10 Signaling, Screening, and Information

Michael Spence

10.1 Introduction

In the past seven years, a variety of models that focus upon the informational aspects of labor markets have been developed. The starting premise of most of these models is that while individuals differ in their abilities with respect to various kinds of jobs, these differences are not immediately evident to the employer, either at the time of hiring, or even soon thereafter. Jobs and income are allocated on the basis of imperfect indicators or surrogates for productive capability or potential. Education has been the focus of much of the discussion, it being one of the bases for entry into job categories and for salary levels. There are, however, other potential sources of information about employees in labor markets. Previous work history, previous salary, the very fact that an individual is in a particular labor market, criminal and service records, medical history are all potential sources of information.

The collection of models is variously referred to as signaling and screening. The literature in the area of signaling attempts several objectives. One is the construction of rigorous models in which the equilibrium content of a potential signal is explained and explored. A second is the identification of the implications of the existence of signaling for market performance and the allocation of individual resources. A third consists of an attempt to identify the empirical magnitude of the signaling effects, if any, especially with respect to education. A fourth area concerns the concept of equilibrium that is employed and some related problems with the existence of equilibria. A fifth broad area deals with the policy implications of signaling and screening. These may include discrimina-

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tion, job mobility, efficiency in the area of education, aspects of the problem sometimes (and misleadingly) referred to as privacy, the effects of training programs, and licensure.

The goals of this paper are to survey and extend in certain areas the analysis of signaling and screening. Except in the earlier sections which are expository, I have tried to set out versions of the models that would lend themselves to empirical testing and the application of data. I have also attempted to explore in a limited way the implications of results of regressions of earnings on education and “ability,” depending upon assumptions about the underlying structure of the model. The paper tries to present a more complete and balanced view of the welfare aspects of signaling. Later sections discuss, sometimes briefly, some of the policy areas that I mentioned above. Conclusions must be tentative, as the empirical work upon which the policy implications are partially based is not yet complete. I am afraid that this latter discussion may appear rather disjointed, perhaps a necessary consequence of the state of development of the subject.

10.2 The Signaling Model and Alternatives

Let me begin by reviewing a variety of models that are in some sense competitive hypotheses. It is convenient to do this with numerical examples, leaving to a later section the problem of putting the models in an empirically more usable form. The main point of this analysis is to convey the idea that at least three rather different models have very similar-looking equilibria, and that, using data one can reasonably be expected to have, they are difficult to distinguish. They differ not in their predictions about resource allocation, but rather in the welfare implications of the pattern of resource allocation that develops. What appears below are three polar cases, the signaling model, the rationing model, and the human capital model. It is possible to have a debate about whether these terms have been used historically to refer to the phenomena I have in mind, and it might be fairer to refer to them as A, B, and C. Those who find the terms misleading can so translate.

Let us assume that there are two groups of people in an employable population. The people have different productivities in different jobs. Each group can invest in education. The costs, monetary and psychic, of education differ from the two groups. For group 1, \( e \) years of education cost \( c_1e \) dollars. For group 2, the cost is \( c_2e \) dollars. It is assumed that \( c_2 < c_1 \). The proportion of people in group 1 is \( a_1 \). The proportion in group 2 is \( a_2 = 1 - a_1 \). Let us assume that there are two jobs. Let \( f_{i,j}(e) \) be the productivity of some of group \( i \) in job \( j \) with education \( e \). There are a variety of assumptions that one can make about the magnitudes of the \( f_{i,j}(e) \). These will appear in the sequence of models that follow. In fact, this is what distinguishes the models from each other.
The models have certain assumptions in common. Employers observe education, but not productivity, directly, and make job decisions on the basis of education. People of a given level of education are offered the job where their expected productivity is highest. Further, they are offered a salary equal to their expected productivity, conditional on the education level. Finally, these expected productivities are accurate. That is to say, the salaries at each education level correspond exactly to the average productivity of people in that education group. Individuals optimally invest in education, given the costs and the above-mentioned salary and job offers. These investment decisions determine the average productivities for each education level, which in turn determine the job and salary offers. I shall assume that individuals maximize income net of signaling costs. Preferences with respect to jobs are ignored, though their introduction would produce no qualitative changes in the results. These then are the common features of the models.

10.2.1 Pure Signaling

Assume that $f_i(e)$ does not depend upon $e$ and further that $f_{i1} = f_{i2} = f_i$ for groups $i = 1, 2$. Define $e^*$ to be a number which satisfies the inequalities

$$\frac{f_2 - f_1}{c_1} < e^* < \frac{f_2 - f_1}{c_2}$$

Here, it is assumed that $f_2 > f_1$, and that $c_2 < c_1$. The equilibrium in the model is as follows. The salary offer is $f_1$ if $e < e^*$, and $f_2$ if $e \geq e^*$. Group 1 rationally invests $e = 0$, while group 1 sets $e = e^*$. Salaries correspond to average productivities because group 2 (at $e = e^*$) has productivity $f_2$ and group 1 (at $e = 0$) has productivity $f_1$. The model is referred to as pure signaling because education does not contribute to productivity and because productivity of all jobs is the same.

The equilibrium is summarized in table 10.1. The single most important property of the equilibrium is that the private and social returns to education differ. As a result, the second group overinvests in education. The optimum would require $e = 0$ for both groups. But then group 2 would not be distinguished. Salary would be $\alpha_1 f_1 + \alpha_2 f_2$ for everyone. That would benefit group 1 but not group 2. In the pure signaling case, education is invested in because it distinguishes people, thereby redistributing (rather than increasing) the product and income.

10.2.2 Pure Human Capital

Assume that $f_{ij}(e) = f(e)$ for both groups and both jobs. Neither the job nor the type of person affects productivity. Let $f(0) = f_1$ and $f(e^*) = f_2$. Assume that $e$ satisfies the inequalities in (1) above. The equilibrium for this model is much like that for the signaling model. It is summarized in table 10.2. The difference between this model and the signaling model
Table 10.1  Signaling

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Schooling</th>
<th>Education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e = 0$</td>
<td>$e = e^*$</td>
<td></td>
</tr>
<tr>
<td>Group 1</td>
<td>$f_1$</td>
<td>$c_1$</td>
<td>$f_1$</td>
</tr>
<tr>
<td>Group 2</td>
<td>$f_2$</td>
<td>$c_2$</td>
<td>$f_2$</td>
</tr>
</tbody>
</table>

Note: Bold face figures indicate equilibrium productivities.

Table 10.2  Human Capital

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Schooling</th>
<th>Education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e = 0$</td>
<td>$e = e^*$</td>
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</tr>
<tr>
<td>Group 1</td>
<td>$f_1$</td>
<td>$c_1$</td>
<td>$f_1$</td>
</tr>
<tr>
<td>Group 2</td>
<td>$f_1$</td>
<td>$c_2$</td>
<td>$f_2$</td>
</tr>
</tbody>
</table>

lies in the off-diagonal terms in the productivity part of the table, and therefore in the welfare implications. Here salary offers at different levels of education reflect different productivities. The equilibrium is efficient, meaning that each person or group invests in the correct amount of education. I assume the choice is restricted to $e = 0$ and $e = e^*$. Education is productive. People invest in it differentially because the costs vary over people. But that is the desired outcome.

There is an alternative version of the model that deserves mention. It could be that $f_{ij}(e)$ does vary with $i$. If the employer could directly observe an individual’s type, or productivity, then he would put a group $i$ person with education $e$ in a job where $j$ maximizes $f_{ij}(e)$. Individuals would then optimize by selecting $e$ to maximize

$$\max_j f_{ij}(e) - c_i e$$

The results would again be efficient.

10.2.3 The Rationing Model

In the previous models, we have concentrated on the type of person and the educational level as a determinant of productivity. Here, we turn to the job. Suppose that $f_{ij}(e) = f_j$ so that productivity depends on the job, but not the type of person or the education level. Assume that the proportion of jobs of type 2 is $a_2$ and that $f_2 > f_1$. The fact that the proportion of jobs of type 2 is $a_2$ is rigged to make the model work. Later, I shall discuss rationing in a more general setting.

The equilibrium in this model is summarized in table 10.3. Once again, it has the observable attributes of the equilibria in the previous two
models. But here education is being used to ration high-productivity jobs so the group with the lower costs of education does not get them. If jobs were randomly assigned to people, output and total incomes would be the same, but the education costs would be avoided. People could be paid either $f_1$ or $f_2$, depending upon the job they draw, or the average $a_1f_1 + a_2f_2$. As in the signaling model, education investment redistributes income without changing the size of the pie.

The rationing model is incomplete without an explanation of the reason why higher-productivity jobs might be scarce, or at least have higher salary offers attached to them. Certainly, the usual notion of expansion and contraction until productivities are equated at the margin has been dropped. It is not difficult to see that certain kinds of jobs in hierarchical productive organizations are not easily duplicated. This would tend to suggest that there is a difficulty in defining the notion of productivity, and perhaps some concomitant arbitrariness in the salaries and incomes attaching to certain kinds of jobs. The salary structure might be maintained for incentive purposes. The idea that salaries in part attach to jobs does not fit into conventional theory easily, and whether or not it is true is an open question. If it is, then the jobs have to be rationed, and costly investments might be one basis for the rationing.

