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Environment in an Overlapping Generations Economy with Endogenous Labor Supply: a Dynamic Analysis

Thomas Seegmuller[*]and Alban Verchère[†]

March 16, 2005

Abstract

We consider an overlapping generations model with environment, where we introduce an elastic labor supply. In this framework, consumers have to choose between consumption, environmental quality and leisure. We establish that several steady states can coexist, even under a Cobb-Douglas technology, and we put in evidence a non monotonic relationship between pollution and per capita income, as suggested by the Environmental Kuznets Curve. Moreover studying local dynamics, we show the existence of deterministic cycles and endogenous fluctuations due to self-fulfilling expectations. In contrast to previous results, the occurrence of such fluctuations does not require a high emission rate of pollution. Finally, we discuss some welfare and policy implications of our results. Especially, we show that a government which would reduce pollution emissions can face a trade-off between an increase of steady state welfare and an intergenerational welfare inequality due to indeterminacy.

JEL classification: C62, E32, Q20.

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Keywords: Environment, Labor supply, Overlapping generations, Multiplicity of steady states, Environmental Kuznets Curve, Indeterminacy, Endogenous cycles.

Résumé

Nous considérons un modèle à générations imbriquées avec environnement, dans lequel nous introduisons une offre de travail élastique. Dans cette économie, les consommateurs ont à choisir entre consommation, qualité de l'environnement et loisir. Nous établissons que plusieurs états stationnaires peuvent coexister, même lorsque la technologie est Cobb-Douglas, et nous mettons en évidence une relation non monotone entre la pollution et le revenu par tête, comme celle suggérée par la Courbe Environnementale à la Kuznets. De plus, étudiant la dynamique locale, nous montrons l'existence de cycles déterministes et de fluctuations endogènes dues aux anticipations autoréalisatrices des agents. Contrairement à de précédents résultats, l'apparition de telles fluctuations ne requiert pas un taux d'émission de pollution élevé. Finalement, nous discutons de certaines implications en termes de bien-être et de politique. Nous montrons en particulier qu'un gouvernement qui voudrait réduire les émissions de pollution peut être confronté à un arbitrage entre augmenter le bien-être à l'état stationnaire et créer une inégalité intergénérationnelle due à l'indétermination de l'équilibre.

1 Introduction

The link between environment and dynamic economics has been studied in a lot of contributions.[1] In this literature, overlapping generations models have been used in order to clearly analyze intergenerational problems (John and Pecchenino (1994), John, Pecchenino, Schimmelpfennig, and Schreft (1995), Pezzey and Toman (2002)). However, most of these papers deal with the question of sustainability. It is why they are only interested in steady states analysis and monotonic convergence.

Recently, some papers have enlighten that more complex dynamics can emerge when one considers the interaction between environment and economic activity. Indeed, considering overlapping generations models where consumers have to choose between consumption and abatement, Seegmuller and Verchère (2004) and Zhang (1999) have shown the existence of deterministic cycles and chaos.

Nevertheless, in these works, labor is constant because its supply is inelastic. However, it is well-known that labor plays an important role on economic dynamics, both on growth and fluctuations. In particular, the choice between leisure and consumption has been exploited in the analysis of endogenous fluctuations. For example, in overlapping generations economies, the occurrence of endogenous fluctuations requires a negatively slopped labor supply in monetary models (Grandmont (1985), Grandmont (1989)), whereas it is based on a sufficiently elastic labor supply when there is capital accumulation (Cazzavillan (2001), Reichlin (1986)).

In this paper, we relate these two types of contributions analyzing the emergence of endogenous fluctuations, the first one which studies the link between economic dynamics and environment and the second one which notably exploits the labor supply elasticity. Considering an overlapping generations economy, we assume that consumers have to choose between consumption, environmental quality and leisure, and share their labor income between savings and abatement, because pollution negatively affects their preferences. This framework allows us to show the coexistence of several steady states, the appearance of a stationary Environmental Kuznets Curve (EKC), the emergence of endogenous fluctuations, and finally some welfare and policy implications.

In other words, considering a utility function characterized by a constant intertemporal elasticity of substitution (CIES), we study uniqueness or multiplicity of steady states and we establish that a multiplicity of steady states can be easily obtained. In contrast to the recent analysis developed

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[1] For a recent survey see Xepapadeas (2003).

by Cazzavillan (2001), such multiplicity appears even if the technology is Cobb-Douglas.

Concerning also the steady state analysis, we show that the pollution-income relationship (PIR) can have a non monotonic shape, namely an EKC. Hence, pollution flow raises for a sufficiently low level of per capita income, before to exhibit a U-turn for greater ones, as it is empirically documented, at least for some pollutants, by Grossman and Krueger (1993), Selden and Song (1994), Shafik (1994) and Carson, Jeon, and McCubbin (1997).

Studying local dynamics, we prove that local indeterminacy and deterministic cycles can occur. It means that not only cyclical deterministic trajectories can appear, but also endogenous fluctuations due to self-fulfilling expectations, under weaker conditions than previous existing works. Indeed, contrary to several contributions, we remark that the existence of indeterminacy and cycles needs neither a weak substitution between production factors (Grandmont, Pintus, and de Vilder (1998), Reichlin (1986), Woodford (1986)), nor increasing returns or imperfect competition (d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (1995), Benhabib and Farmer (1994), Cazzavillan (2001), Cazzavillan, Lloyd-Braga, and Pintus (1998)), Gali (1994)). We can further notice that, in contrast to Seegmuller and Verchère (2004) and Zhang (1999), the occurrence of such fluctuations does not require a high level of the pollution emission rate. Consequently, this paper shows more generally that non monotonic dynamics can easily occur in economies where an environmental dimension is taken into account.

Finally, taken into account the previous results, we deduce some welfare and policy implications. Considering a government that would like to reduce the pollution emission rate per unit of capita, it could face to a trade-off concerning the effect of such a reduction on intergenerational welfare. On one hand, welfare goes up with a lower pollution flow while, on the other hand, it can induce an intergenerational welfare inequality due to the occurrence of fluctuations driven by sunspots, as it is shown by Seegmuller and Verchère (2004) in an one-dimensional model.

The paper is organized as follows. In the next section, we present the model. In section 3, we first establish the existence of a steady state, analyze uniqueness or multiplicity of steady states and then exhibit that every stationary solution is characterized by an EKC. In section 4, we study local dynamics and provide interpretations of our findings. In section 5, we give some welfare and policy implications of our results. Finally, we present concluding remarks in section 6, whereas several technical results are given in the Appendix.

