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# REAL AND MONETARY DISTURBANCES IN AN EXCHANGE-RATE UNION

Richard C. Marston

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# Real and Monetary Disturbances in an Exchange-Rate Union

## ABSTRACT

This paper investigates how a small country fares in an exchange-rate union if that country is subject to real and monetary disturbances originating at home and abroad. By joining a union, the country can fix the exchange rate between its currency and the currency of another country or countries. The paper asks whether or not fixing this exchange rate helps to modify the effects of disturbances on the domestic economy.

This question is investigated within a model consisting of an aggregate demand equation dependent upon the terms of trade, an aggregate supply equation in which labor supply is responsive to the general price level, and a financial equation that determines the exchange rate of the domestic currency relative to one of two foreign currencies (the other being determined by triangular arbitrage). Aggregate supply behavior varies depending upon whether wages respond to prices with a lag or are indexed to current changes in the general price level. Because the small country model cannot be used by itself to analyze the effects of foreign disturbances, the paper introduces models of two foreign countries with the same analytical structure as the domestic country model. Foreign disturbances are studied in two stages, first within the foreign model, then within the domestic model.

The analysis shows that one of the most important factors determining the effects of the union is the degree of wage indexation in the domestic economy. The greater the degree of indexation, the less difference there is between output variation in the union and in a flexible regime. Apart from wage behavior, two other factors are important: the sources of the disturbances and the pattern of trade. Contrary to common belief, the case for a union is not necessarily strengthened if disturbances primarily originate outside the union and if the domestic country trades primarily with other members of the union.

Richard C. Marston
The Wharton School
University of Pennsylvania
Philadelphia, Pennsylvania 19104
(215) 243-7626

Visiting Professor (Spring 1981)
Ecole Superieure des Sciences
Economiques & Commerciales
BP 105
95021 Cergy-Pontoise Cedex
France

Although the current period is often characterized as one of flexible exchange rates, many currencies are tied together in joint floats against other currencies, forming exchange-rate unions within a system of flexible rates. This paper investigates how a small country fares by joining such an exchange-rate union.(1)

The country is assumed to be buffetted by real and monetary disturbances originating at home and abroad. By joining the union, this country is able to fix the exchange rate between its currency and the currencies of the union countries. The central question addressed in the paper is whether or not fixing this exchange rate helps to modify the effects of disturbances on the domestic economy.(2) The specific sources of the distubances clearly matter in determining the impact of the union and so also does the pattern of trade between home and foreign countries. The paper will investigate two intuitively plausible propositions about these factors : that the case for a union is stronger when disturbances primarily originate outside the union and when the home country trades primarily with other countries in the union. The paper shows that neither proposition holds in general.

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(1) The terminology here is potentially confusing. In this study, the term exchange-rate union refers to an arrangement in which member countries of the union maintain fixed exchange rates between member currencies, but with each country retaining its own central bank with control over its national monetary policy. This limited type of union, which Corden (1972, p. 3) calls a "pseudo-exchange-rate union", is to be distinguished from a "complete exchange rate union", or monetary union, with a single central bank and a union-wide currency.

(2) Tower and Willett (1975) provide a comprehensive survey of the literature on optimum currency areas which examines the conditions necessary for successful exchange-rate or monetary unions ; McKinnon (1963) and Mundell (1961) are two early contributions to this literature. Among other studies are Allen (1976), Corden (1972), and Ingram (1973). The method of analysis used in the present study is more similar to that of the internal-external balance literature, of which Mundell's (1963) study of fixed and flexible rates is the best known.

The macro model used to address these issues consists of three basic equations : an aggregate demand equation dependant upon the terms of trade as well as foreign output, an aggregate supply equation in which labor supply is responsive to the general price level, and a financial equation that determines the exchange rate of the domestic currency relative to one of the two foreign currencies (the other being determined by triangular arbitrage). The paper shows to what extent aggregate supply and demand behavior are important in determining the effects of the union. Supply behavior varies depending upon whether wages respond to prices with a lag or are indexed to current changes in the general price level, while demand behavior is particularly sensitive to price elasticities.

The small country model has the advantage of analytic simplicity, but by itself gives misleading results when foreign disturbances are examined. The model can be used to show, for example, that variations in foreign income due to foreign disturbances anywhere abroad unambiguously lead to greater domestic output variance in the union. But without a model of the two foreign regions, the union countries and the rest of the world, it is difficult to determine the <u>total</u> effects of the underlying foreign disturbances, since these disturbances affect the domestic economy through a variety of channels. For example, in addition to changing foreign output, a foreign demand disturbance concentrated in one of the two foreign regions will change the terms of trade between the foreign goods imported by the small country and the exchange rate between the two foreign currencies. The total effects of this foreign disturbance thus may be quite different from the effects of a change in foreign income alone.

To study foreign disturbances, the paper introduces a model of the two foreign trading regions with the same analytical structure as the small country model. This model determines the exchange rate and terms of trade between the two regions as well as the prices, interest rates, and output in each region. Foreign disturbances are studied first within this model, then are traced through the domestic model to determine the effects on the small country.

The first section of the paper introduces the small country model, and the next two sections successively examine the effects of disturbances within the one country and extended models.

## I. OUTLINE OF THE MODEL

Country 1 is a small country which has economic ties with two countries : Country 2 is the potential partner of Country 1 in an exchange-rate union, while Country 3 represents the rest of the world. Country 1 has important trade and financial ties with the other two countries ; the relative importance of these economic ties will be an issue in the analysis below. Country 1 is assumed to be too small to influence conditions in either foreign country.(3) The two foreign countries affect Country 1 through their outputs ($Y_t^2$, $Y_t^3$), the prices of their goods ($P_t^2$, $P_t^3$), both expressed in logarithms, and interest rates ($r_t^2$, $r_t^3$).

In this three country world, there are three exchange rates to be determined. If the franc is the currency of Country 1, the mark the currency of Country 2 and the dollar the currency of Country 3, then the three exchange rates are as follows (all expressed in logs) :

(3) The two foreign countries could represent blocs of countries with fixed exchange rates within each bloc, in which case Country 1 does not need to be small relative to individual countries in each bloc. Country 2, for example, might represent a group of countries in an existing union (e.g., the European Monetary System) and Country 3 represent a second group of countries tied to the dollar.

$X_t^1$ : the franc price of the dollar,

$X_t^2$ : the mark price of the dollar, and

$X_t^{12} = X_t^1 - X_t^2$ : the franc price of the mark.

Figure 1 illustrates the relationships among the three currencies. $X_t^2$ is exogenously determined for Country 1, while a second exchange rate is determined by triangular arbitrage.

A. Demand Behavior

Unless purchasing power parity is assumed, demand behavior in a country trading with two other countries is inherently complicated. Three national prices are involved as well as at least two exchange rates. We begin by defining the prices of the three countries' goods expressed in francs :

$P_t^1$ : price of Country 1's good,

$P_t^3 + X_t^1$ : price of Country 3's good in francs,

$P_t^2 + X_t^{12}$ : price of Country 2's good in francs.

It is convenient for later analysis to measure Country 2's prices relative to Country 3. Define the terms of trade between Countries 2 and 3 as :

$$T_t = P_t^2 - (P_t^3 + X_t^2).$$

Then the price of Country 2's good in francs can be written as $P_t^3 + T_t + X_t^1$.