### Table 10.3 Rationing

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Schooling Cost</th>
<th>Salary</th>
<th>Education Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>$f_1$</td>
<td>$c_1$</td>
<td>$f_1$</td>
<td>0</td>
</tr>
<tr>
<td>Group 2</td>
<td>$f_1$</td>
<td>$c_2$</td>
<td>$f_2$</td>
<td>$c_2e^*$</td>
</tr>
</tbody>
</table>

The information carried by signals in the market can be socially productive if it improves the quality of decisions with respect to jobs or training. Assume that $f_{ij}$ does not depend on $e$. But let us assume that $f_{12} = f_{21} = g$, and that $f_{22} > f_{11} > g$. Under these considerations, there is a benefit to ensuring that group 1 ends up in job 1 and group 2 ends up in job 2. The equilibrium involves the same investments in education and incomes as in the pure signaling model. It is summarized in table 10.4. As before, the incentive for investing in education is to distinguish different types of people. But, the information is productive, because the types of people are differentially productive. Without the signal, the best that could be accomplished is the maximum of $a_1 f_{11} + a_2 g$ and $a_1 g - a_2 f_{22}$, both of which are lower than the output $a_1 f_{11} + a_2 f_{22}$ realized with the signal. Of course, the signals have a cost and the cost does not necessarily justify the increase in output. The best signaling outcome occurs when
$e^* = (f_{22} - f_{11})/c_1$. In that case, the signaling equilibrium increases net income (net, that is, of signaling costs) if

$$g < a_2[(1 - c_2/c_1) f_{22} + (c_2/c_1)f_{11}]$$

The value of the signals, therefore, decreases as $g$ increases, and increases with $a_2, f_{22}, f_{11},$ and $c_2/c_1$. I should add that in doing this calculation, I have assumed that $a_1 f_{11} + a_2 g$ is larger than $a_1 g + a_2 f_{22}$. A similar formula holds for the opposite assumption.

Notice that if $f_{22} = d_{11}$, the signaling equilibrium could not be sustained. Group 2 would set $e = 0$ and hence not be distinguished. In this case, it would benefit everyone if group 2 were paid $w_2 > f_{22} = f_{11}$ and if, as a result, group 1 were paid $w_1 < f_{11} = f_{22}$. In fact, this kind of noncompetitive salary could be advantageous even if $f_{22} < f_{11}$, at least over a certain range.

10.2.5 Inducing Efficient Investment in Education

The signaling effect can be beneficial in providing the correct incentives to invest in education, when the latter is productive. Assume that $f_1(e) = f_2(e)$, so that jobs are not relevant. Assume further that $f_1(0) = f_2(0)$ and that $f_1(e^*) < f_2(e^*)$. Let $f(e^*) = a_1 f_1(e^*) + a_2 f_2(e^*)$, the average of the productivities at $e = e^*$. Finally, assume that because of education costs, it is efficient for group 1 so to invest. The equilibrium is depicted in table 10.5. Once again, it has the properties discussed under previous models. Moreover, the equilibrium is efficient.

However, there is another possible type of equilibrium. Suppose that the salary offer for $e^*$ were $f(e^*)$, the average. This might induce group 1 to invest in the signal, in which case we would have an equilibrium, but it would be inefficient. Group 1 would be overinvesting. Similarly, such a salary offer might induce group 2 to set $e = 0$. That also would be an

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<tbody>
<tr>
<td>$f_{11}$</td>
<td>$g$</td>
<td>$c_1$</td>
<td>$f_{11}$</td>
</tr>
<tr>
<td>$f_{22}$</td>
<td>$g$</td>
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<tr>
<td>$f_{11}$</td>
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<td>$f_{22}$</td>
<td>$g$</td>
<td>$c_2$</td>
<td>$f_{22}$</td>
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</tbody>
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<table>
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<tr>
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<th>Productivity</th>
<th>Schooling</th>
<th>Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{11}$</td>
<td>$g$</td>
<td>$c_1$</td>
<td>$f_{11}$</td>
</tr>
<tr>
<td>$f_{22}$</td>
<td>$g$</td>
<td>$c_2$</td>
<td>$f_{22}$</td>
</tr>
</tbody>
</table>
equilibrium and would be inefficient in the other direction. The point here is that education signals productivity and thus provides the returns that induce efficient levels of investment in education—or at least it can.

10.2.6 Some General Remarks

One could go further with these models. There is no reason to assume that the signaling, human capital, and rationing effects are mutually exclusive. In fact, they are unlikely to be. The point I want to make is that market outcomes with quite different normative properties appear similar, in terms of the observable variables, education levels, and average productivities or outputs. To distinguish among them, one has to observe the off-diagonal entries in the tables above. These productivities result when groups have levels of education and jobs that differ from the equilibrium levels.

10.3 Signaling, Human Capital, and Ability

In this section, I should like to set forth a version of the model containing signaling and human capital that can be adapted to empirical work on the determinants of individual productivity. Let $y$ be the number of years of schooling and let $n$ be the ability that is relevant to the determination of individual productivity. Productivity is determined by education and ability according to the function

$$ s = ny^\alpha $$

Here, $s$ denotes productivity.

Education is costly, and the costs vary over individuals. Let us assume that $y$ years of schooling cost a person of type $z$, $C(y, z) = y/z$ dollars, or at least that this is the monetary equivalent of the cost. Productivity is to be interpreted as an average and discounted output over the expected life of the individual. The present value of salary is $W(y)$. Individuals select schooling to maximize $W(y) - y/z$ by setting

$$ W'(y) = \frac{1}{z} $$

The parameters $n$ and $z$ are distributed in the employable population. To complete the model, we must characterize the joint distribution of $n$ and $z$. The assumption is that

$$ n = \frac{z^\varepsilon}{K(\varepsilon)} u $$

where $u$ is independent of $z$ and has a mean of one. Further, it is assumed that $K(\varepsilon)$ is the expected value of $z^\varepsilon$, so that
\[ K(\varepsilon) = E(z^\varepsilon) \]

As a result, the expected value of \( n \), given \( z \), is

\[ N(z) = \frac{z^\varepsilon}{K(\varepsilon)} \]

The unconditional expected value of \( n \) is one. The parameter \( \varepsilon \) is the elasticity of the conditional mean with respect to \( z \). Thus we can vary the power of the signal by varying \( \varepsilon \), and the noisiness by varying the dispersion of \( u \).

The model can now be completed. In equilibrium for every \( y \),

\[ (2) \quad E(s|y) = W(y) \]

From (1) \( y \) is determined by \( z \). Thus (2) becomes

\[ (3) \quad y^\alpha N(z) = W(y) \]

or simply

\[ (4) \quad \frac{y^\alpha}{K(\varepsilon)} [W'(y)]^{-\varepsilon} = W(y) \]

The solution to this first-order differential equation gives the family of equilibrium salary schedules. I shall assume that when \( n = 0 \), \( y = 0 \). This is the equilibrium with the least overinvestment in education (see Riley 1975a for a discussion). Thus the solution, upon integrating (4), is

\[ W(y) = \left( \frac{1 + \varepsilon}{\alpha + \varepsilon} \right)^{\varepsilon} K(\varepsilon)^{-\frac{1}{1+\varepsilon}} y^{\frac{\alpha + \varepsilon}{1+\varepsilon}} \]

This is the equilibrium salary schedule in the market. The elasticity of salary with respect to schooling is \( (\alpha + \varepsilon)/(1 + \varepsilon) \). It contains a term involving \( \varepsilon \), the elasticity of productivity with respect to schooling. The signaling effect and \( \varepsilon \) are the same thing. Signaling occurs when ability and the costs of schooling are negatively correlated, or, more precisely, when the expected value of ability falls as education costs rise.

The model can be solved for investment in schooling, productivity, and net income, for each type of individual. An individual is characterized by \( n \) and \( z \) or, equivalently by \( u \) and \( z \). Table 10.6 reports these quantities for the signaling case (i.e., the situation in which productivity is not directly observed by employers) in column 1. Schooling and net income do not depend upon \( y \). Productivity does. The average value of productivity is formed by setting \( u = 1 \) in the productivity figure. That is equal to the gross income (before schooling costs) for individuals of type \( z \).

In column 2 of table 10.6, I have reported, for comparative purposes, the values of the relevant variables for the case where market information
is perfect, so that schooling does not serve as a signal. Here, $u$ appears in all the expressions, because investment in schooling depends upon $n$.

The third column reports the ratio of the first two columns. It is the second column divided by the first. I want to focus on the third column for the moment.

There is a tendency to overinvest in schooling. The term $\left[\frac{(\alpha + \alpha \varepsilon)}{(\alpha + \varepsilon)}\right]^{1/1-\alpha}$ is less than 1. Thus the ratio is less than one until $u$ gets above its mean which is one. However $u^{1/1-\alpha}$ is convex in $u$. Therefore those with high levels of $u$ may underinvest in schooling by large amounts. This is worrying, since these are talented people who happen to have high schooling costs. If there were a mechanism for identifying them, the efficiency of the market could be improved. Figure 10.1 depicts the relation between the ratio $R$ and $u$.

Note that if $u=1$ so there is no noise, then everyone overinvests in schooling, which is the conventional conclusion. Here I want to emphasize that there is the further problem of the distribution of investments associated with variations in productivity, not correlated with schooling costs.

The second row of table 10.6 contains productivity figures. The constant term in the ratio is again less than one. However the shape of $u^{\alpha/1-\alpha}$ depends on $\alpha$. It can be convex or concave. If $\alpha$ is small, the ratio as a function of $u$ is concave. The productivity of those with high $u$ will not differ from the optimum by as much as it will if $\alpha$ is large. Thus the performance of output depends in part on the elasticity of productivity with respect to education. Figure 10.2 illustrates the relationship for the case $\alpha < 1/2$.

