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« **A strategic model of complex
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A strategic model of complex networks formation[*]

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Abstract. This paper introduces a spatialized variation of the Connections model of Jackson and Wolinski (1996). Agents benefit from their direct and indirect connections in a communication network. They are arranged on a circle and bear costs for maintaining direct connections which are linearly increasing with geographic distance. In a dynamic setting, this model is shown to generate networks that exhibit the *small world* properties shared by many real social and economic networks.

Key words : Strategic Network Formation, Pairwise Stability, Small World, Monte Carlo Simulations.

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1. Introduction

There is an increasing consensus in the economic literature to recognize that network structures significantly influence the outcomes of many social and economics activities. In the meantime, networks are often strategically shaped by the participating agents in a game theoretic fashion (Jackson and Wolinski, 1996 ; Bala and Goyal, 2000). The growing interest for network formation appears in the recent surge of models encompassing various contexts such as job-contact networks (Calvó-Armengol, 2003), oligopolies and R&D collaborations (Goyal and Moraga, 2001 ; Goyal and Joshi, 2003), buyer-seller networks (Kranton and Minehart, 2000), innovation networks (Carayol and Roux, 2003), etc[1].

This literature has not yet dedicated much attention to the full characterization of the endogenous networks and most importantly to the conditions that may lead to the emergence of more complex and realistic networks. Indeed, the networks observed and discussed in most models are much too regular as compared to real social and economic networks which have been extensively explored and characterized in Sociology and Physics. It has been shown that the average distance between any two agents[2] in most social networks is remarkably short while agents remain highly clustered[3,4]. Networks that share these two structural properties are said to be *small worlds* à la Watts and Strogatz (1998).

In this paper, we show that the strategic approach to network formation can lead to the emergence of networks that share such structural properties. Our larger aim is to understand which economic conditions of network formation are consistent with the emergence of such networks[5]. For these purposes, we introduce a strategic model of network formation built on a simple variation of the connections model of Jackson and Wolinski (1996). Myopic self-interested agents benefit from other agents with whom they are directly or indirectly connected in a communication network. The longer the distance in the network the weaker the spillover. Moreover, we consider that agents bear costs for direct connections which are linearly increasing with geographic distance in a similar fashion than Johnson and Gilles (2000) who refer to a linear world while our agents are arranged on a circle. With this simple specification, we obtain in a dynamic setting (close to the one developed in Jackson and Watts, 2002) and for a wide range of parameters values, endogenous pairwise stable networks which exhibit both high local clustering and some distant connections. We demonstrate that these networks have the small worlds characteristics.

The paper is organized as follows. Section 2 presents basic formal definitions. Section 3 is

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[1] For a survey of models of network formation, refer to Jackson (2004).

[2] The distance between two agents is computed as the minimal number of inter-individual connections between them.

[3] Such properties have for instance been evidenced on web sites links or coauthorship of scientific papers (e.g. Albert and Barabási, 1999 ; Newman, 2001).

[4] Most real networks also share some other structural properties such as skew degree distribution or short diameter (Albert et Barabási, 1999, 2002).

[5] This aim is shared by several previous papers : Carayol and Roux (2003), Galeotti, Goyal, and Kamphorst (2004) and Jackson and Rogers (2005).

devoted to the static features of the model and to the presentation of analytical results on pairwise stability and efficiency. In Section 4 we introduce the dynamic stochastic process and the results obtained in this framework. The proofs are deferred to Section 5.

2. Basic notions

2.1. Graphs

Consider a finite set of n agents, $N = \{1, 2, ..., n\}$ with $n \geq 3$, and let i and j be two members of this set. Agents are represented by the nodes of a non-directed graph the edges of which represent the links between them. The graph constitutes the relational network between the agents. A link between two distinct agents i and $j \in N$ is denoted ij. A graph g is a list of non ordered pairs of connected and distinct agents. Formally, $\{ij\} \in g$ means that ij exists in g. We define the complete graph $g^N = \{ij \mid i, j \in N\}$ as the set of all subsets of N of size 2, where all players are connected with all the others. Let $g \subseteq g^N$ be an arbitrary collection of links on N. We define $G = \left\{ g \subseteq g^N \right\}$ as the finite set of all possible graphs between the n agents.

Let $g' = g + ij = g \cup \{ij\}$ and $g'' = g - ij = g \backslash \{ij\}$ be respectively the graph obtained by adding ij and the one obtained by deleting ij from the existing graph g. The graphs g and g' are said to be *adjacent* as well as the graphs g and g''. For any g, we define $N(g) = \{i \mid \exists j : ij \in g\}$, the set of agents who have at least one link in the network g. We also define $N_i(g)$ as the set of neighbors agent i has, that is : $N_i(g) = \{j \mid ij \in g\}$. The cardinal of that set $\eta_i(g) = \#N_i(g)$ is called the *degree* of node i. The total number of links in the graph g is $\eta(g) = \#g$.