The general price level in Country 1 can be expressed as a weighted average of the prices of the three goods :

$I_t^1 = a_{11} P_t^1 + a_{12}(P_t^3 + T_t + X_t^1) + a_{13} (P_t^3 + X_t^1)$, where $a_{1j}$ is the

expenditure weight for country j's good.(4)

There are two relative prices affecting demand in Country 1, the price of Country 1's good relative to each of the foreign goods. The demand for domestic output can be expressed as a function of these relative prices, output in the two foreign countries, the real rate of interest, and a stochastic factor with mean zero and serially uncorrelated(5):

$$(1) \quad Y^1_t = g_0 + g_{p2}(P^3_t + T_t + X^1_t - P^1_t) + g_{p3}(P^3_t + X^1_t - P^1_t)$$

$$g_{y2}\, Y^2_t + g_{y3}\, Y^3_t - g_r\left[r^1_t - ({}_tEI^1_{t+1} - I^1_t\right] + u^{d1}_t.$$

A rise in foreign prices relative to the domestic price is assumed to increase aggregate demand as is a rise in foreign output.(6) A rise in the real interest rate is assumed to reduce aggregate demand.(7) In the case of perfect substitution between foreign and domestic goods, where $g_{pj}$ becomes infinite in size, this aggregate demand equation reduces to the familiar purchasing power parity relationship.

B. Supply Behavior

When a country imports many of the products it consumes, two prices are important for supply decisions, the price of domestic output ($P^1_t$) and the general price level ($I^1_t$) based on foreign and domestic prices and foreign exchange rates.

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(4) When expressed in levels, this price index has a geometric form with weights $a_{11} + a_{12} + a_{13} = 1$.

(5) In the appendix this demand function is derived from a more standard demand function, and the coefficients, $g_{pi}$ and $g_{yi}$, are expressed in terms of conventional income and price elasticities.

(6) We assume that the Marshall-Lerner condition holds, so that the price coefficients are positive. (See the appendix).

(7) ${}_tEI^1_{t+1}$ is Country 1's general price level at period t+1 expected at period t, so $({}_tEI^1_{t+1} - I^1_t)$ is the expected change in that price level. ${}_tEI^1_{t+1}$ is assumed to be formed rationally from the rest of the model.

Output decisions are responsive to nominal wages relative to the price of domestic output, while wages are sensitive to the general price level. In many modern economies, moreover, wages are at least partially indexed to current changes in the general price level. As a result, supply as well as demand can respond to current changes in foreign prices and exchange rates.

The model chosen to represent this supply behavior consists of a production function of the Cobb-Douglas form and a wage equation based on the partial or complete indexation of wages.[8] The wage equation assumes that labor ties its wage to the general price level in an attempt to maintain a constant value of purchasing power despite frequent changes in prices and exchange rates. Because of lags in labor contracts, the current wage is based on last period's expectation of current prices. Wages may be indexed, however, to protect the wage earner from unexpected changes in the price level[9] :

$$(2a) \qquad Y_t^1 = (1 - c')\, L_t^1 + c_0'$$

$$(2b) \qquad W_t^1 = {}_{t-1}E\, I_t^1 + b(I_t^1 - {}_{t-1}E\, I_t^1).$$

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(8) As with the other structural equations in the model, the supply equations were kept relatively simple. Since this model is used to represent all three countries (as modified below) any additional complexities are multiplied by a factor of three.

(9) For studies of wage indexation behavior, see Fischer (1977), Gray (1976), Modigliani and Padoa-Schioppa (1978), and Sachs (1979), the latter two dealing with open economies. Flood and Marion (1980) provide an interesting analysis of how indexation behavior may respond to the choice of exchange rate regime. Here we have assumed that the degree of indexation has been fixed institutionally, leaving optimal indexation behavior in a union for later analysis.

The coefficient, b, the indexation parameter, typically varies between zero (no indexation) and one (full indexation).[10]

If domestic producers choose an output level where the marginal product of labor is equal to their real wage, $W_t^1 - P_t^1$, then output can be expressed as follows :

$$(2) \quad Y_t^1 = c_0 + c(P_t^1 - I_t^1) + c(1 - b)(I_t^1 - {}_{t-1}E\, I_t^1),$$

where $c = (1 - c')/c'$

$$c_0 = (\log c_0')/c' + c \log(1-c')$$

Whatever the degree of indexation in the economy, output depends upon the price of domestic output as well as the general price level. With full indexation, only current prices matter, but with partial indexation output responds to unanticipated changes in the general price level through the differential impact of changes in prices on the real wages faced by producers and workers, respectively. Notice that when indexation is complete (b=1), equal increases in $P_t^1$ and $I_t^1$ leave supply unaffected. When indexation is less than complete, however, equal increases in $P_t^1$ and $I_t^1$ increase supply since the real wages faced by producers fall.

C. Financial Behavior

The three countries have two financial assets each : money and bonds (the latter bearing interest rate $r_t^i$). To keep the financial sector simple, however, the three bonds are assumed to be perfect substitutes so that their expected returns expressed in the same currency are equal.[11]

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(10) Note that there are some countries (notably Italy) where indexation has been greater than 100 %. See Modigliani and Padoa-Schioppa (1978).

(11) For a more general study of financial behavior in an exchange-rate union, see Marston (1980 b). In that study where the effects of various financial disturbances are analyzed, foreign and domestic bonds are assumed to be imperfect substitutes.

$$r_t^1 = r_t^3 + ({}_tE\, X_{t+1}^1 - X_t^1) ,$$

$$r_t^2 = r_t^3 + ({}_tE\, X_{t+1}^2 - X_t^2) .$$

The demand for money is expressed as a function of domestic prices and output as well as the interest return,

$$(3) \quad M_t^1 = P_t^1 - k_1(r_t^3 + {}_tE\, X_{t+1}^1 - X_t^1) + k_2\, Y_t^1 + k_0 ,$$

where for convenience the expected franc return on Country 3's bond replaces Country 1's interest rate.

The behavior of the money supply depends upon the exchange rate regime. Under flexible exchange rates, the money supply is assumed to be exogenously determined as follows :

$$M_t^1 = \bar{M}^1 + u_t^{m1} .$$

The current money supply is equal to a base level plus a random term, where the latter has mean zero and is serially uncorrelated. The supply of money in the exchange-rate union is described below.

D. Foreign Behavior

All foreign variables in the model with the exception of $X_t^1$ are exogenously determined by the small country assumption. We express each foreign variable as a constant plus a random variable where the latter is the innovation in that variable for period t.

$$P_t^i = \bar{P}^i + u_t^{pi} ,$$

$$Y_t^i = \bar{Y}^i + u_t^{yi} ,$$

$$X_t^2 = \bar{X}^2 + u_t^{x2} ,$$

$$T_t = \bar{T} + u_t^T ,$$

$$r_t^i = \bar{r}^i + u_t^{ri} , \quad i = 2,3.$$

In the third part of the paper, the random variables will be expressed in terms of the underlying foreign disturbances.(12) Until then, these variables are assumed to have zero mean, to be serially uncorrelated, and to be uncorrelated with each other.(13)

E. Description of the Two Exchange Rate Regimes

The exchange rate union will be compared with a regime of flexible exchange rates where no exchange market intervention occurs. The flexible regime is discussed first.

Flexible Exchange Rates :

The basic equations of the model, (1) - (3), determine three domestic variables, $Y_t^1$, $P_t^1$, and $X_t^1$, as functions of all the stochastic and nonstochastic variables. ($X_t^{12}$ is then determined by triangular arbitrage). To facilitate comparison between the two regimes, equations (1) and (2) are first solved for $Y_t^1$ and $P_t^1$ as functions of $X_t^1$ and the exogenous variables. The resulting expressions, equations (I) and (II) in Table 1, describe aggregate demand and supply behavior in both exchange-rate regimes.(14) The stochastic variables influencing $Y_t^1$ and $P_t^1$ include the domestic aggregate demand disturbance, $u_t^{d1}$, as well as the random components of $X_t^1$ and the foreign variables.

