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FIXED PRICE VERSUS SPOT PRICE CONTRACTS: A STUDY IN RISK ALLOCATION

A. Mitchell Polinsky

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ABSTRACT

This paper is concerned with the risk-allocation effects of alternative types of contracts used to set the price of a good to be delivered in the future. Under a fixed price contract, the price is specified in advance. Under a spot price contract, the price is the price prevailing in the spot market at the time of delivery. These contract forms are examined in the context of a market in which sellers have uncertain production costs and buyers have uncertain valuations. The paper derives and interprets a general condition determining which contract form would be preferred when the seller and/or the buyer is risk averse. In addition, an example is provided in which a spot price contract with a floor price is superior both to a "pure" spot price contract and a fixed price contract.

A. Mitchell Polinsky
Stanford Law School
Stanford University
Stanford, CA 94305

1. Introduction

When a seller and a buyer enter into a contract for the future delivery of some good, they can set the price to be paid in several ways. For example, they can specify the price in advance, which will be called a fixed price contract. Or they can agree to the price prevailing in the spot market for the good on the date of delivery, which will be referred to as a spot price contract.[1/] This paper is concerned with the effects of these two contract forms on the allocation of risk between the parties when at least one of them is risk averse.[2/] (A hybrid form -- a spot price contract with a floor price -- also will be discussed.)

Variations of these contract forms are widely used in practice. For example, in a survey of members of the National Association of Purchasing Management, it was found that 90% used fixed price contracts, 65% used "price at delivery" contracts (what I am calling spot price contracts), 50% used renegotiated price contracts (price adjustments only in unusual circumstances), 39% used "escalator clause" contracts (price affected by increases or decreases in the costs of specific inputs), and 20% used "cost plus" contracts.[3/] Contracting practices in specific industries also illustrate the variety of contract forms used. For example, contracts for the sale of nuclear reactors have, at different times, been of the "turn-key" (fixed price) form and of the cost-plus form.[4/] Similarly, contracts for the sale of natural gas, petroleum coke, and coal have included fixed price contracts and several different kinds of contracts with variable prices, including some with price floors and ceilings.[5/]

To begin to see how fixed price and spot price contracts allocate risk, consider a simple example (the details of which are discussed in section 4 below). Suppose there are a large number of sellers with identical, but uncertain, production costs in an industry with a flat supply curve. Because of the firms' cost uncertainty, the supply curve also is uncertain. And because the supply curve is flat, the equilibrium price in the spot market equals the realized value of the firms' costs (regardless of the demand curve). Suppose further that there are many buyers (also firms) whose valuations are not uncertain.

In this example, a spot price contract would insure a seller against risk for the following reason. If that seller's costs are high, so are all other sellers' costs, and so also is the supply curve and the spot price. The increase in revenue from the higher spot price exactly offsets the increase in production costs. Thus, in this example, a spot price contract provides perfect insurance for the seller against production cost uncertainty. A fixed price contract would leave all of the risk of production cost uncertainty on the seller.

However, a fixed price contract would insure a buyer against risk in this example. This is because, assuming the value of the good to the buyer is fixed, a fixed price contract would guarantee the buyer a certain level of profits. A spot price contract would cause the buyer's profits to be uncertain.

Thus, in this example, a spot price contract insures the seller and a fixed price contract insures the buyer. A spot price contract will be chosen over a fixed price contract if and only if the seller is more risk averse than the buyer.

The main contribution of this paper is to derive and interpret the condition determining whether a spot price contract or a fixed price contract is superior in a model in which the following complications are added: First, it is assumed that the industry supply curve may be rising (rather than flat), and that the seller's uncertain production cost is positively, but imperfectly, correlated with shifts in the industry supply curve. Second, it is assumed that the buyer's valuation and the industry demand curve are uncertain and that the buyer's valuation is positively, but imperfectly, correlated with shifts in the industry demand curve. Note that, if the supply curve is rising, the equilibrium spot price will depend on fluctuations of both the industry supply curve and the industry demand curve.

In this more general framework, a spot price contract will still tend to insure the seller against production cost uncertainty. The reason is similar to that discussed in the previous example, although the upward slope of the industry supply curve and the less than perfect correlation between the seller's costs and shifts in the industry supply curve reduces the value of a spot price contract as insurance against production cost uncertainty.[6/]

However, as noted, the spot price also will fluctuate because of shifts in the industry demand curve. A fixed price contract would insure the seller against these demand-side uncertainties. Thus, in general, neither contract can protect the seller against both supply-side and demand-side risk.

The results regarding the buyer are the mirror image of those regarding the seller. A spot price contract will tend to insure the buyer against valuation uncertainty, while a fixed price contract will insure the buyer against supply-side uncertainties.

The preceding discussion shows that neither contract form is best in terms of risk allocation in all circumstances. Whether a spot price or a fixed price contract is preferred depends on: the parties' relative aversion to risk; the magnitudes of the supply-side and demand-side uncertainties; the degree of correlation between the seller's costs and shifts in the industry supply curve; the degree of correlation between the buyer's valuation and shifts in the industry demand curve; and the slopes of the supply and demand curves.

