NBER WORKING PAPER SERIES

HOUSING, CONSUMPTION, AND ASSET PRICING

Monika Piazzesi
Martin Schneider
Selale Tuzel

Working Paper 12036
http://www.nber.org/papers/w12036

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2006

Emails: piazzesi@uchicago.edu, martin.schneider@nyu.edu, and tuzel@marshall.usc.edu. We are particularly indebted to Lars Hansen and Stephen Schaefer. We thank an anonymous referee for comments that greatly improved this paper and the editor, Bill Schwert. We would also like to thank Peter Bossaerts, John Campbell, John Cochrane, Darrell Du.e, John Heaton, Charlie Himmelberg, Per Krusell, Martin Lettau, Sydney Ludvigson, Dino Palazzo, Jonathan Parker, Dick Roll, Steve Ross, Pedro Santa-Clara, Ken Singleton, and Bill Zame. We also thank seminar participants at Caltech, NYU, Northwestern University, Princeton University, Stanford University, Stockholm University, University of Illinois, UC Berkeley, UCLA, UC San Diego, USC, University of Texas at Austin, University of Wisconsin, Yale University, the Siena Conference 2003 in honor of Michael Brennan and the AEA 2004 meetings. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

Housing, Consumption, and Asset Pricing
Monika Piazzesi, Martin Schneider and Selale Tuzel
NBER Working Paper No. 12036
February 2006
JEL No. G0

ABSTRACT

This paper considers a consumption-based asset pricing model where housing is explicitly modeled both as an asset and as a consumption good. Nonseparable preferences describe households' concern with composition risk, that is, fluctuations in the relative share of housing in their consumption basket. Since the housing share moves slowly, a concern with composition risk induces low frequency movements in stock prices that are not driven by news about cash flow. Moreover, the model predicts that the housing share can be used to forecast excess returns on stocks. We document that this indeed true in the data. The presence of composition risk also implies that the riskless rate is low which further helps the model improve on the standard CCAPM.

Monika Piazzesi
Graduate School of Business
University of Chicago
5807 S. Woodlawn
Chicago, IL 60637
and NBER
piazzesi@uchicago.edu

Martin Schneider
NYU and FRB Minneapolis
Economics Department
269 Mercer Street, 7FL
New York, NY 10003
martin.schneider@nyu.edu

Selale Tuzel
University of Southern California
Marshall School of Business
Finance and Business Economics Dept.
Hoffman Hall - 701, MC-1427
701 Exposition Blvd., Ste. 701
Los Angeles, CA 90089-1427
tuzel@marshall.usc.edu

I Introduction

Real estate is an important asset that pays off housing services, a major consumption good. Nevertheless, existing literature on consumption-based asset pricing has paid no particular attention to housing. Indeed, the standard CCAPM approach works with preferences defined over a single aggregate consumption good that lumps together housing services with other "nondurables and services." It is also common to identify a claim to all future consumption, including housing services, with equity.

This paper explores the simplest consumption-based asset pricing model that reserves an explicit role for housing. A representative agent consumes housing services and a numeraire (non-housing) consumption good, both of which can be purchased in frictionless markets. In addition to a claim to future numeraire, the agent is endowed with a housing stock that provides housing services. We calibrate this model to US consumption data and derive predictions for asset prices. We find that it delivers a simple explanation for the long horizon predictability of excess stock returns.

The standard CCAPM focuses on investors' concern with *consumption risk* – asset prices are driven by changes in the conditional distribution of a single factor, aggregate consumption growth. However, actual consumption-savings decisions depend not only on the uncertain overall size of future consumption bundles, but also on their uncertain composition, for example between housing and other consumption. This *composition risk* takes center stage in the present paper – changes in the expenditure share on housing emerge as a second factor that drives asset prices.

In the standard model, investors' concern with consumption risk implies that stock prices move with the business cycle. In recessions, investors expect higher future consumption and try to sell stocks today to increase current consumption. This intertemporal substitution mechanism drives down stock prices in bad times. In our model, investors' concern with composition risk implies that recessions are perceived as particularly severe when the share of housing consumption is low. In severe recessions, a new intertemporal substitution mechanism thus increases the downward pressure on stock prices.

Stock price movements generated by this new mechanism are not only larger, but also qualitatively more realistic than those generated by the standard CCAPM. On the one hand, they occur at frequencies that are much lower than business cycle frequencies, as do stock price movements in the data. The reason our model predicts low frequency swings in stock prices is that the housing share changes slowly over time, so that severe recessions are rare. On the other hand, a concern with composition risk generates price movements in the absence of news about future cash flow or dividends. Indeed, stock prices are volatile in our model even if dividend growth is close to unforecastable, as it is in the data.

Investors' concern with composition risk also suggests a simple explanation for observed long-horizon predictability of *excess* stock returns. Indeed, severe recessions lead to drops in stock prices – and hence increases in expected capital gains – that are not accompanied by large increases in the riskless interest rate. This is because severe recessions typically go along with an increase in the conditional volatility of the housing share. But an increase in composition risk strengthens investors' precautionary savings motive. For riskfree assets, precautionary saving thus mitigates downward pressure on prices caused by the intertemporal substitution mechanism. As a result, bond prices – and hence interest rates – move less than stock prices. Precautionary savings in the face of composition risk also implies that the riskfree rate should be on average lower than what the CCAPM predicts – composition risk thus helps resolve the riskfree rate puzzle.

Our model rationalizes why standard financial indicator variables that involve normalized stock prices, such as the price-dividend ratio and the price-earnings ratio, help forecast excess stock returns. At the same time, it predicts that the expenditure share on housing should forecast excess stock returns. We document that this is indeed the case in the data. This result is remarkable because the housing share is a macroeconomic aggregate. In contrast to other common predictor variables, it is not constructed from stock prices themselves. We show that the forecasting power of the housing share increases with the forecast horizon, as does that of the price-dividend ratio. According to our model, this is because high frequency noise due to changes in numeraire consumption growth becomes less relevant at long horizons, where composition risk considerations matter relatively more.

Composition risk plays a subordinate role in the standard CCAPM, because the empirical implementation of that model relies on aggregate price and quantity indices from the National Income and Product Accounts (NIPA). It is thus implicitly assumed that NIPA statisticians correctly model investors' preferences over housing services and other consumption. In the present paper, we explicitly model preferences over multiple goods: we work with power utility over a CES quantity index that aggregates housing and other consumption. With nonseparable utility, a concern for composition matters for asset pricing because housing consumption affects the marginal utility of numeraire (non-housing) consumption. The resulting pricing kernel is closely tied to macroeconomic data and tightly parameterized. In particular, it depends on the discount factor, the coefficient of relative risk aversion, and the *intra*temporal elasticity of substitution ε between housing and other consumption.

Measuring the real quantity of housing services is difficult. Readily available measures such as square footage only reflect one input into the production of housing services, and the aggregation of inputs involves difficult quality judgments. In fact, a number of recent studies, including the Boskin Commission Report (Boskin et al. 1996), have argued that NIPA real housing quantities are grossly mismeasured. For us, this measurement issue creates two problems. First, we cannot obtain a reliable estimate of the intratemporal elasticity directly from quantity data. Second, it is not desirable to specify the forcing process of the model in terms of real consumption - or, equivalently, real dividends from the two trees. Such a process would have to be estimated using real housing services data, which is likely to produce misleading results for asset pricing.

However, we show that the pricing kernel of a multi-good asset pricing model can be written in terms of the consumption of one of the goods (in our case, non-housing consumption) as well as the *expenditure shares* of the other goods. Data on aggregate housing expenditure is arguably more reliable than data on real housing consumption since its construction involves fewer quality judgments. We thus take as our forcing process the joint distribution of non-housing consumption growth and the expenditure share on housing. The asset pricing properties of the model can then be fully characterized without recourse to quantity data, avoiding the second problem above. In addition, asset prices are informative about the value of the intratemporal elasticity, which helps with the first problem.

Quantitatively, the model generates a sizeable and volatile equity premium together with a low and smooth riskless rate, and it replicates predictability regressions based on the price dividend ratio and the housing share well. We obtain these results for either of two parameterizations. First, we set the intratemporal elasticity to 1.25. This value is close to the point estimate from a cointegrating regression with NIPA data. We also choose high values for the coefficient of relative risk aversion and the discount factor, 16 and 1.24, respectively. As a second parameterization, we use risk aversion of 5 and a discount factor of .99 – standard values in the literature – and set the intratemporal elasticity to 1.05.

Under both parameterizations, the asset pricing moments are essentially the same; in particular, the equity premium is 3.5 percent, the volatility of excess stock returns is about 11 percent and the riskfree rate has a mean of 1.8 percent as well as a volatility of less than 1 percent. In the second case, the premium is thus sizeable and the riskfree rate is low although risk aversion and the discount factor are low and there is no idiosyncratic risk. This is because the volatility of "true" aggregate consumption growth – that is, changes in the unobservable ideal quantity index implied by preferences – is about 5 times larger than the volatility of NIPA consumption growth. In contrast, in our first case, model-implied and NIPA consumption volatility are roughly the same.

We conclude that introducing composition risk helps understand why excess returns are predictable and also makes a partial, but quantitatively relevant, contribution to resolving the volatility and equity premium puzzles. As in previous studies, a high equity premium must be due either to high risk aversion or to high perceived risk. In our context, high perceived risk means high composition risk, which translates into high volatility of the unobservable "true" aggregate consumption process. Such volatility is compatible with smooth consumption expenditure in the data. Importantly, though, whatever the source of premia and volatility, the mechanism for predictability described above operates, as long as the intratemporal elasticity of substitution is above one. Severe recessions (in which the housing share falls) then lead to drops in stock prices that are not associated with bad news about dividends or increases in the riskless interest rate.

The paper proceeds as follows. Section II discusses related work. Section III presents the model and derives our pricing equations. Section IV documents key properties of the data. Section V specifies the forcing process for the model and documents properties of equilibrium returns. The

Appendix contains additional results.

II Related Work

This paper is the first to derive the effects of housing on asset prices in a general equilibrium model. Existing general equilibrium models with housing include Davis and Heathcote (2005), who explore the business cycle implications of an RBC model with a construction sector, and Ortalo-Magne and Rady (2006), who analyze an overlapping generations model to study prices and volume in the housing market. None of these papers is concerned with financial assets. Portfolio choice with exogenous returns in the presence of housing is considered by Cocco (2005), Flavin and Yamashita (2002), and Flavin and Nakagawa (2005).

Consumption-based asset pricing models traditionally assume that there is a single consumption good. In the standard model, equity is represented by a single "tree," the "fruit" of which corresponds to aggregate dividends. The one-good assumption is also maintained in models that - like ours - feature multiple trees, such as Menzly, Santos, and Veronesi (2004) or Cochrane, Longstaff, and Santa-Clara (2005). The distinctive feature of our model is that fruit from two trees are not perfect substitutes in the utility function. This assumption is natural since one of our trees represents the housing stock that provides a unique fruit, namely housing services.

Eichenbaum, Hansen, and Singleton (1988) and Jagannathan and Wang (1996) show that non-separable utility over consumption and leisure does not help explain mean asset returns. Santos and Veronesi (2005) show that the ratio of consumption to labor income forecasts stock returns. However, their pricing kernel is the same as in the standard model, because utility is separable in consumption and leisure. Their result therefore does not arise from composition risk as we have defined it.

Dunn and Singleton (1986), Eichenbaum and Hansen (1990) and Heaton (1993, 1995) consider the consumption Euler equation when utility depends on services from consumer durables. They show that adding consumer durables does not help understand the level of the equity premium. In a more recent contribution to this literature, Yogo (2006) shows that, conditional on high risk aversion, a model with consumer durables can account for time variation in the equity premium, well as the size and value premia. The definition of durables in these papers does not include real estate, while our paper focuses exclusively on real estate. Moreover, we would like to address the volatility puzzle, which leads us to determine asset prices endogenously from our model.

A key difference between real estate and other durables is that NIPA provides a direct measure of service flow for the former, whereas it only reports expenditure on the latter. This unique role of housing services data is also recognized in the literature on home production. For example, Benhabib, Rogerson, and Wright (1991), Greenwood, Rogerson, and Wright (1995), and McGrattan, Rogerson, and Wright (1997) consider models with nonseparable preferences over a home- and a market-produced good. The home-produced good contains housing services, with housing capital as one of the inputs. These papers are interested in the production side, especially the allocation of labor between the home and market production sectors. In the present paper, our focus on asset pricing leads us to abstract from the production side.

The pricing kernel implied by our model is driven by a persistent, heteroskedastic state variable, the housing share. In this respect, our pricing kernel resembles that in Campbell and Cochrane (1999). These authors propose a model in which agents consume a single good, but want to "catch up with the Joneses." Their pricing kernel depends on what they call the consumption-surplus ratio, a parametric function of past aggregate consumption, the parameters of which are inferred from asset market data. The consumption-surplus ratio is persistent and heteroskedastic, which is important for the model to tightly match stock return dynamics. While our model does not perform as well as the Campbell-Cochrane model, our pricing kernel is arguably more closely tied to macro data. Since the housing share is observable, we can estimate persistence and heteroskedasticity directly.[1]

Our results confirm the findings in Cochrane (1991, 1996) who investigates real estate investment as a pricing factor in a production-based approach. Cochrane (1991) documents that real-estate investment growth predicts stock returns. Cochrane (1996) finds that real-estate investment growth

[1] Another difference is that expenditure shares are bounded. As a result, marginal utility in our model is bounded above by the standard expression $c_t^{-1/\sigma}$, where σ is the elasticity of intertemporal substitution and c_t is numeraire consumption. This is in contrast to the Campbell-Cochrane model, where marginal utility increases without bound as the consumption-surplus ratio goes to zero.

matters for the cross section of stock returns. Kullmann (2002) confirms the latter result with alternative real-estate measures. Moreover, the important component in real-estate investment is residential real estate, not commercial real-estate investment (Cochrane 1996, Table 9 on page 615). These findings support our approach of introducing real estate using a consumption-based view, where residential real estate matters to consumers.

Our model incorporates a minimal amount of frictions – the representative agent benchmark we consider obtains when there are complete financial markets, a perfect rental market for housing, and no borrowing constraints. However, recent work by Lustig and Nieuwerburgh (2005) suggests that the effects we stress are relevant also when there are more frictions. Retaining the assumptions of complete markets and a perfect rental market, these authors provide an aggregation result for economies in which collateral constraints prevent perfect risk sharing. They show that the aggregate expenditure share on housing enters the pricing kernel the same way as in our benchmark economy. The new feature of their model is that the pricing kernel also contains a term that depends on the wealth distribution. This latter term – due to incomplete risk sharing, as in Constantinides and Duffie (1996) – further improves the performance of the model.

III Model

A. Setup

There is a large number of identical agents. Preferences over aggregate consumption take the standard form

$$E\left[\sum_{t=0}^{\infty}\beta^{t}u\left(C_{t}\right)\right], \tag{1}$$

where

$$u\left(C_{t}\right)=\frac{C_{t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}},$$

and σ is the *intertemporal* elasticity of substitution. For low values of σ, agents are unwilling to substitute aggregate consumption over time.

Aggregate consumption itself is a quantity index that aggregates two goods, housing services, or shelter, s_t, and non-housing consumption c_t, defined as consumption of all nondurables and services except housing services:

$$(2) \qquad C_t = g\left(c_t, s_t\right) := \left(c_t^{\frac{\varepsilon-1}{\varepsilon}} + \omega s_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The parameter ε represents the *intratemporal* elasticity of substitution between housing services and non-housing consumption. For high values of ε, agents are willing to substitute the two goods within each period. The two goods become perfect substitutes as $\varepsilon \to \infty$ and perfect complements as $\varepsilon \to 0$.[2] Taking the limit as $\varepsilon \to 1$ yields the Cobb-Douglas form. If $\varepsilon = \sigma$, utility is separable.

