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David B. Gordon
Eric M. Leeper

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ABSTRACT

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David B. Gordon
Clemson University
Department of Economics
222 Sirrine Hall
Clemson, SC 29634
gd@clemson.edu

Eric M. Leeper
Indiana University
Department of Economics
105 Wylie Hall
Bloomington, IN 47405
and NBER
eleeper@indiana.edu

The Price Level, the Quantity Theory of Money, and the Fiscal Theory of the Price Level

David B. Gordon and Eric M. Leeper*

1. Introduction

Two largely independent views of price level determination currently coexist. The first view stems from the venerable quantity theory of money. The second view, based on extensions of Sargent and Wallace's (1981) "Unpleasant Monetarist Arithmetic," is the fiscal theory of the price level.[1] In their starkest forms, the quantity and fiscal theories are distinguished by which nominal government liability is sufficient to determine the price level: in the quantity theory it is the money supply and in the fiscal theory it is the quantity of government bonds. Debate between the two views echoes Hicks's (1939) characterization as a "sham dispute" the old argument over whether the rate of interest is determined in the loan market or the money market. As he put it, with "price level" in place of his original "rate of interest": "The problem is not one of determining [the price level] *in vacuo*, but it is really the general problem of price-determination in an economy where … [the price level] is therefore a constituent part of the general price-system" (p. 153). This paper argues that for the reasons Hicks lays out it is counterproductive to choose between the two theories of price level determination.

Hicks (1939) embeds the determination of the overall level of prices in the general problem of portfolio choice and asset valuation. His perspective found formal voice in the work of Friedman (1956), Tobin (1961) and Brunner and Meltzer (1972).[2] Because those authors treat

* The authors thank Jon Faust for helpful suggestions and Matthew Shapiro for raising questions this paper attempts to answer.

[1] Recent work includes Leeper (1991, 1993), Sims (1994, 1997, 1999), Woodford (1994, 1995, 1998a, 1998b, 2001), Bergin (2000), Cochrane (1999, 2001a, 2001b), Daniel (2001a, 2001b), Dupor (1997), Mikek (1999), and Schmitt-Grohe and Uribe (2000). Important antecedents to the recent work include Sargent and Wallace (1981), Begg and Haque (1984), Aiyagari and Gertler (1985), Sargent (1986), and Auernheimer and Contreras (1990, 1993).

[2] Other classic analyses include Hicks (1935), Friedman (1948), Tobin (1969, 1980), and Brunner and Meltzer (1993).

the demand for money symmetrically with the demands for other assets, the equilibrium price level emerges from the valuation of all assets jointly. That treatment suggests that in a modern economy, with a wide array of real and nominal assets that vary in return characteristics and forms of substitutability, it is futile to try to value one type of asset separately from the rest of the portfolio. Nonetheless, the portfolio choice problems emphasized in earlier analyses often receive scant attention. Asset substitutions are frequently omitted or ignored in both monetary theory and empirical estimates of money demand.[3]

We consider price level determination from the perspective of portfolio choice. Arbitrages among money balances, bonds, and investment goods determine their relative demands. Returns to real balance holdings (transactions services), the nominal interest rate, and after-tax returns to investment goods determine the relative values of nominal and real assets. Since expectations of government policies ultimately determine the expected returns to both nominal and real assets, monetary and fiscal policies jointly determine the price level. We focus on circumstances in which all sources of fiscal financing distort portfolio choices. Special cases of the fiscal and monetary policies considered produce the quantity theory of money and the fiscal theory of the price level.

Both the quantity and the fiscal theories, as popularly presented, employ a conventional money demand function of the form:

$$\frac{M^d}{P} = h(i, y), \tag{1}$$

where P is the price level, i, the nominal interest rate, is the opportunity cost of money, and y is a scale variable. While empirical specifications like (1) are often justified on grounds of long-run stability [see, for example, Friedman (1959), Meltzer (1963), and Lucas (1988)], theoretical justifications are more involved: they inform our view of the "sham dispute" and form the basis of this paper.

Expression (1) springs from the arbitrage between money and nominal bonds when the nominal interest rate suffices to capture the substitutions between these assets. Missing from (1) is the substitution between real and nominal assets—the "Tobin (1965) effect"—that expected

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[3] Important exceptions that do adopt the broad asset substitution perspective of Hicks are Bryant and Wallace (1979, 1984) and Wallace (1981, 1989), among others. Also relevant is the "New

monetary and fiscal policies might induce. If those substitutions are also important for determining the demand for money, the nominal interest rate does not adequately summarize the opportunity cost of money and (1) does not adequately summarize the determinants of the price level.

Combining a portfolio choice perspective with rational expectations further complicates the analysis of price level determination. The price level depends fundamentally on mutually consistent combinations of current and expected future policies. Current policies affect prices both directly and indirectly through changes in expectations of future policies. Current polices may constrain future policy options, if current policies imply debt obligations or future expenditure commitments. Similarly, expected future policies feedback to influence the set of current policies that can be chosen. To answer policy questions, one must consider current and future policies jointly. When any taxes distort portfolio choices, the exact policies implemented matter for the price level.

A key implication of mutually consistent current and future policies is that changes in current fiscal policy must (i) change the real value of government liabilities and therefore change future policies; (ii) force the price level to adjust, in which case the money market will clear only if the money supply adjusts to satisfy the demand for real money balances; or (iii) some combination of (i) and (ii). The ultimate effect on the price level depends on exactly which policies adjust.

From the portfolio choice perspective, the quantity theory and the fiscal theory emerge under assumptions on policy that differ in the degree to which monetary policy is constrained to satisfy fiscal financing needs. If monetary policy and government debt play no role in financing real government expenditures, then the quantity theory completely describes the path of the price level. The price level is then independent of fiscal policy. But it is not independent of debt. The level of debt service affects real money balances through its influence on future money creation.

The fiscal theory of the price level arises as an extension of unpleasant monetarist arithmetic when fiscal obligations together with fixed future policies constrain monetary policy. A current tax cut cannot be financed by higher real debt because future policies are fixed. Instead, any expansion in nominal debt must bring forth a proportional increase in the price level that leaves the real value of debt unchanged. But higher prices raise the demand for nominal money

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Monetary Economics" associated with Fama (1980), Hall (1982, 1983) and Wallace (1983), balances. To restore equilibrium, the supply of money must expand passively to return real money balances to their original level. Fiscal financing needs determine the money stock.

The portfolio choice perspective does admit pure fiscal effects on the price level that are independent of the money stock. A bond-financed tax cut that generates higher expected taxes reduces the expected return to capital and induces substitution from real to nominal assets, including money. This Tobin effect reduces current nominal spending and the price level. When monetary policy does not accommodate fiscal finance, a tax cut has opposite effects on nominal demand from those under the fiscal theory.