In the next section the basic framework is described, including the general condition determining which contract form is preferred. In section 3, this condition is interpreted by examining some special cases. In section 4, an example is presented in which a spot price contract with a floor price dominates both a "pure" spot price contract and a fixed price contract. And in section 5, some concluding remarks are made.[7]

2. Basic Framework

This section presents the basic framework. First, the spot market equilibrium is described. Then the seller's and buyer's utility under spot price and fixed price contracts is stated. Finally, the general condition determining whether a spot price contract is preferred to a fixed price contract is derived.

The industry supply curve in the spot market is assumed to be of the form:

$$P = k_1 + k_2 Q + \Gamma, \tag{2.1}$$

where P is the price of the good, Q is industry output, $k_1 > 0$ and $k_2 \geqq 0$ are positive constants, and Γ is a random variable with mean zero and variance $\sigma^2(\Gamma)$.[8/] Similarly, the industry demand curve is assumed to be of the form:

$$P = k_3 - k_4 Q + \Omega, \tag{2.2}$$

where $k_3 > 0$ and $k_4 > 0$ are constants and Ω is a random variable with mean zero and variance $\sigma^2(\Omega)$. The random variables Ω and Γ are assumed to be independent.[9/]

Setting supply equal to demand and solving for the equilibrium spot price gives:[10/]

$$P^* = \bar{P} + \lambda_1 \Gamma + \lambda_2 \Omega, \tag{2.3}$$

where

$$\bar{P} = \frac{k_1 k_4 + k_2 k_3}{k_2 + k_4}, \quad \lambda_1 = \frac{k_4}{k_2 + k_4}, \quad \lambda_2 = \frac{k_2}{k_2 + k_4}. \tag{2.4}$$

The corresponding expression for the equilibrium output is omitted because it will not be used below. Industry equilibrium is shown graphically in Figure 1.

Now consider a particular seller (hereafter "the Seller") and a particular buyer ("the Buyer") who are contemplating entering into a contract for the future exchange of one unit of the good, but who do not yet know their respective costs and valuations.[11] The Seller's cost of producing the good is

$$C = c + \gamma, \tag{2.5}$$

where $c > 0$ is a constant and γ is a random variable with mean zero and variance $\sigma^2(\gamma)$. The random variable representing the Seller's cost uncertainty is assumed to be positively correlated with the random variable representing supply curve uncertainty:

$$\rho(\Gamma, \gamma) \equiv \frac{\mathrm{Cov}(\Gamma, \gamma)}{\sigma(\Gamma)\sigma(\gamma)} > 0, \tag{2.6}$$

where $\rho(.)$ is the correlation coefficient, $\mathrm{Cov}(.)$ is the covariance, and $\sigma(.)$ is the standard deviation of the respective arguments.