Let p_t^s and p_t^c denote prices of housing and non-housing consumption respectively. The price p_t^s can be interpreted as rent in a perfect rental market. There are two assets in positive net supply. At date t, a claim to the future stream of non-housing consumption, $\left\{p_{t+j}^c \bar{c}_{t+j}\right\}_{j=1}^{\infty}$, trades at a price q_t^c. Similarly, a claim to the future stream of housing services $\left\{p_{t+j}^s \bar{s}_{t+j}\right\}_{j=1}^{\infty}$ trades at a price q_t^s. The budget constraint is

$$(3) \qquad p_t^c c_t + p_t^s s_t + q_t^c \theta_t^c + q_t^s \theta_t^s = \left(q_t^c + p_t^c \bar{c}_t\right) \theta_{t-1}^c + \left(q_t^s + p_t^s \bar{s}_t\right) \theta_{t-1}^s.$$

where θ_t^c and θ_t^s denote asset holdings. The economy is summarized by the preference parameters β, ω, σ, and ε, as well as stochastic processes $\{\bar{c}_t, \bar{s}_t\}$ for output of the two goods. In equilibrium, we must have $c_t = \bar{c}_t$, $s_t = \bar{s}_t$ and $\theta_t^s = \theta_t^c = 1$. Equilibrium prices are thus a collection of $\{p_t^c, p_t^s, q_t^s, q_t^c\}$ such that the processes of consumption bundles $\{\bar{c}_t, \bar{s}_t\}$ and portfolio holdings $\theta_t^s = \theta_t^c = 1$ maximize utility (1) subject to the budget constraint (3).

Interpretation

We have chosen to focus on the consumption side of housing. As a result, our model only restricts the joint behavior of asset prices and housing consumption; it has nothing to say about quantity

[2] We use standard Hicksian language here: two goods are substitutes if and only if $\varepsilon > 1$. This property can be inferred from data on relative prices and quantities, and has nothing to do with the agent's intertemporal concern for smoothing consumption. Some papers refer to $u_{12} < 0$ as the case where numeraire and shelter are "substitutes", while the case $u_{12} > 0$ is referred as "complements". We refrain from this language here, since the second derivative of the utility function captures both intertemporal and intratemporal tradeoffs.

data from the production side, such as residential investment. Incorporating a richer production structure is an important issue for future research. However, the advantage of our approach is that it is compatible with many different structures on the production side. For example, our approach allows us to abstract from important production-side features such as adjustment costs and indivisibility.

We view housing services as a final good that can be home-produced (by owner-occupiers) or market-produced (by landlords). In either case, the production of housing services involves a variety of different inputs, such as housing capital, time and materials spent on upkeep of the house, access to facilities, and even the nature of neighbors and the number of people living in a house. Among these inputs, some are fixed in the short run, while others can be adjusted quickly at little cost. To model the production side, we would have to take these factor-specific adjustment costs explicitly into account. Here we are only interested in preferences over the final good, the supply of which we take to be exogenous and competitively priced.

This perspective also helps to clarify the nature of fluctuations in housing-services consumption at the individual level. Importantly, these fluctuations should not be thought of as simply fluctuations in square footage or other physical measures of housing capital. After all, housing capital is only one input into the production of housing services. In the short run, the variable inputs listed above are likely to account for a larger part of the volatility. The situation is analogous to the production of non-housing consumption goods, which also involves factors that are difficult to adjust, such as commercial real estate, machines, and equipment.

In the medium run, another important source of shocks to the quantity of housing services is distortionary regulation. For example, rent control effectively distorts the factor mix in the production of housing services. The control caps the price of the final good based on the quantity of a particular input, usually the amount of space. As a result, firms change the factor mix to produce lower quality housing for the given space (see Malpezzi and Turner 2003 for evidence on this effect). This means that the introduction or abolition of rent control can be viewed as shocks to the production side of the economy. Consumers' first-order conditions over the final goods housing services and non-housing consumption hold with or without rent control.

B. Pricing Kernel

To evaluate the model using asset prices and returns quoted in dollars, we need to choose a numeraire. With multiple goods, this choice is not obvious and has important consequences for pricing. Throughout much of the paper, we will use non-housing consumption as the numeraire. We now derive the pricing kernel for this case. The agents' Euler equation implies that the price-dividend ratio v_t of a claim to the nominal dividend stream $\{D_t\}$ solves

$$v_t = E_t \left[M_{t+1} \left(v_{t+1} + 1 \right) \frac{D_{t+1}}{D_t} \frac{p_t^c}{p_{t+1}^c} \right], \tag{4}$$

where dividends are deflated by the price p_t^c of non-housing consumption.

The pricing kernel is the present value of an extra unit of non-housing consumption tomorrow:

$$M_{t+1} = \beta \frac{u'\left(C_{t+1}\right) \ g_1\left(c_{t+1}, s_{t+1}\right)}{u'\left(C_t\right) \ g_1\left(c_t, s_t\right)} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\sigma}} \left(\frac{1 + \omega \left(\frac{s_{t+1}}{c_{t+1}}\right)^{\frac{\varepsilon - 1}{\varepsilon}}}{1 + \omega \left(\frac{s_t}{c_t}\right)^{\frac{\varepsilon - 1}{\varepsilon}}} \right)^{\frac{\sigma - \varepsilon}{\sigma(\varepsilon - 1)}}. \tag{5}$$

The pricing kernel consists of two terms. The first term is familiar from the standard one-good model with power utility. It reflects agents' concern with (numeraire) *consumption risk:* numeraire payoffs are valued more highly in states of the world where numeraire consumption growth is low. The higher the coefficient of relative risk aversion $1/\sigma$, the larger is the effect of consumption risk. If utility over numeraire consumption and other consumption goods is separable ($\varepsilon = \sigma$), the second term in the pricing kernel collapses to 1, and consumption risk alone matters for asset pricing.

When utility is nonseparable, the pricing kernel also reflects consumers' concern with *composition risk*, captured by the second term. Suppose that the intratemporal elasticity of substitution is larger than the intertemporal elasticity ($\varepsilon > \sigma$, or, equivalently, $u_{12} < 0$), the case we consider below. The agent is now more willing to substitute between housing and other consumption within a period than he is to substitute between overall consumption bundles at different points in time. As a result, numeraire is valued highly not only when numeraire consumption tomorrow is lower than today, but also *when the relative consumption of housing services tomorrow is lower than today.*

In other words, numeraire is valued highly in recessions – as in the standard model – but it is valued especially highly in *severe recessions*, when the relative quantity of housing consumption is low. The marginal utility of an extra unit of non-housing consumption is high for severe recession states, because the agent wants to compensate the future shortfall in housing services by substituting non-housing consumption. Consequently, an asset denominated in numeraire (non-housing) consumption is more attractive if it pays out a lot when there is a relative shortfall of housing.

Prices, Quantities, and Expenditure Shares

The pricing kernel (5) involves *real* relative quantities s_t/c_t. However, the price p_t^s and quantity s_t of housing services are difficult to measure. We now show that the pricing kernel can be equivalently written in terms of expenditure shares, for which available data are more reliable, as discussed in Section IV below. We begin with the static first order condition

$$\frac{p_t^c}{p_t^s} = \frac{g_1(c_t, s_t)}{g_2(c_t, s_t)} = \omega^{-1}\left(\frac{c_t}{s_t}\right)^{-\frac{1}{\varepsilon}}. \tag{6}$$

In words, the FOC says that the agent chooses housing and non-housing consumption in each period so that the marginal rate of substitution between the two goods is equal to their price ratio. The FOC thus implies that relative prices and relative quantities move in opposite directions for any value of the elasticity of intratemporal substitution ε.

Multiplying both sides by relative quantities, we obtain the *expenditure ratio*

$$z_t = \frac{p_t^c c_t}{p_t^s s_t} = \omega^{-1}\left(\frac{c_t}{s_t}\right)^{1-\frac{1}{\varepsilon}} = \omega^{-\varepsilon}\left(\frac{p_t^c}{p_t^s}\right)^{1-\varepsilon}. \tag{7}$$

This ratio can take values anywhere between 0 and infinity. In equilibrium, the FOC thus creates a one-to-one relationship between expenditure ratios, relative quantities, and relative prices. The expenditure ratio moves with the relative quantity of non-housing consumption, and against its relative price, if and only if the goods are Hicksian substitutes, that is, $\varepsilon > 1$.

Pricing Kernel in terms of Expenditure Shares

To rewrite the pricing kernel, we also define the *expenditure share on non-housing consumption*

$$\alpha_t = \frac{z_t}{1+z_t} = \frac{p_t^c c_t}{p_t^c c_t + p_t^s s_t}, \tag{8}$$

which is always between 0 and 1. With this definition, some algebra delivers a reformulation of the pricing kernel (5), where the composition risk term depends only on the expenditure share as well as the elasticities ε and σ:

$$M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\sigma}} \left(\frac{\alpha_{t+1}}{\alpha_t}\right)^{\frac{\varepsilon-\sigma}{\sigma(\varepsilon-1)}}. \tag{9}$$

In what follows, we focus on the case $\varepsilon > 1$, where the expenditure share α – like the expenditure ratio z – moves together with relative quantities. A severe recession – a state where the relative consumption of housing is low – is thus associated with a high value of α_{t+1} and a high value of the pricing kernel. We also maintain that intertemporal consumption smoothing is more important than intratemporal smoothing ($\varepsilon > \sigma$) – as before, a severe recession at $t+1$ implies that the pricing kernel is high.

The pricing kernel (9) clarifies the "two factor" structure of the pricing kernel. The standard CCAPM without housing is a one factor model: the pricing kernel depends only on consumption growth, and expected returns therefore depend exclusively on their correlation with consumption growth. With nonseparable utility, the change in the expenditure share emerges as a second factor in our "Housing CCAPM." This composition risk factor drives the asset pricing performance of the model. Indeed, numeraire (non-housing) consumption growth behaves much like NIPA aggregate consumption growth: it is smooth, and its covariance with stock returns (denominated in units of numeraire) is small and positive. With separable utility, tiny values of the intertemporal elasticity σ would thus be needed to generate high equity premia. In Table 1 below, we document that the covariance of stock returns with expenditure share growth $\Delta \ln \alpha_{t+1}$ is negative. This means that stocks have low payoffs during recessions, when non-housing consumption growth is low, and especially low payoffs in severe recessions, when housing consumption is relatively low (and α is high). This generates higher equity premia than under the standard model.

C. Aggregate Consumption as Numeraire

In the previous subsection, we have used non-housing consumption as the numeraire. An alternative is to use aggregate consumption. However, a key feature of our model is that aggregate consumption C_t is not defined according to NIPA conventions, but according to equation (2). This implies that both consumption and inflation series cannot be taken from NIPA but must be constructed from disaggregated data to respect preferences. We now derive the appropriate pricing kernel and inflation series. We then show that this choice of numeraire is less convenient for asset pricing than simply working with non-housing consumption as the numeraire.

With aggregate consumption as the numeraire, the appropriate deflator for nominal dividends is the value of the basket C_t, which is the ideal price index P_t associated[3] with the CES quantity index g:

$$P_t = \left((p_t^c)^{1-\varepsilon} + \omega^{\varepsilon} (p_t^s)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

The new definition of aggregate consumption entails the new true inflation rate P_{t+1}/P_t.

We can express both aggregate consumption growth and true inflation derived from our ideal price index in terms of the (well-measured) inflation and real growth rates of non-housing consumption as well as the expenditure share:

$$\begin{aligned} \frac{C_{t+1}}{C_t} &= \frac{c_{t+1}}{c_t} \left(\frac{a_{t+1}}{a_t} \right)^{\frac{\varepsilon}{1-\varepsilon}} \\ \frac{P_{t+1}}{P_t} &= \frac{p_{t+1}^c}{p_t^c} \left(\frac{a_{t+1}}{a_t} \right)^{\frac{1}{\varepsilon-1}}. \end{aligned} \tag{10}$$

For the *dollar* return $R^{\$i}$ on asset i, the new Euler equation is

$$E_t \left[M_{t+1}^C R_{t+1}^{\$i} \, \frac{P_t}{P_{t+1}} \right] = 1, \tag{11}$$

[3] For any quantity index $g(c,s)$ that is homogenous of degree one, the ideal price index is the expenditure function at utility level one, i.e.

$$\begin{aligned} p(p^c, p^s) &:= \min_{(c,s)} p^c c + p^s s \\ s.t.\ g(c,s) &= 1. \end{aligned}$$

For the optimal consumption bundle (c^*, s^*), we then have $p(p^c, p^s)\, g(c^*, s^*) = p^c c^* + p^s s^*$.

which is based on a new pricing kernel, the present value of an extra unit of aggregate consumption one period ahead. It takes the familiar form

$$M_{t+1}^{C} = M_{t+1}\frac{P_{t+1}}{P_t}\frac{p_t^c}{p_{t+1}^c} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}. \tag{12}$$

Despite their formal similarity, the two Euler equations (4) and (11) point to two reasons why asset pricing in our model will be different from the standard CCAPM. First, consumption growth measured by our quantity index C_t will behave differently from aggregate consumption growth measured by NIPA. Second, our true inflation rate P_{t+1}/P_t will behave differently from the CPI that is usually used to compute real returns. This will have important implications for excess returns, an issue that we turn to next.

Numeraire Inflation and Excess Returns

One advantage of using non-housing consumption as the numeraire is that the inflation rate for non-housing consumption is well-measured and behaves similarly to the CPI. In particular, it is smooth enough to justify the common practice of equating nominal and real excess returns. In contrast, for some parameterizations our constructed true inflation rate P_{t+1}/P_t for the aggregate basket will be too volatile for this practice to be sensible. To see the issue, assume for the moment that the pricing kernel, the dollar return $R^{\$i}$ on asset i and inflation are jointly lognormally distributed. Since $E_t\left[M_{t+1}R_{t+1}^{\$i}\ p_t^c/p_{t+1}^c\right] = 1$ must hold both for asset i and for the riskfree asset with nominal return $R_{t+1}^{\$f}$, we have

$$E_t\left[r_{t+1}^{\$i} - \pi_{t+1}\right] + \frac{1}{2}\text{var}_t\left(r_{t+1}^{\$i} - \pi_{t+1}\right) + E_t\left[m_{t+1}\right] + \frac{1}{2}\text{var}_t\left(m_{t+1}\right) = -\text{cov}_t\left(r_{t+1}^{\$i} - \pi_{t+1}, m_{t+1}\right),$$
$$r_{t+1}^{\$f} - E_t\left[\pi_{t+1}\right] + \frac{1}{2}\text{var}_t\left(\pi_{t+1}\right) + E_t\left[m_{t+1}\right] + \frac{1}{2}\text{var}_t\left(m_{t+1}\right) = \text{cov}_t\left(\pi_{t+1}, m_{t+1}\right),$$

where lower case letters denote logarithms and $\pi_{t+1} = \ln p_{t+1}^c/p_t^c$ is the inflation rate for non-housing consumption. The premium on asset i can then be written as

$$E_t\left[r_{t+1}^{\$i}\right] - r_{t+1}^{\$f} + \frac{1}{2}\text{var}_t\left(r_{t+1}^{\$i} - r_{t+1}^{\$f}\right) = -\text{cov}_t\left(r_{t+1}^{\$i} - r_{t+1}^{\$f}, m_{t+1}\right) + \text{cov}_t\left(r_{t+1}^{\$i} - r_{t+1}^{\$f}, \pi_{t+1}\right).$$

If non-housing inflation, or the CPI, is used to deflate returns, then the last term is small in the data, and the real pricing kernel m_{t+1} can be used to price nominal excess returns $r^{\$i}_{t+1} - r^{\$f}_{t+1}$. In other words, with low inflation volatility, nominal excess returns $r^{\$i}_{t+1} - r^{\$f}_{t+1}$ are a good proxy for the difference between the real return on asset i and a real riskfree asset, the particular excess return that asset pricing models are typically interested in. More generally, this approximation is not accurate.

