NBER WORKING PAPER SERIES

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Working Paper 9970
http://www.nber.org/papers/w9970

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2003

I have benefitted from comments on this and an earlier version of the paper by Fernando Alvarez, Marco Bassetto, Larry Christiano, Elhanan Helpman, Zvi Hercowitz, Sam Kortum, Robert Lucas, Alex Monge, Assaf Razin, Helene Rey, and seminar participants at Northwestern, Wisconsin, Harvard, Columbia GSB, NYU, Princeton, Maryland, Ohio State, the Canadian Macro Study Group, Society of Economic Dynamics, and the National Bureau of Economic Research. I am also grateful to Valerie Ramey for supplying me with the data and the original code from her paper. The views expressed here do not necessarily reflect the position of the Federal Reserve Bank of Chicago or the Federal Reserve System. The views expressed herein are those of the authors and are not necessarily those of the National Bureau of Economic Research.

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NBER Working Paper No. 9970
September 2003
JEL No. E32, D92

ABSTRACT

In his famous monograph, Lucas (1987) put forth an argument that the welfare gains from reducing the volatility of aggregate consumption are negligible. Subsequent work that revisited Lucas' calculation continued to find only small benefits from reducing the volatility of consumption, further reinforcing the perception that business cycles don't matter. This paper argues instead that fluctuations can affect welfare by affecting the growth rate of consumption. I present an argument for why fluctuations can reduce growth starting from a given initial consumption, which could imply substantial welfare effects as Lucas (1987) already observed in his calculation. Empirical evidence and calibration exercises suggest that the welfare effects are likely to be substantial, about two orders of magnitude greater than Lucas' original estimates.

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1. Introduction

In his famous monograph, Lucas (1987) argued that business cycles in the post-War U.S. involved at most negligible welfare losses, thereby challenging the presumption that stabilizing the cyclical fluctuations that persisted during this period would have been highly desirable. His argument can be stated as follows. Consider a representative consumer with a conventional constant-relative risk aversion (CRRA) utility function over consumption streams $\{C_t\}_{t=0}^{\infty}$, i.e.

$$U(\{C_t\}) = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

where $\gamma \geq 0$ measures relative risk-aversion and $\beta < 1$ denotes the rate at which the agent discounts future utility. Suppose this consumer is given a consumption stream

$$C_t = \lambda^t (1 + \varepsilon_t) C_0 \tag{1.1}$$

where λ is a constant greater than one and ε_t is an i.i.d. random variable with mean zero. That is, average consumption starts out at C_0 and grows at rate λ, so that average consumption after t periods will equal $\lambda^t C_0$. Actual consumption will deviate from this average by a factor of $1 + \varepsilon_t$. Using per-capita consumption growth from the post-War era, we can estimate λ and the variance of ε_t. To determine the costs of fluctuations, Lucas asked what constant fraction of each year's consumption the consumer would be willing to give up to avoid fluctuations, i.e. to replace ε_t with zero in each and every period. For reasonable values of γ, the answer turns out to be astonishingly small, less than one-tenth of one percent. By contrast, Lucas calculated that a consumer would sacrifice as much as 20% of his consumption each year when $\gamma = 1$ to increase the average growth rate λ by one percentage point. Thus, Lucas concluded that growth matters, but business cycles, at least of the magnitude that occurred in the U.S. over the post-War period, do not.

Although the calculation above abstracts from several important considerations, its implication that consumption volatility at business cycle frequencies does not matter much for welfare has proven to be quite robust. For example, Imrohoroglu (1989), Atkeson and Phelan (1994), Krusell and Smith (1999), Storesletten, Telmer, and Yaron (2001), and Beaudry and Pages (2001) depart from the representative agent setting and calibrate income streams to individual-level data, assuming only imperfect insurance arrangements across agents. Since agents can still save against future income shocks, the implied costs of business cycles in these papers are small, typically less than 1% of consumption per year, and in some cases are even negative. Obstfeld (1994) maintains a representative agent framework but allows shocks to be persistent rather than i.i.d., and again finds relatively small costs. Lastly, Obstfeld (1994), Campbell and Cochrane (1995), Pemberton (1996), Dolmas (1998), Alvarez and Jermann (1999), Tallarini (2000), and Otrok (2001) examine departures from CRRA utility. With the exception of Campbell and Cochrane and Tallarini, whose findings are challenged in some of the other papers, these experiments again tend to find small costs of fluctuations at business cycle frequencies for reasonable parameter configurations. Thus, it appears that there is limited scope for generating large costs of business cycles from aversion to consumption risk.[1]

The results above would seem to deny that business cycles can ever matter much for welfare. Nevertheless, this paper argues that cycles can be associated with considerable welfare costs, about two orders of magnitude greater than those Lucas originally computed. These costs are not due to consumption volatility *per se*, as maintained in previous work, but to the fact that cyclical fluctuations can affect the economy's long-run growth rate. The motivation for examining growth effects comes from Lucas' original insight that changes in growth rates can have large welfare effects: even if aggregate fluctuations have a modest effect on the rate at which an economy grows, they could in principle have a significant effect on welfare.

The notion that business cycles might lead to large welfare costs because of their effect on growth has been raised in previous work, including Mendoza (1997), Jones, Manuelli, and Stacchetti (1999), Epaulard and Pommeret (2000), and Matheron and Maury (2000). However, these models fail to generate large welfare costs for fluctuations of the magnitude of the post-War U.S. experience. The reason, as I argue below, is that these models do not produce growth effects of the type implicit in Lucas' calculation on the benefits from faster growth. Lucas computed how much an agent would sacrifice to obtain a consumption stream that starts at the *same initial level of consumption* but grows more rapidly, as illustrated by the shift from the solid to the dashed line in the first panel of Figure 1. But in the models cited above, fluctuations affect long-run growth by changing the average amount of resources that are allocated to growth-enhancing activities in equilibrium. If eliminating fluctuations increases average investment, consumption will still grow at a faster rate, but since resources must be diverted to investment, average initial consumption will fall, as illustrated in the bottom panel of Figure 1. Clearly, the welfare implications of a given change in the growth rate will be very different for these two experiments. Previous work has therefore focused on the benefits of allowing agents to reoptimize between present and future consumption, not the benefits of more rapid growth in the sense that Lucas described.

[1] Lucas (2003) surveys the literature on the costs of consumption risk over the business cycle that emerged since his original monograph and reaches a similar conclusion.

This paper reformulates the endogenous growth models above in a way that does give rise to the growth effect illustrated in the top panel of Figure 1. It does this by introducing diminishing returns to investment, which implies that the growth rate will be a concave function of the level of investment. As a result, even if eliminating fluctuations had no effect on average investment, and thus no effect on average initial consumption, it would still lead to more rapid growth by making investment less volatile. Intuitively, eliminating fluctuations reallocates investment from periods of high investment to periods of low investment. Since the marginal return to investment is higher when there is less investment, this allows agents to achieve more growth from the same resources. The first part of the paper formalizes this argument using a familiar model of endogenous growth.

In the remainder of the paper, I assemble various pieces of evidence on the extent of diminishing returns to investment. These consistently suggest that eliminating fluctuations will increase the growth rate of per-capita consumption from 2.0% to about $2.4 - 2.5\%$ if we held average investment fixed. For $\gamma = 1$, this implies a cost of cycles of $8 - 10\%$ of consumption per year, and the cost only rises when I turn to an alternative set of preferences that can better replicate the volatility of aggregate investment over the cycle. While this suggests business cycles are quite costly, it need not follow from this result that stabilization is desirable. This largely depends on the source of aggregate shocks. For example, if cycles are due to productivity shocks, policymakers will only end up making agents worse off by attempting to offset these shocks with countercyclical policy (although agents could potentially be made better off by coordinating to an equally productive but less volatile technology). Thus, this paper does not directly advocate for stabilization; rather, its purpose is to offer a rationale for why business cycles matter. Documenting a large cost of cycles is an important result in its own right given the grave importance people often seem to attach to economic fluctuations despite previous results that suggest they shouldn't, and it opens the door to a role for stabilization, at least in some scenarios, that previous calculations would deny. The basic insights developed here can and should be used to explore environments where policy can play a beneficial role in order to determine whether stabilization policy is ultimately desirable.

The paper is organized as follows. Section 2 develops a model of endogenous growth that allows for diminishing returns to investment. Section 3 attempts to quantify the welfare cost of fluctuations using data on growth. Section 4 examines whether calibrating the endogenous growth model to produce a significant growth effect can accord with other time series data, e.g. investment. Section 5 calibrates the model to data on Tobin's q. Section 6 concludes.

2. Endogenous Growth with Diminishing Returns to Investment

To study the effects of economic fluctuations on growth and welfare, I need a model in which the growth rate is determined endogenously. Towards this end, I use a stochastic AK growth model. This specification has become a staple for modeling endogenous growth under uncertainty, and using it facilitates comparison with the previous literature. The first to analyze this model were Levhari and Srinivasan (1969), who used it to study savings decisions under uncertainty. They in turn solved an infinite-horizon version of a problem that was originally studied by Phelps (1962). Leland (1974) subsequently reinterpreted this model in terms of long-run economic growth. Many authors have since used variations of this basic model to study endogenous growth in the face of aggregate uncertainty.

The economy consists of a representative agent who derives utility only from consumption. To simplify the exposition, I focus on the planning problem for this economy. One can show that the allocation of resources that solves this problem coincides with the equilibrium allocation in a decentralized market economy. Time is discrete, and the agent discounts the future at rate β. For now, I assume per-period utility is isoelastic in consumption, i.e. for a given consumption stream $\{C_t\}_{t=0}^{\infty}$, the utility of the agent is equal to

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{2.1}$$

In evaluating the benefits of growth, Lucas (1987) considered the case where $\gamma = 1$, i.e. log utility, a value that falls within the range of estimates for intertemporal elasticity of substitution found in the literature, e.g. Epstein and Zinn (1991). I similarly use this case as my benchmark for calculating the cost of fluctuations. However, below I argue that this specification for utility does a poor job of matching certain empirical regularities, which motivates me to eventually turn to a more general formulation that includes (2.1) as a special case.

The agent has access to a production technology that converts inputs into consumption goods. The only input for producing consumption goods is capital. Specifically, production is linear in capital, but the number of units that can be produced from a unit of capital fluctuates stochastically over time, i.e.

$$Y_t = A_t K_t$$

where A_t follows a Markov process. Thus, the source of fluctuations in this economy are shocks to productivity. However, as noted by Eaton (1981), we can always reinterpret these shocks as government policy shocks. That is, suppose aggregate productivity is constant over time, but there is a government sector that collects a random fraction τ_t of income in taxes which it uses to finance contemporaneous purchases that do not enter into the agent's utility. This leaves the agent with an income of $Y_t = (1 - \tau_t) AK_t \equiv A_t K_t$.

In what follows, it will prove necessary to impose some regularity conditions on the Markov process A_t. First, I require that for any x, $\text{Prob}(A_{t+1} \leq x \mid A_t)$ is weakly decreasing in A_t. Thus, a higher realization of productivity today is associated with a weakly higher expected productivity next period. This assumption ensures the agent will be better off at higher levels of aggregate productivity. This includes the case where A_t is i.i.d. I further assume that the values of A_t are such that expected discounted utility is well-defined, i.e. the growth rate cannot exceed the discount rate, and the planning problem has an interior solution. Finally, I assume the initial distribution over A_t corresponds to the invariant distribution of the Markov chain (which implicitly assumes such a distribution exists and is unique). I evaluate welfare using this distribution throughout the paper, i.e. the expected utility of the agent is calculated prior to the resolution of uncertainty over the initial level of productivity A_0.