Similarly, the Buyer's value from having the good is

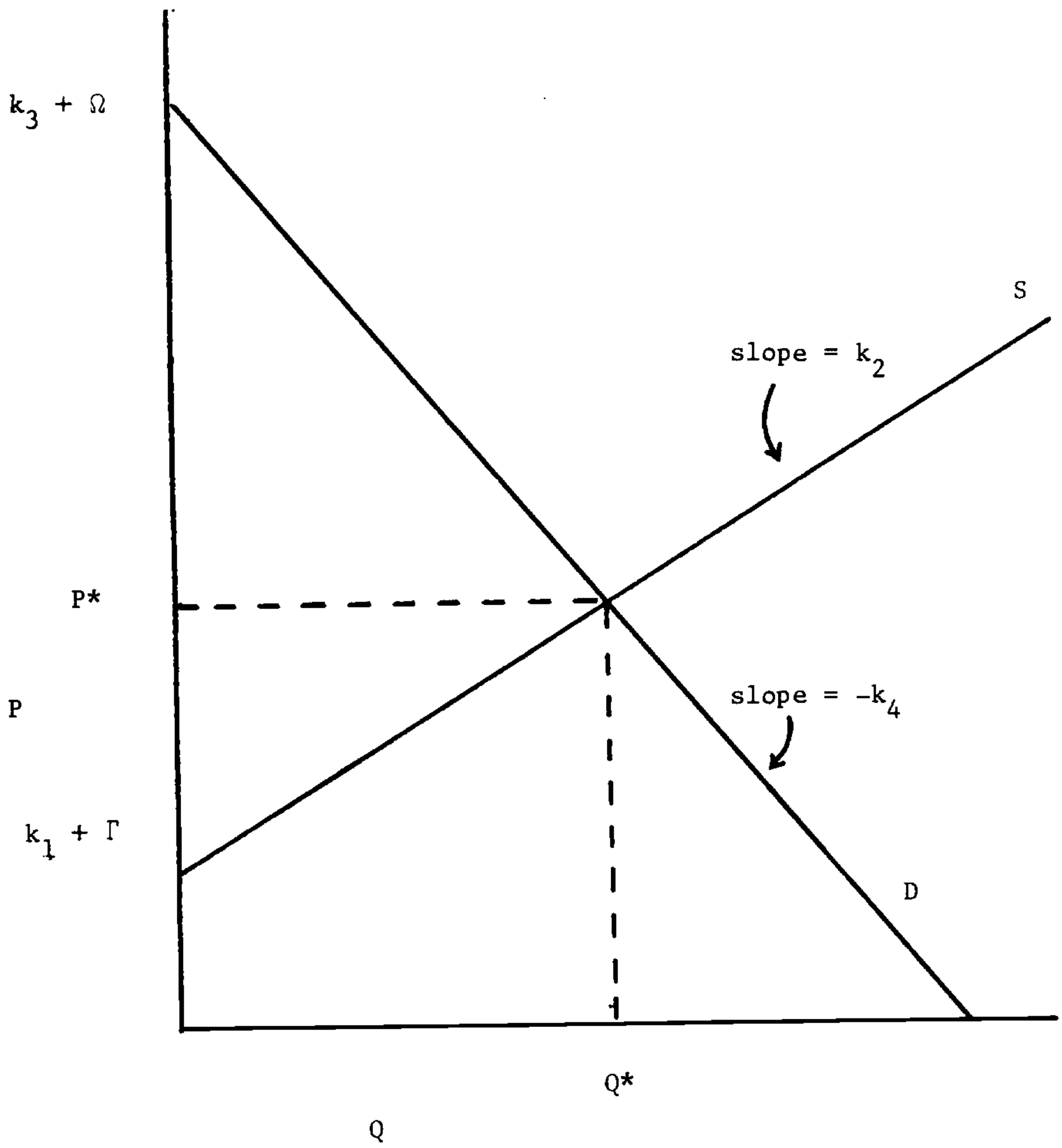
$$V = v + \omega, \tag{2.7}$$

where $v > 0$ is a constant and ω is a random variable with mean zero and variance $\sigma^2(\omega)$. The random variable representing the Buyer's valuation uncertainty is assumed to be positively correlated with the random variable representing demand curve uncertainty:

$$\rho(\Omega, \omega) > 0. \tag{2.8}$$

Figure 1

Spot Market Equilibrium



The Seller's cost and the Buyer's valuation are assumed to be independent.[12/]

The parties are assumed to have the following mean-variance utility functions:

(2.9) Seller: $U = E(\pi) - s\mathrm{Var}(\pi)$,

(2.10) Buyer: $V = E(\pi) - b\mathrm{Var}(\pi)$,

where π is the relevant party's profit and $s \geq 0$ and $b \geq 0$ are constants measuring the Seller's and Buyer's risk aversion.[13/]

The Seller's profit under a spot price contract is

(2.11) $\pi = P^* - [c + \gamma]$.

Using (2.3), (2.11) can be rewritten as:

(2.12) $\pi = (\bar{P} - c) + (\lambda_1 \Gamma + \lambda_2 \Omega - \gamma)$.

Thus, under a spot price contract, the Seller's expected profit is $\bar{P} - c$ and the variance of his profit is $\sigma^2(\lambda_1 \Gamma + \lambda_2 \Omega - \gamma)$. Similarly, the Seller's profit under a fixed price contract with contract price $\tilde{P}$ is:

(2.13) $\pi = \tilde{P} - [c + \gamma]$.

This results in an expected profit of $\tilde{P} - c$ and a variance of profit of $\sigma^2(\gamma)$.

The Buyer's profit under a spot price contract is

(2.14) $\pi = v + \omega - P^*$,

or, using (2.3),

(2.15) $\pi = (v - \bar{P}) + (\omega - \lambda_1 \Gamma - \lambda_2 \Omega)$.

The resulting expected profit is $v - \bar{P}$ and the resulting variance of profits is $\sigma^2(\omega - \lambda_1\Gamma - \lambda_2\Omega)$. Similarly, the Buyer's profit under a fixed price contract with contract price $\tilde{P}$ is:

$$\pi = v + \omega - \tilde{P}, \tag{2.16}$$

leading to an expected profit of $v - \tilde{P}$ and variance of profit of $\sigma^2(\omega)$. The results of the preceding cases are summarized in Table 1.

It is shown in the Appendix that, with mean-variance utility functions, the optimal contract form minimizes the weighted sum of the parties' variances of profits, where the weights are the parties' risk aversion coefficients.[14/] Thus, a spot price contract is preferred to a fixed price contract if and only if:

$$\begin{aligned} &s\sigma^2(\lambda_1\Gamma + \lambda_2\Omega - \gamma) + b\sigma^2(\omega - \lambda_1\Gamma - \lambda_2\Omega) \\ &\quad < s\sigma^2(\gamma) + b\sigma^2(\omega). \end{aligned} \tag{2.17}$$

The left-hand side of (2.17) can be interpreted as the parties' disutility due to the bearing of risk under a spot price contract, while the right-hand side is their comparable disutility under a fixed price contract.

Table 1

Expected Profits and Variances of Profits

(a) Spot Price Contract

	Seller	Buyer
Expected Profit	$\bar{P}-c$	$v-\bar{P}$
Variance of Profit	$\sigma^2(\lambda_1\Gamma+\lambda_2\Omega-\gamma)$	$\sigma^2(\omega-\lambda_1\Gamma-\lambda_2\Omega)$

(b) Fixed Price Contract

	Seller	Buyer
Expected Profit	$\tilde{P}-c$	$v-\tilde{P}$
Variance of Profit	$\sigma^2(\gamma)$	$\sigma^2(\omega)$

3. Interpretation

In order to understand better the condition determining which contract form is preferred, this section will consider several special cases. First, it will be assumed that only one of the parties is risk averse, and then that both are risk averse.