IV Data

We now present the data used in our empirical work and discuss various measurement issues that arise due to new aspects of our model that have to do with housing.

A. Data on Housing Consumption

To measure housing services, we rely on the National Income and Product Accounts (NIPA). For each consumption category, the NIPA tables report 3 different data series: dollar expenditures on the item per period, a price index, and a quantity index. Unfortunately, the construction of both price and quantity indices is based on the CPI rent component that has been criticized heavily by a number of recent studies, including the Boskin Commission Report (Boskin et al. 1996), Prescott (1997), Hobijn (2003) and Gordon and vanGoethem (2004). However, we now argue that the criticism does not affect the NIPA expenditure series.

The source of the NIPA service flow data for housing are surveys. The questionnaires in these surveys ask a group of households about the dollar amount they spend on housing each period. More precisely, renters are asked for the dollar amount spent on rent, while owners are asked for a dollar estimate of how much they would rent their house for.[4] These dollar amounts are summed up and reported in the NIPA tables as expenditure on housing services each period. The survey data for years that NIPA calls "benchmark years" are from the Decennial Census of Housing and the Survey of Residential Finance. These data are supplemented with additional surveys that are

[4]To the extent that owners make mistakes in estimating the rent on their house, these owner-imputed rent numbers contain measurement error. There are studies that show that house owners only make small mistakes on average when it comes to estimating the property value of their house (for example, Goodman and Ittner 1993). We are not aware of similar studies that investigate the accuracy of rent estimates.

conducted more frequently in the other, non-benchmark years. These surveys include the American Housing Survey and the Current Population Survey. For more details, see U.S. Bureau of Economic Analysis (1990, 2002).

The surveys measure expenditures on housing services per period in dollars, $p_t^s s_t$. NIPA statisticians take these dollar numbers and split them up into a price p_t^s and a quantity s_t index. The split is based on rent information provided by the Bureau of Labor Statistics, the agency that computes the Consumer Price Index. The problem with the rent component of the CPI is its treatment of *housing quality.* For example, the Boskin Report documents that most houses today have indoor plumbing, electricity, heating systems, air conditioning, and other amenities that were not around in 1929, when the NIPA tables started. Moreover, the service provided by a house depends on its surroundings, such as location, infrastructure, pollution etc. These surroundings have also changed for the average house, as more and more people move to the south-west and move to the suburbs (Glaeser and Gyourko 2005). The Boskin Report argues that CPI rents do not appropriately take these quality changes into account.

Mismeasurement of the CPI rent component p_t^s also affects the quantity index s_t since it is computed in NIPA by dividing dollar expenditures $p_t^s s_t$ by p_t^s. We conclude that out of the three series $p_t^s s_t$, p_t^s and s_t, expenditure is the only one that is not beset by measurement problems. This motivates the use of expenditure data in the calibration.

Empirical Properties of the Aggregate Expenditure Share

Figure 1 shows the non-housing expenditure share α_t as a black line. (The gray/green line is the dividend-yield on stocks, but we will ignore it for the moment). The plot uses annual data from NIPA Table 2.2 that goes back to 1929, instead of the short post-war quarterly NIPA sample. We see that α_t varies little over time, which means that consumers spend around the same fraction of their total expenditures on non-housing consumption over time. The expenditure share fluctuates around an average value of 82.6 percent, as shown in Table 1, with a standard deviation of 1.5 percent. Figure 1 also shows some large movements in α_t. These movements, and the associated 1.5 percent volatility number, already hint at one property of preferences for the representative consumer, which is that they are *not accurately described as Cobb-Douglas*, since that would imply

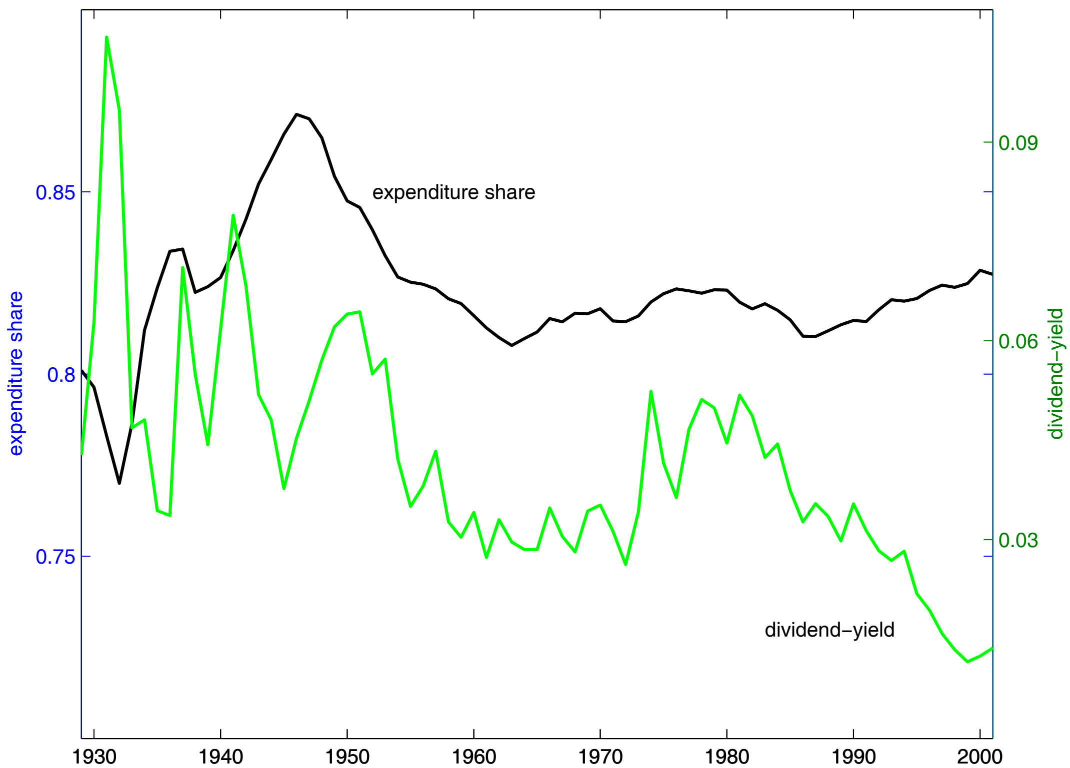

Figure 1: Expenditure share and dividend yield, annual data 1929-2001.

constant expenditure shares. But the volatility is low, which means that ε may not be far from one.

If housing and non-housing consumption are substitutes ($\varepsilon > 1$), movements in the non-housing expenditure share α_t correspond to movements in relative quantities c_t/s_t. Figure 2 shows log relative prices and relative quantities. The plot indicates strong trends in $\ln p_t^s/p_t^c$ and, because of the way the data is constructed, these trends lead to opposite trends in relative quantities $\ln s_t/c_t$. In particular, housing services have become cheaper over time and, as the FOC (6) would predict, more housing services were consumed. Despite these trends, the plot confirms, together with Figure 1, that expenditure shares commove with relative quantities. Indeed, the correlation between the two series is 75 percent. This suggests that ε *is greater than one*.

Another important empirical property of the expenditure share is that even if relative prices and quantities are trending, α_t itself does not trend over time. At the same time, real income per capita has increased dramatically over our sample period. This suggests that the expenditure share

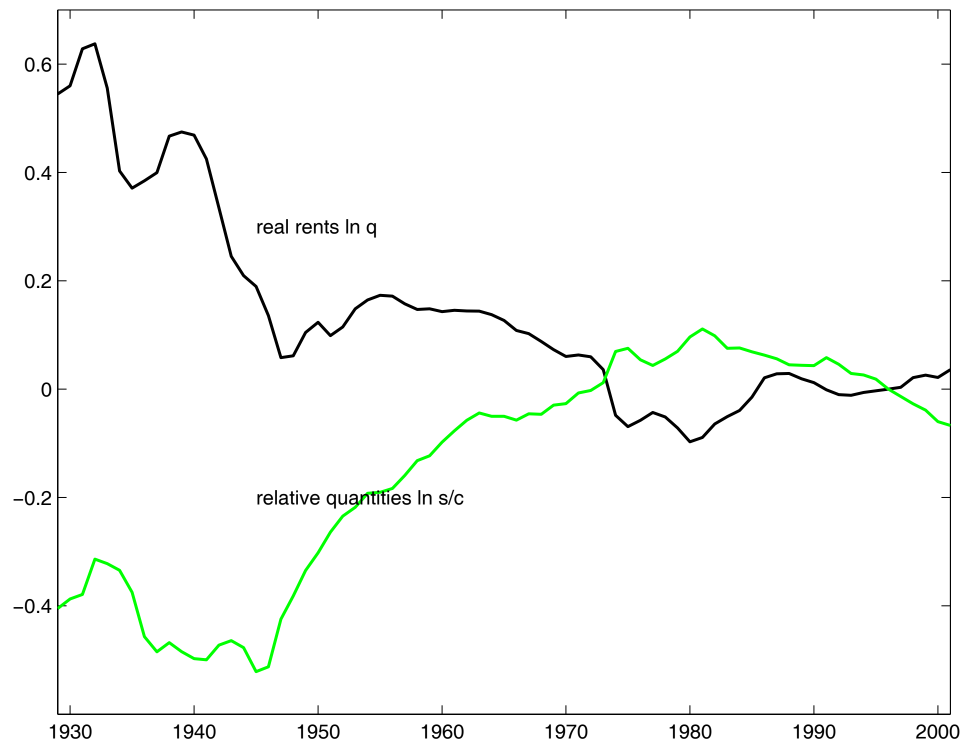

Figure 2: Real rents and relative quantity of housing services, annual data 1929-2001.

does not go up with real income, which means that our homogeneity assumption on preferences does not seem to be at odds with the data. The absence of trends in expenditure shares is also an advantage for econometric work.

The non-housing expenditure share is highly persistent but stationary. Its autocorrelation is 0.965, so that low variations in α_t translate into the low frequency movements that we see in Figure 1. The low frequency is specific to housing and does not obtain for just any good. These empirical properties of α_t imply that composition risk introduces *predictable variations in the pricing kernel (9).* This property is crucial for our asset pricing results.

Microevidence on Expenditure Shares

To investigate the properties of expenditure shares at the microlevel, we use data from the Consumer Expenditure Survey (CEX). In the Appendix, we document that the CEX evidence is remarkably consistent with the aggregate evidence. The expenditure shares on shelter are similar across different groups of households. These groups are classified by income quintile, region of residence, age of the person who rents or owns the house, race, number of persons in the household, housing tenure, and education. For each group, the CEX evidence also suggests that expenditure shares are not volatile over time. This microevidence confirms that the behavior of the aggregate expenditure share is not an artifact of aggregation.

Subsamples

Throughout this paper, we report results for the post-war period as well as for the post-depression period. Figure 1 shows that the behavior of the expenditure share was qualitatively similar during the two periods. In particular, α_t is persistent and positively correlated with the dividend yield on stocks. It is also heteroskedastic: when it is high, it tends to be subject to larger shocks. The sample starting in 1936, rather than in 1947, is informed by particularly large variation in expenditure. This volatility probably shaped agents' perception of composition risk, which makes the post-1936 sample interesting. We also consider the post-war sample to provide a lower bound on the contribution of composition risk.

We do not include the Great Depression in our sample. The reason is that the expenditure share behaved qualitatively very differently during the depression than at any time since then: it fell and then rebounded together with the stock market, and the rebound made it experience a large (positive) shock at a time when it was small. In a post-1929 sample, two of the depression years thus act as large outliers that dominate any empirical averages. This result shows that the Great Depression was accompanied by a shock to housing and stock markets unlike any shock seen since then.

Since we want to apply the standard methodology of calibrating a stationary model to empirical moments, we thus have two options. First, we can specify a data-generating process for the post-1929 sample. This process would have to allow for signs of correlations to flip and conditional variances to change over time in a way to accommodate the special movements of the depression. The problem with this approach is that, since there is only one depression, and only one exit from a depression, many parameters of this process would necessarily be poorly estimated. This poorly estimated process would nevertheless have to be imposed on agents in the model as guiding their expectation formation.

Second, we can leave out the depression from our sample, and specify a data-generating process for the post-1936 period, where the behavior of the key series is qualitatively consistent across subsamples. In effect, we would assume that agents treat the Great Depression as a unique shock, and that the New Deal marks a break in the behavior of U.S. housing and stock markets. In light of the institutional changes introduced in the early 1930s (for example, in the mortgage market), this second option strikes us as more sensible. We thus assume that the agents in our model are like us and consider the Great Depression as caused by a unique shock.

B. Data on Non-housing Consumption

To measure non-housing consumption c_t, we use aggregate consumption of nondurables and services from NIPA Table 7.4. We follow the convention of excluding shoes and clothing, because they may be viewed as durable (see, for example, Lettau and Ludvigson, 2001). However, we *exclude housing services.* Table 1 presents summary statistics of our non-housing consumption series: it grows at an average rate of 2.2 percent and its standard deviation is 1.9 percent per year. For comparison, the penultimate column of Table 1 also reports the corresponding numbers for the conventional consumption growth measure (which includes housing), which are similar. To deflate returns, we also construct the price index p_t^c that exactly corresponds to our definition of non-housing consumption from the NIPA tables. (Details are available upon request.)

For completeness, we also report statistics on the NIPA quantity index on housing services in the last column of Table 1. Again, we want to stress that we do not use this series, because of the quality-judgments and other problems involved in constructing this quantity index. Having said that, several properties of the series are noteworthy. First, the growth rate of NIPA housing consumption $\Delta \ln s_t$ is highly persistent – its autocorrelation is .74 and even goes up to .77 during the postwar sample. The growth rate of non-housing consumption $\Delta \ln c_t$ is much less persistent; its autocorrelation over the two samples is .23 and .40, respectively. Second, both growth rates, $\Delta \ln c_t$ and $\Delta \ln s_t$, are not volatile. Their standard deviations are both around 2 percent and somewhat lower in the postwar sample.

