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EXPECTED INTERRUPTIONS IN LABOR FORCE PARTICIPATION AND SEX RELATED DIFFERENCES IN EARNINGS GROWTH

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ABSTRACT

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The paper analyzes the joint determination of wives' earnings and labor force participation over the life cycle given the interruptions in wives' work careers. The interruptions affect the profitability of the investment in human capital, which in turn determines earnings. The earnings prospects feed back into the participation decision, namely, the decision whether and for how long to drop out of the labor force.

The formal analysis compares the age-earnings profiles of persons who drop out of the labor force with those who do not during the pre- and post-interruption period. The comparison is carried out where interruptions are assumed to be exogenous and when they are endogenous. The effect of productivity at home, the initial stock of human capital and its rental value on the length of the interruption is investigated.

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1. INTRODUCTION

The last decade has witnessed an awakened interest in the role of women in the labor market. Spurred by the increasing share of women in the labor force, economists have focused on the factors that affect women's (and in particular, married women's) labor supply and their compensation (particularly their pay relative to males). Though it has long been recognized that wages and participation are jointly determined, most empirical studies of labor force participation tend to regard wages as an exogenous variable, while studies of the structure of women's wages (and male-female wage differentials) often regard labor force experience (namely, participation in the past) as exogenous. In this paper we examine the implications of the interaction between participation and wages for the interpretation of observed, sex related, differences in earnings

The link binding participation and wages is the individual investment in himself. Investment in human capital on the job is one of the ways for an individual to increase his future wages. Labor force participation is a pre-requisite for this kind of investment. Thus participation affects future wages which in turn affect future participation. Conversely future participation plans determine the utilization of human capital and therefore influences current investment decisions.

The feature that makes the women's experience so unique and different from that of males' is the interruptions (or expected interruptions) in their careers associated with marriage, birth of children and geographical mobility. The interruptions do not merely result in the loss of current earnings; they also affect investment in human capital. Consequently, the difference between the earning of women and men reflects both the loss of human capital due to past interruptions and the lower accumulation of human capital due to expected future interruptions. An understanding of these complex interactions between wages and participation at different stages of the life cycle is crucial for the interpretation of recent findings on sex related differences in earnings growth and on wage discrimination.

An often-noted empirical regularity is the tendency of the sex-related earnings differentials to increase with potential work experience for at least the early phases of the work cycle. While the phenomenon has been observed in both cross-section and longitudinal data (see Johnson and Stafford [1974], Farber [1977], Weiss and Lillard [1978]), a different explanation applies in each case. In cross-section data, as potential experience increases so does the likelihood of past breaks in experience. It is then sufficient to assume that the accumulation of human capital is inhibited by exits from the labor force. In longitudinal data where individuals are observed repeatedly, or alternatively when retrospective data on past exits from the labor force is available, one can control for such differences in work history. The explanation in this case must rely on the unobservable future breaks in labor-force participation. It has been argued (see Mincer and Polachek [1974]) that a woman who expects a shorter work span will invest less in activities that raise her earning capacity. In contrast to the previous explanation, which relies on properties common to all wealth-maximizing models of human capital, the second explanation requires assumptions on the specific form of the production function. The reason is that the slope of the optimal earning profile depends not only on the rate of investment but also on the speed at which investment is reduced. An explanation that merely states that women are likely to invest less in on the job training is incomplete, unless it is also shown that women do not reduce their investment faster than men. A question arises, what are the restrictions on the production function which would enable us to infer the effects of future breaks on earnings growth?

Several authors (e.g. Johnson-Stafford [1974], Oxaca [1977] have attempted to measure the discrimination that women face in the labor market. Attention has been given to wage discrimination (as distinct from barriers to entry) whereby the market awards women with lower returns for "identical" characteristics. Presumably such a phenomenon would be revealed by differences in the level of earnings standardizing for differences in schooling and work experience. However, such comparisons fail to account for the unobserved differences in participation plans. Thus, for the same experience, women may have lower earnings because they invested less in the past, expecting to participate less in the future. One would not ascribe this gap in earnings to discrimination, unless the expected interruptions are caused by discrimination. Two questions arise: can one separate the effects of past and future breaks? How much of the observed difference in the level of earnings can be ascribed to discrimination?

The objective of this paper is to provide a theoretical framework for the analysis of the above issues. A modified version of the model in Blinder and Weiss [1976] is presented in which labor force participation and earnings are jointly determined over the life cycle. In previous works, either wages or participation were taken as exogenous (e.g. Heckman Macurdy [1980], Polachek [1975]). Our analysis is also limited, however, in treating fertility and marriage decisions as exogenous and in ignoring variations in the intensity of labor force participation.

To simplify the exposition, we first assume that labor force interruptions are exogenously determined and fully anticipated. Breaking the link between earnings and participation, we are free to analyze the effect of career interruptions on the earnings structure. Two features of the earnings profiles are examined: the level of earnings and their rate of growth. To separate these issues, we restrict the analysis to models in which either the absolute growth or its rate are independent of the stock of capital. The age-earnings profiles of people who drop out of the labor force are compared with those who do not during the pre- and post-interruption periods. It is often asserted that women have a flatter earnings profile because of their lower rate of investment in human capital. We point out that to make this assertion one has to know not only the level of investment but also its rate of change over time. It is shown that under our assumptions on the nature of the investment production function, an expected interruptions in labor force participation will lead to a reduction in earnings growth if and only if the earning-experience profiles are concave.