Since output is proportional to capital, the time path of output (as well as consumption) depends on the evolution of the capital stock K_t. At date 0, the agent is endowed with some initial amount of capital K_0. Beyond this date, the level of capital depends on the endogenous decisions of the agent. If the agent begins the period with K_t units, a fraction δ of the capital is assumed to depreciate over the period, so that at the beginning of the next period only $(1 - \delta) K_t$ units remain. The agent can add to this stock by setting aside some of the output from the current period and converting it, together with his existing capital, into capital for use in the subsequent period. The technology for producing new capital is characterized by a function $\Phi(I_t, K_t)$ that depends on the amount of output set aside for investment I_t and the existing stock of capital K_t. The function $\Phi(\cdot, \cdot)$ is assumed to be homogenous of degree 1 and increasing in its first argument. Hence, we can rewrite this production function as

$$\Phi(I_t, K_t) = \phi\left(\frac{I_t}{K_t}\right) K_t$$

where $\phi'(\cdot) > 0$. The stock of capital available for production in period $t+1$ is thus

$$K_{t+1} = \left(1 + \phi\left(\frac{I_t}{K_t}\right) - \delta\right) K_t$$

Repeated substitution of the above equation yields the capital stock at date t as a function of the initial capital stock K_0:

$$K_t = \left[\prod_{s=0}^{t} \left(1 + \phi\left(\frac{I_s}{K_s}\right) - \delta\right)\right] K_0 \tag{2.2}$$

The original Levhari and Srinivasan (1969) model is a special case of this model where the function $\phi(\cdot)$ is equal to the identity function and $\delta = 1$. In this case, equation (2.2) reduces to $K_{t+1} = I_t$, i.e. the wealth K_{t+1} the agent holds at the beginning of each period is just his savings from the previous period $I_t = Y_t - C_t$. While several authors have by now departed from the assumption of full depreciation, in line with the interpretation of K_t as physical capital, the assumption that $\phi(\cdot)$ is linear is still commonplace. But following Uzawa (1969), it is just as natural to allow $\phi(\cdot)$ to be concave, implying diminishing returns to investment. Formally, concavity in $\phi(\cdot)$ implies that for a fixed amount of capital, each additional unit of investment will contribute less to the stock of capital. We can alternatively interpret this concavity as adjustment costs denominated in units of capital, so that increasingly more capital is required to merge new investment goods with the existing capital stock. As I discuss in more detail below, empirical evidence suggests there is indeed some curvature in $\phi(\cdot)$ in practice.[2]

To summarize, output in period t is produced using all of the capital available at the beginning of the period. Out of this output, the agent chooses to consume an amount C_t, and uses the remainder $I_t = Y_t - C_t$ to invest in capital for the next period. It will prove convenient to define $c_t = C_t/Y_t$ as the fraction of output the agent consumes and $i_t = I_t/Y_t = 1 - c_t$ as the fraction he sets aside for investment. Using this notation, we can rewrite the agent's consumption stream in a form reminiscent of Lucas' original specification:

$$\begin{aligned} C_t &= c_t A_t K_t \\ &= c_t A_t \left[\prod_{s=0}^{t} \left(1 + \phi(i_s A_s) - \delta\right) \right] K_0 \\ &\equiv \left[\prod_{s=0}^{t} \lambda_s \right] (1 + \varepsilon_t)\, C_0 \end{aligned} \tag{2.3}$$

where $\lambda_s \equiv 1 + \phi(i_s A_s) - \delta$ is the growth rate of the capital stock, $\varepsilon_t \equiv \dfrac{c_t A_t}{c_0 A_0} - 1$ is the deviation of consumption from its trend, and $C_0 = c_0 A_0 K_0$ is the initial level of consumption. Note that if the growth rate of capital λ_s were constant, this consumption stream would simplify to $\lambda^t (1 + \varepsilon_t) C_0$, precisely the form Lucas posited. But this consumption stream now emerges endogenously as an optimal response to the underlying economic environment rather than as an exogenous specification.

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[2] As an aside, diminishing returns also serve to dampen the response of investment to changes in A. This helps to address a common critique of AK growth models, namely that policy changes that ought to affect investment have little effect on growth empirically. While some have argued that policy changes have no effect at all on long-run growth, McGrattan (1998) argues that these effects are apparent in longer time series.

The cost of aggregate fluctuations is defined as the *ex-ante* utility gain for the agent (i.e. prior to the realization of the sequence $\{A_t\}_{t=0}^{\infty}$) if we were to move him from the stochastic environment above to one with the same initial capital stock but where productivity is constant and equal to the unconditional average productivity in the stochastic environment. That is, both environments are equally productive on average, but in one environment productivity fluctuates around its mean. For notational convenience, let an asterisk denote the value of a variable in the counterfactually stable environment. Thus, productivity A_t^* is constant for all dates t and is equal to $E[A_t] \equiv A^*$. Following Lucas, I measure the cost of cyclical fluctuations in terms of compensating variation, i.e. by the additional fraction of consumption per period the agent would require in the stochastic environment to attain the same level of expected utility as in the stable environment.

A few remarks about the interpretation of this cost are in order. If A_t reflects exogenous technology shocks, the above cost is purely hypothetical in the sense that it cannot be avoided by stabilization policy. Although the government can replicate the effects of productivity shocks on the agent through taxes and subsidies, the original allocation is Pareto optimal, and any self-financing scheme that fools him into different consumption and savings choices will only lower his welfare. Hence, the cost of cycles does not represent the gains from any feasible stabilization policy. Still, establishing that this cost is large can explain why individuals often cite business cycles as an important concern. By contrast, if fluctuations in A_t reflect spurious fiscal policy shocks, the cost of cycles can be avoided simply by eliminating arbitrary policy variability, in which case the cost of cycles is also the value to stabilization. However, for this logic to go through, it is important that fluctuations be spurious rather than an optimal response to some other underlying shock.[3] For example, we would not want local governments to even out their expenditures on snow removal over the year. Although smoothing these expenditures over the year would eliminate one source of volatility, it would prevent resources from being allocated to address underlying weather shocks that are seasonal in and of themselves.

Since the utility of the agent depends on the consumption stream he faces, we first need to solve the optimal consumption for a given economic environment. We therefore set up the

[3] Arbitrary fluctuations are not limited to capricious policymaking. Sunspots and coordination failures can also lead to spurious volatility in measured productivity A_t. For example, Benhabib and Farmer (1994) generate sunspots in models with increasing returns to scale, while Shleifer (1986) generates volatile productivity through coordination on the implementation of exogenous technological improvements.

problem of the representative agent:

$$V(K_0, A_0) \equiv \max_{C_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right] \tag{2.4}$$

subject to

1. $K_{t+1} = \left[\phi \left(\frac{A_t K_t - C_t}{K_t} \right) + 1 - \delta \right] K_t$
2. the law of motion for A_t

We can rewrite this problem recursively as

$$\begin{aligned} V(K_t, A_t) &= \max_{C_t} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} + \beta E \left[V(K_{t+1}, A_{t+1}) \mid A_t \right] \right\} \\ &= \max_{c_t \in [0,1]} \left\{ \frac{(c_t A_t K_t)^{1-\gamma}}{1-\gamma} + \beta E \left[V((1 + \phi(i_t A_t) - \delta) K_t, A_{t+1}) \mid A_t \right] \right\} \end{aligned}$$

where recall $i_t = 1 - c_t$.

Using the maximization problem above, I next argue that in the face of aggregate fluctuations, the agent will choose to vary his investment rate $i_t A_t$ over time. Consequently, the growth rate $\lambda_t = 1 + \phi(i_t A_t) - \delta$ will fluctuate with A_t. Establishing this result requires additional technical assumptions, namely that $\phi(\cdot)$ is strictly concave and that it satisfies the boundary conditions $\lim_{x \to 0} \phi'(x) = \infty$ and $\lim_{x \to \infty} \phi'(x) = 0$. With these assumptions, we have the following proposition, whose proof is contained in an Appendix:

Proposition: Suppose Prob$(A_{t+1} \leq x \mid A_t)$ is weakly decreasing in A_t. Then $i_t A_t = \frac{I_t}{K_t}$ is increasing in A_t.

An implication of this proposition is that if we decompose investment or output into a stochastic trend and deviations from trend, we would find that trend growth for both investment and output is higher when these variables are above their trend. Formally, rewrite Y_t and I_t in the same way as C_t in (2.3), i.e.

$$Y_t = \left[\prod_{s=0}^{t-1} \lambda_s \right] \left(1 + \varepsilon_t' \right) Y_0 \tag{2.5}$$

and

$$I_t = \left[\prod_{s=0}^{t-1} \lambda_s \right] \left(1 + \varepsilon_t'' \right) I_0 \tag{2.6}$$

where $\lambda_s = 1 + \phi(i_s A_s) - \delta$ as before, $\varepsilon_t' = \frac{A_t}{A_0} - 1$, and $\varepsilon_t'' = \frac{i_t A_t}{i_0 A_0} - 1$. Then ε_t' and ε_t'' will be positively correlated with λ_t. This will not necessarily be true for consumption, since $c_t A_t = A_t - i_t A_t$ could either increase or decrease with A_t, and hence ε_t in (2.3) could be negatively correlated with λ_t. However, since trend consumption growth and trend output growth are both equal to λ_s, consumption growth will generally be positively correlated with output growth. This observation will be important for interpreting some of the empirical results below.

To understand the welfare implications of cyclical fluctuations, it will help to work through the different ways in which shifting from stochastic productivity $\{A_t\}$ to a constant productivity A^* induces the agent to change his consumption. First, eliminating fluctuations induces the agent to choose a consumption path that does not fluctuate around its trend. Formally, when A_t^* is constant for all t, the agent will choose to consume a constant fraction c^* of output. But this implies the deviation from trend consumption in (2.3) will equal

$$\varepsilon_t^* = \frac{c_t^* A_t^*}{c_0^* A_0^*} - 1 = \frac{c^* A^*}{c^* A^*} - 1 = 0$$

Since the agent is risk-averse, eliminating deviations from trend makes him better off. But when Lucas (1987) computes the implied welfare gain assuming that *all* observed fluctuations in consumption growth reflect deviations from trend, the welfare gains from eliminating such deviations turn out to be negligible for reasonable utility specifications.[4] Simply put, deviations from trend per-capita consumption over the post-War period are not large enough to generate significant costs of cyclical fluctuations for reasonable degrees of risk aversion.

Second, since the agent will choose to set aside a constant fraction i^* of his income for investment when productivity is stable, eliminating fluctuations in aggregate productivity induces the agent to choose a consumption path with a deterministic trend rather than a stochastic trend. Thus, stabilization eliminates both fluctuations *around* trend consumption as well as fluctuations *in* trend consumption. Once again, this will make the agent better off given his aversion to risk. But since fluctuations in trend consumption are permanent, they reduce welfare by more than stationary fluctuations around trend. Still, when Obstfeld (1994) calculates the implied welfare gain under the assumption that *all* observed fluctuations in consumption growth represent i.i.d. permanent shocks to trend, he again finds relatively small welfare costs.

[4] Lucas' calculation assumes deviations from trend are i.i.d., while the model allows deviations to be serially correlated. More persistent shocks would generate larger costs. But as I discuss below, allowing for more persistent shocks to the level of consumption does not lead to substantial costs of fluctuations.