Table 1. Summary Statistics Of Historical Data

	Data Series for Calibration and Model Evaluation								Other NIPA Series	
	$\Delta \ln c_t$	α_t	$\Delta \ln \alpha_t$	$\ln z_t$	r_t^s	r_t^h	r_t^f	$\Delta \ln d_t$	$\Delta \ln C_t$	$\Delta \ln s_t$
mean (%)	2.17	82.6	.01	156.0	6.94	2.52	.75	1.48	2.25	3.85
autocorr.	.23	.965	.56	.964	-.06	.48	.73	.34	.24	.74
Post-war sample										
mean (%)	1.85	82.3	-.09	153.6	7.80	2.09	1.57	1.79	1.98	3.91
autocorr.	.40	.84	.64	.83	.02	.44	.52	.58	.41	.77
Standard Deviations and Correlations										
$\Delta \ln c_t$	1.88									
α_t	.03	1.54								
$\Delta \ln \alpha_t$	.54	.14	.50							
$\ln z_t$	.03	1.00	.14	11.43						
r_t^s	.04	-.02	-.17	-.03	16.56					
r_t^h	.53	.10	.08	.10	.01	2.73				
r^f	.02	-.71	-.42	-.70	.20	.02	3.68			
$\Delta \ln d_t$	.24	.05	-.15	.05	-.02	.28	.16	8.28		
$\Delta \ln C_t$	.98	.07	.51	.07	.004	.53	-.05	.24	1.58	
$\Delta \ln s_t$	-.10	.49	-.21	.41	-.19	-.09	-.48	.04	.04	1.72
Post-war sample: Standard Deviations and Correlations										
$\Delta \ln c_t$	1.46									
α_t	-.45	1.28								
$\Delta \ln \alpha_t$	.34	-.43	.36							
$\ln z_t$	-.46	1.00	-.43	9.39						
r_t^s	.15	.02	-.24	.01	15.36					
r_t^h	.48	-.05	.11	-.05	.05	2.34				
r_t^f	.42	-.63	.03	-.64	.14	-.04	2.86			
$\Delta \ln d_t$	.06	.30	-.32	.31	.20	.16	-.02	5.26		
$\Delta \ln C_t$	.98	-.40	.28	-.41	.10	.48	.33	.08	1.23	
$\Delta \ln s_t$	-.17	.49	-.46	-.48	.50	-.02	-.58	.24	.01	1.67

Note: The summary statistics are computed over the post-depression sample 1936-2001 and over the post-war sample 1947-2001. The data on housing returns are only available until 2000. The middle columns report statistics of NIPA series and returns that are used to calibrate and evaluate the model, while the last two columns consider additional NIPA series. The diagonal numbers are standard deviations, while the numbers below are correlations. Non-housing consumption data $\Delta \ln c_t$ is nondurables and services from lines 6 and 13 from NIPA Tables 2.2 and 7.4, minus shoes and clothing (line 8) and housing services (line 14). The non-housing consumption expenditure share α_t defined in (8) is based on housing expenditures (line 14). The expenditure ratio z_t is defined in (7). Log real stock returns r_t^s, the log real rate r_t^f and dividend growth $\Delta \ln d_t$ are from Robert Shiller's website. Log real housing returns r_t^h are constructed as in equation (21) from NIPA Fixed Asset Tables 2.1, line 68. To deflate returns, we construct our own price index corresponding to our definition of c_t from NIPA Tables 2.2 and 7.4. The growth rate of the bundle $\Delta \ln C_t$ stands for the standard CCAPM measure of consumption growth which includes housing services. The growth rate of housing services $\Delta \ln s_t$ is measured using the NIPA quantity index in Table 7.4 line 14.

C. Financial data

To compare the implications of our model to financial data, we use data on nominal stock prices and corresponding dividends, and the nominal riskfree rate from Robert Shiller's website. Table 1 reports summary statistics for returns, which are deflated with our new inflation rate for only non-housing consumption. Still, the summary statistics for these real returns look familiar. The real returns on stocks have a high mean, 6.9-7.8 percent, and a high volatility, 15.4-16.6 percent. By contrast, the riskfree rate has a low mean, 0.8-1.6 percent, and a low volatility, 2.9-3.7 percent.

To measure returns on housing, we compute returns from the NIPA Fixed Asset Tables, which contain the value of the aggregate housing stock. The Appendix compares our return definition with several alternatives (such as the OFHEO house price index and the National Association of Realtors index), which give similar results. Table 1 shows that the mean real returns on housing are 2.1-2.5 percent, closer in value to the mean riskfree rate than to mean stock returns. The real returns on housing have a low volatility, 2.3-2.7 percent, comparable to the volatility of the riskfree rate. Of course, these numbers can only provide a rough indication, since aggregate house-price indices are smoothed.

V Equilibrium Prices

We now consider asset pricing in an economy where housing and numeraire consumption shocks are the only sources of uncertainty. We calibrate the model based on a VAR for the growth rate and the expenditure share of non-housing consumption. We then compute asset prices for a range of preference parameters.

A. Calibration

As a forcing process, we take the vector $(\Delta \ln c_t, \ln z_t)$, where $z_t = (p_t^c c_t) / (p_t^s s_t)$ is the expenditure ratio defined in (7). We assume that $(\Delta \ln c_t, \ln z_t)$ follows a stationary bivariate VAR with conditionally normal errors. The stationarity of $\ln z_t$ implies that log expenditures on consumption and housing are cointegrated. The intratemporal FOC (6) implies that the same is true for relative quantities and relative prices. We impose restrictions on the VAR to capture three key properties of the data: (i) consumption growth is not forecastable, (ii) the log-expenditure ratio is persistent and past consumption growth does not help forecast it, and (iii) shocks to consumption growth are homoskedastic, while shocks to the log-expenditure ratio are heteroskedastic.

Dynamics of consumption growth and expenditure shares

We assume that consumption growth is i.i.d.,

$$\Delta \ln c_{t+1} = \mu_c + u_{t+1}^c, \tag{13}$$

where the consumption growth shock u_{t+1}^c has mean zero and variance v_c. While the Table 1 shows some positive autocorrelation in the data, Heaton (1993) and others have argued that this autocorrelation may be entirely due to time aggregation. We therefore assume that expected consumption growth μ_c is constant. We also assume that the variance of consumption growth v_c is constant. We set these parameters equal to their sample values from Table 1.

A regression of $\ln z_{t+1}$ on its lagged value and $\Delta \ln c_t$ shows that consumption growth is barely

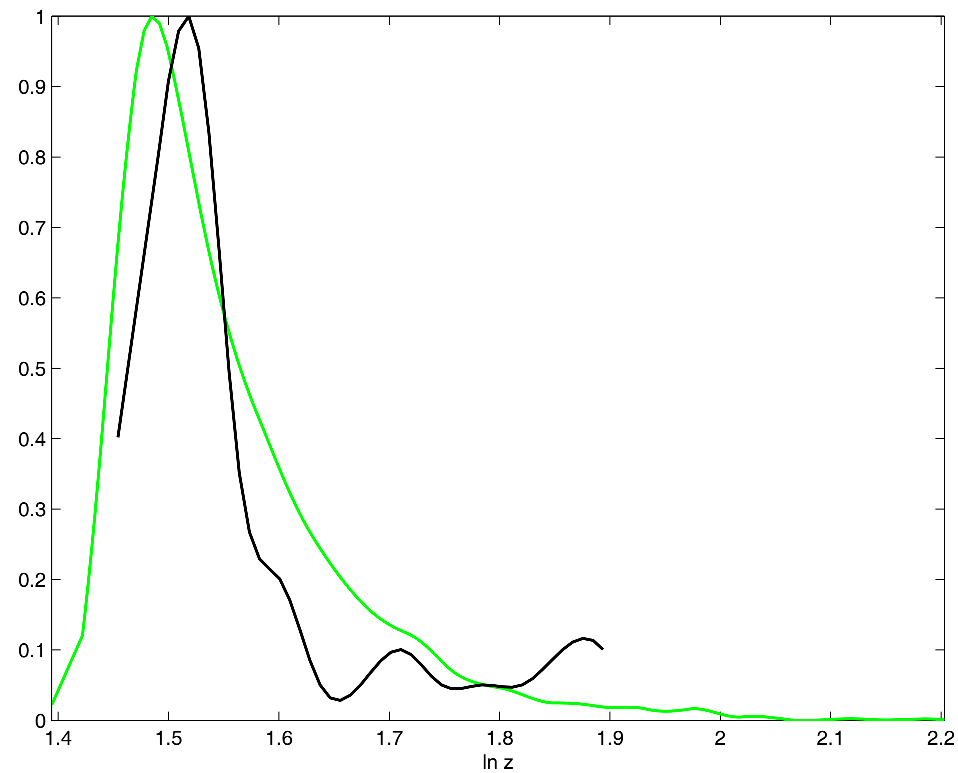

Figure 3: Empirical density of the log expenditure ratio $\ln z$ (black line) and simulated density (gray line).

significant. We therefore specify the log expenditure ratio as the autoregressive process

$$\ln z_{t+1} = (1-\rho)\,\mu_z + \rho \ln z_t + u^z_{t+1}, \tag{14}$$

where u^z_{t+1} has mean zero and conditional variance $v_{z,t}$. The shocks u^c_{t+1} and u^z_{t+1} are conditionally normal. Their correlation is negative in the data, which turns out to have negligible effects on our results. For parsimony, we therefore set the correlation to zero.

The shocks u^z_t to the log expenditure ratio show substantial heteroskedasticity – their variance increases with $\ln z_t$. We specify the conditional variance as

$$v_{z,t} = a_1 \max\{\ln z_t, \overline{z}\} - a_0. \tag{15}$$

The conditional variance is thus linear in $\ln z_t$ except for small $\ln z_t$, where it is constant.

Table 2 reports parameter estimates and t-statistics for the VAR. The estimation is in two steps. The first step estimates μ_z and ρ in equation (14) using ordinary least squares and saves the squared residuals. The second step regresses squared residuals on a constant and $\ln z_{t-1}$ to estimate a_0 and a_1. The precise value of $\overline{z}$ does not matter much in our application; we fix it to match the unconditional variance of $\ln z$. Figure 3 shows the empirical and simulated densities of the estimated process for $\ln z$. The black empirical density is skewed to the left, and this skewness is well captured by the grey density of the simulated data-generating process.

Table 2. Estimates Of Expenditure Ratio Dynamics

μ_z	ρ	a_0	a_1	$\overline{z}$
1.56	0.96	−0.0117	0.0081	1.47
(52.29)	(13.20)	(−3.82)	(3.92)	
		Post-war data		
		−0.0009	0.008	
		(−1.97)	(2.48)	

Note: The parameters are estimated in two steps, as explained in the text. T-statistics in brackets are based on 4 Newey-West lags.

Table 2 shows that the heteroskedasticity of $\ln z_t$ is significant. In particular, the estimate of a_1 is significantly positive, as expected. The estimated process captures the heteroskedasticity in the data well. Intuitively, shocks to $\ln z_t$ are larger in times when the expenditure ratio is high. We can use the FOC (6) to interpret this feature in terms of quantities. If housing and non-housing consumption are substitutes $(\varepsilon > 1)$, times with relatively little housing correspond to times when the volatility of shocks is higher. In other words, times with little housing are times when uncertainty increases.

Long-Lived Assets

To price equity, we specify dividends as

$$\Delta \ln d_{t+1} = k \Delta \ln c_{t+1} + u^d_{t+1}, \tag{16}$$

where k is a constant and u^d_{t+1} is i.i.d. normal with mean zero and variance v_d, independent of all other shocks. Our results are based on $k = 1$ and a variance v_d that matches the variance of dividend growth. The advantage of this specification is that we can allow dividend growth to be more volatile than consumption growth, and we can also allow for imperfect correlation between consumption growth and dividend growth. From Table 1, we get $v_d = 8.28^2 - 1.88^2$ percent for the long sample, $v_d = 5.26^2 - 1.46^2$ percent for the postwar sample. For the long sample, this approach matches the empirical correlation between consumption and dividend growth exactly, while it somewhat overstates the correlation in the postwar sample. Below we will discuss the implications of alternative specifications.

A difference equation for the price-dividend ratio is obtained by plugging the discount factor (9) into the pricing equation. For housing, we calculate an analogous price-dividend ratio by equating the value of the housing stock with the present discounted value of all future housing services, $q_t s_t = c_t \exp(-\ln z_t)$:

$$\text{(17)} \qquad \begin{aligned} \upsilon_t^s &= E_t \left[M_{t+1} \left(\upsilon_{t+1}^s + 1 \right) e^{k\Delta \ln c_{t+1} + u_{t+1}^d} \right] \\ \upsilon_t^h &= E_t \left[M_{t+1} \left(\upsilon_{t+1}^h + 1 \right) e^{\Delta \ln c_{t+1} - \Delta \ln z_{t+1}} \right]. \end{aligned}$$

Conveniently, the solution υ_t^s reduces to the price of a consol bond if $k = 0$ and $v_d = 0$.

The dividend processes of all the assets we want to price can be written as functions of the forcing process $(\Delta \ln c_{t+1}, \ln z_t)$ plus i.i.d. shocks. Given parameters ε and σ and the estimated distribution of the forcing process, we determine asset prices as stationary solutions to the stochastic difference equation (17). Although we have not specified an exogenous endowment process explicitly, the resulting prices are equilibrium prices for an economy summarized by a tuple $\{\beta, \sigma, \varepsilon, \omega, (\bar{c}_t, \bar{s}_t)\}$, as in Section III. Indeed, the intratemporal FOC must hold in any equilibrium of such an economy. We can thus define a jointly stationary and Markov process $(\Delta \ln c_t, \ln(c_t/s_t))$ by (6), for some positive scalar ω.[5] An endowment process $(\bar{c}_t, \bar{s}_t)$ can then be constructed by fixing a time zero level of consumption c_0.

[5] Our approach does not identify the parameter ω. This is not necessary, since the pricing kernel implies that when expenditure data is available, there is no need to know ω in order to fully characterize the asset pricing implications of the model. Of course, this does not mean that ω does not matter for asset pricing. For example, if ω is equal to zero, housing is not valued, and we are back in the one-good case. The point is that expenditure shares already contain the information about ω that is needed. For example, any nonzero amount of expenditures on housing implies that the value of ω cannot be zero.

The pricing kernel for the economy just defined takes the form (9), and its distribution is by construction the same as that of the expenditure-share-based kernel we use in our empirical work. Dividends on our assets can also be expressed in terms of $(\Delta \ln \bar{c}_{t+1}, \ln(\bar{c}_t/\bar{s}_t))$ by (6). Moreover the Markov structure implies that their price-dividend ratios at time t depend only on $(\Delta \ln c_t, \ln(c_t/s_t))$, as well as on the parameters $(\beta, \sigma, \varepsilon, \omega)$ that enter the pricing kernel (9). In other words, the price-dividend ratios in the constructed economy follow the same stochastic difference equation (17) that we use to compute prices below.

Preference Parameters

For the elasticity of intertemporal substitution, we follow Hall (1988) who estimates σ to be around 0.2. Studies based on micro data find values for σ that are somewhat higher, but not by much. For example, Runkle (1991) reports an estimate of 0.45 using micro data on food consumption. Attanasio and Browning (1995) report estimates using CEX data between [0.48,0.67]. We will refer to $\sigma = 0.2$ as our *lo risk aversion* benchmark and then consider higher values.

Estimates of the intratemporal elasticity are more difficult to obtain. The problems with data quality in Section IV imply that direct estimation of the intratemporal FOC (6) is problematic. We therefore report results for a range of ε values. We focus only on values of ε greater than 1. This choice is based on two pieces of evidence. First, this range is suggested by the existing empirical literature. Ogaki and Reinhart (1998) estimate ε with aggregate data on durable consumption. Their Table 2 on page 1091 gives $[1.04, 1.43]$ as a 95 percent confidence interval for ε. The test for unitary elasticity $\varepsilon = 1$ is thus rejected. Papers in the home-production literature also estimate ε to be above one. Benhabib et al. (1991) obtain $\varepsilon = 2.5$ and McGrattan et al. (1997) get 1.75.

Second, we estimate the cointegrating relationship implied by the intratemporal FOC (6) between NIPA quantity and price data for housing services. The idea is that even if these data are mismeasured, they may still provide useful information about long-run trends. The Appendix reports the results of this exercise. The key parameter in the cointegrating relationship is ε, which we estimate to be 1.27 with a standard error of 0.16. We also estimate ε based on Euler equations for excess returns in Subsection F. We obtain $\varepsilon = 1.17$ and $\varepsilon = 1.24$, but these come with huge standard errors. These pieces of evidence suggest that ε is above one.