Next consider net income. The term $(1 + \varepsilon) \left[\frac{(\alpha + \alpha \varepsilon)}{(\alpha + \varepsilon)}\right]^{\alpha/1-\alpha}$ can be shown to be greater than one. The ratio as a function of $u$ is depicted in

![Figure 10.1](image-url)
Table 10.6  Schooling, Productivity, and Net Income with and without Perfect Information

<table>
<thead>
<tr>
<th>Variable</th>
<th>Employer Does Not Observe Productivity Directly</th>
<th>Employer Does Observe Productivity Directly</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Schooling</td>
<td>( \frac{1}{K^{\alpha-1}} \left( \frac{\alpha+\varepsilon}{1+\varepsilon} \right) \frac{1}{1-\alpha} z^{1-\alpha} )</td>
<td>( \frac{1}{\alpha^{1-\alpha}} \frac{1}{z^{1-\alpha}} \frac{1}{K^{\alpha-1}} \frac{1}{u^{1-\alpha}} )</td>
</tr>
<tr>
<td>Productivity</td>
<td>( u \frac{1}{K^{\alpha-1}} \left( \frac{\alpha+\varepsilon}{1+\varepsilon} \right) \frac{\alpha}{1-\alpha} z^{1-\alpha} )</td>
<td>( \frac{1}{u^{1-\alpha}} \frac{1}{\alpha^{1-\alpha}} \frac{1}{z^{1-\alpha}} )</td>
</tr>
<tr>
<td>Income for individuals</td>
<td>( \frac{(1-\alpha)}{(1+\varepsilon)} \frac{1}{K^{\alpha-1}} \left( \frac{\alpha+\varepsilon}{1+\varepsilon} \right) \frac{1}{(1-\alpha)} z^{1-\alpha} )</td>
<td>( \frac{1}{(1-\alpha)} \frac{1}{\alpha^{1-\alpha}} \frac{1}{z^{1-\alpha}} )</td>
</tr>
<tr>
<td>Total net income</td>
<td>( \frac{(1-\alpha)}{(1+\varepsilon)} \frac{1}{K^{\alpha-1}} \left( \frac{\alpha+\varepsilon}{1+\varepsilon} \right) E \left( z^{\frac{\alpha+\varepsilon}{1-\alpha}} \right) (1-\alpha) E \left( u^{\frac{1}{1-\alpha}} \right) \frac{1}{K^{\alpha-1}} \frac{1}{\alpha^{1-\alpha}} \frac{1}{z^{\frac{\alpha+\varepsilon}{1-\alpha}}} )</td>
<td></td>
</tr>
</tbody>
</table>

Figure 10.2  Relative Productivity Levels
the remainder. If the high schooling costs are associated with variables such as family income, the discriminatory implications of imperfect information deserve attention, because they are more extreme than they would be with better information (even if there were no changes in the schooling costs themselves). To put this another way, there is an interaction effect between schooling costs and imperfect job market information.

Figure 10.3 Relative Net Incomes
that imposes very high costs on high-productivity/high-schooling-cost people.

Total net income is unambiguously greater with perfect information because, in addition to the constant term that is positive, $E(u^{1/1-\alpha}) > 1$ because of the convexity of $u^{1/1-\alpha}$.

To summarize, the model captures in analytically tractable form, the signaling effect of schooling. The elasticity of productivity with respect to schooling can be adjusted parametrically. The "correlation" of schooling costs and productivity is also parametrically specified. In addition to the earlier conclusions about overinvestment in education, there are distribu-
tional or discrimination problems associated with high-productivity people whose schooling costs are high, sometimes for reasons extraneous to their abilities.

Thus far, I have been using the market with perfect employer information as the standard. Such a standard may not be attainable. One can ask what the second-best alternative is, and how it compares with the market equilibrium. The second best concedes that productivity is not directly observable, so that investment in schooling cannot be sensitive to $u$. Nevertheless, we can tax the signaling activity so as to change the relationship between $y$ and $z$. If we do this so as to maximize total net income, the results are as reported in column 4 of table 10.6. Column 5 reports the ratio of column 4 to column 1, so that we have a measure of the extent to which the market outcome differs from the imperfect information, second-best outcome.

The results are quite easy to summarize. With imperfect information, we get what would be the optimal levels of investment if $u = 1$, that is, if the signaling effect is removed. But the sensitivity to the random component of productivity is lost. Thus if we compare columns 3 and 5 for total net income, column 3 has two terms that are greater than one—$E(u^{1/1-\alpha})$ and $(1 + \varepsilon) ((\alpha + \alpha \varepsilon)/(\alpha + \varepsilon))^{1/1-\alpha}$. Column 5 for the informationally constrained optimum has only the second term. This represents the gain from preventing average overinvestment in signaling. But the unobserv-
bility of $u$ means that the first term is lost.

The point of this is to illustrate that imperfect information causes two problems. Investment in schooling is insensitive to the full differences across individuals in the return to investment in schooling. Second, there is a tendency to overinvest in schooling on average because the private return contains that component of ability that is correlated with schooling costs. The fact that schooling is a signal means that investment in it is partially sensitive to ability differences. This is good, but the market tends to overdo it; the second-best optimum calculation illustrates this.

If one calculates the return to schooling required to achieve the second-
best optimum, it is
where \( d \) is calculated so as to cause the supply side to break even. The ratio of \( \dot{W}'(y) \) to \( W'(y) \), the slope of the equilibrium schedule, is

\[
\frac{\dot{W}'(y)}{W'(y)} = \left( \frac{\alpha + \beta \varepsilon}{\alpha + \varepsilon} \right)^{\frac{1}{\varepsilon}} < 1
\]

Hence the optimal schedule is flatter than the market equilibrium. It induces lower levels of investment in schooling. We can think in terms of taxing schooling. Let the tax schedule be \(-f + ty\) where \( f \) is a subsidy, and \( t \) is the marginal tax rate. It can be shown that when

\[
t = \left( \frac{\alpha + \beta \varepsilon}{\alpha + \varepsilon} \right)^{\frac{1}{\varepsilon}} - 1
\]

then the market equilibrium is efficient in the second-best, imperfect information sense. Thus the optimal tax schedule for schooling is linear. The subsidy \( f \) is calculated so that the government breaks even. While the marginal tax is positive, the overall tax is negative for lower levels of schooling and positive for higher levels.

There is one related point I should like to make in this context. Consider average net income in the market (column 1, row 4, in table 10.6). It is

\[
N(\alpha, \varepsilon) = (1 - \alpha)K^{\alpha - 1}(1 + \varepsilon)^{\alpha - 1}(\alpha + \varepsilon)^{\alpha - \alpha}E(z^{\alpha - \alpha})
\]

As we have seen, there is a signaling component of the return to schooling that includes overinvestment. On the other hand, with imperfect information, schooling distinguishes people and therefore makes investment sensitive to at least one component of ability. One can ask which is more important. What I want to show is that

\[
\alpha \eta(a + \alpha) - (\alpha + \varepsilon) > 0
\]

so that at least over some range, the beneficial second effect outweighs the overinvestment cost. Or to put it another way, with imperfect information, at least some signaling is beneficial. There is in fact an optimal degree of correlation of \( n \) and \( z \). I will confine myself to showing that inequality 5 holds.

The argument is as follows. Taking logs and differentiating, we have (with \( \beta = \alpha/(1 - \alpha) > 0 \),

\[
\frac{\partial N}{\partial \varepsilon}(\varepsilon, \alpha) = 0
\]
We want to show that the term in square brackets is positive. Let $X = \log z$ and $r(X) = e^X h(e^X)$. It is sufficient to show that

$$\int x e^{\beta X} r(X) dx > \int x r(X) dx \int e^{\beta X} r(X) dx$$

Let $\bar{X}$ be the mean of $X$, that is, $\bar{X} = \int X r(X) dX$.

Then

$$\int x e^{\beta X} r(X) dx = \int x e^{\beta X} r(X) dx + \int (X - \bar{X}) e^{\beta X} r(X) dx$$

which, by the convexity of $e^{\beta X}$, equals

$$\bar{X} \int x e^{\beta X} r(X) + \beta e^{\beta \bar{X}} \int (X - \bar{X})^2 r$$

Thus the term in square brackets is positive for $\beta > 0$, or equivalently $\alpha > 0$. The implication is that net income rises as $\varepsilon$ rises starting from $\varepsilon = 0$.

Note however that when $\alpha = 0$, net income is

$$N(0, \varepsilon) = \frac{1}{K} \cdot \frac{1}{1 + \varepsilon} E(z^\varepsilon) = \frac{1}{1 + \varepsilon}$$

This declines in $\varepsilon$. The reason is that when $\alpha = 0$, the optimal investment in $y$ is zero. Thus the presumed benefit of signaling, making investment sensitive to differences in productivity, is nonexistent when any amount of investment is inefficient, i.e., when $\alpha = 0$.

### 10.4 Estimating the Determinants of Productivity

In view of the preceding models, several potential problems arise in estimating the determinants of productivity using data on earnings and schooling.

The first problem is that for young workers, earnings and productivity may differ because their productivity has not been discovered. If productivity is $ny^\alpha$, earnings in a sample of younger workers might be

$$e = \beta ny^\alpha + (1 - \beta) W(y),$$

where $W(y)$ is the average return to each level of education and $\beta$ is a random variable between zero and one.
If we could observe \( n \) and \( y \), and estimated \( (6) \), the result would be

\[
e = \hat{\beta} ny^\alpha + (1 - \hat{\beta}) W(y),
\]

where \( \hat{\beta} \) is the average value of \( \beta \) in the sample.