A *path* in a non empty graph $g \in G$ connecting i to j, is a sequence of edges between distinct agents such that $\{i_1 i_2, i_2 i_3, ..., i_{k-1} i_k\} \subset g$ where $i_1 = i$, $i_k = j$. The length of a path is the number of edges it contains. Let $i \longleftrightarrow_g j$ be the set of paths connecting i and j on graph g. The set of *shortest paths* between i and j on g noted $i \overset{\frown}{\longleftrightarrow}_g j$ is such that if $\forall k \in i \overset{\frown}{\longleftrightarrow}_g j$; implies that $k \in i \longleftrightarrow j$ and $\#k = \min_{h \in i \longleftrightarrow_g j} \#h$. We define the *geodesic distance* between two agents i and j as the number of links of the shortest path between them : $d(i,j) = d_g(ij) = \#k \in i \overset{\frown}{\longleftrightarrow}_g j$. When there is no path between i and j then their geodesic distance is conventionally infinite : $d(i,j) = \infty$.

An external metrics is also introduced, representing for example the geographic position of agents (Johnson and Gilles, 2000). Such external metrics defines a new distance operator denoted $d'(i,j)$. In our model, we consider that agents are located on a circle (or a ring). Without loss of generality, agents are ordered according to their index, such that i is the immediate geographic neighbor of agent $i + 1$ and agent $i - 1$ but agent 1 and agent n who are neighbors. As a consequence, the geographic distance between any two agents is given by $d'(i,j) = \min\{|i - j|; n - |i - j|\}$.

Several typical graphs can be described. Let $i \neq j \in N$. First of all, the *empty graph*, denoted $g^\emptyset$, is such that it does not contain any links. The *ring* g° is a regular network of order $k = 1$, in which all agents are connected and only connected with their two closest geographic

neighbors. The *chain* g^c, is defined as a connected subset of the ring, that is $g^c \subset g^\circ$ and $\forall i,j \in N(g^c), i \longleftrightarrow_{g^c} j \neq \emptyset$. If $\#g^c = \#g^\circ$ then $g^c = g^\circ$. Let g^{mc} be a maximally connected chain such that $\#g^{mc} = \#g^\circ - 1$. If g^c such that $\#g^c \leq g^{mc}$, there is always one and only one path between two connected agents i and j (the set $i \longleftrightarrow_{g^c} j$ is a singleton). The covering chain of the graph g is a chain g^{cc} such that for all $i,j \in N : i\widetilde{\longleftrightarrow}_{g^\circ}j \subset g^{cc}$ iff $ij \in g$. The *double ring* denoted $g^{2\circ}$ is a regular network of order $k = 2$ such that all agents are only connected with their four closest neighbors. The *triple ring* denoted $g^{3\circ}$ is a regular network of order $k = 3$: All agents are only connected with their six closest neighbors. Finally, a (complete) *star*, denoted $g^\star$, is such that $\#g^\star = n - 1$ and there exists an agent $i \in N$ such that if $jk \in g^\star$, then either $j = i$ or $k = i$. Agent i is called the center of the star. It should be noted that there are n possible stars, since every node can be the center.

2.2. Networks stability and efficiency

In the two-sided network formation game of Jackson and Wolinski (1996), pairs of agents meet and decide to form, maintain or break links. The formation of a link requires the consent of both agents but not its deletion which can emanate from one of them unilaterally. Moreover, agents are myopic which means that they take decisions on the basis of the immediate impacts on their current payoffs. Jackson and Wolinski (1996) introduce the notion of *pairwise stability* which departs from the Nash equilibrium since the process of network formation is both cooperative and non cooperative. A network is said to be pairwise stable if no incentives exist for any two agents to form a link or for any agent to break one of his links.

Formally, let $\pi_i : \{g \mid g \subseteq g^N\} \rightarrow \mathbb{R}$, the payoffs received by agent i from his position in the communication network g, with $\pi_i(\emptyset) = 0$. The definition of the pairwise stability notion follows.

Definition 1. *A network $g \subseteq g^N$ is pairwise stable if :*
    i) for all $ij \in g$, $\pi_i(g) \geq \pi_i(g - ij)$ and $\pi_j(g) \geq \pi_j(g - ij)$, and
    ii) for all $ij \notin g$, if $\pi_i(g + ij) > \pi_i(g)$ then $\pi_j(g + ij) < \pi_j(g)$.