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(12) The foreign variables can be expressed in this simple form because the underlying disturbances are assumed to be of mean zero and to be serially uncorrelated (see Section III).

(13) As a result, ${}_tEP_{t+1}^3 = \bar{P}^3$, and similarly for the other variables.

(14) $\bar{Y}^1$, $\bar{P}^1$ and $\bar{X}^1$ are the reduced form solutions for $Y_t^1$, $P_t^1$, and $X_t^1$ when all disturbances are equal to zero. To obtain the coefficients of $u_t^T$ in Table 1, we have assumed that the shares of the two foreign goods in the price indices are proportional to the respective price elasticities in the aggregate demand equation : $a_{12}/a_{13} = g_{p2}/g_{p3}$. This restriction allows trade to be biased toward one foreign country or the other (see below), but the bias must be equally reflected in the $a_{1j}$'s and $g_{pj}$'s.

Under flexible rates, the franc price of the dollar, $X_t^1$, can be expressed as a function of exogenous variables only by solving all three equations, (1) - (3), for the reduced form. Equation (IIIa) in Table 1 presents the expression for $X_t^1$.

Exchange-Rate Union :

If Country 1 joins an exchange-rate union with Country 2, the monetary authorities must intervene in the exchange market to ensure that $X_t^{12} = X_t^1 - X_t^2$ remains constant. For convenience we assume that the franc price of the mark is initially equal to one (so that the log of this exchange rate, $X_t^{12}$, is initially equal to zero). In that case, intervention keeps $X_t^1 = X_t^2$ at all times. Suppose that the foreign intervention is carried out by the monetary authority of Country 1 which buys (or sells) Country 2's currency and sells (or buys) Country 1's currency to keep $X_t^{12}$ fixed. (The results would be the same if Country 2 carried out this intervention).[15] In that case, equation (3) describing money market equilibrium simply determines the money supply consistent with keeping $X_t^1$ equal to $X_t^2$. In place of equation (IIIa) determining $X_t^1$ on the basis of Country 1's behavior, we have an exogenously determined exchange rate :

$$X_t^1 = X_t^2 = \bar{X}^2 + u_t^{x2}.$$

By joining the union, Country 1 has not only fixed its mark exchange rate, but has effectively surrendered control over its dollar exchange rate.

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(15) The choice of which country to intervene would be important if the intervention took the form of buying or selling foreign bonds (as would be the case if the foreign currency were used as a reserve currency), but here we assume a simple form of intervention with no sterilization of the intervention effects. For an analysis of different types of intervention policy, see Marston (1980 a).

## II. EVALUATION OF THE UNION : DOMESTIC DISTURBANCES

In order to judge the usefulness of the exchange-rate union, we need to formulate appropriate criteria. A variety of criteria are possible since the economic effects of the union have many dimensions. In this paper, we concentrate our attention on one traditional criterion for judging any change in policy, the effect of the policy on the variance of domestic output :

$$V = E \left(Y_t^1 - E(Y_t^1)\right)^2 .$$

For each disturbance in the model, we ask if introduction of the union increases or decreases the variance of output due to that disturbance. Because of the simple stochastic structure described below, moreover, we can determine the relative sizes of the output variances by comparing the increases or decreases in output in response to a single (one period) disturbance.

### A. Domestic Disturbances

The two disturbances to be analyzed are a money supply disturbance and an aggregate demand disturbance. We assume initially that there is no wage indexation in the domestic economy, then consider the effects of full indexation.

The monetary disturbance is particularly easy to analyze given the small country assumption. This disturbance has no effect on output in an exchange-rate union. A monetary expansion, for example, leads to pressure on both franc exchange rates because of the incipient fall in domestic interest rates, but intervention in the exchange market ensures that $X_t^1 = X_t^2$ (where the latter is exogenously determined). The disturbance simply results in an equal and offsetting capital flow.(16) In a flexible

(16) If Country 1 were not small, its dollar exchange rate could change because the monetary expansion together with the accompanying foreign exchange intervention would also significantly affect the money supply in Country 2.

regime, in contrast, a monetary expansion leads to a depreciation and to an increase in output.(17) Thus as in the case where only one foreign country is involved (see Mundell, 1963), domestic monetary disturbances cause variations in output only under flexible exchange rates.

In contrast, an aggregate demand disturbance in Country 1 leads to greater changes in output in an exchange-rate union. An increase in aggregate demand raises domestic output and prices. There is pressure on both franc exchange rates because of the incipient rise in domestic interest rates, but once again intervention in the exchange market ensures that $X_t^1 = X_t^2$. Under flexible exchange rates, in contrast, the increase in output leads to an appreciation of the domestic currency ($X_t^1$ and $X_t^{12}$ fall) which modifies the overall increase in aggregate demand.(18) Thus there is less output variation under flexible rates.

When wage indexation is complete (b=1), however, these familiar results break down. Aggregate demand disturbances or monetary disturbances then have identical effects in the two regimes ; that is, the variance of output is the same regardless of the exchange rate regime. The reason can be seen by examining Figure 2 showing how aggregate demand and supply adjust to these disturbances.

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(17) The effect of $u_t^{x1}$ on $Y_t^1$ in equation (I) can be rewritten for the general case where $0 \leq b \leq 1$) :

$$Y_t^1 = \bar{Y}^1 + \frac{(g_{p2} + g_{p3} + g_r\, a_{11})\; c(1-b)}{Z}\, u_t^{x1},$$

where the coefficient of $u_t^{x1}$ must be positive as long as b is less than one.

(18) Under flexible exchange rates, the direct (positive) impact of $u_t^{d1}$ on $Y_t^1$ in equation (I) is modified by the fall in $u_t^{x1}$ (which reduces $Y_t^1$). Output nonetheless increases even under flexible rates. In contrast, Mundell's study of internal-external balance showed that output did not increase at all in response to a demand disturbance ; given his assumptions that domestic prices were fixed and exchange rate expectations static, only a constant output was consistent with money market equilibrium. In this study, an appreciation of the exchange rate leads to a change in the expected exchange rate and therefore to a change in the domestic interest rate, so output can increase despite a constant money supply.

First consider the monetary disturbance. Under flexible rates, the resulting depreciation leads to an outward shift in the aggregate demand function (to point B) because of the sensitivity of aggregate demand to relative prices and the real interest rate. When wages are fully indexed to the general price level, however, there is a corresponding leftward shift in aggregate supply (to point C) because the depreciation leads to rise in the nominal wage. The price of domestic output accordingly rises further ; in fact, the price of domestic output and the exchange rate increase by the same amount, with the constant terms of trade ensuring that output remains fixed at its original level.(19) In the union, in contrast, no depreciation occurs, so the aggregate supply and demand curves remain at point A. With full indexation of wages, therefore, the monetary disturbances leaves output unaffected in both regimes. The two regimes differ with respect to price behavior, however, since prices increase only under flexible rates.

The aggregate demand disturbance does change real output in both regimes. In the union, the demand disturbances shifts equilibrium to point A'. Under flexible rates, the increase in demand leads to an appreciation of the franc and to a smaller increase in aggregate demand than in the union (to point B'). With complete indexation of wages, however, the appreciation of the exchange rate also raises aggegate supply by lowering real wages faced by producers. Point C' is reached where the

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(19) Under full indexation, the aggregate supply equation becomes a simple function of the terms of trade, while the aggregate demand equation is a function of the terms of trade and the real interest rate (as well as foreign output which is exogenous). Because the expected change in the exchange rate is equal to the expected change in the domestic price as well as in the general price level, there is no change in the real interest rate. The real interest rate could change if the disturbances took a more complicated form, in which case there would be some changes in output (proportional to $g_r$) even under full indexation.

change in output in the same as in the union. Thus, when wages are fully indexed, the variance of output is the same in either regime.