2 The Model

We consider a perfectly competitive overlapping generations model with discret time $t = 1, 2, ..., \infty$ and a constant population normalized to one. A generation of consumers is born at each period and households live two periods. So at each period, a generation of young and a generation of old consumers coexist. When young, the representative consumer supplies labor l_t, which is remunerated at the real wage w_t. He shares his wage earnings between savings, through the purchase of aggregate capital k_t, and environmental maintenance d_t.[2] When old, he rents capital to the firms, earns the real interest factor r_{t+1} and consume the final good c_{t+1}.[3] Moreover, there is a government that levies a constant tax $\tau \in (0, 1)$ on his capital income to finance its public expenditures G_t. The two budget constraints facing by the consumer can be written:

$$k_t + d_t = w_t l_t \tag{1}$$

$$c_{t+1} = (1 - \tau) r_{t+1} k_t \tag{2}$$

In this economy, the environmental quality decreases with respect to the pollution. At period $t + 1$, the level of pollution is given by:

$$P_{t+1} = \alpha k_{t-1} - d_t \equiv P(k_{t-1}, d_t) \text{ , with } \alpha > 0 \tag{3}$$

The pollution, which is always strictly positive, linearly increases with respect to the capital stock inherited from the previous period and is a decreasing function of the environmental maintenance.[4] In this sense, pollution is a consequence of both the secularly accumulation of capital and agents' actions against the negative effects associated to this accumulation. Roughly speaking, the externalities (αk_{t-1}) can be seen as flows from previous activities as well as wastes inherited from the past.

Consumers derive utility from consumption, leisure and environmental quality. Assuming additively separable preferences, the utility function of the representative household is given by:

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[2] As in John and Pecchenino (1994), we consider positive environmental maintenance or pollution abatement ($d_t \geq 0$).

[3] We assume that capital totally depreciates after one period of use.

[4] Note that the pollution P_{t+1} can be interpreted as a flow, or a stock determined by $P_{t+1} = (1 - m)P_t + \alpha k_{t-1} - d_t$, with a natural rate of absorption m equal to 1. Since, in overlapping generations models, the length of period is assumed to be long, the assumption $m = 1$ does not seem to be too restrictive.

$$BU(c_{t+1}/B) - vl_t - P(k_{t-1}, d_t)^{1+\phi}/(1+\phi) \tag{4}$$

where $B > 0$ and $v > 0$ are two scaling parameters, and $\phi > 0$. We can notice that as usually in dynamic macroeconomic models, the disutility of labor is linear (see Hansen (1985)), while the disutility of pollution is increasing and convex. Furthermore, we assume:

Assumption 1 *The function $U(x)$ is continuous for all $x \geq 0$, $\mathcal{C}^n$ for $x > 0$ and n large enough, with $U'(x) > 0$, $U''(x) \leq 0$ and $U'(x) + xU''(x) > 0$.*

The representative consumer maximizes his utility function (4) under the constraints (1), (2) and (3). We deduce the two following equations:

$$U'(c_{t+1}/B)(1-\tau)r_{t+1}w_t = v \tag{5}$$

$$(\alpha k_{t-1} - d_t)^{\phi} w_t = v \tag{6}$$

These two expressions define the consumer choice between leisure, environmental maintenance and future consumption.

The final good is supplied by a representative firm using a constant returns to scale technology. The production is given by $y_t = f(a_t)l_t$, where $a_t = k_{t-1}/l_t$ denotes the capital-labor ratio and $f(a_t)$ the intensive production function. In what follows, we assume:

Assumption 2 *The intensive production function $f(a)$ is continuous for $a \geq 0$, positively valued and differentiable as many times as needed for $a > 0$, with $f'(a) > 0$ and $f''(a) < 0$.*

The producers maximize their profits. Since the economy is perfectly competitive, we obtain:

$$r_t = f'(a_t) \equiv r(a_t) \tag{7}$$

$$w_t = f(a_t) - a_t f'(a_t) \equiv w(a_t) \tag{8}$$

Finally, the government uses capital income taxation $\tau r_t k_{t-1}$ to finance public expenditures G_t. At each period, the budget is balanced, i.e. $G_t = \tau r_t k_{t-1}$. One can further notice that the public good does neither enter the utility function, nor affect the technology, but a amount of G_t can eventually be used to reduce the pollution emission rate α.[5]

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[5] See section 5 below.

Substituting equations (1), (2), (7) and (8) into (5) and (6), we can define an intertemporal equilibrium as follows. An intertemporal equilibrium is a sequence $(a_t, k_{t-1})_{t\geq 1}$ which satisfies:

$$U'((1-\tau)r(a_{t+1})k_t/B)(1-\tau)r(a_{t+1})w(a_t) = v \tag{9}$$

$$k_t = w(a_t)\frac{k_{t-1}}{a_t} - \alpha k_{t-1} + \left(\frac{v}{w(a_t)}\right)^{1/\phi} \tag{10}$$

$$\alpha k_{t-1} > w(a_t)k_{t-1}/a_t - k_t \geq 0 \tag{11}$$

where $k_0 > 0$ is given.

Note that inequalities (11) ensure a strictly positive pollution and a positive environmental maintenance. Taken as given these conditions, equations (9) and (10) define a two-dimensional dynamic system with one predetermined variable, the capital. One can further notice that the environmental dimension of the model implicitly appears into (9), by k_t, which is given by relation (10). Indeed, this last equation means in fact that $k_t = w(a_t)l_t - d_t$ at equilibrium, where $d_t = \alpha k_{t-1} - P_{t+1}$ and $P_{t+1} = (v/w(a_t))^{1/\phi}$. Note that in the limit case without any pollution ($\alpha = 0$), the trade-off between investments in productive capital and environmental maintenance disappears ($d_t = 0$), and equation (10) becomes $k_t = w(a_t)l_t$.

Before studying steady states, we define the following relationships. We note $s(a) \equiv r(a)a/f(a) \in (0,1)$ the capital share in income and furthermore, if $\sigma(a)$ represents the elasticity of capital-labor substitution, $1/\sigma(a) = dlnw(a)/dlna - dlnr(a)/dlna$. Since $w'(a) = -ar'(a)$, we obtain the two following expressions:

$$w'(a)a/w(a) = s(a)/\sigma(a) \quad \text{and} \quad r'(a)a/r(a) = -(1-s(a))/\sigma(a) \tag{12}$$

3 Steady State Analysis

In this section, we first establish the existence of a steady state. Secondly, we analyze the uniqueness and multiplicity of stationary solutions considering an economy where consumer utility is characterized by a constant intertemporal elasticity of substitution (CIES). Finally, using such consumer preferences, we also study the link between the pollution and the level of production at a steady state.

3.1 Existence of a Steady State

A stationary solution of the dynamic system defined by (9), (10) and (11) is given by (a, k) such that:

$$U'((1-\tau)r(a)k/B)(1-\tau)r(a)w(a) = v \tag{13}$$

$$k\left(\alpha + 1 - \frac{w(a)}{a}\right) = \left(\frac{v}{w(a)}\right)^{1/\phi} \tag{14}$$

$$\alpha + 1 > w(a)/a > 1 \tag{15}$$

We can notice that in section 4 we are interested in fluctuations around a steady state. Since environmental maintenance has to be positive at each period, we assume a strictly positive environmental maintenance at the steady state, i.e. $w(a)/a > 1$.