This would understate the contribution of ability, and overstate that of schooling.

A second problem is that we do not measure the relevant kind of ability directly. There are surrogates, such as test scores, that may be used in isolating the effects of schooling and ability on productivity. Let me assume that earnings accurately measure productivity so that the first problem does not arise. The problem then is that we do not measure \( n \).

Using capital letters for logs, and eliminating means and constant terms, the signaling model implies that

\[
N = \epsilon Z + U
\]

and

\[
y = (1 + \epsilon) Z
\]

Then, using the equilibrium relationship

\[
E = (\alpha + \theta) Y + U
\]

where \( \theta = \epsilon (1 - \alpha) / (1 + \epsilon) \), productivity is \( E = N + \alpha Y \).

Suppose there is a measure \( I \) of ability. It can be thought of as a test score. Suppose further that we regress \( E \) on \( Y \) and \( I \):

\[
E = \gamma_Y y + \gamma_I I + \nu
\]

The estimated coefficients, for a large sample are

\[
\hat{\gamma}_I = \frac{\sigma_U}{\sigma_I} \frac{\rho_{IY}}{1 - \rho_{YI}^2}
\]

\[
\hat{\gamma}_Y = (\alpha + \theta) - \frac{\sigma_U}{\sigma_Y} \frac{\rho_{YI} \rho_{IY}}{1 - \rho_{YI}^2}
\]

Now if the variable \( I \) is correlated with \( N \) because of its correlation with \( Z \) and hence \( Y \), so that \( \rho_{IY} = 0 \), then the estimated coefficient of \( I \) will be zero and the coefficient of schooling will be \( \hat{\gamma}_Y = (\alpha + \theta) \). The schooling coefficient picks up the productive effect and the signaling effect of schooling. The variable, \( I \), would capture nothing because it operates through schooling; that is, it is correlated with ability because it is correlated with schooling costs. One can put this more positively. In order for the surrogate for ability not to distort the estimates, it must be positively correlated with that part of the ability variable that is not correlated with schooling or schooling cost variables.
It has been suggested that ability may be productive through its effect on the productivity of schooling. There are several interpretations that can be given to this idea. One is that ability and schooling are complementary inputs, a feature built into the model above. A second might be that the variance of $U$ is small, so that productivity is largely determined by the ease with which the individual acquires education. One difficulty with that interpretation is that there are real cost differences over individuals related to family income and the like, which make it seem unlikely, at least to me, that the variance of $U$ is small or zero. But that aside, a low coefficient on the surrogate ability variable is at least open to the interpretation that the surrogate variable picked up the part of ability that varies systematically with schooling cost, and that as a result its impact was already captured by the schooling variable. The measured ability variables are often test scores, where the tests were designed to predict educational performance. The conclusion that ability is not particularly important does not seem warranted.

If the variable $I$ is perfectly correlated with $U$, so that it precisely captures that part of the variance in ability that is not systematically related to schooling, then the estimated coefficients will be

$$
\gamma_I = \frac{\sigma_U}{\sigma_I} \\
\gamma_Y = (\alpha + \theta)
$$

because $\rho_{IY} = 0$ in that case. Schooling continues to pick up the signaling effect. The magnitude of the coefficient on $I$ will depend upon the relative sizes of the school-related and independent components of the variance in $N$. There is a direct test of this hypothesis. If $U$ and $I$ are perfectly correlated, then $I$ and $Y$ are uncorrelated, because $Y$ and $U$ are uncorrelated. This is testable and, for most measures of ability employed, does not hold.

The formulas above express the estimated coefficients in terms of parameters of the model. They all have unobservables in them. It remains, therefore, to consider what can be estimated with the data, and what one needs to know in order to separate the productivity and signaling effects. The answer to the second question is that one has to know the correlation coefficient between $N$ and $I$. To see this, we can proceed as follows. First, from $E = N + \alpha Y$, and $N = \theta Y + U$, we can rewrite the expression for $E$: $E = (\alpha + \theta) Y + U$. Also, $U$ and $Y$ are uncorrelated, so that immediately

$$\alpha + \theta = \frac{\sigma_{YE}}{\sigma_Y^2}$$
Moreover, we can compute the variance of $U$ and the covariance of $I$ and $U$:

$$\sigma_U^2 = \sigma_E^2 - \frac{\sigma_Y^2}{\sigma_Y^2}$$  
$$\sigma_{IU} = \sigma_{IE} - \frac{\sigma_Y}{\sigma_{YI}}$$

Ability is related to schooling and $U$ by the relationship $N = \theta Y + U$. Thus, the variance of $N$ and the covariance of $N$ and $I$ are

$$\sigma_N^2 = \theta^2 \sigma_Y^2 + \sigma_E^2 - \frac{\sigma_Y^2}{\sigma_Y^2}$$  
$$\sigma_{IN} = \theta \sigma_{YI} + \sigma_{IE} - \frac{\sigma_Y \sigma_{YI}}{\sigma_Y^2}$$

These two relationships contain three unknowns, the variance of $N$, the covariance of $N$ and $I$, and the parameter $\theta$. Any one of these pieces of information would suffice to compute the other two. In particular, it is sufficient to know the correlation coefficient of $N$ and $I$, to compute $\theta$. Once $\theta$ is computed, $\alpha$ follows immediately.

If one could experiment, one would want randomly to assign people education levels, and then observe their subsequent earnings. In terms of the model, that would have the effect of artificially setting $\theta = 0$ and therefore $\varepsilon$ equal to zero. The estimated coefficient of $Y$ would then be $\alpha$ as desired.

10.4.1 Self-Employment

Some people are self-employed and hence not in need of signals of productivity directed at employers. One might argue that the self-employed have to signal to their consumers, as with doctors, dentists, and lawyers. But let us set that aside for the moment. If the self-employed do not have to signal, one might expect the self-employed sector to be a place where the returns to schooling are easier to observe.

Suppose that those who go into the self-employed sector know their own productivities, and invest accordingly. Suppose further, for the moment, that those who enter the self-employed sector are statistically similar to those in the non-self-employed sector. By this I mean the decision to enter the employed or the self-employed sectors is uncorrelated with schooling costs, or ability. I shall relax this assumption shortly. The question that I want to pose is, What will the average productivity at each education level look like in the self-employed sector, and how does that compare with the return in the other sector? The answer is, I think, somewhat surprising.

If people know their own productivity, they maximize $ny^a - y/z$ by setting
Solving for $z$ in terms of $y$ and $u$, we have

$$z = \left( \frac{Ky^{1-\alpha}}{\alpha u} \right)^{1+\epsilon}$$

Therefore, that person will have a productivity, expressed in terms of $y$ and $u$, of

$$E_1 = \frac{1}{1+\epsilon} - \frac{1}{1+\epsilon} y^{\alpha + \theta} u^{1+\epsilon}$$

Thus, the expected or average productivity of people with education level $y$ is

$$w^*(y) = \frac{\epsilon}{1+\epsilon} - \frac{1}{1+\epsilon} y^{\alpha + \theta} E(u^{1+\epsilon})$$

Here, $\theta = \frac{\epsilon(1-\alpha)}{1+\epsilon}$ as in previous sections.

Several points are of interest. First, the elasticity of the average productivity with respect to schooling contains the signaling term $\theta$. The reason is that abilities are correlated with schooling costs and therefore with levels of schooling. It is not, therefore, surprising that that effect should appear in the averages. Second, we can compare the returns in the self-employed and the employed sectors. From a previous section, the return to schooling in the employed sector is

$$w(y) = \left( \frac{1+\epsilon}{\alpha + \epsilon} \right)^{1+\epsilon} K(\epsilon) \frac{1}{1+\epsilon} y^{\alpha + \epsilon}$$

Therefore, the ratio of the two returns of average productivity is

$$\frac{w^*(y)}{w(y)} = \left[ \frac{\alpha + \epsilon}{\alpha(1 + \epsilon)} \right]^\frac{\epsilon}{1+\epsilon} E(u^{1+\epsilon})$$

The first term is greater than one because $\alpha + \epsilon > \alpha + \alpha \epsilon$. On the other hand, the second term is an expected value of a concave function of $u$, which has a mean of one, so that it is less than one. The net effect is ambiguous. No definite relationship exists between the returns in the two sectors. If the variance of $u$ is small, the second term is close to one and the return or average in the self-employed sector is higher than in the other sector. The elasticities of the returns are the same in each sector, independent of the relative magnitudes of the coefficients. The inconclusive results of this type of test do not therefore seem surprising.

If the allocation of people between the self-employed and the non-self-employed sectors is nonrandom, then different results may be obtained.
It is difficult to model the interaction of the two sectors rigorously, because of the complexity of the model. However, one would expect that the people who select themselves into the self-employed sector are those whose ability is high relative to the average, given the costs of education. That is to say, the self-employed are likely to be people who find schooling unattractive, so that it is better to avoid being assessed on the basis of schooling. If this process went on to its logical limit, everyone would be self-employed. But then one would have to take into account changes in productivities due to changes in factor input ratios. This is the complexity that is hard to model. But if the process does not go to its extreme, then those in the self-employed sector will have high abilities relative to education costs, and hence high ability relative to levels of schooling. Therefore, the productivity per unit of schooling will be high in the self-employed sector, though the average levels of schooling may be lower. The kind of selection process will make schooling look more productive in the self-employed sector than in the other sector. That would reflect differences in schooling costs across individuals. But it does not directly test for the presence of the signaling effect.

