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RESIDUAL RISK REVISITED

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ABSTRACT

The Capital Asset Pricing Model in conjunction with the usual market model assumptions implies that well-diversified portfolios should be mean variance efficient and, hence, betas computed with respect to such indices should completely explain expected returns on individual assets. In fact, there is now a large body of evidence indicating that the market proxies usually employed in empirical tests are not mean variance efficient. Moreover, there is considerable evidence suggesting that these rejections are in part a consequence of the presence of omitted risk factors which are associated with nonzero risk premia in the residuals from the single index market model. Consequently, the idiosyncratic variances from the one factor model should partially reflect exposure to these omitted sources of systematic risk and, hence, should help explain expected returns. There are two plausible explanations for the inability to obtain statistically reliable estimates of a linear residual risk effect in the previous literature: (1) nonlinearity of the residual risk effect and (2) the inadequacy of the statistical procedures employed to measure it.

The results presented below indicate that the econometric methods employed previously are the culprits. Pronounced residual risk effects are found in the whole fifty-four year sample and in numerous five year subperiods as well when weighted least squares estimation is coupled with the appropriate corrections for sampling error in the betas and residual variances of individual security returns. In addition, the evidence suggests that it is important to take account of the nonnormality and heteroskedasticity of security returns when making the appropriate measurement error corrections in cross-sectional regressions. Finally, the results are sensitive to the specification of the model for expected returns.

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1. Introduction

The central feature of modern asset pricing theory is the distinction between systematic and diversifiable risk. As a consequence, it is hardly surprising that the notion that expected returns depend on total variance and not merely on exposure to systematic risk has proved to a ubiquitous choice as a maintained alternative hypothesis in empirical tests.[1] What is surprising is the outcome of such tests: there has been no statistically reliable rejection of any modern asset pricing theory in favor of the hypothesis that expected returns depend on total risk.

There is an interesting puzzle implicit in this observation. There is now a large body of evidence indicating that both the equally weighted and value weighted **CRSP** indices are not mean variance efficient and that this finding is due to some extent to omitted risk factors.[2] Yet this is inconsistent with the generally insignificant measured residual risk effects found in the literature--by this logic, measured residual variances should in part reflect the squared factor loadings of omitted risk factors and, hence, should be associated with significant risk premia in large samples.[3] The

[1] Moreover, diversifiable risk may affect equilibrium asset prices even when the notion of systematic risk is meaningful if all investors face substantive transactions costs. See Levy (1978) and Mayshar (1978) for details.

[2] See, for example, Lehmann and Modest (1985) and the papers cited therein. Note that this finding could occur when the static Capital Asset Pricing Model is true but the market portfolio is unobservable and the usual **CRSP** indices are inadequate substitutes. It is also consistent with the implications of multifactor asset pricing theories along the lines of Merton (1973), Cox, Ingersoll, and Ross (1985), and Ross (1976, 1977) in their equilibrium and approximate arbitrage incarnations.

[3] Note that this need not occur. In general, if there is a single omitted risk factor and the average factor loading of individual securities was zero while their values were symmetrically distributed across securities, the residual risk premium would be zero. This possibility would not appear to be empirically relevant given the evidence presented in Lehmann and Modest (1985) regarding own variance effects.

notion that expected returns depend on residual risk as well as systematic risk should be a powerful maintained alternative hypothesis in tests of asset pricing theories. It has not proved to be one.

In fact, the total risk hypothesis has instead provided a consistent foil for investigators interested in making points about the statistical properties of such tests. In early work, Douglas (1968) and Linter (1965) found that average security returns were significantly related to estimates of both individual security betas (computed with respect to an equally weighted index) and total or residual variances, apparently rejecting the mean variance efficiency of the equally weighted index. The justly celebrated Miller and Scholes (1972) investigation documented an important statistical problem with these results: individual security returns are marked by significant positive skewness so that firms with high average returns will typically have large measured total or residual variances as well.

Such measurement error correlations suggest that considerable caution should be exercised when using total or residual variance as an explanatory variable. The need for such caution was substantiated in practice by Fama and MacBeth (1973) who found no residual risk effect when residual variances and betas were estimated in an earlier period to mitigate the Miller-Scholes problem. Similar results were obtained by Roll and Ross (1980) in their tests of the Arbitrage Pricing Theory. Total risk added to the explanatory power of factor loadings, which measure putative exposure to systematic risk, in accounting for mean returns when the relevant sample moments were measured from the same returns but failed to make a significant addition when different observations were used to estimate mean returns, factor loadings, and total variances.

These results have been interpreted differently in other comparisons of the explanatory power of total and systematic risk measures. Friend, Westerfield, and Granito (1978) and Friend and Westerfield (1981) found that both measured betas and residual variances (computed with respect to a variety of indices) typically proved to have insignificant effects in cross-sectional regressions constructed to avoid the Miller-Scholes problem.[4] Similarly, Dhrymes, Friend, and Gultekin (1984) and Dhrymes, Friend, Gultekin, and Gultekin (1984) found that both factor loadings and residual variances yielded insignificant estimates of risk premia in an Arbitrage Pricing context. Taken at face value, these results indicate that neither systematic nor diversifiable risk measures adequately explain average equity returns which perhaps suggests that no reliable risk/return tradeoff is implicit in mean returns.

An alternative interpretation is that there is another serious statistical problem associated with tests involving both systematic and

[4]Friend and Westerfield (1981) performed regressions relating individual and grouped security returns to both contemporaneously estimated betas and residual variances and standard deviations and those estimated in previous periods. The regressions which employed contemporaneous estimates often yielded significant residual risk effects which of course can be ascribed to the skewness effects discussed above. With one exception, no significant measured residual risk effects were found when beta and residual variance estimates were computed in a previous period. The exception occurred when separate regressions were estimated for months when the return on their market proxy exceeded the riskless rate and for those in which the reverse was true. As Friend and Westerfield (1981) correctly observed, this finding could simply reflect the correlation between estimation error in the betas and corresponding residual variances. Such correlations arise in small samples when security returns have skewed distributions.

diversifiable risk measures: multicollinearity.[5] This could arise because of the positive correlation in small samples between the measurement error in sample betas and residual variances when security returns are skewed to the right. Similarly, high correlation between systematic and idiosyncratic risk measures could reflect an underlying association between the corresponding unobservable population moments. The first possibility could account for the inability of Friend and Westerfield (1981) and Dhrymes, Friend, Gultekin, and Gultekin (1984) to find either significant systematic or diversifiable risk effects in their cross-sectional regressions. The second problem could lead to an inability to disentangle these effects even in the absence of measurement error. There is little evidence regarding the relative importance of these two potential causes of collinearity between systematic and idiosyncratic risk measures.

It is of course possible that there is no true residual risk effect or that such an effect is nonlinear. The first possibility was sharply rejected in Lehmann and Modest (1985) who found that portfolios sorted on the basis of previous period total variance had highly significant intercepts, rejecting the mean variance efficiency of the usual market proxies. This finding is consistent with a nonlinear residual risk effect which is surely plausible since idiosyncratic variances should reflect the squared factor loadings of omitted risk factors in the presence of a residual risk effect. There is no direct evidence on the possibility of nonlinearity save for the similarity of results obtained from either residual variance or standard deviation.

[5]Again, this possibility was first analyzed in detail by Miller and Scholes (1972).

