# THE SOCIAL COST OF LABOR, AND
# PROJECT EVALUATION: A GENERAL APPROACH

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ABSTRACT

This paper develops a general methodology for analyzing shadow wage (and

other shadow prices). Our approach is to identify those reduced form

relationships describing the economy which are central to the determination of

the shadow wage, and use these to obtain simple formulae for the shadow wage.

Among the aspects of the economy on which we focus are: (i) the difference

between the domestic and international prices, (ii) the equilibrating mech-

anisms in the economy, (iii) the mechanisms which determine earnings of

industrial and agricultural workers, (iv) the nature of migration, and (vi)

the intertemporal trade-offs and the attitudes towards inequality.

These aspects are modelled in a general manner, which can be specialized

to a number of alternative hypotheses concerning technology, behavioral

postulates, and institutional settings. Most earlier results on the shadow

wages are derived as special cases of our formulae. In addition, we identify

a number of new qualitative results concerning the relationship between the

shadow wage and the market wage.

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# 1. INTRODUCTION

The methodology to evaluate public activities and investments based on shadow prices probably stands out as the most important contribution of economic theory to the practice of economic development in recent decades. A key aspect of this methodology is the determination of the shadow wage, because it influences the most visible, and often controversial, aspects of public activities, namely, the employment created by a public project, and the labor-capital mix of a public investment. This paper examines a number of issues concerning the determination of shadow wage. This analysis, with some modifications, can also be applied to the determination of other shadow prices.

There is a widespread agreement concerning the basic principles of cost-benefit analysis, but there remain considerable disagreements about the appropriate value of shadow wage, and its relation to the market wage. Disagreements among economists arise from two sources: from different assumptions concerning what are the salient features of the structure of the economy (behavioral postulates, technological relationships, etc.), and from different assumptions concerning value judgements. Some of the well known debates on the shadow wage have taken place because of the differences in the assumed intertemporal trade-off of the government (i.e., the valuation of investment versus consumption), and in the assumed migration pattern between the agricultural and industrial sectors. But, as we shall see, there are several other aspects of the economy which critically influence the magnitude of the shadow wage.

Given this sensitivity of the shadow wage, it is desirable to examine this issue within a general framework which is consistent with a number of

alternative hypotheses. We develop such a framework in this paper. While the model presented here is not the most general which might be constructed, it is sufficiently rich so that we have been able to derive almost all important contributions made to date in the literature as special cases. This allows us to identify the precise assumptions which different researchers have made, and to point out the exact sources of disagreements among their results.

In addition, we are able to make some important qualitative statements - which have not been previously noted - concerning the relationship between the shadow wage and the market wage. Many of these statements are robust, i.e., they are valid for a wide range of parameter values. The importance of these qualitative results lies in the fact that obtaining the precise numerical estimates of some of the critical parameters is often inherently difficult.

In our analysis we have emphasized the following aspects of the economy:

(i) The agricultural sector: The shadow wage depends on whether agricultural workers receive the marginal product, the average product, or some other endogenously determined wage. Also, it depends on the technology of agricultural production and on the labor supply behavior of agricultural workers. We present a general model of this sector, which can be specialized to different technological relationships and a variety of hypotheses concerning the determination of agricultural earnings.

(ii) The industrial sector: Many models of the industrial sector have been proposed recently which suggest an important relationship between the wages paid to workers and their output; for example, the wage-productivity model, the wage-quality model, and the labor-turnover model. Here we represent the industrial sector in a general manner that can be specialized to the various specific approaches.

(iii) The migration of labor between sectors: The literature thus far has

focussed on two cases; where there is no endogenous migration or where the migration is governed by a Harris-Todaro type model. We present a general model of migration which subsumes the above two cases. Also, our determination of the shadow wage takes into account a number of general equilibrium effects of endogenous migration which have been ignored in earlier studies.

(iv) Foreign trade environment: Most of the studies on the shadow wage are based on a model of an open economy in which there is no distortion between the domestic and the international prices. Empirical evidence on LDCs, on the other hand, points out that there exist substantial price distortions. We therefore take into account such distortions; it turns out that these distortions may exert a first order effect on the magnitude of the shadow wage. In addition, we examine the case in which the distortions are being set at the socially optimal level, and analyze its implications for the shadow wage. We also consider the case in which the economy is closed to foreign trade.[1]

(v) Government policies and constraints on government behavior: In addition to the intertemporal trade-off mentioned earlier, the evaluation of public projects depends on the interpersonal trade-off (i.e., the social valuation of the income of workers in different sectors relative to that of investment). These value judgements are represented in our formulae through clearly indentifiable parameters.

Another important aspect of government policy is its impact on the equilibrating mechanisms in the economy. More specifically, creation of new employment entails a perturbation in the economy, and the consequences of new employment creation, therefore, depend on how the economy arrives at the new equilibrium.[2] The shadow wage thus depends on the equilibrating mechanisms

which operate in the economy.

How the economy equilibrates, in turn, depends on: what is the set of instruments which the government can potentially control, which of these instruments are left unchanged when the new employment is created, and how the government changes the remaining instruments. There are two circumstances in which the issue of how the economy equilibrates can be ignored; first, if the government does not possess any instrument of control at all, and second, if the government sets every available instrument at its socially optimal level. Given the observed behavior of governments, both of these extremes appear rather suspect. We therefore present a brief assessment of the impact of alternative equilibrating mechanisms.[3]

## 2. THE BASIC MODEL

We consider here a stylized model of an open economy, in which the government exercises its control on the agricultural sector only indirectly, through (at most) the imposition of output taxes and subsidies. The government proposes to undertake a project in the industrial sector which will create new employment. Our objective is to trace the consequences of this employment creation. In the basic model, described below, we assume there is no endogenous migration, the agricultural sector consists of family farms, and the industrial wage is rigid. [4] More general approaches are considered later.

Agricultural Sector: Agricultural sector's population is $N^1$, and A is total agricultural land which is owned equally within the agricultural sector.[5] $a = A/N^1$ is land per worker, and $L^1$ is the number of hours worked by each worker. The production technology exhibits constant returns to scale. We can therefore write: $X \equiv X(A/N^1, L^1) \equiv X(a, L^1)$ as the output of an

agricultural worker. An agricultural worker's consumption of agricultural and industrial goods is denoted by $(x^1, y^1)$. Surplus of agricultural good per agricultural worker is $Q = X - x^1$. Relative price of agricultural good in terms of industrial good is denoted by p. An agricultural worker's budget constraint is

(1)      $y^1 = pQ \equiv p[X(a, L^1) - x^1]$

An agricultural worker chooses $x^1$, $y^1$, and $L^1$, subject to the above budget constraint, to maximize his utility. The resulting level of utility will depend on p and $N^1$, and it is represented by the indirect utility function: $v^1 \equiv v^1(p, N^1)$. Then

(2)      $\dfrac{\partial v^1}{\partial p} = \lambda^1 Q > 0$, and $\dfrac{\partial v^1}{\partial N^1} = - \lambda^1 p X \varepsilon_{Xa}/N^1 < 0$

where $\varepsilon_{Xa} = \dfrac{\partial \ln X}{\partial \ln a}$ is the elasticity of agricultural output per worker with respect to land per worker, and $\lambda^1$ is (positive) marginal utility of income in sector i.