The second part of the paper relaxes the assumption that labor force interruptions are exogenous. The labor force participation decision depends on current wages, the prospects of further investment in human capital, and productivity in the home sector. We begin by noting that planned temporary (as opposed to permanent) withdrawals are not consistent with fixed market conditions and fixed productivity at home. It is, therefore, necessary to introduce some time or age dependence into the model. We introduce explicit dependence of productivity at home on age. It is assumed that increases in home productivity occur in jumps associated with the birth of children, with productivity declining as the children grow older. It is shown that a necessary condition for women who have left the labor force to return to work is that the rate of decline of home productivity exceeds the rate of depreciation of human capital. The probability and the duration of withdrawal are greater the higher is the productivity at home and the lower is the initial stock of human capital and its rental value. As noted by Strober and Quester [1977] and Johnson and Stafford [1977] discrimination against women in the labor force is a possible cause for their longer withdrawals. Thus, wage discrimination can affect not only the level of earnings but also the patterns of investment and earning growth through its effect on the length of career interruptions.

The last section of the paper draws the implications of our analysis for the interpretation of sex related differences in earnings.

2. A MODEL OF EARNINGS AND LABOR FORCE PARTICIPATION

2a. Description of the Maximization Problem

Consider an individual who plans a life time earnings and participation program under conditions of certainty and perfect capital markets. At each point in time the individual may either participate in the labor force or not. Let $P(t)$ be the index of labor force participation such that $P(t) = 1$ if the individual is in the labor force or in school and $P(t) = 0$ otherwise. Earnings capacity in the labor market depends on past participation and investment patterns. Actual earnings depend on current earning capacity and current investment. Let $RK(t)$ be the earnings capacity of the individual where K is the stock of human capital and R its rental value. Let $y(t)$, $0 \leq y(t) \leq 1$, be the proportion of earnings potential that is realized in the form of actual earnings. Thus, if the individual participates, his observed earnings, Y, equals yRK. By giving up part of his current earning capacity, the individual improves his future earnings' potential according to the production function:[1/]

$$\dot{K} = Pf(y,K) - \delta K \quad , \quad f_y < 0, \quad f_K \geq 0 . \tag{1}$$

where δ denotes the depreciation rate of human capital.

The value of time during periods of nonparticipation is denoted by $Q(t)$. Productivity at home is assumed exogenous, that is, independent of the participation and investment history of the individual.[2/] It may depend, however, on age, number and age of children, health and similar factors, and may vary, therefore over the life cycle.

The individual's maximization problem is:

$$(2) \qquad \max_{P(t)=0,1,0\leq y(t)\leq 1} \int_0^T e^{-rt}[PRKy + (1-P)Q]dt$$

$$\text{s.t. (1) and } K(0) = K_0$$

where T is the planning horizon and r the market rate of interest.

The necessary conditions which the optimal program must satisfy are:

$$(3) \qquad P(K + \psi f_y) \begin{cases} = 0 & \text{if} \quad 0 < y < 1 \\ \leq 0 & \text{if} \quad y = 0 \\ \geq 0 & \text{if} \quad y = 1 \end{cases}$$

$$(4) \qquad Ky + \psi f - Q/R \begin{cases} \geq 0 & \text{iff} \quad P = 1 \\ < 0 & \text{iff} \quad P = 0 \end{cases}$$

$$(5) \qquad \dot{\psi} = (r + \delta)\psi - P(y + \psi f_K) \quad , \quad \psi(T) = 0$$

where the function $\psi(t)$ is the shadow price of human capital divided by R.

Condition (3) determines the optimal value allocation of earning potential between current earnings and investment. If the individual participates, $P = 1$, then an interior maximum requires that the opportunity costs of y in terms of current earnings (represented by K) equals the value of marginal product in terms of future earning potential (represented by ψf_y). If the individual does not participate $(P = 0)$ y is indeterminant. Participation need not imply positive earnings, since the individual may specialize in "schooling" $(y = 0)$.

The participation decision described in condition (4) involves the comparison of home productivity with the full value of market oriented activities consisting of realized earnings and the value of investment. One may participate even if the current wage is below the productivity at home provided that the imputed value of the increase in future earnings capacity is sufficiently high. The participation decision is a function of past work history (embodied in K) and future work plans (reflected in ψ).

Condition (5) describes the development of the marginal value of human capital along the optimal path. When the individual is out of the labor force $(P = 0)$, the shadow price of human capital is increasing (see also Polachek [1975]). This reflects the profitability of shifting investment from the time just prior to the exit to the time just after entry, thus saving the depreciation and interest costs associated with unutilized human capital.

We describe the solution to conditions (3) to (5) in two stages. In subsection 2b, we consider the case in which labor force participation is exogenously determined and condition (4) is therefore not binding. The problem is then reduced to the analysis of the effects of past and anticipated exogenous interruptions in labor force participation on the development of earnings over the life cycle. We consider separately in sub-section 2c the case in which labor force participation is endogenous.

2b. Exogenous Breaks in Labor Force Participation

We wish to compare the optimal earnings profiles of two (otherwise identical) individuals: A, who participates continuously ($P(t) = 1$ for all $t \varepsilon [0,T]$); and B who expects a single interruption in labor-force participation such that $P(t) = 0$ for $t \varepsilon [t', t'']$ and $P(t) = 1$ for $t \varepsilon [0,t')$ and $t \varepsilon (t'',T]$. The difference in the earnings profiles of the two individuals prior to t' can be ascribed to differences in their anticipations while the difference after t'' can be ascribed to differences in their history. The history of the system is captured by the accumulated amount of human capital K, while anticipations for the future are summarized in the shadow price ψ. In general, K and ψ affect both the level and growth of earnings, and it is difficult to separate the roles of past (potentially observed) and future (unobservable) interruptions in labor force participation. There is, however, a special class of models in which expected interruptions are revealed by the choices of the individuals with respect to their earnings growth. That is, the slope of the earning profile is independent of the level of human capital. There are two such specifications:

(6) $$f(K,y) = g[K(1 - y)] \quad , \quad g' > 0, \quad g'' < 0, \quad g(0) = 0$$

and

(7) $$f(K,y) = Kh(y) \quad , \quad h' < 0, \quad h'' < 0, \quad h(1) = 0$$

The form (6) imposes Ben-Porath's [1967] "neutrality" hypothesis. In this case the slope of the earning profile, i.e., the absolute growth in earning, is, in the absence of depreciation, independent of K. The form (7), due to Blinder and Weiss [1976], leads to the independence from K of the slope of the log earning profile (i.e. the rate of growth in earnings).[3/]

Consider the formulation (6) first: The optimal path is characterized by the following conditions:

(3') $$P[1 - \psi g'(K(1 - y))] = 0 \qquad \text{if } 0 < y < 1$$

(5') $$\dot{\psi} = \psi[r + \delta] - P \qquad \text{if } 0 < y \leq 1$$

$$= \psi[r + \delta - Pg'(K)] \qquad \text{if } y = 0$$

$$\psi(T) = 0$$

For the specification (6) one can show that during periods of participation, if $\dot{\psi}$ is ever positive it remains positive thereafter.[4/] Since during periods of nonparticipation, $P = 0$, ψ is increasing, it follows that during periods of participation, $P = 1$, ψ must decrease monotonically. Otherwise the condition $\psi(T) = 0$ will be violated. If we denote the shadow price for the two individuals by $\psi_A(t)$ and $\psi_B(t)$ respectively, then it follows from (3') and (5') that person A and person B face the same shadow price of human capital after the interruption ($\psi_A(t) = \psi_B(t)$ for $t \in (t'', T]$), but person A has a higher shadow price prior to the interruption ($\psi_A(t) > \psi_B(t)$ for $t \in [0,t')$).

The slope of the earning profile during periods with positive earnings can be written as:

(8) $$\dot{Y} = \frac{r + \delta - g'}{\psi\, g''} + g - \delta K \qquad \text{if } 0 < y < 1, \quad P = 1$$

$$= - \delta Y \qquad \text{if } y = 1, \quad P = 1$$

It is seen from (8) and (3') that if $\delta = 0$, the slope depends only on ψ. In this case, individuals A and B will have the <u>same</u> slope for their earnings profile (i.e., the same absolute growth in earnings) during the interval $(t'', T]$. In the period preceding t', the slope of B's profile will be lower if and only if the (absolute) curvature index $\frac{-g''}{g'}$ is decreasing with the rate of investment. This is seen from:

$$(9) \qquad \frac{d\dot{Y}}{d\psi} = - [(g'')^2 - g'''g'] \frac{(r\psi - 1)}{[\psi g'']^3}, \quad \text{when } \delta = 0, \quad 0 < y < 1, \quad P = 1.$$

The concavity of the earning-experience profiles during the period of on-the-job investment requires that $\frac{d\dot{Y}}{d\psi} > 0$. Thus a specification of the production function that yields a strictly concave earning-experience profile also implies a lower earnings growth for B prior to the withdrawal from the labor force at time t' (specifically, this is true if one adopts the commonly used homogeneity assumption, namely, that $g(\ )$ has a constant elasticity). A border-line case in which the slope of both earnings profiles is the <u>same</u> is when $g(\ )$ is exponential, i.e. $\dot{K} = a - ce^{-b[K(1 - y)]}$.

When the rate of depreciation is positive, the slope of the earning profile depends also on the level of human capital, and therefore on initial conditions. Since B has a lower rate of investment prior to t', he will have a lower level of K, and thus a <u>higher</u> slope for his earning profile during the interval $(t'',T]$. During the period prior to t', the lower accumulation and the shorter horizon may have opposing effects on the slope of the earning profile and the outcome appears ambiguous.

In empirical comparisons of men's and women's earnings, it is common to compare the logs of earning profiles, that is, to focus on earnings growth rates.

However, under specification (6) the effect of an expected interruption on the rate of growth in earning is unclear, since both Y and $\dot{Y}$ are likely to be lower during the phase preceding t'. Let us turn, therefore, to specification (7) which places direct restrictions on the growth rate of earnings. The optimal path is now characterized by the conditions:

$$(3'') \qquad P[1 + \psi h'(y)] = 0 \qquad \text{if} \quad 0 < y < 1$$

$$(5'') \qquad \psi(T) = 0 \quad \text{and} \quad \dot{\psi} = (r+\delta)\psi - P[y+\psi h(y)]$$

As in the previous case, $\dot{\psi}$ is increasing when $P = 0$ and decreasing elsewhere.[5/] Also $\psi_A(\ ) = \psi_B(t)$ for $t \in (t'',T]$ and $\psi_A(t) > \psi_B(t)$ for $t \in [0,t'')$.

A convenient property of the specification (7) is that during periods of positive earnings $\frac{\dot{Y}}{Y}$ is determined uniquely by ψ according to the relation:

$$(10) \qquad \frac{\dot{Y}}{Y} = \frac{1}{E_{h'}}[-(r+\delta) + y(E_h + E_{h'} + 1)] - \delta \quad \text{if} \quad 0 < y < 1, \quad P = 1$$

$$= -\delta \qquad \text{if} \quad y = 1, P = 1$$

where we define:

$$(11) \qquad E_h = -\frac{h'(y)y}{h(y)} > 0 \quad \text{and} \quad E_{h'} = \frac{h''(y)y}{h'(y)} > 0$$

Thus, for the interval $(t'',T]$ both individuals will have the <u>same</u> slope for their log earning profile. In the period prior to t', B will have a lower earnings growth if and only if the (relative) curvature index $\frac{h''y}{h}$ is increasing with y. This follows from:

$$\text{(12)} \qquad \frac{d(\frac{\dot{Y}}{Y})}{d\psi} = \frac{1}{(E_{h'})^2} \frac{(h')^2}{h''}((r + \delta) - h(E_h + 1))\frac{d}{dy} E_{h'}. \quad \text{If } 0 < y < 1,\ P = 1$$

Again, we note that if a specification is chosen such that log earning experience profiles are concave during the period of positive on-the-job investment $(0 < y < 1)$ then the effect of expected interruption is to reduce earnings growth. A border-line case in which the slope of both log earning profiles will be the <u>same</u> is when $h'(y)$ is of constant elasticity, i.e., $h(y) = \beta(1 - y^{\alpha})$.