Some versions of the fiscal theory, for example Cochrane (1999, 2001b), start with the observation that the equilibrium value of current government debt must equal the present value of expected net-of-interest government surpluses. With all future policies held fixed, the real value of government debt is constant, and any change in the current level of debt must be associated with a proportional change in the price level. We show that this seemingly appealing inference is misleading. First, as Cochrane observes, the restriction that debt sells at par is critical. If government debt sells at a discount, an expansion of the quantity of bonds need not increase the market value of the issue. Second, even if bonds sell at a discount, an expansion in the real liabilities carried into the future is in general inconsistent with the government budget constraint under fixed future policies. The impact of this policy change ultimately depends on how the increased debt liabilities are financed.

Thirty years ago the profession was unclear about the precise policy implications of the quantity theory of money.[4] Today confusion centers on the fiscal theory of the price level. Several authors have criticized its logical foundations and at least three papers have tried to explain the fiscal theory.[5] One reason the fiscal theory may be poorly understood is that much of the literature focuses on fine theoretical points. Among these points are: (i) the appropriate interpretation of the government's budget constraint; (ii) price level indeterminacy and how the economy behaves off equilibrium paths; and (iii) how important money and monetary policy are under the fiscal theory. Moreover, expositions of the fiscal theory typically rely on a narrow set

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surveyed by Cowen and Kroszner (1987).

[4] See discussions in Gordon (1974) and Stein (1976).

[5] Buiter (1998, 2000) and McCallum (2001) criticize and Carlstrom and Fuerst (1999), Kocherlakota and Phelan (1999), and Christiano and Fitzgerald (2000) explain.

of monetary and fiscal policies. Lost to these theoretical points and specialized policies is a straightforward discussion of price level determination in a general setting with a rich array of assets and arbitrary policies. This paper presents such a setting and argues that none of these theoretical points are central to understanding the role of fiscal policy in determining the price level. The combination of portfolio balance, distorting taxes, and mutually consistent current and future policies can produce effects of macro policies that differ dramatically from those emphasized in existing literature.

2. A Simple Model of Portfolio Choice

We consider a standard Ramsey-Cass-Koopmans growth model combined with a transactions sector that provides a substitute for the transactions services of money. Private agents may hold a nominal substitute for money, government bonds, and a real substitute, capital. This range of substitutions suffices to provide a richer perspective on price level determination than appears in much of the recent literature. Direct taxes are put on an equal footing with inflation taxes—both distort portfolio choices. Equilibrium is a mapping from current and expected future policy variables to portfolio choices and, consequently, equilibrium asset values and the price level.

2.1 The Model

The model consists of a representative household, two firms—one producing goods and one producing transactions services—and a government.

The gross physical assets of the economy at date *t*, $f(k_{t-1})$, are allocated to consumption, c_t, capital, k_t, or government purchases, g_t. The aggregate resource constraint each period is

$$f(k_{t-1}) \geq c_t + k_t + g_t, \tag{2}$$

with $c_t \geq 0$, $k_t \geq 0$, and $g_t \geq 0$.[6] The function $f(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable.

[6] Complete depreciation of capital simplifies many of the expressions without affecting the characteristics of the equilibrium that concern us. For a version of the model with partial depreciation of capital, see Gordon, Leeper and Zha (1998).

Two types of representative firms rent factors of production from households and then sell their outputs back to households. Goods producing firms rent *k* from households at rental rate *r* and pay taxes levied against sales of goods. Firms choose *k* to solve

$$\max\ D_{Gt} = (1-\tau_t) f(k_{t-1}) - (1+r_t)k_{t-1}, \tag{3}$$

taking the tax rate, τ, and *r* as given.

Transactions service producing firms rent labor, *l*, from households at wage rate *w* and sell transactions services, $T(l)$, to households at price P_T. The function $T(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable. Firms choose *l* to solve

$$\max\ D_{Tt} = P_{Tt} T(l_t) - w_t l_t, \tag{4}$$

taking P_T and *w* as given.

The household owns the firms and receives factor payments, so its income at the beginning of period *t* is

$$I_t = (1+r_t)k_{t-1} + D_{Gt} + w_t l_t + D_{Tt}, \tag{5}$$

where D_G and D_T are dividends received from the goods-producing and transactions-producing firms.

Households use money balances and transactions services to acquire goods. Transactions services purchased from the financial sector at time *t* execute the fraction $T_t \in [0,1]$ of private expenditures on goods. Choices of money and services must satisfy the finance constraint:

$$\underbrace{\frac{M_{t-1}}{P_t}}_{\substack{\text{value of}\\ \text{transactions}\\ \text{performed}\\ \text{with money}}} + \underbrace{T_t(c_t + k_t)}_{\substack{\text{value of}\\ \text{transactions}\\ \text{performed}\\ \text{with services}}} \geq \underbrace{c_t + k_t}_{\substack{\text{value of}\\ \text{private}\\ \text{transactions}}}, \tag{6}$$

where M_{t-1} is nominal money balances carried into period *t* and P_t is the price level at *t*.[7] Transactions services may be thought of as a clearinghouse, money market mutual funds, or credit cards, although our specification abstracts from any institutional details. In advanced economies, where most transactions involve the financial sector but do not involve cash directly, *T* may be close to unity on average. Holding resources devoted to the financial sector fixed, the

[7] Including investment goods in the finance constraint, as in Stockman (1981), is substantive. Excluding investment goods implies that the acts of investing or reallocating investments do not generate any demand for money or for transactions services.

constraint implies that doubling the value of transactions doubles the value of transactions performed with services by doubling the size of each transaction. It also implies that the marginal product of transactions services increases with the value of transactions performed.[8]

Preferences are defined over consumption and leisure. The current period utility function, $U(\cdot)$, is time-separable, strictly increasing in both arguments, strictly concave, and continuously differentiable. Households are endowed with one unit of time each period and choose *c, k, l, T, M,* and *B,* nominal bonds, to solve[9]

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t), \qquad 0<\beta<1 \tag{7}$$

subject to the budget constraint

$$c_t + k_t + \frac{M_t + B_t}{P_t} + P_{Tt}T_t \leq I_t + \frac{M_{t-1} + (1+i_{t-1})B_{t-1}}{P_t}, \tag{8}$$

and the finance constraint, (6), with $0 \leq l_t \leq 1$, P_t, P_{Tt}, i_t and τ_t taken as given, and the initial conditions $\left(k_{-1}, M_{-1}, (1+i_{-1})B_{-1}\right)$.[10] Total government expenditures, *g*, are financed by printing money, *M*, and selling nominal bonds, *B*, which pay a net nominal interest rate of *i*, and levying a proportional tax rate, τ, against net output. The government's budget constraint is

$$\tau_t f(k_{t-1}) + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1+i_{t-1})B_{t-1}}{P_t} = g_t. \tag{9}$$

Let the state of the economy at the beginning of each period be given by the resources available to the private sector and expected sequences of future policies. At time *t* the state is

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[8] The specification is in the spirit of Tobin with Golub's (1998) characterization of Baumol's (1952) and Tobin's (1956) theories of cash inventories, where economies of scale "result from costs that are as large for hundred-dollar transactions as for million-dollar transactions" [Tobin with Golub (1998), p. 49]. The specification is also in keeping with Friedman's (1956) view of the myriad ways that households and firms may create substitutes for money in transactions.

