NBER WORKING PAPER SERIES

DIVIDEND YIELDS AND STOCK RETURNS: A TEST FOR TAX EFFECTS

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Working Paper No. 649

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

March 1981

This paper is based upon my Ph.D. dissertation at the University of Chicago. I am thankful to the members of my committee; G. Constantinides, E. Fama, J. Ingersoll, A. Madensky, M. Miller (chairman), and M. Scholes. I owe a particularly important debt to Merton Miller for his help and encouragement. Finally, I would like to thank the participants in the Finance workshop at Ohio State University and the University of Chicago. Special thanks are due to Steve Buser, Roger Gordon, Gailen Hite and E. Han Kim. The research reported here is part of the NBER's research program in Taxation. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #649
March 1981

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ABSTRACT

This paper examines the empirical relation between stock returns and dividend yields. Several equilibrium pricing models incorporating differential taxation of dividends and capital gains are nested as systems of time series regressions. Estimates of these models and tests of parameter restrictions implied by the models are conducted within the context of Zellner's seemingly unrelated regression. It is concluded that the data fail to support these models as well as the hypothesis that dividends are neutral. The inability to distinguish between these competing hypotheses suggests the need for further research before definitive conclusions are reached regarding the tax impacts of dividends.

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By

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Introduction

The effect of dividends on stock returns is a major issue in finance, with a rich historical background. It has been argued (see Graham and Dodd [1951]) that low yield stocks require higher returns because investors value dividends more than retained earnings. Miller and Modigliani [1961] demonstrated that in a world of no transaction costs, equal tax treatment of dividends and capital gains, and investor rationality, the dividend policies of firms have no impact on the welfare of security holders (given investment policies). Miller and Modigliani pointed out, however, that the heavier taxation of dividends than capital gains might lead to higher before-tax returns on shares with high dividend yields. But they warned that such a relation would be weakened and possibly completely offset by clientele effects. One important class of investors, pension funds, pay no taxes and, therefore, have no reason to require higher returns on dividend paying stocks. Other investors, notably corporations and casualty insurance companies, face lower statutory tax rates on dividends than capital gains. On the other hand, some investors consuming wealth may find the tax penalties of dividend-paying stocks offset by lower transaction costs if part of their returns are received as dividends. Recently, even the proposition that dividends are taxed at higher rates than capital gains has been challenged. Miller and Scholes [1978] have pointed to features of the tax code and currently available financial instruments that investors could use to blunt any tax disadvantages of dividends.

Attempts to establish which of these hypothesized relations between dividends and stock returns holds empirically have not resulted in a consensus.

Early tests of the general effects of dividends, such as Friend and Puckett [1964], relied on cross sectional regressions of common stock prices on dividends per share, and various alternative measures of earnings or retained earnings per share. Friend and Puckett found little evidence of yield effects, but their tests are subject to serious limitations. Black and Scholes [1974] also tested for the general impact of dividends on common stock returns, using time series methods that avoided many of the estimation difficulties of the earlier cross sectional regressions of stock prices on dividends and retained earnings. On the basis of their test, Black and Scholes [1974] found themselves unable to reject the hypothesis that dividend yields have no significant effect on stock returns.

Several authors have tested for the tax effects of dividends by comparing returns on cum and ex days. Elton and Gruber [1970], in a well known study, claim documentation of both a tax effect and a dependency between tax rates of marginal investors and dividend yields (a clientele effect reflected in asset prices). With a longer sample period and more appropriate statistical procedure, Black and Scholes [1973] are unable to confirm either of these conclusions. However, Black and Scholes do find evidence of unusual behavior in security returns for several days surrounding ex dividend days.

Recent work, notably by Litzenberger and Ramaswamy [1979] and Blume [1978], has sought to improve the efficiency of the Black-Scholes [1974] tests. Both papers claims to have found evidence supporting the importance of dividends. Litzenberger and Ramaswamy, at least, argue that the importance of dividends is best explained by differential taxation of dividends.

In this paper I present a different estimation method that avoids some of the shortcomings of the Litzenberger-Ramaswamy and Blume tests. I conclude that the data fail to support the Litzenberger-Ramaswamy conjecture of a return differential that can be attributed to heavier taxation of dividends than capital gains.[1] In fact, while dividends seem to matter in a statistical sense, the evidence does not support any simple economic relation between dividends and capital gains. In particular, the effects of dividends are not uniform across all securities.

Review of Related Works

This review concentrates on the papers of Black and Scholes [1974], Litzenberger and Ramaswamy [1979], and Blume [1978]. These papers represent the state of the art with respect to testing the hypothesis of interest. All are consistent with modern theories of security pricing and use estimation techniques that avoid many of the problems associated with earlier cross-sectional studies.

Black and Scholes [1974]

Black and Scholes [1974] compare two models of the relation between dividends yields and expected returns:

$$E(\tilde{R}_i) = \gamma_0 + \beta_i[E(\tilde{R}_m) - \gamma_0] + \gamma_1 (\delta_i - \delta_m)/\delta_m \tag{1}$$

and

$$E(\tilde{R}_i) = \gamma_0 + \beta_i[E(R_m) - \gamma_0] \tag{2}$$

where $E(\tilde{R}_i)$ is the expected return of security i, β_i is the covariance of security i's return with the return of the market portfolio divided by the variance of the return of the market portfolio, δ_i is the dividend yield of security i, δ_m is the dividend yield of the market portfolio, $E(\tilde{R}_m)$ is the expected return of the market portfolio, and γ_0, γ_1 are parameters of the pricing equation.

Black and Scholes estimate γ_1, as the average return of the minimum variance portfolio satisfying the restrictions,

$$\sum_{i=1}^{N} \omega_i = 0, \quad \sum_{i=1}^{N} \omega_i \beta_i = 0, \quad \sum_{i=1}^{N} \omega_i \delta_i = \delta_m \tag{3}$$

where N is the number of assets and ω_i is the proportion of asset i in the portfolio. To reduce the impact of errors in variables, Black and Scholes solve (4) with twenty-five portfolios constructed from all New York Exchange stocks that were listed for at least five years. These portfolios are constructed to maintain dispersion in both beta and dividend yield. The ij elements of the covariance matrix of portfolio returns is approximated as $\hat{\beta}_i \hat{\beta}_j \hat{\sigma}_m^2$ for $i \neq j$ and $\hat{\beta}_i^2 \hat{\sigma}_m^2 + \hat{\sigma}_i^2$ if $i = j$ where $\hat{\beta}_i$ is the estimated betas of portfolio i, $\hat{\sigma}_m^2$ is the estimated variance of the equally weighted index of all New York Stock Exchange common stocks and $\hat{\sigma}_i^2$ is the estimated residual variance of portfolio i in the regression of portfolio i's return on the returns of the equally weighted index.

Black and Scholes estimate γ_1 as the average monthly return of the portfolio. For their overall sample period, January, 1939 to December, 1966, the average return is .09 percent. During three ten year subperiods, the γ_1 estimates range from .02 percent to .16 percent. None of the estimates are statistically significant.

<u>Litzenberger and Ramaswamy [1979]</u>

Litzenberger and Ramaswamy begin by deriving an after-tax capital asset pricing model of the form,

$$E(\tilde{R}_i) - r_f = a + b\,\beta_i + c(d_i - r_f) \tag{4}$$

where r_f is the risk-free rate of interest, d_i is the dividend yield of security i, and a, b, c are parameters of the pricing equation. Litzenberger and Ramaswamy assume that the capital gains tax is zero and dividends are taxed as ordinary income. The parameter c of equation (5) represents an average of tax rates, less a shadow price reflecting any increase in investors' ability to borrow due to an additional dollar of dividends.[2] If dividends income has no effect on investors' ability to borrow, c will represent only tax effects. The other parameters of their model have values,

$$a = E(\tilde{R}_{z*}) - r_f$$

$$b = E(\tilde{R}_m) - E(\tilde{R}_{z*}) - c(d_m - r_f)$$

where $E(\tilde{R}_{z*})$ is the expected return on a zero beta portfolio, with a dividend yield equal to the risk-free rate of interest $E(\tilde{R}_m)$ is the expected return of the market portfolio, and d_m is the dividend yield of the market portfolio. The model (5) is a more general form of Brennan's [1970] after tax pricing equation,

$$E(\tilde{R}_i)-r_f = b' \beta_i + \tau(d_i - r_f) \qquad (5)$$

where

$$b' = E(\tilde{R}_m) - r_f - \tau(d_m - r_f)$$

and τ represents a tax differential between dividends and capital gains.[3] While (4) and (5) differ from the Black and Scholes model, a portfolio satisfying the constraints of Black and Scholes would have an expected return equal to cd_m or τd_m.

In estimating equation (4), Litzenberger and Ramaswamy consider the stochastic version of their model,

$$\tilde{R}_{it} = E(\tilde{R}_{it}) + \tilde{u}_{it}$$

$$\tilde{R}_{it} - r_{ft} = a + b\,\beta_{it} + c(d_{it} - r_{ft}) + \tilde{u}_{it} \qquad (6)$$

for securities i = 1,2 ...N and periods t = 1, 2 ...T.[4] If all the β_i's, d_{it}'s, and r_{ft}'s in (6) could be estimated without error, the model could be estimated by pooling cross sectional regressions over time. Litzenberger and Ramaswamy use three different, although closely related techniques to estimate (6).

The first technique is a regression of individual securities' risk premiums (returns less than one month T-Bill rate) on a constant, the securities' betas estimates in the previous sixty months, and a measure of anticipated dividend yield of the securities for the month less the one month T-Bill rate. The securities' betas are estimated with the following regression,

$$\tilde{R}_{i\tau} - r_{r\tau} = a_{i\tau} + \beta_{i\tau}\,(\tilde{R}_{m\tau} - r_{f\tau}) + \tilde{e}_{i\tau} \qquad (7)$$

for securities i = 1,2 ...N and τ = t-60 ... t-1, where $R_{m\tau}$ is the return in month τ of a value weighted index of all New York Stock Exchange common stocks. The measure of anticipated dividends for any month equals the taxable dividends paid in the month if the dividends were announced prior to the beginning of the month. If the taxable dividends were unannounced and nonrecurring, anticipated dividends were assigned a value of zero and if dividends were unannounced but recurring, anticipated dividends were set equal to the last recurring payment.[5]

The dividend yield is anticipated dividends divided by the closing price of the previous month.

The second technique uses the same regression but with the risk premiums, constant, estimated betas, and anticipated dividends yields standardized by the estimated standard errors of the securities' beta estimates. Litzenberger and Ramaswamy claim that this estimator is generalized least squares if the disturbances in (7) are uncorrelated across securities, and, indeed, if all variables were observable, the claim would be true.

The third estimation technique used by Litzenberger and Ramaswamy is motivated by the errors in variables problem associated with individual securities' beta estimates. The estimation technique includes an explicit adjustment for errors in the beta estimates. Litzenberger and Ramaswamy argue that their method of dealing with errors in variables is superior to the portfolio grouping procedure of Black-Scholes because it does not destroy cross sectional dispersion. Moreover, the authors claim that the estimator is consistent and is a maximum likelihood estimator if the joint distribution of security returns is normal. These are remarkable claims since the errors in variables problem has severely limited the ability to make reliable inferences in financial economics as well as economics in general. Litzenberger and Ramaswamy's adjusted estimator is analyzed in Appendix A and it is shown there that this estimator is neither consistent nor is it maximum likelihood.

Litzenberger and Ramaswamy derive estimates from each of these techniques by averaging estimates over time. The standard errors of the estimates are estimated from the time series of monthly estimates. The resulting estimates of c are all quite close. During the sample period January, 1936 to December, 1977, they are .227, .234, and .236 for the OLS, GLS, and adjusted estimator. The t-values of these estimates relative to zero are 6.33, 8.24, and 8.62 for the OLS, GLS, and adjusted estimator.[6] The adjusted estimator is computed for six subperiods. Litzenberger and Ramaswamy note that during the overall sample period of Black and Scholes, January, 1936 to December, 1966, the dividend yield of the market was .048 per year. Interpreting the Black-Scholes γ_1 as equivalent to cd_m, they infer an estimate of c equal to .225 from the Black and Scholes estimate of γ_1 (.0009 x 12/.048). On the basis of this calculation, they claim to have documented roughly the same effect as Black and Scholes, but in a more precise way.[7]

Blume [1978]

Blume [1978] proposes the following model,

$$\tilde{R}_{it} = a_t + b_t \beta_{it} + c_t d_{it} + \tilde{u}_{it} \tag{8}$$

for i = 1, 2 ... N and t = 1, 2 ...T. The coefficients of (8) are each assumed to be normally, independently, and identically distributed with means $\bar{a}$, $\bar{b}$, and $\bar{c}$ and variances σ_a^2, σ_b^2, and σ_c^2, respectively. Equation (8) closely resembles the stochastic version of Litzenberger and Ramaswamy's model, equation (6). The major differences are supression of the risk-free rate and the interpretation of random coefficients period by period.

Blume estimates (8) by regressing quarterly returns of twenty-five portfolios on a constant, the estimated portfolio betas, and a measure of the portfolios' quarterly dividends yield. Portfolios are used to reduce the errors in variables associated with estimated betas. Like Black and Scholes, Blume attempts to maintain dispersion in both the portfolio betas and dividend yields.

The portfolio betas are estimated with sixty months of data prior to the quarterly estimates. Although monthly data are available, Blume uses quarterly returns to avoid smudging of tax effects across ex and non-ex months. Blume also uses a time series of quarterly estimates in calculating an average estimate and the standard error of the average estimate.

Blume estimates the model for the period January, 1936 to December, 1976 and for four non-overlapping ten year subperiods. The average quarterly estimate of $\bar{c}$ for the overall period is positive and significant at the 95 percent significance level. In two remaining subperiods the estimates are positive, but not significant at the 95 percent level. Blume does not interpret these results as being consistent with differential taxation of dividends relative to capital gains, although this is presumably the interpretation Litzenberger and Ramaswamy would draw from a positive $\bar{c}$.[8] Instead, he argues that $\bar{a}$ is not constant across securities and is positively related to dividend yield so that the estimate of $\bar{c}$ is simply proxying for the variable intercept. Blume attempts to investigate this alternative but the tests are not well motivated and the connection between the tests and the conjectured relationship is unclear.

Alternative Tests of the Tax Effects of Dividends

In this section I will argue that the authors previously discussed have overlooked important testable properties of models which assume differential taxation of dividends.

Litzenberger and Ramaswamy

Recall that Litzenberger and Ramaswamy's model takes the form,

$$E(\tilde{R}_{it}) - r_{ft} = a + b\,\beta_i + c(d_{it} - r_{ft}) \tag{4}$$

where r_{ft} is the riskless rate of interest in period t and d_{it} is the dividend yield of security i in period t. The stochastic version of their model is,

$$\tilde{R}_{it} = E(\tilde{R}_{it}) + \tilde{u}_{it}$$

$$\tilde{R}_{it} - r_{ft} = a + b\,\beta_i + c(d_{it} - r_{ft}) + \tilde{u}_{it} \tag{7}$$

The authors assume that the parameters a, b, and c are constant and they use five years of data to estimate betas. Implicitly, then, they assume that betas are constant for five year periods. Combining the assumptions of constant parameters and betas with the stochastic version of their model results in the equations,

$$\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_1 d_{it} + \gamma_2 r_{ft} + \tilde{u}_{it} \tag{9}$$

$$\gamma_{oi} = a + b\beta_i$$

$$\gamma_1 = -\gamma_2 = c$$

where (9) holds for all securities and periods consistent with the assumption of constant betas. The intercept of (9) is free across securities, but constant over sixty month periods for each security.