For later use, we define $\varepsilon_{Qp} = \dfrac{\partial \ln Q}{\partial \ln p}$, and $\varepsilon_{Qa} = \dfrac{\partial \ln Q}{\partial \ln a}$ as elasticities of surplus per agricultural worker with respect to its price, and with respect to land per agricultural worker. The sign of $\varepsilon_{Qp}$ is not predictable theoretically from the usual restrictions on the utility and the production functions, but the available empirical evidence indicates that $\varepsilon_{Qp} > 0$, which we maintain throughout the paper.[6] $\varepsilon_{Qa}$ depends on the scarcity of agricultural land. If land is not scarce, then $\varepsilon_{Qa} = 0$, and $\varepsilon_{Xa} = 0$. For brevity in interpreting our results, we assume throughout that $1 > \varepsilon_{Qa}$, i.e., land is moderately scarce. Parallel interpretations can be worked out if this

is not the case.

<u>Industrial Sector</u>: Industrial population is $N^2$, and an industrial worker supplies $L^2$ hours of work which are fixed due to technological and other considerations. An industrial worker's consumption of agricultural and industrial goods is denoted by $(x^2, y^2)$, and w is the wage income in terms of industrial good. The budget constraint of an industrial worker is given by

$$(3) \qquad px^2 + y^2 = w$$

An industrial worker chooses $x^2$ and $y^2$ to maximize his utility. The resulting utility level depends on p, w, and $L^2$. In the ensuing discussion we suppress the dependence on $L^2$, and write the indirect utility as: $v^2 \equiv v^2(p, w)$. Then

$$(4) \qquad \frac{\partial v^2}{\partial w} = \lambda^2 > 0, \text{ and } \frac{\partial v^2}{\partial p} = - \lambda^2 x^2 < 0$$

We define $\epsilon^2_{xp} = - \frac{\partial \ell n x^2}{\partial \ell n p}$ , and $\epsilon^2_{xw} = \frac{\partial \ell n x^2}{\partial \ell n w}$ as the elasticities of an industrial worker's consumption of agricultural good with respect to its price, and with respect to income. These elasticities are positive because consumption goods are normal.

The output of an industrial worker is denoted by $Y \equiv Y(k, L^2)$, where $k = K/N^2$ is capital stock per industrial worker, and K is the total industrial capital stock. There are both private and public firms in the industrial sectors. All firms pay the same wage to their workers and the profits of private firms are entirely taxed away.

Market Equilibrium: N is the total population, and

(5)      $N = N^1 + N^2$

The supply of industrial good is used either for consumption or for
investment, I. Hence

(6)      $I = N^2 Y + M_y - N^2 y^2 - N^1 y^1$

where $M_y$ is the net import of industrial good. Similarly, the balance between
the supply and demand of agricultural good requires

(7)      $N^1 0 + M_x = N^2 x^2$

where $M_x$ is the net import of agricultural good. Finally, the foreign trade
balance is given by

(8)      $PM_x + M_y = 0$

where P denotes the international relative price of the agricultural good. P
is fixed under the small country assumption, but this can be easily relaxed.

For later use, we obtain an alternative expression for investment.
Substitution of (1), (3), (7), and (8) in (6) yields

(9)      $I = N^2(Y - w) + (P - p)N^1 0 + (p - P)N^2 x^2$

That is, investment equals the retained part of the industrial output (after

deducting industrial wage payment) and the net revenue from tariff on foreign trade.

Equilibrating Mechanism: Creation of industrial employment changes the sectoral populations which, in turn, alters the demand and supply of agricultural good. An equilibrating change must therefore occur to bring back the balance between supply and demand, (7). The social impact of employment creation thus depends on the particular equilibrating change which occurs. In much of the paper, we assume that the (foreign) trade quantities, $M_x$ and $M_y$, change to maintain the equilibrium, i.e., the government does not change its tariff policy. An alternative equilibrating policy is examined in a later section.[7]

## 3. DETERMINATION OF THE SHADOW WAGE

### 3A. Shadow Wage in the Basic Model

We begin by defining an additive Bergson-Samuelson welfare function

$$(10) \qquad \psi = N^1 W(V^1) + N^2 W(V^2)$$

where W is concave and increasing in V. If $\delta$ is the social value of the marginal investment, then the current value of the aggregate social welfare is given by the Hamiltonian

$$(11) \qquad H = \psi + \delta I$$

in which I is given by (9).

If the shadow wage is denoted by s, then

$$(12) \qquad s = - \frac{1}{\delta} \frac{\partial H}{\partial N^2} \Big|_{N^2 Y}$$

In the above, the industrial good is taken as the numeraire. Industrial output is kept unchanged because the fruits of industrial employment creation should not be counted while calculating its cost.[8]

An explicit expression for (12) is derived from (11).

$$(13) \qquad s = w - \frac{1}{\delta} [w^2 - w^1] - \frac{\beta^1}{\delta} pX\varepsilon_{Xa} - (p - P)Z, \text{ where}$$

$$(14) \qquad Z = Q(1 - \varepsilon_{Qa}) + x^2 > 0$$

In obtaining the above, we have used (2), (4), and (5), and defined $W^1 = W(V^1)$, and $\beta^1 = \frac{\partial W}{\partial V^1} \lambda^1$. $\beta^1$ is social value (weight) of a marginal increase in the income of a worker in sector i.

Each of the four terms above represents a distinct social effect of moving an agricultural worker to the industrial sector. The first term is the direct cost of the wage payment to the worker. Naturally, a larger wage implies a larger shadow wage. The second term captures the change in the welfare of the worker who has moved. The third term represents the effect of reduced congestion on agricultural land. Specifically, a migrant worker releases land area a, which adds $pX\varepsilon_{Xa}$ to the income of those remaining in the agricultural sector. A higher congestion on agricultural land, therefore, corresponds to a lower shadow wage.

The last term captures the general equilibrium effect of employment creation on the demand and supply of agricultural good. This can be seen as

follows. The agricultural surplus decreases directly by $Q$ because now there is one less agricultural worker. The agricultural surplus increases indirectly, on the other hand, by an amount $Q\varepsilon_{Qa}$ because of the extra land which has now become available to those in the agricultural sector. Also, the newly arrived industrial worker consumes $x^2$ of the agricultural good.

The net shortfall in the supply of the agricultural good is therefore $Z$, as in (14), which is met through increased imports. Employment creation thus leads to increased net agricultural imports. The loss (or gain) in the government revenue then is $(p - P)Z$, which is the last term in (13).