[9] One can imagine the representative household composed of a worker/shopper pair. Each member of the household is endowed with a unit of time each period and specializes in the production of a specific commodity. The worker supplies labor inelastically to the goods producing firm and the shopper supplies labor elastically to the transactions services producing firm. The worker's labor supply is unity and the shopper's labor supply is l.

[10] The operator *E* in (7) denotes equilibrium expectations of private agents over future policy. Although we focus on perfect foresight equilibria, we employ the notation E_t to distinguish between current and past policies (dated *t* and earlier) and future policies (dated $s > t$).

$z_t = \left(k_{t-1}, M_{t-1}, (1+i_{t-1})B_{t-1}, \{E_t\rho_j, E_t\tau_j, E_t s^g_j\}_{j=t}^{\infty}\right)$, where ρ is money growth, $\rho_t = M_t / M_{t-1}$, and s^g is the government-spending share, $s^g_t = g_t / f(k_{t-1})$. Rational expectations require that expected policies are consistent with equilibrium. A perfect foresight competitive equilibrium is a set of sequences $\{c_t, k_t, l_t, T_t, M_t, B_t, P_t, P_{Tt}, i_t, r_t, w_t, \tau_t, g_t\}_{t=0}^{\infty}$ such that given the $\{P_t, P_{Tt}, i_t, r_t, w_t, \rho_t, \tau_t, g_t\}$ sequences, the $\{c_t, k_t, l_t, T_t, M_t, B_t\}$ sequences solve the firms' and the household's optimum problems, clear markets, and satisfy the government's budget constraint at each date.[11]

Functional Forms

To obtain an explicit characterization of the model's equilibrium, we specialize the model by assuming the following functional forms for the production functions and for preferences:

$$f(k_{t-1}) = k_{t-1}^{\sigma}, \qquad 0 < \sigma \le 1 \tag{10}$$

$$T(l_t) = 1 - (1-l_t)^{\alpha}, \qquad \alpha > 1 \tag{11}$$

$$U(c_t, 1-l_t) = \ln(c_t) + \gamma \ln(1-l_t), \qquad 0 < \gamma < \alpha. \tag{12}$$

First-Order Conditions

The firms' first-order conditions are:

$$k_{t-1}: \qquad 1 + r_t = \sigma(1-\tau_t)\, k_{t-1}^{\sigma-1}, \tag{13}$$

$$l_t: \qquad w_t = \alpha(1-l_t)^{\alpha-1} P_{Tt}. \tag{14}$$

Letting φ be the lagrange multiplier for the budget constraint and λ be the multiplier for the finance constraint, the household's first-order conditions are:

$$c_t: \qquad \varphi_t + \lambda_t = \frac{1}{c_t} + \lambda_t T_t, \tag{15}$$

$$l_t: \qquad \frac{\gamma}{1-l_t} = w_t \varphi_t, \tag{16}$$

$$T_t: \qquad \varphi_t P_{Tt} = \lambda_t (c_t + k_t), \tag{17}$$

[11] We prefer this conventional definition of equilibrium over the non-standard one employed by Kocherlakota and Phelan (1999) in their explanation of the fiscal theory. Their definition includes the puzzling description of policy behavior as simultaneously taking the price level sequence as given and potentially choosing a sequence of nominal interest rates.

M_t : $$\frac{\varphi_t}{P_t} = \beta E_t\left[\frac{\varphi_{t+1}+\lambda_{t+1}}{P_{t+1}}\right], \tag{18}$$

B_t : $$\frac{\varphi_t}{P_t} = \beta(1+i_t)E_t\left[\frac{\varphi_{t+1}}{P_{t+1}}\right], \tag{19}$$

k_t : $$\varphi_t + \lambda_t = \lambda_t T_t + \beta E_t(1+r_{t+1})\varphi_{t+1}. \tag{20}$$

Expressions (18)-(20) price assets and (17) prices transactions services. Interconnections among the asset pricing equations make it clear that price level determination is only one component of a general portfolio choice problem. Only under special circumstances that effectively decouple (18) and (19) from (20) can nominal assets be priced independently of the real asset.[12]

2.2 The Equilibrium

We characterize the equilibrium at time *t* in terms of two policy expectations functions, (μ_t, η_t), current government claims to goods, s_t^g, and beginning-of-period assets, $\left(k_{t-1}, M_{t-1}, (1+i_{t-1})B_{t-1}\right)$. A solution maps expectations of policy into portfolio choices, where those expectations are restricted to policy paths consistent with market clearing, the resource constraint, and budget constraints. The functions (μ_t, η_t) summarize everything agents need to know to form rational expectations about the equilibrium of the economy.

As detailed below, η and μ capture the portfolio balance effects of expected policies. η, a product of solving the first-order condition for capital, measures the direct tax distortion on investment as well as the extent to which government expenditures are financed by taxing output. A rise in expected future taxes lowers η and lowers investment. μ, a product of solving the first-order condition for money, reflects expected inflation and thus the expected return on nominal assets. Higher expected inflation lowers μ and lowers the demand for money.

Although in general the equilibrium depends on the entire expected paths of future policies, it is convenient to focus on circumstances in which the economy is in a stationary equilibrium for dates $s > t$ but starts from some other equilibrium at *t*. Assume:

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[12] Necessary and sufficient conditions for an optimum are (6), (8), (9), (13)-(20) plus transversality conditions for M_t/P_t, B_t/P_t, and k_t.

$$\begin{aligned} \rho_{t+j} &= \rho_F, & \forall j > 0 \\ \tau_{t+j} &= \tau_F, & \forall j > 0 \\ s^g_{t+j} &= s^g_F, & \forall j > 0. \end{aligned} \tag{21}$$

Define the private savings rate as $s_t = k_t / (c_t + k_t)$. In equilibrium that rate is[13]

$$\frac{1}{1-s_t} = \eta_t, \tag{22}$$

and (the inverse of) the velocity of money, $1-T_t$, is

$$(1-T_t)\left[\frac{1}{1-s_t} - \frac{\gamma}{\alpha}\right] = \frac{\mu_t}{\rho_t}, \tag{23}$$

where

$$\eta_t(\overset{(-)}{\tau_F}, \overset{(+)}{s^g_F}) = \frac{1 - \sigma\beta \dfrac{\gamma}{\alpha}\left(\dfrac{1-\tau_F}{1-s^g_F}\right)}{1 - \sigma\beta\left(\dfrac{1-\tau_F}{1-s^g_F}\right)}, \tag{24}$$

and

$$\mu_t(\overset{(-)}{\rho_F}) = \left(\frac{\beta \dfrac{\gamma}{\alpha}}{1 - \beta/\rho_F}\right). \tag{25}$$

Signs appearing in parentheses above the function arguments in (24) and (25) denote partial derivatives of the functions with respect to future values of the policy variables.