B. Numerical Results

Panel A of Table 3 reviews properties of the standard CCAPM. We report first and second moments of annual equity premia, consol premia, and the riskless rate, all in logarithms. We consider two parameterizations, which will be compared to two benchmark versions of the housing model below. In the case of *lo risk aversion*, the coefficient of relative risk aversion is $1/\sigma = 5$, and the discount factor is $\beta = .99$. For the *hi risk aversion* case, we set the coefficient of relative risk aversion to $1/\sigma = 16$, and we also make the agent more patient $\beta = 1.24$. The two cases imply roughly the same equity premium and riskfree rate.

As is standard in the literature, aggregate consumption growth and dividend growth are i.i.d. lognormal processes of the forms (13) and (16) with $k = 1$. Therefore, the riskfree rate and the price-dividend ratio are constant. Moreover, expected excess returns are constant and therefore not predictable. Panel A of Table 3 also illustrates other familiar problems with the CCAPM. The CCAPM predicts a high riskfree rate of almost 12 percent – the riskfree rate puzzle – as well as an equity premium of less than 60 basis points, or 0.6 percent – the equity premium puzzle. In addition, the volatility of stock returns in the model is too small; this is the volatility puzzle. For example, $\sigma\left(er^{s}\right) = \sqrt{k^{2} \times v_{c} + v_{d}}$ is 1.6 percent when we assume that dividends equal consumption as in Mehra and Prescott (1985), so that $k = 1$ and $v_{d} = 0$. In Table 3, the volatility of stock returns is higher than that, 8.2 percent, because we allow for orthogonal shocks to dividend growth, $v_{d} \neq 0$.

Panel B of Table 3 reports the same financial moments for the model with housing, together with first and second moments of annual housing returns. We compute the model for candidate values of the intratemporal substitution ε above one. In particular, Panel B emphasizes two benchmark cases in bold-face. In the first benchmark case, *hi perceived risk*, the elasticity of intratemporal substitution ε is set close to the Cobb-Douglas case. Equation (10) shows that true aggregate consumption - the quantity index implied by preferences - becomes more volatile as ε goes to one. We combine $\varepsilon = 1.05$ with lo risk aversion ($1/\sigma = 5$ and $\beta = 0.99$). The second benchmark case has *hi risk aversion* ($1/\sigma = 16$ and $\beta = 1.24$) as well as an intertemporal elasticity of substitution $\varepsilon = 1.25$ that is close to the point estimate ($\varepsilon = 1.27$) from Appendix C.

Table 3. Model-Implied Moments Of Returns (%)

Panel A. Standard CCAPM

$1/\sigma$	β	ε	$E\left(r^f\right)$	$E\left(er^s\right)$	$E\left(er^b\right)$	$E\left(er^h\right)$	$\sigma\left(r^f\right)$	$\sigma\left(er^s\right)$	$\sigma\left(er^b\right)$	$\sigma\left(er^h\right)$
5	0.99		11.9	0.1	0.0		0.0	8.2	0.0	
16	1.24		11.1	0.6	0.0		0.0	8.2	0.0	

Panel B. Housing-CCAPM

$1/\sigma$	β	ε	$E\left(r^f\right)$	$E\left(er^s\right)$	$E\left(er^b\right)$	$E\left(er^h\right)$	$\sigma\left(r^f\right)$	$\sigma\left(er^s\right)$	$\sigma\left(er^b\right)$	$\sigma\left(er^h\right)$
				Homoskedastic Shocks: $v_{z,t} = \text{constant}$						
5	0.99	1.05	2.2	6.5	5.7	7.6	5.0	18.9	14.9	19.9
16	1.24	1.25	2.2	5.3	4.5	6.3	4.1	17.9	14.0	18.5
				Heteroskedastic Shocks: $v_{z,t} = a_1 \max\{\ln z_t, \bar{z}\} - a_0$						
5	0.99	1.25	11.1	1.0	0.3	0.8	0.7	8.6	2.0	5.0
		1.10	9.0	1.4	0.8	1.5	0.9	9.1	3.4	6.8
		1.05	**1.8**	**3.5**	**2.5**	**3.7**	**0.9**	**11.4**	**5.9**	**10.1**
		1.04	−2.3	6.0	4.4	5.6	1.8	14.9	9.1	13.6
3		1.05	5.2	1.8	1.1	1.6	0.9	10.4	5.3	9.0
4			4.2	2.5	1.7	2.7	0.6	10.7	5.5	9.7
6			−1.9	5.7	4.1	5.6	2.2	13.8	4.1	12.5
7			−4.1	9.8	5.1	6.2	3.7	18.8	10.1	13.8
16	0.99	1.25	25.5	1.1	0.1	2.4	0.5	8.4	0.1	4.5
	1.24	**1.25**	**1.8**	**3.5**	**2.5**	**3.9**	**0.5**	**11.9**	**6.6**	**10.5**
				Post-war Results						
5	0.99	1.05	7.5	3.0	2.7	3.1	7.5	17.0	15.2	16.7
	1.05		1.0	3.7	3.2	3.8	7.5	21.4	19.4	20.9
16	1.24	1.25	2.9	2.9	2.4	3.1	6.4	17.3	15.6	16.8
	1.26		1.8	3.1	2.4	3.1	6.4	18.1	16.4	17.6

Panel C. Properties of Aggregate Bundle for different ε

	Long Sample				Post-war Sample			
For ε-value:	1.04	1.05	1.10	1.25	1.04	1.05	1.10	1.25
$\mu\left(\Delta \ln C\right)$	2.0	2.0	2.1	2.1	4.4	3.9	3.0	2.4
$\rho\left(\Delta \ln C\right)$	.59	.60	.62	.56	.62	.62	.62	.56
$\sigma\left(\Delta \ln C\right)$	12.0	9.5	4.7	2.2	9.0	7.2	3.7	1.8

The two benchmark cases in Panel B deliver exactly the same mean riskfree rate and equity premium. The model generates a low and smooth riskfree rate with a mean of 1.8 percent and a volatility of 0.9 percent. The equity premium is high and excess stock returns are volatile; their mean is 3.5 percent and their volatility above 11.4 percent. In contrast, the consol premium in the model is smaller and smoother. Its mean is 2.5 percent and its volatility is below 6.6 percent. The model also does a reasonable job on housing returns in both cases. The mean housing premium is roughly 3.7 percent with a volatility of roughly 10.1 percent.

Panel C reports properties of model-implied aggregate consumption. With *hi perceived risk*, the volatility of the aggregate bundle in equation (10) is 9.5 percent in the long sample. The aggregation method used by NIPA produces an aggregate bundle with lower volatility of 1.6 percent (from Table 1). The higher volatility perceived by agents in our model is partly due to autocorrelation: the agent perceives the bundle to have a first autocorrelation of 60 percent, while NIPA aggregation methods result in a bundle that with only 23 percent autocorrelation. Panel C also shows that with *hi risk aversion*, the true consumption bundle behaves more like the NIPA bundle.

Figure 4 plots asset prices as a function of the single state variable, the log expenditure ratio $\ln z_t$. The figure – and all other figures in this paper – is based on *hi perceived risk* and equation (16) with $k = 1$ and a volatility v_d of orthogonal dividend shocks that matches the volatility of dividend growth. All prices of long-lived assets are decreasing in $\ln z_t$, with stock prices showing the most sensitivity.[6] The model thus correctly predicts the positive comovement of α_t and the dividend-yield on stocks in Figure 2. Importantly, the movements in the stock price are not due to changes in expected dividend growth, because consumption growth is i.i.d.. As a result, stock price movements are mostly driven by news about future discount rates, captured by α_t, rather than news about future dividends. This means that composition risk generates the "right type" of volatility, in line with the empirical findings of Cochrane (1994).

Decreasing the elasticities of substitution ε and σ increases the impact of composition risk. Table 3 illustrates both cases. As ε goes to one, the equity premium increases, the average riskless rate decreases, and all asset prices become more volatile. A lower σ has a similar effect on premia

[6]Kinks in the price function for low values of $\ln z_t$ occur since the conditional variances of the innovations become constant as $\ln z_t$ drops below $\bar{z}$. The results are not sensitive to the choice of $\bar{z}$.

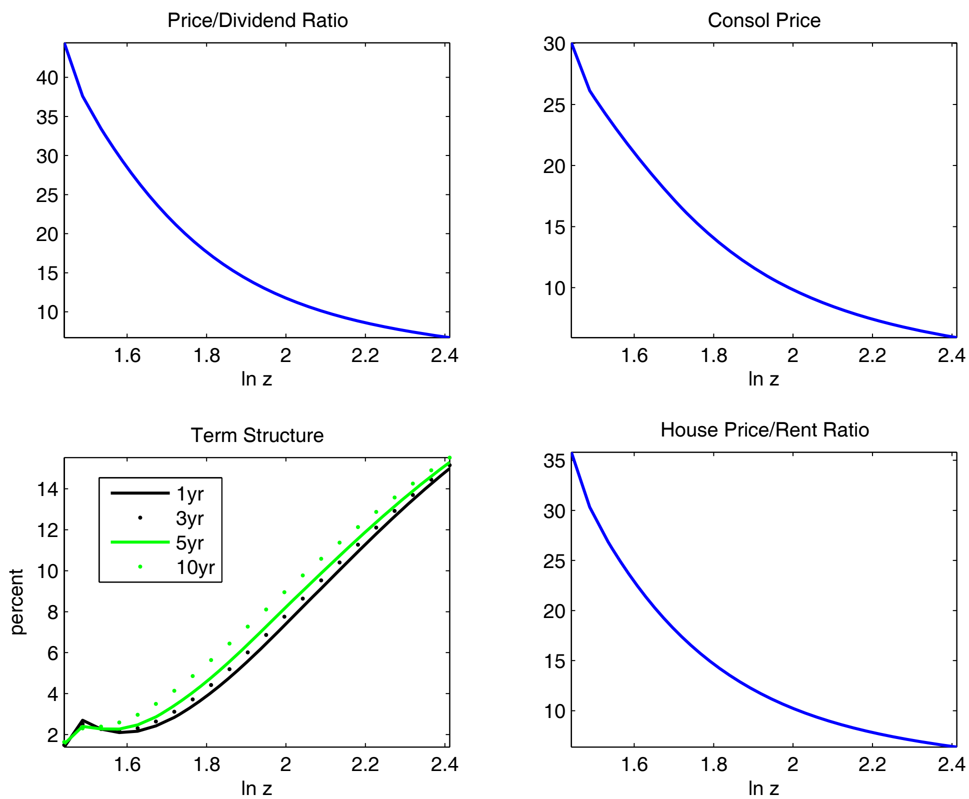

Figure 4: Asset Prices as a Function of $\ln z_t$.

and asset-price volatility, since it also increases the exponent $(\varepsilon - \sigma)/\sigma(\varepsilon - 1)$ in the pricing kernel (9). However, the parameter σ leaves the properties of the aggregate bundle and its price index unchanged. Lowering σ thus has the standard effect of increasing the riskless rate since agents who like to substitute consumption push up the riskless rate in a growing economy. To keep the riskfree rate low, we need to make the agent more patient. This can be accomplished with a higher discount factor β.

C. Volatility and Premia

Our numerical results are based on the nonlinear pricing kernel (9). To gain intuition about the unconditional moments reported in Table 3, it is helpful to linearly approximate the log kernel. We write $M_{t+1} = \beta e^{-(1/\sigma)\Delta \ln c_{t+1} + [(\varepsilon-\sigma)/\sigma(\varepsilon-1)]\Delta \ln \alpha_{t+1}}$ and linearize $\Delta \ln \alpha_{t+1}$ around the point

$z_{t+1} = z_t$. We thus obtain[7]

$$M_{t+1} \approx \beta \exp\left(-\frac{1}{\sigma}\Delta \ln c_{t+1} + \frac{(\varepsilon - \sigma)}{\sigma(\varepsilon - 1)}(1-\alpha_t)\Delta \ln z_{t+1}\right). \tag{18}$$

Riskfree Rate

Conditional normality of $(\Delta \ln c_t, \ln z_t)$ and the approximation (18) also lead to a convenient formula for the riskless interest rate:

$$\begin{aligned} r_{t+1}^f \approx & -\ln\beta + \frac{1}{\sigma}\mu_c - \frac{1}{2\sigma^2}v_c \\ & + (1-\alpha_t)\left\{-\frac{(\varepsilon-\sigma)}{\sigma(\varepsilon-1)}(1-\rho)(\mu_z - \ln z_t) - \frac{1}{2}\left[\frac{\varepsilon-\sigma}{\sigma(\varepsilon-1)}\right]^2(1-\alpha_t)v_{z,t}\right\}. \end{aligned} \tag{19}$$

The effect of consumption risk on the riskfree rate is familiar and is captured by the first line of the equation. If consumption is expected to grow, agents try to borrow, pushing the interest rate up. If consumption growth becomes more uncertain, agents try to engage in precautionary saving, pushing the interest rate down. Both effects are stronger the more agents want to smooth consumption (low σ). Since consumption is not very volatile, the precautionary savings effect is small for reasonable values of σ; this is the riskfree rate puzzle. Also, consumption risk does not lead to time variation in interest rates, because consumption growth is not forecastable and its variance is constant over time.

The effect of composition risk on the interest rate is represented by the expression in braces. The presence of composition risk implies that the riskfree rate is lower on average. This is because agents worry about composition risk and therefore attempt to save more on average. Formally, suppose the expenditure ratio $\ln z_t$ is equal to its unconditional mean μ_z. The first term in braces then collapses to zero, while the second term is negative, since α_t is always smaller than 1. Precautionary savings induced by the volatility $v_{z,t}$ of shocks to the expenditure ratio thus pushes the riskfree rate down. Therefore, *composition risk helps resolve the riskfree rate puzzle.*

[7] The approximation is not exact. In particular, it masks the fact that in the nonlinear kernel, correlation with consumption growth will be also be weighted differently as α_t changes. This effect will be made explicit by picking a different linearization point. For example, one could assume an AR(1) process for $\ln z_t$ and linearize around the conditional mean. The current approximation is simpler and is sufficient to interpret the computational results, which are based on the true nonlinear kernel.

Composition risk also leads to variation in the riskless rate. There are two effects at work. First, there is a new intertemporal substitution effect: agents try to borrow in severe recessions, when the expenditure ratio $\ln z_t$ is high and housing consumption is relatively low. In severe recessions, agents correctly expect that better times are ahead, because the expenditure ratio will revert to its mean. In equation (19), the intertemporal substitution effect is captured by the first term in braces: the interest rate increases when $\ln z_t$ is higher than its unconditional mean μ_z.

A second effect is that the strength of the precautionary savings motive varies over time with the amount of composition risk. Agents worry more about composition risk in severe recessions, when shocks to the expenditure ratio are larger. Indeed, by (15), an increase in $\ln z_t$ goes along with an increase in the conditional variance $v_{z,t}$ and thereby increases the second term in braces. As a result, agents try to save more in severe recessions and push the interest rate down. The precautionary savings effect thus counteracts the intertemporal substitution effect and *reduces the volatility of the riskless rate.*[8]

At the same time, heteroskedasticity does not fully neutralize the intertemporal substitution effect. This is because the impact of heteroskedasticity is diminished as the non-housing share rises: the second term is multiplied by $(1-\alpha_t)$. Indeed, as $\alpha_t \to 1$, the precautionary savings effect will vanish faster than the intertemporal substitution effect. This implies that the interest rate for high α_t will be higher than its mean. At least for high α_t, we can thus expect an interest rate function that is increasing in α_t. Figure 3 and Figure 4 confirm the intuition that the riskless rate is very stable in the part of state space which has highest probability. Indeed, it is non-monotonic in this area, as a result of the counteracting precautionary savings and intertemporal smoothing effects.

