\theta} + T)T^2 < 0$ since $\bar{\theta} > T$. In this case, we will have

$$\frac{\partial\tau}{\partial\sigma_\theta^2} > 0. \qquad \text{(A.13)}$$