The equilibrium capital stock is

$$k_t = \left(1 - \frac{1}{\eta_t}\right)(1-s^g_t) f(k_{t-1}), \tag{26}$$

and equilibrium real money balances are

$$\frac{M_t}{P_t} = \left(\frac{\mu_t}{\eta_t - \gamma/\alpha}\right)(1-s^g_t) f(k_{t-1}). \tag{27}$$

In (27), μ substitutes for the nominal interest rate, as arbitrage between nominal assets implies

[13] Appendix A solves the model.

$$\mu_t = \beta \frac{\gamma}{\alpha}\left(1 + \frac{1}{i_t}\right). \tag{28}$$

Equilibrium nominal interest rates depend only on expected future money growth.

3. Price Level Determination

In the equilibrium outlined above expectations of government policies determine the returns on money holdings, bonds, and real investments, and therefore determine the price level. This section derives the implications for the price level of various policies and explores the tradeoffs between current and expected policies that are consistent with equilibrium.

3.1 Policy Expectations and the Price Level

The velocity equation, (27), determines the price level, given the policy expectations functions μ_t and η_t. Equilibrium real balances are

$$\frac{M_t}{P_t} = \Delta_t (1 - s_t^g) f(k_{t-1}), \tag{29}$$

where

$$\Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha}, \tag{30}$$

with

$$\Delta_t(\overset{(-)}{\rho_F}, \overset{(+)}{\tau_F}, \overset{(-)}{s_F^g}) = \frac{\beta \frac{\gamma}{\alpha}}{1 - \gamma/\alpha}\left[\frac{1 - \sigma\beta\left(\frac{1-\tau_F}{1-s_F^g}\right)}{1 - \beta/\rho_F}\right]. \tag{31}$$

μ and η capture three distinct aspects of the influence of policy expectations on the price level. The first works through μ, the marginal value of end-of-period real money balances, which is ubiquitous in dynamic monetary models. All else equal, changes in μ imply changes in expected inflation and the rate of return on money holdings, producing the direct effects of monetary policy. Expectation of a higher rate of money growth depreciates the real value of money, lowers μ, induces substitution away from money, and raises the equilibrium price level.

η captures a Tobin effect through two interdependent impacts of expected policies. One impact is a direct tax distortion, which alters the private return on real assets. To isolate this effect, consider the impact of higher expected future taxes, holding future money growth and government-spending shares fixed. Further suppose that debt is identically zero and, in order to focus on substitution effects, that the revenues collected through higher distorting taxes are rebated as a lump sum. Higher future tax rates reduce the expected return on investment and induce agents to substitute from capital into consumption. A lower expected return on capital also induces substitutions into nominal assets, including money, and produces the Tobin effect. With the current money stock fixed, higher money demand drives down the price level today.

A second impact comes from η's summary of the composition of expected fiscal financing in terms of the relative sizes of the real and the inflation tax bases. Higher η reflects an increase in expected nominal liability creation, a rise in the inflation tax base, and a reduction in the role of real taxation in financing government expenditures. This tradeoff can be seen heuristically from an alternative expression for the terms $(1-\tau)/(1-s^g)$ that appear in the definition of η in (24). A transformation of the government budget constraint yields:

$$\frac{1-\tau_t}{1-s_t^g} = 1 + \frac{\left(M_t - M_{t-1} + B_t - (1+i_{t-1})B_{t-1}\right)/P_t}{(1-s_t^g)f(k_{t-1})}, \quad t \geq 0. \tag{32}$$

Terms in $(1-\tau)/(1-s^g)$ reflect the fraction of private resources absorbed by the acquisition of new nominal liabilities issued by the government. Higher η indicates an expected shift in future financing that expands the inflation tax base and contracts the real tax base. By reflecting the relative sizes of the two tax bases, changes in η generate an expected inflation effect that is not embedded in the nominal interest rate. Whenever policies change η, the conventional money demand expression in (1) will mispredict the impacts of future policies on the price level.

3.2 Equilibrium Policies

The preceding discussion focused on how changes in rates of return on real and nominal assets affect the price level and how the rates of return relate to government policies. Well-posed policy questions require complete specification of current and future policies. Even standard questions like "What are the impacts of an increase in the expected growth rate of money?" or "What are the effects of an increase in the expected tax rate?" must be linked to other policies to satisfy the equilibrium conditions and clear the government budget constraint.

Suppose expectations are initially consistent with equilibrium, and that agents expect future government expenditures to be financed by money creation or tax receipts. Absent some other change in government expenditures or taxes, an increase in the growth rate of money (a decline in μ) increases seigniorage in the future, and neither current nor future government budget constraints are satisfied. Similarly, a rise in the expected tax rate (a fall in η) is consistent with equilibrium only if it is associated with an appropriate expected change in future money growth, government spending, or debt creation.

Not only do current policies constrain future policy options, but also variations in fiscal financing in the future require changes in current policy. Any change in expected future policy that affects the current price level will alter the real values of current changes in money and debt. As a result, the government budget constraint will continue to be satisfied only under specific circumstances.

To analyze how jointly consistent combinations of changes in current and future policy affect the price level, we must consider

which policy combinations are consistent with equilibrium given current expectations (μ and η); and

how current policy changes affect the set of future policies that are consistent with equilibrium—that is, which policy expectations are rational.

The government budget constraint in period t yields policy combinations that satisfy 0 given current expectations:

$$\left[\frac{\rho_t - 1}{\rho_t} + \left(\frac{B}{M}\right)_t - \frac{(1+i_{t-1})}{\rho_t}\cdot\left(\frac{B}{M}\right)_{t-1}\right]\Delta_t = \frac{s_t^g - \tau_t}{1-s_t^g}, \tag{33}$$

where $(B/M)_s \equiv B_s/M_s$ and Δ_t, defined in (30), summarizes the given expected policies.[14]

We now characterize which future (date F) policies satisfy 0, given the state of government indebtedness at date t. The current state of indebtedness is captured by the bond-to-money ratio at the end of period t, $(B/M)_t$. Assume that future interest liabilities are correctly anticipated at time t and that the bond-to-money ratio is constant at $(B/M)_F$ in the future stationary equilibrium. Re-labeling variables dated $t+1$ with an "F" subscript and imposing a stationary equilibrium yields:

[14] Details of the derivations underlying (33) and (34) appear in Appendix B.

$$\Delta_t = \left(\frac{s_F^g - \tau_F}{1 - s_F^g}\right)\left[\frac{1}{\left(\frac{B}{M}\right)_F - \frac{1}{\beta}\left(\frac{B}{M}\right)_t + \left(\frac{\rho_F - 1}{\rho_F}\right)}\right]. \tag{34}$$

Given government indebtedness at the end of period *t*, expression (34) describes the tradeoffs among future policies consistent with equilibrium and a given set of current policies; that is, it reports policies that satisfy (37) with Δ_t constant.[15]

To deduce the impacts on the price level of changes in monetary and fiscal policy, the contemplated policies must satisfy relationships (33) and (34). Those relationships imply that standard questions about the impacts of policy changes or changes in expectations may not be consistent with equilibrium now or in the future, and must be reinterpreted after being coupled with policies that jointly determine the equilibrium.