Risk Premia

The expected return r^i on an asset in excess of the riskless rate is now approximately

$$E_t\left(r^i_{t+1}\right) - r^f_{t+1} + \frac{1}{2} var_t(r^i_{t+1}) \approx \frac{1}{\sigma} cov_t\left(\Delta \ln c_{t+1}, r^i_{t+1}\right) - (1-\alpha_t)\frac{(\varepsilon-\sigma)}{\sigma(\varepsilon-1)} cov_t\left(\Delta \ln z_{t+1}, r^i_{t+1}\right). \tag{20}$$

[8] As mentioned in Section II, Campbell and Cochrane (1999) also rely on heteroskedasticity. Their estimates of the heteroskedasticity parameters are, however, based on moments of asset prices – the volatility of the riskfree rate. Our estimates from Table 2 are only based on macroeconomic data.

The risk premium on any asset depends on the conditional covariance of its return with two factors, non-housing consumption growth and the change in the expenditure ratio. The conditional covariance of returns and non-housing consumption growth is small. However, Figure 4 has shown that the prices of long-lived assets such as stocks and consols move opposite to the composition risk factor $\ln z_t$, so that returns are negatively correlated with $\Delta \ln z_t$. In light of the second term on the right hand side of (20), this is exactly what is needed to generate additional premia due to composition risk. In addition, leverage and growth in dividends implies that stock prices are more volatile than consol prices. As a result, the equity premium is larger and more volatile than the consol premium.

The model also implies that expected excess returns vary over time. This is illustrated in Figure 5, which plots the model-implied dividend yield and expected returns over the post-war period. It is apparent that the dividend yield is a slow-moving state variable which forecasts returns. This is consistent with recent empirical evidence, which we compare more closely with the model's implications in Subsection D. Moreover, the model predicts that α_t, which is highly correlated with the dividend yield, should also be a good forecasting variable. This is indeed the case in the data, as documented in Subsection E.

Price Volatility of Long–Lived Assets

To understand the role of discount rate news for prices, it is useful to first abstract from heteroskedasticity. We can then use (18) to define a state-dependent discount rate $\delta_{t+1} = -\ln M_{t+1}$ and rewrite the price-dividend ratio (17) as

$$\begin{aligned}
\upsilon^s(\ln z_t) &= E_t\left[e^{-\left(\delta_{t+1}+k\Delta \ln c_{t+1}+u^d_{t+1}\right)}\left(1+\upsilon^s(\ln z_{t+1})\right)\right] \\
&= E_t\left[\sum_{j=1}^{\infty} \exp\left(-\sum_{i=1}^{j}\delta_{t+i}\right)\exp\left(\sum_{i=1}^{j} k\Delta \ln c_{t+i}+u^d_{t+i}\right)\right] \\
&= \sum_{j=1}^{\infty} \beta^j e^{\left((k-1/\sigma)\mu_c+\frac{1}{2}(k-1/\sigma)^2 v_c+\frac{1}{2}v_d\right)j} E_t\left[\exp\left(-\frac{(\varepsilon-\sigma)}{\sigma(\varepsilon-1)}\sum_{i=1}^{j}(1-\alpha_{t+i})\Delta \ln z_{t+i}\right)\right] \\
&= \sum_{j=1}^{\infty} \upsilon^*_j(k)\, w_j(\ln z_t)\,.
\end{aligned}$$

Here $\upsilon^*_j(k)$ is the present discounted value, adjusted for consumption risk and normalized by current

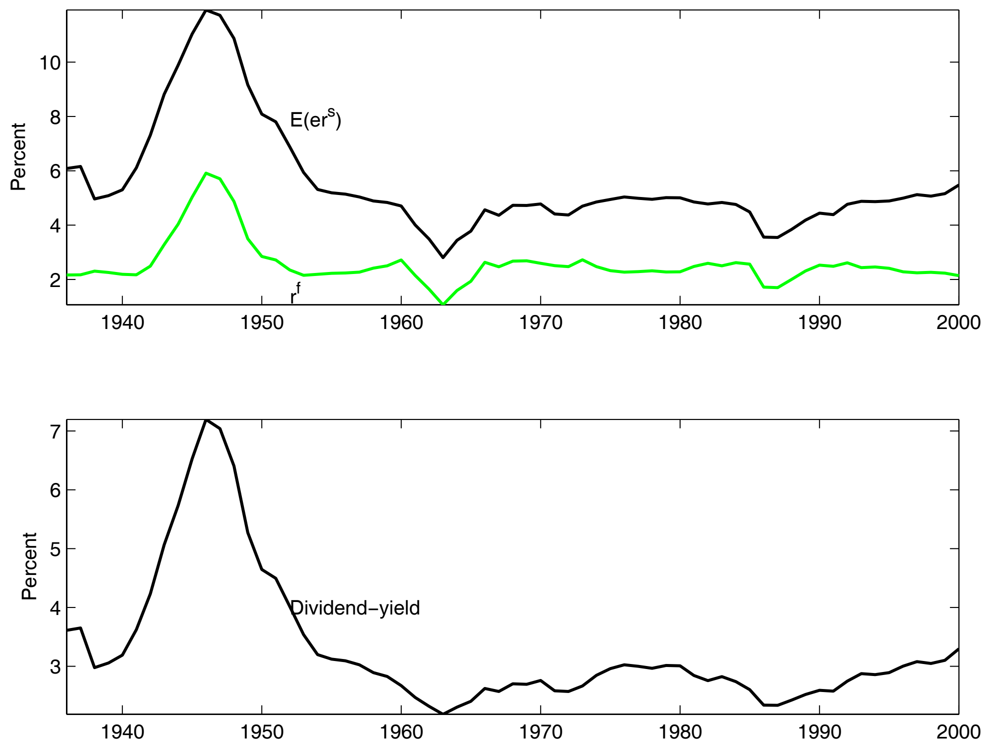

Figure 5: Model-implied expected excess returns and dividend yield.

dividends, of a claim to dividends in period $t+j$. In other words, it is the price of such a claim in the standard CCAPM divided by current dividends. With i.i.d. consumption, this ratio is constant, which amounts to a version of the volatility puzzle. In our Housing CCAPM, volatility is induced by the new *discount factor for composition risk* $w_j(\ln z_t)$. Innovations to $\ln z_t$ provide news about current and future discount rates δ_{t+j}. In line with the empirical findings of Cochrane (1994), it is this type of news, not news about dividends, that accounts for most changes in prices.

The formula also clarifies the relationship between stock and consol prices. On the one hand, as long as υ_j^* is increasing in k, the price-dividend ratio is larger and more volatile than the consol price. The random factor $w_j(\ln z_t)$ affects consols and stocks in the same way; differences across assets exist only to the extent that there are differences in $\upsilon_j^*(k)$. The latter is increasing if $\mu_c > \frac{1}{2}(1/\sigma - k)\, v_c + \frac{1}{2} v_d / (1/\sigma - k)$. An increase in k increases both mean growth and dividend risk. If risk aversion and consumption risk are not too high, the mean effect dominates and the price-dividend ratio goes up. On the other hand, mean reversion in the non-housing ratio $\ln z_t$ implies that both prices move opposite to $\ln z_t$. Unfortunately, the discount factor $w_j(\ln z_t)$ is not available in closed form, since z_t itself is a nonlinear function of α_t. However, numerical results for the homoskedastic case (not shown) deliver a shape much like Figure 4. If $\ln z_t$ is large, then $\Delta \ln z_{t+1}$ is negative with high probability, that is, housing is expected to grow faster than expenditure on numeraire. This makes saving in numeraire terms relatively less attractive and hence lowers asset prices.

With heteroskedasticity, bad times are associated with more composition risk. This is captured by the fact that $v_{z,t}$ goes up with $\ln z_t$, which has two effects. First, it dampens the response of prices to a unit change in the log expenditure ratio $\ln z_t$. The reason is that an increase in risk encourages precautionary savings, so that agents discount the future less. Therefore, prices fall by less than in the homoskedastic case. Second, the size of the typical shock is larger when $\ln z_t$ is high. This tends to increase the conditional volatility of returns when $\ln z_t$ is high. On net, the variance of returns and their associated premia thus tend to be higher in the heteroskedastic case.

Figure 4 also shows that our measure of the value-rent ratio for the housing stock varies responds much less to a change in z_t than the price-dividend ratio. The reason is that, while an increase in $\ln z_t$ increases discount rates, it also increases the expected growth rate of housing dividends, $\Delta \ln c_{t+1} - \Delta \ln z_{t+1}$. Since the non-housing ratio reverts to its mean, a high value today predicts an increase in housing expenditure in the future. This increases the current value of the housing stock, partly offsetting the increase in discount rates. In our model, houses are thus less risky than stocks and command a lower premium.[9] The price dynamics also suggests the share of housing in total wealth, $c_t z_t v_t^s / \left(c_t v_t^s + c_t z_t v_t^s\right) = (1-\alpha_t)\, v_t^h / \left(v_t^h + v_t^s\right)$ as another candidate variable for forecasting returns. We do not pursue this implication in the current paper, since it would require data on wealth.

Post-war Performance

To see how the model behaves over the post-war sample, we report results for the two benchmark parameterizations in the last four rows of Panel B in Table 3. Composition risk still leads to

[9] Housing returns computed from the model according to equation (17) not only measure the returns on one unit of housing, but also include the value of new housing. Housing returns computed for Table 1 do not include the value of new housing. When we do include the value of new housing, mean returns go up by 2.5 percentage points. The standard deviation of housing returns is unchanged.

substantial average excess returns on stocks. The model also still generates high volatility of stock returns. However, a larger portion of these effects is now simply attributable to a term premium – compensation for holding a long maturity asset – as opposed to an equity premium. The reason is that the model generates more volatility in the riskfree rate when we calibrate to post-war data, and term premia increase to compensate for this higher volatility.

To understand the increase in $\sigma\left(r^f\right)$, consider the parameter estimates in Table 2. Over the long sample, the estimates show substantial time variation in the volatility of the expenditure share α_t. In particular, the volatility of α_t increases during bad times, when the expenditure share α_t is high. This higher risk generates a motive for precautionary savings, which pushes down the riskfree rate. In the shorter post-war sample, there is still evidence of heteroskedasticity in the expenditure share α_t. Table 2 shows that the parameter a_1, which governs the dependence of the conditional variance on $\ln z_t$, is still estimated to be significant. The point estimate of a_1, however, is much smaller than the long-sample estimate. When we calibrate the model to post-war data, agents thus have less reason to save in bad times, which increases the risk-free rate during bad times relative to good times. This time variation in the risk-free rate leads to higher volatilities $\sigma\left(r^f\right)$ of around 7 percent in Panel B.

Another, less important, difference between the model implications over the two samples is that the mean of the riskfree rate is lower over the long sample. The reason is that there have been large shocks to the expenditure share in the first half of this century, which increase our point estimate of the unconditional volatility of the expenditure share relative to the post-war experience. This higher average composition risk leads to more precautionary savings, and thus a lower risk-free rate on average. Comparing our benchmark results in Panel B to the post-war results reveals the effect of this decrease in unconditional volatility on $E\left(r^f\right)$. This effect can easily be counteracted, however, by increasing the discount factor β to values above one. The results with *hi perceived risk* show that the model still generates a low average short rate for $\beta = 1.05$, while the same is true for $\beta = 1.26$ with *hi risk aversion.*

Alternative Dividend Specifications

To investigate how the specification of dividends in equation (16) affects our results, we consider the following three cases:

a. dividends are equal to consumption plus i.i.d. shocks, $k = 1$ and $v_d > 0$. The parameter v_d is picked to match the volatility of dividend growth.

b. dividends are equal to consumption, $k = 1$ and $v_d = 0$.

c. dividends are equal to a levered up version of consumption, $k > 1$ and $v_d = 0$. The constant k is picked to match the volatility of dividend growth.

The results in all our tables are based on case a, which is arguably the most realistic specification, since it captures both the relative volatility and the correlation between consumption growth and dividend growth. We now report some results for Cases b and c because they frequently appear in the literature. Case b is the specification of Mehra and Prescott (1985). Case c, due to Campbell (1986) and Abel (1989), captures the fact that dividend growth is more volatile than consumption growth, but assumes that the two are perfectly correlated. At our parameter values, Table 1 implies that $k = 8.28/1.88 = 4.4$ for the long sample and $k = 5.26/1.46 = 3.6$ for the postwar sample.

We find that the specification of dividends matters for the size of the equity premium and the volatility of equity returns implied by the model. Specifically, cases a and b imply roughly the same equity premium – the short rate is unaffected by the specification of dividends and average stock returns in c. are only slightly lower.[10] However, equity premia in case c are more than twice as large as the numbers reported in Table 3. Intuitively, the agent does not worry about shocks to dividends u_d that are orthogonal to consumption: since these shocks do not represent systematic risk, they do not increase the equity premium. The amount of systematic risk in dividend growth is roughly the same in cases a and b, because $k = 1$ in both specifications. In contrast, the amount of systematic risk in case b is much higher, because the entire volatility of dividend growth is due to shocks that are perfectly correlated with consumption growth, and so the equity premium is higher. Of course, any shocks to dividend growth – whether systematic or not - increase the volatility of stock returns, and so the volatility of stock returns in cases a and c is much higher than in case b.

[10] To see why, it is useful to consider a solution v_{t+1}^s to the difference equation (17) without dividend shocks, $v_d = 0$. The price-dividend ratio with shocks, $v_d \neq 0$, is larger, because it solves the same equation, but with a higher discount factor. Since the logarithm is concave, the mean of $\ln(v_{t+1}^s + 1) - \ln(v_t^s)$ is lower and thus average stock returns are lower. However, the difference between cases a. and b. is small – always lower than 30 basis points, or .3 percentage points, for the models considered in the various rows of Table 3.

D. Predicting Excess Returns with the Dividend-Yield

The model implies that excess returns are predictable. In particular, excess returns on stocks are predictable by the dividend yield. This can be read off Figure 4, which shows that the dividend yield is a function of the stationary variable $\ln z_t$. The dividend yield inherits the persistence and mean reversion from $\ln z_t$. If the dividend yield is high, it predicts a lower dividend yield and thus higher price-dividend ratio in the future. Together with i.i.d. dividend growth and a smooth riskfree rate, a high dividend yield therefore predicts high excess returns. Intuitively, expected excess returns are higher in severe recessions (times when $\ln z_t$ is high), because investors demand higher compensation for composition risk in those times.