What we have then is a conundrum--no linear residual risk effect has been reliably measured in the literature despite strong theoretical reasons and some empirical evidence for presuming its existence. There are several plausible explanations for the missing effect. One possibility is the inadequacy of the econometric procedures for measuring the residual risk effect. In particular, the existing literature has used only inefficient grouping procedures to mitigate the harmful effects of measurement error in both the systematic and diversifiable risk variables.[6] These harmful effects include both the usual attenuation bias associated with measurement error and the additional problems created by positive correlation between the measurement error in systematic and nonsystematic risk variables which results from positive skewness in individual security returns. Moreover, no study has yet confronted the possibility of collinearity between the true idiosyncratic risk and systematic risk exposure of individual firms. A detailed and comprehensive reexamination of the empirical relevance of the residual risk factor and its role as a maintained alternative hypothesis in asset pricing theory tests seems clearly warranted.

The purpose of this investigation is to remedy these omissions and solve the puzzle of the absence of a reliably measured residual risk effect. The next section details the statistical procedures employed here to correct for measurement error and mitigate the effects of potential true collinearity between firm betas and residual variances. The third section examines the

[6]Friend and Westerfield (1981) did adjust individual security betas with the Vasicek (1973) and Blume (1975) empirical Bayes procedure for shrinking the estimates toward their common mean of unity. They did not correct for measurement error in the sample idiosyncratic variances.

extent to which previous failures to find a residual risk effect can be reasonably attributed to the use of inappropriate and inadequate estimation procedures in the presence of measurement error and to the presence of true collinearity between risk measures. The final section provides concluding remarks.

2. Statistical Methods

The basic premise underlying this study is that the inability to find a reliably measured residual risk effect may well reflect the inadequacy of the statistical procedures employed in the previous literature. There are three potential weaknesses in the approaches taken in the existing literature: (1) the potential loss in estimation efficiency associated with the use of ordinary least squares procedures instead of weighted or generalized least squares; (2) inadequate corrections for measurement error in the systematic and unsystematic risk measures; and (3) the possible deleterious consequences of collinearity between the true systematic risk exposure and idiosyncratic risk of individual firms. This section is devoted to a discussion of these issues.

The market model will serve as the basic model for systematic risk throughout the paper. The single index market model is a logical choice in this context because of the large and persuasive body of evidence which suggests its inadequacy as a comprehensive model for the systematic risk of equity securities. As a consequence, we should expect to find a significant residual risk effect to the extent that multifactor equilibrium and approximate arbitrage pricing models better characterize expected security returns. Hence, differences in the results yielded by alternative statistical procedures presented below ought to reflect their comparative merits in actual practice.

As a consequence, risk measurements are taken from:

$$(1) \qquad R_{it} = \alpha_{it} + \beta_{it}R_{mt} + \varepsilon_{it}$$

where R_{it} is the percentage return of security i in month t, R_{mt} is the return on a market index in month t, β_{it} is the usual market beta of security i in month t, α_{it} is the market model intercept in month t, and ε_{it} is the idiosyncratic disturbance term. The random disturbance ε_{it} has zero mean, finite variance σ^2_{it} , and is uncorrelated with R_{mt}. No restrictions are imposed on the correlations among the idiosyncratic disturbances of different firms. Naturally, β_{it} will measure systematic risk exposure and σ^2_{it} will reflect residual risk.

The goal of the exercise is to measure the degree to which systematic and idiosyncratic risk affect expected returns. As a consequence, the model for expected returns is:[7]

$$(2) \qquad E[R_{it}] = \gamma_{ot}[1 - \beta_{it}] + \gamma_{1t}\beta_{it} + \gamma_{2t}\sigma^2_{it} + \nu_{it}$$

[7]Previous studies are roughly evenly split between the use of the residual standard deviation σ_{it} and the residual variance σ^2_{it}. In order to conserve space, the residual variance σ^2_{it} will be used to measure residual risk throughout the body of the paper. The appendix, which is available from the author on request, contains all of the empirical findings corresponding to those reported in Section 3 obtained after substituting σ_{it} for σ^2_{it}. This substitution alters none of the conclusions reached in Section 3.

If the market proxy is mean variance efficient, then γ_{ot} is the expected return on the efficient portfolio whose returns are uncorrelated with R_{mt}, γ_{1t} is the expected value of R_{mt}, and both γ_{2t} and ν_{it} are zero. If not, γ_{2t} measures the residual risk effect and ν_{it} reflects the remaining pricing error associated with the market proxy.[8]

The basic problem with the model for expected returns is that none of the variables in equation (2) is directly observed and so our procedures must utilize imperfect measures of the relevant parameters. Following conventional practice, equation (1) can be used to obtain estimates of α_{it}, β_{it}, and σ_{it}^2 given appropriate assumptions regarding the variation in these parameters. Two assumptions will be considered below. The first is that these parameters are constant and so can be estimated efficiently by ordinary least squares. Under this assumption, the estimates of α_{it}, β_{it}, and σ_{it}^2 will be obtained from the application of ordinary least squares to equation (1) for months t-1 throughout t-60 and then related to individual security returns in month t to mitigate the Miller-Scholes problem.[9] The second assumption is that the

[8]Obviously, equation (2) is not a new asset pricing model. If the chosen market proxy is not mean variance efficient, the premium γ_{2t} simply reflects the correlation between expected returns and σ_{it}^2 that might reasonably be expected. The variable ν_{it} is defined to be that part of expected returns not explained by a constant, β_{it}, or σ_{it}^2.

[9]There is an apparent internal inconsistency in this strategy since, under the constant parameter assumption, better estimates can be obtained by using observations after month t as well. By using only five years of data, we are implicitly assuming that α_{it}, β_{it}, and σ_{it}^2 are varying over time. Of course, the other implicit assumption is that the intertemporal variation in these parameters is sufficiently slow and muted that the constant parameter assumption is a reasonable approximation. Evidence on this hypothesis is presented below. Following Fama and MacBeth (1973), an alternative view is that this formulation corresponds to using the market model in a normative or predictive manner.

relevant parameters vary over time. In this setting, we will still employ these ordinary least squares estimates but must interpret them differently. We will assume there is sufficient structure on the temporal variation in α_{it}, β_{it}, and σ^2_{it} so that the ordinary least squares estimates may reasonably be interpreted as estimates of the mean values of these parameters.[10]

The estimates of α_{it}, β_{it}, σ^2_{it} obtained from these first pass time series regressions can be used to estimate the expected return model (2). The combination of (1) and (2) yields:

$$R_{it} = \gamma_{ot}[1-\beta_{it}] + \beta_{it}[\gamma_{1t} + R_{mt} - E[R_{mt}]] + \gamma_{2t}\sigma^2_{it} + \nu_{it} + \varepsilon_{it} \tag{3}$$

Two aspects of equation (3) are noteworthy. The first is that the dependent variable R_{it} is observable and the independent variables β_{it} and σ^2_{it} can be estimated from the relevant time series regressions. The second concerns the presence of the observable variable R_{mt} in the coefficient on β_{it} (i.e. γ_{1t} + $R_{mt} - E(R_{mt})$).

This second observation suggests two plausible cross-sectional regression equations for measuring the relevant risk premia. By analogy with Fama and

[10]The basic requirements are that there is no covariation between these parameters and the risk premia, that the unconditional means of α_{it}, β_{it}, and σ^2_{it} exist, and that their correlations with α_{it-k}, β_{it-k}, and σ^2_{it-k} diminish sufficiently rapidly as k goes to infinity. This will occur, for example, when α_{it}, β_{it}, and σ^2_{it} are drawn from a covariance stationary stochastic process or when they have no autocorrelation but time varying variances and higher order moments. There is very little serial correlation in monthly returns which suggests that autocorrelation in these parameters should be of little concern. In contradistinction, considerable evidence suggests the presence of heteroskedasticity in monthly market model regressions.