Following Cazzavillan, Lloyd-Braga, and Pintus (1998) and Aloi, Dixon, and Lloyd-Braga (2000), we ensure in what follows the existence of a normalized steady state $(a, k) = (1, 1)$ by choosing appropriate values of the two scaling parameters $B > 0$ and $v > 0$. In order to do that, we assume:

Assumption 3 $\alpha + 1 > w(1) > 1$.

Under Assumptions 1-3, there exists a unique solution $v^* > 0$ to the equation:

$$v^* = (\alpha + 1 - w(1))^{\phi} w(1) \tag{16}$$

Assume that $lim_{x\to+\infty}U'(x) < v^*/((1-\tau)r(1)w(1)) < lim_{x\to 0}U'(x)$. Then, taken as given v^*, there is a unique B^* which satisfies:

$$U'((1-\tau)r(1)/B^*) = v^*/((1-\tau)r(1)w(1)) \tag{17}$$

Proposition 1 *Assuming that* $lim_{x\to+\infty}U'(x) < v^*/((1-\tau)r(1)w(1)) < lim_{x\to 0}U'(x)$ *and Assumptions 1-3 are satisfied, then* $(a, k) = (1, 1)$ *is a steady state of the dynamic system (9)-(10) if and only if* v^* *and* B^* *are the unique solutions of (16) and (17).*

3.2 Uniqueness and Multiplicity of Steady States in a CIES Economy

In Proposition 1, we have established the existence of a steady state $(a, k) = (1, 1)$. However, this steady state is not necessarily unique. In this section, assuming that Proposition 1 is satisfied and considering a constant intertemporal elasticity of substitution (CIES) economy, we analyze the uniqueness or multiplicity of stationary solutions. We notably prove that it can exist two steady states even if the technology has an unit elasticity of capital-labor substitution (Cobb-Douglas).

Since we consider a CIES economy, we assume:

Assumption 4 *$U(X) = X^{1-u}/(1-u)$, with $u \in (0, 1)$.*

One can easily verifies that this utility function satisfies Assumption 1. A steady state is a solution (a, k) such that:

$$(B^*)^u(1-\tau)^{1-u}r(a)^{1-u}w(a)k^{-u} = v^* \tag{18}$$

$$k\left(\alpha + 1 - \frac{w(a)}{a}\right) = \left(\frac{v^*}{w(a)}\right)^{1/\phi} \tag{19}$$

and which satisfies the inequality (15). Equation (18) defines a function $k = G(a)$ and equation (19) a function $k = H(a)$, with:

$$G(a) = B^*(1-\tau)^{(1-u)/u}r(a)^{(1-u)/u}w(a)^{1/u}/(v^*)^{1/u} \tag{20}$$

$$H(a) = \frac{(v^*)^{1/\phi}}{w(a)^{1/\phi}(\alpha + 1 - w(a)/a)} \tag{21}$$

Studying the uniqueness or multiplicity of steady states requires to analyze the number of stationary solutions a to the equation $F(a) \equiv G(a)/H(a) = 1$.

In what follows, we note $\underline{a}$ such that $w(\underline{a})/\underline{a} = 1 + \alpha$ and $\overline{a}$ such that $w(\overline{a})/\overline{a} = 1$. Furthermore, we assume:

Assumption 5 *$s(1) < 1/2$ and $\sigma(a) > s(a)$ for all $a \in (a_0, a_1)$, with $a_0 < \underline{a}$ and $a_1 > \overline{a}$.*

The first assumption means that capital share in income is smaller than one half at the steady state $(a, k) = (1, 1)$. The second one is not too restrictive and is empirically founded. Among others, Duffy and Papageorgiou

(2000) show that the elasticity of substitution between capital and labor take values in a neighborhood of the unit case, greater and smaller values than one being admissible.[6] It means that $w(a)/a$ is a strictly decreasing function for all $a \in (a_0, a_1)$, where a_0 can be arbitrarily close to $\underline{a}$ and a_1 arbitrarily close to $\overline{a}$. Since $w(a)/a$ belongs to $(1, 1+\alpha)$, a belongs to $(\underline{a}, \overline{a})$. Moreover, under Assumption 3, we have $\underline{a} < 1 < \overline{a}$, which also implies that $\sigma(1) > s(1)$.

Now, we study the number of solutions of the equation $F(a) = 1$ when $a \in (\underline{a}, \overline{a})$. Using equations (16), (17), (20) and (21), we have:

$$\begin{aligned} &F(a) = I(a)\frac{\alpha + 1 - w(a)/a}{\alpha + 1 - w(1)} \ , \\ &\text{with } I(a) = \left(\frac{r(a)}{r(1)}\right)^{(1-u)/u} \left(\frac{w(a)}{w(1)}\right)^{1/u + 1/\phi} \end{aligned} \tag{22}$$

We can easily verify that $F(1) = 1$. From the definition of $\underline{a}$, we deduce that $F(\underline{a}) = 0$. Moreover, we have:

$$F(\overline{a}) = I(\overline{a})\frac{\alpha}{\alpha + 1 - w(1)} \tag{23}$$

We can notice that $F(\overline{a})$ decreases with respect to α. Furthermore, when $I(\overline{a}) \geq 1$, $F(\overline{a}) > 1$ for all α. On the contrary, when $I(\overline{a}) < 1$, $F(\overline{a}) = 1$ for $\alpha = \alpha^*$, with:

$$\alpha^* = \frac{w(1) - 1}{1 - I(\overline{a})} \tag{24}$$

Hence, $F(\overline{a}) > 1$ for $\alpha < \alpha^*$ and $F(\overline{a}) < 1$ for $\alpha > \alpha^*$. In what follows, we note $\sigma = \sigma(1)$ and $s = s(1)$. Then, differentiating (22) evaluated at $a = 1$ and using relations (12), we can determine:

$$\frac{F'(1)}{F(1)} = \frac{1}{\sigma}\left[s(1 + 1/\phi) + (\sigma - s)\frac{w(1)}{\alpha - w(1) + 1} - \frac{1-u}{u}(1 - 2s)\right] \tag{25}$$

We can easily establish that $F'(1)/F(1) < 0$ if $u/(1-u) < \gamma_{uT}$, $F'(1)/F(1) = 0$ if $u/(1-u) = \gamma_{uT}$ and $F'(1)/F(1) > 0$ if $u/(1-u) > \gamma_{uT}$, where γ_{uT} is given in the Appendix.

From these findings, we can deduce the main results concerning the number of steady states:

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[6] As we will see, we will also need this assumption in sections 3.3 and 4.