Table 4. Predicting Excess Returns With The Dividend-Yield

Horizon	Model				Long Sample			Post-war Sample		
	hi perceived risk		hi risk aversion							
(years)	slope	R^2	slope	R^2	slope	t-stat	R^2	slope	t-stat	R^2
1	0.15	0.05	0.14	0.06	0.11	2.51	0.07	0.11	2.25	0.08
2	0.29	0.09	0.27	0.10	0.21	2.15	0.12	0.21	1.95	0.12
3	0.41	0.13	0.39	0.15	0.24	1.70	0.11	0.24	1.53	0.11
4	0.51	0.16	0.50	0.18	0.32	1.63	0.13	0.32	1.41	0.11
5	0.60	0.19	0.59	0.21	0.49	2.49	0.19	0.52	2.26	0.17

Note: We report regression results of log excess stock returns $\sum_{j=1}^{n} r_{t+j}^{s} - r_{t+j}^{f}$ on a constant and the log dividend yield $\ln 1/v_t^s$ for $n = 1,\ldots,5$ years. The "Model" columns contain the average slope and R^2 over 50,000 simulated samples with 65 observations. The model with *hi perceived risk* is the bold-face parameterization in Table 3 with $\varepsilon = 1.05$, $\beta = 0.99$, and $1/\sigma = 5$. The model with *hi risk aversion* is the bold-face parameterization in Table 3 with $\varepsilon = 1.25$, $\beta = 1.24$, and $1/\sigma = 16$. The "Long Sample" columns run regressions with historical 1936-2001 data, and the "Post-war Sample" columns use 1947-2001 data. T-statistics are based on Newey-West standard errors to correct for overlapping observations.

To see how much predictability the model generates, we simulate 50,000 sample paths at the two sets of benchmark preference parameters. For each simulated path, we regress excess returns on a constant and the dividend yield. We also run the same regression with historical data. The "Model" columns in Table 4 report average slope coefficients and average R^2 based on the simulated data with *hi perceived risk* and *hi risk aversion*, respectively. The results indicate that the slope coefficient is positive and increasing, as we vary the forecasting horizon from 1 to 5 years. The R^2 also go up with the forecasting horizon. The "Long Sample" and "Post-war Sample" columns report the corresponding results based on historical data. The model-implied regression coefficients are between 0.14 and 0.60. The empirical regression coefficients are comparable, between 0.11 and 0.52. The model also does a good job in matching the R^2; the model-implied R^2 are within 2-4 percentage points of their empirical counterparts.

E. Predicting Excess Returns with Expenditure

Interestingly, the model also implies that a macroeconomic variable – the expenditure share α_t – should be a good forecasting variable. Intuitively, the model implies that α_t is high in severe recessions, when expected excess returns are high. To investigate this implication of the model, we again simulate 50,000 samples from the model at the two sets of benchmark preference parameters. For each simulated sample path, we regress log excess stock returns on a constant and the log expenditure share $\ln \alpha_t$. The "Model" columns in Panel A of Table 5 report the average slope coefficient and the average R^2 from these regressions. The results show that the expenditure share predicts excess returns with a positive sign. The slope coefficient increases from 2.0 to 9.3 as the forecasting horizon increases from 1 to 5 years. The 5-21 percent R^2 are comparable to the 5-21 percent R^2 in Table 4 based on the dividend-yield, and they also rise with the horizon.

The "Long Sample" columns run the corresponding regression results with historical data over the whole sample, while the "Post-war Sample" results use the post-war period. We can see that both the 1.6-10.7 slope estimates and the 2-22 percent R^2 are comparable to those in the model. Panel B of Table 5 reports the results from regressing historical excess returns on both the expenditure share and the dividend yield. The results indicate that the expenditure share outperforms the log dividend yield, especially over longer forecasting horizons. The t-statistics of the expenditure-share coefficient are larger, while the slope coefficients and R^2 from the univariate regression in Panel A remain almost intact. Like many macroeconomic models, returns in our setup are driven by only a few shocks. This implies that the two variables are highly correlated, and so it does not make sense to run this horse race in the simulated data.

Predictability of excess returns – both in our model and in the data – crucially depends on whether the price-dividend ratio is stationary. In our model, price-dividend ratios inherit their the persistence properties of the log expenditure ratio $\ln z_t$. From Table 1, we know that $\ln z_t$ is highly persistent; its autocorrelation is .964 estimated over the long sample and .83 over the postwar sample. In what follows, we discuss theoretical and statistical reasons to believe that the log expenditure ratio is stationary, and evidence that suggests that the predictability results in Table 5 do not suffer from small-sample bias.

TABLE 5. PREDICTING EXCESS STOCK RETURNS WITH THE EXPENDITURE SHARE

PANEL A. REGRESSIONS ON EXPENDITURE SHARE

Horizon	Model				Long Sample			Post-war Sample		
	hi perceived risk		hi risk aversion							
(years)	slope	R^2	slope	R^2	slope	t-stat	R^2	slope	t-stat	R^2
1	2.00	0.05	2.40	0.06	1.36	1.47	0.02	1.42	1.68	0.03
2	3.80	0.09	4.55	0.10	3.30	2.03	0.07	3.68	2.24	0.08
3	5.42	0.13	6.50	0.14	5.01	2.40	0.14	6.25	3.21	0.20
4	6.88	0.16	8.25	0.18	6.58	2.84	0.18	8.63	3.95	0.28
5	8.19	0.19	9.83	0.21	8.44	3.65	0.22	10.73	4.92	0.30

PANEL B. REGRESSION ON EXPENDITURE SHARE AND DIVIDEND-YIELD

	Long Sample					Post-war Sample				
Horizon	$\ln 1/v_t^s$		$\ln \alpha_t$			$\ln 1/v_t^s$		$\ln \alpha_t$		
(years)	slope	t-stat	slope	t-stat	R^2	slope	t-stat	slope	t-stat	R^2
1	0.10	2.04	0.43	0.44	0.07	0.10	1.84	0.50	0.13	0.08
2	0.17	1.60	1.75	1.11	0.14	0.16	1.39	2.14	0.75	0.14
3	0.15	1.01	3.65	1.77	0.18	0.10	0.66	5.30	2.11	0.21
4	0.16	0.86	5.08	2.29	0.20	0.06	0.30	8.09	3.04	0.28
5	0.28	1.49	5.87	2.64	0.26	0.15	0.81	9.24	3.43	0.31

NOTE: Panel A reports regression results of log excess stock returns $\sum_{j=1}^{n} r_{t+j}^s - r_{t+j}^f$ on a constant and the log expenditure share $\ln \alpha_t$ for $n = 1,\ldots,5$ years. The "Model" columns contain the average slope and R^2 over 50,000 simulated samples with 65 observations. The model with *hi perceived risk* is the bold-face parameterization in Table 3 with $\varepsilon = 1.05$, $\beta = 0.99$, and $1/\sigma = 5$. The model with *hi risk aversion* is the bold-face parameterization in Table 3 with $\varepsilon = 1.25$, $\beta = 1.24$, and $1/\sigma = 16$. The "Long Sample" columns run regressions with historical 1936-2001 data, and the "Post-war Sample" columns use 1947-2001 data. T-statistics are based on Newey-West standard errors to correct for overlapping observations. Panel B reports regression results of $\sum_{j=1}^{n} r_{t+j}^s - r_{t+j}^f$ on a constant, $\ln \alpha_t$, and the log dividend yield $\ln 1/v_t^s$.

To see the theoretical reasons, consider a model where $\ln z_t$ is a random walk. In this setup, the probability that the process $\ln z_t$ will stay within some finite range $[\ln \underline{z}, \ln \overline{z}]$ forever is zero. This implies that the probability that the expenditure share α_t will stay within the finite range $[\underline{\alpha}, \overline{\alpha}]$ with $\underline{\alpha} = \underline{z}/(1+\underline{z})$ and $\overline{\alpha} = \overline{z}/(1+\overline{z})$ is zero as well. Economically, this means that expenditures on housing will either become negligible or dominant over time – both cases are implausible. Even in finite samples, the random-walk specification implies that there is a high probability of observing α_t-values outside the range of values $A = [.81, .87]$ ever observed historically. For example, simulations show that $\Pr(\alpha_t \notin A \text{ for some } t \leq 100) = 90\%$, if we assume that $\ln z_t$ is a random walk.

To investigate the statistical evidence for stationarity, we test for a unit root by conducting a series of augmented Dickey-Fuller (ADF) tests. We use Schwarz and Akaike criteria to select the maximum lag length k of lagged difference terms in the ADF test equation. For the full sample, k equals 1 and 9, respectively, and we reject the null of a unit root even at the 1% level. Of course, the evidence against a unit root is weaker in the postwar sample. We are not able to reject the null at conventional test sizes in this shorter sample.

The persistence of the expenditure share raises the concern that the predictability regressions in Table 5 give biased results in small samples. Stambaugh (1999) derives a formula for the bias for the slope coefficient in univariate regressions. The formula expresses the bias as some multiple b times the small-sample bias in the autoregressive coefficient of the right-hand side (RHS) variable, which is typically negative. The multiple b is the ratio of the covariance of the innovations of RHS and LHS variables divided by the RHS innovation variance.[11] For the log expenditure share used in Table 5, we estimate b to be equal to -2.0, -1.3, -0.1, -2.9, and 5.2 estimated over the long sample at the 1, 2, 3, 4 and 5 year horizon, respectively.

This suggests that the bias is small. For example, at the 1 year horizon, we estimate the autoregressive coefficient of $\ln \alpha_t$ to be .96. The downward bias in this estimate is at most 0.04. Therefore, the bias in the slope coefficient is below 0.08, which is small relative to the slope coefficient of 1.36 in Table 5. Similar results obtain for the other horizons; in fact, the positive b estimate for the

[11]To be precise, Stambaugh (1999) considers the regressions: $y_t = \alpha + \beta x_{t-1} + u_t$ and $x_t = \theta + \rho x_{t-1} + v_t$. Stambaugh's Proposition 4 derives the following expression for the small sample bias: $E[\hat{\beta} - \beta] = b \times E[\hat{\rho} - \rho]$, where $b = \text{cov}(u_t, v_t)/\text{var}(v_t)$. To compute the bias for longer horizons n, we take the errors u_t and v_t from the regression equations $y_t = \alpha + \beta x_{t-n} + u_t$ and $x_t = \theta + \rho x_{t-n} + v_t$ estimated with data sampled at dates $1, 1+n, 1+2n, \ldots$, so that there is no overlap.

5-year horizon even biases against finding predictability. Intuitively, the reason for this finding is that, unlike the dividend yield and other commonly used predictor variables, the expenditure share is a macroeconomic variable and thus covaries less with returns. This lower covariance helps avoid small sample bias.

As an alternative check, we ran the predictability regressions with non-overlapping data. To be precise, we ran log excess stock returns $\sum_{j=1}^{n} r_{t+j}^{s} - r_{t+j}^{f}$ on a constant and the log expenditure share $\ln \alpha_t$ for $t = 1, 1+n, 1+2n, \ldots$ and $n = 1, \ldots, 5$ years. The resulting slope coefficient estimates are 1.4, 1.8, 5.4, 8.0, and 7.6 with t-statistics of 1.5, 1.2, 2.8, 3.9 and 3.7. Although concern about small-sample bias will always remain, these different pieces of evidence show that the results in Table 5 are not obviously biased.

F. GMM based on Euler equations

Up to now, we have solved for returns implied by the model at a range of values for the preference parameter ε. It is also possible to estimate ε from Euler equations based on return data. We consider unconditional Euler equations $E\left[\left(R_{t+1}^{i} - R_{t+1}^{f}\right) M_{t+1}\right] = 0$ based on the pricing kernel (9). We fix the coefficient of relative risk aversion at $1/\sigma = 5$ and estimate ε using GMM with excess stock returns ($i = s$). This approach reflects our prior that risk aversion should be low. (The value for the discount factor β does not matter for excess returns). The resulting estimate for ε is 1.17 and its 95% confidence interval is $[1.014, \infty)$. To compute this confidence interval, we use the fact that the GMM objective function J_T multiplied by the number of observations T is χ^2 distributed under the hull hypothesis. Specifically, we evaluate the GMM objective function $J_T(\varepsilon)$ for different values of ε and determine the parameter region for which $T \times J_T(\varepsilon)$ is smaller than its 5% critical value. The usual GMM standard errors turn out to be huge, independently of the number of lags in the Newey-West weighting matrix.

When we add housing returns ($i = s, h$) as a second moment, the estimate is 1.24 and its 95% confidence interval is $[1.015, \infty)$. The J-test statistic is 0.31, which is smaller than the 5 percent $\chi^2(1)$ critical value, 3.84, so we fail to reject the model. To summarize, the GMM estimation results are not very informative, but at least the point estimates are roughly consistent with the values we used earlier — ε is above 1.

VI Conclusion

We introduce an equilibrium model for asset pricing with housing. Agents care about the composition of a consumption basket that contains shelter and other goods. We calibrate the model to data on non-housing consumption and housing expenditures. Compared to the standard CCAPM, our model implies higher equity and housing premia, higher stock return volatility, a lower riskfree rate which is not volatile, and lower bond premia. It also predicts that the dividend-yield and the non-housing expenditure share α_t forecast future excess stock returns. We document that the expenditure share α_t predicts excess stock returns in the data better than does the dividend yield. This is particularly interesting, because – contrary to common predictor variables – α_t is not based on asset market data.

References

[1] Abel, Andrew B. (1999). "Risk Premia and Term Premia in General Equilibrium." *Journal of Monetary Economics* 43, pp. 3-33.

[2] Attanasio, Orazio P. and Guglielmo Weber (1995). "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey." *Journal of Political Economy* 103, pp. 1121-57.

[3] Benhabib, Jess, Richard Rogerson, and Randall Wright (1991). "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy* 99, pp. 1166-1187.

[4] Boskin, Michael J., Eller R. Dulberger, Robert J. Gordon, Zvi Griliches, and Dale Jorgenson (1996). "*Toward a More Accurate Measure of the Cost of Living.*" Final Report to the Senate Finance Committee from the Advisory Commission to Study the Consumer Price Index. Washington: Senate Finance Committee, December 4.

[5] Campbell, John Y. (1986). "Bond and Stock Returns in a Simple Exchange Model." *Quarterly Journal of Economics* 101, pp. 785-804.

[6] Campbell, John Y. and John H. Cochrane (1999). "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107, pp. 205-51.

[7] Caplin, Andrew, Sewin Chan, Charles Freeman, and Joseph Tracy (1997). *Housing Partnerships.* MIT Press, Cambridge, Massachusetts.

[8] Case, Karl E. and Robert J. Shiller (1989). "The Efficiency of the Market for Single-Family Homes." *American Economic Review* 79, pp. 125-37.

[9] Cocco, Joao (2005). "Portfolio Choice in the Presence of Housing." *Review of Financial Studies* 18, pp. 535-567.

[10] Cochrane, John H. (1991). "Production-Based Asset Pricing and the Link between Stock Returns and Economic Fluctuations." *Journal of Finance* 46, pp. 209-37.

[11] Cochrane, John H. (1996). "A Cross-Sectional Test of an Investment-Based Asset Pricing Model." *Journal of Political Economy* 104, pp. 572-621.

[12] Cochrane, John, Francis Longstaff and Pedro Santa-Clara (2005). "Two Trees." Working paper, University of Chicago.

[13] Constantinides, George and Darrell Duffie (1996). "Asset Pricing with Heterogeneous Consumers." *Journal of Political Economy* 104(2), pp. 219-40.

[14] Davis, Morris and Jonathan Heathcote (2005). "Housing and the Business Cycle." *International Economic Review* 46, pp. 751-784.

[15] Dunn, K. and Kenneth Singleton (1986). "Modeling the Term Structure of Interest Rates under Non-Separable Utility and Durability of Goods." *Journal of Financial Economics* 17, pp. 27-55.

[16] Eichenbaum, Martin and Lars Peter Hansen (1990). "Estimating Models with Intertemporal Substitution using Aggregate Time Series Data." *Journal of Business and Economic Statistics* 8, pp. 53-69.

[17] Eichenbaum, Martin, Lars Peter Hansen, and Kenneth Singleton (1988). "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty." *Quarterly Journal of Economics* 103, pp. 51-78.