MacBeth (1973) and Litzenberger and Ramaswamy (1979), one possible formulation is:

$$R_{it} = \lambda^*_{ot} + \lambda^*_{1t}\beta_{it} + \lambda^*_{2t}\sigma^2_{it} + u_{it} \tag{4}$$

where the cross-sectional regression coefficients λ^*_{ot} , λ^*_{1t} , and λ^*_{2t} have expected returns γ_{ot}, γ^*_{1t}, and γ_{2t}, respectively, and where the composite risk premium γ^*_{1t} is equal to $\gamma_{1t} - \gamma_{ot}$. All previous investigations of residual risk effects have used a variant of equation (4).

A second natural form follows from the null hypothesis that R_{mt} is the return on a mean variance efficient portfolio. In this eventuality, the risk premium γ_{1t} is equal to $E[R_{mt}]$ which simplifies matters considerably. By analogy with Black, Jensen, and Scholes (1972) and Blume and Friend (1973), a second plausible cross-sectional regression equation is:

$$R_{it} - \beta_{it}R_{mt} = \lambda_{ot}[1 - \beta_{it}] + \lambda_{2t}\sigma^2_{it} + v_{it} \tag{5}$$

where the cross-sectional regression coefficients λ_{ot} and λ_{2t} have expected returns δ_{ot} and δ_{2t}, respectively, and where δ_{ot} is equal to γ_{ot} and δ_{2t} is zero under the null hypothesis. To the best of my knowledge, this formulation has not been employed previously in the residual risk context.

In the more conventional formulation given by equation (4), the additional parameter γ^*_{it} is estimated and a further test of the mean variance efficiency of the market proxy may be obtained by comparing $\gamma^*_{1t} + \gamma_{ot}$ with the sample mean of R_{mt} since these risk premia sume to $E[R_{mt}]$ under the null hypothesis. Unfortunately, this putative advantage must be weighed against the cost of the potential decrease in the precision with which the risk premia γ_{ot}, γ_{1t}^{*}, and γ_{2t} are measured. The potential loss in estimation efficiency might be expected to be quite severe given the high degree of collinearity that might reasonably be expected among 1, β_{it}, and σ_{it}^2. Section 3 presents results obtained from this approach.

The employment of equation (5) largely avoids the collinearity problem in precisely the circumstances where the use of equation (4) is disadvantageous. For example, if λ_{ot} and λ_{2t} were computed from the ordinary least squares[11] projection of $R_{it} - \beta_{it}R_{mt}$ on $1 - \beta_{it}$ and σ_{it}^2 their expected returns δ_{ot} and δ_{2t} satisfy:

(6)
$$\begin{bmatrix}\delta_{ot}\\ \delta_{2t}\end{bmatrix} = \begin{bmatrix}\gamma_{ot} + E[R_{mt}] - \gamma_{1t}\\ \gamma_{2t}\end{bmatrix} + [1 - R^2]^{-1}\begin{bmatrix}k_1\\ k_2\end{bmatrix}[\gamma_{1t} - E[R_{mt}]]$$

where R^2 is the coefficient of determination from the regression of $1 - \beta_{it}$ on σ_{it}^2 times the ratio of the standard deviation of σ_{it}^2 to that of $1 - \beta_{it}$ and the constants k_1 and k_2 depend on R^2 and the means and dispersions of $1 - \beta_{it}$ and σ_{it}^2 across firms.

When the proxy is mean variance efficient, δ_{2t} will be zero and both γ_{ot} and δ_{2t} will be measured with greater precision than the corresponding estimates from equation (4). In addition, so long as γ_{1t} is substantially different from $E[R_{mt}]$, the coefficient δ_{2t} will be nonzero. Moreover, when

[11]The values of δ_{ot} and δ_{2t} depend on which method of projection is used (i.e. on the assumed statistical properties of v_{it}). The basic expression remains unchanged when generalized least squares is employed except that the coefficients R^2, k_1, and k_2 defined below will depend on the weighting matrix used in the projection.

the collinearity between $1 - \beta_{it}$ and σ^2_{it} is severe (i.e. when R^2 is far from zero), the residual risk premium δ_{2t} will be far from zero as well. Hence, the excess return formulation given by equation (5) might reasonably be expected to provide a substantially more powerful test for the presence of a residual risk effect, especially when multicollinearity is severe. The results in Section 3 bear this out.

The analysis thus far has largely ignored problems of estimation. The avenue most often taken in the residual risk literature is to ignore the distinction between the true values of β_{it} and σ^2_{it} and their associated estimated values and estimate equation (4) by ordinary least squares with the time series estimates of β_{it} and σ^2_{it} replacing the corresponding population moments. Of course, the application of ordinary least squares in this fashion leads to biased and inefficient estimates of the relevant risk premia. Matters can be improved in two ways: (1) the use of generalized least squares instead of ordinary least squares and (2) consideration of measurement error corrections.

Generalized least squares estimation will, in principle, lead to more efficient estimates of the risk premia. This advantage is difficult to gauge in practice for two reasons. The main problem is that the error terms u_{it} in equation (4) and v_{it} in equation (5) reflect both the true idiosyncratic disturbance ε_{it} and the pricing error ν_{it} as in equation (3). The idiosyncratic disturbance covariance matrix can, in principle, be estimated from the time series estimates of the market model disturbances but the pricing errors have an unknown covariance matrix which depends on the relation of the inefficient proxy to the efficient frontier. The second problem is that the number of firms in each cross-section greatly exceeds the number of time series observations (60 months) and so the residual covariance matrix cannot be estimated in fact without the imposition of further restrictions.

These two difficulties will be confronted in a conventional manner. Under the hypothesis that the market proxy is mean variance efficient, the pricing errors ν_{it} are identically zero so the use of the idiosyncratic disturbance covariance matrix of the market model disturbances is appropriate for generalized least squares estimation. In addition, the covariances of the market model disturbances will be ignored so that the generalized least squares estimates reported below are better characterized as weighted or diagonal generalized least squares estimates with the estimated idiosyncratic variances used as weights.[12]

The final estimation problem involves corrections for measurement error in $\hat{\beta}_{it}$ and $\hat{\sigma}^2_{it}$. The approach taken here follows the analysis in Litzenberger and Ramaswamy (1979) with two modifications: (1) consideration of the effects of measurement error in $\hat{\sigma}^2_{it}$ as well as in $\hat{\beta}_{it}$ and (2) extension to the excess return regression formulation (5) to complement the analysis along the lines of equation (4). Throughout this discussion, attention will be confined to the diagonal generalized least squares version of the analysis to conserve

[12]The alternative is to specify a structure for the residual covariance matrix <u>a priori</u>. Two possibilities are zero correlations across industries and unrestricted covariances within industries (with industries defined by the Standard Industrial Classification system) or a factor model. Both strategies would greatly complicate the analysis. The present approach was followed by Litzenberger and Ramaswamy (1979) in their analysis of dividend effects. See Shanken (1983) for an analysis of the statistical properties of diagonal generalized least squares estimates.

space. Both ordinary and generalized least squares formulations will be employed below.