Much of the existing literature on shadow wage has ignored this general equilibrium effect by making the assumption that there is no price distortion i.e., $p = P$. Empirical studies indicate, however, that not only is this assumption incorrect but, in fact, there are extremely large differences between domestic and international prices in most developing economies.[9] Further, if one were to assume that the government is setting the prices at the socially optimal levels, then the optimal prices, in general, will entail a price distortion.[10]

A simple example might help in establishing the practical importance of price distortions. Suppose the domestic price of food is twice its international price, and the workers spend roughly half of their income on food. Then, assuming that investment is highly scarce, i.e., $\delta \to \infty$, and that the workers' earnings in the two sectors are roughly equal, we find from (13) that the shadow wage is half of the market wage. In contrast, the shadow wage equals the market wage if the general equilibrium effect is ignored. Quite plausible parameters therefore show that the magnitude of shadow wage will be substantially erroneous if the general equilibrium effects are not taken into account.

### 3B. Special Cases

Many of the results which have been prominent in the literature - and some simple results which have not previously been noted - can be obtained as special cases of the formula derived above, (13). The specializations entail various specific assumptions concerning technology and the nature of government policy.

(i) Extreme scarcity of capital: In this case, $\delta$ is very large, and

$$(15) \qquad s \approx w - (p - P)Z$$

If investment is highly scarce, then the shadow wage is higher (lower) than the market wage if the domestic price of the agricultural good is lower (higher) than its international price.

(ii) No price distortions: A direct implication of (13) is as follows. In the absence of price distortions, the shadow wage is less than the market wage, so long as industrial workers are better-off than agricultural workers.

(iii) No price distortion and utilitarianism: Utilitarianism implies $W^1 = V^1$, and $\beta^1 = \lambda^1$. Denote the value of the marginal product of an agricultural worker by $g$. That is, $g = pX_L L^1$. Constant returns to scale in agricultural production implies $pX\varepsilon_{X_a} = pX - g$. Then (13) can be written as

$$(16) \qquad s = w - \frac{1}{\delta} [v^2 - v^1] - \frac{\lambda^1}{\delta} (pX - g)$$

The above corresponds to a result of Stern (1972), and it was also obtained by Newbery (1972).

Note that the remaining special cases also employ the assumption of no price distortions, in addition to the specific assumptions mentioned below.

(iv) Output maximizing society: If a society maximizes the level of its aggregate output without distinguishing between investment and consumption, or between consumption of workers in different sectors, then this is a special case of our formulation in which $W^1$ equals the income of a worker in the ith sector, and $\delta$ equals one. [11] That is, $W^1 = pX$, $W^2 = w$, and $\beta^1 = \delta = 1$. Therefore, from (13)

$$(17) \qquad s = g$$

That is, the shadow wage equals the value of the marginal product of an agricultural worker. This was one of the earliest views on shadow wage determination. Further, this view implies a zero shadow wage, if the marginal product of agricultural labor is zero.[12]

(v) Society does not care about the agricultural sector: The terms with the superscript 1 drop out of (13), and as a result

$$(18) \qquad s = w - \frac{1}{\delta} W^2$$

Sen (1968) employed the above assumption to derive two of the earlier results on the shadow wage. His results can be obtained as special cases of (18). First, assume that the society is maximizing output [as in the special case (iv) above]. That is, $W^2 = w$, and $\delta = 1$. Then the shadow wage is zero. Second, assume that the investment is highly scarce. Then the shadow wage equals the market wage. As is well known, the latter result will hold even if the society cared about the agricultural sector [see expression (13)].[13]

## 4. LABOR MOBILITY, INDUSTRIAL WAGE, AND INDUSTRIAL PRODUCTIVITY

### 4A. A General Model

Unemployment and endogenous mobility of labor across sectors are common features of many developing economies. We propose here a general model of labor mobility across different groups of workers. This model, as we shall see, subsumes the existing models of labor mobility. The utility level of an unemployed worker is denoted by $V^u$, and the number of unemployed is denoted by $N^u$. For simplicity, transfer arrangements from the employed to the unemployed workers are ignored here, and it is assumed that the unemployed workers have a fixed level of utility.

The agricultural population is expressed as a (reduced form) function of the relative price and industrial employment.

$$(19) \qquad N^1 = N^1(p, N^2)$$

Obviously then, the level of unemployment is obtained also as a function of $p$ and $N^2$, since

$$(20) \qquad N = N^1 + N^2 + N^u$$

Next, consider the determination of industrial wage. As we shall see later, there are a number of alternative hypotheses concerning how the industrial wage is determined. To obtain an integrated view of the implications of these hypotheses on the shadow wage, we represent the industrial wage schedule through a reduced form function. [14]

(21)    $w = w(p, N^2)$

It is worth noting that the above representation of labor mobility and industrial wage schedule is far more general than it appears, and it can handle a wide variety of interdependence among economic variables. [15,16] Also, note that by postulating (21), we are implicitly assuming that the government cannot perfectly control the level of industrial wage, or, that if it is choosing an optimal wage, then the optimal wage depends on other variables in the economy.

Finally, many recent theories have suggested that the productivity of industrial workers might depend on other variables in the economy such as the industrial wage, the rate of unemployment, etc. Such a dependence can, in general, be represented as

(22)    $Y = Y(k, L^1, p, N^2)$

where the first argument of the function Y continues to represent the direct effect of the size of industrial work force on industrial output (which is not counted while calculating the shadow wage) while the last two arguments reflect all of the indirect effects. [17]

An advantage of using general functions like (19), (21), and (22) is that the resulting shadow wage formulae are quite free of the precise nature of institutions in the economy. As we shall see below, these functions are easily specialized to represent specific hypotheses concerning labor mobility, productivity effects, and industrial wage determination.

For later use, we define the following elasticities. From (22), $\varepsilon_{Ye} = \dfrac{d\ln Y}{d\ln N^2}$ is the elasticity of per worker industrial output with respect

to the fourth argument in the function (22). Given that the price is fixed in the present model, this elasticity captures all of the indirect effects of the perturbation in the economy on the industrial output. From (19), $n = -\dfrac{dN^1}{dN^2}$ is the number of workers who leave the agricultural sector if one industrial job is created. Finally, from (21), $\varepsilon_{we} = \dfrac{d\ell nw}{d\ell nN^2}$ is the elasticity of industrial wage with respect to industrial employment.

The relevant Lagrangian is given by (9) and (11) in which

$$(23) \qquad \psi = N^1 W(V^1) + N^2 W(V^2) + (N - N^1 - N^2)W(V^u)$$

The shadow wage is obtained as

$$(24) \qquad s = w - \frac{1}{\delta}[W^2 - W^u] + \frac{\phi}{\delta} n - (p - P)Z + (1 - \frac{\beta^2}{\delta}) w\varepsilon_{we} - Y\varepsilon_{Ye}$$

where

$$(25) \qquad \phi = W^1 - W^u - \beta^1 pX\varepsilon_{Xa}, \text{ and}$$

$$(26) \qquad Z = 0(1 - \varepsilon_{0a})n + x^2(1 + \varepsilon_{xw}^2 \varepsilon_{we})$$

The expression (24) reduces to (13) if $n = 1$, $\varepsilon_{Ye} = 0$, and $\varepsilon_{we} = 0$. That is: If the level of unemployment is left unchanged by the creation of industrial employment, if the industrial wage is fixed, and if there are no indirect effects on the industrial output, then the shadow wage is the same as that derived in the basic model.