Somewhat larger costs can arise if we assumed persistent rather than i.i.d. shocks to trend, as is allowed under the more general Markov structure of A_t. But empirically, consumption growth exhibits fairly low persistence, as documented among others by Christiano, Eichenbaum, and Marshall (1991).[5] This reaffirms Lucas' original observation, namely that even though eliminating fluctuations makes it easier for the agent to smooth consumption, the volatility of per-capita consumption over the cycle is so small that this welfare gain is negligible.

Lastly, eliminating fluctuations in productivity in this environment can induce the agent to maintain a differently-sloped consumption profile from the one in the stochastic environment. Here, we need to distinguish between changes in the consumption profile that are due to changes in the *volatility* of investment and those that are due to changes in the *level* of investment. To separate the two, suppose we were to eliminate fluctuations in A_t but force the agent to save a constant fraction i^* such that the average investment-to-capital ratio is the same as in the stochastic environment, i.e. i^* solves

$$i^* A^* = E\left[i_t A_t\right]$$

Given the concavity in $\phi(\cdot)$ and the results in the proposition, this level of investment will be associated with a higher average growth rate, since for any non-degenerate distribution of $i_t A_t$,

$$\begin{aligned} 1 + \phi\left(i^* A^*\right) - \delta &= 1 + \phi\left(E\left[i_t A_t\right]\right) - \delta \\ &> 1 + E\left[\phi\left(i_t A_t\right)\right] - \delta \end{aligned}$$

Thus, merely eliminating volatility in investment will lead to more rapid growth, even if the average amount of resources set aside for investment remains unchanged. If we then allow the agent to freely choose his investment optimally and he chooses some $i^* \neq E\left[i_t A_t\right]/A^*$, the long-run growth rate will change further. The direction of the change depends on whether the agent desires higher or lower investment in the absence of shocks. Both scenarios are possible for different parameter values. The effect of eliminating fluctuations in A_t on growth can thus be decomposed into two parts: the part due to eliminating the volatility of investment around its original average, and the part due to changes in average investment in response to moving to a more stable environment.

[5] Bansal and Yaron (2001) argue instead that consumption growth is persistent. They assume consumption growth follows an ARMA process $g_{t+1} = \rho g_t + \eta_t - \omega \eta_{t-1}$ where η_t is i.i.d. If ω is close to ρ, it will be hard to distinguish this process from white-noise even when ρ is large, which could lead us to erroneously infer growth is not highly autocorrelated if we fail to take into account the moving-average term. Bansal and Yaron estimate $\rho = 0.95$ and $\omega = 0.85$ from dividend growth (which is the same as consumption growth in their model), and use this to argue the costs of business cycles are large. However, ARMA models for consusmption growth data, such as Lewbel (1994), fail to find high values of ρ.

To see why it is important to distinguish between these two effects, note that by forcing the agent to keep the average investment-capital ratio unchanged, we leave his expected initial consumption unchanged, since

$$\begin{aligned} c^* A^* K_0^* &= (A^* - i^* A^*) K_0^* \\ &= [E(A_t) - E(i_t A_t)] K_0 \\ &= E(c_t A_t) K_0 \end{aligned}$$

Thus, eliminating fluctuations and forcing the agent to maintain the same average investment allows him to attain a consumption path that starts at the same level on average but grows more rapidly. Lucas (1987) provides some calculations on just how much the agent would be willing to sacrifice to achieve this scenario. For $\gamma = 1$, he reports the agent would be willing to sacrifice 20% of consumption each year to increase the growth rate of consumption by one percentage point. By contrast, if the growth rate is higher because the average investment rate $i^* A^*$ is higher, initial consumption will be lower, since $C_0^* = (A^* - i^* A^*) K_0^*$ is decreasing in $i^* A^*$. The two channels are therefore associated with welfare changes of very different magnitudes (and possibly of different signs) for a given change in the average growth rate. For example, the agent might optimally choose a lower average investment rate, $i^* A^* < E(i_t A_t)$, even though it leads to lower growth. Increasing growth by undertaking more investment does not automatically raise welfare as it does in Lucas' thought experiment.[6]

Since previous authors assume $\phi(\cdot)$ is linear, they allow fluctuations to affect growth only through changes in the level of investment. Consequently, those who previously allowed for growth effects have found only small costs. This is not surprising: if eliminating fluctuations increases growth by increasing investment, the gain the agent reaps from more rapid consumption growth will be largely offset by the loss he incurs from a lower initial consumption. To generate more substantial welfare gains of the magnitude suggested by Lucas' calculations on the value of growth, it is necessary that cycles retard growth for a fixed average investment, which is precisely what happens under diminishing returns.

In sum, the key to generating large welfare costs of cyclical fluctuations appears to be the presence of diminishing returns to investment, i.e. a concave function $\phi(\cdot)$. In the next several

[6] A similar distinction arises in models where growth is driven by learning-by-doing rather than factor accumulation, e.g. Ramey and Ramey (1991). In these models, eliminating fluctuations can simultaneously raise both current output and future output growth. The inherent tradeoff in these models is not between present and future consumption, but between present leisure and present (and future) consumption. Once again, we would want to distinguish between changes in the growth rate due to changes in the *level* of output, which require sacrificing initial leisure, and those due to changes in the *volatility* of output, which do not.

sections, I argue that empirical evidence supports the notion of diminishing returns and thus a large cost of business cycles. At first glance, this result might seem puzzling: if investment volatility has such an adverse effect on welfare, why wouldn't the agent smooth investment at the expense of consumption volatility, especially if consumption volatility is so inconsequential? But this intuition is misleading. Previous literature has only established that the consumption volatility we observe over the cycle is inconsequential. But the consumption volatility that would be required to counterfactually stabilize investment over the cycle may be substantially larger than the volatility of consumption we actually observe over the cycle. As the proposition above demonstrates, it is in fact optimal to vary investment when productivity fluctuates. Ultimately, the costs of aggregate fluctuations come not from the fact that investment is volatile *per se*, but from the fact that the underlying stochastic environment forces the agent to choose a volatile path for investment. As such, the agent might be significantly better off if the shocks that caused him to behave this way were eliminated.

3. Quantitative Analysis: Evidence from Growth Data

Since the cost of fluctuations above stems from the effects of fluctuations on growth, it seems natural to begin with evidence on growth rates themselves to quantify this cost. In this section, I pursue two different approaches to exploiting such data to gauge the effects of fluctuations on growth. First, I use a reduced-form approach that exploits variation in volatility and growth across observations to predict the counterfactual growth rate in the U.S. if aggregate volatility were set to zero. This approach has the virtue that it does not require estimating returns to investment. Second, I make use of estimates of diminishing returns together with the distribution of consumption growth over the post-War period as an alternative approach to estimating the same counterfactual growth rate.

3.1. Reduced-Form Estimates

In computing the cost of fluctuations, recall that cycles affect growth in two ways: they make investment fluctuate around its average level, and they can change the average level of investment itself. For the model above, eliminating the volatility of investment while holding average investment fixed provides a lower bound on the cost of business cycles. This is because investment is Pareto optimal, so allowing agents to also change the level investment can only make them better off. In what follows, then, I abstract from the effects of fluctuations on the average level of investment. This provides me with a lower bound on the cost of cycles, at least when investment is socially efficient. If investment is inefficient, my approach could potentially overstate (or understate) the true cost of aggregate fluctuations. However, the welfare effects of changing average investment are typically much smaller than the effects of reducing growth for a given initial level of consumption, so it seems unlikely that abstracting from them will significantly overstate the true cost of cycles. Moreover, there is some evidence (which I cite below) that macroeconomic volatility does not appear to be related to the level of investment, suggesting these effects are likely to be of minor significance in any event.

Suppose we had cross-section or time-series data on growth rates, investment, and macroeconomic volatility. We could use this data to estimate average growth as a function of volatility and other variables, i.e. to estimate

$$E[\lambda] = f(X, \sigma) \tag{3.1}$$

where $E[\lambda]$ denotes average growth over a given time period, σ is a measure of aggregate volatility over this same period, and X denotes other variables that affect average growth. If X includes the average level of investment over the relevant time period, we can use the function $f(\cdot,\cdot)$ to project average growth to the case where $\sigma = 0$ but average investment is held fixed. Conveniently, previous authors have already carried out such an exercise. For example, Ramey and Ramey (1995) estimate a linear specification of (3.1) using cross-country data. They estimate a model of the form

$$\begin{aligned} \Delta \ln y_{it} &= \mu\sigma_i + \theta X_i + \varepsilon_{it} \\ \varepsilon_{it} &\sim N\left(0, \sigma_i^2\right) \end{aligned} \tag{3.2}$$

where $\Delta \ln y_{it}$ denotes the growth rate of real GDP per capita in country i and year t, the vector X_i is a set of country-specific explanatory variables, and the error term ε_{it} is distributed normally with a variance σ_i^2 specific to each country.[7] This specification assumes that in any given year, the growth rate in each country varies from its historical mean by a normally distributed error term ε_{it}, where the historical mean $\mu\sigma_i + \theta X_i$ depends on the size of the standard deviation of the error term for the country in question.

The model in (3.2) can be estimated by maximum likelihood. Ramey and Ramey estimate (3.2) for a sample of 92 countries using observations between 1962 and 1985, as well as a

[7] Although the relevant growth rate for calculating the welfare cost of fluctuations involves the growth rate of consumption, the two series grow at the same rate in the long-run in the model constructed above. Thus, the fact that Ramey and Ramey use the growth rate of output per capita does not pose a problem for my analysis.

subset of 24 OECD countries using observations between 1952 and 1988. They find that after conditioning on average investment, growth and volatility are negatively related. The coefficient μ on the volatility term σ_i is significantly different from zero at conventional significance levels. Their point estimate for μ varies across samples and specifications between -0.1 and -0.9, although estimates appear clustered around -0.2. Since the standard deviation of per-capita output growth in the U.S. is 2.5%, using the point estimate of -0.2 suggests that eliminating aggregate shocks altogether should increase the growth rate by half a percentage point, from 2.0 to 2.5% per year. Applying Lucas' estimate that an agent would sacrifice approximately 20% of consumption for a 1 point increase in growth when $\gamma = 1$, we obtain a cost of aggregate fluctuations of at least 10% of consumption per year, two orders of magnitude greater than Lucas' original estimate. It is also noteworthy that Ramey and Ramey find that controlling for the investment to output ratio has no effect on the point estimate for μ; the negative correlation between growth and volatility does not operate through differences in average investment shares. This accords with additional evidence they present that the investment share of output does not vary systematically with the volatility of GDP growth across countries. Thus, macroeconomic volatility appears to be correlated with growth directly and not through its effect on investment.