10.5 The Rationing Model

In an earlier section, I mentioned that jobs may contribute to productivity and that they might be scarce. If they are scarce, for whatever reason, then they will be allocated on some basis, and schooling or some other costly characteristic like experience or age might serve the purpose. Part of the return will then appear to be to the characteristic which serves as a basis for rationing. And for the individual, the return will be real as well as perceived. I should like to set out the rationing model somewhat more precisely, to illustrate it with an example, and to show that the rationing effect can be added to the signaling-human capital model discussed previously.

Suppose that there is a spectrum of jobs, indexed by their productivity $J$. For the time being, $J$ is the only determinant of productivity. The distribution of jobs is $G(J)$, this being the left-hand cumulative. As before, the marginal cost of education is $1/z$, and the cumulative distribution of $z$ is $H(z)$. The problem is to find an equilibrium income function $W(y)$. Given the income function, individuals select $y$ to maximize $W(y) - y/z$, so that

$$W'(y) = 1/z$$

In addition, incomes must correspond to productivities, so that if a person with schooling $y$ receives job $J$

$$W(y) = J$$
We now have to associate the $J$'s and the $y$'s. Schooling is used to ration the jobs. The people who have schooling of $y$ or less are paid $W(y)$ or less. They therefore have productivities of $W(y)$ or less. Their number is therefore $G[W(y)]$. On the other hand, the same group has marginal costs of schooling equal to or greater than $z = 1/W'$. Thus, there are $H(1/W')$ of them, counting through the education cost distribution. For these two different tallies to be consistent, it must be true that

$$(7) \quad G(W) = H(1/W')$$

for every level of $y$ actually observed. That defines the equilibrium income function. As in the signaling model, there are many such functions, but I shall not dwell on that here. Differentiating (7), we have

$$W'' = -(W'')^2 \frac{G'}{H'} < 0$$

so that $W(y) - y/z$ is concave, and the first order condition $W' = 1/z$ in fact yields a maximum. This holds for any distributions $G$ and $H$.

The two distributions $G$ and $H$ determine the income function in the rationing model. As can be seen from (7), the formula is quite simple. It may be useful to illustrate the equilibrium with an example. Suppose that

$$G(J) = 1 - TJ^{-\beta} \text{ and } H(z) = 1 - Nz^{-\gamma}$$

Here, $T$, $N$, $\beta$, and $\gamma$ are parameters of the distributions. It then follows that the equilibrium income function satisfies the equation

$$W' = \left( \frac{T}{N} \right)^\frac{1}{\gamma} W^\beta \gamma$$

Its solution, assuming that $W(y) = 0$ at $y = 0$, is

$$W(y) = C y^{\beta + \gamma}$$

where the constant is $C = [(\beta + \gamma)/\gamma]^{(\beta + \gamma)} (T/N)^{1/(\beta + \gamma)}$. An increase in $T$ raises $C$ because it increases the number of high-productivity jobs. An increase in $N$ lowers $C$ because it increases the mean level of marginal costs of education. Similarly, as $\beta$ gets large, the variance in $J$ falls, and, assuming a compensating change in the mean of $J$, the return to education falls, eventually reaching zero.

10.5.1 Signaling, Rationing, and Human Capital

Rationing need not function by itself. It can simply augment the returns to the signal in the market without rationing. The following model, which integrates the earlier model of signaling and human capital
and the model above, will serve to illustrate. Productivity is determined jointly by ability, schooling, and the job according to

\[ s = nJy^\alpha \]

The income schedule is \( W(y) \). The equilibrium is defined by three relations. Schooling is rationally invested in, so that

\[ W' = 1/z \]

Jobs are rationed so that

\[ G(J) = H(z) \]

This connects \( J \) and \( z \). And, finally, salaries are equal to expected productivities. Assume that the mean of \( n \), conditional on \( z \), is \( z^\epsilon[K(\epsilon)] \) as before. Then we have

\[ W(y) = y^\alpha J \frac{z^\epsilon}{K(\epsilon)} \]

Combining these conditions, and assuming \( G \) and \( H \) are as in the numerical example above, we have

\[ W(y) = \frac{y^\alpha}{K(\epsilon)} (W')^{-\epsilon} G^{-1}(1/W') \]

or

\[ W(y) = \frac{y^\alpha}{K(\epsilon)} \left( \frac{T}{N} \right)^{\frac{1}{\beta}} (W')^{-(\epsilon + \frac{\gamma}{\beta})} \]

This again defines the equilibrium income function. The solution to (8) is

\[ W(y) = D y^{1 + \phi} \]

where \( D \) is a constant and \( \phi = \epsilon + (\gamma/\beta) \). Notice that if \( \beta \) is large, there is no variance in \( J \) and the rationing effect is missing. Then we have the previous model with just signaling and human capital. Rationing simply adds to the elasticity of the return to schooling.

I do not want to pursue the rationing model further here. It does not accord easily with existing microeconomic theory, and, like some of the other effects that have been discussed, it is not established empirically. On the other hand, it does seem to me that it is worth pursuing empirically and theoretically. Some things of considerable importance are rationed. I have in mind places in college, places in several different kinds of professional schools, places in legislatures and congresses, and so on. Its greatest applicability may be to organizations with elements of hierarchical structure that are not easily eliminated by the forces of supply and demand for jobs.
10.6 Contingent Contracts and the Avoidance of Signaling Costs

It has been argued that under certain conditions, employers can elicit information from potential employees by offering them a menu of contracts whose rewards are contingent upon subsequent discovery of their productive capabilities. (See Salop and Salop 1976). These devices can replace costly signals and reduce the costs of transmitting information, borne by individuals. That is to say, they are in everyone's interest, when they can function effectively. The analysis of contingent contracts has been carried out for discrete groups. But discrete numerical examples often have some special features. I should like, therefore, to develop a general model of self-selection with contingent contracts, using a large number or continuum of people of the type employed in the preceding analysis of signaling.

The idea behind self-selection through contingent contracts is relatively straightforward. It is assumed that individuals know their productivities in advance. Employers learn individual productivities after a period whose length is less than the full period of employment. Individuals are then induced to reveal their productive capability when they select a contract. The menu of contracts, somewhat roughly, consists of a set of opportunities to defer income now in favor of higher incomes later, contingent upon high productivity. The analytic task is to show that such a menu can be part of an equilibrium, and to explore what the properties of the equilibrium intertemporal wage or salary contracts are.

Let the productivity of an individual be $s$. Productivities are distributed in the population. Let $s^*$ be the lowest value of $s$. Contingent contracts have two components. One is the initial salary, which depends upon the contract that the individual selects. While it is not necessary for the individual to report a productivity at the time of hiring, it is convenient to have him do so, for the purposes of the analysis. Let $r$ be the productivity that the individual reports. The initial salary is $w(r)$. That salary lasts for a period at the end of which his productivity is discovered. At that point, his salary becomes a number which depends upon his productivity and the reported productivity in the previous period. Denote it by $v(r, s)$. Depending upon the length of time before the productivity is discovered, these two incomes will have different weights. By choosing the period and the discount rate for present value calculations, the present value of the income of a person of type $s$ reporting a productivity $r$ is

$$w(r) + dv(r, s)$$

The factor $d$ represents the discount rate and the relative lengths of the two periods. The present value of income will be denoted $T(r, s)$, so that

$$T(r, s) = w(r) + dv(r, s).$$
There are three equilibrium conditions that must be satisfied. Individuals maximize $T(r, s)$ with respect to $r$, so that

$$T_r(r, s) = w'(r) + dv_r(r, s) = 0$$

Individuals must accurately report $s$, so that $r = s$, or

$$w'(s) + dv_r(s, s) = 0$$

for all $s$. That is one condition. The second is that the present value of the incomes of all people equal the present value of their output or

$$(10) \quad w(s) + dv(s, s) = (1 + d)s$$

This also must hold for all $s$. The third is that future earnings must be sufficient to keep the individual working for the firm he contracted with. Otherwise, he might simply leave and earn $s$ elsewhere. The assumption is that individuals cannot bind themselves to an employer forever. The condition is that

$$v(s, s), \geq s$$

for all $s$.

The first question is, What are the feasible equilibrium contracts? Here I will discuss a class of them and their properties. Differentiating (10) we have

$$w'(s) + d[v_r(s, s) + v_s(s, s)] = (1 + d)$$

But because $w'(s) + dv_r(s, s) = 0$, this reduces to

$$(11) \quad v_s(s, s) = (1 + d)/d$$

Consider the class of functions

$$v(r, s) = K r^a s^b - \text{constant}$$

Members of that class will satisfy (11) provided that $K = (1 + d)/ad$ and $a + b = 1$. Let the constant be $w^*/d$. Then

$$v(r, s) = \frac{(1 + d)}{ad} r^a s^{1-a} - w^*/d$$

With that assumption, $w(r)$ becomes, from (10)

$$w(r) = w^* - (1 + d)(1/a - 1)r$$

The present value of total income is

$$T(r, s) = \frac{(1 + d)}{a} r^{1-a} s^a - (1 + d)[1/(a - 1)]r$$

Maximizing with respect to $r$, we have

$$T_r = (1 + d)[(1 - 1/a) + [1/(a - 1)](s/r)^a] = 0$$
The solution to this equation is \( r = s \), so that individuals do accurately report their productivity. Moreover,

\[
T_{rr} = -(1 + d)(1 - a) s^a r^{- (1 + a)} < 0
\]

so that the second-order condition for a maximum is satisfied.