As regard network efficiency, we use the 'strong' notion introduced by Jackson and Wolinski (1996). It relies on the computation of the *total value* of a graph g given by : $\pi(g) = \sum_{i \in N} \pi_i(g)$. A network is then said to be efficient if it maximizes this sum.

Definition 2. *A network $g \subseteq g^N$ is efficient if it maximizes the value function $\pi(g)$ on the set of all possible graphs $\{g \mid g \subseteq g^N\}$ i.e. $\pi(g) \geq \pi(g')$ for all $g' \subseteq g^N$.*

It should also be noticed that several networks can lead to the same maximal total value. For example, if we consider strictly homogenous agents, any isomorphic graph of an efficient network is also efficient.

3. The spatialized connections model

3.1. Model

The model we introduce is a variation of the so called *Connections model* introduced by Jackson and Wolinski (1996). In this model, links represent individuals' relationships (for example,

friendships). In such a context, agents benefit from their direct and indirect connections, through the relational network of their partners. But, the communication is not perfect : the positive externality deteriorates with the relational distance of the connection. Formally, there is a decay parameter which represents the quality of links used for information flows. Moreover, agents bear costs for maintaining direct connections. As a consequence, agents try to maximize the value generated from direct and indirect connections, avoiding superfluous connections. This simple specification of the individual payoffs allows the authors to obtain systematic analytical results on graphs' efficiency and partial results on networks' stability. Nevertheless, the efficient and stable network structures they discuss are very simple and somehow unrealistic (complete network, empty network, complete star).

In order to obtain emerging networks which tend to correspond to the empirically observed social or economic networks, we let payoffs depend on the geographic positions of agents on the circle. Formally, the net profit received by any agent i is given by the following expression :

$$\pi_i\left(g_t\right) = \sum_{j \in N \setminus i} \delta^{d(i,j)} - c \sum_{j : ij \in g_t} d'\left(i, j\right) \tag{1}$$

where $d(i,j)$ is the geodesic distance between i and j. $\delta \in \left]0; 1\right[$ is the decay parameter and $\delta^{d(i,j)}$ gives the payoffs resulting from the (direct or indirect) connection between i and j. It is a decreasing function of the geodesic distance since δ is less than unity. Notice that if there is no path between i and j, $d(i,j) = \infty$ and thus $\delta^{d(i,j)} = 0$. The second part of the right-hand side of the equation describes the costs of direct links. $d'(i,j)$ gives the geographic distance between any two agents on the external metrics (a circle) we consider. Thus the cost of a link increases linearly with the geographic distance separating neighboring agents. Finally, the positive parameter c is the unit costs of links formation. We thus obtain the same payoffs specification as in Johnson and Gilles (2000), but with a circle as an external metrics instead of a line.

3.2. Results

The analytical results obtained on networks efficiency and stability in the simple model described in equation (1) are summed up in the following two propositions.

Proposition 1. *Efficiency.*
i) The empty network $g^{\emptyset}$ is the only efficient network when $c > \delta + \frac{(n-2)}{4}\delta^2$.
ii) If $\delta^2 - \delta^{n-1} < c$, the value of any acyclic graph g is less than its associated covering chain g^c.
iii) Consider three chains $g^c, g^{c\prime}$ and $g^{c\prime\prime}$, if $g^{c\prime} \cap g^{c\prime\prime} = \emptyset$ and $\#g^c \geq \#g^{c\prime} + \#g^{c\prime\prime}$, then $\pi(g^c) > \pi(g^{c\prime}) + \pi(g^{c\prime\prime})$. Moreover a maximal chain g^{mc} (a chain such that $\#g^{mc} = n-1$) is the most efficient positive value chain.

Proposition 2. *Stability.*
i) When $\delta > c$, the empty graph is never pairwise stable. When $\delta < c$, the empty graph $g^{\emptyset}$ is the unique acyclic pairwise stable graph and no network containing a peripheral agent (has only one connection) is pairwise stable. The empty graph is pairwise stable when $c = \delta$.

ii) The star $g^\star$ is not stable either if $c \geq 2/n$ and n is even, or if $c \geq 2/(n-1)$ and n is odd.

iii) Assume n is even. The ring g° is pairwise stable if a) $c > \left(1 - \delta^{\frac{n}{2}}\right) \sum_{i=1}^{i=\frac{n}{2}-1} \delta^i$, b) and for any even x such that $2 \leq x \leq n/2$ then $cx + \delta^{n/2} + \delta - 2\delta^{\frac{n-x}{2}} - 2\left(1 - \delta^{\frac{x}{2}}\right)\sum_{i=1}^{i=\frac{n-x}{2}-1} \delta^i > 0$, and c) for any uneven y such that $2 < y < n/2$, we have $cy + \delta^{n/2} + \delta - \delta^{\frac{y+1}{2}} - \delta^{\frac{n-y+1}{2}} - 2\left(1 - \delta^{\frac{y-1}{2}}\right)\sum_{i=1}^{i=\frac{n-y-1}{2}} \delta^i > 0$.