Proposition 2 *Considering that Assumptions 2-5 are satisfied and that there exists a steady state, namely* $(a,k)=(1,1)$ *(Proposition 1), we have the following.*

(i) If $I(\overline{a}) \geq 1$ *or if* $I(\overline{a}) < 1$ *and* $\alpha < \alpha^*$*, there exists an odd number of steady states. Furthermore, if* $u/(1-u) < \gamma_{uT}$*, there are at least three stationary solutions.*

(ii) If $I(\overline{a}) < 1$ *and* $\alpha > \alpha^*$*, there exists an even number of steady states.*

This proposition establishes that several steady states can coexist. In particular, when $u/(1-u)$ crosses γ_{uT} (i.e. $F'(1)/F(1)$ crosses 0), two steady states can exchange their stability properties or two new stationary solutions can appear in the neighborhood of the steady state $a=1$. These two phenomenon can be related to the results that we will obtain in section 4 (Proposition 4) when we will study the emergence of endogenous fluctuations. Indeed, the first one corresponds to the occurrence of a transcritical bifurcation, whereas the second one corresponds to the occurrence of a pitchfork bifurcation.[7]

We can further notice that considering an overlapping generations model with increasing returns, Cazzavillan (2001) has recently shown that the steady state is always unique when the technology is Cobb-Douglas, whereas there exist two steady states when one considers a small deviation of the elasticity of capital-labor substitution from the unitary case. In contrast to this result, we prove in the following corollary that in our framework, two steady states can coexist even if the technology is Cobb-Douglas.

Corollary 1 *Assume that* $f(a) = Aa^s$ *with* $1/(1-s) < A < (\alpha+1)/(1-s)$ *and* $s < 1/2$*. Then, there exists two steady states when* $(1-u)/u > (s/(1-2s))(1+1/\phi)$ *and* $\alpha > \alpha^*$*.*

Proof. When the production is given by $y = Af(a)l$, with $f(a) = a^s$, then $w(a) = (1-s)Aa^s$. Since $1/(1-s) < A < (\alpha+1)/(1-s)$, we first notice that Assumption 3 is verified. Hence, applying results obtained in Proposition 2, one can establish that there are two steady states if $(1-u)/u > (s/(1-2s))(1+1/\phi)$ and $\alpha > \alpha^*$. ■

These results notably mean that under a Cobb-Douglas technology, the existence of several steady states requires a high intertemporal elasticity of

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[7] For more details about the link between the multiplicity of steady states and the occurrence of a transcritical bifurcation, see Cazzavillan, Lloyd-Braga, and Pintus (1998).

consumption (small u) and/or a strongly convex disutility of pollution (high ϕ). It corresponds to the case where consumer utility is weakly affected by a variation of consumption whereas it is strongly sensitive to pollution variations.

3.3 The Pollution-Income Relationship

The Pollution-Income Relationship (PIR) have received a lot of attention in recent years. From an empirical point of view, several contributions have shown that this relation can take an inverted-U shape, namely the Environmental Kuznets Curve (EKC). The EKC conjecture states that, in the long run, economic growth allows to solve previous environmental matters. It degrades the environment at low levels of income, but as income increases, harmful environmental effects go down. So, according to this hypothesis, environment and natural resources are over-stressed and over-exploited at the beginning of development to sustain the take-off and accelerate economic growth. When the level of development is high enough, the environment becomes more valued by people, and through technical progress and the demand for green goods and technologies, it becomes possible to create wealth with reduced environmental degradation. For example, the seminal paper by Grossman and Krueger (1993) found an EKC for sulphur dioxide (SO_2). Selden and Song (1994) and Carson, Jeon, and McCubbin (1997) exhibit this shape for SO_2, suspended particulate matter (SPM) and oxides of nitrogen (NO_x), whereas Shafik (1994) shows it for SO_2 and SPM.

Some economists have try to find some theoretical foundations of this relationship (see among others John and Pecchenino (1994) and Stokey (1998)). In this section, we prove that our framework is also characterized by such a link between income and environmental quality.[8] In other words, we establish the conditions to obtain a relation close to the EKC between output and environmental quality. Since environmental quality decreases with respect to pollution flow in this model, it means that we will show that at a steady state, production y increases with respect to pollution P for sufficiently weak levels of output and decreases when y is high enough.[9] In order to do that we assume in this section that $U(X)$ satisfies Assumption 4 and the technology is CES, i.e.

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[8] We restrict our attention to steady state analysis, because in this paper, when we will study dynamics, we are interested in the emergence of cycles around the stationary equilibria and not on long-run growth.

[9] Recall that in our framework, y represents both total and per capita income, since we consider representative agents.

$$f(a) = A(sa^{\frac{\sigma-1}{\sigma}} + 1 - s)^{\frac{\sigma}{\sigma-1}}, \quad \text{with} \quad s \in (0,1), \sigma > 0, \sigma \neq 1, A > 0 \tag{26}$$

It means that the real wage is defined by:

$$w(a) = A(1-s)(sa^{\frac{\sigma-1}{\sigma}} + 1 - s)^{\frac{1}{\sigma-1}} \tag{27}$$

and the capital share in income by:

$$s(a) = \frac{s}{s + (1-s)a^{\frac{1-\sigma}{\sigma}}} \tag{28}$$

In order to establish our result, we further consider that Assumption 5 is satisfied and:

Assumption 6

(i) $\frac{1}{1-s} < A < \frac{1+\alpha}{1-s}$;

(ii) $\frac{u}{1-u} < \frac{1}{\sigma}$.

One can notice that the first inequality ensures that $1 < w(1) < 1 + \alpha$ (Assumption 3) and the last one that the utility for consumption is not too concave.

Assuming in the rest of this section that it exists a steady state (a, k), equations (18) and (19) can be rewritten as follows:

$$P = \left(\frac{v}{\beta w(a)} \right)^{1/\phi} \tag{29}$$

$$l = Bv^{-\frac{1}{u}}(1-\tau)^{\frac{1-u}{u}} r(a)^{\frac{1-u}{u}} w(a)^{\frac{1}{u}} a \tag{30}$$

From (29), we deduce that P decreases with respect to a. We then establish our result by showing that y is a decreasing function of a for sufficiently weak values of the capital-labor ratio, whereas it is an increasing function for greater values of a. Using (26), (30) and $y = f(a)l$, we have:

$$\epsilon_{y,a} \equiv \frac{dy}{y}\frac{a}{da} = \frac{1}{\sigma}\left[s(a)\left(1 + \sigma + 2\frac{1-u}{u}\right) + \sigma - \frac{1-u}{u} \right] \tag{31}$$

When $\sigma > 1$, $s(a)$ increases with respect to a, from 0 to 1 when a raises from 0 to $+\infty$, and then, under Assumption 6, $\epsilon_{y,a}$ is strictly negative when a is sufficiently weak, whereas it is strictly positive when a is high enough. Moreover, $\epsilon_{y,a}$ changes of sign for $a = a^*$, with:

$$a^* = \left(\frac{1-s}{s} \frac{\frac{1-u}{u} - \sigma}{1 + 2\sigma + \frac{1-u}{u}} \right)^{\frac{\sigma}{\sigma-1}} \tag{32}$$

Finally, since any steady state value has to satisfy inequalities (15), we have to show that $1 < w(a^*)/a^* < 1 + \alpha$. Using (27) and (32), we obtain:

$$\frac{w(a^*)}{a^*} = As^{\frac{\sigma}{\sigma-1}} \left[\frac{\frac{u}{1-u}(1+2\sigma)+1}{1-\sigma\frac{u}{1-u}} \right] \left[1 + \frac{\frac{u}{1-u}(1+2\sigma)+1}{1-\sigma\frac{u}{1-u}} \right]^{\frac{1}{\sigma-1}} \equiv K(a^*) \tag{33}$$

We can notice that $K(a^*)$ increases with respect to $\frac{u}{1-u}$ and:

$$lim_{\frac{u}{1-u}\to 0} K(a^*) = As(2s)^{\frac{1}{\sigma-1}} \text{ and } lim_{\frac{u}{1-u}\to \frac{1}{\sigma}} K(a^*) = +\infty \tag{34}$$

Under Assumption 6, $sA < 1 + \alpha$ and $(2s)^{\frac{1}{\sigma-1}} < 1$ which imply that $lim_{\frac{u}{1-u}\to 0} K(a^*) < 1 + \alpha$. Hence, there exists some values $\frac{u}{1-u} \in (0, \frac{1}{\sigma})$ such that $1 < w(a^*)/a^* < 1 + \alpha$ is satisfied.

