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INTRODUCTION

This little book grew out of an investigation into the history of interest rates and security prices in the United States since January, 1857. In handling the various series of monthly data prepared during that investigation, the problem arose of how comparisons between the series should be made. The relations between the different monthly series might have been analyzed by many statistical methods. We chose one of the simplest. We "smoothed" each data series in such a manner that relations between the larger [1] movements of the

[1] The object of smoothing the series included in the study of interest rates and security prices was to compare the "cyclical" movements of the various series. We wished to eliminate the multitude of minor movements, as we felt that their presence would obscure the picture of the major movements. For the discussion of the minor movements we relied primarily upon the raw data. However, for purposes other than ours, the investigator might wish to smooth his material in such a manner as to eliminate seasonal fluctuations and at the same time preserve not only the larger but most of the smaller movements of the data. The handling of such a problem is illustrated and discussed in Appendix I.

Any graduation of economic time series must, almost inevitably, be for a particular purpose only. The graduations presented in the study of interest rates and security prices are intended to supplement the data. They are not intended to replace the data. In this, they differ from adjustments made on physical observations in order to eliminate errors of measurement. They also differ from graduations which are intended to estimate the "universe" from a sample. The graduation of a mortality table is of this latter type.

various series became immediately apparent when the smooth curves were plotted on a chart. The particular method of smoothing or "graduation" which we used, eliminates from each series not only minor erratic movements but also monthly seasonal fluctuations. The resulting smooth curves will appear in the study of interest rates and security prices.

A brief chapter on the problem of smoothing was prepared for inclusion in that study. It covered little more than a description of the methods actually used. The manuscript was read by a few fellow statisticians. So many questions were then asked that the chapter was expanded. More questions were asked. Further expansion followed. What was originally designed as little more than a memorandum on a particular method grew into a rather general treatment of the whole problem of smoothing. It soon became apparent that a really simple treatment of even the elements of the subject required more space than a short chapter. As a long chapter on smoothing seemed somewhat of a digression, if included in an investigation of the history of interest rates and security prices, the National Bureau of Economic Research decided to publish this study separately.

An attempt has been made to present this intricate problem in as simple a manner as possible. This introduction is intended for the reader who wants merely a brief description of the nature of the smoothing process accompanied by some illustrations of a few simple graduations and directions for computing them. The more systematic discussion is contained in the body of the book.

A smooth curve may be described as one which does not change its slope in a sudden or erratic manner.[1] Smooth curves are not necessarily representable by simple mathematical equations. Indeed the expression, in its narrower sense, has often been reserved for curves which are not so representable. To suggest an adequate definition of smoothness is difficult, but it is still more difficult to suggest an adequate measure. The most commonly used mathematical measure of smoothness is based on the smallness of the sum of the squares of the third differences of successive points on the curve. This criterion amounts to measuring the smoothness of a curve by measuring how closely successive groups of four consecutive points can be described by second-degree parabolas.[2] Though such a concept may be useful, it certainly is not entirely logical. It implies that no curves are perfectly smooth except straight lines and second-degree parabolas.

[1] The mathematician may feel that this definition of smoothness ties up too definitely to mere second differences. However, do not the words "sudden or erratic manner" imply more than mere second differences?

[2] See note 1, page 54.

The fitting of a mathematical curve to physical observations may be a rational operation; the "smoothing" of economic time series is almost inevitably purely empirical. A mathematical curve fitted to observations made on bodies falling in a vacuum may be a statement of a law; the result of smoothing average monthly rates for Time Money on the New York Stock Exchange can hardly be more than a picture of what such rates would have been had they been unaffected by seasonal and erratic factors. It constitutes no law. However, in spite of the absence of any even hypothetically rational law, smoothing or "graduation"[1] seems useful for many purposes and therefore quite legitimate.