The model (9) makes very precise predictions about securities' returns. First, the model predicts that securities' risk premiums are constant over time, except for changes in the riskless rate and the securities' dividend yields. If the dividend yield of a security, less the riskless rate, is constant over time, the risk premium of the security is also constant. Second, changes in the riskless rate and dividend yields of securities result in the same per unit change in risk premiums for all securities. These very specific implications may be tested with equation (9). Such a test has the further advantage of not requiring estimation of securities' betas because the beta terms are subsumed in the intercept, γ_{0i}. Equation (9) models the tax effects of dividends as a system of time series regressions with restricted parameters. The system includes the joint restriction that $\gamma_1 + \gamma_2$ equal zero for all securities and the γ_1 and γ_2 are equal across all securities.

As noted, the model (9) does not include the return on the market portfolio, but can readily be expanded to include it. The definitions of a, b, and c are,

$$a = E(\tilde{R}_{z*}) - r_{ft}$$

$$b = E(\tilde{R}_m) - E(\tilde{R}_{z*}) - c(d_{mt} - r_{ft})$$

where $E(\tilde{R}_{z*})$ is the expected return of a zero beta portfolio with a dividend yield equal to the riskless rate of interest, and d_{mt} is the dividend yield of the market portfolio in period t. Incorporating these terms into equation (4) and simplifying,

$$E(\tilde{R}_{it}) - r_{ft} = E(\tilde{R}_{z*}) - r_{ft} + [E(\tilde{R}_m) + E(\tilde{R}_{z*}) -$$

$$c(d_{mt} - r_{ft})]\ \beta_i + c(d_{it} - r_{ft})$$

$$= E(\tilde{R}_{z*})(1 - \beta_i) + \beta_i\ E(\tilde{R}_m) - \beta_i\ c(d_{mt} - r_{ft}) +$$

$$c(d_{it} - r_{ft})$$

If,

$$\tilde{R}_{it} = E(\tilde{R}_{it}) + \tilde{u}_{it}$$

$$\tilde{R}_{mt} = E(\tilde{R}_{mt}) + \tilde{u}_{mt}$$

The stochastic version of the model is,

$$\tilde{R}_{it} = E(\tilde{R}_{z*})(1 - \beta_i) + \beta_i \tilde{R}_{mt} - \beta_i c(d_{mt} - r_{ft}) + c(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it} \qquad (10)$$

$$\tilde{\varepsilon}_{it} = \tilde{u}_{it} - \beta_i \tilde{u}_{mt}$$

Equation (10) holds across securities and over time periods consistent with constant betas. Like (9), equation (10) involves restrictions on parameters for each security, as well as restrictions on parameters across securities. The restrictions implied by (10) are more difficult to incorporate than the restrictions of (9), because equation (10) includes restrictions on products of parameters. More important, estimation of (10) requires identification of the market portfolio and this imposes an untestable restriction on the data. Nor can much comfort be taken on the latter count from evidence that suggest particular implications of asset pricing seem to be insensitive to alternative specifications of the market portfolio.[9] The concern of this study is with a particular problem and generalization of results obtained with other problems is inappropriate. Since determining the effects of alternative market portfolios is beyond the scope of this paper, equation (10) is used only as a check on the sensitivity of some of our tests.

Brennan

Brennan's model closely resembles Litzenberger and Ramaswamy's. The model is,

$$E(\tilde{R}_{it}) - r_{ft} = b' \quad \beta_i + \tau(d_{it} - r_{ft}) \tag{5}$$

Using the same notation, the stochastic version of Brennan's model may be expressed as,

$$\tilde{R}_{it} - r_{ft} = b' \quad \beta_i + \tau(d_{it} - r_{ft}) + \tilde{u}_{it} \tag{7a}$$

$$\tilde{R}_{it} = E(\tilde{R}_{it}) + \tilde{u}_{it}$$

It is clear from (6) that if b', τ, and beta can be taken as constant for sixty months, the stochastic version of Brennan's model may be expressed as,

$$\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_1 \, d_{it} + \gamma_2 \, r_{ft} + \tilde{u}_{it} \tag{9a}$$

$$\gamma_{oi} = b' \quad \beta_i$$

$$\gamma_1 = - \gamma_2 = \tau$$

The testable implications of this model are identical to the testable implications of (9). The differences involve only the value of the intercept and can be ignored if interest centers on tax effects.

As was the case for Litzenberger and Ramaswamy, a market return can be explicitly incorporated into Brennan's model. The b parameter of his model is,

$$b = E(\tilde{R}_{mt}) - r_{ft} - \tau(d_{mt} - r_{ft})$$

Substituting this into (6) and rewriting, the stochastic version of the model becomes,

$$\tilde{R}_{it} = r_{ft}(1 - \beta_i) + \beta_i \tilde{R}_{mt} - \beta_i \tau(d_{mt} - r_{ft}) + \tau(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it} \qquad (10a)$$

$$\tilde{\varepsilon}_{it} = \tilde{u}_{it} - \beta_i \tilde{u}_{mt}$$

with properties essentially the same as (10).

Blume

The model Blume estimates is,

$$\tilde{R}_{it} = a_t + b_t \beta_i + c d_{it} + \tilde{u}_{it} \qquad (8)$$

$$a_t = \bar{a} + u_{at}$$

$$b_t = \bar{b} + u_{bt}$$

$$c_t = \bar{c} + u_{ct}$$

The parameters a_t, b_t, and c_t are random over time, but the distribution of the parameters is fixed over time. From our discussion of the previous models, it would seem natural to express Blume's model as,

$$\tilde{R}_{it} = \gamma_{oi} + \gamma_1 d_{it} + \tilde{\nu}_{it} \tag{9b}$$

$$\gamma_{oi} = \bar{a} + \bar{b}\beta_i$$

$$\gamma_1 = \bar{c}$$

$$\tilde{\nu}_{it} = \tilde{u}_{it} + \tilde{u}_{at} + \beta_i \tilde{u}_{bt} + d_{it}\tilde{u}_{ct}$$

Notice, however, that the disturbance of (9b), $\tilde{\upsilon}_{it}$, is a composite disturbance, and one of the terms included in the disturbances depends upon dividend yield, $d_{it}\,\tilde{u}_{it}$. Because dividend yields vary over time, the variance of the disturbances and the covariances across securities are not constant. But because most quarterly dividend yields are small in absolute value (typically ranging from .0 to .02), the departure from standard assumption are of little importance and will be ignored for purposes of estimation. (A more detailed analysis is presented in Appendix B.)

The coefficients $\bar{a}$, $\bar{b}$, and $\bar{c}$ can be readily expressed in terms of the market portfolio. If (8) holds for all assets, it holds for the market portfolio. Since the beta of the market equals one, the market return equals,

$$\tilde{R}_{mt} = \bar{a} + \bar{b} + \bar{c}d_{mt} + \tilde{\nu}_{mt}$$

so that

$$\bar{b} = \tilde{R}_{mt} - \bar{a} - \bar{c}d_{mt} - \tilde{\nu}_{mt}$$

Substituting this expression for $\bar{b}$ into (8) and making use of the definition of $\tilde{\upsilon}_{it}$,

$$\tilde{R}_{it} = \bar{a}(1 - \beta_i) + \beta_i \tilde{R}_{mt} - \beta_i \bar{c}d_{mt} + \bar{c}d_{it} + \tilde{\nu}_{it} - \beta_i \tilde{\nu}_{mt} \tag{10b}$$

Both formulations of Blume's model, (9b) and (10b), closely resemble the empirical formulations proposed for Litzenberger and Ramaswamy, (9) and (10), and Brennan, (9a) and (10a). In particular, the models share the implication that the dividend yield coefficient is common across securities.

Synthesis of Formulations

Two fundamental equations have been proposed for testing the models of Litzenberger and Ramaswamy, Brennan, and Blume. The equations are,

$$\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \gamma_{2i} r_{ft} + \tilde{\varepsilon}_{it} \tag{11}$$

$$R_{it} = \alpha_{oi} + \alpha_{1i} \tilde{R}_{mt} + \alpha_{2i} d_{mt} + \alpha_{3i} r_{ft} + \alpha_{4i} d_{it} + \tilde{\varepsilon}_{it} \tag{12}$$

Table 1 catalogues the parameter values and restrictions on parameter values implied by the alternative model.[10]

Estimating and Testing the Alternative Formulations

Equations (11) and (12) are systems of time series regressions with a regression equation for each security. To estimate the systems properly and to test the model's implications, the cross sectional dependence in security returns must be taken into account. Zellner [1971] has called systems like (11) and (12) "seemingly unrelated regressions" (SUR). The regressions have different independent variables, but are related by the contemporaneous correlation of the disturbances. The disturbances are presumed to be contemporaneously correlated across equations, but the non-contemporaneous covariances and autocovariancies are here all assumed to be zero. The general form of the SUR is,

$$\begin{bmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_k \end{bmatrix} = \begin{pmatrix} x_1 & & \\ & \ddots & \\ & & x_k \end{pmatrix} \begin{bmatrix} \underline{\delta}_1 \\ \vdots \\ \underline{\delta}_k \end{bmatrix} + \begin{bmatrix} \underline{u}_1 \\ \vdots \\ \underline{u}_k \end{bmatrix}$$

$$\underline{y} = x\,\underline{\delta} + \underline{u}$$

$$E\underline{u} = \underline{0}$$

$$E\underline{u}\,\underline{u}' = \begin{pmatrix} \sigma_{11}I_T & \cdots & \sigma_{1k}I_T \\ \vdots & & \\ \sigma_{k1}I_T & \cdots & \sigma_{kk}I_T \end{pmatrix} = \Sigma \otimes I_T$$

TABLE 1

CATALOGUE OF PARAMETER VALUES AND RESTRICTIONS

Equation (11): $R_{it} = \gamma_{oi} + \gamma_{1i}\, d_{it} + \gamma_{2i}\, r_{ft} + \tilde{\varepsilon}_{it}$

Model: Litzenberger and Ramaswamy

Parameter Value: $\gamma_{oi} = a + b\ \beta_i, \gamma_{1i} = c, \gamma_{2i} = 1\text{-}c$

Restrictions: $\gamma_{1i} + \gamma_{2i} = 1, \gamma_{1i} = \gamma_{1j}$ and $\gamma_{2i} = \gamma_{2j}$ for all i and j.

Model: Brennan

Parameter Values: $\gamma_{oi} = b\ \beta_i, \gamma_{1i} = \tau, \gamma_{2i} = 1 - \tau$

Restrictions: $\gamma_{1i} + \gamma_{2i} = 1, \gamma_{1i} = \gamma_{1j}$ and $\gamma_{2i} = \gamma_{2j}$ for all i and j

Model: Blume

Parameter Values: $\gamma_{oi} = \bar{a} + \bar{b}\ \beta_i, \gamma_{1i} = \bar{c}\ , \gamma_{2i} = 0$

Restrictions: $\gamma_{1i} = \gamma_{1j}$ and $\gamma_{2i} = 0$ for all i and j

Equation 12: $R_{it} = \alpha_{oi} + \alpha_{1i}\, R_{mt} + \alpha_{2i}\, d_{mt} + \alpha_{3i}\, r_{ft} + \alpha_{4i}\, d_{it} + \tilde{\varepsilon}_{it}$

Model: Litzenberger and Ramaswamy

Parameter Values: $\alpha_{oi} = E(\tilde{R}_{z*t})(1 - \beta_i), \alpha_{1i} = \beta_i, \alpha_{2i} = -\ c\ \beta_i$

$\alpha_{3i} = \beta_i c - c, \alpha_{4i} = c$

TABLE 1 (Continued)

Restrictions: $\frac{\alpha_{oi}}{1-\alpha_{1i}} = \frac{\alpha_{oj}}{1-\alpha_{1j}}$, $-\alpha_{2i} + \alpha_{3i} = \alpha_{4i}$, $\alpha_{4j} = \alpha_{4i}$

$\alpha_{1i} \ \alpha_{4i} = -\alpha_{2i}$ for all i and j

Model: Brennan

Parameter Values: $\alpha_{oi} = 0$, $\alpha_{1i} = \beta_i$, $\alpha_{2i} = -\tau \ \beta_i$, $\alpha_{3i} = (1-\beta_i) + \beta_i \tau$

$-\tau$, $\alpha_{4i} = \tau$

Restrictions: $\alpha_{oi} = 0$, $\alpha_{1i} \ \alpha_{4i} = -\alpha_{2i}$, $\alpha_{1i} = 1 - \alpha_{3i} + \alpha_{2i} + \alpha_{4i}$,

$\alpha_{4i} = \alpha_{4j}$ for all i and j

Model: Blume

Parameter Values: $\alpha_{oi} = \bar{a} \ (1-\beta_i)$, $\alpha_{1i} = \beta_i$, $\alpha_{2i} = \beta_i \bar{c}$, $\alpha_{3i} = 0$,

$\alpha_{4i} = \bar{c}$

Restrictions: $\frac{\alpha_{oi}}{1-\alpha_{1i}} = \frac{\alpha_{oj}}{1-\alpha_{1j}}$, $\alpha_{1i} \ \alpha_{4i} = -\alpha_{2i}$, $\alpha_{3i} = 0$

$\alpha_{4i} = \alpha_{4j}$ for all i and j where k is the number of equations (securities), T is the number of observations, and I_T is the TxT identity matrix. The Gauss-Markov estimator of the system is,

$$\hat{\underline{\delta}}^* = [x'(\Sigma^{-1} \otimes I_T)\ x]^{-1}\ x'(\Sigma^{-1} \otimes I_T)\underline{y}$$

The estimator of this system exploits the cross sectional information that is ignored by single equation estimators. For example, if the independent variables are orthogonal across equations and if the disturbance are multivariate normal the SUR estimator say for equation (1) equals,[11]

$$\hat{\underline{\delta}}_1^* = \underline{\delta}_1 + (x_1'\ x_1)^{-1} x_1' [\underline{u}_1 - E(\underline{u}_1 \mid \underline{u}_2 \ . \ . \ . \ \underline{u}_k)]$$

Alternatively, if b_i is the multiple regression coefficient for $\underline{u}_i$ when $\underline{u}_1$ is the dependent variable and the remaining k-1 disturbances are the independent variable, the estimator may be expressed as,

$$\hat{\underline{\delta}}_1^* = \underline{\delta}_1 + (x_1'\ x_1)^{-1}\ x_1'(\underline{u}_1 - b_2\underline{u}_2 - b_3 u_3 \ . \ . \ . -b_k\underline{u}_k)$$

The estimator adjusts the disturbances for any linear dependence with the remaining disturbances in the system.

Although the orthogonality condition does not hold for our problem, the spirit of the adjustment is similar. Thus, if the market portfolio can be approximated with the securities included in the system, the loss in efficiency from not including the market portfolio in equation (11) is reduced. More generally, if security returns are linear in other unobservable factors, the seemingly unrelated estimator partially adjust for the excluded variables.