The effect of employment creation on labor mobility and the effect of this on shadow wage can be decomposed into three parts. First, n migrants

from agricultural sector join the pool of unemployed and $\phi$ is the loss in the welfare for each such worker [see (25)]. The welfare loss is thus $\frac{\phi}{\delta}n$ in (24). Second, one of the unemployed workers receives the newly created industrial employment, and the welfare gain due to this is $\frac{1}{\delta}[W^2 - W^u]$. Third, labor mobility influences the shortfall Z in the supply of agricultural good. This can be seen in (26), in which $Q(1 - \epsilon_{Qa}^1)n$ is the decrease in agricultural supply.

The effect of employment creation on the industrial wage is felt through $\epsilon_{we}$. If, for example, the industrial wage increases with industrial employment, then employment creation leads to an increase in the wage payment to intramarginal industrial workers. This, in turn, leads to a gain in the welfare of these workers, but at the expense of resources available for investment. The net of these two effects is represented in the fifth term in (24). The induced change in industrial wage also increases the net shortfall in the supply of agricultural good, as can be seen in the expression (26).

Finally, the loss or gain due to the indirect effects on industrial output is represented by the last term in the right hand side of (24).

4B. <u>Harris-Todaro Migration Model</u>

A special case of our general model of labor mobility is the Harris-Todaro hypothesis [Harris and Todaro (1970)]. Under this hypothesis, a migrant from the agricultural sector may find an industrial job, with probability $N^2/(N - N^1)$, or could become unemployed. Migration continues until the expected utility level of a potential migrant equals the utility level of an agricultural worker. This hypothesis is therefore a special case of (19) in which

$$(27) \qquad NV^1 = N^1V^1 + N^2V^2 + (N - N^1 - N^2)V^u$$

where, it will be recalled, $V^1$ is a function of p and $N^1$, and $V^2$ is a function of p and w. For simplicity, we assume that the industrial wage is fixed. Also that the social welfare function is utilitarian, i.e,

$W(V) = V$ and $\beta^1 = \lambda^1$. While a more general approach is easily possible, the latter assumption enables us to ignore the issue of defining the social welfare over <u>ex ante</u> versus <u>ex post</u> utilities. Finally, to keep $N^u$ positive, we assume that $V^2 > V^1 > V^u$. The above model is not meaningful otherwise.

Perturbing (27), we obtain

$$(28) \qquad n = \frac{N^1(V^2 - V^u)}{N^1(V^1 - V^u) + (N - N^1)\lambda^1 pX\epsilon_{Xa}} > 0$$

Substitution of the above in (24) yields

$$(29) \qquad s = w - \frac{N}{N^1} \frac{\beta^1}{\delta} pX\epsilon_{Xa} n - (p - P)Z$$

where Z is given by (26), in which $\epsilon_{we} = 0$.

On comparing (29) with earlier expressions, (13) and (24), it is clear that the shadow wage now does not depend on the differences in the utilities of different workers. This should not be surprising since all workers have the same expected utility under the present model. Also, note that the dependence of shadow wage on the difference between domestic and international prices, p - P, is quite similar to what we had encountered earlier.[18]

<u>Special Cases</u>: (i) Consider the special case in which there is no congestion on agricultural land, and the domestic price equals the

international price. Then, from (29), <u>the shadow wage equals the market wage</u>, regardless of the society's valuation of investment versus consumption. This well known result [19] reversed the presumption of the earlier literature that the shadow wage was much less than the market wage and that it approached the market wage only when the social value of investment (relative to consumption) was very high.

The basic reason for this result is that the migration in the present model does not change the aggregate level of utility in the economy and, hence, the only effect of employment creation is on investment. This can be seen as follows. The utility level of an agricultural worker is fixed, since $\epsilon_{Xa} = 0$. Similarly, the utility level of an industrial worker is fixed because his wage is fixed. Further, in this model, the workers allocate themselves between sectors to maximize their expected utility. It follows then that the effect of migration on the aggregate expected utility is zero.

Formally, recall that the creation of one industrial job leads to the migration of n workers from agricultural sector. Out of these, (n - 1) join the pool of unemployed, and one worker gets the industrial job. Now, from (28), $n = (v^2 - v^u)/(v^1 - v^u)$. Using this, it follows that the net social gain in utility, $v^2 + (n - 1) v^u - nv^1$, is zero. Since the only effect of creating an industrial job is on investment (which, from (9), is reduced by the market wage), it follows that the shadow wage equals the market wage.[20]

(ii) Stiglitz (1982a) considered a case in which the congestion effect is not ignored, but it is assumed that the society is maximizing the total output. Recalling this special case discussed earlier, one can write $\lambda^1 = \delta = 1$, $v^2 = w$, and $v^u = 0$. Substituting these in (28) and (29), his result is obtained

$$(30) \qquad s = w\alpha / \left[ \frac{N}{N^1} - \frac{(N - N^1)\alpha}{N^1} \right]$$

where $\alpha = 1 - \varepsilon_{Xa}$ is the elasticity of total agricultural output with respect to agricultural population.

## 4C. Alternative Models of Industrial Sector

Many recent models have postulated that the net output (e.g., net of hiring and training costs) in an industrial firm may be a function of, among others, the wage this firm pays, the wages paid by other firms in the industrial sector, and the level of industrial unemployment.[21] In this section, we show how the resulting wage determination mechanisms can be treated as special cases of the general formulation presented earlier. Also, we show how these models can be employed to study the industrial wage determination in many more institutional settings than those considered in the earlier literature.

The general points can be established by examining the wage-efficiency model. This model postulates that the efficiency per work hour, $\lambda$, depends on the industrial wage, i.e., $\lambda = \lambda(w)$, where $\lambda_w \geq 0$ in the relevant range, and $Y = Y(k, \lambda L^2)$. The particular institutional setting which has been studied in the literature is the one in which private firms maximize their profit per worker, and hire workers up to the point where their marginal product equals the wage. This implies that the labor cost per efficiency unit, $\frac{w}{\lambda(w)}$, is minimized, which leads to

$$(31) \qquad \lambda_w = \frac{\lambda}{w}$$

Thus:  The level of the efficiency wage is a fixed technological

parameter. The substitution of $\varepsilon_{we} = 0$ and $\varepsilon_{Ye} = 0$ in (24) yields the

corresponding shadow wage.