To fix matters, let the vector $\hat{\underline{\omega}}^*_{it} = [\ 1\ \hat{\beta}_{it}\ \hat{\sigma}^2_{it}]'$ denote the time series estimates of the population quantities $\underline{\omega}^*_{it} = [\ 1\ \beta_{it}\sigma^2_{it}]'$ and let Ω^*_{it} denote the covariance matrix of the estimation error of these three numbers (where its first row and column are, of course, zero). In this more compact notation, the model given by equation (4) is:

$$(7) \qquad R_{it} = \hat{\underline{\omega}}^{*\prime}_{it}\ \underline{\lambda}^*_t + u_{it}$$

where $\underline{\lambda}^*_t = (\lambda^*_{ot}\ \lambda^*_{1t}\ \lambda^*_{2t})'$. The Litzenberger/Ramaswamy estimator follows from the observation that the deleterious consequences of the measurement error in $\hat{\underline{\omega}}^*_{it}$ can be offset with knowledge of the measurement error covariance matrix Ω^*_{it}. The reason is that the usual covariances between the dependent variable R_{it} and the independent variables $\hat{\underline{\omega}}^*_{it}$ are not biased by measurement error:

$$(8) \qquad E[\sum_{t=1}^{T}[\hat{\underline{\omega}}^*_{it}\ R_{it}]/\sigma^2_{it}\ ] = \sum_{t=1}^{T}[\underline{\omega}^*_{it}\ R_{it}]/\sigma^2_{it}\ ]$$

because the parameters $\hat{\underline{\omega}}^*_{it}$ were measured in the sixty months prior to time t.[13] The problem is that the sums of squares and crossproducts of the sample betas and residual variances equal those of the true values plus the associated measurement error variances and covariances. Fortunately:

[13]This is not correct to the extent that there is serial correlation in the true values of β_{it} and σ^2_{it}. In this case, equation (8) will be a reasonable approximation unless such autocorrelation is pronounced.

$$(9) \qquad E[\sum_{t=1}^{T}[\hat{\underline{\omega}}^*_{it}\,\hat{\underline{\omega}}^{*\prime}_{it} - \Omega^*_{it}]/\sigma^2_{it}] = \sum_{t=1}^{T}[\underline{\omega}^*_{it}\,\underline{\omega}^{*\prime}_{it}]/\sigma^2_{it}$$

where the measurement error covariance matrix helps to disentangle the sample estimates and the true values of the relevant moments. Hence, the Litzenberger/Ramaswamy estimator for the model given by (7) is given by:

$$(10) \qquad \hat{\underline{\lambda}}^*_t = [\sum_{t=1}^{T}[\hat{\underline{\omega}}^*_{it}\,\hat{\underline{\omega}}^{*\prime}_{it} - \hat{\Omega}^*_{it}]/\hat{\sigma}^2_{it}]^{-1}\sum_{t=1}^{T}[\hat{\underline{\omega}}^*_{it} R_{it}]/\hat{\sigma}^2_{it}$$

See Shanken (1983) for a detailed statistical analysis of this class of cross-sectional regression estimators.[14]

Similarly, let the vector $\hat{\underline{\omega}}_{it} = [(1 - \hat{\beta}_{it})\ \hat{\sigma}^2_{it}]'$ denote the time series estimates of the corresponding population moments $\underline{\omega}_{it} = [\ (1 - \beta_{it})\ \sigma^2_{it}]'$ and let Ω_{it} denote the covariance matrix of the estimation error of these two parameters. The estimation of this covariance matrix will be discussed below. Then the excess return model (5) may be conveniently written as:

$$(11) \qquad R_{it} - \hat{\beta}_{it}R_{mt} = \hat{\underline{\omega}}'_{it}\underline{\lambda}_t + v_{it}$$

[14]Shanken (1983) also suggests a modification of this estimator for the zero beta model (omitting the residual risk term) that takes account of the error in estimating Ω^*_{it} when returns are assumed to normally distributed. This estimator would be more complicated in the present setting since the residual variance σ^2_{it} follows a χ^2 distribution. The differences between the appropriate modification of Shanken's estimator and this one are likely to be minimal in these sample sizes, a conjecture which I substantiated in limited experiments with the zero beta formulation provided by Shanken. His finite sample analysis does not extend to the case of nonnormal and heteroskedastic returns considered below.

where $\underline{\lambda}_t = [\lambda_{ot} \lambda_{2t}]'$. By analogy with the results obtained above, note that:

$$(12) \qquad E[\sum_{t=1}^{T} [\hat{\underline{\omega}}_{it} \hat{\underline{\omega}}_{it}' - \Omega_{it}]/\sigma_{it}^2] = \sum_{t=1}^{T} [\underline{\omega}_{it} \underline{\omega}_{it}']/\sigma_{it}^2$$

$$E[\sum_{t=1}^{T} [\hat{\underline{\omega}}_{it} [R_{it} - \hat{\beta}_{it} R_{mt}] - [\underline{\Omega}_{it} R_{mt}]]/\sigma_{it}^2] = \sum_{t=1}^{T} [\underline{\omega}_{it} [R_{it} - \beta_{it} R_{mt}]]/\sigma_{it}^2$$

where $\underline{\Omega}_{it}$ is the two element vector formed from the first column of Ω_{it}.[15] As a consequence, the Litzenberger/Ramaswamy estimator for the excess return model (11) is:

$$(13) \qquad \hat{\underline{\lambda}}_t = [\sum_{t=1}^{T} [\hat{\omega}_{it} \hat{\omega}_{it}' - \hat{\Omega}_{it}]/\hat{\sigma}_{it}^2]^{-1} \sum_{t=1}^{T} [\hat{\underline{\omega}}_{it} [R_{it} - \hat{\beta}_{it} R_{mt}] - \hat{\underline{\Omega}}_{it} R_{mt}]/\hat{\sigma}_{it}^2$$

The final consideration is the choice of the appropriate estimators for the measurement error covariance matrices Ω_{it}^* and Ω_{it}. Following the earlier discussion, two assumptions about β_{it} and σ_{it}^2 will be considered which lead to alternative estimates of the error covariances. The first is that α_{it}, β_{it}, σ_{it}^2 are constant and security returns are jointly normally distributed. In this circumstance, the covariance matrix of the estimation error in these parameters is:

[15] $\underline{\Omega}_{it}$ appears because of the presence of the estimated beta on the left hand side of (11).

$$(14) \qquad V\begin{bmatrix}\hat{\alpha}_{it}\\ \hat{\beta}_{it}\\ \hat{\sigma}^2_{it}\end{bmatrix} = \sigma^2_{it}\begin{bmatrix}(X_t'X_t)^{-1} & \underline{0}\\ \underline{0}' & 2\sigma^2_{it}/(T-2)\end{bmatrix}$$

where $X_t = (\underline{\iota R_{mt}})$, $\underline{\iota}$ is a vector of ones, $\underline{R}_{mt}$ is the vector with the relevant sixty months of observations on R_{mt}, and T is sixty.