Ramey and Ramey's results have been subsequently extended. For example, Martin and Rogers (2000) consider the relationship between growth and volatility across European regions. They estimate μ at -0.274, compared with Ramey and Ramey's estimates of -0.211 and -0.384 for the full-country sample and the OECD sample, respectively. This point estimate suggests a larger growth effect of almost seven-tenths of a percentage point.[8]

As a further check, I examine the same relationship using time-series variation for the U.S. Previous authors, including Zarnowitz and Moore (1986) and Ramey and Ramey (1991), already demonstrated that there is a significant negative correlation between the level and the volatility of output growth in the U.S. over time. However, these studies only report raw correlations rather than point estimates for μ, and neither controls for average investment. I therefore re-examine the time-series data to derive estimates that are comparable to the above specification. I follow Ramey and Ramey (1991) in dividing the data into distinct four-year episodes between presidential elections. This identification assumes that elections play an important role in gen-

[8] Martin and Rogers' methodology is not identical to Ramey and Ramey's. For example, they regress average growth on the unconditional standard deviation of output growth rather than use (3.2). But the explanatory variables account for such a small share of the variation in growth that this is insignificant. Martin and Rogers also do not control for average investment. But as noted above, average investment is uncorrelated with volatility across countries, suggesting that omitting this variable may not be of much consequence either.

erating changes in macroeconomic policy, which in turn affects the level of aggregate volatility.[9] I apply this time-series data to estimate the same model as in (3.2). That is, I assume that the growth rate each quarter is a deviation from the mean growth rate over the relevant four year period, where the deviation is drawn from a normal distribution with standard deviation σ_i specific to that period. Using quarterly data from 1953:1 - 2000:4 on output and gross domestic investment from publicly available Bureau of Economic Analysis (BEA) sources (and converted to per-capita levels using Census Bureau population figures), I constructed a panel dataset of 12 four-year periods, with each panel consisting of 16 quarterly observations. In contrast with the cross-country data, there is now no need to introduce additional control variables such as measures of human capital, which are unlikely to have much explanatory power for a single country within a relatively short time horizon.

The results for the 12 four-year panels are reported in Table 1. The parameter of interest μ is the coefficient on the standard deviation of output growth. The point estimate of μ is negative, although we cannot reject the null hypothesis that $\mu \geq 0$. Its point value is -0.348, which again falls within the range from cross-country data in Ramey and Ramey (1995). I experimented with dropping observations to gauge the robustness of the point estimate of μ. For all of the subsamples I considered, the point estimate for μ remained consistently on the order of -0.2 and -0.3. Thus, various sources of variation that can be used to estimate a reduced-form relationship – countries, regions, and time periods – all suggest that eliminating volatility in the U.S. should increase growth by between half and three quarters of a percentage point when holding average investment fixed.

3.2. Estimates Based on Diminishing Returns

As an alternative approach, we can use data on growth together with actual estimates of diminishing returns to predict the same counterfactual rate at which the economy would grow if we stabilized investment at its mean that the reduced-form estimates above are meant to capture. To be more precise, suppose $\phi(\cdot)$ is given by

$$\phi\left(\frac{I}{K}\right) = \left(\frac{I}{K}\right)^{\psi} \tag{3.3}$$

[9] I also experimented with accounting for the two presidential successions that occurred between elections during this period, but this change proved unimportant. I likewise considered grouping observations by presidential administrations rather than by terms. However, a likelihood ratio test overwhelmingly rejected this specification in favor of one that allows volatility to differ across terms.

where $\psi \in (0,1]$ governs the extent of diminishing returns to investment. Suppose further that we know the distribution of trend per-capita consumption growth, $\lambda_t = 1 + \phi(i_t A_t) - \delta$. By inverting the function $\phi(\cdot)$ in (3.3), we can compute the implied growth rate if we were to stabilize investment at its mean value. Specifically,

$$1 + \phi(E[i_t A_t]) - \delta = 1 + \left(E\left[(\lambda_t - 1 + \delta)^{\frac{1}{\psi}}\right]\right)^{\psi} - \delta \tag{3.4}$$

Thus, if we estimate ψ together with the distribution of the permanent part of consumption growth, we can directly compute what the growth rate would be in the absence of fluctuations if we were to hold average investment fixed.

Turning first to ψ, note that the first order condition of the agent's maximization problem, which is derived in the Appendix, can be rewritten as

$$\phi'(i_t A_t) = \left[\frac{E[\beta V_K(K_{t+1}, A_{t+1})]}{U'(C_t)}\right]^{-1} \tag{3.5}$$

The expression inside the brackets is the ratio of the marginal value of a unit of capital relative to the value of an additional unit of investment. This ratio is commonly referred to as marginal q.[10] For the isoelastic function, we obtain the following relationship between the investment-to-capital ratio and q:

$$\ln\left(\frac{I}{K}\right) = \text{constant} + \frac{1}{1-\psi}\ln q \tag{3.6}$$

Hence, we can recover the curvature parameter ψ from the elasticity of the investment-to-capital ratio with respect to q. Estimates of this elasticity abound in the literature. One of the few papers that explicitly estimates an isoelastic specification in line with (3.6) is Eberly (1997), who estimates investment equations using micro data for various OECD countries. For the U.S., her point estimate is equal to 1.22, with a 95% confidence interval of $[1.08, 1.36]$, which implies $\psi \in [0.07, 0.26]$. If the investment technology is identical in the 11 OECD countries in her sample, we can obtain a more precise point estimate by averaging the estimated elasticity across countries. Weighting by the number of observations in each country, the average elasticity of investment with respect to q is equal to 1.36. This estimate lies within the 95% confidence interval for 10 out of the 11 OECD countries in the sample (including the U.S.). Since the cross-country comparison in Ramey and Ramey (1995) is only valid if the investment technology $\phi(\cdot)$ is the same across countries, the consistency of this elasticity across countries in Eberly's sample is reassuring.

[10] Since the production technologies are all homogeneous of degree 1, marginal q in this economy is equal to average q, as shown in Hayashi (1982). The latter measure is the one frequently used in empirical work.

Other studies that look at the relationship between investment and q in the U.S. yield estimates that are broadly consistent with those reported above. For example, various papers estimate a version of (3.6) that uses levels as opposed to logs. These can still be used to compute a point elasticity at the sample mean to gauge the magnitude of ψ. Few of the estimates that are reported in the literature exceed those reported above, but a significant number purport to find lower elasticities. For instance, in an early contribution to this literature, Abel (1980) reports estimates of this elasticity that range between 0.50 and 1.14, which captures the range of most subsequent estimates. Estimates below 1 are problematic, since they imply a negative value of ψ. However, such low values have been dismissed on the grounds that measurement error in q would bias the estimated elasticity towards 0. For example, Cummins, Hassett, and Oliner (1999) argue that instrumenting for q routinely yields estimates of elasticity above unity. This claim is confirmed in Eberly's dataset: when she estimates (3.6) in levels using the same data, her point elasticity for U.S. firms evaluated at sample means is equal to 0.56 for ordinary least squares, and 1.06 when she instruments for q. An alternative approach to estimating the curvature parameter ψ is pursued by Christiano and Fisher (1998). They use a method of moments approach to estimate various parameter values for their model, including the elasticity of investment with respect to q. Their estimate of elasticity is 1.31, within the range of Eberly's estimates.

Next, to recover the distribution of λ_t from consumption data, recall that consumption growth in the model is given by

$$\Delta \ln C_t = \ln \lambda_t + \Delta \ln (1 + \varepsilon_t)$$

Although we cannot simply plug in the distribution of consumption growth $\Delta \ln C_t$ into our calculation, we can use it to recover the distribution of trend consumption growth λ_t. That is, assuming productivity A_t follows a Markov process, we can estimate the distribution of λ_t using maximum likelihood. Let $P = [p_{ij}]$ be an $N \times N$ matrix that denotes the transition probabilities between the N values A_t can assume. For the model to accord with observed consumption growth, I need to introduce an additional noise term; otherwise, consumption growth can only assume at most N^2 values. I therefore assume that measured consumption growth is given by

$$\Delta \ln C_t = \ln \lambda_t + \Delta \ln (1 + \varepsilon_t) + \eta_t$$

where $\eta_t \sim N\left(0, \sigma^2\right)$ is assumed to reflect measurement error. The parameters of the model that need to be estimated are a set of trend growth rates $\{\ln \lambda_j\}_{j=0}^{N-1}$, a set of first differences in consumption levels $\{\Delta \ln (1 + \varepsilon_{j,j+1})\}_{j=0}^{N-2}$, the transition matrix P, and the variance term σ^2. Using the estimated transition matrix P, we can solve for the invariant distribution over the

N possible states of the world. With this distribution and the estimates for $\ln \lambda_t$, I can then compute expression (3.4) for a given ψ.

For my estimation, I use annual consumption data from the Bureau of Economic Analysis (BEA), divided by population estimates obtained from the Census Bureau. The data spans the years 1951-1998. I constructed the likelihood function recursively following Hamilton (1994). In estimating the likelihood function, I restrict the unconditional expected growth rate $E[\lambda_t]$, evaluated at the invariant distribution of the estimated transition matrix P, to equal 2.0% per year. This accords with conventional estimates for the growth rate of per-capita consumption based on even longer horizons than my sample period. This restriction is not too stringent, since consumption grows at roughly this rate over my sample period. However, because of the finite length of the sample, there is potential for the maximum likelihood estimate to force transient phenomena into estimates of long-run trends. Table 2 reports the maximum likelihood estimates for all the parameters for the case of $N = 2$ and $N = 3$, respectively. The estimates reveal substantial variation in the growth of trend consumption over time, ranging between -0.7% and 4.3% per year. There is no conflict between the finding of a negative growth rate in the data and the fact that the isoelastic specification implies $\phi(x) \geq 0$, since growth can be negative if the depreciation rate δ exceeds $\phi(iA)$. The fact that estimates for $\Delta \ln(1+\varepsilon)$ are negative imply that trend consumption grows more rapidly when consumption is below trend, i.e. the level of consumption falls when aggregate productivity is high. As noted earlier, this could simply reflect a disproportionate shift towards investment when aggregate productivity rises, and is not inconsistent with the fact that consumption growth is procyclical over the long run. By contrast, the model does imply that output and investment grow more rapidly when they are above trend. Estimating a similar decomposition for these two series confirms this prediction, but I omit these results for the sake of brevity.

Using the invariant distribution for consumption growth reported in Table 2, and assuming $\delta = 0.09$ as is standard in the literature, the implied growth rate (3.4) in the two-regime model is given by

$$1 + \left(0.37\,(0.0857)^{\frac{1}{\psi}} + 0.63\,(0.1241)^{\frac{1}{\psi}}\right)^{\psi} - 0.09 \tag{3.7}$$

and in the three-regime model it is given by

$$1 + \left(0.29\,(0.0830)^{\frac{1}{\psi}} + 0.42\,(0.1127)^{\frac{1}{\psi}} + 0.29\,(0.1329)^{\frac{1}{\psi}}\right)^{\psi} - 0.09 \tag{3.8}$$

Figure 2 plots these two values against the investment elasticity $(1-\psi)^{-1}$. Abel's largest estimate for this elasticity, 1.14, implies that stabilizing investment at its average value will raise the growth rate from 2% to 2.78% and 2.91%, respectively. Eberly's point estimate for this elasticity, 1.22, yields more conservative estimates of 2.58% and 2.64% per year, respectively. Christiano and Fisher's estimate of 1.31, implies growth rates of 2.46% and 2.49% per year, respectively. Finally, an elasticity of 1.36, which is Eberly's average estimate across OECD countries, implies growth rates of 2.39% and 2.42% per year, respectively. Thus, for the range of estimates of the investment elasticity reported in the literature, the two- and three-regime models produce similar growth effects, although the three-regime model suggests a slightly larger effect. In either case, cyclical fluctuations appear to lower growth by about $0.4 - 0.9$ percentage points, which is the same range as the one we obtain using the reduced-form approach above. Thus, the two approaches to using growth data are consistent, and both translate into a welfare cost of at least 8% of consumption per year.