We can summarize this result in the following proposition:

Proposition 3 *Under Assumptions 5 and 6, there exists an EKC relationship between y and P at a steady state for appropriate values of $u/(1-u)$.*

4 Local Dynamics

In this section, we analyze the local indeterminacy and the occurrence of endogenous cycles. Furthermore, interpreting our results, we put in evidence that the consumer choice between leisure, environmental maintenance and future consumption has a key role on the occurrence of endogenous fluctuations.

In order to do that, we study local dynamics in the neighborhood of the steady state $(a, k) = (1, 1)$. We first differentiate the dynamic system (9)-(10) around $(a, k) = (1, 1)$. We note $\varepsilon_u \equiv -U''(c/B)(c/B)/U'(c/B) \in (0, 1)$. Using (12) and after some computations, we obtain the trace T and the determinant D of the associated Jacobian matrix, which respectively represent the sum and the product of the two eigenvalues of the characteristic polynomial $P(\lambda) \equiv \lambda^2 - T\lambda + D = 0$:

$$\begin{gathered} T = \gamma_u \frac{s}{1-s} \left[\frac{\alpha - (1+\phi)(w(1)-1)}{\phi} + w(1)\frac{\sigma}{s} \right] + T_1(\alpha), \\ \text{with } T_1(\alpha) = \frac{s}{1-s} + w(1) - \alpha \end{gathered} \tag{35}$$

$$D = (1+\gamma_u)\, D_1(\alpha) \text{ , with } D_1(\alpha) = \frac{s}{1-s}(w(1)-\alpha) \tag{36}$$

with $\gamma_u \equiv \varepsilon_u/(1-\varepsilon_u) \in (0,+\infty)$, which represents an index of the concavity of utility for consumption.

Following Grandmont, Pintus, and de Vilder (1998), we study the local stability of the steady state and the occurrence of bifurcations by analyzing the trace T and the determinant D in the plane (see figure 1). On the line (AC), one eigenvalue is equal to 1 ($P(1)=0$). On the line (AB), one eigenvalue is equal to -1 ($P(-1)=0$). On the segment $[BC]$, the two eigenvalues are complex conjugates and have an unit modulus ($T^2-4D<0$, $D=1$). We deduce that if (T,D) is inside (ABC), the steady state is a sink. When $1-T+D > (<)0$ and $1+T+D < (>)0$, the steady state is a saddle. Otherwise, it is a source. Since one variable is predetermined, the steady state is locally indeterminate if it is a sink and is locally determinate if it is a saddle or a source.

Suppose now that T and D change when a parameter, called the bifurcation parameter, varies continuously. When (T,D) crosses the line (AC), a transcritical or a pitchfork bifurcation generically occurs.[10] When (T,D) crosses the line (AB), one gets a flip bifurcation, i.e. a cycle of period 2 appears around the steady state. When (T,D) crosses the segment $[BC]$, one gets a Hopf bifurcation, i.e. an invariant closed curve appears around the steady state.[11] Moreover, sunspot equilibria can appear around a steady state if it is locally indeterminate. They can also occur in the neighborhood of a cycle of period two if it is locally stable and in a neighborhood of an invariant closed curve if the Hopf bifurcation is supercritical.[12]

We choose γ_u as the bifurcation parameter. Under Assumption 1, γ_u belongs to $(0,+\infty)$. We remark that when this bifurcation parameter varies, (T,D) describes a half-line Δ which starts at $(T_1(\alpha), D_1(\alpha))$ when γ_u tends to 0 and has a slope $S(\alpha)$ defined by:

$$S(\alpha) = \frac{\phi s(w(1)-\alpha)}{\phi(\sigma w(1) - s(w(1)-1)) + s(\alpha - (w(1)-1))} \tag{37}$$

We can study the local stability of the steady state and the occurrence of bifurcations by analyzing the locus $(T_1(\alpha), D_1(\alpha))$ and the slope $S(\alpha)$

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[10] When one eigenvalue crosses the value 1, a saddle, a transcritical or a pitchfork bifurcation generically occurs. However, since we have shown that there exists at least one steady state (Proposition 1), a saddle bifurcation cannot occur.

[11] For further information about local bifurcation theory, one can refer to Grandmont (1988) or Wiggins (1990).

[12] For more details, see Grandmont, Pintus, and de Vilder (1998).