An example of a different institutional setting is the one in which the industrial firms are publicly owned (i.e., the level of industrial employment is publicly determined) and the government directs its public sector managers to maximize their profits (this directive may not always be socially optimal, as we shall soon see). Then the industrial wage is determined through

$$(32) \quad \lambda_w = \frac{1}{Y_L}$$

where $Y_L = \frac{\partial Y}{\partial(\lambda L^2)}$. Clearly in this case the industrial wage depends on the level of industrial employment. Further, $\varepsilon_{Ye} = \varepsilon_{YL}\varepsilon_{\lambda w}\varepsilon_{we}$, where $\varepsilon_{YL} = \frac{\partial \ln Y}{\partial \ln(\lambda L^2)}$ and $\varepsilon_{\lambda w} = \frac{\partial \ln \lambda}{\partial \ln w}$. Substitution of these in (24) yields the required shadow wage.

A particularly important institutional setting is one in which the government sets the wage optimally after taking into account the technological dependence due to the wage-efficiency effects. The optimum wage will thus be chosen to maximize the relevant Hamiltonian. It can be verified that, in general, the optimum wage will depend on the variables in the two sectors. Specifically, the resulting optimal wage schedule will not entail an equalization of the social valuation of investment and the social valuation of the income of an industrial worker, i.e., $\beta^2 \neq \delta$, unlike the standard case in public economics.[22]

Other models of industrial sector need to be discussed only briefly, since the relevant issues in these models are quite similar to those already discussed above. In the wage-quality model, the wage paid by a firm (relative

to other firms) induces a sorting of workers according to their quality.[23]  If

private firms minimize their unit cost of labor in efficiency units then, in

symmetric equilibrium, the wage is given by (31), where $\lambda = \lambda(w, N^u)$.

Similarly, in the labor-turnover model, the training cost to a firm

depends on the quit rate of workers.  If private firms minimize their total

labor cost then, in a symmetric equilibrium, it turns out that the wage

depends on the rate of unemployment and on the number of workers in the

agricultural sector.  With slight modifications, therefore, the above cases,

as well as those arising in alternative institutional settings, can be

accommodated within the general model presented earlier.

In this section, therefore, we have identified the properties of the

industrial sector which are critical for the determination of the shadow

wage.  We have shown how the relevant reduced form equations depicting the

industrial wage schedule can be derived from more primitive technological and

behavioral postulates.  The same reduced form equations and therefore the same

formulae for the shadow wage are consistent with quite different technological

and behavioral assumptions; while the same technological assumptions, in

conjunction with different behavioral postulates, yield markedly different

shadow wages.

## 5. SHADOW WAGE WITH OPTIMAL PRICING

Throughout our analysis, we have stressed the importance of the deviation

between domestic and international prices for the determination of the shadow

wage.  Our results are valid regardless of how the domestic prices are

determined, so long as prices do not change as industrial employment

changes.  It is worthwhile examining how domestic prices would be set if they

were being determined optimally, and to see what this, in turn, implies for shadow wages in an economy in which prices are optimally set.[24]

For brevity, we examine this issue in the context of the Harris-Todaro migration hypothesis, and assume that there are no indirect effects of employment creation on the industrial output. A more general analysis based on (19) and (22) is easily possible. The schedule of industrial wage is given by the general function (21). We obtain the first order condition of the relevant Hamiltonian, given by (9), (11) and (23), with respect to p. This yields

$$(33) \qquad \frac{p - P}{p} = \frac{\frac{\lambda^1}{\delta} N(Q - X\varepsilon_{Xa} m_p) - \frac{N^2 w}{p} \varepsilon_{wp} + M_x}{N^1 Q \, \hat{\varepsilon}_{Qp} + N^2 x^2 \, \hat{\varepsilon}_{xp}^2}$$

In the above expression, we have defined the following elasticities. From (19), $m_p = \frac{d\ell n N^1}{d\ell n p}$ is the elasticity of agricultural population with respect to the relative price and, from (21), $\varepsilon_{wp} = \frac{d\ln w}{d\ln p}$ is the elasticity of industrial wage with respect to price. $\hat{\varepsilon}_{Qp} = \frac{d\ln(N^1 Q)}{dp} = \varepsilon_{Qp} + (1 - \varepsilon_{Qa})m_p$ is the elasticity of total agricultural surplus with respect to its price, and $\hat{\varepsilon}_{xp}^2 = -\frac{d\ln x^2}{d\ln p} = \varepsilon_{xp}^2 - \varepsilon_{xw}^2 \varepsilon_{wp}$ is the own price elasticity of the consumption of agricultural good by an industrial worker, taking into account the induced effect of price on wage. Also, it can be verified from (27) that $m_p > 0$.

The expression (33) yields quite strong results if the induced wage and land congestion effects are negligible. In this case, note from (33) that $p > P$, if $M_x \geq 0$. Also, $p < P$ if $M_x < 0$, and if $\delta$ is very large.

Therefore: <u>The optimal domestic price of agricultural good is higher</u>

than its international price if the country imports this good. The reverse is true if the country exports agricultural good, and if investment is highly scarce.

Putting the above results together with (29), we find that the following holds in an open economy in which the domestic price is being set optimally.

The shadow wage is smaller than the market wage if the country imports agricultural good. The reverse is true if the country exports agricultural good and if investment is highly scarce.

Special Cases: (i) Among the very few studies on the shadow wage which do not assume free trade are those by Dixit, Newbery, and Stern. Newbery (1974) considers a model in which the society maximizes investment, and in which there is no endogenous migration. That is: $n = 1$, $m_p = 0$, and $\delta \to \infty$. Substitution of these in (33) yields[25]

$$(34) \qquad \frac{P - p}{p} = \frac{N^1 Q - N^2 x^2 + \frac{N^2 w}{p} \varepsilon_{wp}}{N^1 Q \varepsilon_{Qp} + N^2 x^2 \hat{\varepsilon}_{xp}^2}$$

(ii) Dixit (1971) and Dixit and Stern (1974) examine a model which entails two additional assumptions over those stated above. First, the industrial wage equals agricultural income, i.e., $w = pX$. Second, the agricultural output does not depend on its price, i.e., $X = X(N^1)$. These two assumptions imply: $\varepsilon_{wp} = 1$, and $\hat{\varepsilon}_{xp}^2 = (X - x^2)\varepsilon_{Qp}/x^2$. Substitution of these in (34) yields the result[26]

$$(35) \qquad \frac{P - p}{p} = \frac{1}{\varepsilon_{Qp}}$$

Expressions (34) and (35), along with the corresponding assumptions, can be inserted into (24) to obtain the resulting shadow wages.

## 6. ALTERNATIVE EQUILIBRATING MECHANISMS

Earlier in the paper, we had argued the importance of equilibrating mechanism in the determination of the shadow wage.[27] We now examine this issue in some detail. Specifically, we have assumed so far that the traded quantities change in response to the creation of industrial employment, while the domestic prices remain unchanged. We now explore the case in which a change in domestic prices equilibrates the economy, while the traded quantities remain unchanged.[28] For simplicity, we assume that there is no effect of the price change on the industrial wage and output and there is no endogenous migration. But, as we have seen earlier, the model can be easily extended to include these effects.