3.3 The Effects of Changing Policy Expectations

3.3.1 Expected Inflation

We have already considered the usual description of the effects of an increase in expected inflation due to an increase in expected money growth. The expected inflation rate rises, the real return on money holdings falls (μ falls), and the current price level rises. If we start from an initial equilibrium in which the government has a positive return from net money and bond growth, this cannot be the total adjustment because the real value of current nominal liability creation has fallen and neither current nor future government budget constraints are satisfied.

The full effect of the change in expected monetary policy depends on the changes in other policy expectations that produce the new equilibrium. A decrease in future taxes or an increase in future government spending generates a substitution away from money and towards real saving. This substitution further increases the current price level. Higher expected money growth directly raises the price level and reduces the real return to current nominal liability creation. If current taxes rise to offset the loss in revenue, there is no additional impact on the price level because private real resources are unchanged. If government spending falls instead,

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[15] We focus on government fiscal obligations in the form of debt, but any expenditure commitments, such as spending programs, Medicare, or Social Security, that constrain future policy options are subject to the same analysis.

private resources increase, raising the demand for money and moderating the effect on the price level.

3.3.2 Tax Policy Expectations

In section 3.1 we considered the direct effects of changes in expected taxes (through η). A fall in the expected future tax rate produces an increase in current capital investment, substitution away from nominal assets, and a rise in the current price level. Again, the resulting fall in government revenues is not consistent with equilibrium, in either the current period or in the future, without some further adjustment in policy. If the fall in expected tax receipts is compensated for by an increase in expected money growth, expected inflation rises and the rise in the current price level is exacerbated. The result is identical to the analysis above of a change in expected money growth with the tax rate adjusting to retain the same level of government spending. Indeed, it is arbitrary to refer to one of the policy expectations as adjusting to a change in the other. The price change results from a shift in the combination of policy expectations that are consistent with equilibrium.

In contrast, suppose the fall in the expected tax rate is associated with an equal decline in expected government spending and no change in money growth. Under the assumption that there is an expected government deficit ($s_F^g > \tau_F$), η_t falls. This change in expectations directly increases the demand for money and decreases the price level. Neither current nor future government budget constraints clear with this policy change alone. A lower current price level increases the real return to money creation. Similarly, the rise in the government-spending share in the future implies that the price level in the future rises, decreasing expected seigniorage (a decline in the seigniorage tax base with no change in the stationary inflation rate reduces seigniorage revenues). With unchanged monetary policy, future taxes must rise more than government spending, moderating the impact on the current price level.

Expected fiscal policy may affect the price level independently of monetary policy. Consider the impact on the price level of the substitution between current and future taxes that a debt-financed tax cut induces. If the sequences of money growth rates and government-spending shares, $\{\rho_t, \rho_F, s_t^g, s_F^g\}$, are fixed exogenously, a bond-financed tax cut at *t* increases government liabilities in the future and necessitates some change in future policy, as (34) indicates. In the spirit of a Ricardian experiment, suppose that future direct taxes rise. Higher τ_F reduces the expected return on physical capital. A lower return on the real asset relative to nominal assets increases the value of the current stock of money and bonds and reduces velocity. With the money stock fixed, the price level at time t falls. A Tobin effect gives debt a natural role in determining the price level, and a role that is independent of the stock of money. This pure fiscal effect from a tax cut reduces nominal demand and the price level.

One consequence of the policy change is that even though money growth is constant, seigniorage revenues rise with the lower price level, due to the impacts of future taxes on the demand for money and equilibrium real money balances. Of course, the change in seigniorage is a consequence of the fiscally induced change in the price level.

4. The Quantity Theory and The Fiscal Theory of The Price Level

Our perspective on price determination emphasizes that the price level depends on jointly consistent current and expected policies. This analysis does not conflict with either the quantity theory of money or the fiscal theory of the price level: each theory arises from particular sets of restrictions on policies.

4.1 The Quantity Theory of Money

The quantity theory is sometimes described as positing strict proportionality between the supply of money and the aggregate price level, and delivering the corresponding neutrality of money. Friedman (1956) rejects this depiction as too extreme and provides a short description of the quantity theoretic perspective. His description includes (a) a demand for money relation that depends on a restricted set of variables and (b) an assumption that the money demand function is stable.[16]

Even in the simple model considered here, (a) is satisfied only if the restricted set of variables includes the parameters of the processes determining policy. In addition, those parameters must be observable. To satisfy (b) we must add the strong assumption that the policy process is "stable" in some well-defined sense. We now explore (a) and (b).

The velocity equation is often expressed in the form of an equilibrium "money demand" function. While not formally a demand function, it has the familiar form of including an index

[16] Friedman also requires a degree of independence between the factors determining the supply and demand for money. We have nothing to add on this point.

of expenditures and a measure of opportunity cost. Although Friedman's (1956) general demand function includes a role for expected inflation that is independent of the nominal interest rate, most monetary analyses include only this rate-of-return influence of expected inflation [for example Laidler (1985)].

Expected inflation affects the demand for money by changing the own return on nominal money balances, which is embedded in the nominal interest rate, *i*. A second expected inflation effect operates through the composition of future fiscal financing, which η summarizes. Changes in η trigger the asset substitutions that generate the Tobin effect. The velocity equation, expressed in terms of the nominal interest rate, is:

$$\frac{M_t}{P_t} = \beta \frac{\gamma}{\alpha} \left(\frac{1+i_t}{i_t} \right) \frac{1}{\eta_t - \gamma/\alpha} (1 - s_t^g) f(k_{t-1}). \tag{35}$$

This expression underscores that the nominal interest rate is generally an inadequate measure of the opportunity cost of money balances. Without auxiliary assumptions that eliminate the Tobin effect and the corresponding substitutions between real and nominal assets, the nominal rate incompletely summarizes the opportunity cost.

Special policy assumptions are necessary for the price level to be determined by monetary policy alone. Those assumptions result in a dichotomy between the nominal and real sides of the economy, but not an independence of the price level from debt. Consider a policy that sets the net-of-interest budget surplus to zero each period, so $\tau_t = s_t^g$ for all *t*. In this special case, the model delivers a simple quantity theory because $\eta_t = (1 - \sigma\beta\gamma/\alpha)/(1 - \sigma\beta)$ for all *t*. This policy removes substitutions between nominal and real assets, so the demand for money stems entirely from arbitrage between bonds and money. Equilibrium real balances reduce to

$$\frac{M_t}{P_t} = h(i_t, c_t + k_t), \tag{36}$$

which is analogous to the specification in (1). Now the price level is independent of fiscal policy, but it is not independent of debt unless debt is zero: money growth must finance the interest obligations, so the level of debt matters for the level of real money balances.[17]

4.2 Representations of Money Demand

Friedman (1956) presents the demand for money as a function of a vector of rates of return on money and alternative forms of wealth. It is evident that stability of the simple money demand relation in (36) requires special assumptions about fiscal policy. We now relax those assumptions to ask whether expectations of fiscal policy summarized by η in (35) can be replaced by a judicious choice of rates of return.