The case for an exchange-rate union thus depends upon whether domestic monetary or demand disturbances are more important. But with respect to both types of disturbances, the difference between regimes diminishes the more highly indexed are wages in the economy.

## III. EVALUATION OF THE UNION : FOREIGN DISTURBANCES

Foreign disturbances affect the domestic economy through a variety of channels :

(a) Changes in foreign output directly affect domestic aggregate demand (proportionally to $g_{y2}$ and $g_{y3}$).

(b) Changes in foreign prices or in the foreign terms of trade induce substitution with the domestic good (depending upon the price elasticities, $g_{p2}$ and $g_{p3}$).

(c) In the union, changes in the mark price of the dollar affect aggregate demand (through those same price elasticities).

(d) Foreign interest rates affect domestic demand indirectly by changing the franc price of the dollar under flexible rates and directly through the real interest rate effect on aggregate demand.

All of these channels are potentially important. For many disturbances, moreover, Country 1's output is pushed in contrary directions.

The net effect of foreign disturbances on Country 1 depends upon which of these channels are most important. That, in turn, depends upon the types of economic disturbances encountered as well as on the pattern of trade. We might expect the union to be more desirable if economic disturbances predominantly originate from outside the union and if the pattern of trade is biased so that Country 1 trades primarily with Country 2. Both of these propositions will be explored in the analysis below.

In order to investigate foreign economic disturbances, we need to specify models of the two foreign countries. To keep the models relatively simple, we adopt the following assumptions :

1. Countries 2 and 3 have identical economic structures. That is, all the structural parameters in the aggregate supply and demand equations and in the financial equations are the same for the two countries. Thus we can specify one national model which can apply to both countries.(20) This assumption does not restrict the nature of Country 1's relationships with the two foreign countries since any asymmetries in Country 1's links with the foreign countries could not affect the latter given the small country assumption. More specifically, Country 1 may choose to trade with one country more than the other, or may be more sensitive to one country's prices or output.

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(20) The countries' price indexes, for example, are mirror images of one another ; the weight of each country's own good in its price index is the same for both countries, $a_{22} = a_{33} = a$. Note that Country 1's prices and output are assumed to have a negligible impact on the foreign countries ; for example, $a_{21}$ and $a_{31}$, the weights of Country 1's good in the foreign price indexes, are assumed to be negligibly small.

2. In both foreign countries, there is complete wage indexation so that equal increases in domestic prices and exchange rates (or foreign prices) leave aggregate supply unaffected. Foreign output is then solely a function of the terms of trade. This assumption considerably simplifies the analysis of the two foreign countries making it easier to show the channels through with foreign disturbances affect Country 1. Even with the simplifying assumption, the effects of foreign aggregate demand disturbances are ambiguous, but we are able to distinguish clearly which factors are important in determining the effects of these disturbances.

We leave for later analysis cases where foreign countries are asymmetric in economic structure or size and cases where they exhibit significant money illusion or lags in adjustment of wages to prices.

A. The Foreign Country Model

The model closely parallels that one specified for Country 1. Differences arise primarily because only two countries are involved, given the small country assumption for Country 1, and because the aggregate supply equation takes a simple form when wage indexation is complete. Each national model can be expressed in three equations (where $i = 2,3$ , $j \neq i$) :

$$(1)' \quad Y_t^i = g_0^f - g_p^f T_t^i + g_y^f Y_t^j - g_r^f \left[ r_t^i - ({}_tE\, I_{t+1}^i - I_t^i) \right] + u_t^{di} ,$$

$$(2)' \quad Y_t^i = c_0^f + c^f (1-a) T_t^i ,$$

$$(3)' \quad \bar{M}^i + u_t^{mi} = P_t^i - k_1 r_t^i + k_2 Y_t^i + k_0 ,$$

where $T_t^2 = T_t = P_t^2 - (P_t^3 + X_t^2)$ ,

$$T_t^3 = - T_t ,$$

$$I_t^2 = a P_t^2 + (1-a)(P_t^3 + X_t^2) ,$$

$$I_t^3 = a P_t^3 + (1-a)(P_t^2 - X_t^2) .$$

Aggregate demand is a function of the terms of trade, $T_t$, foreign output, and the real interest rate as well as a disturbance term, $u_t^{di}$. Aggregate supply is a simple function of the terms of trade. Money demand and supply in the foreign countries are assumed to be identical in form to that of Country 1.(21)

The two country model is naturally complex, but its behavior can be analyzed relatively easily by focusing on two variables, the terms of trade of Country 2 and the price of Country 3's good.(22) The remaining variables can then be expressed in terms of these two.

The terms of trade of Country 2, defined as $T_t = P_t^2 - (P_t^3 + X_t^2)$, can be expressed as a function of the two aggregate demand disturbances, $u_t^{d2}$ and $u_t^{d3}$, plus a non-stochastic term :

$$T_t = \bar{T} + \frac{(u_t^{d3} - u_t^{d2})}{(N^1 - N^2)} ,$$

where $N^1 = - g_p^f - c^f(1-a)(1 + g_y^f + k_2 g_r^f/k_1) + g_r^f(1-a) < 0,$

$$N^2 = g_p^f + c^f(1-a)(1 + g_y^f - k_2 g_r^f/k_1) + g_r^f a > 0.(23)$$

A rise in aggregate demand in Country 3 lowers the terms of trade (since $N^1 - N^2 < 0$), while a rise in aggregate demand in Country 2 raises the terms of trade. With full indexation, monetary disturbances have no effect on the terms of trade.

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(21) More specifically, we assume that the interest and income elasticities of the money demand functions are the same as those in Country 1. The disturbance terms, $u_t^{di}$ and $u_t^{mi}$, are assumed to have mean zero and to be serially uncorrelated.

(22) The price of Country 2's good, alternatively, could have been singled out since the model is symmetric with respect to the two countries. You will recall, however, that in Country 1's model, the foreign prices were defined relative to Country 3's price.

(23) The real interest rate effect on aggregate demand is assumed to be small enough so that $N^1$ and $N^2$ have the signs indicated.

The price of Country 3's good is a function of the aggregate demand disturbances as well as the monetary disturbance in Country 3 :

$$P_t^3 = \bar{P}^3 + \frac{k_1}{g_r^f(1+k_1)} \frac{\left[N^1 u_t^{d2} - N^2 u_t^{d3}\right]}{(N^1 - N^2)} + \frac{u_t^{m3}}{(1+k_1)}$$

A rise in aggregate demand in either country increases the price of Country 3's good (as well as Country 2's good), while an increase in Country 3's money supply also raises this price.

Output in either country is a simple function of the terms of trade as shown in equation (2)'. Note that a rise in the terms of trade increases Country 2's output (because it reduces the real wage faced by producers) but decreases Country 3's output. In fact, the increase in Country 2's output is entirely at the expense of Country 3 when the two countries are identical and each country's output is a function of the terms of trade alone.

The two remaining foreign variables appearing explicitly in Country 1's model, $r_t^3$ and $X_t^2$, can be expressed in terms of $P_t^3$, $T_t$, and the monetary disturbance by solving the money market equilibrium conditions[24] :

$$r_t^3 = \bar{r}^3 + \frac{1}{k_1}\left[(P_t^3 - \bar{P}^3) - k_2 c^f (1-a)(T_t - \bar{T}) - u_t^{m3}\right]$$

$$X_t^2 = \bar{X}^2 - \frac{1}{(1+k_1)}\left[(1 + 2k_2 c^f (1-a))(T_t - \bar{T}) + (u_t^{m3} - u_t^{m2})\right]$$

Monetary disturbances naturally affect both nominal variables, although an equal increase in both money supplies leaves the mark price of the dollar unaffected since there are no asymmetries between the two economies. An

(24) In deriving the expression for $X_t^2$, we have assumed that the interest rates are initially equal, $\bar{r}^2 = \bar{r}^3$.

aggregate demand disturbance affects both variables by changing the terms of trade and, in the case of the interest rate, by changing Country 3's price.