There remains the question of incentive to remain with the firm with whom the contract is made. The condition is that \( v(s, s) \geq s \). Writing this out, we have

\[
w^* \leq \left( \frac{1 + d - ad}{a} \right) s
\]

The right-hand side is at a minimum when \( s = s^* \). Thus the largest value that \( w^* \) can have is

\[
w^* = \left( \frac{1 + d - ad}{a} \right) s^*
\]

When \( w^* \) has this value, then the initial salary of people of productivity \( s = s^* \) is

\[
w(s^*) = w^* - (1 + d)(1/a - 1)s^* = s^*
\]

That is to say, the initial salary of the people with the lowest productivity is \( s^* \), their productivity. If this were not true, then they would have an incentive to report some other productivity, one that corresponded to a higher income, since they are assured \( s^* \) in future periods.

Notice that the highest starting salary is \( w^* \), because \( w(r) \) is declining in \( r \). One might suspect that someone would report \( s^* \) in the first period and then take \( s \) later on. That will not happen for the following reason. To adopt that strategy would be to achieve a present value of income of \( s^* + ds \). By playing the game, one gets \( T(s, s) = (1 + d)s > s^* + ds \). Thus, the strategy is not advantageous.

There are several properties of this kind of equilibrium that are worthy of comment. First, the highest starting salary is \( s^* \). It is paid to those with the lowest productivity. Second, initial salaries decline as a function of productivity. Of course, they are made up in subsequent periods. Third, the rate of decline of starting salaries with productivity can be controlled. The rate of decline diminishes as the parameter \( a \) rises toward one. Therefore, if evening out the income streams is desirable, it can be accomplished to a limited extent with the choice of the function \( v(r, s) \). And presumably the market would move in that direction.

There is another feature of the market that is of interest. Contingent contracts tend to lock people in with the firm that they initially joined. The reason is that they sacrificed income at the start for income later. But that premium for productivity later comes only from the firm that
accepted the contract. This lock-in effect is stronger, the higher the productivity, because the premium increases with productivity. This feature of the equilibrium is not necessarily implausible from an empirical point of view. Intuitively, an individual may have an incentive to remain with the firm he starts with. In fact one can think of the investment in having the firm learn about one's capabilities as a form of specific human capital.

Contingent contracts have been criticized on the ground that the employer has an incentive to renge in the second period. This is a problem, especially when the individual has difficulty proving that his productivity is what he implicitly stated initially, in accepting a contract. However, in most markets reneging may not be a serious problem. Most firms are in business for an extended period. The reputation for reneging on implicit contracts of this type would impair the firm's future ability to hire high-productivity people, with a concomitant high cost.

This discussion of contingent contracts has set signaling aside. The two may interact. In particular, the contingent contracts can eliminate the inefficiency in signaling. Schooling, on the other hand, can make it possible to pay higher-productivity people more than the productivity at the bottom end, thereby reducing but not eliminating the divergences between earnings and productivity. This subject is explored in appendix A.

10.7 Occupational Licensure and Minimum Quality Standards

Occupational licensure has captured the attention of economists and regulators. It is an increasingly pervasive phenomenon, and one which affects almost every consumer in some form or other. It is a complicated subject, and has not received the sort of attention from a theoretical point of view that it deserves (but see Leland 1977). I do not have the space here to do more than set forth some of what seem to me to be the issues, and to suggest how at least some of them might be explored.

There is a tendency for most of us—perhaps with the model of medicine in mind—to think of licensure as the last step in a quality-screening process that is designed to assure competence in a profession. Such screening is thought necessary or desirable because consumers are imperfectly informed about the quality of the services that they receive. Of course, the state of being imperfectly informed has many dimensions: how imperfectly, for how long, how quickly does the consumer learn, and so on.

But licensure is a device which serves many functions, and I should like to pause briefly to comment on some of them. It can simply be the end point of a quality-screening process. It can also be the certification that the individual has acquired a certain amount of human capital and is
thereby likely to be capable of producing reasonably high-quality services. And it can be both of these things together. Certainly, these are aspects of licensure in law, medicine, academia (the license is the Ph.D.), and many other professions.

Licensure is also a device for controlling the behavior of those in the profession. Performance, at least in some industries, is determined not only by ex ante competence, but also ex post effort. The threat of removal of a license in the case of poor performance, combined with some mechanism by which removal takes place, acts as an incentive to maintain performance when consumer perceptions weaken the incentive that would be provided were there perfect information. There are alternative mechanisms for taking care of poor performance. Liability is one. Strongly developed professional norms are another. Ex post control is needed in varying degrees in different professions. Academics presumably have little incentive to do poor research, although it is sometimes argued that permanent licensure in the form of tenure weakens the incentive to produce. On the other hand, there are other objectives than maintenance of job and income, objectives related to professional status and prestige. These function effectively in many professions. Indeed, a stronger professional association and stronger norms in a field like automobile repair, where it appears that fraud is a problem in part of the industry, might serve a useful purpose.

Licensure is used as a control device by the service industries. It is used to control numbers, for good or ill. It is also used to control conduct: rules against price competition and advertising have been cited as potentially adverse prohibitions, based on the ultimate threat of loss of license.

The threat of loss of license, used for whatever purpose, is more or less compelling, depending upon the industry. If entry is relatively easy, the rents not terribly high, and the initial investment in human capital not large, then the threat of removal may not carry much weight. This is especially true if licensure attaches to business and not individuals, so that the business can disappear but the individual reappear with a different corporate suit.

Licensure can serve as an ex post screen for quality (apart from effort). This may be useful if the ex ante screening either does not exist, or is imperfect.

The relative importance, and the welfare effects, of these phenomena vary from industry to industry, and depend upon a host of structural features of the market. Among them are how imperfectly informed consumers are, whether the service industry sells information as well as a service, information upon which the demand for the service is based, the human capital requirements, the degree to which ability varies and is a necessary input, and others.

It would be well beyond my current task to delve into all of these phenomena. I do believe the subject is one of considerable interest and
one requiring additional work with a potentially high payoff for informing regulatory policy. In what follows, I shall focus upon some problems connected with the ex ante quality-screening aspects of the subject, since these bear some resemblance to the earlier models of signaling and rationing.

I want to consider the following constellation of factors in a first pass at the issue of licensure and minimum quality standards. Consumers value quality in a service differently, and are distributed with respect to their valuations of quality. Let quality be denoted by \( n \). Setting a minimum quality standard \( \bar{n} \) would be to screen out suppliers with \( n < \bar{n} \) for some level of \( \bar{n} \). Consumers value quality in dollars according to \( \theta n \), where \( \theta \) is distributed according to \( G(\theta) \). Services of different qualities have prices \( p(n) \) which depend on \( n \). The schedule \( p(n) \) will be determined as part of the equilibrium.

Given \( p(n) \), consumers optimize \( \theta n - p(n) \), by setting

\[
\theta = p'(n)
\]

At least, that is what they would do with perfect information. But I want to assume that the information is imperfect and indeed biased. To be specific, let us assume the quality \( n \) is perceived to be \( a + sn \), where we might expect that \( a > 0 \), and \( s < 1 \). This would make consumers relatively insensitive to quality differences. For reasons which will be apparent shortly, the magnitude of \( a \) is immaterial, while \( s \) is an important parameter.

With imperfect information, the consumer selects \( n \) to maximize \( \theta(a + sn) - p(n) \) by setting

\[
\theta s = p'(n)
\]

If there is a minimum quality standard \( \bar{n} \), so that \( n \geq \bar{n} \), then consumers maximize \( \theta sn - p(n) \), subject to \( n \geq \bar{n} \). The solution to that problem is the following. Let \( \hat{\theta} = p'(\bar{n})/s \). If \( \theta' < \hat{\theta} \), then \( n = \bar{n} \), and if \( \theta > \hat{\theta} \), \( \theta s = p'(n) \).

10.7.1 Model 1

Suppose first that the supply of services at each level of quality \( n \) is potentially unlimited. Let \( w(n) \) be the opportunity cost of being in this market for a server of quality \( n \). The prices will be equal to \( w(n) \) so that \( p(n) = w(n) \). As a result of individual optimization with imperfect information, total net benefits are

\[
W(\bar{n}, s) = \int_{\hat{\theta}}^{\bar{\theta}} \left[ \theta \bar{n} - w(\bar{n}) \right] g(\theta) d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \left\{ \theta n(\theta s) - w[\theta n(\theta s)] \right\} g(\theta) d\theta
\]

where \( \theta s = w'[\theta n(\theta s)] \), and \( g(\theta) \) is the density function for \( \theta \).
Figure 10.4

I want to establish the properties of this net benefit function, and the nature of the optimal quality standard. First, consider the relation between optimal and selected qualities, at each level of $\theta$. The situation is depicted in figure 10.4. Here, $n(\theta)$ is the optimal $n$ given $\theta$, $n(\theta s)$ is the unregulated $n$ given $\theta$ and $s$, and $\bar{n}$ is the minimum quality standard. $\bar{\theta}$ is the upper level of $\theta$. There are three groups. Those below $\bar{\theta}$ purchase too little without regulation, and now purchase too much. Those between $\bar{\theta}$ and $\bar{\theta}$ are closer to the optimum than without regulation. Those above $\bar{\theta}$ are unaffected by regulation. The tradeoff, then, is between the first and second groups. Whether regulation is desirable at all depends on the misperceptions.