If the disturbances of the system are normally, independently, and identically distributed over time, the SUR estimator is the maximum likelihood estimator. The feasible estimator involves an estimated covariance matrix rather than the true one. In general, the estimator has the same asymptotic distribution as the maximum likelihood estimator.[12]

Empirical Results

Two systems of regression equations have been suggested for testing the tax effects of dividends. The systems are,

$$\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i}d_{it} + \gamma_{2i}r_{ft} + \tilde{\varepsilon}_{it} \qquad (11)$$

$$\tilde{R}_{it} = \alpha_{oi} + \alpha_{1i}R_{mt} + \alpha_{2i}d_{mt} + \alpha_{3i}r_{ft} + \alpha_{4i}d_{it} + \tilde{\varepsilon}_{it} \qquad (12)$$

The specific form of (11) for Litzenberger and Ramaswamy and Brennan is,

$$\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i}(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it} \qquad (11a)$$

$$\gamma_{1i} = \gamma_{1j} \text{ for all i and j}$$

for Blume, (11) becomes,

$$\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i}d_{it} + \tilde{\varepsilon}_{it} \qquad (11b)$$

$$\gamma_{1i} = \gamma_{1j} \text{ for all i and j}$$

Equation (11a) and (11b) are estimated for a broad cross section of firms. These firms are classified into systems of securities (equations) on the basis of average dividend yields. These classifications are meant to control for any dependency of effective tax brackets on dividends of the kind reported by Elton and Gruber [1970], and by Litzenberger and Ramaswamy [1980]. If tax induced clienteles effect asset prices, the tax parameters of (11a) and (11b) would not be constant across securities, hence, our tests would be more likely to reject a common effect across securities the larger the dispersion in dividend yields.

Periods of Analysis and Samples of Firms

The empirical tests are conducted over nine sample periods between January, 1926 and December, 1978. The sample periods include substantially different statutory treatment of dividends and capital gains. During the first sample period--January, 1926 to December, 1931--the statuory limit on dividend income was 20 percent and the statutory limit on long term capital gains (two year holding period) was 12.5 percent. Between 1935 and 1940, the statutory limit on dividends increased from 54 to 75 percent, while the statutory limit on long term capital gains (one to ten year holding periods) ranged from 30 to 60 percent. By 1945, the statutory limit on dividend income had risen to 90 percent, and 50 percent of the gain on assets held for more than six months were excluded from taxable income subject to a maximum tax of 25 percent. Between 1945 and 1978, the statutory limit on dividend income decreased to 70 percent, while the limits on long term capital gains have ranged between 25 and 30 percent.[13]

For each of the nine sample periods, systems of thirty securities are formed on the basis of average dividend yields over the sample periods. None of the firms included in the systems have nontaxable cash distributions during the period.

The securities are all sampled from the Center for Research in Security Prices (CRSP) monthly return file and include only those firms with complete data during the period. Each system includes thirty securities.

Data and Definitions of Variables

Monthly returns from the CRSP return file are used in estimating the models. After 1931 the riskless rate of interest is approximated with the one month Treasury Bill rate. Prior to 1931, the shortest Government Bonds with a maturity of at least one month are used to proxy for the riskless rate. The source of this data is Ibbotson and Sinquefield [1977].

The dividend yield variable is actual cash dividends paid in the ex dividend month divided by the closing price in the previous month. Note that this differs from the definitions of Litzenberger and Ramaswamy and Blume (cf. pages 6 and 8). It is unclear on prior grounds which of the definitions is more meaningful. Blume's quarterly measures of dividend yields and returns are not used since our primary interest is detection of tax effects. With this objective, it is unwise to destroy time series variation in dividend yields, i.e., the variation due to dividend yields being zero in months when no dividends are paid.

Estimates and Tests with Equations (11a) and (11b)

The empirical results for equations (11a) and (11b) are reported in Tables 2 through 10. The first column of each table shows the range of average dividend yield for each system. The second column lists the restricted estimates and their asymptotic standard errors in parentheses. The third column shows an "F-statistic" for the hypothesis that γ_{1i} is equal across securities, and the final column lists the approximate probability levels conditional on γ_{1i} being equal for all securities in the system. The "F-statistic" reported in these tables are distributed as F, with degrees of freedom equal to the number of restrictions and thirty times the degrees of freedom per equation if the covariance matrix is known.[14] For an unknown covariance matrix, the asymptotic distribution is chi-squared with the same degress of freedom; however, the F approximation is more conservative with respect to the equality hypothesis.[15, 16, 17]

The constants in equations (11a) and (11b) implicitly include a beta (times a market risk premium). Since other authors have assumed stationary beta for sixty month periods, a dummy variable is included in the estimated equations to allow for shifts in beta. The dummy variable is assigned a value of zero for the first half of the sample periods and one otherwise.[18]

The results of Table 2 thorugh 10 tell a very convincing story. The restriction that the tax parameter γ_{1i} is equal across securities is badly violated for virtually all the sample periods and yield groups. Since the restriction fails to hold, it is not clear what, if any, interpretations can safely be drawn from the restricted estimates. For what it may be worth, the behavior of the restricted estimates across the dividend yield classification does appear roughly consistent with the existence of clientele effects. The restricted estimates do appear to be negatively related to dividend yield, though the relation is not strong or uniform.

TABLE 2

ESTIMATES AND TESTS WITH EQUATIONS (11a) and (11b)
JANUARY, 1926 - DECEMBER, 1931

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i}(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	.96 - 4.00	.451 (.304)	5.006	29,2070	.0001
2	4.04 - 4.76	.529 (.215)	5.099	29,2070	.0001
3	4.80 - 5.26	.261 (.140)	2.529	29,2070	.0001
4	5.28 - 5.79	.952 (.143)	2.861	29,2070	.0001
5	5.80 - 6.42	.670 (.160)	3.177	29,2070	.0001
6	6.43 - 6.99	.174 (.129)	5.327	29,2070	.0001
7	7.00 - 7.53	.917 (.157)	3.724	29,2070	.0001
8	7.58 - 8.32	.266 (.158)	3.643	29,2070	.0001
9	8.35 - 10.64	.890 (.126)	2.553	29,2070	.0001

Equation (11b): $\tilde{R}_{it} = {}_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	.96 - 4.00	.894 (.310)	4.746	29,2070	.0001
2	4.04 - 4.76	.629 (.208)	4.868	29,2070	.0001
3	4.80 - 5.26	.376 (.136)	2.308	29,2070	.0001
4	5.28 - 5.79	1.211 (.142)	3.338	29,2070	.0001
5	5.80 - 6.42	.734 (.160)	3.239	29,2070	.0001
6	6.43 - 6.99	.235 (.131)	5.293	29,2070	.0001
7	7.00 - 7.53	1.035 (.158)	3.611	29,2070	.0001
8	7.58 - 8.32	.331 (.160)	3.329	29,2070	.0001
9	8.35 - 10.64	.839 (.125)	2.647	29,2070	.0001

TABLE 3

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)

JANUARY 1935 - DECEMBER 1940

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimates γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.704 - 4.778	.710 (.190)	4.935	29,2070	.0001
2	4.788 - 5.313	.850 (.153)	1.607	29,2070	.0215
3	5.326 - 5.862	.584 (.154)	2.781	29,2070	.0001
4	5.875 - 6.440	.354 (.123)	2.625	29,2070	.0001
5	6.452 - 7.086	.356 (.127)	3.994	29,2070	.0001
6	7.134 - 8.096	.569 (.126)	4.606	29,2070	.0001
7	8.120 - 9.612	.533 (.116)	4.362	29,2070	.0001
8	9.684 - 14.928	.566 (.130)	3.879	29,2070	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimates γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.704 - 4.778	.715 (.190)	4.932	29,2070	.0001
2	4.788 - 5.313	.851 (.153)	1.593	29,2070	.0235
3	5.326 - 5.862	.582 (.154)	2.760	29,2070	.0001
4	5.875 - 6.440	.353 (.123)	2.626	29,2070	.0001
5	6.452 - 7.086	.358 (.127)	3.977	29,2070	.0001
6	7.134 - 8.096	.569 (.126)	4.613	29,2070	.0001
7	8.120 - 9.612	.535 (.116)	4.360	29,2070	.0001
8	9.684 - 14.928	.566 (.130)	3.874	29,2070	.0001

TABLE 4

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)

JANUARY 1941 - DECEMBER 1945

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	2.336 - 4.400	1.430 (.133)	5.672	29,1710	.0001
2	4.468 - 5.068	.441 (.135)	5.130	29,1710	.0001
3	5.084 - 5.504	.754 (.103)	4.545	29,1710	.0001
4	5.800 - 6.088	.663 (.103)	7.681	29,1710	.0001
5	6.440 - 6.824	.532 (.085)	2.714	29,1710	.0001
6	6.864 - 7.288	.632 (.109)	4.554	29,1710	.0001
7	7.290 - 7.760	.346 (.078)	5.633	29,1710	.0001
8	7.768 - 8.244	.611 (.091)	3.538	29,1710	.0001
9	8.788 - 10.164	.493 (.085)	6.975	29,1710	.0001
10	10.168-11.580	.260 (.083)	5.157	29,1710	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	2.336 - 4.400	1.460 (.133)	5.675	29,1710	.0001
2	4.468 - 5.068	.459 (.135)	5.152	29,1710	.0001
3	5.084 - 5.504	.773 (.102)	4.565	29,1710	.0001
4	5.800 - 6.088	.682 (.103)	7.686	29,1710	.0001
5	6.440 - 6.824	.540 (.085)	2.715	29,1710	.0001
6	6.804 - 7.288	.642 (.109)	4.511	29,1710	.0001
7	7.290 - 7.760	.354 (.078)	5.629	29,1710	.0001
8	7.768 - 8.244	.616 (.091)	3.540	29,1710	.0001
9	8.788 - 10.164	.501 (.085)	6.937	29,1710	.0001
10	10.168 - 11.580	.266 (.083)	5.178	29,1710	.0001

TABLE 5

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)

JANUARY 1946 - DECEMBER 1950

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.760 - 4.880	.845 (.175)	4.835	29,1710	.0001
2	4.888 - 5.436	.996 (.132)	3.236	29,1710	.0001
3	5.452 - 6.060	.248 (.108)	3.792	29,1710	.0001
4	6.068 - 6.468	-.071 (.094)	5.015	29,1710	.0001
5	6.476 - 7.020	.844 (.097)	3.087	29,1710	.0001
6	7.022 - 7.376	.411 (.079)	5.043	29,1710	.0001
7	7.388 - 7.856	.138 (.086)	5.821	29,1710	.0001
8	7.864 - 8.892	.320	4.796	29,1710	.0001
9	8.896 - 9.904	.420 (.083)	7.938	29,1710	.0001
10	9.920 - 13.072	.007 (.076)	2.929	29,1710	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.760 - 4.880	.878 (.176)	4.771	29,1710	.0001
2	4.888 - 5.436	1.009 (.131)	3.325	29,1710	.0001
3	5.452 - 6.060	.257 (.107)	3.801	29,1710	.0001
4	6.068 - 6.468	-.064 (.095)	4.979	29,1710	.0001
5	6.476 - 7.020	.853 (.097)	3.090	29,1710	.0001
6	7.022 - 7.376	.420 (.079)	5.105	29,1710	.0001
7	7.388 - 7.856	.154 (.086)	5.750	29,1710	.0001
8	7.964 - 8.892	.337 (.094)	4.801	29,1710	.0001
9	8.896 - 9.904	.430 (.083)	7.891	29,1710	.0001
10	9.920 - 13.072	.0178 (.076)	2.919	29,1710	.0001

TABLE 6

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)

JANUARY 1951 - DECEMBER 1955

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.248 - 4.420	.424 (.146)	4.856	29,1710	.0001
2	4.564 - 5.185	.355 (.140)	3.742	29,1710	.0001
3	5.200 - 5.648	.669 (.100)	2.332	29,1710	.0001
4	5.652 - 6.026	.325 (.075)	2.511	29,1710	.0001
5	6.026 - 2.348	.523 (.086)	3.781	29,1710	.0001
6	6.360 - 6.680	.023 (.093)	4.535	29,1710	.0001
7	6.696 - 6.972	.0250 (.082)	3.661	29,1710	.0001
8	7.320 - 7.724	.037 (.070)	7.129	29,1710	.0001
9	7.756 - 8.104	.264 (.085)	2.327	29,1710	.0001
10	8.576 - 10.204	.260 (.095)	2.974	29,1710	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.248 - 4.420	.360 (.144)	4.964	29,1710	.0001
2	4.564 - 5.188	.307 (.140)	3.698	29,1710	.0001
3	5.200 - 5.648	.652 (.100)	2.280	29,1710	.0001
4	5.652 - 6.026	.293 (.074)	2.508	29,1710	.0001
5	6.026 - 6.348	.489 (.086)	3.750	29,1710	.0001
6	6.360 - 6.680	.006 (.044)	4.590	29,1710	.0001
7	6.696 - 6.972	-.057 (.081)	3.620	29,1710	.0001
8	7.320 - 7.724	.005 (.069)	7.236	29,1710	.0001
9	7.756 - 8.104	.249 (.085)	2.395	29,1710	.0001
10	8.576 - 10.204	.245 (.095)	2.885	29,1710	.0001

TABLE 7

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)

JANUARY 1956 - DECEMBER 1960

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.256 - 3.764	.198 (.217)	1.146	29,1710	.2700
2	3.788 - 4.472	.348 (.166)	4.282	29,1710	.0001
3	4.449 - 4.836	.309 (.137)	1.567	29,1710	.0281
4	4.837 - 5.160	[illegible]	4.830	29,1710	.0001
5	5.172 - 5.404	-.096 (.139)	2.437	29,1710	.0001
6	5.408 - 5.708	-.133 (.095)	[illegible]	29,1710	.0001
7	5.712 - 6.048	.608 (.106)	2.858	29,1710	.0001
8	6.068 - 6.528	-.140 (.110)	5.747	29,1710	.0001
9	6.572 - 8.632	.454 (.133)	3.912	29,1710	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.256 - 3.764	.022 (.217)	.991	29,1710	.4800
2	3.788 - 4.472	.169 (.162)	4.009	29,1710	.0001
3	4.449 - 4.836	-.076 (.138)	1.582	29,1710	.0256
4	4.837 - 5.160	.822	4.633	29,1710	.0001
5	5.172 - 5.404	-.191 (.137)	2.465	29,1710	.0001
6	5.408 - 5.708	-.229 (.097)	4.301	29,1710	.0001
7	5.712 - 6.048	-.022 (.105)	2.772	29,1710	.0001
8	6.068 - 6.528	-.226 (.108)	5.922	29,1710	.0001
9	6.572 - 8.632	.417 (.134)	3.922	29,1710	.0001

TABLE 8

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)

JANUARY 1961 - DECEMBER 1965

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.040 - 2.480	1.401 (.464)	5.838	29,1710	.0001
2	2.560 - 3.132	1.709 (.298)	3.078	29,1710	.0001
3	3.136 - 3.448	.139 (.201)	5.022	29,1710	.0001
4	3.492 - 3.800	[illegible] ([illegible])	3.794	29,1710	.0001
5	3.808 - 4.096	.486 (.210)	[illegible]	29,1710	.0001
6	4.108 - 4.292	.460 (.193)	[illegible]	29,1710	.0002
7	4.292 - 4.556	-.311 (.179)	2.012	29,1710	.0012
8	4.568 - 4.768	.531 (.182)	2.767	29,1710	.0001
9	4.776 - 5.060	.175 (.143)	5.585	29,1710	.0001
10	5.068 - 5.560	-.311 (.140)	4.516	29,1710	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
1	1.040 - 2.480	1.574 (.466)	4.492	29,1710	.0001
2	2.560 - 3.132	1.695 (.297)	3.059	29,1710	.0001
3	3.136 - 3.448	.160 (.202)	4.827	29,1710	.0001
4	3.492 - 3.800	1.136 (.189)	3.560	29,1710	.0001
5	3.808 - 4.096	.483 (.209)	2.893	29,1710	.0001
6	4.108 - 4.292	.434 (.193)	2.202	29,1710	.0003
7	4.292 - 4.556	-.321 (.179)	1.961	29,1710	.0017
8	4.568 - 4.768	.529 (.182)	2.736	29,1710	.0001
9	4.776 - 5.060	.177 (.143)	5.593	29,1710	.0001
10	5.068 - 5.560	-.313 (.140)	4.433	29,1710	.0001