B. Foreign Monetary Disturbances

Under flexible exchange rates, monetary disturbances abroad have no effect on output (or prices) in Country 1. With full indexation, a foreign monetary disturbance only affects nominal variables abroad. A flexible rate can insulate the home country completely from changes in these variables.(25) This is true regardless of the degree of indexation in Country 1 ; what matters is that the foreign countries be fully indexed.(26)

A general monetary expansion in the two foreign countries, where $u_t^{m2} = u_t^{m3} = u_t^m$, raises the price of Country 3's good (or Country 2's good) by

$$u_t^{p3} = p_t^3 - \bar{p}^3 = u_t^m/(1+k_1),$$

but has no effect on the terms of trade or foreign output as long as the foreign countries are fully indexed. The interest rate in Country 3 (or Country 2) falls by

$$u_t^{r3} = r_t^3 - \bar{r}^3 = (u_t^{p3} - u_t^{m3})/k_1 = - u_t^m/(1+k_1) \ ;$$

in fact, nominal interest rates fall enough to keep real interest rates abroad constant.

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(25) We discuss below, however, an aggregate demand disturbance which has effects only on nominal variables, but for which a flexible rate does not provide effective insulation.

(26) It is interesting to note that the converse is not true. If Country 1 is fully indexed, but the foreign countries are not fully indexed, then foreign monetary disturbances have real effects on Country 1. A general monetary expansion raises output in Country 1 in a flexible exchange-rate regime as well as in an exchange-rate union. But with full indexation in Country 1, the increase in output is identical in both regimes ; that is, the choice between regimes has no effect on the variance of output.

As far as Country 1 is concerned, what matters under flexible rates are the franc prices of the two foreign goods and the franc return on the dollar (or mark) security, since these are the only variables that can affect Country 1's aggregate demand or supply. Both of these remain constant, however, since the franc price of the dollar (or of the mark) falls by

$$u_t^{x1} = X_t^1 - \bar{X}^1 = - u_t^{m3} (1+k_1),$$

thereby completely insulating Country 1 from the disturbance.

In an exchange-rate union, a monetary disturbance in Country 3 still has no effect on Country 1. Flexibility in the mark price of the dollar, which falls by

$$u_t^{x2} = - u_t^{m3}/(1+k_1) = u_t^{x1},$$

simply replaces the franc price of the dollar as the insulating factor.

If the monetary disturbance is in Country 2, however, then in the union the fixed exchange rate with that country's currency ensures that the disturbance is transmitted to Country 1. Consider first the effects of the monetary expansion on the two foreign countries. The price of Country 2's good rises while its interest rate falls :

$$u_t^{p2} = u_t^{m2}/(1 + k_1) = - u_t^{r2}.$$

The mark price of the dollar rises enough to insulate Country 3 from the disturbance : $u_t^{x2} = u_t^{m2}/(1 + k_1)$ ; as a result, Country 3's price and interest rate remain fixed.

In terms of the franc, Country 1's currency, the prices of both foreign goods rise in the union : the franc price of Country 2's good rises because its mark price rises, and the franc price of Country 3's good rises because of the depreciation of the mark (and hence the franc) relative to the dollar.

In addition, Country 1's interest rate falls. So output in Country 1 rises[27] :

$$u_t^{y1} = \frac{c(1-b)(g_{p2} + g_{p3} + g_r a_{11})}{Z} \quad \frac{u_t^{m2}}{(1+k_1)} > 0.$$

Thus, if there is a union between Countries 1 and 2, monetary disturbances originating in Country 2 are transmitted to Country 1.

C. Aggregate Demand Disturbances

Demand disturbances generally do affect Country 1 under both flexible exchange rates or the union. How a country fares in the two regimes depends upon the specific form which the disturbance takes. We will consider three types of demand disturbances because they all provide insight into the effects of the union :

(1) A shift in demand from the products of Country 3 to those of Country 2. The shift in demand affects Country 1 primarily by changing the terms of trade and shifting output (and hence demand for Country 1's products) from one foreign country to another. This disturbance serves to illustrate the importance of the pattern of trade between Country 1 and the other countries.

(2) A generalized increase in demand in both countries. This disturbance, in contrast, shows why price sensitivity in Country 1 is important, since aggregate demand disturbances originating abroad have different effects on the franc exchange rate depending upon Country 1's price elasticities.

(3) An increase in demand which is concentrated in one country more than the other, a third case combining the first two to show the importance of where the disturbance occurs.

(27) Country 1's interest rate falls because with a fixed rate between the franc and mark, $r_t^1$ must decline with $r_t^2$. Only if there is full wage indexation in Country 1 (b=1) does this purely nominal disturbance leave Country 1's output unaffected in the union. The flexibility of wages and prices then makes up for the fixity of the exchange rate.

All of these disturbances have the effect of changing interest rates abroad. If the real interest rate effect on aggregate demand in Country 1 is large enough, however, even a generalized demand expansion abroad will have ambiguous effects on Country 1's output since the resulting rise in foreign interest rates runs counter to other effects on the foreign demand expansion. To avoid this additional ambiguity, we assume in this section that the real interest rate effect in Country 1 is zero ($g_r = 0$). Under this assumption, higher foreign interest rates can still have a significant effect on Country 1's output, but indirectly by changing exchange rates.

Equation (I) can then be written in the following simplified form :

$$(I)' \quad Y_t^1 = \bar{Y}^1 + \frac{c(1 - ba_{11})}{Z}\left[u_t^{d1} + g_{y2}u_t^{y2} + g_{y3}u_t^{y3}\right]$$

$$+ \frac{g_{p3}\, c(1-b)}{Z}(u_t^{p3} + u_t^{xi}) + \frac{g_{p2}\, c(1-b)}{Z}(u_t^{p3} + u_t^{T} + u_t^{xi}),$$

where $u_t^{xi} = u_t^{x1}$, flexible rates,

$= u_t^{x2}$, exchange-rate union,

$Z = g_{p2} + g_{p3} + c(1 - ba_{11})$.

Note that any aggregate demand disturbance originating aboard affects Country 1's output either through changes in foreign output or changes in the (franc) prices of foreign goods.

1. Shift in Demand

Suppose that there is a shift in demand to Country 2's products from those of Country 3 with

$$u_t^{d2} = s_t, \qquad u_t^{d3} = -s_t, \qquad s_t > 0.$$

Then the terms of trade of Country 2 rise by :

$$u_t^T = T_t - \bar{T} = -\ 2\ s_t/(N^1 - N^2) > 0.$$

Output rises in Country 2 and falls in Country 3 proportionately to the change in the terms of trade. Interest rates rise in Country 2 but fall in Country 3 ; as a result, the mark appreciates by :

$$u_t^{x2} = \frac{-[\ 2k_2c^f(1-a) + 1]}{(1 + k_1)}\ u_t^T < 0.$$

Since the two foreign economies are identical, this shift disturbance moves each economy in equal but opposite directions.