One can easily verify that $\partial W/\partial \theta = 0$. Thus the maximum of $W(\bar{n}, s)$ with respect to $\bar{n}$ occurs when

$$\frac{\partial W}{\partial \bar{n}} = F(\bar{\theta}) \left[ E(\bar{\theta}) - w'(\bar{n}) \right]$$

$$= F(\bar{\theta}) \left[ E(\bar{\theta}) - s \bar{\theta} \right] = 0$$

where $E(\bar{\theta})$ is the expected value of $\theta$ given that $\theta \leq \bar{\theta}$. Thus the first order condition for a maximum is

$$E(\bar{\theta}) = s \bar{\theta}$$
One has to be a little careful about second-order conditions, as we shall see in a moment.

Note first that when \( s = 1 \), \( \tilde{\theta} \) must be zero, or whatever the bottom end of the \( \theta \) spectrum is. That means no regulation. Second, since \( E(\tilde{\theta}) \) must cross \( s\tilde{\theta} \) from above at the maximum, an increase in \( s \) reduces \( \tilde{\theta} \) and hence the desired level of minimum quality. There can, however, be sharp jumps in the desired levels of regulation. Consider the case where \( G(\theta) \) is uniform on \([0, 1]\). In that case,

\[
\frac{\partial W}{\partial \tilde{n}} = \tilde{\theta}^2 \left[ \frac{1}{2} - s \right]
\]

Its sign is therefore determined by \( s - (1/2) \). If \( s > 1/2 \), \( \partial W/\partial \tilde{n} < 0 \) and no regulation is optimal. If \( s = 1/2 \), \( \partial W/\partial \tilde{n} = 0 \) and it does not matter. And if \( s < 1/2 \), \( \partial W/\partial \tilde{n} > 0 \), and the minimum standard should be above the unregulated level of \( n \) at \( \theta = 1 \), the upper end. The optimal \( \tilde{n} \) maximizes \((1/2)\tilde{n} - w(\tilde{n})\), and satisfies \( w'(\tilde{n}) = 1/2 \). Everyone selects the minimum quality.

In general, the situation is depicted in figure 10.5. The line \( \tilde{\theta} \) lies above \( E(\tilde{\theta}) \). For \( s \) close enough to one, \( s\tilde{\theta} \) also lies above \( E(\tilde{\theta}) \), and \( \tilde{\theta} = 0 \) is optimal. When \( s = s_1 \), there are two local maxima. Either could be optimal. As \( s \) falls, the second local maximum moves toward \( \tilde{\theta} \). For \( s \) small
enough, $s\theta$ lies below $E(\theta)$. Then $\tilde{\theta} = \bar{\theta}$ is optimal and $\bar{n}$ is selected to maximize $E(\tilde{\theta})\bar{n} - w(\bar{n})$, where $E(\tilde{\theta})$ is just the mean of $\theta$.

The lessons of this analysis are two. The desirability of regulation and the appropriate level of the minimum quality standard increase with the extent to which consumers underestimate quality differentials. And, second, with the right kind of regulation, the greater the underestimates by consumers, the larger the fraction of people at the minimum quality standard.

Note also that a considerable amount of information is required to set the minimum quality. It includes costs $w(n)$, the distribution of $\theta$, and the level of $s$ or an assessment of the misperceptions. An alternative strategy of informing consumers might seem desirable. Indeed, it would be in the interest of at least the upper end of the quality spectrum on the supply side, because it would increase their business. It is also in the interest of the low end of the quality spectrum if regulation is the alternative. The gainers from misperceptions and regulation are those in the middle of the quality spectrum.

10.7.2 Model 2

The preceding model assumed that the supply of services at each level of $n$ is unlimited. When that is not true, the results change. They are reported in appendix B for the reader who may be interested.

10.8 The Privacy Issue

Related to the theoretical work on signaling and screening are a collection of policy issues that are often (and somewhat misleadingly) referred to as privacy issues. These are issues related to the acquisition, storage, and use of personal data about individuals, to screen them in various markets. There are legal and moral issues of privacy. But often, too, there is an important set of questions concerning how collections of information about individuals affect their opportunities in various markets. Job markets and credit markets are conspicuously central in these discussions. Policy in this area must consider not only what information an individual has a right to keep private, but also how the exercise of that right affects his own opportunities and those of others. Distinguishing among two or more things is, after all, a symmetric relation. If you effectively distinguish yourself from me, we are, as it were, effectively distinguished, even though I might have preferred to remain undistinguished in the relevant respect. In an economic context, when an individual exercises an option either to distinguish himself, or not to be distinguished, at least on some criteria, the exercise of that option generates externalities which affect the performance of the market.
In the short space available, I cannot explore these issues completely. But I do want to make one point with respect to solutions to the privacy problem that involve optional signals, having to do with the "copyright" solution to the problem.

In view of the vastly reduced costs of collecting and transmitting personal information it has been suggested that the law should either assert or reaffirm the individual's property right in information about himself. That is to say, the individual is to be regarded as having a copyright on this information. He or she must be consulted before it is transmitted or reproduced. One virtue of such a proposal is that it provides a semiautomatic mechanism for detecting and eliminating errors. A potential defect is its administrative cost (some would regard this as a virtue—perhaps ignoring the difficult question of who would or could bear the cost).

But in addition to errors and costs, there is the central question of whether voluntary control over signals is likely to affect substantially the signals that are in fact used. I believe the answer that it will not is closer to the truth than the opposite conclusion. This is not an argument that copyright is a poor policy, but only that it may be ineffective in dealing with certain kinds of problems. In particular, it may be ineffective in preventing people from being forced to reveal personal information about themselves, and from having certain opportunities foreclosed as a result.

Consider a simple case. Individuals are asked to state their criminal records (if any) and to authorize the institution that is inquiring to check it. Individuals have the right to refuse. They may even have the right to be "considered" for a job or for credit without the answer. If everyone refused to answer, or were compelled to answer, or if the questioner were compelled not to ask, then everyone would be treated as the average. But those with no records are likely to have an incentive to answer. Given that they answered, those with arrests for misdemeanors have an incentive to answer, since they are at the top of the remainder. And, thus, with the incentives operating through those at the top of the remaining undifferentiated group, the question is answered. Refusal to answer amounts to an answer. So the right to refuse confers little benefit, either in privacy, or in opportunities in the market.

Signals that people invest in are subject to similar incentives. If a certain job or salary has an educational prerequisite, those who can afford it, or who derive sufficient benefits from education to make it worthwhile, will invest. If the initial position were one in which everyone had the same or similar educations, or appeared to, there would still be an incentive for certain people to invest further.

In the case of the criminal records, I have tried to avoid the questions of whether arrest records are legitimate, informative, or desirable sources
of information. Whether they are informative is an empirical question to which I do not know the answer. The point I want to make is simply that copyrights in personal information may not solve the privacy or the market opportunity problems, if indeed there are such problems. To alter substantially the performance of the market requires collective action, though not necessarily governmental action.

In general, the incentives for signaling come from the upper end of the spectrum down. To stop this process would require collective action, since the individual incentives are to signal and be screened.

Appendix A

Contingent Contracts and the Time Profile of Earnings

In the signaling model, schooling, in addition to its contribution to an individual's productivity, served to distinguish the person by his or her ability. This occurs because the generalized costs of schooling were negatively correlated with ability and because employers could not observe productivity directly. The result was in some cases overinvestment in schooling in the sense that the private return exceeds the direct contribution to productivity.

It has been objected to the schooling-as-a-signal model that, while productivity or productive potential may not be directly observable for young workers, it will become observable to the employer over time. That raises the possibility that employers will defer wages and salaries until productivity is observed, and then reward people directly on the basis of productivity.

That by itself is not a very interesting possibility, because individuals would not necessarily be sorted out in the earlier years on the basis of their productive potential. And there may be private and social benefits to screening at the early stages of employment. These benefits may result from improved resource allocation to on-the-job training, or from better job placement. But the argument goes further to assert that there may be screening. Potential employees are not offered one deferred salary contract but several. In choosing from the menu of contingent contracts, they will signal their productive potential. Thus the information transfer at the time of hiring that occurs when schooling is a signal may also occur with selection via contingent contracts.

Contingent contracting is a possible market response to the inefficiency associated with overinvestment in schooling in the signaling case. Since the present value of earnings in the contingent contract regime is a
function of *actual* productivity, the ability component of the return to education is removed. As a result, efficient levels of investment in schooling can be sustained in an equilibrium.

However, as we have seen, contingent contracts have an interesting property. They cause the time path of earnings to diverge from the path of productivity, individually and on average. Therefore, if one adopts the view that contingent contracts are a likely market response to signaling inefficiency, one would also expect that earnings would not accurately reflect the profile of the individual's productivity over time.

The purpose of this section is to illustrate how contingent contracts screen people without distorting the educational investment decision, and then to determine the extent of the divergence between earnings and productivity. Generally, earnings rise more rapidly than productivity over time. Thus if one were to use earnings as a proxy for productivity in estimating the relative contributions of schooling and on-the-job learning to productivity, one would overestimate the latter and underestimate the contribution of schooling.