So far, we have discussed only one aspect of the optimal lifetime earnings profiles, namely their slope. The implications for earnings growth must, however, be tested jointly with the implications concerning the level of earnings and the period of schooling. Under both specifications (6) and (7), the <u>level</u> of earnings must be lower for individual B during the interval $(t'',T]$. This results from the fact that during this phase, the investment rates of A and B are identical, while B's stock of human capital is lower. During the phase prior to t' the level of B's earnings may exceed A's for a while since B invests a smaller proportion of his lower earning capacity. Both models predict a shorter period of specialization (in which $y = 0$ and therefore $Y = 0$) for individual B.[6/]

Finally, wage discrimination does not affect the investment in human capital and earning growth as long as the length of the interruption in the woman's work career is exogenously given. Discrimination, in our case, affects the cost of investment and the return to the same extent. It lowers the level of earnings but does not affect earning growth.

2c. Endogenous Withdrawals from the Labor Force

The pattern of exits and entries into the labor force depends on the development over time of productivity at home and in the market. We wish to present some restrictions on $Q(t)$ which will limit the number of exits and entries and determine their pattern. In this analysis we shall restrict ourselves to the specification (7). We utilize some special properties of the optimal solution which hold under specification (7). These are described in the following two lemmas.

Lemma 1: Let V denote the maximal current value of market activities, i.e. $V = \max_{0 \le y \le 1} R(Ky + \psi Kh(y))$, then

$$- \delta \quad \frac{\dot{V}}{V} \le r. \tag{13}$$

Proof. Differentiation of V with respect to time yields:

$$\dot{V} = R\{[y + \psi h(y)]\dot{K} + K[1 + \psi h'(y)]\dot{y} + Kh(y)\dot{\psi}\} \tag{14}$$

Using conditions (3") and (5") we obtain:

$$\dot{V} = - \delta V + (r+\delta)RK\psi h(y) = rV - RKy(r+\delta) \tag{15}$$

Lemma 2: Let $E(t,K(t))$ denote the present value (evaluated at t) of earnings associated with the optimal program. Then,

$$E = RK\psi \tag{16}$$

That is ψ, which, by definition, is the marginal value of K (divided by R) also equals the average value (divided by R). (See also Blinder & Weiss[1976,p.456]).

Proof. Using (5") we find that:

$$\frac{d}{dt}(\psi K) = K\dot{\psi} + \psi\dot{K} = r\psi K - PKy \tag{17}$$

Multiplying both sides of (17) by $Re^{-r(\tau-t)}$, and integrating from t to T, using $\psi(T) = 0$, we obtain:

$$RK\psi = \int_t^T e^{-r(\tau-t)} RPKy \, d\tau \tag{18}$$

Let $Q(t)$ be differentiable. We can then prove the following propositions:

Proposition 1: At a point of entry into the labor force, productivity at home must increase at a rate which is less than the market interest rate, $\frac{\dot{Q}}{Q} < r$.

Proof. At a point of entry we must have $Q = V$ and $\dot{Q} < \dot{V}$. Therefore $\frac{\dot{Q}}{Q} < \frac{\dot{V}}{V} \leq r$.

Proposition 2: At a point of exit from the labor force, productivity at home cannot decrease at a rate which exceeds the depreciation rate of human capital, $\frac{\dot{Q}}{Q} > -\delta$.

Proof. At a point of exit we have $Q = V$ and $\dot{Q} > V$. Therefore $\frac{\dot{Q}}{Q} > \frac{\dot{V}}{V} \geq -\delta$.

Proposition 3: If a point of exit is followed by a point of entry, then at the point of re-entry, productivity at home must decline at a rate which exceeds the depreciation of human capital.

Proof. Let there be an exit at time t_o followed by re-entry at t_1.

If the re-entry point is chosen optimally then, due to (16) it must maximize:

$$W(t_1) = \int_{t_o}^{t_1} e^{-r(t-t_o)} Q(t)dt + e^{-r(t_1-t_o)} \psi(t_1)K(t_1)R \tag{19}$$

where $K(t_1) = K(t_o)e^{-\delta(t_1-t_o)}$, $K(t_o)$ is taken as given and all other possible switching points remain constant.

The first and second order derivatives of $W(t_1)$ are:

$$W'(t_1) = e^{-r(t_1-t_o)}[Q(t_1) - V(t_1)] \tag{20}$$

$$W''(t_1) = -rW'(t_1) + e^{-r(t_1-t_o)}[\dot{Q}(t_1) + \delta V(t_1) - RK(t_1)h(y)\dot{\psi}(t_1)] \tag{21}$$

An interior solution for t_1 requires $W'(t_1) = 0$ and $W''(t_1) < 0$. Since $\dot{\psi}(t_1) < 0$ it is necessary that $\dot{Q} + \delta V(t_1) = \dot{Q} + \delta Q(t_1) < 0$.