TABLE 9

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)
FEBRUARY, 1966 - NOVEMBER, 1971

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i}(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistics	Degrees of Freedom	Probability Level
1	.28 - 1.12	7.064 (1.19)	4.725	29,2010	.0001
2	1.56 - 1.84	3.159 (.511)	2.903	29,2010	.0001
3	2.04 - 2.28	2.163 (.480)	3.575	29,2010	.0001
4	2.52 - 2.64	.070 (.389)	2.050	29,2010	.0009
5	2.96 - 3.12	.526 (.298)	3.159	29,2010	.0001
6	3.68 - 3.88	.637 (.243)	2.603	29,2010	.0001
7	4.24 - 4.40	.332 (.228)	1.939	29,2010	.0020
8	4.84 - 5.08	.087 (.171)	3.507	29,2010	.0001
9	5.28 - 5.56	-.092 (.099)	3.122	29,2010	.0001
10	5.96 - 9.32	.162 (.108)	5.334	29,2010	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + {}_{1i}d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistics	Degrees of Freedom	Probability Level
1	.28 - 1.12	5.688 (3.63)	5.468	29,2010	.0001
2	1.56 - 1.84	2.626 (.509)	3.156	29,2010	.0001
3	2.04 - 2.28	1.949 (.482)	3.741	29,2010	.0001
4	2.52 - 2.64	-.218 (.380)	2.061	29,2010	.0008
5	2.96 - 3.12	.313 (.296)	3.123	29,2010	.0001
6	3.68 - 3.88	.549 (.294)	2.487	29,2010	.0001
7	4.24 - 4.40	.253 (.228)	2.139	29,2010	.0094
8	4.84 - 5.08	.061 (.169)	3.659	29,2010	.0001
9	5.28 - 5.56	-.083 (.099)	3.082	29,2010	.0001
10	5.96 - 9.32	.148 (.107)	5.222	29,2010	.0001

TABLE 10

ESTIMATES AND TESTS WITH EQUATIONS (11a) AND (11b)
JANUARY, 1972 - DECEMBER, 1978

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i}(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistics	Degrees of Freedom	Probability Level
1	.43 - 1.57	3.904 (.702)	1.394	29,2430	.0800
2	1.61 - 2.20	1.963 (.486)	3.293	29,2430	.0001
3	2.50 - 2.81	1.760 (.317)	1.945	29,2430	.0019
4	3.42 - 3.66	1.176 (.329)	2.976	29,2430	.0001
5	3.92 - 4.26	2.705 (.262)	2.176	29,2430	.0003
6	4.72 - 4.95	.903 (.223)	3.276	29,2430	.0001
7	5.21 - 5.64	.907 (.194)	1.271	29,2430	.1515
8	5.88 - 6.24	.466 (.166)	2.432	29,2430	.0001
9	6.76 - 7.45	.253 (.120)	3.087	29,2430	.0001
10	8.29 - 11.82	.095 (.068)	5.113	29,2430	.0001

Equation (11b): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i}d_{it} + \tilde{\varepsilon}_{it}$

System	Average Annual Dividend Yield (Percent)	Restricted Estimate γ_1	F-Statistics	Degrees of Freedom	Probability Level
1	.43 - 1.57	2.725 (.680)	1.561	29,2430	.0300
2	1.61 - 2.20	1.094 (.462)	2.503	29,2430	.0001
3	2.50 - 2.81	1.199 (.316)	2.345	29,2430	.0001
4	3.42 - 3.66	.925 (.330)	2.726	29,2430	.0001
5	3.92 - 4.26	2.615 (.266)	2.049	29,2430	.0008
6	4.72 - 4.95	.768 (.223)	3.185	29,2430	.0001
7	5.21 - 5.64	.823 (.197)	1.404	29,2430	.0745
8	5.88 - 6.24	.378 (.165)	2.283	29,2430	.0001
9	6.76 - 7.45	.196 (.121)	3.190	29,2430	.0001
10	8.29 - 11.82	.061 (.068)	5.205	29,2430	.0001

Model Misspecification

Equation (11a) imposes the restriction that the coefficients on the riskless rate of interest equal 1-c, and (11b) totally excludes the riskless rate of interest. The work of Fama and Schwert [1977] suggests that stock returns are negatively related to the riskless rate of interest. Since equation (11a) and (11b) fail to incorporate this behavior, it is possible that our tests have failed to hold because the effect of the riskless rate is not properly specified. It is true, of course, that the models of Litzenberger and Ramaswamy, Brennan, and Blume make specific statements about the impact of the riskless rate on expected returns. However, if the shortcomings of their models are due to misspecification of the way the riskless rate and not dividends enters the relation, tax effects may be present and masked by the riskless rate of interest.

It is worthwhile, therefore, to separately explore these two effects. Equation (11),

$$\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \gamma_{2i} r_{ft} + \tilde{\varepsilon}_{it} \tag{11}$$

allows the riskless rate of interest to be free. Referring to Table 1, on pages 20 and 21, we see that (11) implies that $\gamma_{1i} = \gamma_{1j}$ and $\gamma_{2i} = \gamma_{2j}$ for all three models, that $\gamma_{1i} + \gamma_{2i} = 1$ for Litzenberger and Ramaswamy and Brennan, and that $\gamma_{2i} = 0$ for Blume. With equation (11) we thus can separate the hypothesis that γ_{1i} is equal across securities from the other hypotheses.

Equation (11) is estimated with a sample selected from the securities included in the Dow Jones Industrials during the January, 1926 to December, 1978 period. The sample always consists of thirty securities but the composition of the sample changes over time. For purposes of comparison, the same thirty securities used for estimating (11) are also used to estimate (11a) for each of the nine periods. These results are reported in Table 11. Table 12 reports test statistics, restricted estimates, and unrestricted estimates for three of the periods, along with their standard errors.

From the results of Tables 11 and 12, it appears that our earlier conclusions are insensitive to the specification of the riskless rate. Generally, the restricted estimates of equations (11) and (11a) are quite close and our inferences regarding the equality of γ_{1i} across equations are unaffected. As for the riskless rate, the hypothesis of a common coefficient appears to be a reasonable approximation for periods after 1960. Prior to 1960, we are generally able to reject this hypothesis. The unrestricted estimates of Table 13 show a wide variation in all sample periods but are predominately negative, consistent with the Fama-Schwert findings. Given the observed variation, it is surprising that the hypothesis of a common effect of the riskless rate is not rejected in all periods. But the smaller variation in the riskless rate makes precision of these estimates much lower than that of estimates for the dividend yield coefficient.

An additional check on the sensitivity of the test to model specification can be obtained from equation (12) which differs from (11) by inclusion of the market return and dividend yield as separate variables (cf. Table 1). These estimates are reported in Table 13 for the same time periods and for the sample of Dow Jones Industrials. Including the market return and yield as separate variables seems to have little impact on the results. The probability levels, restricted estimates, and unrestricted estimates are very similar to those of equation (11).

Definition of Anticipated Dividend Yields

The results presented in Tables 2-13 provide little support for the simple tax effect models of dividends. We can reject <u>any</u> model predicting a common coefficient on dividend yield, including the case of neutrality of dividends. It is possible that our results are due to the perfect foresight definition of dividend yields. Many dividend payments are announced during the ex month, and therefore, announcement and tax effects may be convoluted in our estimates of the dividend yield coefficients. There is, of course, no reason to suppose that such announcement effects would be the same across securities or across time for the same securities and because of this, our tests may unfairly reject the hypothesis of a common dividend yield effect. To control for this possibility, we consider a definition of anticipated dividend yields that relies only upon information announced prior to the ex month, namely, the definition of Litzenberger and Ramaswamy. This choice allows us to make direct comparisons to their results and may also be regarded as a naive definition from the set of possible definitions, i.e., many alternative definitions relying upon information announced prior to the ex month will have forecasting errors that lie somewhere between those of the perfect foresight definition and the Litzenberger and Ramaswamy definition.

TABLE 11

ESTIMATES AND TESTS WITH EQUATION (11a)

DOW JONES 30

Equation (11a): $\tilde{R}_{it} - r_{ft} = \gamma_{oi} + \gamma_{1i} (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}$

Time Period	Restricted Estimate γ_1	F-Statistic	Degrees Of Freedom	Probability Level
January 1926 - December 1931	.327 (.150)	5.875	29,2070	.0001
January 1935 - December 1940	.332 (.128)	4.145	29,2070	.0001
January 1941 - December 1945	.518 (.066)	7.675	29,1710	.0001
January 1946 - December 1950	.272 (.074)	5.992	29,1710	.0001
January 1951 - December 1955	-.028 (.077)	2.735	29,1710	.0001
January 1956 - December 1960	-.413 (.130)	6.025	29,1710	.0001
January 1961 - December 1965	.203 (.216)	4.229	29,1710	.0001
January 1966 - December 1971	.552 (.255)	1.040	29,2070	.4069
January 1972 - December 1978	.555 (.127)	2.152	29,2430	.0004

TABLE 12

ESTIMATES AND TESTS WITH EQUATION (11): PERFECT FORESIGHT DEFINITIONS OF DIVIDEND YIELD

Equation (11): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \gamma_{2i} r_{ft} + \tilde{\varepsilon}_{it}$

DOW JONES 30

Restrictions

	January 1926 - December 1931			January 1935 - December 1940			January 1941 - December 1945		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	4.712	2.837	2.728	4.278	1.533	1.575	7.393	1.850	2.978
Degrees of Freedom	29,2040	29,2040	30,2040	29,2040	29,2040	30,2040	29,1680	29,1680	30,1680
Probability Level	.0001	.0001	.0001	.0001	.0327	.0247	.0001	.0040	.0001
Restricted Estimates	.656	6.702		.358	62.933		.490	132.799	
Standard Errors	.152	2.173		.129	33.664		.068	22.066	

	January 1946 - December 1950			January 1951 - December 1955			January 1956 - December 1960		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	6.004	1.738	1.627	2.590	1.967	2.095	6.106	2.262	2.847
Degrees of Freedom	29,1680	29,1680	30,1680	29,1680	29,1680	30,1680	29,1680	29,1680	29,1680
Probability Level	.0001	.0089	.0176	.0001	.0017	.0005	.0001	.0002	.0001
Restricted Estimates	.225	-.918		.073	-5.990		-.587	-9.437	
Standard Errors	.073	7.769		.094	3.470		.134	2.553	

	January 1961 - December 1965			January 1966 - December 1971			January 1972 - December 1978		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F. Statistics	4.094	1.276	1.276	1.158	1.190	1.510	2.058	1.166	1.159
Degrees of Freedom	29,1680	29,1680	30,1680	29,2040	29,2040	30,2040	29,2400	29,2400	30,2400
Probability Level	.0001	.1459	.1459	.2564	.2228	.0377	.0008	.2480	.2528
Restricted Estimates	.171	-6.627		.364	-9.502		.510	1.591	
Standard Errors	.220	7.524		.249	3.998		.128	2.161	

TABLE 12 (CONT'D)

DOW JONES 30

Unrestricted Estimates of $_{1i}$ and $_{2i}$ With Equation (11)

Security Number	January 1961 - December 1965				January 1966 - December 1971				January 1972 - December 1978			
	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$
1	- 4.579	20.625	.439	1.237	-24.834	13.419	-1.465	1.226	-10.512	7.303	1.449	1.086
2	44.178	28.733	2.401	3.410	-19.433	18.320	3.353	1.749	- 4.579	7.143	- .207	1.436
3	-27.730	26.818	- .785	1.656	- 8.706	12.329	1.661	1.427	- 5.011	4.689	.249	.485
4	- 6.194	15.957	.414	.630	-14.985	12.885	-1.432	1.614	- 2.923	4.307	.340	.447
5	- 8.207	16.013	.167	.942	-11.207	10.711	.834	1.068	- 4.690	3.390	.560	.382
6	3.804	23.640	- .997	.670	-13.894	16.328	-2.134	1.914	- 1.612	7.971	- .341	.648
7	33.989	36.414	5.323	3.815	- 2.249	9.947	- .074	1.229	-14.086	10.117	-1.507	1.818
8	- 8.527	17.193	.694	.841	-10.942	7.931	- .835	1.112	- 3.503	6.379	-1.605	.968
9	16.218	19.581	-1.047	1.356	- 5.830	9.256	.152	1.011	-10.531	6.418	1.150	1.264
10	- 2.992	24.784	1.171	.979	-14.037	13.276	1.545	2.175	- 1.928	7.038	2.248	1.125
11	- 5.965	15.360	-1.441	.649	- .674	9.156	-1.229	1.099	- 8.160	4.500	.576	.533
12	22.231	20.867	1.279	1.323	- 1.000	16.330	-1.338	1.422	-11.949	6.161	1.148	.842
13	-31.835	20.481	1.814	1.621	.322	15.530	2.021	1.492	2.945	6.714	.630	.903
14	2.236	19.191	-2.141	.597	-30.154	17.676	- .582	2.286	- 6.124	3.193	- .442	.355
15	-26.110	23.360	1.907	1.539	-18.726	12.994	.729	1.339	- .820	6.362	- .684	.695
16	-20.736	24.947	- .657	1.411	13.856	12.041	- .590	1.464	- 4.824	7.151	- .757	.988
17	-11.022	20.418	- .258	.951	-10.159	13.400	- .215	1.421	- 4.676	7.615	- .763	.923
18	-20.424	24.781	- .515	1.252	- 3.519	16.643	-2.950	1.765	- 1.835	6.876	- .034	1.023
19	- 2.305	20.286	3.041	1.057	- 7.663	12.694	2.004	1.340	- 3.920	7.561	- .778	1.074
20	19.547	25.086	.248	3.214	- 7.742	8.331	.585	.865	- 7.833	6.608	1.851	1.806
21	4.408	19.602	4.841	1.373	-29.813	17.037	.525	1.804	2.588	6.210	2.674	.964
22	- 4.834	21.152	- .219	1.344	-33.345	15.574	- .734	1.677	- 5.622	4.784	2.043	1.157
23	-13.620	21.884	2.954	2.085	- 6.653	17.525	2.160	1.719	-10.017	6.013	2.311	1.139
24	- .305	16.350	-2.278	.927	- 6.796	12.896	.630	1.302	- 9.642	6.296	- .358	.723
25	-18.010	19.013	-1.521	1.022	- 8.112	11.406	.089	1.398	- 5.290	5.584	- .156	.419
26	- 1.085	20.875	- .083	1.232	-12.751	10.376	- .677	1.027	- 3.606	6,535	2.293	.620
27	- 4.097	26.091	1.584	.774	-24.266	19.388	6.012	2.272	1.497	6.897	- .028	.737
28	14.852	29.741	3.153	1.789	- .792	13.103	.794	1.399	-13.154	7.479	-1.037	1.156
29	25.357	26.062	3.288	1.655	6.834	21.984	4.272	2.962	-42.861	16.157	2.764	2.142
30	-23.936	27.020	.456	1.510	-12.856	11.973	1.386	1.210	- .333	7.568	1.432	.730

TABLE 13

ESTIMATES AND TESTS WITH EQUATION (12): PERFECT FORESIGHT DEFINITION OF DIVIDEND YIELD

Equation (12): $\tilde{R}_{it} = \alpha_{oi} + \alpha_{1i} \tilde{R}_{mt} + \alpha_{2i} d_{mt} + \alpha_{3i} r_{ft} + \alpha_{4i} d_{it} + \tilde{\epsilon}_{it}$