In the hands of a person who is thoroughly acquainted with all aspects of the data, freehand (or French curve) smoothing would seem to have much to recommend it. The most commonly advanced argument for freehand or graphic smoothing is that it saves time. As a matter of fact, one of the chief weaknesses of the method is the amount of time required—if the smoothing is to attain the object desired. If a curve be required which shall (1) be smooth, (2) give a good fit, and (3) eliminate seasonal fluctuations, the

[1] The terms *smoothing* and *graduation* are commonly thought of as not quite synonymous with *curve fitting*. The expression *curve fitting* should, perhaps, be reserved for the fitting to data of a curve representable by a mathematical equation.

amount of time used in correcting and recorrecting the freehand curve often becomes prohibitive. Moreover, most of the work must be done by the investigator himself. It is not the type of operation which can be delegated to a computer. Finally, the investigator himself may easily go astray. Judgment is, at best, a variable quantity. Unless the mathematical checks are so detailed that the freehand fitting practically amounts to a mathematical fitting by successive approximations, the investigator may easily describe the same data by distinctly different smooth curves if he does the fitting twice—with a month's time between operations. Most persons are incapable of good freehand smoothing. I do not hesitate to say, after having worked with many hundreds of students, that any fairly good mathematical method will, in at least nine cases out of ten, give better results than any method which requires much judgment.

Perhaps the best theoretical case for freehand smoothing can be made when there are reasons for suspecting that the underlying ideal curve is itself not smooth. If the underlying curve have cusps or be discontinuous, any continuous "smoothing" —whether mathematical or freehand—will, of course, somewhat obscure such characteristics. But the freehand method can easily smooth by parts—introducing any cusps or discontinuities which the investigator may wish in the "smooth" curve. For example, quotations for Call Money Rates or Bond Yields, before, during and after a financial panic, might be smoothed "up the hill" and "down the hill" but not over the top of the hill. Of course, one of the dangers in such a procedure, for purposes of comparison between various series, is that different draftsmen, or even the same draftsman at different times, will vary in their judgment as to when "over the top of the hill" should not be smoothed. Theoretically, freehand smoothing is ideal. Practically, it is a little like the faith of a mystic. It is conclusive evidence to the recipient of the vision alone.[1]

The simplest of purely mathematical methods of smoothing data is to take a moving average of the data and center that moving average. For example, a moving average, each value of which is the average of seven consecutive observations (which are equally spaced in time), may be used as a smoothed or theoretical value for the observa-

[1] Smoothing for purposes of comparing various series with one another is often useful even when the smoothing process has not described both series in a completely satisfactory manner. For example, the use of any time unit implicitly involves the use of one of the crudest and least adequate of mathematical smoothing processes—the simple moving average. To compare the annual production of pig iron during successive *years* with the annual volume of bank clearings during the same successive *years* is to compare selected points (twelve months apart) on the *12-months* moving average of the *monthly* production of pig iron with corresponding selected points on the *12-months* moving average of the *monthly* volume of bank clearings.

tion which is fourth in the list of the seven used in obtaining the particular moving average value.[1] Such a method of smoothing involves only extremely easy computation. It has, however, serious drawbacks. The resulting curve is seldom very smooth and it will not give a perfect fit to data except in ranges which can be adequately described by a straight line.[2] For example, a simple moving average, if applied to data whose underlying trend is of a second-degree parabolic type, falls always *within* instead of *on* the parabola. If applied to data whose underlying trend is of a sinusoidal type, it falls too low at maximum points and too high at minimum points. When applied to such data it cuts off tops and bottoms, usually resulting in a decidedly poor fit.

In general, if a type of smoothing be desired which shall, when applied to monthly data, eliminate seasonal and erratic fluctuations and at the same time give a smooth curve adequately describing the remaining cyclical and trend factors, something much more delicate than a simple 12-months

[1] If a 12-months moving average of monthly data be taken, any regular seasonal fluctuation in the data will, of course, be eliminated. Such a 12-months average should logically be centered between the sixth and seventh months. If a 2-months moving average of this 12-months moving average be taken, such average may be centered at the seventh month.