Alternatively, α_{it}, β_{it}, and σ^2_{it} may be presumed to vary over time and returns need not be normally distributed. Under mild conditions, the market model estimates of these parameters may be interpreted as estimates of their mean values over the sixty monthly observations. Unfortunately, it is not generally possible to evaluate the covariance matrix of these estimates in small samples. However, the analysis in White (1984) and Hansen (1982) permits the computation of the asymptotic covariance matrix of these estimates. The large sample covariance matrix approximation employed here allows for nonnormality and heteroskedasticity in returns but does not take account of serial correlation in β_{it} and σ^2_{it}.[16] This expression is given by:

[16]Limited experimentation was conducted with the corresponding version which allows for both serial correlation and conditional heteroskedasticity. This involves adding the products of the autocovariances of the idiosyncratic disturbances with the sum of the squared sample mean market return and its autocovariances to (15). Not surprisingly, the results agreed to several significant digits in virtually all cases. Of course, this is to be expected given the modest magnitude of the autocorrelations of individual monthly security returns.

$$(15)\qquad V\begin{bmatrix}\hat{\alpha}_{it}\\ \hat{\beta}_{it}\\ \hat{\sigma}^2_{it}\end{bmatrix} = \begin{bmatrix}[X_t'X_t]^{-1}\sum_{t=1}^{T} \underline{x}_t\underline{x}_t'\,\varepsilon^2_{it}X_t'X_t]^{-1} & T^{-1}\,[X_t'X_t]^{-1}\sum_{t=1}^{T}\underline{x}_t\varepsilon^3_{it}\\ T^{-1}\sum_{t=1}^{T}\underline{x}_t'\varepsilon^3_{it}\,[X_t'X_t]^{-1} & T^{-2}\sum_{t=1}^{T}[\varepsilon^2_{it}-\sigma^2_{it}]^2\end{bmatrix}$$

where $\underline{x}_t$ is the vector formed from row t of X_t.

3. Data and Empirical Results

All data used in this paper were taken from the Center for Research in Security Prices (**CRSP**) monthly returns file. These data consist of monthly percentage returns, inclusive of dividends and capital gains and adjusted for stock splits and dividends, for all common stocks traded on the New York Stock Exchange as well as equally weighted and value weighted indices of their returns. The data employed here run from January 1926 to December 1984.

The basic inputs into the analysis are the time series estimates of β_{it} and σ^2_{it} as well as ordinary and adjusted estimates of their standard errors. For each month from January 1931 to December 1984, individual security returns were taken from the **CRSP** monthly file for all firms that were listed for that month and for the preceding five years (i.e. sixty months) as well. The average number of firms meeting the criterion for inclusion ranged from 468 for 1931 to 1935 to 1240 for the final subperiod 1976-1980. The average number of firms included in the cross-sectional regressions was just under 900 for the entire sample.

The preceding five years of data were then used to estimate the market model (i.e. equation (1)) for all of these securities to obtain the necessary estimates of $_{it}$ and σ^2_{it}. These regressions were performed with both the equally weighted and value weighted **CRSP** indices employed as proxies for R_{mt}. The estimates $^2_{it}$ and the time series means and variances of R_{mt} over this period were inserted into equation (14) to obtain the usual OLS covariance matrix of $\hat{\beta}_{it}$ and $\hat{\sigma}^2_{it}$. In addition, the estimated variances $\hat{\sigma}^2_{it}$ and market model residuals $\hat{\varepsilon}_{it}$ along with the time series of the proxy returns R_{mt} were inserted into equation (15) to obtain the adjusted covariance matrix for $\hat{\beta}_{it}$ and $\hat{\sigma}^2_{it}$ which provides a large sample correction for heteroskedasticity and nonnormality. This procedure yielded time series estimates of β_{it}, σ^2_{it}, Ω^*_{it}, and Ω_{it} for 648 months, running from January 1931 to December 1984, for numerous individual securities.

Table 1 provides some information about the estimation error in these estimates in the form of sample averages of the error covariance matrices $\hat{\beta}_{it}$ and $\hat{\sigma}^2_{it}$ averaged over the individual securities for the entire fifty-four year sample and for ten five year subperiods as well. While the averages themselves are not terribly informative, the differences between the ordinary and adjusted covariance matrices are interesting. Over the whole sample, the adjusted measurement error variancs of the sample betas were approximately 18% larger for the equally weighted index and 20% greater for the value weighted index with especially large differences in the first fifteen years of the sample. The magnitude of the ratio of the adjusted and ordinary measurement error variances of the residual variances is much more striking--more than 945% for the equally weighted index and more than 889% for the value weighted index. Moreover, the ratio of these average variances was in excess of 225% in each subperiod for both indices. This is an obvious consequence of the leptokurtosis of individual security returns. In addition, the adjusted correlation between the measurement errors in $\hat{\beta}_{it}$ and $\hat{\sigma}^2_{it}$ were typically on the order of .2 for both indices. The magnitude of these differences suggests that the residual risk effect obtained with the ordinary measurement error correction might differ markedly from that yielded by the adjusted one.

The first results reported are for the raw return model (4). For each market proxy, the cross-sectional regression:

$$R_{it} = \lambda^*_{ot} + \lambda^*_{1t}\hat{\beta}_{it} + \lambda^*_{2t}\hat{\sigma}^2_{it} + u_{it} \tag{16}$$

was performed for each month from January 1931 to December 1984 using the estimation methods described in Section 2 and the time series estimates described above. Table 2 summarizes the evidence obtained using the equally weighted index as the market proxy while Table 3 provides the corresponding information for the value weighted index.

Each table describes the sample behavior of the time series of cross-sectional regression coefficients λ^*_{ot}, λ^*_{1t}, and λ^*_{2t} obtained from the six estimation methods for the entire sample. The tables summarize both the central tendencies and correlations among these cross-sectional regression coefficients. To this end, they report the sample mean and standard deviation for each coefficient along with its t statistic and corresponding marginal significance level (i.e. the probability of obtaining a t statistic at least that large when the corresponding mean is truly zero) for each estimation method. In addition, the sample correlations between the time estimates of λ^*_{ot}, λ^*_{1t}, and λ^*_{2t} are reported for each estimation method along with their marginal significance levels. Finally, the sample correlation between λ^*_{1t} and R_{mt} and its marginal significance level provide indirect evidence on the precision with which the market risk premium is measured.[17]

The results for the equally weighted index reported in Table 2 and for the value weighted index provided in Table 3 generally conform to the predictions outlined earlier. Moving from ordinary to generalized least squares or from conventional estimation to measurement error correction results in decreases in the mean of λ^*_{ot} and increases in the mean of λ^*_{1t} as well as similar movements in the corresponding t statistics. This is not surprising and merely provides the evidence in the residual risk context corresponding to the results obtained by Litzenberger and Ramaswamy (1979) in their analysis of dividend effects. What is more interesting is the behavior of the residual risk coefficient λ^*_{2t}--its mean also increases in statistical significance (though not necessarily in magnitude) with the change to more efficient estimation methods and corrections for measurement error in $\hat{\beta}_{it}$ and $\hat{\sigma}^2_{it}$. The only unexpected result concerns the two measurement error corrections which provide quite similar estimates of risk premia instead of the superior performance expected from the adjusted measurement error correction.[18]

The correlations among the cross-sectional regression coefficients λ^*_{ot}, λ^*_{1t}, and λ^*_{2t} indicate the presence of substantial collinearity among the true values of β_{it} and σ^2_{it} and a vector of ones (i.e. the intercept).[19] The

[18]Two additional points are worth noting. The first is that the GLS estimate of λ^*_{2t} with no measurement error correction is an order of magnitude larger than the other estimates. The second point is that it is significant at conventional levels in the value weighted regressions. Results not reported here indicate that this is entirely attributable to the January effect documented in Table 7.

[19]This might be expected given the evidence provided by Warga (1985) concerning the sample collinearity among these variables.

correlations among these coefficient estimates are small though sometimes significant at conventional levels for both OLS and GLS estimation with no measurement correction. Similarly, the OLS estimates with both measurement error corrections exhibit moderately large correlations. The striking results concern the generalized least squares estimates with both measurement error corrections--these yielded correlations in excess of .99 between λ_{1t}^* and λ_{2t}^* for both market proxies. Moreover, these correlations remain above .94 for all five year periods.