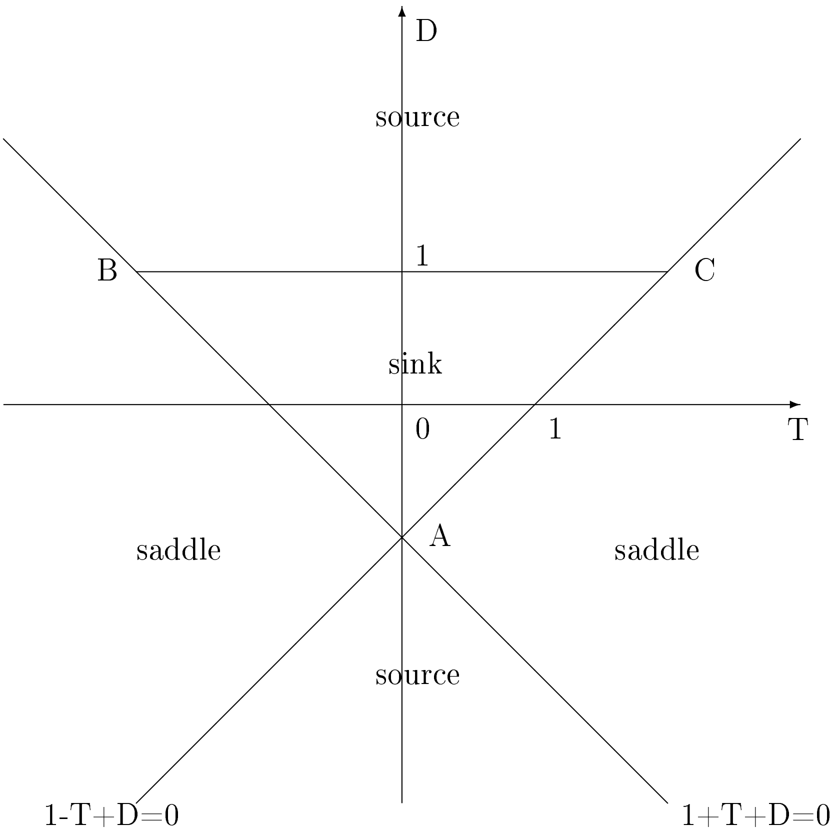

Figure 1: The Geometrical Method

of the half-line Δ. In particular, we will discuss our results in function of α. Recall that $w(1) > 1$ because we always consider a strictly positive maintenance at the steady state and $\alpha > w(1) - 1$ because the pollution is also strictly positive (Assumption 3). We deduce that α can take any value between $w(1) - 1$ and $+\infty$. Furthermore, we assume that Assumption 5 is satisfied. From a theoretical point of view, we are not interested in the case where $\sigma < s$ because it is well-known that endogenous fluctuations can occur under such an assumption in perfectly competitive overlapping generations economies without environmental dimension (see Reichlin (1986)). However, they cannot emerge when $\sigma > s$.[13]

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[13] Using our framework, the overlapping generations economy without an environmental dimension is obtained considering the limit case where $\alpha = 0$ and $w(a) = a$. It means that pollution and enviromental maintenance are both equal to 0 at a steady state. Furthermore, the dynamics of the economy without environment are defined by:

$$U'((1-\tau)r(a_{t+1})k_t/B)(1-\tau)r(a_{t+1})w(a_t) = v \tag{38}$$

We can first notice that when α varies, the locus $(T_1(\alpha), D_1(\alpha))$ describes a half-line Δ_1 which has a slope equal to $s/(1-s) \in (0,1)$ (see figures 2 and 3). When α tends to $w(1)-1$, we have:

$$\begin{aligned} T_1(w(1)-1) = 1/(1-s) \ , \ D_1(w(1)-1) = s/(1-s) \\ \text{and } 1 - T_1(w(1)-1) + D_1(w(1)-1) = 0 \end{aligned} \tag{42}$$

It means that $(T_1(w(1)-1), D_1(w(1)-1))$ is on the line (AC) between the horizontal axis and the point C. When α increases, $T_1(\alpha)$ and $D_1(\alpha)$ decreases and tends to $-\infty$ when α tends to $+\infty$. In particular, $D_1(\alpha) = 0$ when $\alpha = w(1) \equiv \alpha_0$ and $(T_1(\alpha), D_1(\alpha))$ crosses the line (AB) when $\alpha = w(1) + 1 \equiv \alpha_{F_1}$.

Under Assumption **??**, the slope $S(\alpha)$ decreases with respect to α. It means that the half-line Δ makes a clockwise rotation around Δ_1. In particular, we have:

$$\begin{aligned} S(w(1)-1) =& s/[\sigma w(1) - s(w(1)-1)] \in (0,1) \ , \ S(\alpha_0) = 0 \\ & \text{and } S(+\infty) = -\phi < 0 \end{aligned} \tag{43}$$

When $\phi \leq 1$, $S(\alpha)$ is always greater than -1, whereas when $\phi > 1$, $S(\alpha)$ can be smaller than -1. In particular, $S(\alpha) = -1$ if $\alpha = \alpha_{F_2}$, where:

$$\alpha_{F_2} = \frac{\phi(\sigma w(1) + s) - s(w(1)-1)}{s(\phi - 1)} > \alpha_{F_1} \tag{44}$$

From these findings, we deduce that the line Δ goes through the point A for $\alpha = \alpha_I$, with $\alpha_{F_1} < \alpha_I (< \alpha_{F_2})$.[14]

$$k_t = w(a_t) k_{t-1}/a_t \tag{39}$$

Then, for $\sigma(a) > s(a)$, one can easily prove that there exists a unique steady state if $lim_{a \to +\infty} w(a)/a < 1 < lim_{a \to 0} w(a)/a$. Concerning now local dynamics, equations (35) and (36) can be written:

$$T = \gamma_u \frac{\sigma}{1-s} + \frac{1}{1-s} > 0 \tag{40}$$

$$D = (1 + \gamma_u) \frac{s}{1-s} > 0 \tag{41}$$

We deduce that $1+T+D > 0$ and $1-T+D = \gamma_u \frac{s-\sigma}{1-s} < 0$ if $\sigma > s$. It means that under Assumption **??**, the steady state is a saddle and endogenous fluctuations cannot occur in the overlapping generations economy without environment. One can further notice that the existence of a constant tax rate on capital income does not affect local dynamics.

[14] For more details about α_I, see the Appendix.

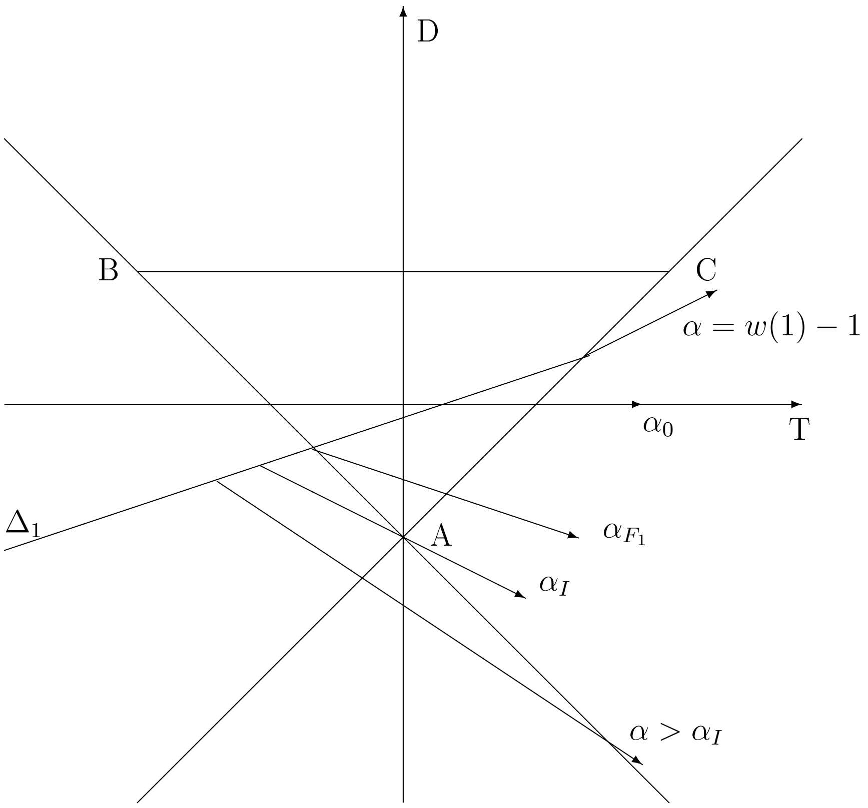

Figure 2: $0 \leq \phi \leq 1$

Hence, when $\alpha = w(1) - 1$, $(T_1(\alpha), D_1(\alpha))$ is on the line (AC) between the horizontal axis and the point C. Moreover, the half-line Δ has a slope belonging to $(0, 1)$ and is on the right side of (AC). Then, the steady state is always a saddle. However, it is a limit case where the level of pollution is equal to zero.