4. Quantitative Analysis: Calibration

The previous section provided estimates of the cost of cyclical fluctuations based only on growth data. In this section, I examine whether a growth effect of the magnitude implied in the previous section can be reconciled with other macro data. More precisely, since the growth rate ultimately depends on investment, I examine whether a large cost of cycles can be reconciled with empirical evidence on aggregate investment. That is, I calibrate the process for $\{A_t\}$ in the model from Section 2 to match the empirical volatility of trend growth reported in Table 2, and then check whether the path for investment given this process is consistent with the actual behavior of investment over the cycle.

I begin my analysis using the standard CRRA utility in Section 2. This function involves two parameters: the discount rate β and the inverse intertemporal elasticity of substitution γ. Following Lucas, I set β to 0.95 and γ to 1. On the production side, I need to specify the Markov process for $\{A_t\}$ that governs the production of consumption goods. To simplify matters, I assume A_t follows a two-state Markov process, where the transition probabilities between the two states are taken from the two-regime model in Table 2. Since the previous section suggests that the economy would likely grow at a rate of 2.5% in the absence of fluctuations, I calibrate the mean of A_t, denoted by $\overline{A}$, to deliver a growth rate of 2.5%. Given the transition probabilities, I then set $A_1 = (1 + x)\overline{A}$ and $A_0 = \left(1 - \frac{0.633}{0.367}x\right)\overline{A}$ to ensure that the average of $\{A_t\}$ is $\overline{A}$, and I choose x to match the standard deviation of trend consumption growth λ_t in Table 2 of 1.85 percentage points.

Lastly, I need to parameterize the technology for producing investment goods. I maintain the isoelastic form for $\phi(\cdot)$ in (3.3). To set ψ, I could have used estimates of the elasticity of investment with respect to q as before. However, the model suggests another way of selecting this parameter. Consider the deterministic steady state of the model, i.e. where $x = 0$. As previously observed by Jones, Manuelli, and Stachetti (1999), if we set $\psi = 1$ and choose $\overline{A}$ to match the empirical average growth rate, the consumption share of output will only equal 0.32. Yet empirically, consumption accounts for about two thirds of output. Jones, Manuelli, and Stacchetti argue that this discrepancy can be resolved by counting a fraction of investment in the model as consumption, under the pretext that investments in human capital such as education and health care expenditures are counted as private consumption in national income accounts. However, between 1951 and 1998, total expenditures for these two categories account for at most 20% of consumption, and in earlier years account for only 6% of total consumption, far less than the nearly 50% share needed to reconcile with the data.[11] An alternative interpretation is that constant returns to investment provide overly powerful incentives for agents to invest. This suggests we can use the fact that consumption accounts for about two thirds of income to determine the extent of diminishing returns. Formally, I calibrate ψ and $\overline{A}$ to match a growth rate of 2.5% and a consumption share of output of 0.67. This yields a value of $\psi = 0.234$, or an elasticity of investment with respect to q of 1.31, on target with estimates of this elasticity described in the previous section.

Once all of the parameter values are assigned, we can solve the model for different values of x and choose the one for which the standard deviation of trend consumption growth is 1.85 percentage points. A brief discussion of how I solve the model is contained in the Appendix. The relevant value of x turns out to be 0.374. At this value, the average growth rate along the equilibrium path is 2.13% per year, despite the fact that the deterministic steady state growth rate is calibrated to 2.5% per year. The lower bound on the cost of fluctuations is actually a little larger than the amount an individual would pay to increase the growth rate from 2.13% to 2.5%, since the increase in growth from eliminating volatility in investment is offset by the fact that average investment $E[i_t A_t]$ falls slightly once we eliminate fluctuations (i.e. the absence of fluctuations will lead the agent to re-optimize between present and future consumption towards slightly higher present consumption). But the calibration confirms my previous results that suggest cyclical fluctuations will lower the long-run average growth rate

[11] Alternatively, some human capital expenditures might not be counted in either GDP or investment, in which case the measured investment share would be smaller than its true value. Jones and Manuelli explore this possibility in subsequent work, but they find that only a negligible fraction of such expenditures is likely to be missing from GDP when returns to investment are assumed constant.

by about 0.4 percentage points relative to the non-stochastic environment.

That said, the calibration also reveals that the investment path implied by the model is much less volatile than what we actually observe over the cycle. Along the optimal path, the investment share of output i_t varies between 0.280 and 0.350. This implies a standard deviation of 10%, only two-thirds of the empirical counterpart of 14%. A more dramatic illustration of this discrepancy involves the volatility of investment relative to the volatility output, a measure often emphasized in the real business cycle literature. If we compare the standard deviation of $i_t A_t$ to the standard deviation of A_t, the fact that i_t is not very volatile implies the standard deviation of detrended investment is only 11% larger than the standard deviation of detrended output. But the actual standard deviation of investment is $2.5 - 3$ times as large as that of output. Thus, if we introduce empirically plausible degrees of curvature in the investment sector, investment in the model winds up being too smooth along the equilibrium path when we calibrate the model to match U.S. data. To put it another way, the model suggests that if investment volatility were really so costly, aggregate investment should be much less volatile than its empirical counterpart. Agents in the model try to avoid varying investment over time given the large cost of investment volatility, and the only reason growth rates fluctuate so much in the calibration is that output is assumed to be so volatile that it is hard to find resources to smooth investment over time without making consumption overly volatile.

At first glance, the above analysis suggests there is contradiction inherent within the idea that growth effects lead to a large cost of business cycles: if cycles are so costly, agents will act to undo that cost. However, one has to be careful about using the above model to draw this conclusion. To appreciate why the model might be problematic, recall that the first-order condition for (2.4) is given by

$$\phi'\left(\frac{I_t}{K_t}\right) = q^{-1}$$

where q is the ratio of the market value of equity to the replacement value of physical capital. As such, the volatility of investment in the model is intimately related to the volatility of asset prices. But it is well known that in the standard real business cycle model, asset prices are far too smooth in comparison with the data. Formally, let q_0 denote the value of q_t when $A_t = A_0$ and similarly for q_1. For the CRRA utility above with $\gamma = 1$, the ratio of q across the two regimes is given by

$$\frac{q_1}{q_0} = \frac{\lambda_1}{\lambda_0}\frac{C_0}{C_1} \tag{4.1}$$

where $\lambda = 1 + \phi(iA) - \delta$ denotes the trend growth rate for each respective level of productivity. Since $\dfrac{\lambda_1}{\lambda_0} \approx 1$ and consumption is not very volatile for plausible parameter values, q will hardly vary over the cycle, and thus neither will investment. But empirically, q is highly volatile, as will be discussed in more detail in the next section. It is therefore unreasonable to expect a model which does a notoriously poor job of accounting for the volatility of asset prices to generate plausible investment volatility when investment inherently responds to these prices.

In fact, several authors have recently argued that if we modify the real business cycle model so that it can match empirical evidence on asset prices, investment will be *overly* volatile unless we allow for some diminishing returns to investment. This was demonstrated, among others, by Jermann (1998), Christiano and Fisher (1998), and Boldrin, Christiano, and Fisher (2001). These papers all set out to modify the traditional real business cycle model so that it can accord with the data on asset prices. Building on insights from the finance literature, they suggest replacing (2.1) with a more general function

$$U(\{C_t\}) = \sum_{t=0}^{\infty} \beta^t \frac{(C_t - bC_{t-1})^{1-\gamma} - 1}{1-\gamma} \tag{4.2}$$

for some $b \geq 0$. Note that the CRRA utility is just a special case of this function where $b = 0$. But if we allow b to be sufficiently different from 0, marginal utility will be quite volatile even when consumption is relatively smooth. This translates into a relatively volatile series for q, which provides strong incentives to vary investment over time. In fact, in the absence of diminishing returns, investment is too volatile, and all three papers above argue that some friction in the investment good sector such as adjustment costs must be introduced to moderate the volatility of investment. Jermann (1998) provides a particularly nice intuition for why business cycle models that are capable of matching asset prices require both habit formation and adjustment costs: "With no habit formation, marginal rates of substitution are not very volatile, since people do not care very much about consumption volatility; with no adjustment costs, they choose consumption streams to get rid of volatility of marginal rates of substitution. They have to both care, and be prevented from doing anything about it."[12]

Since the aforementioned work on incorporating asset prices into real business cycle models abstracts from growth considerations, we need to verify that the same intuition carries over to endogenous growth settings. After all, agents might have more incentive to keep investment

[12] An alternative modification of the RBC model that also generates more volatile asset prices is the generalization to a multi-country setting, as in Baxter and Crucini (1993). In this case, asset prices are volatile not because marginal rates of substitution are volatile, but because dividends are. Specifically, production will tend to be concentrated in the country with the highest productivity, so owning equity in a country-specific technology will yield low dividends when that country is not the leader. Given the implied volatility of asset prices, Baxter and Crucini similarly need to introduce adjustment costs for investment not to be too volatile.

stable if it affects long-run growth. I therefore re-evaluate the model using (4.2) as my utility specification. I continue to set $\gamma = 1$, which previous authors have argued is a reasonable value. I calibrate the habit parameter b so that along the equilibrium path, investment will be 2.5 times as volatile as output, in line with the estimate provided in Jermann (1998). In choosing the parameter ψ, note that the intertemporal Euler equation – which is provided in the Appendix – implies that the steady-state growth rate does not depend on b. Since the CRRA utility function is just a special case of (4.2) in which $b = 0$, it follows that we would still require $\psi = 0.234$ for the model to yield an average consumption share of two thirds.[13] Solving the model is now more difficult, since there is an additional state variable in the form of lagged consumption C_{t-1}. I therefore resort to a linear approximation of the intertemporal Euler equation to solve the model. I provide a brief discussion of my approach in the Appendix.

The value of b that generates the requisite relative volatility in investment is $b = 0.7$. For this value, i_t ranges between 0.12 and 0.42 (although these are extreme values; i_t no longer assumes a two point distribution as in the CRRA case). This value of b is consistent with what previous researchers have argued is necessary to accord with data on the equity premium. For example, Boldrin, Christiano, and Fisher (2001) estimate $b = 0.73$ in calibrating their RBC model to accord with the empirical equity premium. Christiano and Fisher (1998) estimate a somewhat lower value of $b = 0.6$, while Jermann (1998) estimates a somewhat higher value of $b = 0.82$. Thus, if we were to impose the same preference parameters that previous researchers have argued can help to explain asset price volatility, the implied relative volatility of investment would be consistent with what we observe in the data. Moreover, the average growth rate in equilibrium is now 2.05%, even as the non-stochastic steady state remains at 2.5%, suggesting fluctuations reduce growth by as much as 0.45 percentage points. The average investment rate $E[i_t A_t]$ is approximately the same as the non-stochastic steady state investment rate $i^* A^*$, so that nearly all of the effect of fluctuations on growth is due to diminishing returns.