The good news is that the use of more efficient estimation procedures which correct for measurement error in the cross-sectional regressions yields a highly significant residual risk effect. The bad news is that this effect is indistinguishable from the overall market effect as measured by the coefficient on beta (i.e. λ_{1t}^*)! This finding suggests that the concerns recorded in Miller and Scholes (1972) and echoed in Section 2 regarding collinearity among one, β_{it}, and σ_{it}^2 were clearly warranted. The analysis in Section 2 also suggested that the excess return formulation (5) can potentially mitigate these problems.

As a consequence, Tables 4 and 5 provide the results obtained from this alternative formulation for the equally weighted and value weighted indices, respectively. For each market proxy, the cross-sectional regression:

$$(17) \qquad R_{it} - \hat{\beta}_{it}R_{mt} = \lambda_{ot}(1 - \hat{\beta}_{it}) + \lambda_{2t}\hat{\sigma}_{it}^2 + v_{it}$$

was performed for each month from January 1931 to December 1984 using the various estimation methods and the time series estimates of β_{it} and σ_{it}^2. Once again, each table describes the sample behavior of the time series of cross-sectional regression coefficients λ_{ot} and λ_{2t} obtained with the six estimation methods for the entire sample by their sample means, standard deviations, t statistics, and their associated marginal significance levels. In addition, the sample correlations between the time estimates of λ_{ot} and λ_{2t} as well as their correlations with the corresponding estimates from the raw return model (i.e. λ^*_{ot} and λ^*_{2t}) are reported for each estimation method along with their marginal significance levels. The correlation between λ_{2t} and λ^*_{1t} is provided as well to indicate the presence of any collinearity between the beta effect reported in Tables 2 and 3 and the residual variance effects obtained from this alternative formulation.

These results indicate that the model given by equation (17) provides reliable estimates of the residual risk effect when more efficient estimation methods are coupled with appropriate measurement error corrections. The measured effect is substantially more significant when generalized least squares estimation is employed instead of ordinary least squares. In addition, generalized least squares estimates of the residual risk effect λ_{2t} exhibit modest correlations with the beta effect λ^*_{1t} reported above, suggesting that this formulation did, in fact, mitigate the effects of collinearity between the true values of β_{it} and σ^2_{it}. More importantly, generalized least squares estimation coupled with the adjusted measurement error correction yields a substantially more significant measured residual risk effect than either the conventional measurement error correction or no measurement error correction.[20] This finding appears to justify the concerns about heteroskedasticity discussed in Section 2 and suggests the usefulness of the adjusted measurement error correction in actual practice.

Tables 6 and 7 examine the degree of nonstationarity of the residual risk effect in three dimensions for the equally weighted and value weighted indices, respectively. First, the means λ_{ot} and λ_{2t} as well as the marginal significance levels of their t statistics are reported for ten five year subperiods. Second, these means and marginal significance levels are provided both with and without the inclusion of the cross-sectional regression coefficients computed in Januaries due to the well-documented seasonality in stock returns. Finally, the tables offer two measures of serial correlation in the measured risk effect--a serial correlation coefficient and the usual χ^2 statistic for the joint significance of the first twelve autocorrelations along with their marginal significance levels. Table 6 provides the first order autocorrelation coefficient while Table 7 gives the twelfth order one. These quantities are provided for only the excess return model (5) estimated by generalized least squares with the adjusted measurement error correction in order to conserve space.

Table 6 reveals remarkably little evidence of nonstationarity in the measured residual risk effect associated with the equally weighted index. The subperiod sample means of λ_{2t} differ substantially but the variances are so large that it is likely that they are not significantly different at conventional levels. The subperiod means differ surprisingly little when computed with and without January returns. Finally, they exhibit remarkably little serial correlation. The first twelve autocorrelations are jointly significant at the 5% level for only the 1936 to 1940 subperiod while the ten subperiod χ^2 statistics are jointly insignificant at conventional levels.[21]

However, the first order serial correlation coefficients are significant at 5% level for two subperiods (1961-65 and 1976-80). It is likely that precise measurement of any nonstationarity of the residual risk effect would require an explicit model of time variation in the systematic and idiosyncratic risk measures.

By contrast, Table 7 reveals considerably nonstationarity in the residual risk effect measured with respect to the value weighted index. The most striking evidence consists of the substantial and highly significant differences in the mean values of λ_{2t} computed with and without January returns. Moreover, the first twelve autocorrelations are jointly significant in four out of ten five year subperiods at the 5% level while the ten subperiod χ^2 statistics are jointly significant below the .5% level.[22] The source of this measured autocorrelation is not in the first order serial correlations, none of which are significant at the 10% level, but rather is in the large measured autocorrelations at lag twelve reported in Table 7. This is, of course, consistent with the January effect implicit in Table 7. These results are not surprising since the value weighted index is basically an index of returns on large firms and the January seasonal is a small firm effect.

[21]The χ^2 statistics for each subperiod are independent under the null hypothesis. Since sums of independent χ^2 statistics are distributed χ^2 as well, the joint significance of the subperiod autocorrelations can be tested by examining the sum of the χ^2 statistics reported in Table 6. Their sum is 129.25 and is distributed as χ^2 with 120 degrees of freedom which has a marginal significance level of .2659.

[22]Their sum is 165.29 and is distributed as χ^2 with 120 degrees of freedom under the null hypothesis of no autocorrelation with a marginal significance level of .0030.

Finally, it is worth emphasizing that the subperiod results reported in Tables 6 and 7 reflect a pronounced residual risk effect irrespective of any possible nonstationarities. The ten subsample mean values of λ_{2t} are jointly significantly different from zero at conventional significance levels for both indices and with and without the inclusion of January returns. In addition, several of the subsample means have marginal significance levels below the conventional 5% and 1% levels. The use of generalized least squares estimation of equation (17) coupled with the adjusted measurement error correction produced reliable estimates of a substantive residual risk effect which had a decided effect on security returns in many subperiods.

4. Conclusion

This paper had a simple motivation. There is considerable evidence that the residuals from the single index market model contain factors which are asssociated with nonzero risk premia. Consequently, the idiosyncratic variances from the one factor model should partially reflect exposure to these omitted sources of systematic risk and, hence, should help explain expected returns. There are two plausible explanations for the inability to obtain statistically reliable estimates of linear residual risk effect in the previous literature: (1) nonlinearity of the residual risk effect and (2) the inadequacy of the statistical procedures employed to measure it. The econometric methods employed previously are more plausible culprits since linearity is probably a reasonable first order approximation. Hence, the search for a residual risk effect provides a natural laboratory for the investigation of the efficacy of alternative statistical procedures for measuring risk premia.

This laboratory provided an apt setting for this investigation and yielded considerable information regarding the anatomy of the residual risk effect. Three conclusions warrant special mention. The first is that the absence of a reliably measured residual risk effect in the previous literature appears to be a direct consequence of the inappropriate estimation procedures employed there. In particular, generalized least squares estimation coupled with correction for measurement error in the sample betas and residual variances yields a pronounced residual risk effect. Second, there appears to be considerable collinearity among a vector of ones and the true values of the betas from the one factor model and the associated idiosyncratic variances. As a consequence, it is important to utilize the excess return formulation (5) to more efficiently estimate the residual risk effect. Finally, there appears to be relatively little nonstationarity in the residual risk effect measured with respect to the equally weighted index but there is a pronounced January seasonal in that yielded by the value weighted index. Of course, this is unsurprising since this residual risk effect is likely to reflect the small firm effect since the value weighted index is a large firm index.