When $w(1) - 1 < \alpha \leq \alpha_{F_1}$, $(T_1(\alpha), D_1(\alpha))$ is inside (ABC) and the slope of the half-line Δ is smaller than 1 and greater than -1. The half-line Δ only crosses the line (AB).

When $\alpha_{F_1} < \alpha < \alpha_I$, $(T_1(\alpha), D_1(\alpha))$ is on the left side of (AB) and the half-line Δ crosses the lines (AB) and (AC) above A.

When $\phi \leq 1$ and $\alpha > \alpha_I$ or when $\phi > 1$ and $\alpha_I < \alpha < \alpha_{F_2}$, $(T_1(\alpha), D_1(\alpha))$ is on the left side of (AB) and the half-line Δ crosses the lines (AB) and (AC) below A.

Finally, when $\phi > 1$ and $\alpha > \alpha_{F_2}$, $(T_1(\alpha), D_1(\alpha))$ is on the left side of (AB) and the slope of Δ is smaller than -1. Then, the half-line Δ only crosses the line (AC) below the point A.

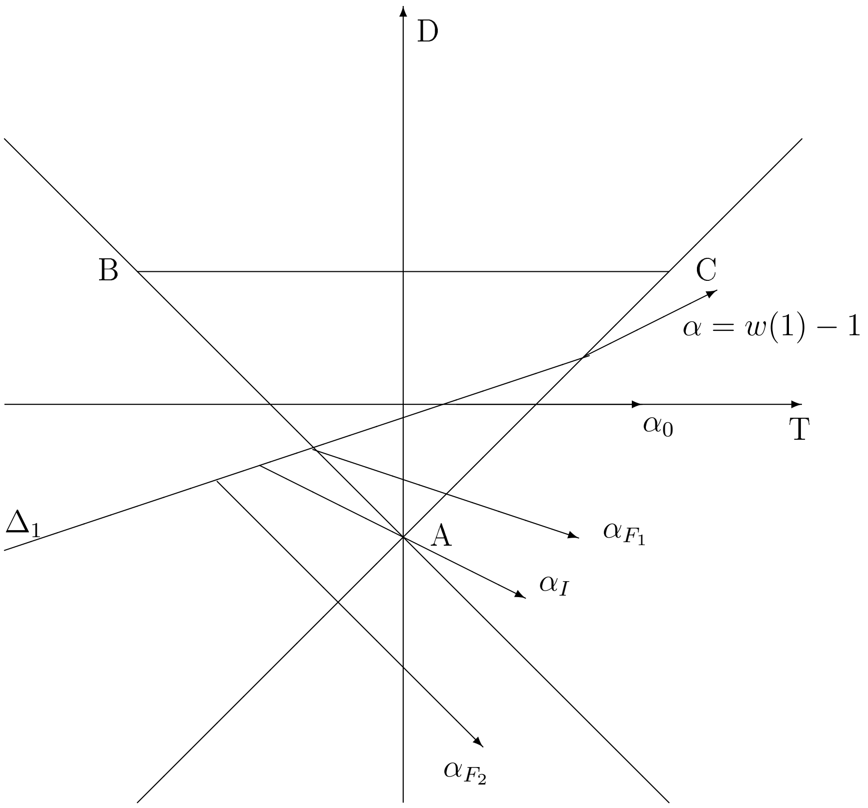

Figure 3: $\phi > 1$

From these geometrical findings, we can deduce the following proposition (see also figures 2 and 3):

Proposition 4 *Assume that Assumptions 1-3 and 5 are satisfied and that there exists a steady state (Proposition 1). Then, the following generically holds.*

(i) If $w(1) - 1 < \alpha \leq \alpha_{F_1}$, the steady state is a sink when $\gamma_u < \gamma_{uT}$, a transcritical or a pitchfork bifurcation generically occurs when $\gamma_u = \gamma_{uT}$ and the steady state is a saddle when $\gamma_u > \gamma_{uT}$.

(ii) If $\alpha_{F_1} < \alpha < \alpha_I$, the steady state is a saddle when $\gamma_u < \gamma_{uF}$, a flip bifurcation generically occurs when $\gamma_u = \gamma_{uF}$, the steady state is a sink when $\gamma_{uF} < \gamma_u < \gamma_{uT}$, a transcritical or a pitchfork bifurcation generically occurs when $\gamma_u = \gamma_{uT}$ and the steady state is a saddle when $\gamma_u > \gamma_{uT}$.

(iii) *If* $\phi \leq 1$ *and* $\alpha > \alpha_I$ *or if* $\phi > 1$ *and* $\alpha_I < \alpha < \alpha_{F_2}$*, the steady state is a saddle when* $\gamma_u < \gamma_{uT}$*, a transcritical or a pitchfork bifurcation generically occurs when* $\gamma_u = \gamma_{uT}$*, the steady state is a source when* $\gamma_{uT} < \gamma_u < \gamma_{uF}$*, a flip bifurcation generically occurs when* $\gamma_u = \gamma_{uF}$ *and the steady state is a saddle when* $\gamma_u > \gamma_{uF}$*.*

(iv) *If* $\phi > 1$ *and* $\alpha > \alpha_{F_2}$*, the steady state is a saddle when* $\gamma_u < \gamma_{uT}$*, a transcritical or a pitchfork bifurcation generically occurs when* $\gamma_u = \gamma_{uT}$ *and the steady state is a source when* $\gamma_u > \gamma_{uT}$*.*

The values γ_{uT} *and* γ_{uF} *are given in the Appendix.*

This proposition establishes that endogenous fluctuations can emerge in the economy through the occurrence of local indeterminacy and flip bifurcations. In particular, indeterminacy and then sunspot equilibria appear as soon as α is not too high and γ_u is weak enough. It means that in contrast to previous results analyzing the existence of cycles in dynamic models taking into account the link between economy and environment (see Seegmuller and Verchère (2004), Zhang (1999)), the emission rate of pollution has not to be too high. Furthermore, as in Cazzavillan (2001) and Reichlin (1986), the utility function for consumption must not be strongly concave.[15]

We can further notice that endogenous fluctuations can occur when capital and labor are not weak substitutes as it is often required (see among others Grandmont, Pintus, and de Vilder (1998), Reichlin (1986) and Woodford (1986)). Our results do not depend any more on the existence of increasing returns or imperfect competition (d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (1995), Benhabib and Farmer (1994), Cazzavillan (2001), Cazzavillan, Lloyd-Braga, and Pintus (1998)), Gali (1994)).

We interpret the occurrence of endogenous fluctuations by explaining how non monotonic and cyclical trajectories emerge due to consumers' choices between consumption, environmental quality and leisure. In this way, first recall the following relations:

I.1. $k_t + d_t = w_t l_t$;

I.2. $P_{t+1} = \alpha k_{t-1} - d_t$;

I.3. $P_{t+1}^{\phi} w_t = v$.