To generate a standard deviation for the growth rate λ_t of 1.85 percentage points, we now need to set $x = 0.165$, less than half the value required under CRRA utility. Although this still implies rather large shocks to productivity, the marginal product of capital which A_t is meant to capture will in practice depend on labor and utilization rates, and will thus be more volatile

[13] Carroll, Overland, and Weil (2000) examine a related model in which growth does depend on the degree of habit. But they use a different specification where agents care about the ratio between consumption and habit, not the difference between them. If utility is logarithmic in the ratio of consumption to habit, the steady state growth rate is again independent of habit. Since Carroll *et al* assume greater curvature in utility than the log case, they find that habit affects growth. If I were to use their preferences, agents would have even greater incentive to invest in growth, requiring a smaller value of ψ to match the consumption share of output.

than measured total factor productivity. To match the relative volatility of investment using a less volatile process for A_t, we would need for the average investment share of output i to account for less than a third, e.g. by allowing for some fraction of output that is not consumed to be allocated to government spending. Ironically, this would only be possible if we were to lower ψ to further discourage incentives for investment. As such, we can modify the model to replicate the volatility of growth rates with less volatile exogenous shocks, but only if the effect of fluctuations on growth is assumed to be even larger. This would be consistent with some of the larger reduced-form estimates above of three quarters of a percentage point, but as I discuss in the next section, such low values of ψ are probably implausible.

Why does habit formation allow for such volatile investment despite the presence of diminishing returns? Consider a negative productivity shock which lowers households' incomes. Consumers are reluctant to decrease their consumption immediately – habit formation implies they will want to do it gradually. Thus, on impact, households will try to sell off their assets to finance consumption. This causes asset prices to fall, which signals to firms that they should cut back on accumulating capital. Eventually, if the recession continues, consumers will get habituated to low consumption, they will not be as driven to get rid of their assets, and investment will begin to recover. This overshooting explains why investment is so volatile. By contrast, under the standard CRRA utility, households do not need to let their consumption fall gradually, so there is no equivalent drop in asset prices and investment.

In sum, once we modify the endogenous growth model to allow for more volatile asset prices, it is possible to reconcile the presence of diminishing returns to investment with the fact that investment is so volatile empirically. Essentially, we need to introduce an additional motive to smooth consumption that offsets the incentive for agents to smooth investment. Such motives are not altogether implausible given the large premium on risky equity, which suggests that in practice agents are averse to too much consumption volatility. However, once we modify preferences so that agents are more willing to smooth their consumption, we need to confirm such a growth effect is still costly to the agent. It turns out that a one percentage point reduction in growth is actually more costly under (4.2) than under the CRRA preferences with the same γ. Intuitively, habit formation implies that agents care even more about growth, since what they care about is how much *more* they consume relative to past consumption. To compute the lower bound on the cost of cycles described above, consider how much an agent would be willing to sacrifice to increase the growth rate by 0.45 percentage points in the absence of fluctuations, as the above calibration suggests would happen. That is, given two consumption streams $C_t = \lambda^t C_0$ and $C'_t = (\lambda + .0045)^t C_0$, by what constant scaling factor should we adjust the first consumption stream to deliver the same utility as C_t'? For $\lambda = 1.02$ and $\gamma = 1$, the scaling factor μ solves the equation

$$\sum_{t=1}^{\infty} \beta^{t-1} \ln \left((1+\mu) \left(\frac{1.0200}{1.0245} \right)^{t-1} \frac{1.0200 - b}{1.0245 - b} \right) = 0 \tag{4.3}$$

When $b = 0$, which corresponds to the CRRA case, $\mu = 0.097$, i.e. an individual would sacrifice 9.7% of his consumption each year for consumption to grow more rapidly. When $b = 0.7$, the corresponding μ is given by 0.107, or 10.7% per year. Allowing for habit formation therefore serves to increase the cost of business cycles in several ways: it makes agents care more about consumption risk; it necessitates greater diminishing returns to investment, which in turn implies fluctuations will lower average growth by a larger amount; and it causes agents to care more about the growth rate, and thus to suffer more from the effects of fluctuations on growth.

It should be emphasized that the large cost of business cycles under habit formation does not simply reflect the fact that agents are more averse to the volatility of consumption over the cycle. In fact, as discussed in the Introduction, previous work such as Otrok (2001) has argued that even with non-separable preferences such as (4.2), the cost of consumption volatility should be viewed as small. This is because they compute the cost of business cycles using the actual volatility of per-capita consumption over the cycle, which is quite small. By contrast, the cost of cycles due to their effects on growth is proportional to the cost of the larger *counterfactual* consumption volatility that would be required to keep investment constant over time (since if the cost of cycles were less than this cost, agents would never tolerate volatile investment). In other words, the large cost of fluctuations above comes from their effects on growth, not just the increased curvature of the utility function.

5. Quantitative Analysis: Estimates Based on q Data

The previous section demonstrated that investment can be volatile even with adjustment costs as long as q is sufficiently volatile. Since q plays such an essential role, I now propose a different approach to calibration that uses shocks to q as the primitive instead of deriving them from some other shock such as fluctuations in productivity. Given a process for q, we can use (3.5) to derive $\frac{I}{K}$, from which we can derive the growth rate $\lambda = 1 + \phi(I/K) - \delta$. Although q is determined endogenously, abstracting from the fundamentals that determine q is legitimate, since the true source of fluctuations is not directly relevant for gauging the effect of fluctuations on growth beyond its implications for q. As such, the question of whether cyclical fluctuations have a significant effect on growth can be analyzed separately from the question of what shock is responsible for fluctuations in q. Similarly, given a particular utility function, we can directly calculate the welfare cost associated with this growth effect independently of the source of these fluctuations. However, as already noted, the source of fluctuations will be important for determining whether there is a role for stabilization policy.

I begin with a discussion of q data. One of the first to construct a series on aggregate q was Summers (1981). He constructed two annual series from 1931 to 1978, one that is equal to the value of equity divided by the value of physical capital, and another which amends this ratio to account for taxes and depreciation allowances that should affect the incentives of firms to undertake investment. Restricting attention to the post-War period, the average value of his measure of regular q is 0.96 with a standard deviation of 24%, while the average value of the tax-adjusted q series over the same period is 1.92 with a standard deviation of 38%. However, the data used in constructing these series has been subsequently revised by the BEA. Blanchard, Rhee, and Summers (1993) reconstruct the series for standard q in light of BEA revisions, and extend the original series up to 1990. Their estimate of average of q for the post-War period falls to 0.64, and the implied standard deviation rises slightly to 29%. Bernanke, Bohn, and Reiss (1988) construct a quarterly series of tax-adjusted q for the period between 1947 and 1983. Their average for tax-adjusted q is equal to 1.41, with an annual standard deviation of 37%. Thus, while Summers' original series tend to overstate the levels of q, they provide fairly reliable measures of the volatility of aggregate q that are confirmed in subsequent studies.

Assuming a two-regime model, we can translate the standard deviation of q into a ratio $\frac{q_1}{q_0}$ that we can then use to calibrate the effect of fluctuations on growth. Once again, let $q_1 = (1+x)\,\overline{q}$ and $q_0 = \left(1 - \frac{0.633}{0.367}x\right)\overline{q}$, so that $E(q_t) = \overline{q}$. A simple computation reveals that the standard deviation of q_t in percentage terms, denoted sd, is equal to $\sqrt{\frac{0.633}{0.367}}x$. Hence, the ratio $\frac{q_1}{q_0}$ is given by

$$\frac{q_1}{q_0} = \frac{1+x}{1-\frac{0.633}{0.367}x} = \frac{1+\sqrt{\frac{0.367}{0.633}}sd}{1-\sqrt{\frac{0.633}{0.367}}sd}$$

For a standard deviation of 29%, it follows that $\frac{q_1}{q_0} = 1.97$. If the standard deviation is instead 38%, as in the tax-adjusted series, the ratio will be equal to 2.56.

Using the fact that $\phi(i_t A_t) = \lambda_t - 1 + \delta$, and the fact that the first order condition implies that $\phi'(i_t A_t) = q_t^{-1}$, we can use the ratio of q over the cycle above together with data on trend growth rate λ over the cycle to pin down the extent of diminishing returns ψ. In particular,

$$\left(\frac{\lambda_1 - 1 + \delta}{\lambda_0 - 1 + \delta}\right)^{\frac{1}{\psi}} = \frac{i_1 A_1}{i_0 A_0} = \left(\frac{q_1}{q_0}\right)^{\frac{1}{1-\psi}} \tag{5.1}$$

so that the ratio $\frac{q_1}{q_0}$ and λ_0 and λ_1 are enough to solve for ψ. Intuitively, the more volatile is q for a given volatility in the growth rate λ, the more diminishing returns there must be to keep growth rates in line with the data. This is because the lower is ψ, the less investment responds to variation in q, and the less growth responds to variation in investment. Using Table 2, we can assign $\lambda_1 = 1.034$ and $\lambda_2 = 0.996$. When the ratio of q is 1.97, the value of ψ which solves (5.1) is given by 0.35, which implies an elasticity of investment with respect to q of $(1-\psi)^{-1} = 1.54$. Based on Figure 2, eliminating the volatility of investment around its mean should then increase the growth rate from 2% to 2.3%. This estimate is a little lower than the estimates reported above. However, if the ratio q_1/q_0 were instead set to 2.56, the value of ψ that solves (5.1) is equal to 0.28, which is on par with some of the more conservative estimates reported above. In particular, this value of ψ implies an elasticity of investment with respect to q of 1.39, which by Figure 2 suggests that eliminating fluctuations will increase the growth rate from 2% to 2.4%. Thus, calibrating the model directly to q points to a growth effect that is a little smaller than suggested in the previous two sections, although it is still quite substantial. For example, using (4.3), we can deduce that an agent with conventional log utility would be willing to pay at least 6.4% of consumption each year to avoid the lower growth associated with cycles when $\psi = 0.35$ (or 7.0% when we set the habit parameter b to 0.7), and 8.5% of consumption when $\psi = 0.28$ (respectively, 9.2% when $b = 0.7$).

Note that we can use (5.1) not only to pin down ψ, but also to back out the volatility of investment over the cycle. That is, once we estimate the curvature parameter ψ that reconciles q and λ_t, we can use it to solve for the ratio $i_1 A_1 / i_0 A_0$, which reflects the degree to which investment at its peak exceeds its value during troughs. Figure 3 illustrates how the implied ratio of investment rates over the cycle varies with ψ. When $\psi = 0.35$, the value we get from using the standard q series above, the investment ratio $i_1 A_1 / i_0 A_0 = 2.85$. This value is reasonable given that the investment share of output i_t ranges between 10% and 20% over the post-War period, so $i_1 / i_0 = 2$ is plausible, and the investment share of capital $i_t A_t$ will be more volatile than i_t given that A_t is positively correlated with i_t. When we instead consider $\psi = 0.28$, as suggested by the tax-adjusted q series, the ratio of investment rates over the cycle rises to 3.69. This value is rather large, but given that investment is so small relative to the capital stock, even small differences in investment will appear large when translated to ratios, so that this ratio is not entirely unreasonable. However, Figure 3 suggests that for still lower values of ψ, investment has to be incredibly volatile to accord with the variability of λ_t over time. For example, for $\psi = 0.18$, which accords with Eberly's (1997) point estimate for the elasticity of investment with respect to q of 1.22, investment at its peak must be almost 8 times as large as at its trough to account for the time series of λ_t. For $\psi = 0.12$, in line with Abel's (1980) most generous point estimate for this elasticity, investment at its peak must be over 21 times as large as at its trough. Both of these seem highly implausible. Thus, while values of ψ that are associated with growth effects of 0.3 and 0.4 percentage points can account for the volatility of growth rates without requiring wild investment swings over the cycle, the same cannot be said for values of ψ that are associated with a larger effect on growth.