[2] Each point on a 12-months moving average, for example, is the middle point of a straight line fitted to 12 observations by the method of least squares.

moving average must be used. The 12-months moving average of any adequate graduation should approximate, or be a relatively good fit to, the 12-months moving average of the original data. In other words, the smooth curve should not itself be a 12-months moving average of the data, but a curve whose 12-months moving average is similar to the 12-months moving average of the data.[1]

Charts IV and V illustrate this characteristic. Chart IV shows a set of data (ninety-seven consecutive months of Call Money Rates on the New York Stock Exchange) and two smoothings or "graduations." The two graduations are (1) a 12-months moving average of the original data, and (2) a 43-term smooth curve fitted by a formula which we have used throughout our study of interest rates and security prices.[2] It is immediately

[1] Whenever the 12-months moving average of the data exactly equals the corresponding 12-months moving average of the smooth curve, the sum of 12 consecutive ordinates of the smooth curve, of course, exactly equals the sum of 12 consecutive ordinates of the data.

[2] This 43-term smooth curve is calculated as follows: Take a 5-months moving total of a 5-months moving total of an 8-months moving total of a 12-months moving total of the data. To the results apply the following simple weights: +7, −10, 0, 0, 0, 0, 0, 0, +10, 0, 0, 0, 0, 0, 0, −10, +7. Divide the final results by 9600. See pages 73, 74, 75.

Though the above procedure may seem complicated, it can easily be followed by any computer capable of taking a 12-months moving average. It takes about three and a half times as long to compute as a simple 12-months moving average.

apparent that the 43-term graduation is much *smoother* than the 12-months moving average. The difference in *goodness of fit* is shown in Chart V where a 12-months moving average of the data is compared with a 12-months moving average of each of the graduations. The reader will notice that the 12-months moving average of the 43-term graduation gives a much better fit to the 12-months moving average of the data than does the 12-months moving average of the 12-months moving average graduation. The 43-term graduation not only is much smoother but gives a much better fit.

As 43 months are needed to obtain one point on the 43-term graduation, there are necessarily 21 months at each end of the data which are not covered by the smooth curve, just as 6 months at each end of the data are not covered by the 12-months simple moving average. Such smoothings have to be extended, if they are to cover the entire range of the data. In the Call Money illustration above, this difficulty has been overcome by using data of the period before January 1886 and data of the period after January 1894. In the study of interest rates and security prices, each series was smoothed for the entire period January 1857 to date. Extension backwards in time was accomplished by using in each case the best data obtainable for the preceding 21 months. Forward in time, the graduations might have been mathematically extended without using any further data, real or hypothetical.[1] However, the easiest, and generally the best, way to extend a graduation forward in time is to apply it to hypothetical extrapolated data. This method has been used in the study of interest rates and security prices. The difficulties and dangers of such a procedure are distinctly less than the difficulties and dangers involved in freehand or other extrapolation of the smooth curves themselves. The tail end of any curve has necessarily a large probable error, and thoroughly adequate results—which would be likely to check with later data, when received—are generally quite improbable. This is just as true of graduations such as the Whittaker-Henderson, which need no extrapolation, as of graduations which require extrapolation. Moreover, mathematical extrapolation does not solve this difficulty.

The reader must not suppose that the particular 43-term formula emphasized in this book is presented as any final word on the subject of smoothing. It is primarily a method which is adapted to graduating monthly data in such a manner as to eliminate seasonal and erratic fluctuations and at the same time save all trend and the non-seasonal cyclical swings. It is not laborious. However, if the reader wishes to reduce the computation still

[1] For the details of the procedure to be used for such mathematical extension of the graduation, see pages 113, 114, 115.

further and yet obtain as good results as possible under such circumstances, he may use a simpler formula involving fewer steps. An example of such a formula would be: Take a 4-months moving total of a 7-months moving total of the data. Subtract a 16-months moving total of the data. Take a 3-months moving total of a 12-months moving total of the result. Divide by 432.[1] Twenty-nine observations are required to obtain one point on the graduated curve. The amount of computation is less than that involved in calculating the 43-term graduation. The formula will give comparatively good fits to a large range of sine curves.[2] It gives a comparatively good fit to our Call Money data, as may be seen by reference to column 14 of the table in Appendix VIII. It eliminates 12-months seasonal fluctuations. It is a fairly good formula for the investigator who wants a simple substitute for a 2-months moving average of a 12-months moving average. It takes little more than twice as long to compute as such a 2-months moving average of a 12-months moving average.