The pairwise stability approach leads us to results limited to the far ends of the spectrum of parameters values.

4. Dynamic networks formation

4.1. The perturbed stochastic process of network formation

The dynamic process can be described as follows. At each time period t, two agents i and $j \in N$ are randomly selected. If the two agents are directly connected, they can jointly decide to maintain their relation or unilaterally decide to sever the link between them. If they are not connected, they can jointly decide to form a link or renounce unilaterally. Formally, those two situations are the following :

i) if $ij \in g_t$, the link is maintained if $\pi_i(g_t) \geq \pi_i(g_t - ij)$ and $\pi_j(g_t) \geq \pi_j(g_t - ij)$. Otherwise, the link is deleted.

ii) if $ij \notin g_t$, a new link is created if $\pi_i(g_t + ij) \geq \pi_i(g_t)$ and $\pi_j(g_t + ij) \geq \pi_j(g_t)$, with a strict inequality for one of them.

The stochastic process introduced here can be conceptualized as a Markov process in which the state of the system at time t (with $t = 0, 1, 2, ...$) is given by the graph structure $g_t \in G$. The evolution of the system $\{g_t, t \geq 0\}$ can be described as a discrete-time stochastic process with (finite) state space G.

Following Jackson and Watts (2002), we then introduce small random perturbations ε which invert agents' right decisions in creating, maintaining or deleting links. These perturbations may be understood as mistakes or as mutations. For small but non null values of ε, it can be shown that the discrete-time Markov chain becomes irreductible and aperiodic and has thus a unique corresponding stationary distribution (μ^ε). Such perturbed stochastic processes are said to be ergodic. Intuitively ergodicity implies that it is possible to transit directly or indirectly between any chosen pair of states in a potentially very long period of time. It allows the long run state of the system to become independent of its initial conditions.

Usually, the modeler let $\varepsilon \to 0$ (once the long run is reached) in order to restrict the number of states selected in the long run. State z is said to be a stochastically stable state (Young, 1993) if it has a non null probability of occurrence in the stationary distribution : $\lim_{\varepsilon \to 0} \mu^\varepsilon(z) > 0$. In the network formation context, Jackson and Watts (2002) show that stochastically stable networks are either pairwise stable or part of a closed cycle (of the unperturbed process). In practice the precise computation of the stochastically stable networks requires the identification

of all the recurrent classes of the unperturbed process (Young, 1998) which, in the network context, are likely to be extremely numerous. To make that point clear, think that there is a recurrent class for each pairwise stable network and that in models such as the connections model or the spatialized connections model presented in Section 3, possibly thousands of networks are pairwise stable. Therefore we propose a slightly different regime for the perturbation process. We let the error term decrease in time according to the following simple rule :

$$\varepsilon^t = \left\{ \begin{array}{ll} \varepsilon & \text{if } t < T \\ 1/t & \text{otherwise} \end{array} \right. , \tag{2}$$

with $\varepsilon > 0$ the initial noise and T some finite time. This rule ensures that the noise does affect the dynamics while it decreases down to zero when time increases with $\lim_{t \to \infty} \varepsilon^t = 0$. It also preserves the ergodicity property of the system. Notice that this property is interesting since it renders numerical experiments more tractable in order to examine with good confidence the long run behavior of the system (Vega-Redondo, 2005). Therefore we use Monte Carlo experiments to approximate the unique limiting stationary distribution (of networks) of the perturbed dynamic process presented above. The unit costs is fixed as $c = 2/n$ as a normalization device that might account for an inverse proportionality between the costs and the size of the network which is set to $n = 20$ agents. All experiments are performed with randomly drawn values of δ over its value space $]0, 1[$. The experiments are stopped at $t = 10,000$, date after which the process is proven to have almost surely stabilized on a given pairwise stable state[6]. If not, the process still goes until it reaches one.