The hypothesis that expected returns depend on both systematic and residual risk ought to provide a sharp analytical knife for testing the validity of modern asset pricing theories in which expected returns depend solely on systematic risk exposure. This paper has demonstrated that the residual risk hypothesis is a tool well-suited to this purpose only when wielded appropriately. However, when the model for expected returns is estimated with proper econometric methods, the residual risk hypothesis can serve as a powerful maintained alternative hypothesis.

TABLE 1

Descriptive Statistics for Subperiods (1931-1985)

Period	Index	Average Number of Firms	Average Variance of β_{it} (OLS)	Average Variance of σ_{it}^2 (OLS)	Average Variance of β_{it} (adj)	Average Variance of σ_{it}^2 (adj)	Average Covariance between β_{it} & σ_{it}^2 (adj)
1931-35	EW	468	.02330	.0000868	.05084	.0007415	.0013617
	VW		.04494	.0001100	.08797	.0009105	.0026168
1936-40	EW	615	.02211	.0000795	.05703	.0008973	.0029104
	VW		.05707	.0001000	.11610	.0010878	.0052991
1941-45	EW	707	.02663	.0000274	.06332	.0001956	.0015995
	VW		.07042	.0000402	.11725	.0003798	.0032054
1946-50	EW	779	.03264	.0000045	.03666	.0000277	.0003685
	VW		.06808	.0000059	.06416	.0000394	.0004639
1951-55	EW	913	.04458	.0000010	.04529	.0000024	.0000454
	VW		.06296	.0000011	.05692	.0000027	.0000537
1956-60	EW	947	.06258	.0000009	.06506	.0000028	.0000497
	VW		.06278	.0000011	.05944	.0000031	.0000530
1961-65	EW	944	.05056	.0000012	.04511	.0000055	.0000563
	VW		.06568	.0000013	.05635	.0000057	.0000652
1966-70	EW	967	.04843	.0000016	.04456	.0000040	.0000445
	VW		.08360	.0000018	.08013	.0000045	.0000300
1971-75	EW	1077	.03265	.0000025	.03571	.0000058	.0000533
	VW		.06325	.0000030	.07089	.0000076	.0000532
1976-80	EW	1240	.02958	.0000034	.03908	.0000077	.0000739
	VW		.06225	.0000047	.09913	.0000112	.0001571
1931-84	EW	892	.03781	.0000128	.04463	.0001121	.0004340
	VW		.06356	.0000166	.07641	.0001476	.0007214

TABLE 2

Estimation of: $R_{it} = \lambda_{0t}^{*} + \lambda_{1t}^{*} \hat{\beta}_{it} + \lambda_{2t}^{*} \hat{\sigma}_{it}^{2} + u_{it}$

Market Proxy: Equally Weighted Time Period: 1931-1984

Estimation Method

		Conventional		Measurement Error Correction			
				Ordinary		Adjusted	
Coefficient	Statistic	OLS	GLS	OLS	GLS	OLS	GLS
λ_{0t}^{*}	Mean	.01009	.00739	.00903	.00645	.00894	.00616
	(Std.Dev.)	.03940	.03136	.04033	.03318	.03983	.03319
	t statistic	6.52	6.00	5.70	4.95	5.71	4.72
	(p-value)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
λ_{1t}^{*}	Mean	.00410	.00481	.00545	.00713	.00514	.00748
	(Std.Dev.)	.05951	.06693	.07923	.07846	.08836	.08212
	t statistic	1.75	1.83	1.75	2.31	1.48	2.32
	(p-value)	.0802	.0678	.0806	.0210	.1394	.0207
λ_{2t}^{*}	Mean	-.05234	.14148	-.10756	.00798	-.12896	.00828
	(Std.Dev.)	1.70867	2.35146	2.15349	.08057	2.79436	.08351
	t statistic	-.78	1.53	-1.27	2.52	-1.17	2.52
	(p-value)	.4358	.1261	.2040	.0119	.2405	.0119
$\rho(\lambda_{0t}^{*},\lambda_{1t}^{*})$	Correlation	.214	.030	-.196	-.228	-.372	-.304
	(p-value)	<.0001	.4488	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{0t}^{*},\lambda_{2t}^{*})$	Correlation	-.019	-.064	.098	-.233	.096	-.313
	(p-value)	.6351	.1011	.0128	<.0001	.0150	<.0001
$\rho(\lambda_{1t}^{*},\lambda_{2t}^{*})$	Correlation	.056	.091	-.276	.994	-.253	.995
	(p-value)	.1513	.0207	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{1t}^{*},R_{mt})$	Correlation	.852	.867	.832	.906	.814	.912
	(p-value)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001

TABLE 3

Estimation of: $R_{it} = \lambda_{0t}^{*} + \lambda_{1t}^{*} \hat{\beta}_{it} + \lambda_{2t}^{*} \hat{\sigma}_{it}^{2} + u_{it}$

Market Proxy: Value Weighted Time Period: 1931-1984

		Estimation Method					
		Conventional		Measurement Error Correction			
				Ordinary		Adjusted	
Coefficient	Statistic	OLS	GLS	OLS	GLS	OLS	GLS
λ_{0t}^{*}	Mean	.01108	.00778	.01018	.00650	.00977	.00608
	(Std.Dev.)	.03821	.03118	.04081	.03388	.04516	.03513
	t statistic	7.38	6.35	6.35	4.89	5.51	4.41
	(p-value)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
λ_{1t}^{*}	Mean	.00189	.00247	.00260	.00516	.00251	.00557
	(Std.Dev.)	.04551	.05081	.06613	.06471	.07472	.06794
	t statistic	1.06	1.24	1.00	2.03	.86	2.09
	(p-value)	.2907	.2162	.3175	.0427	.3920	.0372
λ_{2t}^{*}	Mean	.02704	.26658	.01209	.00621	.02301	.00656
	(Std.Dev.)	1.90711	2.73349	2.20584	.06749	2.86816	.07019
	t statistic	.36	2.48	.14	2.34	.20	2.38
	(p-value)	.7182	.0133	.8891	.0194	.8382	.0177
$\rho(\lambda_{0t}^{*},\lambda_{1t}^{*})$	Correlation	.218	-.055	-.339	-.353	-.562	-.436
	(p-value)	<.0001	.1657	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{0t}^{*},\lambda_{2t}^{*})$	Correlation	.097	.034	.171	-.347	.222	-.432
	(p-value)	.0133	.3928	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{1t}^{*},\lambda_{2t}^{*})$	Correlation	.108	.150	-.174	.992	-.219	.993
	(p-value)	.0058	<.0001	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{1t}^{*},R_{mt})$	Correlation	.804	.839	.795	.837	.781	.838
	(p-value)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001

TABLE 4

Estimation of: $R_{it} - \hat{\beta}_{it} R_{mt} = \lambda_{0t}(1 - \hat{\beta}_{it}] + \lambda_{2t} \hat{\sigma}_{it}^2 + v_{it}$