Can (35) be converted into a simple and stable money demand relation if it is expressed in terms of a real return and expected inflation, as Friedman argues? Because the real interest rate described by r includes transactions costs, no simple relation links it to the nominal rate and the expected inflation rate. A simple Fisher relation can be obtained by adjusting the real rate for transactions costs. Let r_{t+1}^* denote the net real rate of return on capital between t and $t+1$ after adjusting for transactions costs. The net-of-transactions-costs real intertemporal marginal rate of transformation is given by

$$\varphi_t = \beta E_t (1 + r_{t+1}^*) \varphi_{t+1}. \tag{37}$$

Simplification yields the simple Fisher equation

$$1 + i_t = E_t (1 + \pi_{t+1})(1 + r_{t+1}^*). \tag{38}$$

Combining this simple Fisher equation with (35) yields an equivalent version of (35) in terms of expected inflation and the real return on capital adjusted for transactions costs:

$$\frac{M_t}{P_t} = \beta \frac{\gamma}{\alpha} \left(\frac{1}{E_t (1 + \pi_{t+1})(1 + r_{t+1}^*) - 1} + 1 \right) \frac{1}{\eta_t - \gamma/\alpha} (1 - s_t^g) f(k_{t-1}). \tag{39}$$

This money demand relation is stable only under certain assumptions about fiscal policy.

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[17] Of course, with lump-sum taxes in place of the distorting tax, the quantity theory and Ricardian equivalence hold under the wider range of policies described in Leeper (1991) as "active monetary" and "passive fiscal" policies [see also Woodford (1999) or Cochrane (2001b)]. The convention of using lump-sum taxes in association with the quantity theory is consistent with Friedman's (1976, p. 311) "Marshallian approach to theory," as opposed to "Tobin's Walrasian approach."

An alternative expression involving the real rate of return, *r*, may be obtained using the capital Euler equation, (20):

$$1+i_t = E_t(1+\pi_{t+1})(1+r_{t+1})\left(\frac{\eta_t - \gamma/\alpha}{\eta_t}\right). \tag{40}$$

Using (40) yields another equivalent, although somewhat more complicated equilibrium money demand relation in terms of expected inflation, the expected return on capital, and expected fiscal variables:

$$\frac{M_t}{P_t} = \beta\frac{\gamma}{\alpha}\left(\frac{1}{E_t(1+\pi_{t+1})(1+r_{t+1})\left(1-((\gamma/\alpha)/\eta_t)\right)-1}+1\right)\frac{1}{\eta_t - \gamma/\alpha}(1-s_t^g)f(k_{t-1}). \tag{41}$$

Effects of expected fiscal policies continue to appear separately through η.[18] Brunner and Meltzer (1972) specify a money demand function that includes the expected real return on real assets, an argument closely related to η.

There has also been extensive discussion about the appropriate scale variable in a money demand relation. While it may be more informative because of relative stability, measurement, or information problems to use one specification or another, there is no theoretical basis for the choice. It is logically equivalent to express the equilibrium demand for real balances in terms of private expenditures yields, as in (35), or in terms of consumption, as in

$$\frac{M_t}{P_t} = \beta\frac{\gamma}{\alpha}\left(\frac{1+i_t}{i_t}\right)\frac{\eta_t}{\eta_t - \gamma/\alpha}c_t. \tag{42}$$

On a theoretical level there is no basis for distinguishing among these various representations of equilibrium M/P. They may have some descriptive value in terms of the nature of the substitutions at work, but they are not structural relationships. Moreover, because expectations of policy enter as arguments in all these relationships, strong assumptions are necessary for these relationships to be stable over time.[19]

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[18] Adding a long-term nominal interest rate, as Meltzer (1963) and Lucas (1988) do, still does not eliminate the Tobin effect from the money demand function. Arbitrage between the one-period bond and the long-term bond implies that the equilibrium long rate also depends only on expected money creation.

[19] Lucas (1988) makes a similar observation about the money demand function he derives by presenting agents with a portfolio problem.

Friedman argues that if asset substitutions are very stable, the price level can be determined without analyzing all the margins relevant in the general specification. This argument may be without fault on a theoretical level. It may also be consistent with some broad empirical regularities. But our analysis is another example of circumstances under which it is misguided to treat an empirically stable relationship involving real money balances as identifying structural elements of the economy.

4.3 "Unpleasant Monetarist Arithmetic" as a Fiscal Theory

Sargent and Wallace (1981) develop an environment in which fiscal policy drives inflation. In their example, limitations on tax receipts result in a fiscal responsibility for the monetary authority. As Sargent (1986) emphasizes, this perspective places responsibility for controlling inflation equally on monetary and fiscal authorities.

The potential for unpleasant arithmetic arises in the present model when considering an open market operation. Consider a bond sale with $M_t + B_t$ fixed. In addition, fix the sequence $\{s_t^g, s_F^g\}$ and the current value of τ_t. Because the open-market sale raises future government indebtedness, it is clear that some policy must change in the future. With government-spending shares fixed exogenously, either future taxes, τ_F, or future money growth, ρ_F, must rise to accommodate the increase in debt service. Unpleasant monetarist arithmetic arises when individuals expect higher money growth in the future, which lowers the expected return on money (lower μ), decreases money demand, and raises the price level. This effect is counteracted by the direct negative effect on the price level from the decrease in M_t due to the open-market sale. The ultimate effect on the price level can go either way, and depends on how much future money growth must adjust following an open-market operation to be consistent with equilibrium. The change in future money growth, in turn, depends on the bond-money ratio at the time of the nominal asset exchange. To see this, note that the proportion by which $(B/M)_t$ changes from a given open-market operation varies with the initial bond-money ratio. And the larger the change in B/M, the greater is the change in debt service and, therefore, money growth and inflation in the future.

Price level changes from open-market operations depend on the corresponding changes in policy expectations, and on the state of government indebtedness.[20] State-dependence of the price effects of changes in money supply produced by normal central bank procedures arises from fiscal considerations frequently overlooked in conventional monetary analyses.

4.4 The Canonical Fiscal Theory Exercise

Like the quantity theory, the fiscal theory can be understood as eliminating all substitutions between real and nominal assets, and emerges as a special case of the general analysis above. What we term the "canonical" fiscal theory exercise stems from the policy assumptions in Leeper (1991), Sims (1994), and Woodford (1995), among many others.[21] This exercise restricts asset substitution to occur only between current nominal assets.

Assume all future policies, (ρ_F, τ_F, s_F^g), and s_t^g are fixed. These policies make η_t constant and equilibrium real money balances again reduce to (1). They also peg the nominal interest rate. Under these assumptions, consider the impact of a bond-financed tax cut, so τ_t falls and B_t rises. Can this be equilibrium? The government budget constraint (34) implies it cannot. If current money growth is unchanged, then future government liabilities, summarized by $(B/M)_t$, rise. Some future policy must change. There is only one equilibrium consistent with unchanged future policies; that equilibrium arises when current money growth expands to prevent $(B/M)_t$ from rising when nominal debt expands. Expression (33) yields the monetary adjustment required when τ_t is reduced, if future government liabilities are to remain unchanged. Note that future money growth is held fixed. The current monetary expansion required to maintain equilibrium is exactly enough so that the increase in future seigniorage (because the *level* of money supplied is now higher) at the fixed rate of monetary growth just suffices to pay for the increased debt service. To determine the effect on the equilibrium price level, note that fixing $(s_t^g, \rho_F, \tau_F, s_F^g)$ pegs the nominal interest rate and determines a constant level of real money

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[20] Sargent and Wallace (1981) also have state-dependent price effects from open-market operations.