4.2 Network indicators

Several indicators are used in order to provide a synthetic characterization of the structural properties of networks. We first compute the density of the network as follows :

$$\hat{\eta}(g) = \eta(g)/n. \tag{3}$$

This indicator hence corresponds to the average degree in the network. We also compute the average distance (or average path length) of (directly or indirectly) connected agents. It is given by :

$$d(g) = \frac{\sum_i \sum_{j \neq i} d(i,j) \times 1\{i \leftrightarrow_g j \neq \emptyset\}}{\#\{ij \,|i \neq j, i \leftrightarrow_g j \neq \emptyset\}}, \tag{4}$$

with $\#\{\cdot\}$ denoting the cardinal of the set defined into brackets and $1\{\cdot\}$, the indicator function that is equal to unity if the condition is verified and zero otherwise. The average clustering (or average cliquishness as it is often referred to in Physics) indicates the extent to which neighborhoods of connected agents overlap. It is given by :

$$c(g) = \frac{1}{n} \sum_{i \in N} \sum_{jl} \frac{1\{j, l \in N_i(g); j \neq l; j \in N_l(g)\}}{\#\{lj \,|j \neq l; j, l \in N_i(g)\}}. \tag{5}$$

It is the frequency with which agents' neighbors are also neighbors together.

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[6] See Carayol and Roux (2003) for more precise time series analysis of the network formation process.

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The last two indicators presented above are affected by the density of the network (the first indicator) that is likely to vary with δ. Therefore, these indicators are somehow biased and we must find an efficient control for density. We propose to build, for each generated network, *control random graphs* which have exactly the same number of agents and connections (thus the same density). Such networks are simply built by allocating the given number of edges to randomly chosen pairs of agents (Erdös and Rény, 1960). For each given number of edges and agents, the four above mentioned indicators are then numerically computed for $1,000$ of such randomly drawn networks. The average of the observations is used as the control value. For instance, instead of looking at $c(g)$, where g is a given stable network, we will look at the *ratio* $c(g)/c(g^{rd})$, where $c(g^{rd})$ is the average clustering of neighborhood sizes over a set of $1,000$ random networks that have exactly the same density as g ($\#g^{rd} = \#g$). Each of the indicators is corrected using the corresponding ratio. This method allows us to analyze the structural properties of networks for the different values of δ while controlling for their density.

4.3 Results : When do small worlds emerge ?

Let first examine how the network density $\hat{\eta}(g)$ is affected by δ. As shown in Figure 1, the density is null when $\delta \leq c = 0.1$: the process converges to the empty graph which is pairwise stable for such values of the parameters (see Proposition 2). However when δ becomes close to 0.1, some non empty networks begin to be selected. These networks are somehow in a phase transition between the empty graph and networks for which agents have in average two connections (density equal to unity). This configuration appears to be mostly selected for $0.1 \leq \delta \leq 0.2$. The density of the network then increases up to nearly $\hat{\eta}(g) = 3$ for $\delta \approx 0.55$. Then it deceases to slightly below unity when δ reaches its maximal value 1.
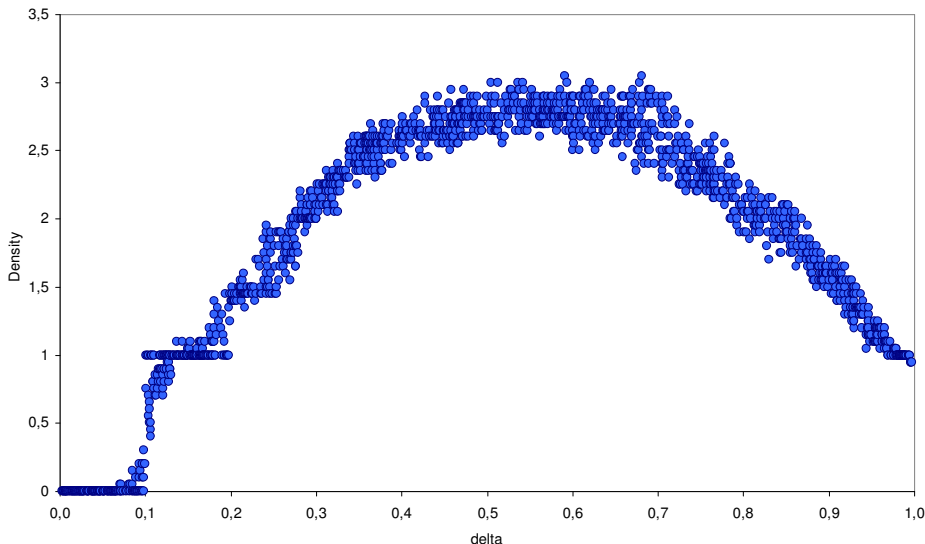

Figure 1. *Network density $\hat{\eta}(g)$ when $\delta \in]0;1[$.*

Behind the dots of Figure 1, one can find networks with many different structures. This diversity calls for a rigorous statistical analysis of the structural properties of emergent networks.