DOW JONES 30

Restrictions

	January 1926 - December 1931		January 1935 - December 1940		January 1941 - December 1945	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	2.6312	4.5874	1.027	3.146	3.830	6.495
Degrees of Freedom	29,2010	29,2010	29,2010	29,2010	29,1650	29,1650
Probability Level	.0001	.0001	.4258	.0001	.0001	.0001
Restricted Estimates	-.997	.652	12.608	.702	6.577	.656
Standard Errors	.543	.156	5.670	.122	4.283	.068

	January 1946 - December 1950		January 1951 - December 1955		January 1956 - December 1960	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	2.534	3.593	1.681	2.477	3.254	3.071
Degrees of Freedom	29,1650	29,1650	29,1650	29,1650	29,1650	29,1650
Probability Level	.0001	.0001	.0133	.0001	.0001	.0001
Restricted Estimates	-1.995	.396	1.105	-.003	3.247	-.484
Standard Errors	2.078	.083	1.438	.103	.680	.152

	January 1961 - December 1965		January 1966 - December 1971		January 1972 - December 1978	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	2.109	3.293	1.259	1.836	1.018	2.286
Degrees of Freedom	29,1650	29,1650	29,2010	29,2010	29,2370	29,2370
Probability Level	.0005	.0001	.1610	.0044	.4387	.0001
Restricted Estimates	-1.891	-.194	-2.163	.578	1.670	.854
Standard Errors	.884	.262	1.930	.315	.757	.175

TABLE 13 (CONT'D)

DOW JONES 30

Unrestricted Estimates of α_{3i} and α_{4i} With Equation (12)

Security Number	January 1961 - December 1965				January 1966 - December 1971				January 1972 - December 1978			
	$\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$	$\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$	$\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$
1	.783	9.809	- .003	3.615	- 4.726	8.577	-1.462	3.995	.396	5.048	5.153	1.684
2	16.869	14.542	29.933	11.452	- 6.236	13.714	3.607	1.805	2.376	6.601	.481	1.624
3	.754	12.163	- .965	3.058	- 1.662	9.043	2.688	1.478	.742	3.616	.221	.900
4	- 4.227	8.183	.269	.649	-15.616	11.456	-1.989	6.468	1.131	3.696	.367	.533
5	- 8.274	6.446	3.936	1.425	- 1.029	7.913	-2.108	1.463	.032	2.808	1.349	1.021
6	12.211	10.916	- .026	.712	- 8.667	12.161	-2.376	2.675	8.663	6.510	- .448	.974
7	- 5.551	19.450	- 2.354	6.292	- 6.783	8.638	-1.462	1.938	5.988	8.930	1.746	1.533
8	- 6.501	6.557	2.157	2.320	- 3.076	6.303	8.578	3.388	5.185	4.800	3.709	2.106
9	25.496	8.022	- .814	1.907	- .587	6.902	-1.597	2.143	- 1.677	4.883	.045	2.140
10	11.844	12.648	.284	.983	- 8.897	10.222	.550	2.162	4.850	6.160	2.346	1.121
11	-16.990	7.261	- 8.267	1.857	3.374	7.580	-1.452	3.799	- .886	3.593	2.477	1.304
12	15.944	9.289	3.212	1.398	10.417	10.743	-2.644	1.569	- 1.794	3.375	.776	.791
13	-16.391	9.366	3.224	5.060	1.928	11.893	4.210	1.747	12.268	5.179	2.490	1.956
14	- 3.736	8.126	- 2.942	1.332	-20.012	13.279	4.695	2.903	- .695	4.218	- .828	.600
15	- 2.989	9.810	7.865	4.239	-16.585	9.938	- .805	1.444	6.647	4.856	-1.513	2.152
16	- 9.519	10.039	16.760	3.223	- 1.004	8.496	- .327	5.111	3.619	6.006	-1.279	.992
17	.773	11.033	1.131	1.044	1.148	9.881	2.737	5.543	2.402	6.040	- .036	.930
18	- 8.430	9.951	3.323	3.745	.729	10.865	-9.446	3.665	8.893	5.220	2.888	2.258
19	6.731	9.945	3.476	3.344	3.425	9.395	- .300	1.343	2.897	6.516	-2.998	2.611
20	19.532	10.316	- 9.774	6.888	.400	6.895	.127	.886	3.297	5.018	9.611	5.011
21	14.316	9.820	2.290	2.572	-10.870	10.536	-6.466	6.199	11.017	4.372	5.050	1.119
22	- 7.950	9.016	- 2.051	1.537	-30.788	12.012	1.585	2.165	1.637	3.432	1.067	1.152
23	- 8.168	10.509	.319	6.811	4.578	11.606	.802	1.849	- 2.670	4.692	.013	2.416
24	- 5.193	8.294	- 3.558	1.669	2.264	9.260	2.152	1.442	- 2.208	5.386	.309	1.734
25	-15.095	8.930	- 1.808	1.031	3.202	7.927	2.454	5.938	1.749	4.201	.295	.434
26	9.887	8.257	- 3.001	3.431	- 4.279	7.414	.745	1.058	6.469	4.516	2.602	.632
27	9.534	9.449	4.128	1.770	-11.124	15.546	6.411	2.409	10.953	5.767	.938	.806
28	23.174	16.651	- 1.287	4.268	6.929	8.793	-3.027	1.450	- 5.737	6.735	- .566	2.250
29	30.376	12.454	3.302	4.237	14.148	17.070	1.652	4.917	-26.738	14.084	4.877	2.390
30	1.674	13.104	2.963	1.685	- 2.167	8.906	1.191	1.297	7.737	6.085	1.248	.811

TABLE 14

ESTIMATES AND TESTS WITH EQUATION (11): LITZENBERGER AND RAMASWAMY DEFINITION OF DIVIDEND YIELD

Equation (11): $\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} d_{it} + \gamma_{2i} r_{ft} + \varepsilon_{it}$

DOW JONES 30

Restrictions

	January 1926 - December 1931			January 1935 - December 1940			January 1941 - December 1945		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	4.708	2.819	3.052	2.404	1.551	1.467	3.678	2.505	4.400
Degrees of Freedom	29,2040	29,2040	30,2040	29,2040	29,2040	30,2040	29,1680	29,1680	30,1680
Probability Level	.0001	.0001	.0001	.0001	.0391	.0492	.0001	.0001	.0002
Restricted Estimates	-.168	8.338		.022	23.239		-.0411	148.063	
Standard Errors	.156	2.177		.011	.681		.020	19.108	

	January 1946 - December 1950			January 1951 - December 1955			January 1956 - December 1960		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	3.364	1.548	1.578	3.040	1.866	2.029	5.124	2.563	3.099
Degrees of Freedom	29,1680	29,1680	29,1680	29,1680	29,1680	30,1680	29,1680	29,1680	30,1680
Probability Level	.0001	.0166	.0244	.0001	.0036	.0009	.0001	.0001	.0001
Restricted Estimates	.040	1.981		.052	-6.088		-.207	-7.730	
Standard Errors	.045	7.798		.077	3.446		.124	2.545	

	January 1961 - December 1965			January 1966 - December 1971			January 1972 - December 1978		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	1.546	1.373	1.357	1.808	1.241	1.012	2.276	1.112	1.200
Degrees of Freedom	29,1680	29,1680	29,1680	29,2040	29,2040	30,2040	29,2400	29,2400	30,2400
Probability Level	.0321	.0895	.0946	.0053	.1764	.4480	.0001	.3111	.2099
Restricted Estimates	.531	-2.977		-.295	-.809		.573	-3.086	
Standard Errors	.216	7.399		.174	4.215		.132	2.244	

UTILITIES

Restrictions

	January 1926 - December 1931			January 1935 - December 1940			January 1941 - December 1945		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	4.391	1.376	1.457	4.584	2.074	2.089	4.092	2.931	2.871
Degrees of Freedom	29,2040	29,2040	29,2040	29,2040	29,2040	30,2040	29,1680	29,1680	30,16[illegible]
Probability Level	.0001	.0874	.0524	.0001	.0007	.0005	.0001	.0001	
Restricted Estimates	-.276	4.859		.085	-64.049		-.062	26.453	
Standard Errors	.077	2.332		.067	39.038		.062	17.188	

	January 1946 - December 1950			January 1951 - December 1955			January 1956 - December 1960		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	2.213	1.758	1.7956	2.452	1.092	1.7138	3.871	1.148	1.65[illegible]
Degrees of Freedom	29,1680	29,1680	30,1680	29,1680	29,1680	30,1680	29,1680	29,1680	30,16[illegible]
Probability Level	.0002	.0078	.0053	.0001	.3369	.0096	.0001	.0001	.003[illegible]
Restricted Estimates	-.046	-8.363		.436	-13.064		.241	-10.702	
Standard Errors	.052	3.463		.059	3.174		.099	2.657	

	January 1961 - December 1965			January 1966 - December 1971			January 1972 - December 1978		
	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$	$\gamma_{1i} = \gamma_{1j}$	$\gamma_{2i} = \gamma_{2j}$	$\gamma_{1i} + \gamma_{2i} = 1$
F-Statistics	3.200	2.565	2.648	3.387	1.063	1.026	10.800	1.135	1.3[illegible]
Degrees of Freedom	29,1680	29,1680	29,1680	29,2040	29,2040	30,2040	29,2400	29,2400	30,[illegible]
Probability Level	.0001	.0001	.0001	.0001	.3747	.4275	.0001	.2529	[illegible]
Restricted Estimates	.088	-18.354		.196	4.609		.050	-2.743	
Standard Errors	.132	9.276		.067	3.974		.043	1.862	

TABLE 14 (CONT'D)

DOW JONES 30

Unrestricted Estimates of γ_{1i} and γ_{2i} With Equation (11)

Security Number	January 1961 - December 1965				January 1966 - December 1971				January 1972 - December 1978			
	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$
1	- 7.107	20.612	-1.701	1.152	-21.985	13.085	.673	1.176	-10.532	7.303	1.339	1.029
2	46.963	28.369	3.277	1.652	.793	11.980	- .594	1.986	- 4.617	7.142	- .321	1.459
3	-25.165	26.565	- .817	1.664	4.110	8.362	-1.114	.729	- 5.008	4.689	.280	.466
4	- 4.745	16.072	1.236	1.047	- .756	8.568	- .059	.743	- 2.918	4.307	.329	.447
5	- 5.495	15.706	2.004	.937	3.480	7.139	- .711	.733	- 4.723	3.390	.527	.391
6	3.674	23.842	2.589	1.591	- 9.401	10.104	- .239	.622	- 1.665	7.972	- .476	.654
7	26.786	36.573	.848	2.702	-28.990	13.808	1.325	1.359	-14.108	10.162	-1.103	1.135
8	- 7.814	17.032	1.977	.977	-11.154	7.919	- .542	1.029	- 4.096	6.360	-2.055	1.203
9	15.467	19.749	.771	1.341	- 7.939	8.232	-1.297	3.113	-10.303	6.409	3.417	2.083
10	- 1.704	25.263	-1.357	1.313	- 5.890	12.763	6.080	1.910	- 1.878	7.039	2.496	1.130
11	2.656	15.237	1.804	.890	- 6.198	7.966	- .881	.656	- 8.181	4.500	.515	.533
12	22.511	21.000	.413	1.277	-18.038	10.442	- .796	1.348	-11.967	6.163	1.208	.840
13	-34.182	20.409	.431	1.055	5.113	8.695	.057	1.198	2.973	6.714	.708	.910
14	12.255	19.470	- .116	1.255	- 5.368	8.521	- .131	.512	- 7.258	5.219	- .332	.482
15	-32.136	23.930	1.682	1.669	- 7.076	10.296	-1.428	1.698	- .832	6.364	- .624	.707
16	-14.701	24.575	4.382	1.446	5.185	9.965	1.291	1.569	- 4.783	7.151	-1.611	1.099
17	- 8.080	20.085	3.304	1.351	- 9.980	10.330	-1.197	.966	- 4.690	7.610	- .813	.940
18	-19.732	24.858	.281	1.856	- 3.957	10.832	-1.717	.952	- 1.761	6.882	.234	1.063
19	-11.247	20.910	-1.963	.945	1.168	10.502	.793	1.312	- 8.875	7.561	-1.030	1.095
20	19.127	24.952	- .298	1.289	-14.330	9.187	2.303	2.577	- 7.716	6.614	2.316	1.944
21	- 2.811	20.479	-1.105	1.098	1.159	11.225	- .146	2.151	2.546	6.210	2.585	.971
22	- 3.799	21.149	1.779	.759	- 4.054	7.843	1.611	1.300	- 5.573	4.979	1.730	1.193
23	-18.876	21.867	.213	1.309	-13.191	8.607	2.269	2.017	-10.195	5.968	3.818	1.265
24	8.059	16.723	- .684	1.282	-10.185	8.531	- .210	.684	- 9.635	6.296	- .342	.727
25	-13.247	19.023	1.241	.830	- 5.287	8.626	.034	.544	- 5.292	5.582	- .209	.430
26	.877	20.830	1.108	.906	- 5.792	8.594	- .572	.615	- 1.314	6.492	2.325	.627
27	- 8.107	26.607	.542	1.597	- 1.695	9.568	- .143	.514	1.509	6.897	.033	.736
28	2.021	29.483	-1.553	2.580	-14.786	14.621	3.761	2.091	-13.088	7.479	-1.272	1.153
29	16.848	26.585	-2.410	1.848	-18.593	10.694	5.415	1.673	-20.931	8.079	1.382	1.063
30	-27.124	27.222	2.406	2.164	-13.221	12.067	.330	1.510	- .348	7.567	1.423	.736