With the two foreign countries responding <u>symmetrically</u> to the disturbance, the effect on Country 1 clearly depends upon its pattern of trade with the two countries. First, consider the neutral case where Country 1's trade is evenly balanced between Country 2 and Country 3, so that the output and price coefficients (or elasticities) in equation (I)' are equal, $g_{y2} = g_{y3}$, $g_{p2} = g_{p3}$.[28] In this case, the shift in demand has no effect on Country 1's output under flexible rates. The foreign output term in equation (I)' is zero since $u_t^{y2} = -\ u_t^{y3}$, so the shift in demand has no direct output effect on Country 1's aggregate demand. In addition, it can be shown that the franc prices of goods from the two countries move in equal and opposite directions as follows :

Country 3's good : $u_t^{p3} + u_t^{x1} = -\ u_t^T/2$,

Country 2's good : $u_t^{p3} + u_t^{x1} + u_t^T = u_t^T/2$.

This change in relative prices also has opposing effects on aggregate demand. So Country 1's output is insulated from the disturbance.

(28) Country 1's trade is evenly balanced in the sense that its exports (and imports) to the foreign countries are equal. The income coefficients also depend upon the income elasticities of demand for Country 1's good in the two foreign countries, while the price coefficients also depend upon the export and import price elasticities. (See the Appendix). We assume these are equal for the two countries throughout the discussion in this section, although the trade pattern could alternatively be interpreted in terms of asymmetries in these underlying elasticities.

Under flexible rates, the franc appreciates relative to the dollar, but by less than the mark appreciates since the franc depreciates relative to the mark. (Recall that the shift in demand towards Country 2's products raises Country 2's interest rate and lowers Country 3's interest rate). In fact, the appreciation of the franc is exactly half as large as that of the mark :

$$u_t^{x1} = -\frac{[2\ k_2 c^f (1 - a) + 1]}{2(1 + k_1)}\ u_t^T = u_t^{x2}/2.$$

In the union, the franc is tied to the mark, so it appreciates more than under flexible rates (i.e., twice as much). The franc prices of both foreign goods fall, so output in Country 1 must fall in the union, and Country 1 is accordingly worse off.

For similar reasons, Country 1 fares better under flexible rates when trade is biased towards Country 3 ($g_{y3} > g_{y2}$, $g_{p3} > g_{p2}$). In that case, the direct effect of the shift in foreign output is to reduce Country 1's aggregate demand since the output of its closest trading partner, Country 3, drops. Since the appreciation of the franc in the union is greater than under flexible rates, Country 1's output falls even further in the union than under flexible rates. The greater appreciation associated with the union adds to the deflationary effects of the shift in demand.[29]

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(29) This result holds even if there is no foreign output effect (ie, the output coefficients are equal, $g_{y2} = g_{y3}$), since the price effects of the disturbance are also greater in the union than in the flexible regime. Under flexible rates, the price effects on $Y_t^1$ can be simplified to :

$$c(1-b)(g_{p2} - g_{p3})k_1\ u_t^T/(-2B).$$

If $g_{p3} > g_{p2}$, this expression must be negative. In the union, the franc appreciates even more than under flexible rates, thereby adding to the deflationary price effects of the shift in demand.

If Country 1's trade is biased towards Country 2 ($g_{y2} < g_{y3}$, $g_{p2} < g_{p3}$), which may be the more likely case for a country joining a union, then the direct output effect is positive. That is, the shift in output towards Country 2 tends to raise Country 1's output. Now the greater appreciation of the franc in the union than under flexible rates may help to stabilize Country 1's output.(30) So the union may be superior to flexible rates when trade is biased towards Country 2. If the price elasticities ($g_{pj}$'s) are large enough so that the price effects in equation (I') dominate the output effect, however, then once again the greater appreciation of the franc in the union becomes a drawback. Country 1's output then varies more in the union than under flexible rates.(31)

2. General Increase in Demand

There is no longer the same presumption in favor of flexible rates when the demand disturbance is a general one,

$$u_t^{d2} = u_t^{d3} = u_t^{d}.$$

(30) In the union, equation (I)' can be written :

$$Y_t^1 = \bar{Y}^1 + c(1 - ba_{11})(g_{y2} - g_{y3})\, c^f(1-a)\, U_t^T/Z - g_{p3}\, c(1-b)u_t^T/Z$$

$$- (g_{p2} + g_{p3})\, c(1-b)\left[k_2\, c^f(1-a) - k_1/2\right] u_t^T/Z(1+k_1).$$

The output effect is positive if $g_{y2} > g_{y3}$, while the price effects are negative (assuming that $k_2\, c^f(1-a) > k_1/2$) because of the appreciation of the franc. Since the price effects run counter to the output effect, one may neutralize the other thus helping to stabilize Country 1's output.

(31) In equation (I'), the price terms increase in (absolute) value relative to the foreign output term as the price elasticities ($g_{p2}$, $g_{p3}$) increase.

Flexible rates are superior to the union only when price elasticities are high.

If demand increases (or decreases) by the same amount in both countries, there is no change in the terms of trade nor in the mark price of the dollar. Since foreign output is a function of the real wage, which is constant as long as the terms of trade remain fixed, output in both countries is constant.(32) So this demand disturbance has effects only on nominal quantities. Domestic prices in both countries rise :

$$u_t^{p3} = (k_1/g_r^f)u_t^d / (1 + k_1) = u_t^{p2} > 0.$$

Similarly, both rates of interest rise :

$$u_t^{r3} = u_t^{p3}/k_1 = u_t^{r2}.$$

In the exchange-rate union, there is no change in the franc price of the dollar since the mark price of the dollar is fixed. But the rise in foreign prices increases output in Country 1 with the increase proportional to the price elasticities, $g_{p2} + g_{p3}$. (See equation (I')).

Under flexible rates as well, Country 1's output is increased. Whether output is increased more or less under flexible rates depends upon whether the franc appreciates or depreciates. There are two influences on the exchange rate : higher foreign interest rates lead to depreciation of the franc, while higher domestic interest rate due to the increase in domestic transactions lead to an appreciation. How much the domestic interest rate increases depends upon the price elasticities, $g_{p2}$ and $g_{p3}$, since higher price elasticities imply greater increases in domestic output. The franc appreciates if the product of these price elasticities and the income

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(32) If indexation were less than complete in the foreign countries, this type of demand disturbance would lead to increases in output as well as prices, with the relative importance of changes in output increasing as the degree of indexation fell.

elasticity of money demand exceeds unity, and depreciates if the product is less than unity[33] :

$$u_t^{x1} \lesseqgtr 0 \quad \text{as} \quad k_2(g_{p2} + g_{p3}) \gtreqless 1$$

If the franc appreciates, then domestic output increases less under flexible rates than in the union. A depreciation, on the other hand, causes domestic output to increase more under flexible rates. Thus in the presence of this general demand disturbance, the union increases or decreases the variance of output depending upon whether $k_2(g_{p2} + g_{p3})$ is greater or less than one.[34]

3. Increase in Demand Concentrated More in One Country

Shifts in demand between the two foreign countries and general increases in demand abroad have very different effects on Country 1, but in each case the choice between flexible rates and the union is relatively simple. Unfortunately, actual demand disturbances that occur seldom fall solely in one

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(33) In the union (when $u_t^{x1}$ is constant), the foreign interest rate rises by $u_t^{r3} = u_t^{p3}/k_1$, whereas the domestic interest rate rises by

$$u_t^{r1} = \frac{u_t^{p1} + k_2\, u_t^{y1}}{k_1} = \left[1 + c(1-b)\,(k_2\, g_{p2} + k_2\, g_{p3} - 1)/Z\right] \frac{u_t^{p3}}{k_1} .$$

If $k_2(g_{p2} + g_{p3}) > 1$, then the domestic interest rate rises more than the foreign interest rate ; thus under flexible rates, the franc appreciates relative to the dollar and mark. But if $k_2\,(g_{p2} + g_{p3}) < 1$, the franc depreciates.