3.1 Seller Risk Averse, Buyer Risk Neutral

In this case, condition (2.17) reduces to:

$$(3.1) \qquad \sigma^2(\lambda_1\Gamma + \lambda_2\Omega - \gamma) < \sigma^2(\gamma).$$

It will be useful to examine this condition when either supply-side uncertainty or demand-side uncertainty is absent.

Suppose first that the only source of uncertainty is with respect to the demand curve; therefore, $\sigma^2(\Gamma) = 0$ and $\sigma^2(\gamma) = 0$. Then (3.1) becomes

$$(3.2) \qquad \sigma^2(\lambda_2\Omega) < 0,$$

implying that a fixed price contract is preferred to a spot price contract. This makes sense intuitively. If the Seller's production costs are fixed, then the Seller bears no risk if his revenue is fixed, as it would be under a fixed price contract. A spot price contract would make his revenue uncertain because the spot price would vary with fluctuations in the demand curve. Thus, <u>a fixed price contract insures the Seller against demand-side uncertainty</u>.

Now suppose that the only sources of uncertainty are on the supply side, both with respect to the Seller's production costs and with respect to the supply curve; therefore, $\sigma^2(\Omega) = 0$. Now (3.1) becomes:

$$(3.3) \qquad \sigma^2(\lambda_1\Gamma - \gamma) < \sigma^2(\gamma).$$

After some manipulation, this can be rewritten as:

$$(3.4) \qquad \rho(\Gamma,\gamma) > \frac{k_4}{2(k_2 + k_4)} \frac{\sigma(\Gamma)}{\sigma(\gamma)}.$$

Clearly, this condition may or may not be satisfied. Therefore, with respect to supply-side uncertainty, either a spot price contract or a fixed price contract may be preferred.

To understand the circumstances in which each contract form would be preferred, consider the terms in (3.4). Everything else equal, the higher is $\rho(\Gamma,\gamma)$--the (positive) correlation between the Seller's production cost and the industry supply curve--the more likely a spot price contract is preferred to a fixed price contract. The reason, suggested in the introduction, is easy to see. Suppose, for example, that the Seller's costs turn out to be high. Since $\rho(\Gamma,\gamma) > 0$, the industry supply curve, and hence the spot price, is likely to be high as well. Therefore, <u>a spot price contract can be viewed as a form of insurance for the Seller against production cost uncertainty</u>. High costs will tend to be associated with high revenue, and vice versa.[15/]

This conclusion needs to be qualified for the following reason. As can be seen from (3.4), if $\sigma(\Gamma)/\sigma(\gamma)$--the ratio of the standard deviation of the industry supply curve to the standard deviation of the Seller's costs--is sufficiently high, then a fixed price contract would be preferred to a spot price contract regardless of the magnitude of $\rho(\Gamma,\gamma)$. Under these circumstances, the implicit insurance provided by the spot price contract is "too much of a good thing." For example, if the Seller's costs rise, the spot price is likely to rise by so much more that the Seller's profits become more variable rather than less variable under a spot price contract. Thus, in order for a spot price contract not to "overinsure" the Seller in this sense, the variance of the industry supply curve must not be too large relative to the variance of the Seller's costs.[16/]

One final factor needs to be taken into account. Everything else equal, the extent to which shifts in the industry supply curve lead to changes in the spot price depends on the slopes of the industry supply and demand curves. This is accounted for in (3.4) by the term $k_4/2(k_2 + k_4)$. For example, the flatter the demand curve (i.e., the lower is k_4), the smaller the impact on the spot price of a shift in the supply curve. Consequently, a spot price contract would be more likely to be superior because the problem of "overinsuring" would be less likely.

To summarize: A fixed price contract insures the Seller against demand-side uncertainty, while a spot price contract tends to insure the Seller against production-cost uncertainty (although it might overinsure him). Therefore, when the Seller is risk averse and the Buyer is risk neutral, which contract form is preferred will depend on the relative importance of the two sources of uncertainty (unless the spot price contract overinsures the Seller against production cost uncertainty, in which case a fixed price contract would be preferable).

3.2 Buyer Risk Averse, Seller Risk Neutral

In this case, condition (2.17) reduces to:

$$(3.5) \qquad \sigma^2(\omega - \lambda_1\Gamma - \lambda_2\Omega) < \sigma^2(\omega),$$

or, equivalently,

$$(3.6) \qquad \sigma^2(\lambda_1\Gamma + \lambda_2\Omega - \omega) < \sigma^2(\omega).$$

Note that (3.6) is identical to (3.1) with ω--the Buyer's valuation uncertainty--substituted for γ--the Seller's production-cost uncertainty. Thus, the results of section 3.1 apply here in reverse.

If the only source of uncertainty is with respect to the supply curve, (3.6) becomes

$$(3.7) \qquad \sigma^2(\lambda_1\Gamma) < 0,$$

implying that a fixed price contract is preferred to a spot price contract. For reasons analogous to those discussed earlier, <u>a fixed price contract insures the Buyer against supply-side uncertainty</u>.