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Unrestricted Estimates of γ_{1i} and γ_{2i} With Equation (11)

ity er	January 1961 - December 1965				January 1966 - December 1971				January 1972 - December 1978			
	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$	$\hat{\gamma}_{2i}$	$\hat{\sigma}_{2i}$	$\hat{\gamma}_{1i}$	$\hat{\sigma}_{1i}$
	-15.202	19.444	-1.704	1.122	- 1.348	10.315	- .392	.663	- 2.635	4.894	-1.788	.461
	- 7.625	16.023	.487	.667	3.369	7.139	.010	.270	- 5.026	3.390	- .081	.181
	-16.731	19.139	- .814	.867	1.248	7.955	- .072	.480	- 6.053	4.693	-1.193	.279
	-12.565	16.017	.175	.807	- 1.335	7.803	- .154	.700	- 6.679	5.957	- .478	.462
	-37.563	20.196	3.403	1.287	- 4.714	9.637	.877	.935	- 7.588	7.047	-1.448	.541
	1.932	18.175	- .541	1.160	.055	7.689	.371	.789	- 4.781	4.802	.310	.362
	3.879	20.624	.843	.606	- .567	9.292	.806	.265	- 2.290	4.891	- .326	.165
	- 2.183	19.495	- .356	.669	.961	7.084	- .098	.208	- 2.367	4.489	.237	.138
	2.025	15.257	-1.356	.830	- 7.228	9.125	.368	.438	.375	5.000	.765	.339
	- 9.236	16.245	- .213	.750	- 6.892	7.101	21.449	17.858	-12.177	9.010	- .724	.498
	-39.992	23.346	.759	1.127	- 3.561	8.005	- .532	.557	- 7.411	4.774	-1.703	.253
	7.983	22.128	-1.031	1.359	- 2.003	8.756	2.363	.583	- 6.557	4.702	.506	.320
	- 6.454	17.810	1.952	.854	- 2.370	6.819	.411	.562	-10.949	6.359	- .916	.282
	-14.682	21.641	2.558	1.290	10.578	9.964	- .651	1.056	- 4.109	7.889	- .144	.651
	- 2.431	23.339	3.653	1.978	2.195	9.046	1.292	1.189	- 5.978	7.368	-1.056	.799
	-13.567	20.018	1.789	1.581	3.479	8.960	1.400	.731	- 2.409	4.040	2.152	.380
	-22.550	20.089	- .680	1.189	4.970	10.000	- .077	.678	- 1.756	5.815	.130	.400
	-17.838	20.152	.234	.489	.907	8.636	.180	.285	- 6.269	6.907	.245	.204
	- 8.115	14.667	-1.618	.786	- 4.658	6.203	- 1.693	.806	- 7.108	4.958	- .196	.299
	-29.854	18.602	- .375	.808	.945	7.798	1.126	.426	- 3.737	5.102	.096	.309
	-13.065	17.255	-1.585	.883	- 3.470	10.120	1.651	.930	- 1.193	4.238	- .120	.392
	-11.508	21.615	-1.609	1.002	- 4.554	7.077	- .256	.448	- 1.398	3.260	-2.010	.301
	-21.070	16.349	-1.938	.802	- 2.059	7.200	- .833	.528	- 3.885	4.315	-1.177	.248
	-30.118	21.152	.090	.538	4.619	7.648	- .773	.232	- 1.602	4.683	.501	.153
	-12.169	16.581	1.233	.716	- 4.960	6.217	- .383	.432	9.243	4.665	-1.036	.277
	-58.521	19.104	.928	.431	- .065	10.280	.421	.226	- 1.359	5.524	.514	.112
	-16.472	17.099	.117	.822	- 5.297	8.620	1.105	.811	- 5.471	5.719	- .147	.304
	-28.111	16.556	-2.388	.702	1.798	11.871	- .343	1.223	.184	4.924	.579	.346
	- 4.818	20.915	1.770	1.078	- .576	11.106	- .448	.619	- 8.172	5.011	.667	[illegible]
	-44.070	15.729	2.085	.981	3.992	8.090	-1.317	.757	- 8.860	4.397	.063	.415

TABLE 15

ESTIMATE AND TESTS WITH EQUATION (12): LITZENBERGER AND RAMASWAMY DEFINITION OF DIVIDEND YIELD

Equation (12): $\tilde{R}_{it} = \alpha_{oi} + \alpha_{1i} \tilde{R}_{mt} + \alpha_{2i} d_{mt} + \alpha_{3i} r_{ft} + \alpha_{4i} d_{it} + \varepsilon_{it}$

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Restrictions

	January 1926 - December 1931		January 1935 - December 1940		January 1941 - December 1945	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	2.912	3.043	1.184	2.706	5.417	2.563
Degrees of Freedom	29,2010	29,2010	29,2010	29.2010	29,1650	29,1650
Probability Level	.0001	.0001	.229	.0001	.0001	.0001
Restricted Estimates	-1.213	-.045	12.167	.0252	1.969	-.029
Standard Errors	.539	.157	5.682	.0106	3.955	.016

	January 1946 - December 1950		January 1951 - December 1955		January 1956 - December 1960	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	2.205	1.562	1.698	2.470	3.638	3.079
Degrees of Freedom	29,1650	29,1650	29,1650	29,1650	29,1650	29,1560
Probability Level	.0003	.0291	.0001	.0001	.0001	.0001
Restricted Estimates	-1.622	-.012	-.270	-.027	3.903	.005
Standard Errors	2.190	.041	1.598	.075	.714	.108

	January 1961 - December 1965		January 1966 - December 1971		January 1972 - December 1978	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	1.680	2.093	1.026	1.667	1.040	2.329
Degrees of Freedom	29,1650	29,1650	29,2010	29,2010	29,2370	29,2370
Probability Level	.0134	.0006	.4282	.0144	.4071	.0001
Restricted Estimates	-1.494	.499	2.902	.410	1.755	.863
Standard Errors	.974	.181	1.008	.230	.750	.165

UTILITIES

Restrictions

	January 1926 - December 1931		January 1935 - December 1940		January 1941 - December 1945	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	1.388	5.536	1.201	4.006	2.472	3.2611
Degrees of Freedom	29,2010	29,2010	29,2010	29,2010	29,1650	29,1650
Probability Level	.0818	.0001	.1303	.0001	.0001	.0001
Restricted Estimates	3.630	-.295	-5.850	.165	23.384	.058
Standard Errors	1.154	.080	16.356	.065	10.604	.061

	January 1946 - December 1950		January 1951 - December 1955		January 1956 - December 1960	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	1.616	1.096	1.029	2.286	2.354	4.419
Degrees of Freedom	29,1650	29,1650	29,1650	29,1650	29,1650	29,1650
Probability Level	.0205	.3310	.0001	.0001	.0001	.0001
Restricted Estimates	-9.454	-.044	.325	-3.991	.777	.343
Standard Errors	4.899	.052	.062	2.550	2.462	.119

	January 1961 - December 1965		January 1966 - December 1971		January 1972 - December 1978	
	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$	$\alpha_{3i} = \alpha_{3j}$	$\alpha_{4i} = \alpha_{4j}$
F-Statistics	1.266	3.522	1.275	2.565	1.286	7.571
Degrees of Freedom	29,1650	29,1650	29,2010	29,2010	29,2370	29,2370
Probability Level	.1564	.0001	.1488	.0001	.1409	.0001
Restricted Estimates	-8.902	.013	9.589	-.164	.034	-.011
Standard Errors	2.302	.147	2.715	.070	1.579	.042

TABLE 15 (CONT'D)

DOW JONES 30

Unrestricted Estimates of α_{3i} and α_{4i} With Equation (12)

Security Number	January 1961 - December 1965 $\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$	January 1966 - December 1971 $\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$	January 1972 - December 1976 $\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$
1	7.285	10.[illegible]	-1.235	.994	4.244	9.412	2.811	2.490	.402	5.[illegible]46	5.178	1.70
2	[illegible]	15.[illegible]	2.951	1.685	8.508	[illegible]	5.442	4.652	2.455	[illegible]	.639	[illegible]
3	.358	12.083	.308	1.565	9.616	6.500	- .906	2.569	.735	[illegible]	.[illegible]16	[illegible]
4	- [illegible].331	8.597	.587	.828	1.062	1.653	.159	.861	1.070	3.[illegible]	.443	[illegible]
5	- [illegible].723	7.254	1.024	.802	9.418	4.819	.533	1.864	- .363	2.804	.906	[illegible]
6	[illegible].033	10.480	5.403	1.339	3.348	6.912	- .586	1.507	8.733	6.510	- .384	[illegible]
7	[illegible]5.139	21.025	- .130	2.320	-14.203	9.713	3.112	1.820	- 6.494	8.994	1.832	[illegible]
8	- [illegible]	2.447	1.096	.830	- 1.736	6.311	1.427	2.584	6.480	4.949	4.637	2.8[illegible]
9	[illegible].834	8.078	2.602	1.276	[illegible].029	5.784	-2.053	4.516	.127	4.898	13.290	5.4[illegible]
10	[illegible].001	13.366	- .567	1.246	7.941	9.864	5.941	1.778	5.133	6.160	2.166	1.14
11	[illegible].534	8.211	1.471	.824	.583	6.245	2.209	2.242	- 1.136	3.591	2.095	1.2[illegible]
12	[illegible].376	9.559	1.448	1.273	.245	7.456	-2.226	1.307	- 1.829	3.374	.439	.39
13	- [illegible].909	9.500	.902	.864	10.046	6.299	1.297	1.681	12.314	5.177	2.575	1.94
14	- [illegible].132	9.475	- .549	1.002	7.283	4.859	1.961	.847	- 1.657	4.259	- .501	.74
15	[illegible].207	9.[illegible]	3.546	1.584	3.673	7.390	6.739	6.318	6.659	4.859	-1.436	2.1[illegible]
16	- [illegible].618	11.407	4.053	1.231	10.698	7.376	3.541	3.470	3.379	6.[illegible]	-1.427	1.0[illegible]
17	[illegible].116	12.551	2.671	1.275	- .398	8.144	-1.084	.937	2.572	6.[illegible]	- .027	.94
18	[illegible].536	10.134	3.051	1.706	7.353	6.664	3.133	3.636	9.070	5.209	3.350	2.36
19	[illegible].306	10.506	-1.122	.813	8.064	7.131	8.111	5.406	2.934	6.507	-3.436	2.53
20	14.812	11.523	- .163	1.040	- 4.182	5.734	.099	8.023	4.311	5.050	14.086	5.77
21	14.798	9.814	- .859	1.026	- 3.944	8.201	-3.585	3.411	10.906	4.375	4.959	1.1[illegible]
22	-15.405	9.185	1.838	.722	4.833	6.055	1.957	1.525	1.635	3.454	1.117	1.16
23	-13.990	11.767	-1.145	1.136	.999	5.925	-6.391	6.905	- 1.845	4.649	7.203	3.4[illegible]
24	- [illegible].842	8.486	.377	1.239	- 1.518	5.978	- .193	2.037	- 2.265	5.396	.227	1.73
25	- [illegible].258	9.[illegible]	.892	.631	2.189	5.713	- .107	.582	1.771	4.200	.266	.44
26	10.588	9.410	.176	.854	7.054	5.184	- .483	.699	9.061	4.462	2.652	.63
27	7.208	9.482	2.743	1.522	6.547	6.383	.400	.674	10.998	5.767	1.039	.80
28	[illegible].117	18.413	.233	2.221	- 6.192	10.854	-1.888	3.261	- 5.772	6.733	- .779	3.35
29	19.098	14.385	-2.376	1.588	- 2.920	7.603	10.440	3.034	-12.840	7.042	2.497	1.19
30	- 1.353	13.224	2.901	1.993	1.764	8.564	.592	1.630	7.698	6.089	1.256	.806

UTILITIES

Unrestricted Estimates of α_{3i} and α_{4i} With Equation (12)

Security Number	January 1961 - December 1965 $\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$	January 1966 - December 1971 $\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$	January 1972 - December 1976 $\hat{\alpha}_{3i}$	$\hat{\sigma}_{3i}$	$\hat{\alpha}_{4i}$	$\hat{\sigma}_{4i}$
1	- 7.9[illegible]	8.634	- 5.664	3.994	6.620	7.256	.691	1.404	2.637	4.102	.229	.576
2	- 8.787	6.689	1.246	.947	9.479	4.814	.398	.765	-1.331	2.279	- .164	.471
3	- 7.134	9.382	.837	1.006	8.561	5.872	- .475	.487	-2.228	3.974	- .875	.293
4	-15.[illegible]	8.564	- .805	1.005	4.232	5.792	-1.031	.725	1.522	5.396	- .617	.459
5	-15.469	10.052	.878	1.541	1.678	7.528	1.356	1.003	-1.267	6.145	-1.176	.532
6	- 6.[illegible]	9.111	-3.506	1.377	6.201	5.815	-1.230	.838	- .168	4.266	- .492	.365
7	- 5.[illegible]	9.933	- .876	.752	5.805	6.677	.534	.279	1.563	4.535	- .443	.166
8	- .185	10.171	-1.192	.617	7.449	4.791	- .520	.235	2.280	3.959	- .114	.143
9	- 6.50[illegible]	7.162	.358	.975	2.451	6.469	- .056	.486	5.802	3.931	.937	.347
10	-12.036	7.470	-1.051	.735	1.240	5.218	5.727	13.785	-4.950	8.268	- .125	.634
11	-22.790	10.003	16.985	3.058	.711	5.990	- .478	.897	-3.014	4.290	-1.233	.427
12	.019	10.335	- .875	1.356	2.348	6.339	2.613	.632	- .836	3.997	[illegible]	.320
13	- 7.900	8.797	2.217	.940	3.843	5.085	.568	.595	-4.729	5.6[illegible]	-1.[illegible]6	.27[illegible]
14	- 7.94[illegible]	9.518	1.811	1.344	16.458	7.709	- .022	1.282	.856	6.876	-2.571	.999
15	9.233	10.525	.575	3.585	10.946	6.610	5.360	3.452	- .434	6.521	-2.795	2.14[illegible]
16	- 8.305	10.345	2.368	1.950	8.800	7.018	1.483	.837	1.228	3.571	2.226	.435
17	- 8.537	10.047	-3.372	1.489	13.197	7.479	- .709	.790	4.712	5.066	- .665	.469
18	-14.979	8.990	1.400	.529	9.142	6.404	- .025	.315	1.868	5.698	.050	.216
19	- 7.947	7.024	- .952	.632	- .623	4.764	-1.309	.736	-2.573	4.402	.055	.407
20	-13.809	8.485	-2.856	2.641	4.560	6.069	1.358	1.216	1.338	4.474	.439	.342
21	- 9.060	7.457	- .525	.963	4.378	7.395	.889	1.033	2.241	3.757	.332	.400
22	-12.675	9.436	- .639	1.050	.939	5.546	- .377	.460	1.055	2.902	-1.721	.[illegible]
23	-13.227	7.560	.424	.903	3.945	5.155	-1.156	.573	.803	3.515	-1.1[illegible]	.[illegible]
24	-25.7[illegible]	10.921	.644	.591	9.691	5.786	- .648	.269	2.339	4.195	[illegible]	.1[illegible]
25	-13.1[illegible]	7.591	4.854	1.306	.246	4.415	-1.098	.662	-5.222	[illegible]	-1.[illegible]	.487
26	-27.981	9.671	.305	.509	6.985	8.160	.313	.242	3.730	4.742	[illegible]	.129
27	-18.179	7.542	3.122	2.364	1.442	6.350	- .627	1.369	1.218	[illegible]	[illegible]	.414
28	-14.018	7.950	-2.144	.812	11.193	8.224	- .250	1.236	4.951	[illegible]	.6[illegible]	.363
29	- 3.356	8.686	.476	1.002	8.722	7.950	.314	.607	-2.851	[illegible]	.433	.267
30	-20.528	8.815	4.173	1.664	9.747	5.866	.45	.955	-4.369	[illegible]	.586	.781

Using Litzenberger and Ramaswamy's definition of anticipated dividend yields, equations (11) and (12) are estimated with two groups of thirty securities: the Dow Jones Industrials sample and a sample of utilities and railroads. The utility-railroad sample provides a check against peculiarities of the Dow Jones Industrials, and this sample can be presumed to have been identified by investors as high-yield stocks. The test with these securities are conducted for the same periods as the previous tests are reported in Tables 14 and 15. These results indicate that the earlier conclusions regarding the dividend yield coefficient are basically unaffected by the definition of anticipated dividend yields. The F-statistics and restricted estimates change somewhat, but the changes, on balance, are not large enough to alter our conclusions. Further, the utility-railroad sample exhibits the same pattern as our earlier samples of high-yield stocks, and the variation of the unrestricted estimates for the Dow Jones sample is similar to those earlier under the perfect foresight definition. In sum, the failure to find a common tax effect cannot plausibly be attributed to an obscuring of the tax effect by announcement effects impounded in the perfect foresight definition.

Dividend Yields and Shifts in Expected Returns

Tables 2 through 15 reveal a complex relation between expected returns and dividend yields. The unrestricted estimates highlight this complexity. In many respects, our results are unsettling since we have rejected the two simplest models of the impact of dividends: neutrality and common tax effects. In fact, the data tell us both of these are bad approximations. But not why.