[18] Fama, Eugene F. and Kenneth R. French (1997). "Size and Book-to-Market Factors in Earnings and Returns." *Journal of Finance* 50, pp. 131-155.

[19] Flavin, Marjorie and Shinobu Nakagawa (2005). "A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence." Working paper, UC San Diego.

[20] Flavin, Marjorie and Takashi Yamashita (2002). "Owner-Occupied Housing and the Composition of the Household Portfolio." *American Economic Review*, March, pp. 345-62.

[21] Glaeser, Edward L. and Joseph Gyourko (2005). "Urban Decline and Durable Housing." *Journal of Political Economy* 113, pp. 345-375.

[22] Goodman, John L. and John B. Ittner (1993). "The Accuracy of Home Owner's Estimates of House Value." Economic Activity Section Working Paper No. 131, Board of Governors, Washington.

[23] Gordon, Robert J. and Todd vanGoethem (2004). "A Century of Housing Shelter Prices: How Big is the CPI Bias?" Working paper, Northwestern University.

[24] Greenwood, Jeremy, Richard Rogerson, and Randall Wright (1995). "Household Production in Real Business Cycle Theory." *Frontiers of Business Cycle Research* (Editor, Thomas F. Cooley) Princeton University Press.

[25] Heaton, John (1993). "The Interaction between Time-Nonseparable Preferences and Time Aggregation." *Econometrica* 61, pp. 353-85.

[26] Heaton, John (1995). "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications." *Econometrica* 63, pp. 681-717.

[27] Hobijn, Bart (2003). "On Both Sides of the Quality Bias in Price Indices." Working Paper, New York Federal Reserve.

[28] Jagannathan, Ravi and Zhenyu Wang (1996). "The Conditional CAPM and the Cross-Section of Expected Returns." *Journal of Finance* 51, pp. 3-53.

[29] Kullmann, Cornelia (2002). "Real Estate and its Role in Asset Pricing." Working paper, UBC.

[30] Lettau, Martin and Sydney Ludvigson (2001). "Consumption, Aggregate Wealth and Expected Stock Returns." *Journal of Finance* 56, pp. 815-49.

[31] Lustig, Hanno and Stijn Van Nieuwerburgh (2005). "Housing Collateral, Consumption Insurance and Risk Premia." Working paper, UCLA and NYU.

[32] Malpezzi, Stephen and Bengt Turner (2003). "A Review of Empirical Evidence on the Costs and Benefits of Rent Control." Working Paper, University of Wisconsin.

[33] McGrattan, Ellen R., Richard Rogerson, and Randall Wright (1997). "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy." *International Economic Review* 38, pp. 267-290.

[34] Mehra, Rajnish and Edward Prescott (1985). "The Equity Premium Puzzle." *Journal of Monetary Economics* 15, pp. 145-61.

[35] Menzly, Lior, Tano Santos and Pietro Veronesi (2004). "Understanding Predictability." *Journal of Political Economy* 112, pp. 1-47.

[36] Ogaki, Masao and Carmen M. Reinhart (1998). "Measuring Intertemporal Substitution: The Role of Durable Goods." *Journal of Political Economy* 106, pp. 1078-98.

[37] Ortalo-Magne, Francois and Sven Rady (2006). "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints." Forthcoming *Review of Economic Studies.*

[38] Prescott, Edward (1997). "On Defining Real Consumption." *Review of Federal Reserve Bank of Minneapolis*, May/June, pp. 47-53.

[39] Runkle, David E. (1991). "Liquidity Constraints and the Permanent-Income Hypothesis: Evidence from Panel Data." *Journal of Monetary Economics* 27, pp. 73-98.

[40] Santos, Tano and Pietro Veronesi (2005). "Labor Income and Asset Returns." Forthcoming *Review of Financial Studies.*

[41] Stambaugh, Robert (1999). "Predictive Regressions." *Journal of Financial Economics* 54, pp. 375-421.

[42] U.S. Bureau of Economic Analysis (1990). "Personal Consumption Expenditures." Methodology Paper No 6. Washington, DC: U.S. Government Printing Office.

[43] U.S. Bureau of Economic Analysis (2002). "Updated Summary NIPA Methodologies." http://www.bea.gov/bea/ARTICLES/2002/10October/1002UpdatedMeth.pdf

[44] Weil, Pierre (1989). "The Equity Premium Puzzle and the Riskfree Rate Puzzle." *Journal of Monetary Economics* 24, pp. 401-21.

[45] Yogo, Motohiro (2006). "A Consumption-Based Explanation of Expected Stock Returns." Forthcoming *Journal of Finance* 61(2), April.

Appendix

A. Microevidence on Expenditure Shares

We use data from the Consumer Expenditure Survey (CEX) to obtain some microevidence on expenditure shares. Table A1 reports summary statistics of the housing expenditure share across different groups of households. The groups are classified by income quintile, region of residence, age of the person who rents or owns the house, race, number of persons in the household, housing tenure, and education. For renters, the data on housing expenditures just measures rent. For homeowners, the data measures actual expenditures on shelter (such as mortgage interest and charges, maintenance, repairs, insurance, property taxes and other expenses) and do not include expenditures on household operation, housekeeping supplies etc.

TABLE A1. MICROEVIDENCE ON EXPENDITURE SHARES FROM THE CEX

	Income Quintiles					Regions			
	1st	2nd	3rd	4th	5th	Northeast	Midwest	South	West
mean	17.8	20.0	18.0	16.4	16.9	19.9	16.2	15.8	20.4
std	1.0	1.0	1.0	0.8	0.9	1.8	1.0	0.7	0.9
	Age							Race	
	<25	25-34	35-44	45-54	55-64	65-74	75+	Hispanic	Non-Hisp.
mean	18.9	20.0	18.8	16.7	15.5	15.6	17.5	20.3	18.5
std	1.0	0.8	1.1	1.4	1.3	0.9	0.9	0.5	0.5
	Number of Persons					Home		Race	
	1	2	3	4	5+	Owner	Renter	Black	Non-Black
mean	21.6	17.1	17.0	17.3	17.2	16.4	21.8	18.9	17.7
std	1.0	0.8	1.1	1.1	0.8	1.0	1.1	1.3	0.9
	Education								
	I	II	III	IV	V	VI	VII	VIII	
mean	18.2	17.7	18.2	19.9	19.9	19.9	18.1	18.5	
std	0.5	0.5	0.5	0.6	0.5	0.8	0.4	0.4	

NOTE: Annual data 1984-2002 from the CEX. The series of 5+ persons per households starts in 1988. The series on hispanics/non-hispanics starts in 1994. The education series start in 1996. The levels correspond to the following: I = less than highschool, II = high school graduate, III = associate degree, IV = college degree, V = Bachelor's degree, VI = Master, professional doctorate, VII = less than college graduate, VIII = high school with some college.

Table A1 reports average housing expenditure shares together with their standard deviations over time (in brackets). The data are the available annual CEX series for the years 1984-2002. Table A1 suggests that average expenditure shares across different household characteristics are very similar. For example, poorer households do not seem to spend much more on housing than richer households. The lowest income quintile spends 17.8% on housing, while the highest quintile spends 16.9%. This finding is further supported by the fact that education levels do not seem to matter much for expenditure shares. The households with less than a highschool degree spend 18.2% on housing, while households with higher degrees (such as masters and doctorates) spend 19.9%. These facts suggest that the homogeneity assumption on preferences is not contradicted by these data. Another interesting finding is that older households do not seem to spend much less on housing. For example, households that are 75 years and older spend 17.5%, whereas the youngest households spend 18.9 and 20%.

By and large, the differences in average expenditure shares in Table A1 seem small. Interestingly, the housing expenditure shares do not vary much over time. Most standard deviations in Table A1 are below 1% per year. The highest standard deviation is the 1.8% number in the Northeast.[12] These values are amazingly consistent with the standard deviation of the aggregate expenditure share in Table 1.

To summarize, the CEX evidence does not reveal large differences in expenditure shares across different groups of households. For each group, the CEX evidence also suggests that these expenditure shares are not volatile over time. This microevidence therefore confirms the aggregate evidence from Section IV – preferences are different from Cobb-Douglas, but ε is still close to 1.

[12] It is tempting to interpret these standard deviations as standard errors for average expenditure shares. This is, however, not appropriate, since it ignores CEX measurement error within groups.

B. Data on Housing Returns

This appendix defines our NIPA-based measure of house prices and compares it with returns based on alternative measures. We define housing returns according to the NIPA tables as follows. The real housing value $p_t^h h_t$ is recorded in NIPA Fixed Asset Tables 2.1, line 68. This series computes the nominal housing value using the current value method which measures the current market value of the assets (as opposed to the historical value method, which measures the book value of assets.) The series records the year-end value of residential housing structures. To include the value of land, we assume that land prices are perfectly correlated with the price of structures. Using Census data, we estimate that the value of the land is 36% of the total housing value. We therefore adjust houses prices to $p_t^h / (1 - 0.36)$. The dividends on housing are rent payments during that year, $q_t s_t$. We follow Flavin and Yamashita (2002) and assume that maintenance roughly equals depreciation, so that we need to subtract $\delta p_{t-1}^h h_{t-1}$ from dividends. We also subtract net real property tax payments $(1 - 0.33) \times 0.025 \times p_{t-1}^h h_{t-1}$, where the marginal tax rate is assumed to be 33% and the property tax rate is assumed to be 2.5%. The real housing return is thus

$$\frac{\left(p_t^h h_t + q_t s_t\right) / h_t}{\left(p_{t-1}^h h_{t-1}\right) / h_{t-1}} - \delta - (1 - 0.33) \times 0.025. \tag{21}$$

The summary statistics in Table 1 are based on this definition of returns.

Davis and Heathcote (2005) use the price index for new residential investment from NIPA Table 7.6, line 38, as measure of house prices p_t^h. This series is a chain-type price index for investment in private residential structures starting in 1947, and it does also not include the value of land. This is the index that mimics our index the closest among all the indexes; the correlation of price changes between this index and our house price index is 0.80.

An alternative price index is provided by the Office of Federal Housing Enterprise Oversight (OFHEO). Starting in 1975, this index tracks the changes in the value of single family homes through repeat sales using the mortgage transaction data provided by Fannie Mae and Freddie Mac. The OFHEO index reflects the cost of structures and land, simultaneously controlling for the quality of the house. The series, however, does not go back very far. The correlation of price changes with our index is 0.71 over the 25 years where we have data on the OFHEO index.

The National Association of Realtors (NAR) publishes indexes that report median house prices starting early 1960s. The Bureau of the Census also reports median and average sale prices of houses sold in the United States since 1963. These indexes do not control for quality of the median house. The Census Bureau also publishes constant-quality price indexes that do not include the value of the land, but correct for the quality problem. These indexes are also available starting early 1960s.

Flavin and Yamashita (2001) use PSID data on house prices to estimate the housing returns over the 1968-1992 period. Unlike the other house price measures we just discussed, PSID house price data is at the homeowner level. Returns can therefore be computed for individual houses. There is, however, no rent data accompanying house prices in the PSID. Flavin and Yamashita therefore compute real housing returns as:

$$\frac{p_t^h + \overline{r_f}\; p_{t-1}^h + \tau \text{Propertytax}_t}{p_{t-1}^h} = \frac{p_t^h}{p_{t-1}^h} + \overline{r_f} + 0.33 \times 0.025, \tag{22}$$

where $\overline{r_f}$ denotes the average real short term interest rate, the personal tax rate τ is set to be 33% and property tax rate is set to 2.5%. Flavin and Yamashita set $\overline{r_f}$ to be 5% which seems too high in our sample. We compute $\overline{r_f}$ using our data.

There are two main reasons that make our house price measure (21) superior to other alternatives in our analysis. First, ours is the only measure that goes back until the 1930s. Second, we have rents (housing expenditures) that correspond to our house price series. Table B1 reports summary statistics on individual housing returns from Flavin and Yamashita (2001, Table 1A) and our aggregate housing returns series. We compute aggregate housing returns using Flavin and Yamashita's (FY) return definition (22), and using our definition based on rent data (21).

Table B1 shows that average returns on individual housing are more than 3 times as high as those on aggregate housing. The difference in standard deviations is even more striking. Returns on individual houses are more than 5 times as volatile as returns on the U.S. housing stock as a whole. The last columns in Table B1 shows that rent data only matters little for the volatility of aggregate returns.

Table B1. Various Measures Of Real Returns On Housing

	FY definition (Eq.22) 1968-1992		Our definition (Eq.21) 1968-1992
	PSID data	Our data	Our data
mean	6.59	1.80	1.97
std	14.24	2.74	2.81

Note: This table reports mean and standard deviation of real housing returns. The first column reports the findings in Flavin and Yamashita (2001, Table 1A), the second column is (22) evaluated with our house price index. The third column is (21) evaluated with our house price index. Returns are deflated using the price index that corresponds to our definition of non-housing consumption c_t.

Table B2 presents real returns on housing using the FY definition of returns with different house price indexes that are discussed before. The mean returns on housing are around 2-3% for all indexes and time periods, and the standard deviation of returns are in the 1.5-3% range except the last column. In the last column, housing return statistics are calculated for each state separately using OFHEO state level house price indexes and then averaged. Going from the aggregate to the state level, the volatility of housing returns almost doubles. Idiosyncratic housing returns are still more than 2 times as volatile as state-level housing returns.

Table B2. Real Returns On Housing Using The FY Definition

	Our data	DH data	OFHEO data	
			aggregate	state level
	1947-2000	1948-2000	1975-2000	1975-2000
mean	1.96	2.00	2.82	2.51
std	2.21	1.70	3.19	5.86

Note: This table reports mean and standard deviation of real housing returns using the FY definition (22). The first column is based on our house price index. The second column is based on the price index for new residential investment as used in Davis and Heathcote (2005). The third and fourth columns are based on the OFHEO price indexes at aggregate and state levels, respectively. Returns are deflated using the price index that corresponds to our definition of non-housing consumption c_t.

C. Cointegration of Real Rents and Relative Housing Quantity

The first-order condition (6) relates the relative quantity of housing consumption s_t/c_t to the relative price of housing consumption p_t^s/p_t^c. The key parameter in this relationship is the intratemporal elasticity of substitution. To estimate ε, we can take logs of the FOC and derive the cointegrating equation,

$$\ln \frac{s_t}{c_t} = \text{constant} - \varepsilon \ln \frac{p_t^s}{p_t^c} + \text{error}, \tag{23}$$

between log relative quantities and log real rents.

Table C1 presents the results of this exercise. The Johansen-test for cointegration of $\ln s_t/c_t$ and $\ln p_t^s/p_t^c$ strongly rejects the null of no integration. (We allow for linear trends in the data, and include 2 lags.) Over the full sample, the estimate of ε implied by the estimated cointegrating equation is 1.27, greater than 1, indicating that housing s_t and non-housing consumption c_t are substitutes. The 0.16 standard errors indicate that ε is not likely to be below one. In other words, we find that the utility function is not likely to be Cobb-Douglas. Over the post-war sample, the estimate of ε is 0.77, below 1. The 0.22 standard errors are, however, larger over this sample.

Table C1. Estimation Of Intratemporal Elasticity

LR		ε	
21.75	[20.04]	1.27	(0.16)
Post-war Sample			
21.41	[20.04]	0.77	(0.22)

Note: The first two columns reports the likelihood ratio of the Johansen-test for cointegration and the corresponding 1% critical value in square brackets. The last two columns reports ε from the cointegrating equation (23) and the standard errors in round brackets. The estimates are obtained using the full sample 1936-2001 and the post-war sample 1947-2001.