We can rewrite the expression for investment, (9), as

$$(36) \qquad I = N^2(Y - w) + (p - P)M_x$$

which, along with (10), defines the Hamiltonian (11). Denoting the shadow wage by $s^P$, we have

$$(37) \qquad s^P = - \frac{1}{\delta} \left[ \frac{\partial H}{\partial N^2} \bigg|_{N^2 Y} + \frac{\partial H}{\partial p} \frac{dp}{dN^2} \right]$$

The second term above is the indirect loss in social welfare, because $\frac{dp}{dN^2}$ is the change in price which will keep (7) in balance.

A perturbation of (7) yields

$$(38) \qquad \frac{dp}{dN^2} = \frac{pZ}{N^1 Q \varepsilon_{Qp} + N^2 x^2 \varepsilon_{xp}^2}$$

Where Z is given by (14), and it is positive. Therefore: <u>An increase in the price of agricultural good will accompany industrial employment creation if a change in the domestic prices is the equilibrating mechanism.</u>

Next, from the definition of aggregate social welfare, we obtain

$$(39) \qquad \frac{\partial H}{\partial N^2}\bigg|_{N^2 y} = -\delta w + [w^2 - w^1] + \beta^1 pX\varepsilon_{Xa}, \text{ and}$$

$$(40) \qquad \frac{\partial H}{\partial p} = N^1 Q\beta^1 - N^2 x^2 \beta^2 + \delta M_x$$

Finally, by substituting (38), (39), and (40) in (37), we obtain a direct expression for shadow wage.

$$(41) \qquad s = w - \frac{1}{\delta}[w^2 - w^1] - \frac{\beta^1}{\delta} pX \, \varepsilon_{Xa} - \frac{[\beta^1 - \beta^2 + M_x(\delta - \beta^2)/N^1 Q] pZ}{\delta[\varepsilon_{Qp}^1 + \varepsilon_{xp}^2 + M_x \varepsilon_{xp}^2 /N^1 Q]}$$

As one would expect, the only difference between the two comparable shadow wages, (41) and (13), is due to the equilibrating mechanism.

Expression (41) yields strong results if the land congestion effect is negligible, and if the quantity of agricultural good exported (or imported) is small in comparison to the agricultural surplus. That is, if $\varepsilon_{Xa} \approx 0$, and $M_x/N^1 Q \approx 0$. To obtain the result, first note that $\frac{\delta\beta}{\delta V} < 0$, if we make a reasonable assumption that the private marginal utility of income is lower at a higher level of utility. This implies

$$(42) \qquad w^2 \gtrless w^1, \text{ and } \beta^2 \lessgtr \beta^1, \text{ if } V^2 \gtrless V^1$$

It follows from (41) that: <u>if a change in domestic prices is the</u>

equilibrating mechanism ___, then the shadow wage is larger (smaller) than the market wage, so long as agricultural workers are better off (worse off) than industrial workers. Further, the result within the bracket holds even if land congestion effect is not negligible.

The intuition behind these results is quite clear. If agricultural workers are worse off, then industrial employment creation yields a direct welfare gain to those who receive the newly created jobs, and it yields an indirect welfare gain due to an increase in the price of agricultural good. These two effects lead to a reduction in the shadow wage.

Finally, in the polar case of highly scarce investment, (41) yields

$$(43) \qquad s = w - \left( \frac{M_x}{N^1 Q \varepsilon_{Qp} + N^2 x^2 \varepsilon_{xp}^2} \right) pZ$$

In this case, whether the shadow wage is higher or lower than the actual wage depends simply on whether the country exports or imports agricultural good.

It is easily seen from (41) and (13) that these two shadow wages will differ in general. This point has general validity regardless of the specific model which one uses, and the reason for this is easy to understand. Alternative equilibrating mechanisms affect various agents in the economy differently.[29] The net social impact therefore is different for different equilibrating mechanisms which, in turn, leads to different shadow wages.

Special Case: Note that the above analysis can be easily adapted to a closed economy. The only relevant difference for a closed economy is that $M_x = 0$ in (36). Naturally, therefore, the shadow wage is given by (41) when $M_x = 0$. It is obvious then that all of the results noted above hold for a closed economy as well.

## 7. FURTHER EXTENSIONS

(i) Our assumption that the agricultural sector consists of family farms might appear to be restrictive. We briefly discuss here a more general model of agricultural income determination. Denote the schedule of rural wage per hour as $w^1 = w^1(p, a)$. The income of an agricultural worker is thus $w^1 L^1$. If the agricultural rent is entirely appropriated by the government, then the investment equation (9) is replaced by

$$(44) \quad I = N^2(Y - w) + (P - p)N^1 Q + (p - P)N^2 x^2 + N^1(pX - w^1 L^1)$$

where the last term represents the rent from agricultural sector. [30]

If agricultural workers are price takers in the labor market [31], then the shadow wage in the basic model will be

$$(45) \quad s = w - \frac{1}{\delta}[W^2 - W^1] - (p - P)Z + (1 - \frac{\beta^1}{\delta})w^1 L^1 \varepsilon^1_{wa}$$

$$+ (1 - \varepsilon_{Lw}\varepsilon^1_{wa})(pX_L - w^1)L^1$$

where $\varepsilon^1_{wa} = \frac{d \ln w^1}{d \ln a}$, and $\varepsilon_{Lw} = \frac{\partial \ln L^1}{\partial \ln w^1}$ are, respectively, the elasticity of

the wage rate with respect to the land per worker, and the elasticity of labor supply with respect to the wage rate.

The first three terms in (45) are already familiar. The fourth term represents the net social cost (i.e., cost minus benefit) of the induced effect on wage rate due to the extra land area, a, released by the migrant

worker. The final term represents the net change in agricultural rent from having one less agricultural worker. Clearly, the expression (45) can be specialized to different institutional settings. Further, if there are wage-productivity type effects of workers' earnings on the agricultural output, then these effects will have to be taken into account.

Special Case: An important institutional setting is the one in which agricultural workers are paid their marginal product, i.e., $w^1 = pX_L$. Then the last term in (45) drops out, and $\varepsilon^1_{wa} = \eta/(1 + \eta\varepsilon_{Lw})$,

where $\eta = - X_{LL}L^1/X_L > 0$. Now consider an economy in which investment is highly scarce and there is no price distortion. Then (45) yields

$$(46) \qquad s = w + w^1 L^1 \varepsilon^1_{wa}$$

The conventional result that the shadow wage equals the market wage when investment is highly scarce, therefore, does not hold in this case.

In fact: if agricultural workers are paid their marginal product, if there is no price distortion, and if investment is highly scarce, then the shadow wage is higher than the market wage so long as the labor hours supplied by agricultural workers are fixed, or they are positively related to the wage rate.

(ii) We have emphasized above that the shadow wage depends on the equilibrating mechanisms within the economy. There are two other possibilities which might be mentioned in this context.