Before performing such a task, we provide in Figure 2 some intuitions on the typical networks shapes obtained for several values of the parameter δ. When $0.09 \lesssim \delta \leq 0.2$, it is the ring g° which emerges most often. So agents not only have two neighbors in average, but all are connected to their two closest geographic neighbors. Numerical computations from Proposition 2 show that the ring is indeed pairwise stable when $\delta \in [0.0909, 0.2] \cup [0.975, 0.998]$. When δ is equal to 0.3, networks in which all agents are connected to their four closest geographic neighbors are likely to emerge. Such situation corresponds to the double geographic ring $g^{2\circ}$. At the other far end of the spectrum, the networks tend to become maximal chains (g^{mc}). When δ approches unity, direct and indirect connections are likely to provide the same wealth, and thus overlapping connections become redundant. Within these two extremes, when $0.35 \leq \delta \leq 0.7$, we find structurally distinguishable configurations characterized by the conjunction of : i) a prevalence of local connections, ii) the existence of some "short cuts". Such typical structure are known in the literature as small worlds in the sense of Watts and Strogatz (1998).
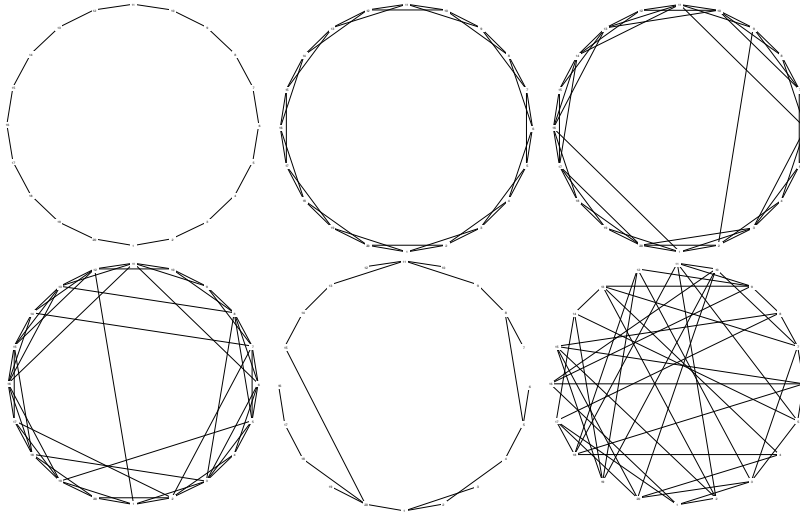

Figure 2. *Limit typical stable networks selected by the stochastic process in the simple distributed innovation model. These networks have been obtained as follows : (top) $\delta = 0.15$, $\delta = 0.3$, $\delta = 0.35$; (bottom) $\delta = 0.7$, $\delta = 0.98$. The last network has been generated with the simple connections model of Jackson and Wolinski (1996) with $\delta = 0.7$ and $c = 0.5$.*

Such small world configuration is usually established if the two following conditions are verified : $c(g)/c(g^{rd}) >> 1$ and $d(g)/d(g^{rd}) \approx 1$. In words, a small world is a network which is both highly clustered and the average distance of which is close to the one of random graphs of the same density. Random networks are taken as points of comparison because they are known to exhibit very low average path length. These ratios are plotted in Figure 3. From there it appears that the average distance of emerging networks becomes close to unity when δ reaches 0.35, and then stays on this value until $\delta \approx 0.9$. The average clustering ratio also decreases quite sharply with $\delta > 0.2$. Nevertheless, the clustering of emergent networks remains significantly higher than their corresponding random networks, at least until $\delta < 0.7$. Therefore, we can conclude that the small world configurations is observed for the whole region characterized by $\delta \in [0.35, 0.7]$.