In sum, even if we are unsure about the exact source of aggregate fluctuations, we can use data on q to estimate the effects of these fluctuations on growth without having to take a stand on the precise nature of the shocks that drive these fluctuations. The data reveal that cycles reduce the growth rate by between 0.3 and 0.4 percentage points, depending on how exactly q is measured. This is a little less than the estimates of between 0.4 and 0.9 percentage points suggested in the previous sections. But even if cycles reduce growth by only 0.3 percentage points, they will make agents significantly worse off.

6. Conclusion

This paper considers a cost of aggregate fluctuations that is due not to consumption volatility, but to the effects of fluctuations on growth. In a sense, it closes a circle that began with Lucas (1987), who argued that growth matters for welfare while business cycles do not; if business cycles can affect the rate of economic growth, they can matter after all. My analysis suggests that eliminating fluctuations can increase the growth rate by a little under half a percentage point without affecting average initial consumption. This produces a cost of business cycles 100 times larger than what Lucas computed based on the costs of consumption risk alone.

The key to obtaining such large costs is the presence of diminishing returns to investment. A variety of indicators point to the presence of significant diminishing returns in the data: the significant negative relationship between growth and volatility across countries, regions, and time periods for a *fixed* average investment; the low estimated elasticity of investment with respect to q both in the U.S. and in other OECD countries; the high share of consumption in aggregate output, which suggests low incentives to undertake investment; and the relatively large volatility of q compared to more modest volatility of per-capita consumption growth. All of these consistently point to a curvature parameter ψ on the order of $0.25 - 0.35$, which suggest that cycles reduce the average growth rate by between 0.3 and 0.5 percentage points. In a previous version of this paper, I argued that there is similar evidence in favor of substantial diminishing returns to R&D, which serves as the engine of growth in alternative models. Thus, large costs are likely to arise in a variety of growth models, although the fact that investment is not always efficient in some of these models requires further work to investigate the welfare effects of changes in the level of investment. More generally, the introduction of inefficiencies into the growth process that are absent in the AK framework above raises various interesting questions, such as the timing of growth-enhancing activity, which I have begun to explore in other work. Regardless of what costs of business cycles ultimately come out of such models, the analysis above certainly suggests that ignoring the effects of fluctuations on growth can lead to incorrect conclusions regarding the welfare implications of these fluctuations.

Lastly, while the arguments presented here suggest that business cycles may be quite costly, it bears repeating that it does not immediately follow from this that stabilization policy is inherently desirable or could avoid these underlying costs. This conclusion hinges on the precise nature and source of aggregate fluctuations, and the degree to which government policy can truly eliminate them. This paper opens the door to large welfare effects of fluctuations, and the growth channel it explores can and should be used to examine policy in the multitude of models that have been devised to explain business cycle fluctuations.

Table 1: The Relationship between Growth and Volatility in U.S. time-series data

Dependent variable: real per-capita GDP growth

	U.S. 12 Four-year Panels
Intercept	0.0065 (0.0055)
Average I/Y	0.0132 (0.0292)
μ	-0.3484 (0.2216)
Log likelihood	651.99
# of panels	12
# of obs	192

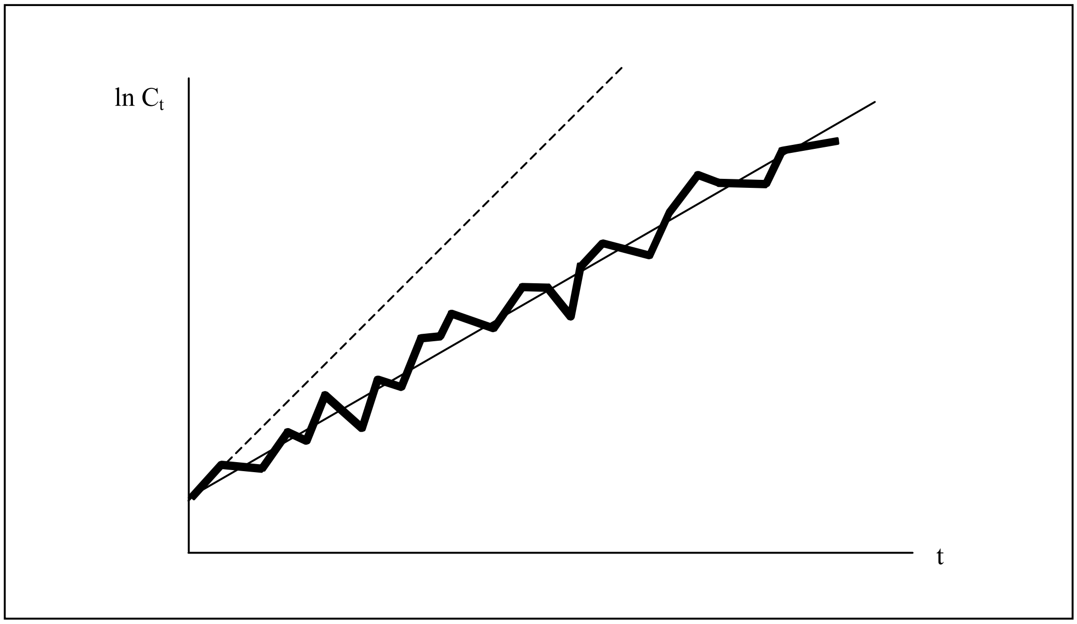
Data source: U.S. quarterly real GDP and real gross domestic investment (in 1996 chained dollars) are taken from BEA estimates for 1953:1 - 2000:4. Real GDP is divided by U.S. population, where population each quarter was interpolated geometrically from annual population figures taken from the U.S. Census Bureau for July 1st of each year. Panels correspond to presidential terms, so each panel consists of 16 quarterly observations. μ denotes the coefficient on the standard deviation term in equation (3.2) in the text. The method of estimation is maximum likelihood. Numbers in parentheses denote asymptotic standard errors.

Table 2: Maximum Likelihood Estimates for Markov Model of Consumption Growth

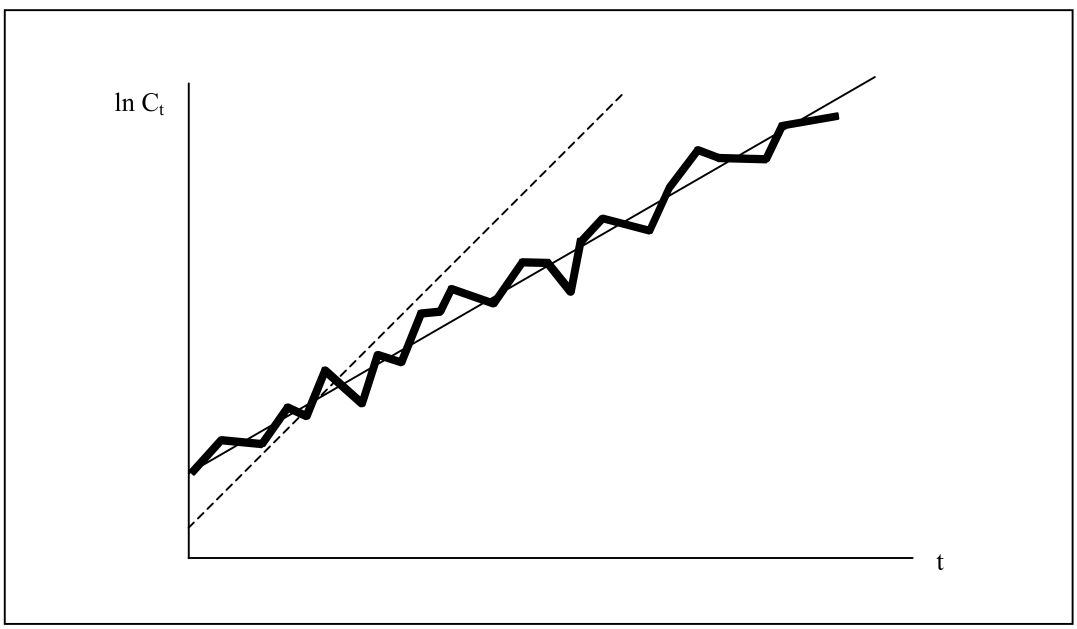
2 regime model			3 regime model		
	coef	se		coef	se
p_{01}	0.5369	0.1543	p_{01}	0.1110	0.1356
p_{10}	0.3117	0.1113	p_{02}	0.3936	0.1326
			p_{10}	0.2308	0.1057
			p_{12}	0.1812	0.1086
			p_{20}	0.1730	0.1317
			p_{21}	0.4724	0.1892
$\ln \lambda_0$	-0.0043	--	$\ln \lambda_0$	-0.0070	--
$\ln \lambda_1$	0.0341	0.0066	$\ln \lambda_1$	0.0227	0.0033
			$\ln \lambda_2$	0.0429	0.0039
$\Delta\ln(1+\varepsilon_{01})$	-0.0250	0.0049	$\Delta\ln(1+\varepsilon_{01})$	-0.0174	0.0045
			$\Delta\ln(1+\varepsilon_{12})$	-0.0201	0.0031
σ^2	0.00024	0.00009	σ^2	0.00010	0.00003

Implied Invariant distribution		Implied Invariant distribution	
State	Prob	State	Prob
0	0.37	0	0.29
1	0.63	1	0.42
		2	0.29

Data sources: real consumption per capita between 1950-1998 is taken from the Economic Report of the President. p_{ij} denotes the transition rate between states i and j. $\ln\lambda_j$ corresponds to the expression $\phi(iA)+1-\delta$ in the model. Since the estimation constrains the average growth rate to equal 2.0% per year, only N-1 growth rates are estimated in the N regime model. $\ln\Delta\varepsilon_{ij}$ denotes the change in the level of consumption for a transition rate between states i and j. σ^2 denotes the variance of the measurement error term η. The implied invariant distribution is computed using point estimates for the transition matrix p_{ij}. Standard errors on coefficients correspond to asymptotic standard errors computed using the expected score method.