[21] Brunner and Meltzer (1972) present an early version of the fiscal theory in an environment with a money demand function similar to ours.

balances. The new higher level of M_t, together with (27), yield the new higher equilibrium price level.[22]

The fiscal theory contrasts with the bond-financed tax cut examined in section 3.3.2. That bond-financed tax cut was pure fiscal policy in the sense that it was independent of the stock of money. It also reduced nominal spending and the price level. An essential aspect of the fiscal theory is that the current money stock must adjust to clear the money market.[23] This makes a tax cut expand nominal demand and raise the price level.

Designating either the fiscal policy change or the response of monetary policy as the source of the price level change is completely arbitrary. But as in unpleasant arithmetic, monetary policy is constrained by the government's fiscal obligations.

4.5 Taking the Government Budget Constraint Seriously

Does the intertemporal government budget constraint determine the price level? Some expositions of the fiscal theory may make it seem so. Cochrane (1999, 2001b), for example, focuses attention on the government's intertemporal budget constraint:

$$\frac{B_t}{P_t} = \text{Expected present value of future primary surpluses.} \tag{43}$$

Woodford (1995, p. 12) writes that "…it is often more useful to think of [the government's present value budget constraint, (43)] rather than [equilibrium real money balances, (29)] as the equilibrium condition that determines the equilibrium price level at date *t*." This leads to direct inferences based on (43) such as, lower expected surpluses or higher current nominal debt raise the price level. These seemingly appealing inferences are misleading for two reasons.

First, as Cochrane (2001a) correctly observes, (43) assumes that government bonds sell at par. If instead bonds sells at price q_t, then the left-hand-side of the equation is $q_t B_t / P_t$. Now higher B_t may raise the price of bonds, leaving the real market value of debt, $q_t B_t / P_t$, and the price level unchanged. We agree with Cochrane that (43) is really a "valuation equation" for nominal government debt.

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[22] Again, with lump-sum taxes, a range of policies is consistent with the fiscal theory. Leeper (1991) labels them "active fiscal" and "passive monetary" policies.

Second, even if one incorporates Cochrane's observation, higher B_t and unchanged $q_t B_t / P_t$ and P_t cannot be an equilibrium. The real liabilities of the government have increased in the future (future debt service increased). Some future policy must change. The ultimate effect on the price level depends on which policy the private sector expects will adjust.

5. Concluding Remarks

This paper takes a portfolio choice perspective in which the price level depends generically on all current and expected future monetary and fiscal policies. Generic dependence of the price level comes from the presence of distorting taxes and an insistence that current and future policies be jointly consistent with equilibrium.

This perspective also delivers a money demand function that depends on the nominal interest rate, a scale variable, and expected tax and government spending policies. Expected fiscal policies cannot be replaced by a vector of real rates of return, expected inflation, a long-term interest rate, or other typical arguments of money demand.

The quantity and fiscal theories emerge under restrictions on policy behavior that eliminate substitutions between real and nominal assets. When, in addition, neither money creation nor debt expansion finances fiscal expenditures, a conventional quantity theory of money emerges. When all future policies are fixed and the current money stock adjusts passively to satisfy money demand at the pegged nominal interest rate, the fiscal theory of the price level is reproduced.

Although the required policy restrictions are severe, they are revealing about conditions under which either the quantity theory or the fiscal theory is likely to prevail. During periods when direct tax revenues approximately cover government spending and monetary policy is conducting routine open-market operations, the quantity theory should hold well. On the other hand, periods when the central bank supports bond prices (as it might during wars), should find that fiscal policy plays a larger role in determining the price level.

These restrictions also point toward circumstances when the quantity theory and the fiscal theory are likely to breakdown. Those circumstances include periods when expectations of fiscal

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[23] Models with a single asset make the logical point that the fiscal theory can be independent of money: Woodford's (1998b, 1998c) "cashless limit"; Sims's (1997) model with nominal bonds and no money; Cochrane's (2001b) model with zero overnight money demand.

policy change in important ways, triggering substitutions between real and nominal assets. The Tobin effect should be important for large changes in macro policies, including tax reforms, big new spending initiatives, or changes in the inflation target pursued by the central bank. These circumstances call for a more general money demand specification than commonly appears in macro models.

An insistence that either the quantity theory or the fiscal theory governs the price level is extremely limiting. Those theories are powerful in their simplicity but require strong assumptions on policy behavior that are unlikely to hold generally. We have offered a broader perspective that admits the two theories as special cases. The combination of portfolio choice and mutually consistent current and expected macro policies applies generally, with only a small sacrifice in simplicity.

Appendix A: Solving The Model

This appendix describes how to solve the model given the first-order conditions (13)-(20), the aggregate resource constraint, (2), and the finance constraint, (6).

Define the private savings rate as $s_t = k_t/(c_t + k_t)$ and the government's share of output as $s_t^g = g_t/f(k_{t-1})$. The equilibrium savings rate is given by the solution to the Euler equation for capital, which implies the difference equation:

$$\frac{1}{1-s_t} = \sigma\beta E_t\left[\frac{1-\tau_{t+1}}{1-s_{t+1}^g}\left(\frac{1}{1-s_{t+1}}\right)\right] + E_t\left[1-\sigma\beta\frac{\gamma}{\alpha}\cdot\frac{1-\tau_{t+1}}{1-s_{t+1}^g}\right]. \tag{A.1}$$

The solution to (A.1) is

$$\frac{1}{1-s_t} = \eta_t, \tag{A.2}$$

where

$$\eta_t \equiv E_t\sum_{i=0}^{\infty}(\sigma\beta)^i d_i^\eta\left[1-\sigma\beta\frac{\gamma}{\alpha}\cdot\frac{1-\tau_{t+i+1}}{1-s_{t+i+1}^g}\right], \quad d_i^\eta = \prod_{j=0}^{i-1}\left(\frac{1-\tau_{t+j+1}}{1-s_{t+j+1}^g}\right), \quad d_0^\eta = 1. \tag{A.3}$$

The Euler equation for money is solved analogously. Letting $\rho_t = M_t/M_{t-1}$ denote the growth rate of the money supply, the Euler equation is

$$(1-T_t)\left[\frac{1}{1-s_t}-\frac{\gamma}{\alpha}\right] = \beta\frac{1}{\rho_t}E_t\left\{(1-T_{t+1})\left[\frac{1}{1-s_{t+1}}-\frac{\gamma}{\alpha}\right]+\frac{\gamma}{\alpha}\right\}. \tag{A.4}$$

The solution is:

$$(1-T_t)\left[\frac{1}{1-s_t}-\frac{\gamma}{\alpha}\right] = \frac{\mu_t}{\rho_t}, \tag{A.5}$$

where

$$\mu_t \equiv \beta\,\frac{\gamma}{\alpha}E_t\sum_{i=0}^{\infty}\beta^i d_i^\mu, \qquad d_i^\mu \equiv \prod_{j=0}^{i-1}\frac{1}{\rho_{t+j+1}}, \qquad d_0^\mu \equiv 1. \tag{A.6}$$

This delivers expressions (22) and (23) in the text.