(34) An analogous condition would apply to a small country choosing between fixed and flexible exchange rates (i.e., where only one foreign country is involved). The above condition resembles the Marshall-Lerner

category, but more often take the form of expansions or contractions of aggregate demand that are highly correlated across countries but are concentrated more in one foreign country or the other. Such demand disturbances can either strengthen or weaken the case for an exchange-rate union depending upon which country experiences the greater change in demand.

The analysis of this more general case, however, can draw on the first two cases, since this disturbance can be regarded as a combination of the first two. Suppose that both countries experience a demand expansion, but that the expansion is greater in Country 2 (or 3). Then the two disturbances take the form,

$$u_t^{d2} = u_t^d \ (\overset{+}{-}) \ s_t,$$

$$u_t^{d3} = u_t^d \ (\overset{-}{+}) \ s_t,$$

where each disturbance includes a common element ($u_t^d$) modified by the shift term ($s_t$).(35)

The general increase in demand (represented by the $u_t^d$ factor) raises output in Country 1 under both exchange rate regimes. Under which regime output expands more depends again on the price elasticities. Suppose that $k_2(g_{p2} + g_{p3}) = 1$, so that the general increase in demand has the same effect on output in the union as under flexible rates. Then the choice between regimes depends upon which country is more subject to the demand disturbance.

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(34) ... condition, which requires that the sum of the elasticities of export and import demand exceed 1. These elasticities, however, are those of the aggregate demand function as a whole ; in fact, as the Appendix shows, the Marshall-Lerner condition is required in order for these elasticities to be positive. When indexation is less than complete in the two foreign countries, the effect of foreign demand disturbances depends not only upon these price elasticities but also upon the relative degree of indexation in the domestic and foreign countries.

(35) We assume that $s_t$ is proportional to $u_t^d$, $s_t = \bar{s}\, u_t^d$, where $\bar{s}$ is a fraction less than one so that $u_t^{d3} = (1-\bar{s})\, u_t^d > 0$.

Consider the case where trade is balanced between Country 1 and the two foreign countries (the neutral case considered in Section 1), but where the increase in demand is more concentrated in Country 2. Then strangely enough Country 1 is better off in the exchange-rate union -- tying itself to the country with greater demand disturbances. That is simply because changes in relative prices counter the effects of the general expansion.(36) With the disturbance more concentrated in Country 2, the terms of trade of that country rise and the mark appreciates. Under flexible rates, the franc also appreciates relative to the dollar, but by less that the mark. By joining the union, Country 1 ties the franc to the mark and finds its output increasing less because of the dampening effect of the mark's appreciation.

If Country 1's trade is biased toward Country 2, however, then the advantage of a union over flexible rates is diminished. This paradoxical result arises because the pattern of trade helps to determine how much the franc appreciates relative to the dollar under flexible rates. The franc appreciates more under flexible rates when trade is biased toward the country with the greater expansion of output than when trade is balanced, since Country 1's interest rate rises more in the former case. So the advantage of moving to a union, where the franc is tied to the appreciation mark, is correspondingly diminished.

When trade is balanced but the increase in demand is more concentrated in Country 3, in contrast, Country 1 is worse off in the union. Now the mark depreciates more than the franc. So joining the union further increases the effect of the disturbance on Country 1's output.

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(36) We assume, however, that the demand shift is not large enough to lower Country 1's output in either regime.

Thus the net impact of this more general disturbance depends not only on price elasticities, but on the location of the disturbance, with output variation being smaller in the union when the disturbance is concentrated in the union country. And the pattern of trade is also important, but greater trade with the union country can actually weaken the case for a union. This disturbance serves particularly well to illustrate the pitfalls in easy generalizations about how the source of economic disturbances or the pattern of trade affect the case for a union.

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The form which a foreign demand disturbance takes clearly determines how desirable an exchange rate union would be. Only in the case of a demand shift between countries is there a definite presumption in favor of one regime or the other, but that presumption is in favor of flexible rates. In the second case, that of a general increase in demand, the union is superior only if prices elasticities are low. In the final case, where the disturbance is concentrated more in one country than the other, the result also depends upon where the disturbance is concentrated as well as upon the pattern of trade. If foreign demand disturbances are a major source of concern, the case for or against an exchange-rate union will have to be based on a variety of considerations including the type of disturbance typically encountered, the pattern of trade with union and non-union countries, and the price sensitivity of the domestic economy.

## IV. CONCLUSION

This paper has shown how a union would affect a country subject to monetary and aggregate demand disturbances originating at home or abroad. How much difference a union makes depends first of all on domestic wage behavior. Any disturbance has an identical effect on the home country under flexible rates or in an exchange-rate union if wages are fully indexed to the general price level. Short of full indexation, the case for a union is stronger if monetary disturbances originate at home rather than abroad and weaker if domestic demand disturbances are important. But the advantage of one regime over another diminishes the closer is the country to full indexation.

Apart from wage behavior, two other factors are important : the pattern of trade and the sources of disturbances. But a pattern of trade biased toward the potential partner in the union does not always bolster the case for a union. Such a pattern of trade will not modify in any way the effect of domestic disturbances as long as Country 1 is small. In the case of some foreign demand disturbances, moreover, trade weighted toward the union country actually reduces the advantages of a union. The sources of economic disturbances also play a complex role in the choice of regimes. With respect to foreign monetary disturbances, the case for a union is weaker, as might have been expected, when the monetary disturbance is concentrated in the union country. But with respect to foreign demand disturbances, the case for a union is actually strengthened when the demand disturbance is larger in the union country. The effects of a union are indeed complex, with easy generalizations tenuous at best.

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APPENDIX: THE AGGREGATE DEMAND FUNCTION

The foreign output and price coefficients in the aggregate demand equation play a major role in the analysis of foreign disturbances. This appendix derives the aggregate demand equation from a more traditional equation (in level form) and interprets the output and price coefficients in terms of this more traditional model.

We begin with an expression relating output to consumption, exports, and imports:

$$(A1) \quad Q^1 = C^1(Q^1) + EX^2(Q^2,R) + EX^3(Q^3,R) - R\ IM^2(Q^1,R) - R\ IM^3(Q^1,R)$$

All variables are expressed in level form with the time subscripts omitted. $Q^i$ and $C^i$ are domestic output and spending in Country i, respectively, while $EX^i$ is the demand for exports of Country 1 by Country i and $IM^i$ is the demand for imports from Country i by Country 1. To simplify the analysis, we assume that there are no changes in the terms of trade between the two foreign countries and that the dollar prices of the two foreign goods are initially equal. Thus R can represent the terms of trade between Country 1 and either foreign country. We also assume that trade is initially balanced with each foreign country so that $EX^i = R\ IM^i$.

Define the following elasticities (where the partial derivative with respect to the first (second) argument of a function has the subscript 1 (2)):

$h_i$ = income elasticity of foreign demand for Country 1's good,
$= EX^i_1 (Q^i/EX^i)$,

$n_{fi}$ = price elasticity of foreign demand for Country 1's good,
$= EX^i_2 (R/EX^i)$,

$n_i$ = price elasticity of domestic demand for Country i's good,
$= -IM^i_2 (R/IM^i)$.