Notice that the Pontryagin necessary conditions (3) to (5) imply only $W'(t_1) = 0$. The second order condition $W''(t_1) < 0$ is an additional requirement for optimality which becomes relevant if the solution to (3) to (5) is not unique. Indeed, due to the multiplicative appearance of K, P and y, the maximand in (2) need not be concave. In order to gain further insight into the nature of this difficulty, assume that Q(t) is a constant and consider the phase diagram in Figure 1. Suppose that there is a solution to conditions (3) to (5) such that $P = 0$ for $t \in [0, t_1)$ and $P = 1$ for $t \in [t_1,T]$. This solution is presented in the phase diagram by trajectory II. Existence of such a solution implies that there are at least two other solutions, one with $P = 1$ for $t \in [0,T]$ and another with $P = 0$ for $t \in [0,T]$ which also satisfy the necessary conditions (3) to (5). These are presented in Figure 1 by the trajectories I and III respectively. Lemma 2 allows us to compare the value of the objective function associated with each of the three programs. From (21) we see that $\dot{Q}(t) = 0$ implies that $W''(t_1) > 0$ for all t_1 in which $W'(t_1) = 0$.

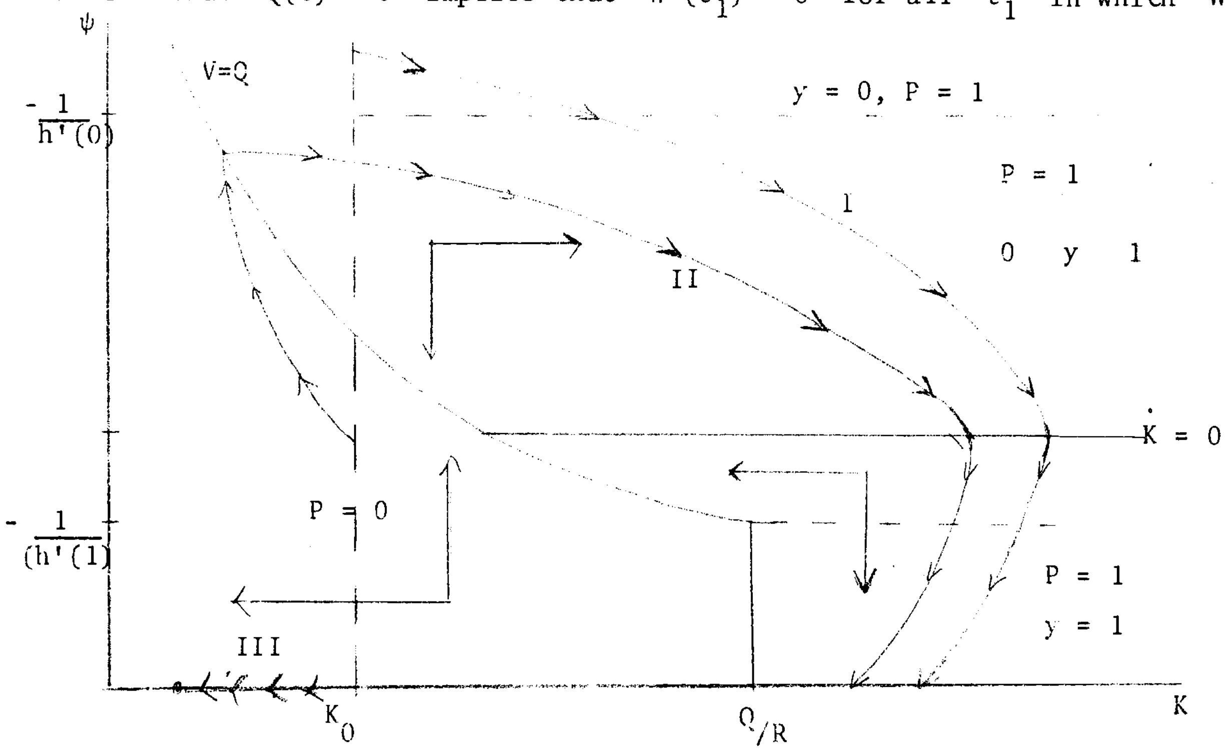


FIGURE 1

Thus either of the "corner" solutions $t_1 = 0$ or $t_1 = T$ is superior to a plan with re-entry at $0 < t_1 < T$.