First, it is possible in some cases that a government manages its instruments of control in a conflicting manner such that some of the markets do not clear. As an extreme example, if a government creates industrial

employment without allowing any other variable to change, then a shortage of food might emerge in cities. The government may then attempt to remove this shortage through non-price methods such as rationing. The derivation of the shadow wage in such cases will have to take into account the non-price methods which will be employed to arrive at the final quantity balances.

Second, it is possible that the economy is in a temporary equilibrium such that some of the prices are rigid and some of the markets, other than that for labor, are characterized by excess supply or demand. In such cases, it is necessary to base the shadow wage derivation on an explicit model of the short term equilibrium.[32]

(iii) The models we have analyzed can be easily enlarged to include additional instruments of policy. For example, if the government can maintain different relative prices in the agricultural and the industrial sectors, denoted by p and q respectively, then it can be shown that the shadow wage in the basic model is given by (13), provided we replace the last term in (13) by

$$(40) \qquad - (p - P) \, Q(1 - \varepsilon_{0a}^{1}) - (q - P)x^{2}$$

The intuition is obvious. The government's gain or loss due to the general equilibrium effect on the demand and supply of agricultural good is valued differently in the two sectors. Further, it can be verified that if the government is setting the price optimally, then neither of these two prices would, in general, correspond to the international price [see Sah and Stiglitz (1983a)].

(iv) The present analysis can be easily adapted to study the optimal location of population. The condition for optimality is given by $\frac{\partial H}{\partial N^{2}} = 0$, which yields

(48) $$Y_L L^2 = s$$

That is, the marginal product of an industrial worker should equal the shadow wage. It is worthwhile pointing out here that the expression (48) has sometimes been taken as the definition of shadow wage in the literature. This presumes that the population is already located in an optimal manner, which is a rather strong assumption to make.

## 8. CONCLUSIONS

While the importance of using shadow prices and wages in the evaluation of public projects has been widely recognized in LDCs, the correct magnitude of shadow wage — and its relationship to the market wage — have remained controversial. In particular, the earliest studies on the shadow wage focussed on the widespread unemployment (open or disguised) in LDCs, and inferred from this that the opportunity cost of hiring new industrial workers was low. Sen and Marglin (among others) though agreeing that the opportunity cost — in the sense of forgone output — might be low, contended that the shadow wage might nonetheless be high; to pay an additional worker required diverting resources from (relatively more valuable) investment to consumption. If the investment was highly scarce, then the shadow wage equaled the market wage.

This view, in turn, was criticized by Harberger and Stiglitz for ignoring the induced migration from the agricultural sector to the industrial sector. If the workers earned fixed wages in the two sector, then under the Harris-Todaro hypothesis that the expected wage in the industrial sector equals the

agricultural wage, they showed that the shadow wage was equal to the market wage, regardless of the relative social valuation of investment. These earlier studies thus identified two of the important determinants of the shadow wage - the nature of intertemporal trade-off and endogenous migration.

In this paper, we present a general framework for shadow wage determination which, while incorporating these issues, deals explicitly with a number of aspects which have not received the attention they deserve. These include: (i) the differences between domestic and international prices, (ii) the equilibrating mechanisms in the economy which determine, for example, whether the general equilibrium impact of industrial employment creation is to increase the relative price of the agricultural good or to increase its net import, (iii) the mechanisms which determine the earnings of agricultural and industrial workers, and (iv) the consequences of industrial employment creation on those who remain in the agricultural sector, e.g., through what we identify as congestion effects and price effects. Many of these aspects have first order effects on the magnitude of shadow wage; to ignore them would lead to misleading results.

Our approach has been to identify those reduced form relationships for describing the economy which are central to the determination of the shadow wage. As we have shown, it is easy to specialize our general model to different technological assumptions (e.g., the nature of production relationships in the agricultural and industrial sectors) and institutional settings, as well as to different behavioral hypotheses. For example, our general model of migration can be specialized to the standard Harris-Todaro case, and our representation of industrial wage determination can be specialized to the wage-efficiency and the labor-turnover models.

We have specifically shown how most earlier results on the shadow wage

can be obtained as special cases of our formulae for the shadow wage. Also, our general approach enables us to identify a number of new qualitative results concerning the relationship between the shadow wage and the market wage. Further, our general formulae have the virtue of analytical simplicity so that they provide an integrated view of the critical determinants of the shadow wage.

There are several limitations of our analysis which we have pointed out in the paper. In particular, we have examined only a limited set of rigidities; we have not considered, for example, the possibility that markets other than that for labor may not clear. Also, the models of migration and the models for the determination of workers' earnings which we have considered are essentially static. It is possible for example that the brunt of the effects of employment creation in one period are felt in the future. We plan to address these questions in a sequel to this paper.

1.  It is obvious that both an open and a closed economy are two polar
    representations of a variety of regimes which exist in LDCs.  Sah and
    Stiglitz (1983a) have examined many such regimes in the context of inter-
    sectoral pricing.  The same approach is applicable here but, for brevity,
    we do not pursue it.

2.  It is important to observe that the term 'equilibrium' does not
    necessarily imply a conventional Walrasian equilibrium.  It also denotes
    temporary equilibria of the kind that have been recently investigated [for
    example, Solow and Stiglitz (1968), Bennasey (1975), and Malinvaud
    (1977)].

3.  Before beginning our analysis, it might be useful to clarify our usage of
    the term shadow wage.  The shadow wage here is a summary statistic which
    sums up all of the changes in the economy (due to the creation of
    industrial employment), multiplied by the social marginal evaluations of
    each of these changes.  This definition is, in general, distinct from
    another summary statistic, opportunity cost of labor, often employed in
    the literature, which calculates the net change in the aggregate output
    due to employment creation.  This distinction becomes explicit in the
    later analysis.

4.  The assumption of a fixed (real) industrial wage is often justified on the
    basis of certain unspecified institutional constraints.  But, as we shall

see, fixed industrial wage is consistent with a particular mechanism of wage determination based on the wage-productivity effect.

5.  Throughout the paper, superscripts $i = 1$ and 2 denote the agricultural and the industrial sectors respectively.

6.  See Sah and Stiglitz (1982) for a discussion of the evidence.

7.  Of course, the government can employ a combination of equilibrating mechanisms. The resulting shadow wage for any such combination, however, can be studied by examining their separate effects.

8.  We exclude only the direct contribution, however. Thus, if industrial employment creation has indirect repercussions on the industrial output (e.g., because of a change in workers' efficiency or hours worked) then this indirect effect should not be excluded. Such situations arise later in the paper.

9.  See Peterson (1979), and Bale and Lutz (1979), for example.

10. There are some special cases, stressed by Diamond and Mirrlees (1971), in which there is no restriction on the government's ability to impose commodity and factors taxes, which predict that free trade is the optimal policy. This is not the optimal policy, however, in the more general cases examined below and in Sah and Stiglitz (1983a).

11. It is this particular assumption which defines the opportunity cost of

labor. The opportunity cost of labor thus is a special case of shadow wage when society is maximizing the aggregate output. It is obvious from above that the opportunity cost is a well-defined concept only when relative prices are fixed, or equivalently, there is only one good in the economy. In more general cases, as we shall see later, the relative prices will not be fixed.