This 29-term formula falls an appreciable distance *outside* the parabola $y = x^2$, when applied

[1] This formula may also be applied as follows: Take a 14-months moving total of the data with the following simple weights: — 1, 0, 0, 0, + 1, + 1, + 1, + 1, + 1, + 1, 0, 0, 0, — 1. Take a 3-months moving total of a 3-months moving total of a 12-months moving total of the results. Divide by 432.

[2] See formula number 14 in Appendices IV, VII and VIII.

to points on that parabola.[1] If the investigator prefers a formula falling approximately on the parabola $y = x^2$, he may use the 29-term formula described on page 59. If he furthermore insists upon an absolutely irreducible minimum of labor, he must use a formula with a poorly shaped weight diagram. For example, a 27-term formula, which will eliminate monthly seasonal fluctuations, and give a distinctly better fit and smoother graduation than a 12-months moving average, but not as good a fit or smooth a graduation as can be obtained by using more complicated formulas, may be applied as follows: Take a 16-months moving total of the data with the following simple weights: $-1, 0, 0, 0, +1, +1, +1, +1, +1, +1, +1, +1, 0, 0, 0, -1$.[2] Take a 12-months moving total of this weighted 16-months moving total. Divide each of the final results by 72.[3] It is seldom advisable to use such an extremely simple formula.[4] A very little more labor will give distinctly better results.[5]

[1] This is usually an advantage with cyclical data.

[2] The reader will, of course, note that for calculation the middle set of units is treated as a simple 8-months moving total.

[3] If applied to points on the parabola $y = x^2$, the graduation will fall $\frac{1}{6}$ of a unit inside the parabola.

[4] Except in the case of the measurement of average seasonal fluctuations by means of operations on the deviations of the data from a graduated curve. For that purpose it is not necessary that the graduation be more than roughly adequate. See Appendix I.

[5] For example, the use of the following 27-term formula: Take a 10-months moving total of the data with the following simple weights: $-1, 0, 0, +1, +1, +1, +1, 0, 0, -1$. Take a 7-months

If the elimination of seasonal fluctuations is of very minor importance and if the short time fluctuations of the data are not too large (when compared with the amplitude of the longer time cyclical fluctuations), some of the well-known third-degree parabolic [1] summation formulas may be used without introducing any large element of erratic fluctuation.[2] For example, Kenchington's 27-term formula,[3] if applied to such a series as Railroad Stock Prices, gives fairly good results. If the investigator desires to follow his data somewhat more closely than Kenchington's formula permits, he may use such a formula as Spencer's

moving total of a 12-months moving total of the results. Divide by 168. The resulting graduation will be distinctly smoother and the fit better than with the slightly less laborious 27-term formula described in the text. The graduation falls $1\frac{5}{6}$ outside the parabola $y = x^2$.

[1] By a "third-degree parabola" we mean an equation of the form

$$y = A + Bx + Cx^2 + Dx^3$$

[2] Third-degree parabolic formulas always introduce some element of error in cyclical material. See pages 49, 50.

[3] A 5-months moving total of a 7-months moving total of an 11-months moving total of a 7-months moving total with the following simple set of weights: $-1, 0, +1, +1, +1, 0, -1$. Result divided by 385.

A 27-term formula which is easier to compute than Kenchington's and which gives appreciably closer fits to sine curves of short periods, with almost as close fits to sine curves of long periods, may be applied as follows: Take an 11-months moving total of the data with the following simple weights: $-1, 0, 0, +1, +1, +1, +1, +1, 0, 0, -1$. Take an 8-months moving total of a 10-months moving total of the results. Divide by 240. The weight diagram is only a little less smooth than Kenchington's. When applied to the parabola $y = x^2$ the formula gives a curve falling $\frac{1}{6}$ inside.

21-term formula.[1] However, the Spencer 21-term formula is very poorly adapted to smoothing such a series as Call Money Rates, where there is a large seasonal fluctuation and where the short-time fluctuations are large when compared with the cyclical fluctuations.

The advantages and disadvantages of the 43-term formula emphasized in this book and of a number of other methods of fitting are discussed in the text. For certain particular types of problem, the Whittaker-Henderson method of graduation, when judiciously used, is almost ideal. The Henderson method of computing this smooth curve is one of the most elegant contributions which have ever been made to the literature of the subject.

[1] See pages 51, 52, 53.