If the only sources of uncertainty are on the demand side, (3.6) can be written as:

$$(3.8) \qquad \rho(\Omega,\omega) > \frac{k_2}{2(k_2 + k_4)} \frac{\sigma(\Omega)}{\sigma(\omega)}.$$

The interpretation of this condition is analogous to that of (3.4): <u>A spot price contract can be viewed as a form of insurance for the Buyer against valuation uncertainty</u>. However, if it overinsures the Buyer, a fixed price contract may be preferred.

3.3 Both Parties Risk Averse

In this section limiting values of the slope of the industry supply curve will be considered when both parties are risk averse.

Suppose first that the industry supply curve is flat; i.e., $k_2 = 0$. Then $\lambda_1 = 1$ and $\lambda_2 = 0$ (see (2.4)), and the condition determining whether a spot price contract is preferred to a fixed price contract, (2.17), becomes:

$$(3.9) \qquad s\sigma^2(\Gamma - \gamma) + b\sigma^2(\omega - \Gamma) < s\sigma^2(\gamma) + b\sigma^2(\omega).$$

After some manipulation, (3.9) can be rewritten as

$$(3.10) \qquad b < s\left\{2\rho(\Gamma,\gamma)\frac{\sigma(\gamma)}{\sigma(\Gamma)} - 1\right\}.$$

Since the expression in braces may be positive or negative, either a spot price or a fixed price contract may be preferred.

Condition (3.10) can be interpreted in terms of the results in sections 3.1 and 3.2. First note that when the supply curve is flat, only supply-side uncertainty matters since shifts in the industry demand curve will not affect the spot price. From the Seller's perspective,[17/] recall that a spot price contract is preferred with respect to supply-side uncertainty unless it "overinsures" the Seller; the condition for a spot price contract to be preferred was given by (3.4), which, when $k_2 = 0$, can be rewritten as:

$$(3.11) \qquad 2\rho(\Gamma,\gamma)\frac{\sigma(\gamma)}{\sigma(\Gamma)} - 1 > 0.$$

Note that the left-hand side of (3.11) is identical to the expression in braces in (3.10). From the Buyer's perspective, recall that a fixed price contract is preferred with respect to supply-side uncertainty because it fully insures the Buyer against this source of risk. Therefore, (3.10) can be given the following interpretation. If the Seller prefers a fixed price contract too--that is, if the expression in braces is negative--then (3.10) clearly implies that a fixed price contract will be superior to a spot price contract. However, if the Seller prefers a spot price contract--that is, if the expression in braces is positive--then whether a spot price or a fixed price contract is superior depends on the risk aversion of the Seller relative to that of the Buyer and on the extent to which a spot price contract insures the Seller against production cost uncertainty (as measured by the magnitude of the expression in braces in (3.10)).

Now suppose that the slope of the industry supply curve approaches infinity. Then λ_1 approaches zero, λ_2 approaches unity, and (2.17) becomes:

$$(3.12) \qquad s\sigma^2(\Omega - \gamma) + b\sigma^2(\omega - \lambda_2\Omega) < s\sigma^2(\gamma) + b\sigma^2(\omega).$$

This condition can be rewritten as:

$$(3.13) \qquad s < b\left\{2\rho(\Omega,\omega)\,\frac{\sigma(\omega)}{\sigma(\Omega)} - 1\right\}.$$

The interpretation of (3.13) is analogous to the interpretation of (3.10). In the limit, as the slope of the industry supply curve approaches infinity, only demand-side uncertainty matters.[18] With respect to this source of uncertainty, the Seller prefers a fixed price contract, whereas the Buyer prefers a spot price contract unless it overinsures him. Therefore, if the Buyer prefers a spot price contract, then which contract form is superior depends on the strength of this preference and the relative aversion to risk of the parties, whereas if the Buyer prefers a fixed price contract, then that contract form will be superior.

Other special cases easily can be worked out. Note, for example, that the results when the slope of the industry demand curve approaches (minus) infinity are identical to those when the slope of the supply curve is zero (since λ_1 approaches unity and λ_2 approaches zero). Similarly, when the slope of the demand curve approaches zero, the results are identical to those when the slope of the supply curve approaches infinity.

4. Spot Price Contracts with Floor Prices: An Example

There are many other types of contracts that could be considered by the parties in order to better allocate supply-side and demand-side risks. One additional contract form will be examined in this section--a spot price contract with a floor price. Under this contract, the Buyer pays the Seller the spot price or a prespecified floor price, whichever is greater. Since the floor price can be made arbitrarily low, this contract form can approximate a "pure" spot price contract. Moreover, if the floor price is set sufficiently high and the Buyer is made to pay the spot price or the floor price, whichever is higher, less a positive constant, a spot price contract with a floor price can also approximate a fixed price contract. Clearly, therefore, a spot price contract with a floor price cannot do worse than the other two contract forms. This section presents an example in which it does better.