Appendix B: The Government Budget Constraint

This appendix uses the model's equilibrium together with the government budget constraint to obtain the *equilibrium* connections among current and future policies. Those connections are used in section 3.2 where particular policy rules are studied.

Using expression (27) for equilibrium real balances, the government budget constraint in period t can be written as

$$1+\left(\frac{M_t - M_{t-1} + B_t - (1+i_{t-1})B_{t-1}}{M_t}\right)\Delta_t = \frac{1-\tau_t}{1-s_t^g}, \tag{B.1}$$

where

$$\Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha}. \tag{B.2}$$

Equivalently,

$$\left[\frac{\rho_t - 1}{\rho_t} + \frac{B_t}{M_t} - \frac{(1+i_{t-1})}{\rho_t}\cdot\frac{B_{t-1}}{M_{t-1}}\right]\Delta_t = \frac{s_t^g - \tau_t}{1-s_t^g}. \tag{B.3}$$

Given expectations of policy, as embodied in Δ_t, (B.3) reports the tradeoffs that exist in equilibrium among current policies, given the initial conditions summarized by $(1+i_{t-1})\left(B_{t-1}/M_{t-1}\right)$.

We now seek to characterize the tradeoffs that exist among future policies given the current state of government indebtedness. Combine the equilibrium nominal interest obtained from the arbitrage between money and bonds,

$$\mu_t = \beta\frac{\gamma}{\alpha}\left(1+\frac{1}{i_t}\right), \tag{B.4}$$

with the recursive representation of μ,

$$\mu_t = \beta\left[\frac{\gamma}{\alpha} + E_t\frac{\mu_{t+1}}{\rho_{t+1}}\right], \tag{B.5}$$

to obtain

$$1+i_{t-1} = 1+\frac{\gamma/\alpha}{E_{t-1}\left(\mu_t/\rho_t\right)}. \tag{B.6}$$

Substitute (B.6) into (B.3) and push the dating forward one period to obtain

$$\frac{1}{\rho_{t+1}}\left[\frac{B_{t+1}}{M_t}-\left(1+\frac{\gamma/\alpha}{E_t\left(\mu_{t+1}/\rho_{t+1}\right)}\right)\frac{B_t}{M_t}-1\right]+1=\frac{s_{t+1}^g-\tau_{t+1}}{\Delta_{t+1}(1-s_{t+1}^g)}. \tag{B.7}$$

Under the assumption of perfect foresight, policy choices at t+1 are known at t, so the equilibrium nominal interest rate adjusts to those choices. Then (B.7) may be rewritten as

$$\left[\frac{B_{t+1}}{M_{t+1}}-\frac{1}{\rho_{t+1}}\left(1+\frac{(\gamma/\alpha)\rho_{t+1}}{\mu_{t+1}}\right)\frac{B_t}{M_t}-\frac{1}{\rho_{t+1}}+1\right]\Delta_{t+1}=\frac{s_{t+1}^g-\tau_{t+1}}{1-s_{t+1}^g}. \tag{B.8}$$

Denote future values by an " F " subscript, consistent with the assumptions about policy in (21), and denote current values by a " t " subscript. If B/M is constant in the future at the ratio $(B/M)_F$, so nominal debt grows at the same rate, ρ_F, as money, then substituting for Δ_{t+1} and imposing future values,

$$\mu_{t+1}\cdot\left[\left(\frac{B}{M}\right)_F-\frac{1}{\rho_F}\left(1+\frac{\rho_F-\beta}{\beta}\right)\left(\frac{B}{M}\right)_t+\left(1-\frac{1}{\rho_F}\right)\right]=\frac{s_F^g-\tau_F}{1-s_F^g}\cdot(\eta_{t+1}-\gamma/\alpha), \tag{B.9}$$

which reduces to

$$\mu_{t+1}\cdot\left[\left(\frac{B}{M}\right)_F-\frac{1}{\beta}\left(\frac{B}{M}\right)_t+\left(1-\frac{1}{\rho_F}\right)\right]=\frac{s_F^g-\tau_F}{1-s_F^g}\cdot(\eta_{t+1}-\gamma/\alpha). \tag{B.10}$$

Substitute for μ_{t+1} and η_{t+1} from (25) and (24) to obtain

$$\frac{\beta\frac{\gamma}{\alpha}}{1-\gamma/\alpha}\left\{\frac{1}{\rho_F-\beta}\left[\rho_F\left(\left(\frac{B}{M}\right)_F-\frac{1}{\beta}\left(\frac{B}{M}\right)_t\right)+\left(\rho_F-1\right)\right]\right\}=\frac{s_F^g-\tau_F}{(1-s_F^g)-\sigma\beta(1-\tau_F)}. \tag{B.11}$$

Expression (B.11) can be rewritten in the form

$$\frac{\beta\frac{\gamma}{\alpha}}{1-\gamma/\alpha}\left\{\frac{1}{\rho_F-\beta}\left[\left(1-s_F^g\right)-\sigma\beta\left(1-\tau_F\right)\right]\right\}=\left(s_F^g-\tau_F\right)\left[\rho_F\left(\left(\frac{B}{M}\right)_F-\frac{1}{\beta}\left(\frac{B}{M}\right)_t\right)+\left(\rho_F-1\right)\right]^{-1}. \tag{B.12}$$

Given the state of government indebtedness at t, as summarized by $(B/M)_t$, with which the economy enters the stationary equilibrium in periods $s>t$, (B.12) characterizes the tradeoffs among future policies in equilibrium.

Using (B.2), together with (25), (24), and (B.12),

$$\frac{\Delta_t}{\rho_F}(1-s_F^g) = \frac{\beta\frac{\gamma}{\alpha}}{1-\gamma/\alpha}\left[\frac{(1-s_F^g)-\sigma\beta(1-\tau_F)}{\rho_F-\beta}\right]$$
$$= \frac{s_F^g-\tau_F}{\rho_F\left(\left(\frac{B}{M}\right)_F-\frac{1}{\beta}\left(\frac{B}{M}\right)_t\right)+(\rho_F-1)}, \tag{B.13}$$

or

$$\Delta_t = \left(\frac{s_F^g-\tau_F}{1-s_F^g}\right)\cdot\frac{1}{\left(\left(\frac{B}{M}\right)_F-\frac{1}{\beta}\left(\frac{B}{M}\right)_t\right)+\left(\frac{\rho_F-1}{\rho_F}\right)}. \tag{B.14}$$

(B.3) and (B.14) deliver expressions (33) and (34) in the text.

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