Market Proxy: Equally Weighted Time period: 1931-1984

Estimation Method

		Conventional		Measurement Error Corection			
				Ordinary		Adjusted	
Coefficient	Statistic	OLS	GLS	OLS	GLS	OLS	GLS
λ_{0t}	Mean	.01036	.00714	.00958	.00643	.00461	.00607
	(Std.Dev.)	.04078	.03226	.04165	.03360	.53739	.03367
	t statistic	6.47	5.64	5.86	4.87	.22	4.59
	(p-value)	<.0001	<.0001	<.0001	<.0001	.8273	<.0001
λ_{2t}	Mean	.01709	.03693	.01119	.03913	-.03264	.05996
	(Std.Dev.)	.65309	.41099	.68617	.41632	9.65920	.43830
	t statistic	.67	2.29	.42	2.39	-.09	3.48
	(p-value)	.5055	.0225	.6782	.0170	.9315	.0005
$\rho(\lambda_{0t},\lambda_{2t})$	Correlation	.058	-.052	.112	-.014	.994	.091
	(p-value)	.1378	.1901	.0042	.7131	<.0001	.0202
$\rho(\lambda_{0t},\lambda_{0t}^*)$	Correlation	.987	.994	.989	.998	.119	.997
	(p-value)	<.0001	<.0001	<.0001	<.0001	.0023	<.0001
$\rho(\lambda_{2t},\lambda_{2t}^*)$	Correlation	.851	-.255	.833	-.061	.142	.061
	(p-value)	<.0001	<.0001	<.0001	.1192	.0003	.1231
$\rho(\lambda_{2t},\lambda_{1t}^*)$	Correlation	-.012	.019	-.278	-.036	-.086	.089
	(p-value)	.7665	.6280	<.0001	.3602	.0294	.0238

TABLE 5

Estimation of: $R_{it} - \hat{\beta}_{it} R_{mt} = \lambda_{0t}(1 - \hat{\beta}_{it}) + \lambda_{2t} \hat{\sigma}_{it}^2 + v_{it}$

Market Proxy: Value Weighted Time period: 1931-1984

		Estimation Method					
		Conventional		Measurement Error Correction			
				Ordinary		Adjusted	
Coefficient	Statistic	OLS	GLS	OLS	GLS	OLS	GLS
λ_{0t}	Mean	.00672	.00729	.00637	.00703	.00861	.00809
	(Std.Dev.)	.03423	.03090	.04024	.03437	.18872	.03840
	t statistic	5.00	6.00	4.03	5.21	1.16	5.37
	(p-value)	<.0001	<.0001	<.0001	<.0001	.2460	<.0001
λ_{2t}	Mean	.18461	.35530	.18425	.35348	.16048	.43231
	(Std.Dev.)	2.08722	2.69237	2.19197	2.83778	5.95960	3.08392
	t statistic	2.25	3.36	2.14	3.17	.69	3.57
	(p-value)	.0247	.0008	.0327	.0016	.4933	.0004
$\rho(\lambda_{0t},\lambda_{2t})$	Correlation	.004	.135	.058	.180	.815	.289
	(p-value)	.9097	.0006	.1407	<.0001	<.0001	<.0001
$\rho(\lambda_{0t},\lambda_{ot}^*)$	Correlation	.786	.988	.842	.972	.332	.925
	(p-value)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{2t},\lambda_{2t}^*)$	Correlation	.866	.747	.823	.269	.360	.282
	(p-value)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
$\rho(\lambda_{2t},\lambda_{1t}^*)$	Correlation	.134	.110	-.042	.210	-.062	.219
	(p-value)	.0006	.0049	.2869	<.0001	.1169	<.0001

TABLE 6

Residual Risk Effect by Subperiods (1931-1980)

Estimation of: $R_{it} - \hat{\beta}_{it} R_{mt} = \lambda_{0t}(1 - \hat{\beta}_{it}) + \lambda_{2t} \hat{\sigma}_{it}^2 + v_{it}$

Market Proxy: Equally Weighted
Estimation Method: Generalized Least Squares with Adjusted Measurement Error Correction

		λ_{0t}		λ_{2t}		Serial Correlation	
Period	Statistic	All	Except January	All	Except January	between λ_{2t} & λ_{2t-1}	$\chi^2(12)$[1]
1931-35	Mean	-.00555	-.00560	-.06547	-.07005	-.247	14.19
	(p-value)	.3959	.4165	.4141	.4192	.0557	.2887
1936-40	Mean	.00000	-.00067	-.04652	-.05376	-.201	21.17
	(p-value)	1.000	.9160	.1882	.1588	.1195	.0479
1941-45	Mean	.00768	.00909	.10025	.06975	-.071	6.95
	(p-value)	.0554	.0335	.0020	.0109	.5823	.8609
1946-50	Mean	.00603	.00681	.08400	.08774	-.164	13.41
	(p-value)	.0804	.0616	.0564	.0589	.2040	.3400
1951-55	Mean	.00971	.00926	-.02428	-.02727	-.021	17.14
	(p-value)	<.0001	<.0001	.4493	.4183	.8708	.1444
1956-60	Mean	.01414	.01341	-.01117	-.02468	.031	6.82
	(p-value)	<.0001	<.0001	.7882	.5831	.8102	.8693
1961-65	Mean	.00742	.00788	.23358	.19869	.374	15.34
	(p-value)	.0375	.0313	<.0001	.0007	.0038	.2234
1966-70	Mean	.00371	.00393	.03046	.03767	-.003	7.16
	(p-value)	.3890	.3960	.6785	.5986	.9815	.8469
1971-75	Mean	.00537	.00489	.25573	.30730	-.136	14.85
	(p-value)	.2887	.3570	.0109	.0008	.2921	.2497
1976-80	Mean	.00093	.00171	-.00057	-.02860	-.271	12.22
	(p-value)	.8314	.7138	.9876	.4241	.0358	.4282

[1] Box-Ljung statistic for joint significance of first twelve autocorrelations.

TABLE 7

Residual Risk Effect by Subperiods (1931-1980)

Estimation of: $R_{it} - \hat{\beta}_{it} R_{mt} = \lambda_{0t}(1 - \hat{\beta}_{it}) + \lambda_{2t} \hat{\sigma}_{it}^2 + v_{it}$

Market Proxy: Value Weighted

Estimation Method: Generalized Least Squares with Adjusted Measurement Error Correction

Period	Statistic	λ_{0t} All	λ_{0t} Except January	λ_{2t} All	λ_{2t} Except January	Serial Correlation between λ_{2t} & λ_{2t-12}	$\chi^2(12)$[1]
1931-35	Mean	.00168	-.00203	.92879	.62662	-.001	14.49
	(p-value)	.8234	.7977	.0201	.1089	.9938	.2705
1936-40	Mean	-.00017	-.00333	.16512	-.00721	.144	5.24
	(p-value)	.9771	.5953	.4845	.9763	.2647	.9495
1941-45	Mean	.01137	.00943	1.14776	.68100	.433	22.88
	(p-value)	.0164	.0558	.0009	.0048	.0008	.0288
1946-50	Mean	.00717	.00716	.00206	-.30432	.202	15.32
	(p-value)	.0108	.0153	.9943	.2583	.1177	.2244
1951-55	Mean	.00810	.00729	-.59196	-.92948	.058	8.86
	(p-value)	<.0001	.0003	.1916	.0442	.6532	.7148
1956-60	Mean	.01339	.01195	-.01981	-.66188	.344	22.75
	(p-value)	<.0001	<.0001	.9642	.0457	.0077	.0299
1961-65	Mean	.00905	.00840	.56766	.23226	.231	8.02
	(p-value)	.0157	.0269	.1207	.5035	.0736	.7836
1966-70	Mean	.00483	.00314	.82801	.22811	.195	21.70
	(p-value)	.2655	.4900	.0786	.5608	.1309	.0410
1971-75	Mean	.00630	.00159	.28545	-.48637	.356	39.32
	(p-value)	.3513	.7971	.6167	.2368	.0058	<.0001
1976-80	Mean	.00701	.00401	.53540	.25434	.095	6.71
	(p-value)	.2985	.5552	.2006	.5371	.4618	.8762

[1] Box-Ljung statistic for joint significance of first twelve autocorrelations.

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