(a) Increased growth from reduced volatility of investment



(b) Increased growth from higher average investment

Figure 1: Consumption Paths under Endogenous Growth

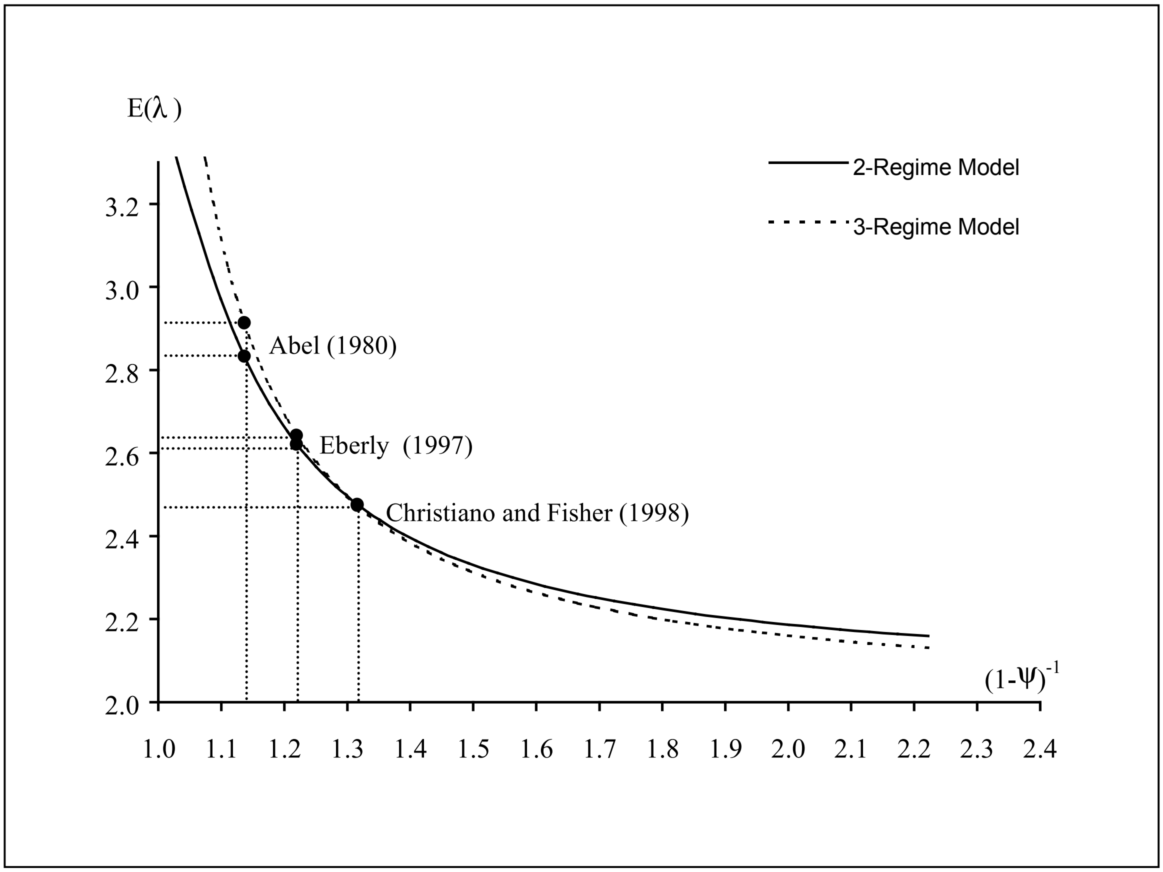


Figure 2: Predicted growth rate after stabilization
for a given investment elasticity

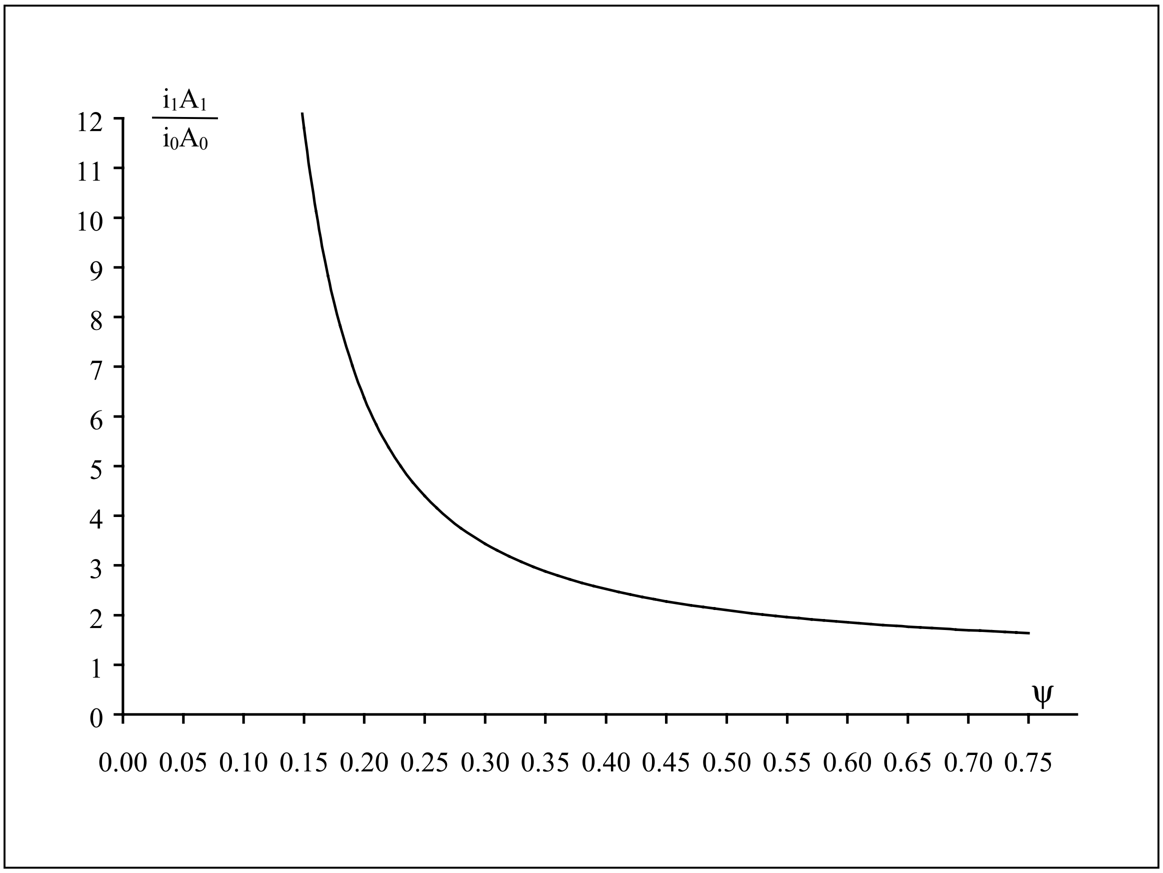


Figure 3: The volatility of investment volatility required to match the volatility of growth rates as a function of ψ

Appendix

Proof of Proposition: We guess that the value function $V(K_t, A_t)$ assumes the form

$$V(K_t, A_t) = \frac{a(A_t) K_t^{1-\gamma}}{1-\gamma}$$

It is easy to confirm that this multiplicatively separable function conforms to the Bellman equation. Ignoring the constant term in the utility function, we can rewrite the above Bellman equation as

$$a(A_t) = \max_{c_t \in [0,1]} \frac{(c_t A_t)^{1-\gamma}}{1-\gamma} + \beta E\left[a(A_{t+1}) \mid A_t\right] \left[\phi((1-c_t) A_t) + 1 - \delta\right]^{1-\gamma}$$

Note that for any function $V(\cdot,\cdot)$ which is increasing in both arguments, the expression

$$\max_{c_t} \left\{ \frac{(c_t A_t K_t)^{1-\gamma}}{1-\gamma} + \beta E\left[V((\phi((1-c_t) A_t) + 1 - \delta) K_t, A_{t+1}) \mid A_t\right] \right\}$$

must be increasing in A_t and K_t given that Prob$(A_{t+1} \le x \mid A_t)$ is weakly decreasing in A_t. The standard fixed-point argument establishes that the solution to the Bellman equation $V(\cdot,\cdot)$ is increasing in both arguments. Hence, if $V(K, A) = \dfrac{a(A) K^{1-\gamma}}{1-\gamma}$, it follows that $a(A)$ is increasing in A. This in turn implies that the conditional expectation $E\left[a(A_{t+1}) \mid A_t\right]$ is increasing in A_t.

Using the first order condition of the Bellman equation, we have

$$(c_t A_t)^{-\gamma} = \beta E\left[a(A_{t+1}) \mid A_t\right] \left[\phi((1-c_t) A_t) + 1 - \delta\right]^{-\gamma} \phi'((1-c_t) A_t)$$

We can rearrange this equation to obtain

$$\phi'(i_t A_t) = \frac{1}{\beta E\left[a(A_{t+1}) \mid A_t\right]} \left(\frac{\phi(i_t A_t) + 1 - \delta}{A_t (1-i_t)}\right)^{\gamma}$$

Let $x = iA$. Then we can rewrite the first order condition as

$$\phi'(x) = k(A) \left(\frac{\phi(x) + 1 - \delta}{A - x}\right)^{\gamma}$$

where $k(A)$ is a positive constant and $k'(A) < 0$. Since $\phi'(x)$ is decreasing in x and $\left(\dfrac{\phi(x) + 1 - \delta}{A_t - x}\right)^{\gamma}$ is increasing in x, there exists at most one x which solves this equation. Existence then follows from the limit conditions $\lim_{x\to 0}\phi'(x)=\infty$ and $\lim_{x\to\infty}\phi'(x)=0$. If we rewrite this equilibrium condition as $f\equiv\phi'(x)-k(A_t)\left(\frac{\phi(x)+1-\delta}{A-x}\right)^{\gamma}=0$, the fact that $f_x<0$ and $f_A>0$ imply that as A rises, x must also rise to maintain $f=0$, which establishes the claim. ■

Solving the model for CRRA utility: From the proof of the proposition above, we know that the value function $V(K,A)$ has the form $\frac{a(A)K^{1-\gamma}}{1-\gamma}$. Substituting into the Bellman equation yields the following equation, one for each possible realization of A_t:

$$\frac{a(A)}{1-\gamma}K^{1-\gamma}=\frac{(c(A)AK)^{1-\gamma}}{1-\gamma}+\beta\frac{E\left[a(A_{t+1})\mid A_t=A\right]}{1-\gamma}\left[(\phi([1-c(A)]A)+1-\delta)K\right]^{1-\gamma}$$

Taking the first order with respect to $c(A)$ on the RHS of the above yields the following Euler equation for $c(A)$:

$$(c(A)A)^{-\gamma}=\beta E\left[a(A_{t+1})\mid A_t\right]\left[\phi((1-c(A))A)+1-\delta\right]^{-\gamma}\phi'((1-c(A))A)$$

Thus, we have two non-linear equations for each pair of variables $c(A),a(A)$. Solving the model amounts to solving this system of non-linear equations,with two equations for each value A_t can assume. ■

Intertemporal Euler Equation under Habit: The intertemporal-Euler equation for a general utility function $U(\{C_t\})$ is given by

$$U_{c,t}=E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}U_{c,t+1}\right]$$

where $U_{c,t}$ denotes the marginal utility of U with respect to C_t and λ_t denotes the Lagrange multiplier on the constraint $K_{t+1}=\left(1+\phi\left(\frac{I_t}{K_t}\right)-\delta\right)K_t$. For (4.2), we have

$$U_{c,t}=(C_t-bC_t)^{-\gamma}-b\beta(C_{t+1}-bC_{t+1})^{-\gamma}$$

while adjustment costs imply that

$$\frac{\lambda_{t+1}}{\lambda_t}=\frac{\phi'(I_t/K_t)}{\phi'(I_{t+1}/K_{t+1})}\left(1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)+\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{C_{t+1}}{K_{t+1}}\right)$$

Define $\widehat{c}_t=\frac{C_t}{K_t}$. Then we can rewrite the Euler equation as

$$E_t\left[v\left(\widehat{c}_{t-1},\widehat{c}_t,\widehat{c}_{t+1},\widehat{c}_{t+2},A_{t-1},A_t,A_{t+1}\right)\right]=0$$

where the function v can be obtained by substitution. The deterministic steady state is the constant value $\widehat{c}$ which solves

$$v\left(\widehat{c},\widehat{c},\widehat{c},\widehat{c},\overline{A},\overline{A},\overline{A}\right)=0$$

and one can show that if we set $\widehat{c}_{t-1}=\widehat{c}_t=\widehat{c}_{t+1}=\widehat{c}_{t+2}$, the coefficient b drops out. To solve for the non-steady state dynamics, I use a linear approximation of $v(\cdot)$ at the deterministic steady state by evaluating the relevant derivatives of v with respect to each of its arguments, and then use this to derive a linear approximation of the policy rule which is given by $\widehat{c}_{t+1}=\alpha_0\widehat{c}_t+\alpha_1 A_t+\alpha_2 A_{t-1}$ for some coefficients α_0, α_1, and α_2 that I need to solve for. ■

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