Unsurprisingly, the relational network both correlates with the geographic metrics (clustering is achieved in local space) and exhibits some distant connections.
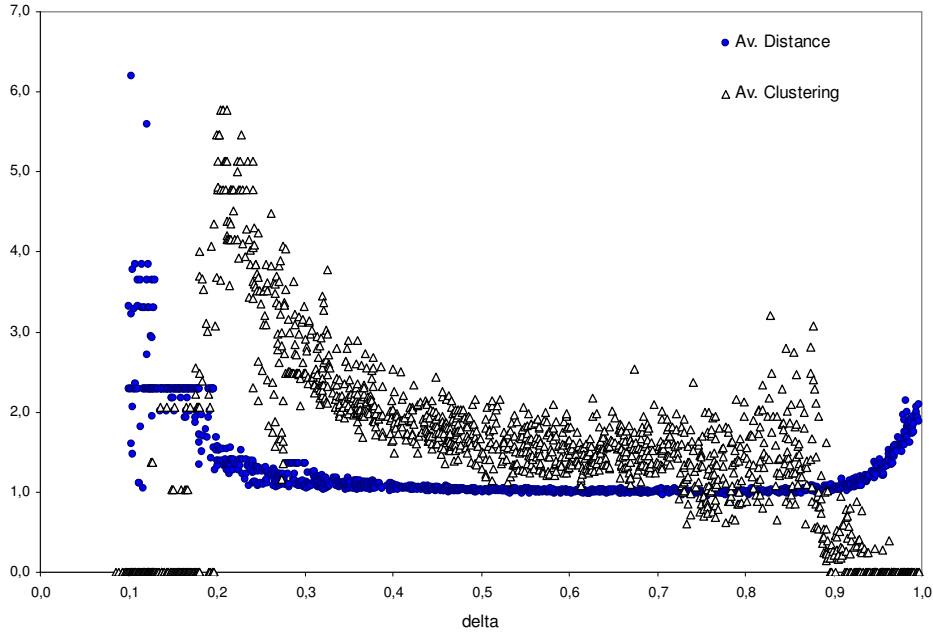

Figure 3. *Average distance and average clustering when $\delta \in \,]0; 1[$.*

5. Proofs

5.1 Proof of Proposition 1

i) The proof uses the following steps. We give an upper bound expression for the connected network value of k links. We show that this expression is at its maximum when $k = n - 1$ links. Next, we show that under the given condition, that expression is negative. Since the value of the empty graph is zero, it is the efficient network under the condition.

An upper bound value of a network of $k > n - 1$ links may be given by :

$$\pi^{\max}(g \,|\#g = k) = 2k\delta + [1/2n(n-1) - k]\,\delta^2 - 2kc$$

This expression considers that all agents that are not directly connected, benefit from each other just as they were at relational distance 2. It also assumes that bonds costs are minimal (as it is connecting immediate geographic neighbors). Now assume that $k = n - 1$. Thus the upper bound network value becomes :

$$\pi^{\max}(g \,|\#g = n - 1) = 2(n - 1)\left(\delta + 1/2n\delta^2 - 2c\right) \tag{A1}$$

To know whether we should consider cases where $k > n - 1$, let us see how the upper expression of the network value behaves when we add a link : The maximal value with k $(k > n - 1)$ links minus the max value with $k + 1$ links is equal to :

$$\pi^{\max}(g \,|\#g = k + 1) - \pi^{\max}(g \,|\#g = k) = 2\delta - \delta^2 - 2c$$

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which is independent of k and strictly negative when $c > \delta$. Thus there is no interest in adding a new link from the beginning that is from $k = n - 1$.

On the other hand if $k < n - 1$, the network is not connected, then no node can benefit from all others. For instance if $k = n - 2$, then the agents are associated to at least two connected components. Since the two components are isolated, the total value of the graph is equal to the sum of the value of the two components. Let us assume that it is possible to connect the two components by adding a bond at distance 1 (as it is assumed in the π^{max} expression). Then the value of this bond adds to the total value more than any of the other ones did previously while it costs at most the same. Thus the value of the graph with $n - 2$ links is negative if the one of the connected graph composed of $n - 1$ links is also negative.

Thus expression (A1) gives the maximal value of the network. It is negative when : $c > \delta + \frac{(n-2)}{4}\delta^2$. This completes the proof.$\square$

Part ii) Any non empty acyclic graph is a tree or a set of disjoint trees (with potentially some isolated agents). A tree of m nodes has always $k = m - 1$ links. A tree of $m + u - 1$ nodes generates more utility than a graph composed of two distinct trees of m and u nodes, that is because with the same number of links, it generates one more direct and several more indirect connections. Thus we can restrict our analysis to connected acyclic graphs which are necessarily trees.

One can get two different types of trees given the following definition.

Definition 3. *A network g is said to exhibit regional overlap if $\exists i \in N(g)$ such that for an arc $jh \in g, i \in N(g^{cc})$, with g^{cc} the covering chain of g.*

a) Let us first consider the trees for which there is no *regional overlap*.