The example is a special case of the model described in section 2. It is characterized by the following assumptions. First, the industry supply curve is flat: $k_2 = 0$. Second, the random variable representing the Seller's production cost uncertainty is identical to the random variable representing supply curve uncertainty: $\gamma = \Gamma$. (This common random variable will be referred to as γ.) Together, these two assumptions have a natural economic interpretation; they describe a competitive industry in long-run equilibrium in which all of the firms' costs are identical, but uncertain. Third, the supply-side uncertainty is binary:

$$(4.1) \qquad \gamma = \begin{cases} -ra, & \text{with probability } (1-r), \\ -ra + a, & \text{with probability } r, \end{cases}$$

where $0 < r < 1$ and $a > 0$ are constants. Note that $E(\gamma) = 0$ and

$$(4.2) \qquad \sigma^2(\gamma) = r(1-r)a^2.$$

Given these three assumptions, the equilibrium spot price is

$$(4.3) \qquad P^* = \bar{P} + \gamma = \begin{cases} \bar{P} - ra, & \text{with probability } (1-r), \\ \bar{P} - ra + a, & \text{with probability } r, \end{cases}$$

where $\bar{P}$ is a constant. Fourth, there is no demand-side uncertainty.

Since the analysis of different contract forms in this example parallels the analysis in the preceding two sections, only a few details concerning the spot price contract with a floor price will be discussed here. From (4.3), the low value of the spot price is $\bar{P} - ra$ and the high value is $\bar{P} - ra + a$. Therefore, let the floor price be $\bar{P} - ra + f$, where $0 \leqq f \leqq a$. If $f = 0$, the spot price contract with a floor price is equivalent to a pure spot price contract and if $f = a$, it can be made equivalent to a fixed price contract by having the Buyer pay the floor price less a positive constant. (Since a reduction of the contract price by a constant would not affect the variance of either party's profit, the possibility of such an adjustment will be ignored in the remainder of this section.)

The Seller's profit under a spot price contract with floor price $f > 0$ is, using (4.1) and (4.3):

$$\pi = P^* - [c + \gamma] \tag{4.4}$$

$$= \begin{cases} \bar{P} - c + f, & \text{with probability } (1-r), \\ \bar{P} - c, & \text{with probability } r. \end{cases}$$

The variance of the Seller's profit is $r(1-r)f^2$. Similarly, the Buyer's profit is:

$$\pi = \begin{cases} v - \bar{P} + ra - f, & \text{with probability } (1-r), \\ v - \bar{P} + ra - a, & \text{with probability } r. \end{cases} \tag{4.5}$$

The variance of the Buyer's profit is $r(1-r)(a-f)^2$.

As noted in section 2, the optimal contract minimizes the weighted sum of the variances of profits, where the weights are the parties' risk aversion coefficients. Thus, the optimal floor price, $\bar{P} - ra + f^*$, can be determined by minimizing

$$sr(1-r)f^2 + br(1-r)(a-f)^2 \tag{4.6}$$

over f. This leads to:[19/]

$$f^* = \left[\frac{ab}{s+b}\right]. \tag{4.7}$$

Inserting (4.7) into (4.6) gives the minimum of the weighted sum of the variances of profits under a spot price contract with a floor price:

$$\left[\frac{sb}{s+b}\right] r(1-r)a^2. \tag{4.8}$$

Under a pure spot price contract the weighted sum of the variances of profits is

$$[b]r(1-r)a^2, \tag{4.9}$$

while under a fixed price contract it is

(4.10) $[s]r(1-r)a^2.$

Note that, as between a pure spot price contract and a fixed price contract, the former is superior if and only if the Seller is more risk averse than the Buyer.

A comparison of (4.8) through (4.10) shows that when both parties are risk averse the spot price contract with the optimal floor price is preferable both to a pure spot price contract and to a fixed price contract since

(4.11) $$\left[\frac{sb}{s+b}\right] = \left[\frac{s}{s+b}\right] b = \left[\frac{b}{s+b}\right] s$$

is less than both b and s when $s > 0$ and $b > 0$. This result can be explained intuitively. In the example, a pure spot price contract fully insures the Seller against production cost uncertainty, leaving all of the risk on the Buyer. A fixed price contract does just the opposite. When both parties are risk averse, it is better to share the risk, which can be accomplished by a spot price contract with a floor price.

There is a simple way in this example to measure the advantage of a spot price contract with a floor price over the other two contracts. Expressions (4.8) through (4.10) represent the disutility to the parties from the bearing of risk. In each case, the term in brackets multiplies the variance of the Seller's production cost (see (4.2)). Therefore, the ratio of the risk-bearing costs under a spot price contract with an optimal floor price to the risk-bearing costs under the other two contracts equals the ratio of sb/(s+b) to s or to b. For example, suppose the Buyer and the Seller are equally risk averse. Then the risk-bearing costs under a spot price contract with an optimal floor price are exactly half of the risk-bearing costs under either a pure spot price contract or a fixed price contract. Or, for example, suppose the Buyer is twice as risk averse as the Seller (i.e., b = 2s). Then a fixed price contract is superior to a pure spot price contract, but a spot price contract with an optimal floor price reduces risk-bearing costs by a third from what they would be under a fixed price contract.

5. Concluding Remarks

Although many simplifying assumptions have been made in this paper, the principal observations seem quite general: A spot price contract tends to insure a seller against production cost uncertainty and a buyer against valuation uncertainty (although it may "overinsure" them). A fixed price contract insures a seller against demand-side uncertainties and a buyer against supply-side uncertainties. Thus, which contract form will be preferred by the parties depends on their relative aversion to risk and the magnitudes of the supply-side and demand-side uncertainties.