The purpose of the model is to investigate the extent to which earnings are deferred from the first period to the second. Individual productivity is $S(n, y)$ where $n$ varies continuously in the employable population. First-period earnings are $w(y)$. In the second period, they are $(1 + P) S(n, Y) - W_{by}$, where $c(y, n)$ is the cost of $y$ years of schooling to a person of type $n$. Here I assume that schooling costs depend directly on $n$. Let $y^*(n)$ be the optimal $y$ for each $n$, and $n^*(y)$ its inverse. Let $G(n) = \max_y (1 + P) S(n, y) - c(y, n)$

The default strategy (the worker stays with the original employer for only the first period, and then goes on the open market) yields benefits of $\bar{W}(y) + \beta S[S(n, y) - c(y, n)]$

Thus in a market equilibrium

(A1) $\bar{W}(y) \leq G(n) - \beta S(n, y) - c(y, n)$

for all $y$ and $n$. In particular, the upper bound on $\bar{W}(y)$ is

$\bar{W}(y) = \min_n [G(n) - \beta S(n, y) + c(y, n)]$

This upper bound has two properties of interest. First, from the definition of $\bar{W}(y)$
Moreover, the $n$ that minimizes the right-hand side of (A1) is not $n^*(y)$. To see that we note that the minimizing condition is

$$(1 + \beta) S_n(y^*(n), n) - c_n(y^*, n) - \beta S_n(y, n) + c_n(y, n) = 0$$

If the solution is $n = n^*(y)$, we would have

$$S_n(y, n) = 0$$

but $S_n > 0$ so that this cannot happen. Therefore the inequality in (A2) is strict, and for everyone in the first period, earnings fall short of productivity. In fact the minimizing $n$ is less than $n^*(y)$ since $S_n > 0$.

It follows that the slope of the earnings schedule is

$$\bar{W}'(y) = c_y(y, n(y)) - \beta S_y(y, n(y))$$

$$> c_y(y, n^*(y)) - \beta S_y(y, n^*(y))$$

$$= S_y(y, n^*(y)) > 0$$

One concludes that the slope of the earnings schedule as a function of $y$ in the first period is (a) positive, and (b) greater than the marginal product of schooling in that period.

It is perhaps worth noting that nothing in the argument above relies on the assumption that schooling costs depend on $n$, the attribute that determines productivity. Thus the divergence of earnings and productivity over time will occur with implicit contingent contracts even when schooling costs are the same for everyone, or, in general, when they vary randomly with respect to ability. It is also to be noted that the divergence of earnings from productivity creates an incentive for people to stay with the firms that they begin with. The reason is that in the second period, earnings exceed productivity, and hence what the older worker can command on the open market.

In view of these results, a few remarks about the use of earnings data seem in order. Investment in schooling is efficient. However, the slope of the schedule of lifetime earnings (in present value terms) as a function of education overstates the productivity of schooling, because it contains the ability component. It seems worth emphasizing that this holds in spite of the appropriateness of the educational investment from an efficiency point of view. The intertemporal earnings profiles are steeper than the productivity profiles. Therefore, if earnings are taken as equal to productivity, and the time slope (with suitable controls for discounting) taken as the result of the acquisition of human capital on the job, then the return to that capital will be overestimated. In the signaling situation without contingent contracts, this problem does not arise. Earnings and produc-
tivity at each level of schooling are equal (at least on average) at each time in the life cycle. Of course there can be overinvestment in schooling because the private return contains the ability effect.

Appendix B

Quality Standards with a Limited Supply of Services

In the model in section 10.7, the supply of services at each level of quality was potentially unlimited. In this second model, the availability of quality is limited, and the supply is distributed according to $F(n)$.

The equilibrium occurs when consumers are distributed over suppliers properly, that is, proportionately. This equilibrium is much like that in the educational rationing model. However, here price is the rationing instrument. Given $p(n)$ consumers with $\theta \leq p'(n)/s$, select quality $n$ or less. There are $G(p'/s)$ such consumers. The fraction of suppliers at quality $n$ or less is $F(n)$. Thus in an equilibrium

$$F(n) = G\left(\frac{p'}{s}\right)$$

for all $n$. This defines the equilibrium schedule $p(n)$, up to a constant. I want to incorporate quality standards or licensure at the outset. That requires the following modification. If $n$ is restricted to be equal or greater than $\bar{n}$, the relevant distribution of $n$ is $[F(n) - F(\bar{n})]/[1 - F(\bar{n})]$. Therefore, with the quality standard, the equilibrium is defined by the two relations

\begin{align*}
(B1) & \quad (A) \quad \theta = \frac{p'(n)}{s} \\
(B) & \quad G(\theta) = \frac{F(n) - F(\bar{n})}{1 - F(\bar{n})}
\end{align*}

Given $\bar{n}$, the level of quality purchased by people of type $\theta$ does not depend on $s$, the parameter that determines perceptions. The price schedule does depend on $s$, and adjusts to make the equilibrium relation (B1) hold. Let

$$H(\theta, \bar{n}) = F^{-1} \left\{ [1 - F(\bar{n})] G(\theta) + F(\bar{n}) \right\}$$

The equilibrium condition (B1) is then

\begin{align*}
(B2) & \quad n = H(\theta, \bar{n}) \\
\end{align*}

Let $w(\theta) = \theta n - p(n)$. Taking the derivative with respect to $\theta$, we have
Let \( \bar{\theta} \) be the highest level of \( \theta \), and assume it exists. Integrating (B3) we have

\[
 w(\theta) = w(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \left[ v(1-s) \frac{\partial H}{\partial \theta} + H \right] d\nu 
\]

This tells us the level of dollar benefits for a person of type \( \theta \) in the market. The number \( w(\bar{\theta}) \) is determined by the position (as opposed to the slope) of \( p(n) \). That in turn is determined by the alternate opportunities of suppliers with different levels of \( n \). Let those alternative salaries or incomes be \( w(n) \). Let \( \bar{n} \) be the highest level of \( n \). I shall assume that \( p(\bar{n}) = w(\bar{n}) = \bar{w} \). This means that the level of prices is set by the highest-quality suppliers. More generally, one assumes \( p(n) \geq w(n) \) for all \( n \), and then computes the level of the schedule from that. For now, this set of constraints is assumed to bind at \( \bar{n} \).

Given that assumption

\[
w(\bar{\theta}) = \bar{\theta} \bar{n} - \bar{w}
\]

We can compute the total net benefits by adding up over the \( w(\theta) \). They are

\[
 w(\theta) = \bar{\theta} \bar{n} - \bar{w} - \int_{0}^{\bar{\theta}} g(\theta) \int_{0}^{\bar{\theta}} \left[ v(1-s) \frac{\partial H}{\partial \theta} + H \right] d\nu d\theta
\]

Several things can be inferred from (B4). First, net benefits are an increasing function of \( s \). The reason is not hard to locate. The parameter \( s \) determines the slope of the price function. Recall that

\[
p'(n) = s G^{-1} \left[ \frac{F(n) - F(\bar{n})}{1 - F(\bar{n})} \right]
\]

But by hypothesis, the upper end of \( p(n) \) is fixed. Therefore, a reduction in \( s \) flattens out the price function as shown in figure 10.6. Thus, if the upper end of the \( n \) spectrum locates \( p(n) \), consumers lose and incomes in the profession rise as consumer perceptions of quality differences are blunted. The reverse holds if the bottom end of the quality spectrum locates \( p(n) \). Then the situation is shown in figure 10.7. Intermediate cases are possible. In general, as \( s \) falls, the likelihood that the upper end
Figure 10.6

Figure 10.7
will be the binding constraint increases. These remarks hold even if consumers overestimate marginal quality differences, i.e., if \( s > 1 \).

A second point relates to the optimal minimum quality standard. Taking the derivative of net benefits with respect to \( n \), we have

\[
\frac{dw}{dn} = -\int_0^\infty \left[ \theta (1 - s) \frac{\partial^2 H}{\partial \theta \partial n} + \frac{\partial H}{\partial n} \right] G(\theta) \; d\theta
\]

Where \( s = 1 \), so that there are no misperceptions,

\[
\frac{dw}{dn} = -\int_0^\infty \frac{\partial H}{\partial n} G(\theta) \; d\theta < 0,
\]

because \( \partial H/\partial n > 0 \). Therefore, with no misperceptions, quality regulation makes consumers worse off.\(^4\)

To do a complete job on the welfare analysis, one would have to specify the costs of overloading some suppliers at the expense of others. I do not have the space to do that here. Nor is the nature of the quality screening developed in detail. The role of training or education as part of that process of screening and the acquisition of specific human capital could also be developed.

Notes

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1. The argument is as follows. When \( \varepsilon = 0 \), the expression in the text has the value, one. Taking logs and differentiating with respect to \( \varepsilon \), we have

\[
\frac{1}{1-\alpha} \left( \frac{\varepsilon}{1+\alpha \varepsilon} + \frac{\varepsilon}{\alpha + \varepsilon} \right) > 0
\]

Thus the term is increasing in \( \varepsilon \). It follows that for \( \varepsilon > 0 \), the expression is greater than one.

2. By taxing the signaling activity, we can determine \( y(z) \), the relationship between schooling costs, and investment in schooling. The objective then is to maximize

\[
E \left[ n y(z)^\alpha - \frac{y}{z} \right] = \int E \left[ \frac{\varepsilon x^e}{K} y(z)^\alpha - \frac{y}{z} \right] h(z) \; dz
\]

The standard calculus of variations solution is the maximum of the integrand with respect to \( y \) for each level of \( z \). It is

\[
y(z) = \alpha \left( \frac{1}{1-z} \right)^{1-\alpha} \frac{1+\varepsilon}{K^{\alpha-1}}\]

That, and the other variables are what is reported in table 10.6, column 4.

3. By second best, we mean simply the most efficient outcome attainable when \( n \) or \( u \) cannot be observed directly.
4. From the definition of $H(\theta, \bar{y})$, we have

$$\frac{\partial H}{\partial \bar{y}} = \frac{1}{F([1 - F(\bar{y})] F(\theta) + F(\bar{y}))} F'(\bar{y}) [1 - G(\theta)]$$

which is nonnegative because $G(\theta) \leq 1$.

References