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[15] One can also notice that the constant tax rate on capital income has no influence on local dynamics. See Guo and Harrison (2004) for a similar result in the optimal growth model.

Assume now that one deviates from the steady state by an increase of the capital stock k_{t-1}. Then, taken into account I.2., young consumers expect that future pollution P_{t+1} will increase and reduce the level of their utility. Consequently, following such expectation, they increase their labor supply l_t (because from I.3. the real wage decreases), and they reallocate their savings from capital accumulation k_t to environmental maintenance d_t (see I.1.). Since capital stock decreases, the next generation of consumers will expect a weaker pollution flow P_{t+2} and then, by the reverse mechanism than the one described above, both l_{t+1} and d_{t+1} will go down and k_{t+1} will raise; and so on successively along the fluctuations.

Finally, one can observe that the greater is α, the most volatile is d_t, which can be a source of instability of the steady state and promote the determinacy of the equilibrium. Hence, it explains why endogenous fluctuations emerge for a not too high level of α.

5 Welfare and Policy Implications

In this section, we use the preceding results to derive some implications concerning the consumer welfare and a policy which would like to reduce pollution emissions.

Indeed, the government can decide to spend a fixed amount of its budget $\widetilde{G} < G_t$ to finance different kinds of environmental policies to reduce the level of intergenerational externality through a decrease of the emission rate α. For example, the government can sustain the adoption of environmental friendly technologies characterized by lower levels of pollutant emissions (water and liquid waste treatment, solid waste recycling, particule filters, low-use energy technologies, etc).

Now, we stress that the conclusions established in Proposition 4 have also some welfare implications. In order to enlighten them, we can notice the two following elements.

First, one can see that consumer utility evaluated at a steady state increases when the pollution emission rate (α) decreases, for a not too concave utility function of consumption.[16]

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[16] For example, using the CIES specification of section 3.2 and equations (4), (18) and (19), the welfare W of a young consumer evaluated at the steady state $(a, k) = (1, 1)$ is equal to:

$$W = \frac{[\alpha + 1 - w(1)]^{\phi}}{1 + \phi} \left[\frac{\phi + u}{1 - u} - \alpha - \phi w(1) \right] \tag{45}$$

One can show that $dW/d\alpha < 0$ if the following condition is satisfied:

Secondly, we have established that the occurrence of fluctuations due to self-fulfilling expectations requires a not too high pollution emission rate ($\alpha < \alpha_I$). Moreover, using (47), we can deduce that γ_{uT} increases with respect to α. It means that fluctuations driven by agent beliefs occur more easily for higher α as soon as $\alpha < \alpha_I$, whereas they no more occur when $\alpha > \alpha_I$. Since along fluctuations, utility of successive generations have generically not the same level, it puts in evidence the existence of an intergenerational welfare inequality.[17]

Consequently, an environmental policy such as those described above which would aim to reduce pollution flows, through a decrease of α, has to be taken carefully. Indeed, authorities would face to the following trade-off. On one hand, a reduction of α increases welfare at the steady state but, on the other hand, it can induce an intergenerational welfare inequality as soon as α crosses and becomes smaller than α_I. On the contrary, such a trade-off is no more relevant if α is already smaller than α_I before the policy is effective, because a greater decrease of α implies that indeterminacy occurs less easily.[18]

6 Conclusion

In this paper, we have considered an overlapping generations model with an environmental dimension, where labor is elastically supplied. In this framework, consumers have then to choose not only between consumption and environmental quality, but between consumption, environmental quality and leisure.

We can summarize the main results as follows. We have first established that it can exist a multiplicity of steady states, even in the case of a Cobb-Douglas technology. Secondly, we have exhibited that stationary solutions are characterized by an inverted-U relationship between pollution and per capita income, namely EKC. Thirdly, we have shown that not only deterministic cycles can emerge, but also endogenous fluctuations due to self-fulfilling expectations, under weaker conditions than those obtained in previous works.

$$\frac{u}{1-u} < w(1) - 1 + \frac{1}{\phi}(\alpha + 1 - w(1)) \tag{46}$$

In other words, at the steady state $(a, k) = (1, 1)$ and for $\alpha > w(1) - 1$, consumer welfare increases as soon as the emission rate of pollution decreases, if $u/(1-u) < w(1) - 1$.

[17] An example of such intergenerational inequality has clearly been established in a more simple framework by Seegmuller and Verchère (2004).

[18] Indeed, when α is smaller than α_I, indeterminacy occurs for $\gamma_u < \gamma_{uT}$ and γ_{uT} varies in the same sense than α.

One implication of our results is that they allow us to enlighten a trade-off concerning the effect of a reduction of pollution flow on consumer welfare. Indeed, a government that would try to reduce the pollution emission rate by unit of capital could face the following contradiction: on one hand, it could push-up consumer welfare at steady state, but on the other hand, create an intergenerational welfare inequality.

Finally, considering the results of section 3.3 and 4 which show first, that there is an EKC relationship between per capita income and pollution which can also be seen as a dynamic long-run relation, and secondly, that endogenous fluctuations can occur around this trend, then one can make the following conjecture. The dynamic relationship between per capita income and pollution flow could be quite different than the one suggested by the EKC, i.e. with a not necessarily inverted-U-shaped curve but rather a more complex pattern. Such intuitive remark is sustained by some empirical studies which establish the existence of "extended-N" relationships (see for example Shafik (1994) on river pollution, or Grossman (1995), Panayotou (1997) and Barrett and Graddy (2000) on SO_2).

Appendix

1. The value γ_{uT}

γ_{uT} is such that $F'(1)/F(1) = 0$ or $1 - T + D = 0$. Using equations (25) or (35)-(36), one obtains:

$$\gamma_{uT} = \frac{(1-2s)(\alpha - w(1) + 1)}{w(1)(\sigma - s) + s(1+\phi)/\phi(\alpha - w(1) + 1)} \tag{47}$$

2. The value γ_{uF}

γ_{uF} is such that $1 + T + D = 0$. Using equations (35) and (36), one obtains:

$$\gamma_{uF} = \frac{\phi(\alpha - w(1) - 1)}{\phi(\sigma w(1) + s) - s(w(1) - 1) - s\alpha(\phi - 1)} \tag{48}$$

2. The value σ_I

σ_I is such that $S(\alpha) = (1 + D_1(\alpha))/T_1(\alpha)$. Using (35), (36) and (37), σ_I is the unique solution of the following equation:

$$a_1\alpha^2 + b_1\alpha + c_1 = 0 \tag{49}$$

with

$$\begin{aligned}
a_1 &= s\left(\phi + \frac{s}{1-s}\right) \\
b_1 &= \frac{s}{1-s}[w(1)[\phi(\sigma + s - 2) - 2s] + 2s - 1] \\
c_1 &= \phi s w(1)\left[w(1) + \frac{s}{1-s}\right] \\
&- \left[\frac{s}{1-s}w(1) + 1\right][(\sigma\phi - s(1+\phi))w(1) + s(1+\phi)]
\end{aligned} \tag{50}$$

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