The analysis in this paper can be used to help explain contracting practices in different industries. Consider, for example, the uranium industry, in which the sellers usually are private firms and the buyers frequently are public utilities (using the uranium to produce electricity). It would seem reasonable to assume that the sellers are more risk averse than the buyers with respect to fluctuations in the price of uranium (since the price of uranium constitutes a small fraction of the utilities' cost of producing electricity and they usually can pass input price changes through to consumers).

The contracting practices in the uranium industry changed during the early 1970's. Before then, fixed price contracts were the norm. Afterwards, spot price contracts were used more frequently (often with a floor price). Assuming for simplicity that the buyers (public utilities) are risk neutral, this change would have to be explained by the risks borne by the sellers. As noted earlier, sellers would prefer spot price contracts with respect to production cost uncertainty (unless such contracts overinsure them, which does not seem likely in this context) and fixed price contracts with respect to demand uncertainty. About the time that contracting practices began to change in the uranium industry, there was a significant increase in production cost uncertainty due to the effects of environmental and mine safety regulations passed in the late 1960's and early 1970's. Thus, the change in contracting form might be explained by these regulatory changes.

Appendix

The following result will be demonstrated in this appendix: Suppose the parties' utility functions are of the mean-variance form, the sum of their expected profits is constant, and lump-sum transfers can be made between them. Then one situation is Pareto superior to another if and only if the weighted sum of the parties' variances of profits, where the weights are the respective coefficients in the parties' utility functions, is lower under that situation.

The notation used will be adapted from the text. Let $\bar{\pi}_{i_j}$ represent the expected profit of party i (i = S,B) in situation j (j = 1,2). Then the assumption that the sum of the parties' expected profits is constant becomes:

$$\text{(A.1)} \qquad \bar{\pi}_{S_1} + \bar{\pi}_{B_1} = \bar{\pi}_{S_2} + \bar{\pi}_{B_2}.$$

Let $\sigma^2_{i_j}$ represent the variance of profit of party i in situation j, and let $k_j \gtreqless 0$ represent a lump-sum transfer from S (the Seller) to B (the Buyer) in situation j.

First it will be shown that if the weighted sum of the variances of profits is lower under one situation, then that situation can be made Pareto superior to the other situation by appropriate lump-sum transfers. Without loss of generality, suppose the weighted sum of the variances is lower under situation 1:

$$\text{(A.2)} \qquad s\sigma^2_{S_1} + b\sigma^2_{B_1} < s\sigma^2_{S_2} + b\sigma^2_{B_2}.$$

Also without loss of generality, suppose the Seller's utility is the same in both situations:

$$\bar{\pi}_{S_1} - s\sigma^2_{S_1} - k_1 = \bar{\pi}_{S_2} - s\sigma^2_{S_2} - k_2. \tag{A.3}$$

The Buyer's utility in situation 1 is:

$$\bar{\pi}_{B_1} - b\sigma^2_{B_1} + k_1. \tag{A.4}$$

Solving for k_1 from (A.3) and substituting the resulting expression into (A.4) gives:

$$(\bar{\pi}_{S_1} + \bar{\pi}_{B_1} - \bar{\pi}_{S_2}) - (s\sigma^2_{S_1} + b\sigma^2_{B_1} - s\sigma^2_{S_2}) + k_2. \tag{A.5}$$

From (A.1), the expression in the first set of parentheses equals $\bar{\pi}_{B_2}$. Therefore, (A.5) can be written as:

$$\begin{aligned} &\bar{\pi}_{B_2} - (s\sigma^2_{S_1} + b\sigma^2_{B_1} - s\sigma^2_{S_2}) + k_2 \\ &> \bar{\pi}_{B_2} - b\sigma^2_{B_2} + k_2, \end{aligned} \tag{A.6}$$

where the inequality follows from (A.2). Hence, the Buyer's utility is higher in situation 1.

Now it will be shown that if one situation is Pareto superior to another, the weighted sum of the variances of profits is lower under that situation. Without loss of generality, suppose situation 1 is Pareto superior to situation 2 and that the Seller's utility is strictly higher in situation 1:

$$\bar{\pi}_{S_1} - s\sigma^2_{S_1} - k_1 > \bar{\pi}_{S_2} - s\sigma^2_{S_2} - k_2, \tag{A.7}$$

$$\bar{\pi}_{B_1} - b\sigma^2_{B_1} + k_1 \geqq \bar{\pi}_{B_2} - b\sigma^2_{B_2} + k_2. \tag{A.8}$$

Adding (A.7) and (A.8) gives:

(A.9) $$(\bar{\pi}_{S_1} + \bar{\pi}_{B_1}) - s\sigma^2_{S_1} - b\sigma^2_{B_1} > (\bar{\pi}_{S_2} + \bar{\pi}_{B_2}) - s\sigma^2_{S_2} - b\sigma^2_{B_2}.$$

From (A.1), the terms in parentheses cancel. Multiplying the resulting expression by -1 gives the desired result.