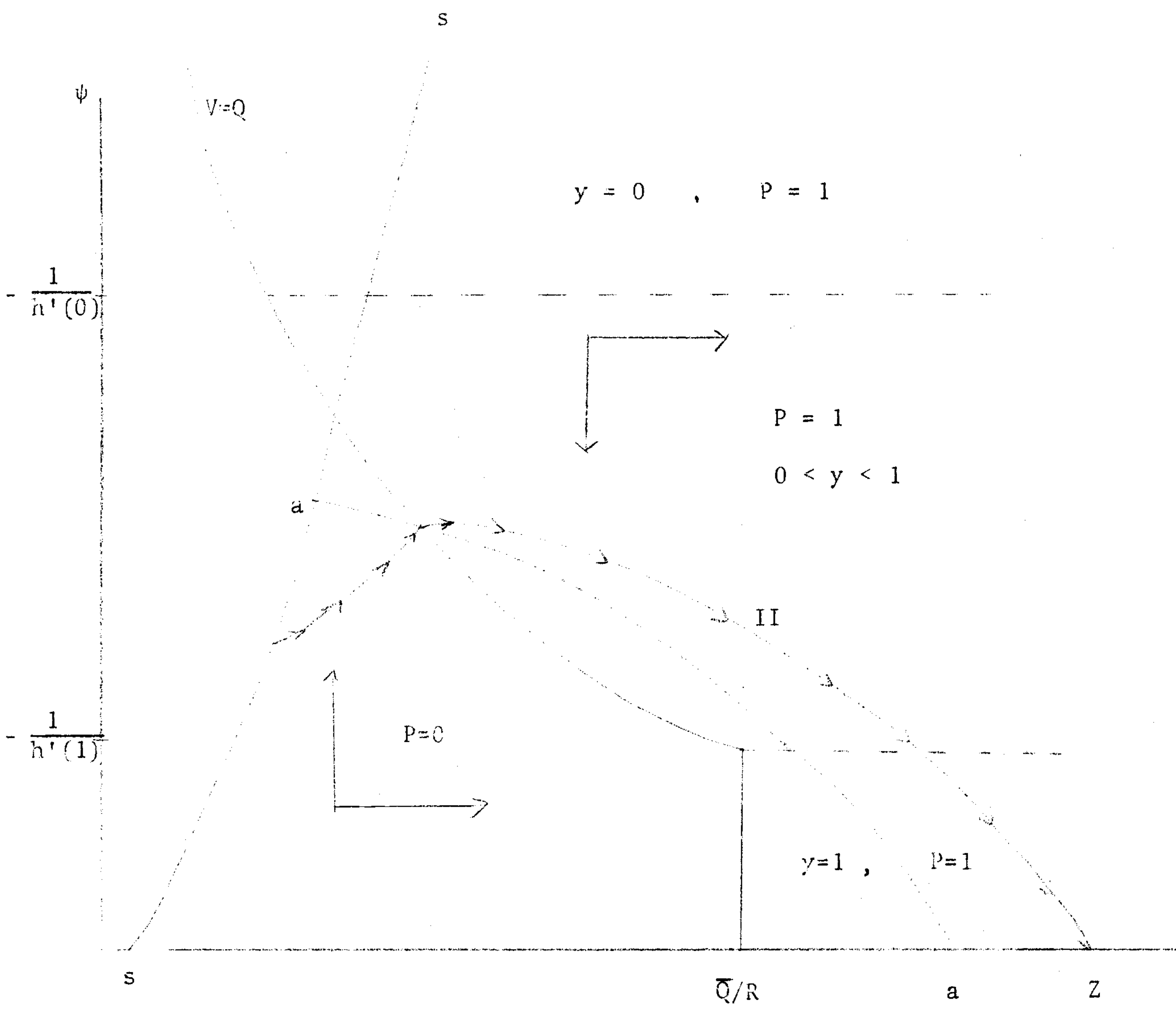
An important corollary of our analysis is that constant productivity at home cannot generate the exit and re-entry patterns of married women. When $Q(t)$ is a constant independent of age, workers will choose either to work throughout their life (with a possible final "retirement" phase) or not to participate at all. The choice between these alternatives depends on the relative magnitudes of Q and RK_0 and on the potential gains from investment in human capital.

It seems likely that productivity of men at home is relatively low and independent of age; their optimal plan is therefore to participate continuously in the labor force. For a married woman (or one who expects to marry) it is more plausible to assume that Q increases at early ages when family size increases and decreases at later ages as children grow up. Moreover, $Q(t)$ may have jumps at points at which birth occur. There is thus a potential for planned exits and entries into the labor force. Let us assume, for simplicity that productivity at home has a single jump, at $t = t_o$, and depreciates at a constant rate thereafter. That is

$$(22) \qquad Q(t) = \begin{cases} 0 & \text{for } 0 \leq t < t_o \\ \overline{Q}e^{-\rho(t-t_o)} & \text{for } t_o \leq t \leq T \end{cases}$$

If we further assume that $\rho > \delta$, then, due to Proposition 2, an exit from the labor force may occur only at the point of discontinuity t_o, and, due to Proposition 3, an interior solution with re-entry at some point $t_o < t_1 < T$ may be optimal. There will be at most, one episode of a planned departure from the labor force. The remainder of this section is devoted to the analysis of this expected departure.

Defining $Z(t) = K(t)e^{\rho(t-t_o)}$ we can use a phase diagram similar to the one used before to describe the solutions for equations (3)to (5) during the phase $[t_o,T]$. For any given departure of length $t_1 - t_o$ one can use (3") and (5") to determine the shadow price of human capital $\psi(t)$ at the point of re-entry t_1 and the point of exit t_o. Given the demand price of human capital at t_o, one can solve for the optimal accumulation from time 0 to t_o, $K(t_o)$ and the stock of human capital at the point of re-entry $K(t_1) = K(t_o)e^{-\delta(t_1-t_o)}$ and obtain the corresponding values of $Z(t_o) = K(t_o)$ and $Z(t_1) = K(t_o)e^{(\rho-\delta)(t_1-t_o)}$. The line ss in Figure 2 describes the level of human capital that will be supplied at t_o for any given shadow price $\psi(t_o)$. Its positive slope reflects the fact that, due to (3") the rate of accumulation of human capital is increasing in ψ. The pairs $Z(t_1)$, $\psi(t_1)$ associated with different prespecified values of re-entry time, t_1, are represented in Figure 2 by the curve aa. For any prespecified value t_1 the optimal trajectory satisfying (3") and (5") must start on ss at time t_o, and cross aa at time t_1. The trajectory corresponding to the optimal value of t_1 satisfying conditions (3"), (5") and (4) must pass through the intersection of aa with $Q = V$ locus. The crucial assumption for the purpose of comparative statics is that



F I G U R E 2

aa has a smaller slope (in absolute value) at the point of intersection than the locus $Q = V$. This assumption guarantees that an interior solution represented in Figure 2 by the trajectory II is the only one satisfying the necessary conditions and is truly optimal.[7/]

Consider now the effect of an increase in $\overline{Q}$. This will shift the locus $V = Q$ to the right. The new intersection with aa will be at a lower level of $\psi(t_1)$, implying an increase in t_1 and the duration of the departure from the labor force.[8/] If the increase in Q is sufficiently large, aa and the locus $V = Q$ do not intersect, i.e., no re-entry occurs. Similarly, an increase in the initial stock of human capital K_0 shifts the ss and aa curves to the right. Since the intersection of aa with the $V = Q$ locus will be at a higher level of $\psi(t_1)$, t_1 must be reduced. (If aa intersects ss above the point where the locus $V = Q$ cuts ss, the person will not withdraw altogether.) Thus, women with a higher initial stock of human capital will plan shorter withdrawals from the labor force. Their earning profiles prior to the withdrawal will be steeper.