In that situation, any link ij in g generates a cost equal to its covering chain (Definition 3) while it generates less utility since less agents are thus directly or indirectly connected. This applied for all bonds that exhibit no regional overlap. Thus the value of such network is always below the one of its associated covering chain.

b) Consider now connected trees which exhibit some regional overlap. In that situation, each link ij of g such that $d'(i,j)$ generates an extra cost of $2c$ as compared to its covering chain, while it generates at most a gross extra value of $2\delta^2 - 2\delta^{n-1}$. Thus $\pi(g) < \pi(g^c)$ if $c > \delta^2 - \delta^{n-1}$.$\square$

Part iii) The proof of the first part of the Proposition is trivial since g^c costs as much as $g^{c\prime}$ and $g^{c\prime\prime}$, while it brings more utility due to more indirect connections. As regard, each new node added to the chain costs as much as the preceding ones while it always brings more value due to more indirect connections. Thus, if the maximal chain g^{mc} has a positive value, then it is always the most efficient chain.$\square$

5.2 Proof of Proposition 2

Some part of the Proofs need the following definition.

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Definition 4. *A network structure g is said to be payoff-regular, iff $\forall i \in N : \pi_i(g) = \pi(g)/n$.*

Among the typical networks defined above, g°, $g^{2\circ}$ and $g^{3\circ}$ are payoff-regular.

Part i) When $\delta > c$, the proof is trivial : two geographic neighbors always have interest in forming a connection. When $\delta < c$, it is easy to show that the empty network is always stable. Being on the empty net, no agent has any interest in forming a link even with his direct -geographic- neighbors since this connection will cost him always more than the (direct) gross payoff it may bring to him. Moreover, as showed by Jackson and Wolinski (1996), in such a situation, stability implies no *loose end*, that is no agent i is connected to only one other agent j. That is because j will always find interest in severing this connection. Thus, as noticed by Johnson and Gilles (2000), since all acyclic networks but the empty graph always have loose ends (among which the star net), the empty network is the only acyclic pairwise stable network.

Part ii) In such a situation, the center of the star is never interested in maintaining a link with his most distant neighbor. If n is even, he is at distance $n/2$. Thus, this link costs him $cn/2$. If $c \geq 2/n$, then this connection costs to the center star at least $(2/n)(n/2) = 1$, which is more than his gross utility which is simply $\delta < 1$. Same reasoning applies when n is odd, with $c \geq 2/(n-1)$.

Part iii) Since the ring is a structure that is payoff-regular (Definition 4) we can restrict our attention to any one agent to account for its stability.

If one agent i severs a link, he will then be at the far end of a maximal chain $g^{mc} = g^\circ - ij$. One may compute the extra net value of such a decision (for even values of n, without loss of generality) :

$$\pi_i\left(g^\circ\right) - \pi_i\left(g^\circ - ij\right) = -c + \sum_{i=1}^{i=n/2-1} \delta^i - \sum_{i=n/2+1}^{i=n-1} \delta^i,$$

Thus, no agent will have an incentive to sever a link if $c > (1 - \delta^{n/2}) \sum_{i=1}^{i=n/2-1} \delta^i$.

If one agent adds a new link to the ring, the extra costs will varies with the geographical distance between him and his new neighbor say j. Assume n is even and let x be that distance $x = d'(i,j)$ which ranges from 2 to $n/2$. The payoffs are then given by :

$$\pi_i\left(g^\circ + ij\right) = -(2+x)c + 2\sum_{i=1}^{i=\frac{x-1}{2}} \delta^i + 2\sum_{i=1}^{i=\frac{n-x-1}{2}} \delta^i - \delta + \delta^{\frac{x+1}{2}} + \delta^{\frac{n-x+1}{2}}$$

if x is uneven, and by :

$$\pi_i\left(g^\circ + ij\right) = -(2+x)c + 2\sum_{i=1}^{i=\frac{x}{2}} \delta^i + 2\sum_{i=1}^{i=\frac{n-x}{2}} \delta^i - \delta,$$

if x is even.

The net payoffs i gains from the ring are greater to this new situation when :

$$\pi_i\left(g^\circ\right) - \pi_i\left(g^\circ + ij\right) = xc + \delta^{n/2} + \delta - \delta^{\frac{x+1}{2}} - \delta^{\frac{n-x+1}{2}} - 2\left(1 - \delta^{\frac{x-1}{2}}\right)\sum_{i=1}^{i=\frac{n-x-1}{2}}\delta^i,$$

if x is uneven, and when :

$$\pi_i\left(g^\circ\right) - \pi_i\left(g^\circ + ij\right) = cx + \delta^{n/2} + \delta - 2\delta^{\frac{n-x}{2}} - 2\left(1 - \delta^{\frac{x}{2}}\right)\sum_{i=1}^{i=\frac{n-x}{2}-1}\delta^i,$$

if x is even.

The proposition follows. $\square$

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