Footnotes

*/ Stanford University and National Bureau of Economic Research. This paper grew out of work I did in 1980 for the defendants in Westinghouse Electric Corporation v. Rio Algom Limited, et al. (an antitrust suit by Westinghouse against domestic and foreign uranium producers). The actual writing of the paper, as well as the derivation of most of the results in the present version, occurred after the case was settled in 1981. Work on the paper during the summer of 1982 was supported by the Stanford Legal Research Fund, made possible by a bequest from the Estate of Ira S. Lillick and by gifts from Roderick E. and Carla A. Hills and other friends of the Stanford Law School. Helpful comments were provided by Lucian Bebchuk, Jeffrey Perloff, Ivan P'ng, Michael Riordan, William Rogerson, Steven Shavell, Edward Sherry, and participants in seminars at Berkeley and Stanford.

1/ If the spot market is competitive, which is consistent with what will be assumed in section 2 below, then a spot price contract is equivalent to transacting in the spot market. Although there are reasons why the parties might prefer to enter into a contract rather than to transact in the spot market (such as reduced transaction costs from dealing with the same person over time), these reasons are not incorporated into the model to be analyzed.

2/ Both the seller and the buyer will be presumed to be firms (although the assumption that the buyer is a firm is not essential to the analysis). The assumption of risk aversion in the case of firms has both theoretical and empirical support.

See, for example, Amihud and Lev (1981) and Marcus (1982).

3/ See Long and Varble (1978). Although many of the respondents obviously utilized more than one type of contract, the use of multiple contract forms will not be considered in this paper.

4/ See, for example, Burness, Montgomery, and Quirk (1980).

5/ See, for example, Pierce (1982), Goldberg and Erickson (1982), and Joskow (1985).

6/ There is also the possibility, discussed in section 3 below, that a spot price contract will "overinsure" the seller, in which case a fixed price contract could be preferable with respect to production cost uncertainty.

7/ Although, to my knowledge, the problem addressed in this paper has not been studied previously, there is much work on risk allocation that is related in one way or another. Of general relevance are articles that consider risk allocation issues in specific contractual settings--such as employment contracts, defense procurement, and products liability. Apparently, the first article of this sort was the study by Cheung (1969) of employment contracts in agriculture.

There is also a large literature on the behavior of the firm under uncertainty that is complementary to the problem addressed here. Especially relevant are those papers which consider what fraction of a firm's output it should sell forward at a fixed price rather than in the spot market. An early classic in this literature is McKinnon(1967) and more recent examples include Feder, Just, and Schmitz (1980) and

Holthausen (1979). Perloff (1981) applies this framework to the excuse doctrine in contract law. This literature does not, however, consider the risk aversion of buyers.

Probably of most relevance to the present analysis is the book by Newbery and Stiglitz (1981) on commodity price stabilization schemes and the paper by Shavell (1976) on deferred compensation schemes. However, in the Newbery-Stiglitz framework, if a firm's output were fixed, then a fixed price contract (perfect commodity price stabilization) would eliminate all risk. This is not the case in my analysis. The general results in the Shavell paper are consistent with those presented here, although he does not consider the specific contract forms and market structure that I do. Also in the spirit of the present study is the paper by Sebenius and Stan (1982) analyzing profit sharing, royalties, and fixed fees, although they assume that one of the parties is risk netural.

8/ Obviously, Γ must be such that $P > 0$. (Analogous comments apply to the other random variables defined below.)

9/ This assumption may be reasonable in some contexts but not others. It would be appropriate, for example, if demand uncertainty is due to fluctuations in the real income of consumers while production cost uncertainty is due to regulatory changes or the vagaries of the weather.

10/ Since the focus in this paper is on the contract choice of a single seller and a single buyer who treat the spot price as exogenous, it is not necessary in deriving (2.3) to explicitly take account of the behavior of all buyers and sellers in the market (although it is necessary to assume that enough choose to trade in the spot market).

11/ It will be assumed that the realized values of the uncertain costs and benefits are such that the parties will want to complete the contract in the way originally contemplated by them--that is, by having the Seller produce the good (as opposed to securing one unit in the spot market) and by having the Buyer keep the good (as opposed to reselling it in the spot market).

12/ The comment in note 9 above is applicable here too. However, in a paper analyzing the effects of demand-side and supply-side uncertainties on the output exchanged in a bilateral trading situation, Weitzman (1981) has suggested why the relevant random variables might be negatively correlated.

13/ Although mean-variance utility functions are widely used because of their simplicity, their justification requires some well-known assumptions. See, for example, Newbery and Stiglitz (1981, Ch. 6).

14/ The conditions assumed in the Appendix include that the sum of the parties' expected profits is constant and that lump-sum transfers can be made between them. That the sum of the parties' expected profits is constant across contract forms is clear from Table 1. That lump-sum transfers can be made between the parties follows from their ability to adjust the contract price by a constant amount. Since such an adjustment would not affect the variance of either party's profit, it is not explicitly taken into account in the analysis.

15/ Note that this implicit insurance need not be perfect in order for a spot price contract to be preferred.

16/ A possible response to this problem would be to make the contract price only partly dependent on the spot price.

I am indebted to Michael Riordan for suggesting this point.

17/ I will use phrases like "from the Seller's perspective" or "what the Seller would prefer" to refer to what the parties would jointly choose if the Seller were risk averse and the Buyer were risk neutral. Similar phrases will be used with respect to the Buyer.

18/ Note that if shifts in the industry supply curve were horizontal rather than vertical, then supply-side uncertainty would matter even when the supply curve is perfectly inelastic.

19/ The second-order condition for a minimum is satisfied.

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