We have seen that when labor force interruptions are exogenous, wage discrimination had no effect on earning growth. This is no more true when the length of the interruption is a decision variable. The effect of a reduction in the rental rate for human capital, R, is identical in the present model to that of an increase in $\overline{Q}$. Wage discrimination results in flatter earnings profiles because it increases the length of career interruptions.

3. IMPLICATIONS FOR SEX RELATED EARNINGS DIFFERENCES

The discussion in this paper highlights the strong interdependency between lifetime plans of labor-force participation, the level of earnings, and earnings growth. Differences in the level of earnings reflect both past and future participation plans. We identified, however, models in which differences in earnings growth reflect differences in participation plans. Observing two groups with similar growth rate we may conclude that future plans and thus current investment patterns are similar. We can then ascribe differences in wage levels to differences in earning capacity. Differences in earning capacity not explained by differences in experience or schooling may be ascribed to discrimination provided that past departures from the labor force are viewed as exogenous. If withdrawals from the labor force are partially endogenous, standardization for differences in work history will underestimate the full effect of discrimination on earnings differences. Findings by Farber [1977] suggest that the male-female difference in earnings growth is more pronounced at early ages and tends to vanish later. Thus the natural period in the life cycle for sex related comparisons in earnings is rather late, e.g. the 40-50 age group, contrary to the procedure suggested by Johnson and Stafford [1974] who compare initial salaries.

If women expect longer breaks in their work career, their reduced investment in human capital should not be confined exclusively to investments on the job. Schooling will also be reduced. If one admits the possibility that the interruptions may be due to discrimination, a standardization for schooling in comparing male-female earnings differences yields an underestimate of the full impact. One may also question the legitimacy of the comparison of earnings of women and men with the same level of schooling on the grounds of selectivity bias. Expecting an interrupted work career, women will invest in schooling as much as men, only if they are more efficient investors.[9/]

Expected withdrawals from the labor force, whether caused by discrimination or 'objective' factors, discourage investments in increased earning potential. Investment, however, is not directly observable and the question is; What are the empirical manifestations of the reduction in investment? It is not true that earnings growth is uniformly lower for individuals who choose to invest less. However, for the special case of separable production functions where earnings growth depends only on the level of investment, one obtains an a priori consistency test: any two measured characteristics that can be assumed to have the same effect on investment will have similar effect on earnings growth. Thus, if earnings growth declines with experience, women will have lower growth in earnings than men. This conclusion is consistent with evidence from panel data that the earnings growth rates decrease with experience and are lower for women after adjustment is made for past breaks in experience (see Mincer and Polachek [1974], Weiss and Lillard [1978], Gustafson [1980]), and that women who reported expectations for labor force participation have higher growth in earnings (Sandell and Shapiro [1980]).

The introduction of endogenous departures from the labor force enriches the models of wage growth considerably. The separation between factors that affect the level and growth in earnings, introduced by the simplifying specification of the production function, disappears. In particular, differences in the rental rate and in the initial stock of human capital affect schooling and earnings growth. Discrimination reflected in a lower rental rate for women induces longer planned withdrawals from the labor force. Thus discrimination reveals itself not only in the wage level which women may receive for the same work experience and the same education but, to some extent, in their lower wage growth as well. Findings by Mincer and Ofek [1980] indicate that higher schooling levels are associated with shorter durations of the interruption in labor force participation. Such a relation contributes to a positive interaction between education and earnings growth. One would, therefore, expect the schooling experience interaction to be stronger among women than among men with similar work history. Our preliminary tests using Israeli data fail to support this hypothesis.

The labor force participation model that emerges from our analysis is quite different from the simple view of women as marginal workers moving in and out of the labor force as new wage opportunities arise. Instead, the woman is viewed as choosing between two occupations; work at home and work in the market. The accumulation of human capital, or learning-by-doing in the market sector, is a strong deterrent to occupational mobility (see Weiss [1971]). Even if one extends our model to allow unexpected events, specialization is quite likely. It has been recently argued (Heckman Willis [1977]) that post marital labor force experience of women has a J shaped distribution, a large fraction of women working only a very small part of their time after marriage and a large fraction working through most of their lifetime. Our model provides a rationale for this presumed heterogeneity in behavior. By this explanation, even a symmetric distribution of characteristics such as the initial stock of human capital, K_o, and home productivity, Q, may generate a J shaped distribution of post marital (or more accurately, post birth) labor force participation. A certain fraction of women will never drop out of the labor force since at the time of birth their full value of market activities exceeds their home productivity. The distribution of the other women by the duration of their withdrawal from the labor force depends on the parameters RK_o, ψ_o, r, δ, ρ and t_o. Because of the finite lifetime we may observe a bunching at the other extreme end of the distribution due to truncation (i.e., women who are expected to reenter the labor force at a point $t_1 > T$ never return).

FOOTNOTES

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1. Strictly speaking (1) is a description of the equilibrium wage structure rather than of production technology (see Rosen [1972]).

2. This assumption ignores accumulation of capital in home activities and transferability of capital across activities (see Weiss [1971]). The assumption that human capital acquired at the market has only limited use at home is quite plausible for on-the-job training and for high levels of schooling (e.g. Ph.D Degree). Empirical findings of a positive effect of schooling on labor force participation suggest a bias in schoolings towards market activities.