12. The marginal product of an agricultural worker in this paper is the number of hours a worker works times the marginal product of one working hour. This is to be distinguished from the notion of the marginal product of an agricultural worker implicit in the surplus labor hypothesis [Lewis (1954)]. This hypothesis, though influential in the literature, entails certain institutional details which hitherto have not been carefully examined. Sah and Stiglitz (1983a) construct an explicit model of such a labor surplus agricultural sector, but we have not explored its implication on the shadow wage.

13. Dixit (1968) studied a model without an agricultural sector, but with a reserve army of unemployed in the industrial sector. Members of this army are subsidized by their working colleagues, such that everyone consumes the same amount, $wN^2/N$. The indirect consumption gain to the population from a job creation is $w$, which corresponds to $pX - g$ in an expression such as (16). Further, the society is utilitarian. It follows that the corresponding shadow wage is a special case of (16) such that:

$$s = w[1 - \frac{\lambda}{\delta}] .$$

14. Khan (1980) employs a similar representation of industrial wage in the

context of a trade model.

15. Specifically, let $N^1$ depend on all of the variables in the economy, i.e.,
$N^1 = \hat{N}^1(p, w, N^u, N^2)$.  Similarly, in general, $w = \hat{w}(p, N^1, N^u, N^2)$.
These two expressions and (20), then, yield (19) and (21) under the
conditions which allow the use of the implicit function theorem.  If the
economy has a wider set of variables then the above representations will
have to be expanded.  As an example, if there are different prices in the
two sectors, then both of these prices will appear as arguments of (19)
and (21).

16. The general model of migration proposed in this paper can be extended to
an economy in which there are several regions which differ from one
another in terms of resources (e.g., land, population, and skills of
worker), institutions (e.g., how workers' earning are determined), and tax
regimes (i.e., workers in these regions face different prices).

17. The dependence of industrial output on other variables in the economy is
implicit in (22), through an argument similar to that in footnote 15.
Also, note that (22) is a summation over firms' production functions; each
of which can be written (in the symmetric equilibrium) as:
$Y^f \equiv Y^f(k^f, L^2, p, N^2)$, where the superscript f denotes a firm, $N^2$
represents the sectoral employment, and $k^f$ reflects the firm's
employment.  In a more general model, $Y^f$ will be a function of the entire
distribution of industrial wages.

18. Note that the expressions based on the Harris-Todaro model, such as (29)

and those to be derived later, are somewhat more general than they appear. This is because the only property of the migration model which has been actually used in deriving these expressions is that the social welfare can be represented by $NV^1$. The resulting expressions therefore will hold for <u>any migration mechanism</u>, so long as the society focusses its attention only on the welfare of agricultural workers.

19. See Stiglitz (1971, 1974), Harberger (1971) and Heady (1981), among others.

20. This result can be looked at in a different way in an output maximizing society in which the earnings (consumption) of a worker in both sectors are fixed and, therefore, a change in investment is the same as a change in output. The impact on aggregate output of creating an industrial job in this economy is the output of one agricultural worker times the number of such workers who migrate. Under Harris-Todaro hypothesis, this product is just equal to the industrial wage. To see this in the simplest case in which everyone is risk-neutral, note that: $V^1 = pX$, $V^2 = w$, and $V^u = 0$. (28) then yields: $n = \frac{w}{pX}$. The loss in output is thus: $npX = w$. For a discussion of this simple case, see Stiglitz (1971, 1974). Our analysis in this paper is, of course, much more general and does not depend on these restrictive assumptions.

21. See Stiglitz (1971, 1974, 1982a, 1982b), and references therein.

22. For illustration, consider a special case in which there is no price distortion, and there is no endogenous migration. The socially optimal wage then is given by: $\beta^2/\delta = 1 - Y_L \lambda_w$. Clearly, therefore, $\beta^2$ will

not equal $\delta$, in general. Next, consider two further special cases in which the social weights are exogenously specified. First, if the society maximizes investment, then the optimal wage is the same as in (32), i.e., what it would be if the public industrial firms are instructed to maximize their profits. Second, if the society maximizes output, then $\lambda_w = 0$, which implies a sufficiently high industrial wage such that the effects of wage on productivity do not exist anymore. It follows in the present case, then, that an output maximizing society will pay a higher industrial wage than an investment maximizing society, or an economy with private industrial firms.

23. Wage-quality models have some additional complexity [See Stiglitz (1982b)]. For example, if earnings vary across agricultural workers of different abilities, then this needs to be taken into account.

24. The shadow wage with optimally set prices is the same as what it would be if an adjustment in prices is the equilibrating mechanism. We examine this case later.

25. Newbery however makes a mistake in defining the investment equation. He leaves out the last term, $(p - P)N^2 x^2$, in the expression (9). His formulae for the optimal price and the shadow wage are therefore incorrect.

26. Newbery (1974) arrives at the same result by making a second error. His expression corresponding to (34) is $(P - p)/p = (N^1 O + \frac{N^1 w}{p} \varepsilon_{wp})/N^1 O \varepsilon_{Op}$, because he erroneously leaves out the other terms. He then arrives at (35)

by substituting $\varepsilon_{wp} = 0$, instead of $\varepsilon_{wp} = 1$ as it should correctly be.

27. Blitzer, Dasgupta and Stiglitz (1981) raise this issue in the context of the shadow foreign exchange rate in an open economy in which there is a trade imbalance in the pre-project situation, and the government considers different policies for removing the additional imbalance created by a project. What we show here is that the issue of equilibrating mechanisms is much more general.

28. Equilibrating mechanism is not always a matter of choice for the government. For example, there are often restrictions on the quantities that a country can export or import. If such is the case, then the relevant shadow wage is the one which is derived below.

29. Specifically, in the present case, recall that the net import of the agricultural good increases if an adjustment in traded quantities is the equilibrating mechanism. This in turn increases (reduces) investment if the domestic price of agricultural good is higher (lower) than its international price, while the welfare of workers is left unchanged. In contrast, the price of agricultural good increases if an adjustment in the domestic prices is the equilibrating mechanism. This in turn helps agricultural workers, hurts industrial workers, and increases (decreases) investment if the agricultural good is being imported (exported).

30. A further generalization will involve an explicit representation of the entire distribution of landowners and workers, and the possibility that the agricultural rent cannot be taxed or that it can be taxed only partly.

31. This distinction is important.  For example, an agricultural sector consisting of family farms which we discussed earlier is, in general, not a special case of the present model.  This is because the family farm workers are not price takers.  With slight modification, however, one could examine the implications of a general earning's schedule under alternative institutional settings.

32. See Roberts (1982), and Marchand, Mintz and Pestieau (1983), for example.  These papers consider a single sector economy, which has limited applicability in LDCs with features associated with dualism.  For a more complete analysis, see Sah and Stiglitz (1983b).

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