One possible explanation is that dividend yields are proxying for other factors that affect returns. Imagine, for example, a world where dividends are "steady" in the sense of not being adjusted immediately by firms to maintain constant dividend yields. A decrease in the price of a firm's shares, on average, will thus imply a higher dividend yield and an increase in price will, on average, imply a lower dividend yield. For leveraged firms, changes in stock prices may also result in changes in the riskiness of the firm's stock. Unless firms make compensating adjustments in leverage, a decrease in the price of common stock implies a higher leverage ratio and an increase a lower leverage ratio. As a result, both dividend yields and expected returns may be increasing or decreasing simultaneously. Stated differently, equations have been omitted, namely, the ones determining expected returns, and dividend yields are proxying for the omitted equations.

To see the possible impact of this relation, consider equation (11). The intercept of equation (11) may be expressed as,

$$\tilde{\gamma}_{oit} = \bar{\gamma}_{oi} + \tilde{\nu}_{it}$$

where $\tilde{\upsilon}_{it}$ is the difference between the expected return of security i in period t and its average during the sample period (ignoring taxes). Substituting this expression into eqution (11),

$$\tilde{R}_{it} = \bar{\gamma}_{oi} + \gamma_{1i} d_{it} + \gamma_{2i} r_{ft} + \tilde{\varepsilon}_{it} + \tilde{\nu}_{it} \qquad (11c)$$

The disturbance term in equation (11) consists of the true disturbance plus the deviation of the expected return in period t from its average during the sample period.

Our previous discussion suggests that, on average, $\tilde{\upsilon}_{it}$ and d_{it} are both negatively related to stock prices and, therefore, positively related to each other. Further, the relation is not easily modeled. The best that we can do is present evidence that establishes the plausibility of the connection. Nevertheless, this evidence had important implications for intercepting tests of the relation between dividend yields and expected returns (and perhaps also other anomalies of asset pricing).

To test for this relationsip, we generalize equation (11) to,

$$\tilde{R}_{it} = \gamma_{oi} + \gamma_{1i} r_{ft} + \gamma_{2i} d_{it} + \gamma_{3i} d_{it-1} + \gamma_{4i} d_{it-2} + \tilde{\varepsilon}_{it} \qquad (11d)$$

where d_{it} is the dividend yield in month t. The Litzenberger and Ramaswamy definition of anticipated dividend yields is used in estimating (11d). In effect, (11d) includes the dividend yield terms of (11) in both the cum and ex months. Since the cum month dividend yield is taken from the previous ex month, there can be no information effects in the coefficient γ_{3i} and a fortiori in the two-month lag coefficient γ_{4i}. Clearly, there are no tax effects of the kind hypothesized by Litzenberger and Ramaswamy in these coefficients since the dividend has already been paid. If the ex month is included, the cum month effects are purely statistical artifacts from their point of view. The tests conducted with (11d) are tests of no effect of current or lagged dividend yields. If both lagged and current dividend yields turn out to be non-zero, there is reason to believe that dividend yields are proxying for shifts in expected returns. The results of these tests for the Dow Jones Industrial sample are reported in Table 16.

TABLE 16

TEST OF CUM AND EX MONTH EFFECTS

Equation (11d): $\tilde{R}_{it} = \gamma_{0i} + \gamma_{1i} r_{ft} + \gamma_{2i} d_{it} + \gamma_{3i} d_{it-1} + \gamma_{4i} d_{it-2} + \tilde{\varepsilon}_{it}$

Restrictions

	January 1926 - December 1931			January 1935 - December 1940			January 1941 - December 1945		
	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$
F-Statistics	3.948	2.920	3.316	3.313	4.184	4.404	1.953	1.655	2.326
Degrees of Freedom	30,1980	30,1980	30,1980	30,1980	29,1980	29,1980	30,1620	29,1620	29,1620
Probability Level	.0001	.0001	.0001	.0001	.0001	.0001	.0016	.0034	.0001

	January 1946 - December 1950			January 1951 - December 1955			January 1956 - December 1960		
	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$
F-Statistics	2.522	3.686	1.584	4.304	2.762	3.558	2.367	2.334	2.501
Degrees of Freedom	30,1620	30,1620	30,1620	30,1620	30,1620	30,1620	30,1620	30,1620	30,1620
Probability Level	.0001	.0001	.0236	.0001	.0001	.0001	.0001	.0001	.0001

	January 1961 - December 1965			January 1966 - December 1971			January 1972 - December 1978		
	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$	$\gamma_{2i} = 0$	$\gamma_{3i} = 0$	$\gamma_{4i} = 0$
F-Statistics	1.496	3.209	1.986	2.740	2.828	3.149	5.392	4.680	4.994
Degrees of Freedom	30,1620	30,1620	30,1620	30,1980	30,1980	30,1980	30,2340	30,2340	30,2340
Probability Level	.0415	.0001	.0012	.0001	.0001	.0001	.0001	.0001	.0001

As can be seen both the lagged and current dividend yields appear to be non-zero in all time periods. It would be difficult, on the basis of these results, to conclude either that the current values dominate the lagged value or vice-versa. The current and lagged values both contribute to the explanation of expected returns, thus warning again of the perils of attaching too much economic significance to the observed relations between expected returns and dividend yields.[19]

Conclusions

The purpose of this paper has been to test certain hypotheses on the relation between dividend yields and expected stock returns, using a methodology that is more powerful in a number of respects that those that have so far been used in that context. The method employs systems of time series regressions with the competing hypotheses taking the form of cross equation restrictions.

The major finding of this paper is that the relation between dividend yields and stock returns is not constant across securities. In almost all cases examined, the hypothesis that dividends have a common effect on expected returns can be decisively rejected. This conclusion is inconsistent with the tax effect models of Litzenberger and Ramaswamy and Brennan or the similar model of Blume. But the results are also inconsistent with the hypothesis that dividends have no effect on expected returns. Stated differently, the tests show a statistically significant relation between yields and returns, but one that is not well described either by dividend neutrality or by tax effect models. The unresolved question is what does explain the results? The observed structure of dividend payments suggests that dividend yields may be proxying for changes in the expected return of securities over time. This possibility was tested by including the lagged dividend yield in cum months along with ex month dividend yield. The cum month dividend yields have no impact on investors' tax liabilities and have no informaitonal content since they are lagged values. Nevertheless, the cum month dividend yields appear to be as important statistically as the ex month dividend yields. The cum month results emphasize the need for greater caution in interpreting the dividend tests here and more generally. They should especially discourage attempts to appeal to existing empirical tests for justifying either the dividend policies of particular firms or the security selection policies of portfolio managers.

APPENDIX A

ERRORS IN VARIABLES

ERRORS IN VARIABLES: A GENERAL DISCUSSION

Errors in estimation of beta are critical for two of the estimation techniques used by Litzenberger and Ramaswamy. To analyze the effect of errors in beta estimates on the estimate of c, consider the general model,

$$\underline{y} = \tilde{X}\,\underline{\phi} + \underline{u} \tag{A1}$$

with observational errors in the independent variables,

$$X = \tilde{X} + V$$

The model in the observed variables is,

$$\underline{y} = X\,\underline{\phi} + \underline{u} - V\,\underline{\phi}$$

and the least squares estimator is,

$$\hat{\underline{\phi}} = (X'X)^{-1}\,X'\,(X\underline{\phi} + \underline{u} - V\underline{\phi})$$

$$= \underline{\phi} + (X'X)^{-1}\,X'\,(\underline{u} - V\underline{\phi})$$

Assume that the disturbances in equation (A1) are independent of the independent variables in a probability limit sense, i.e.,

$$\underset{N\to\infty}{\text{plim}}\ \frac{X'\underline{u}}{N} = 0$$

Further, assume that

$$\operatorname*{plim}_{N\to\infty} \frac{\tilde{X}'V}{N} = 0$$

$$\operatorname*{plim}_{N\to\infty} \frac{\tilde{X}'\tilde{X}}{N} = \tilde{M}$$

$$\operatorname*{plim}_{N\to\infty} \frac{X'X}{N} = M$$

where $\tilde{M}$ and M are finite positive definite matrices and,

$$\operatorname*{plim}_{N\to\infty} \frac{V'V}{N} = \Sigma$$

is finite. With these assumptions the probability limit of the least squares estimator is,

$$\operatorname*{plim}_{N\to\infty} \hat{\underline{\phi}} = \underline{\phi} + \operatorname*{plim}_{N\to\infty} \left(\frac{X'X}{N}\right)^{-1} \operatorname*{plim}_{N\to\infty} \frac{X'\underline{u}}{N} - \operatorname*{plim}_{N\to\infty} \left(\frac{X'X}{N}\right)^{-1}$$

$$\operatorname*{plim}_{N\to\infty} \left(\frac{\tilde{X}'V}{N} + \frac{V'V}{N}\right) \underline{\phi} = \underline{\phi} - M^{-1}\,\Sigma\underline{\phi} \qquad \text{(A2)}$$

since the inverse of $\frac{X'X}{N}$ is a continuous function which does not depend on N.

For sake of discussion, possible errors in d_{it} and r_{ft} are ignored. Equation (A2) may be used to evaluate the effect of errors in beta on the estimates of c. First, note that for Litzenberger and Ramaswamy X, V, X'X and V'V correspond to,

$$X = \begin{vmatrix} 1 & \hat{\beta}_1 & d_1 - r_f \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & \hat{\beta}_N & d_n - r_f \end{vmatrix} \qquad V = \begin{vmatrix} 0 & v_{\hat{\beta}_1} & 0 \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & v_{\hat{\beta}_N} & 0 \end{vmatrix}$$

$$X'X = \begin{vmatrix} N & \sum_{i=1}^{N} \hat{\beta}_i & \sum_{i=1}^{N} (d_i - r_f) \\ \sum_{i=1}^{N} \hat{\beta}_i & \sum_{i=1}^{N} \hat{\beta}^2 & \sum_{i=1}^{N} \hat{\beta}_i (d_i - r_f) \\ \sum_{i=1}^{N} (d_i - r_f) & \sum_{i=1}^{N} \hat{\beta}(d_i - r_f) & \sum_{i=1}^{N} (d_i - r_f)^2 \end{vmatrix}$$

$$V'V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sum_{i=1}^{N} v_{\beta_i}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\hat{\beta}_i$ is the estimated beta of security i and $v_{\hat{\beta}_i}$ is the estimation error of $\hat{\beta}_i$. Suppose that,

$$\operatorname*{plim}_{N \to \infty} \sum^{N} v^2_{\beta i}/N = \Theta$$

then,

$$\Sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Sigma \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ \Theta b \\ 0 \end{bmatrix} \qquad \text{(A3)}$$

From equation (A3), it follows that the degree of inconsistency of Litzenberger and Ramaswamy's estimators will depend only upon the second column of M^{-1}. For $\hat{c}$, it will only depend upon the last element of the second column. To evaluate the effect of $\hat{c}$, note that M equals,

$$M = \begin{bmatrix} 1 & M_1\ (\underline{\beta}) & M_1\ (\underline{d}-\underline{\ }r_f) \\ M_1\ (\underline{\beta}) & M_2\ (\underline{\beta}) & M_1\ (\underline{\beta d}-\underline{1}r_f) \\ M_1\ (\underline{d}-\underline{1}r_f) & M_1\ (\underline{\beta d}-\underline{1}r_f) & M_2\ (\underline{d}-\underline{1}r_f) \end{bmatrix}$$

where $M_i(\underline{\beta})$ is the ith moment of the estimated beta around zero, $M_1(\underline{d}-\underline{1}r_f)$ is the ith moment of the dividend yields less the risk free rate around zero and $M_i(\underline{\beta d}-\underline{1}r_f)$ is ith cross moment around zero. The (3,2) element of the inverse of M is,

$$m^{32} = -\ |M|^{-1}\ [M_1(\underline{\beta}d-\underline{1}r_f) - M_1(\underline{\beta})\ M_1(\underline{d}-\underline{1}r_d)]$$

$$= -\ |M|^{-1}\ M_1(\underline{\beta}-\bar{\beta}\ \underline{1}\ \underline{d}-\bar{d}\underline{1})$$

where $M_1(\underline{\beta}-\bar{\beta}\underline{\iota}d-\bar{d}\underline{\iota})$ is the first cross moment around the respective first moments of betas and dividend yields. From equation (A2) and (A3) it follows that,

$$\underset{N\to\infty}{\text{plim}}\ \hat{c} = c-[-|M|^{-1} M_1(\underline{\beta}-\bar{\beta}\underline{\iota}\underline{d}-\bar{d}\underline{\iota})]\Theta b \qquad \text{(A4)}$$

$$= c+ \quad |M|^{-1} M_1(\underline{\beta}-\bar{\beta}\underline{\iota}\underline{d}-\bar{d}\underline{\iota}) \quad \Theta b$$

By assumption, M is positive definite so that $|M| > 0$. The parameter b is a market after tax risk premium. If $b>0$, it follows that

$$\underset{N\to\infty}{\text{plim}}\ \hat{c}-c \begin{cases} > 0 \text{ if } M_1(\underline{\beta}-\bar{\beta}\underline{\iota}\underline{d}-\bar{d}\underline{\iota}) > 0 \\ < 0 \text{ if } M_1(\underline{\beta}-\bar{\beta}\underline{\iota}\underline{d}-\bar{d}\underline{\iota}) < 0 \\ = 0 \text{ if } M_1(\underline{\beta}-\bar{\beta}\underline{\iota}\underline{d}-\bar{d}\underline{\iota}) = 0 \end{cases}$$

It is easy to imagine circumstances where the cross moment around the mean is not equal to zero. If the cross moment is positive, the Litzenberger and Ramaswamy estimate will be too large in a probability limit sense. If dividend yields are negatively related to beta, the probability limit of the estimate will be less than c. While the magnitude of this effect is unknown for the Litzenberger and Ramaswamy sample, a serious suspicion is cast on the results obtained with the 'OLS' and 'GLS' estimation techniques.

ERRORS IN VARIABLES: LITZENBERGER AND RAMASWAMY'S ADJUSTMENT

Litzenberger and Ramaswamy propose an adjusted estimator. To analyze this estimator, consider again the model shown in equation (A7) with errors in variables. The least squares estimator of this model may be restated as,

$$\hat{\underline{\phi}} = (\tilde{X}'\tilde{X} + V'\tilde{X} + \tilde{X}'V + V'V)^{-1} (\tilde{X}'\tilde{X}\underline{\phi} + \tilde{X}'\underline{u} + V'\tilde{X}\underline{\phi} + V'\underline{u})$$

Taking probability limits and utilizing the previous assumptions,

$$\underset{N\to\infty}{\text{plim}}\ \hat{\underline{\phi}} = \left(\underset{N\to\infty}{\text{plim}} \frac{\tilde{X}'\tilde{X}}{N} + \underset{N\to\infty}{\text{plim}} \frac{V'\tilde{X}}{N} + \underset{N\to\infty}{\text{plim}} \frac{\tilde{X}'V}{N} + \underset{N\to\infty}{\text{plim}} \frac{V'V}{N}\right)^{-1}$$

$$\left(\underset{N\to\infty}{\text{plim}} \frac{\tilde{X}'\tilde{X}}{N}\underline{\phi} + \underset{N\to\infty}{\text{plim}} \frac{\tilde{X}'\underline{u}}{N} + \underset{N\to\infty}{\text{plim}} \frac{V'\tilde{X}}{N}\underline{\phi} + \underset{N\to\infty}{\text{plim}} \frac{V'\underline{u}}{N}\right)$$

$$= (\tilde{M} + \Sigma)^{-1} \tilde{M}\underline{\phi} \qquad \text{(A5)}$$

If Σ were known, it would be possible to adjust X'X by subtracting Σ.