Also, define d = sum of the marginal propensities to save and to import by Country 1 = $(1 - C_1^1) + IM_1^2 + IM_1^3$. Then equation (A1) can be written in terms of percentage changes as follows:

$$\frac{dQ^1}{Q^1} = \frac{1}{d}\left[h_2 \frac{EX^2}{Q^1}\left(\frac{dQ^2}{Q^2}\right) + h_3 \frac{EX^3}{Q^1}\left(\frac{dQ^3}{Q^3}\right) + (n_{f2} + n_2 - 1)\frac{EX^2}{Q^1}\left(\frac{dR}{R}\right)\right. \tag{A1}'$$

$$\left. (n_{f3} + n_3 - 1)\ \frac{EX^3}{Q^1}\ \left(\frac{dR}{R}\right)\right].$$

Thus the coefficients of the aggregate demand equation in the text, which is expressed in logarithms, can be written:

$$g_{yi} = h_i \left(\frac{EX^i}{Q^1}\right) / d,$$

$$g_{pi} = (n_{fi} + n_i - 1)\left(\frac{EX^i}{Q^1}\right) / d.$$

The <u>relative</u> sizes of the $g_{yi}$ coefficients depend upon the <u>income elasticities</u> in the two foreign countries ($h_i$) as well as the <u>share</u> of exports to country i as a fraction of total domestic output. The relative sizes of the $g_{pi}$ coefficients depend upon the underlying <u>price elasticities</u> as well as the share of exports. The Marshall-Lerner condition for the trade balance between Country 1 and Country i is

$$n_{fi} + n_i - 1 > 1,$$

a condition which is necessary for $g_{pi}$ to be positive.

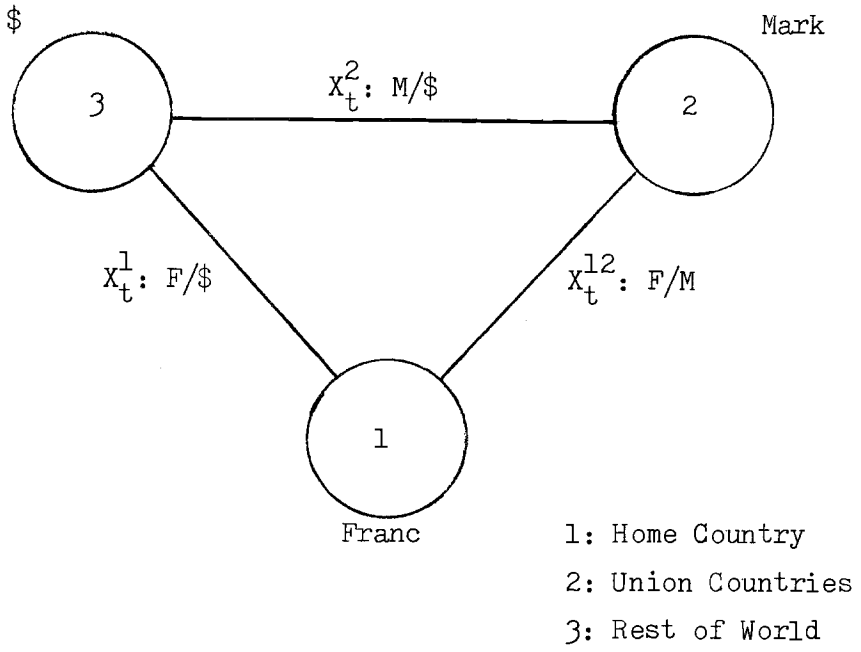
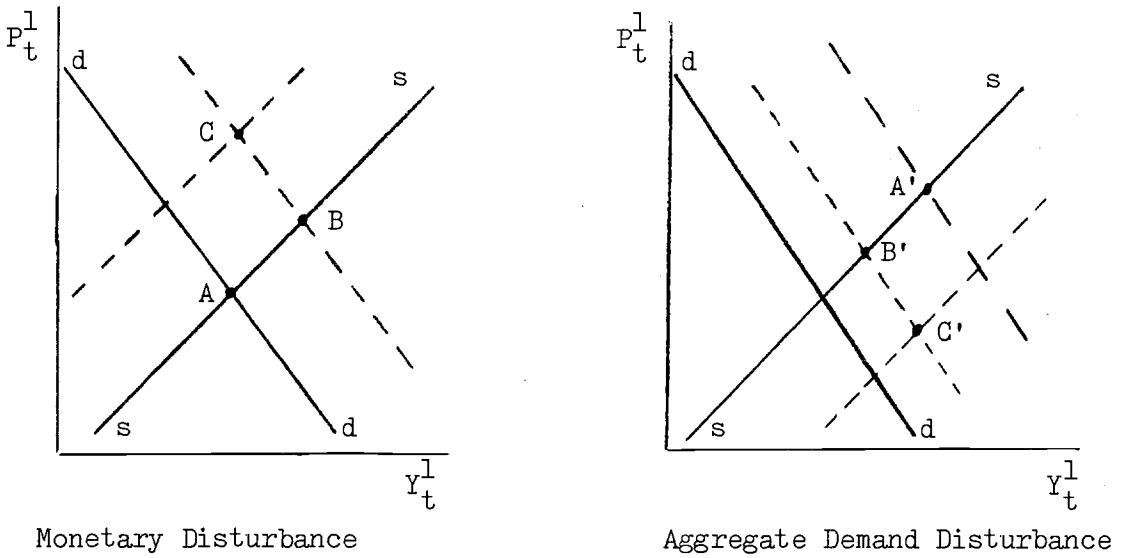



Figure 1. Currencies and Exchange Rates




Monetary Disturbance

Aggregate Demand Disturbance

Figure 2. Domestic Monetary and Aggregate Demand Disturbances

## Table 1: Country 1's Model

$$(I)\quad Y_t^1 = \bar{Y}^1 + \frac{c(1 - b\,a_{11})}{Z}[u_t^{d1} + g_{y2}u_t^{y2} + g_{y3}u_t^{y3}] + \frac{(g_{p3}c(1 - b) - g_r\,c\,a_{13})}{Z}[u_t^{p3} + u_t^{x1}]$$
$$+ \frac{(g_{p2}\,c(1 - b) - g_r c\,a_{12})}{Z}[u_t^{p3} + u_t^{x1} + u_t^{T}] - \frac{g_r\,c(1 - b\,a_{11})}{Z}[u_t^{r3} - u_t^{x1}].$$

$$(II)\quad P_t^1 = \bar{P}^1 + \frac{[u_t^{d1} + g_{y2}\,u_t^{y2} + g_{y3}\,u_t^{y3}]}{Z} + \frac{(g_{p3} - g_r\,a_{13} + c\,b\,a_{13})}{Z}[u_t^{p3} + u_t^{x1}] +$$
$$\frac{(g_{p2} - g_r\,a_{12} + c\,b\,a_{12})}{Z}[u_t^{p3} + u_t^{x1} + u_t^{T}] - \frac{g_r}{Z}[u_t^{r3} - u_t^{x1}].$$

(IIIa) Flexible Rates

$$X_t^1 = \bar{X}^1 - \frac{Z}{B}u_t^{m1} + \frac{[1 + k_2c(1 - b\,a_{11})]}{B}(u_t^{d1} + g_{y2}u_t^{y2} + g_{y3}u_t^{y3}) - \frac{[k_1Z + g_r(1 + k_2c(1-b\,a_{11}))]}{B}u_t^{r3}$$
$$+ \frac{[g_{p3}(1 + k_2c(1 - b)) + a_{13}cb - a_{13}g_r(1 + k_2c)]}{B}u_t^{p3}$$
$$+ \frac{[g_{p2}(1 + k_2c(1 - b)) + a_{12}cb - a_{12}g_r(1 + k_2c)]}{B}(u_t^{p3} + u_t^{T}),$$

(IIIb) Exchange-Rate Union

$$X_t^1 = X_t^2 = \bar{X}^2 + u_t^{x2},$$

where $Z = g_{p2} + g_{p3} + g_r\,a_{11} + c(1 - b\,a_{11}) > 0$ and $u_t^{x1} = X_t^1 - \bar{X}^1$.

$$B = -(1 + k_1)Z - c(1 - b)[k_2g_r\,a_{11} + k_2(g_{p2} + g_{p3}) - 1] < 0.$$