3. A more realistic formulation of the Blinder and Weiss function is

$$f(K,y) = K \text{ Max}[h(y), a(1-y)] \tag{7'}$$

The two branches of (7') correspond to investment-on-the-job and in school respectively. We may assume that schooling is the more efficient way of acquiring high levels of growth, i.e. $h(o) < a$, and that on-the-job investment is more efficient for low levels of investment, i.e. $h'(1) > -a$. The linearity of the schooling activity reflects the feasibility of mixing schooling with work. This formulation can capture the discontinuity in investment which appears to occur at the end of the schooling period (see Mincer [1974], p.94). Upon entry into the labor force there is a jump from $y = 0$ to $y = y_o$ where y_o is defined implicitly by

$$y_o = \frac{a - h(y_o)}{-h'(y_o)}$$

4. For intervals in which $\dot{\psi} = (r + \delta)\psi - 1$, if $\dot{\psi}$ is ever positive then is positive and increasing thereafter. This contradicts $\psi(T) = 0$. Hence, in particular, $(r + \delta)\psi < 1$ at the point of exit from the phase in which $y = 0$. Therefore $(r + \delta) < g'(K^*)$ where K^* is the amount of capital accumulated up to this point. Since $g''(\ ) < 0$, it follows that $(r + \delta) < g'(K)$ for all $K_o \leq K \leq K^*$.

5. We assume $h(0) > r + \delta$. For $y = 0$ and $P = 1$ it follows immediately that $\dot{\psi} = \psi[r + \delta - h(0)] < 0$. For $0 < y < 1$ and $P = 1$ we have $\dot{\psi} = \psi[r + \delta - h(y) + yh'(y)]$, since $\frac{\partial}{\partial y}[(-h(y) + yh'(y)] = yh''(y) < 0$, $h(y) - yh'(y) > h(0)$ and $\dot{\psi} < 0$. For $y = 1$ $P = 1$ $\dot{\psi} = (r + \delta)\psi - 1$ and must be negative or else the transverslity condition $\psi(T) = 0$ will be violated. A similar argument holds for specification (7') in footnote 3, except that we assume $a > r + \delta$ and use the fact that $h(1 + E_h) = a$ at y_o.

6. Under specification (6) the period of specification is determined by the condition $\frac{1}{g'(K)} = \psi(t)$. Since $\frac{1}{g'(K)}$ is increasing with age throughout the interval $y = 0$ (this can be shown to hold even if $\delta > 0$) and since $\psi_B(t)$ and $\psi_A(t)$ are both decreasing while $\psi_B(t) < \psi_A(t)$ the specialization phase will end earlier for B. Similarly under specification (7) a switch occurs when $\psi(t) = -\frac{1}{h'(0)}$, again since $\psi_B(t)$ and $\psi_A(t)$ are decreasing, $\psi_B(t) < \psi_A(t)$ and $-\frac{1}{h'(0)}$ is a constant, B will start to have positive earnings at an earlier age. The same is true if we use specification (7') in footnote 3 except that there will be a jump from 0 to y_o when $\psi(t) = -\frac{1}{h'(y_o)}$.

7. Taking the derivative of (19), allowing $k(t_o)$ to vary, we obtain the following second order condition:

$$(21') \qquad \dot{Q}(t_1) + \delta V(t_1) - RK(t_o)h(y)\,\dot{\psi}(t_o) - V(t_1)\,\frac{\partial \ln K(t_o)}{\partial t_1} < 0$$

The slope of the locus $V = Q$, evaluated at t_1, is given by:

$$\frac{d\psi}{dZ} = -\,\frac{[y + \psi(t_1)h(y)]}{Z(t_1)h(y)}$$

The slope of the aa locus, evaluated at t_1, is given by:

$$\frac{d\psi}{dZ} = \frac{\dot{\psi}(t_1)}{(\rho-\delta)Z(t_1) + Z(t_1)\,\dfrac{\partial \ln K(t_1)}{\partial t_1}}$$

Comparing the two slopes it is seen that (21') implies that the slope of aa is smaller in absolute values.

8. A direct proof can be derived from the following inequalities:

$$(23)\quad \int_{o}^{t_o} e^{-rt}R\overset{*}{K}y\,dt + \int_{t_o}^{t_1^*} \overset{*}{Q}\, e^{-(\rho+r)t}dt + \int_{t_1^*}^{T} e^{-rt}R\overset{*}{K}y\,dt > \int_{o}^{t_o} e^{-rt}R\hat{K}y\,dt + $$

$$+ \int_{t_o}^{\acute{t}_1} e^{-(\rho+r)t}\overset{*}{Q}\,dt + \int_{\hat{t}_1}^{T} e^{-rt}R\hat{K}y\,dt$$

$$(24)\quad \int_{o}^{t_o} e^{-rt}R\hat{K}y\,dt + \int_{t_o}^{\hat{t}_1} \hat{Q}e^{-(\rho+r)t}dt + \int_{\hat{t}_1}^{T} e^{-rt}R\hat{K}y\,dt > \int_{o}^{t_o} e^{-rt}R\overset{*}{K}y\,dt + $$

$$+ \int_{t_o}^{t_1^*} \hat{Q}\, e^{-(\rho+r)t}dt + \int_{t_1^*}^{T} e^{-rt}R\overset{*}{K}y\,dt$$

where $\overset{*}{Q} > \hat{Q}$ and $\overset{*}{K}y$ is evaluated along the path optimal for $\overset{*}{Q}$ and $\hat{K}y$ is evaluated along the path optimal for Q. These inequalities hold because the accumulation path $\overset{*}{K}y$ is feasible also at the 'price' $\hat{Q}$ and vice versa. Subtracting (24) from (23) and rearranging yields:

$$(25)\qquad (\overset{*}{Q} - \hat{Q}) \int_{\hat{t}_1}^{t_1^*} e^{(\rho-r)t}dt > 0 \text{ implying, implying } t_1^* > t_1.$$

9. It is the <u>relative</u> efficiency of investment in school and on the job which is relevant for the determination of the schooling period. Women may invest as much in schooling as males despite their shorter horizon because they find that they cannot invest on the job at the same terms of trade as males.

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