Litzenberger and Ramaswamy propose an estimator in the form of (A5). Their GLS estimator is calculated by standardizing the observations by the estimated standard errors of the beta estimates. Because of this, they state that,

$$\Sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and their adjusted estimator takes the form,

$$\underline{\phi}^{*} = \left(\frac{X'X}{N} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)^{-1} \frac{X'\underline{y}}{N}$$

Recall the Litzenberger and Ramaswamy claim that this estimator is both consistent and the maximum likelihood estimator if the joint distribution of security returns is normal.

An Examination of Litzenberger and Ramaswamy's Claim of Consistency

It is obvious that the properties of this adjusted estimator depond upon Σ taking on the value assumed by the authors. If $s_{\hat{\beta}_i}$ is the estimated standard error of $\hat{\beta}_i$, it is necessary that

$$\operatorname*{plim}_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \frac{v^2_{\hat{\beta}_i}}{s^2_{\hat{\beta}i}} = 1$$

for the claims of Litzenberger and Ramaswamy to hold. Suppose that $s_{\hat{\beta}_i} = \sigma_{\hat{\beta}_i}$ for all i and each $v_{\hat{\beta}_i}$ is normal with a mean of zero. Each term in the above sum then would be chi-squared with one degree of freedom. Further, if each of the estimation errors are independent of all others, 1/N times the sum will converge to 1 as N goes to infinity.[20] Thus, Litzenberger and Ramaswamy's claim that

$$\Sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

requires two assumptions: (1) the true standard errors of all the beta estimates must be known, and (2) each of the estimation errors must be independent of all other.

These are by no means trivial assumptions, although the authors fail to mention either of them. Σ is simply written down without any discussion of the required assumptions. Neither of these assumptions has merit. Conceptually, there is no more reason to condition on $s_{\hat{\beta}_i} = \sigma_{\hat{\beta}_i}$ for all i than there is to assume $\hat{\beta}_i = \beta_i$ for all i. The assumption of independent estimation errors, moreover, implies the disturbances of the market model to independent across securities, which is inconsistent with "industry" effects that have been documented.[21] In summary, the assumptions required for Litzenberger and Ramaswamy's adjusted estimator to be consistent are arbitrary and empirically unattractive.

The Maximum Likelihood Claim

Litzenberger and Ramaswamy also claim that their adjusted estimator is maximum likelihood and, indeed, there are conditions where maximum likelihood estimators exist with errors in variables. The most important of these conditions is prior knowledge of some of the parameters. If the distribution of the variables are normal, it is well known that not all the parameters are identifiable.[22] If Σ is known, the other parameters can be identified and the estimator will be maximum likelihood if other conditions hold. In particular, each row of $\tilde{X}$ must be normally, independently, and identically distributed. If $\tilde{X}$ is assumed to be nonstochastic, then each row of the observed variable will have a different mean vector (the value of each row of $\tilde{X}$). But the $\tilde{X}$ matrix is unobservable and there will be T unknown mean vectors instead of one. In short, if $\tilde{X}$ is assumed to be nonstochastic, there will be T-1 more parameters to identify per independent variable. All this must be accomplished with only T observation per independent variable.

In addition, it is required that each row of V and u_i be normally, independently, and identically distributed with a mean vector of zero. Finally, each u_i and each row of $\tilde{X}$ and V must be mutually independent.

The assumptions made by Litzenberger and Ramaswamy are not jointly consistent with these conditions. For purposes of estimating betas, Litzenberger and Ramaswamy assume that the market model holds,

$$\tilde{R}_{it} - r_{ft} = \alpha_i + \beta_i (\tilde{R}_{mt} - r_{ft}) + \tilde{\varepsilon}_{it} \qquad \text{(A6)}$$

In the stochastic version of their model, equation (7), the distribution is defined as,

$$\tilde{u}_{it} = \tilde{R}_{it} - E(\tilde{R}_{it}) \qquad \text{(A7)}$$

substituting equation (A6) into (A7),

$$\tilde{u}_{it} = \beta_i [\tilde{R}_{mt} - E(\tilde{R}_{mt})] + \tilde{\varepsilon}_{it}$$

If β_i, R_{mt}, and $\tilde{\varepsilon}_{it}$ are all normally distributed, $\tilde{u}$ will be the product of two normally distributed random variables, plus a normally distributed random variable. While sums of normals are normal, products of normals are not normal. If $\tilde{u}_{it}$ is normal, then β_i, R_{mt}, and $\tilde{\varepsilon}_{it}$ can not all be normally distributed. If either β_i or $\tilde{\varepsilon}_{it}$ are not normal, the estimator is not maximum likelihood. However, $\tilde{R}_{mt}$ is simply a linear combination of the $\tilde{R}_{it}$ and the normality of the $\tilde{u}_{it}$ implies the normality of $\tilde{R}_{mt}$. Clearly, all these conditions can not be met simultaneously. It follows, then, that Litzenberger and Ramaswamy's assumptions are not consistent with those required for their adjustment estimator to be maximum likelihood.

Besides arbitrarily conditioning on Σ and the inconsistency of Litzenberger and Ramaswamy's assumptions with those required for a maximum likelihood estimator, the authors ignore the fact that betas and standard errors of betas are functions of parameters in the joint distribution of security returns. It simply does not make sense to call estimators of parameters maximum likelihood when all the parameters are not being simultaneously estimated. This point may be illustrated by noting that Litzenberger and Ramaswamy's model implies that unconditional means of security returns are nonstationary due to the tax effects of dividends, $c(d_{it} - r_{ft})$. The beta of security i equals,

$$\beta_i = \frac{E[\tilde{R}_i - E(\tilde{R}_i)]\,[\tilde{R}_m - E(\tilde{R}_m)]}{E[\tilde{R}_m - E(\tilde{R}_m)]^2}$$

In estimating betas, it is necessary to adjust $E(\tilde{R}_{it})$ for the tax effects in period t, but this adjustment requires an estimate of c.[23]

In summary, Litzenberger and Ramaswamy's claims regarding their adjusted estimator are wrong. It has been shown that both the consistency and maximum likelihood properties of this estimator depend upon an arbitrary conditioning argument that the authors fail even to mention. Further, it was shown that the assumptions made by Litzenberger and Ramaswamy are not consistent with those required for their adjusted estimator to be maximum likelihood. Finally, Litzenberger and Ramaswamy ignore the fact that the betas and standard errors of beta are parameters in the joint distribution of security returns. In short, there is no reason to believe that their adjusted estimator is any better than the OLS and GLS estimators they report.

APPENDIX B

BLUME'S RANDOM COEFFICIENTS

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Blume's random coefficients may be modeled,

$$a_t = \bar{a} + \tilde{u}_{at}$$

$$b_t = \bar{b} + \tilde{u}_{bt}$$

$$c_t = \bar{c} + \tilde{u}_{ct}$$

Substituting these expressions into Blume's model,

$$\tilde{R}_{it} = \bar{a} + \bar{b}\ \beta_{it} + \bar{c} d_{it} + \tilde{u}_{it} + \tilde{u}_{at} + \tilde{u}_{bt}\ \beta_{it} + \tilde{u}_{ct}\ d_{it}$$

If betas are presumed to be constant for $t = \tau_1, \tau_1 + 1 \ldots \tau_2$, then

$$\tilde{R}_{it} = \gamma_{i0} + \gamma_1\ d_{it} + \tilde{\nu}_{it} \qquad \text{(B1)}$$

$$\gamma_{i0} = \bar{a} + \bar{b}\ \beta_i$$

$$\tilde{\nu}_{it} = \tilde{u}_{it} + \tilde{u}_{at} + \beta_i\ \tilde{u}_{bt} + d_{it}\ \tilde{u}_{ct}$$

$$\gamma_1 = \bar{c}$$

for i=1, 2, ... N and $t = \tau_1, \tau_1 + 1 \ldots \tau_2 - 1, \tau_2$.

If the disturbances of (B1) are independent and identifically distributed over time, standard techniques may be used to estimate (B1). Independence of the disturbance vector over time is assumed. From the definitions of the terms in the disturbances, it follows that,

$$E\,\tilde{\nu}_{it} = 0$$

$$E\,\tilde{\nu}^2_{it} = \sigma^2_i + 2(\sigma_{ia} + \beta_i\,\sigma_{ib} + d_{it}\,\sigma_{ic}) + \sigma^2_a + 2(\beta_i\,\sigma_{ab} + d_{it}\,\sigma_{ac})$$

$$+ \beta^2_i\,\sigma^2_b + 2\,\beta_i\,d_{it}\,\sigma_{bc} + d^2_{it}\,\sigma^2_c$$

for $i = 1, 2 \ldots N$ and $t = \tau_1, \tau_1 + 1 \ldots \tau_2$. To simplify, assume that $\sigma_{ia} = \sigma_{ib} = \sigma_{ic} = \sigma_{ab} = \sigma_{ac} = \sigma_{bc} = 0$. The variance of $\tilde{\nu}_{it}$ then reduces to,

$$E\,\tilde{\nu}^2_{it} = \sigma^2_i + \sigma^2_a + \beta^2_i\,\sigma^2_b + d^2_{it}\,\sigma^2_c \tag{B2}$$

From (B2) it is clear that the disturbances of (B1) are not homoscedastic. Define the part of the variance of the disturbances that is constant as,

$$\bar{\sigma}^2_i \equiv \sigma^2_i + \sigma^2_a + \beta^2_i\,\sigma^2_b$$

Equation (B2) may be written as,

$$E\,\tilde{\nu}_{it} = \bar{\sigma}^2_i + d^2_{it}\,\sigma^2_c$$

$$= \bar{\sigma}^2_i\,(1 + \lambda_i\,d^2_{it}) \tag{B3}$$

where $\lambda_i = \bar{\sigma}^2_c/\sigma^2_i$. The changes in the variance of $\tilde{u}_i$ over time depends upon $\lambda_i\,d^2_{it}$. Since λ_i is the ratio of one variance to three variances, it is probably reasonable to approximate λ_i with a value of about .33. An average quarterly dividend yield is approximately .01; for a high yielding stock the value would be .02. Thus, in an ex month the variance of the disturbance would be about $\bar{\sigma}^2_i$ (1.000033) for an average firm, and $\bar{\sigma}^2_i$ (1.000132) for a high yielding firm. In a non ex month, the variance would, of course, be $\bar{\sigma}_i^2$. Differences of this size can probably be safely ignored for purposes of estimation.

Turning attention to the stationarity of the covariances across equations, note that,

$$
\begin{aligned}
E\,\tilde{v}_{it}\,\tilde{v}_{jt} = {} & E\tilde{u}_{it}\tilde{u}_{jt} + E\tilde{u}_{it}\tilde{u}_{at} + \beta_j\,E\tilde{u}_{it}\tilde{u}_{bt} + d_{jt}\,E\tilde{u}_{it}\tilde{u}_{ct} + \\
& E\tilde{u}_{at}\tilde{u}_{jt} + E\tilde{u}_{at}^2 + \beta_j\,E\tilde{u}_{at}\tilde{u}_{bt} + d_{jt}\,E\tilde{u}_{at}\tilde{u}_{ct} + \\
& \beta_i\,E\tilde{u}_{bt}\tilde{u}_{jt} + \beta_i\,E\tilde{u}_{bt}\tilde{u}_{at} + \beta_i\,\beta_j\,E\tilde{u}_{bt} + \beta_i\,d_{it} \\
& E\tilde{u}_{bt}\tilde{u}_{ct} + d_{it}\,E\tilde{u}_{ct}\tilde{u}_{jt} + d_{it}\,E\tilde{u}_{ct}\tilde{u}_{at} + d_{it}\,\beta_j \\
& E\tilde{u}_{ct}\tilde{u}_{bt} + d_{it}\,d_{jt}\,E\tilde{u}_{ct} \qquad\qquad (B4) \\
= {} & \sigma_{ij} + \sigma_a^2 + \beta_i\,\beta_j\,\sigma_b^2 + d_{it}\,d_{jt}\,\sigma_c^2
\end{aligned}
$$

where use is made of the assumption of the mutual independence of $\tilde{u}_{at}$, $\tilde{u}_{bt}$, $\tilde{u}_{ct}$, and $\tilde{u}_{it}$ for all i and t. The covariance of security i and j varies over time due to the $d_{it}\ d_{jt}$ term in (B4). This will be approximately the same magnitude as the nonstationary of the variance and, therefore, ignored for purposes of estimation.

FOOTNOTES

1. It is possible that a relation more complicated than the ones tested here could exist. See, for example, Constantinides, G. [1979].

2. Actually c is a weighted average of investors' marginal tax rates where the weights equal the global risk tolerance divided by the sum of the global risk tolerances across all investors, see Litzenberger, R., and Ramaswamy, K., [1979, 171-172].

3. Ibid., p. 2.

4. Ibid., p. 15.

5. Only dividends paid in the last twelve months are used to calculate the unannounced but recurring dividends. See Litzenberger, R., and Ramaswamy, K., [1979, 182].

6. Litzenberger, R., and Ramaswamy, K., [1979, Table 1].

7. The reliability of this calculation is questionable. First, Black and Scholes measure dividend yield for any year as dividends paid in the previous year, divided by the closing price of the previous year. Tax liabilities will depend upon dividends actually paid and not the previous years dividends. A possible differential tax liability will not be proportional to the Black and Scholes measure of dividend yield. Second, Black and Scholes estimate γ_1, using both ex and non-ex months. Since tax effects only occur in ex months, the γ_1 estimates mixes tax effects with other dividend related effects. Finally, Black and Scholes use an equally weighted index and not a value index. In short, the relation between γ_1 estimate and the presumed dividend tax bracket of the marginal investor is by no means clear. The most that can be said is that a positive estimate γ_1 in a Black and Scholes test is consistent with differential taxation of dividends.

8. Blume apparently expected a negative coefficient, a preference by investors for dividends, see Blume, M., [1978, 1].

9. See, for example, Stambaugh, R., [1979].

10. Gordon, R. and Bradford, D. (1979) estimate a model similar to (12). Unfortunately, the authors never test any of the restrictions implied by their model.

11. See Hess, p. [1980], pp. 23-25.

12. Schmidt, P., [1976, 85].

13. The historical statutory rates are taken from Statistics of Income, Individual Income Tax Returns, U.S. Department of Treasury, various issues between 1945-1969.

14. Theil, H., [1971, 313].

15. Ibid., p. 402.

16. Recently, attention has focused on the finite sampling properties of this test statistic. Meisner has reported simulations that would suggest the test statistic is heavily weighted toward rejecting the restriction when the degrees of freedom are small per equation. As expected, as the degrees of freedom grows per equation (23 per equation were examined by Meisner) the bias is substantially reduced. Since the degrees of freedom are quite large here, the large sample aproximation will be assumed to be appropriate. See Meisner, J., [1979] and Laitinen, K., [1978].

17. The F-distribution is more conservative with respect to rejecting the restriction and that approximation is adopted here. See Theil, H., [1971, 402-03].

18. Since none of our hypotheses involve dummy coefficients, these estimates are not recorded.

19. This sample was also used to test for common cum month effects and common differences between cum and ex month effects. In general, these restrictions were rejected at high probability levels.

20. This sum is distributed as chi-squared with N degrees of freedom. 1/N times the sum has a mean of 1 and a variance of 2/N. See Theil, H., [1971, 402].

21. See, for example, King, B. [1966] and Meyers, S., [1973].

22. See Schmidt, P., [1976, 105-112].

23. I am indebted to E. Han Kim for bringing this particular example to my attention.

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